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Single base station hybrid TOA/AOD/AOA localization algorithms with the synchronization error in dense multipath environment

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Abstract

Mobile station (MS) localization in a cellular network is appealing to both industrial community and academia, due to the wide applications of location-based services. The main challenge is the unknown one-bound (OB) and multiple-bound (MB) scattering environment in dense multipath environment. Moreover, multiple base stations (BSs) are required to be involved in the localization process, and the precise time synchronization between MS and BSs is assumed. In order to address these problems, hybrid time of arrival (TOA), angle of departure (AOD), and angle of arrival (AOA) measurement model from the serving BS with the synchronization error is investigated in this paper. In OB scattering environment, four linear least square (LLS), one quadratic programming and data fusion-based localization algorithms are proposed to eliminate the effect of the synchronization error. In addition, the Cramer-Rao lower bound (CRLB) of our localization model on the root mean-square error (RMSE) is derived. In hybrid OB and MB scattering environment, a novel double identification algorithm (DIA) is proposed to identify the MB path. Simulation results demonstrate that the proposed algorithms are capable to deal with the synchronization error, and LLS-based localization algorithms show better localization accuracy. Furthermore, the DIA can correctly identify the MB path, and the RMSE comparison of different algorithms further prove the effectiveness of the DIA.

Keywords: Localization, Synchronization error, Linear least square (LLS), Quadratic programming (QP), One-bound (OB), Multiple-bound (MB)

1 Introduction

The ability to accurately determine the location of mobile station (MS) in a cellular network is a vital component of numerous applications, such as emergency services, commercial service and network optimization [1]. Emergency services are driven by governmental institutions, which need to locate MS in case of emergency calls. Commercial service is named as location-based services (LBSs) that can be developed commercially by the network operators or the application developers to obtain a revenue, such as navigation, location-based advertising, location-sensitive billing, social networks, and inventory tracking. Network optimization is known as the location-aware communication that can be used to improve the communication capacity and network



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efficiency, for example, network management, radio reconfigurable spectrum, and intelligent transportation systems (ITS), etc.

Millimeter wave (mmWave) and massive multiple-input-multiple-output (MIMO) are two promising techniques to be deployed on the next generation wireless network, due to the large available bandwidths and highly directional communication [2]. In addition, the location information is considered as the key factor to perform directional communication by beamforming. In other words, when the location of the user is known, the base station (BS) can steer its transmission to the user, either directly or through a reflected path. Therefore, it is a meaningful work to research accurate location estimate of MS with hybrid range and angle measurements in a cellular network.

1.1 Related works

The two-step localization scenario with low complexity is the most widely used method in localization systems [3], which first extracts the certain signal parameters, and then estimates the location of MS based on those signal parameters. In the first step, signal parameters, such as time of arrival (TOA), angle of departure (AOD), and angle of arrival (AOA) are estimated. The related literatures can be found in [4-8]. In the second step of location estimation, geometric, statistical or optimized methods are utilized depending on the accuracy requirements and system constraints. The work in [9] proposes a hybrid TOA/AOD/AOA localization method to give an iterative nonlinear least square solution through utilizing Taylor-series linearization. However, an initial guess on the location of MS is needed, which cannot guarantee to converge to the global optimal solution. The least square (LS) algorithm is proposed in [10] by analyzing the geometrical relationship among TOA/AOD/AOA measurements, and maximum likelihood (ML) algorithm is presented to jointly estimate the position of MS and scatterers. Through leveraging the bidirectional estimate of TOA and AOA measurements, the work in [11] also proposes an LS algorithm based on one-bound (OB) scattering paths, and a two-step proximity detection method is presented to detect and discard the multiple-bound (MB) scattering path. Under the assumption of the OB scattering environment, a closed-form solution of the location of MS is derived in [12] with the nonlinear geometrical TOA/AOD/AOA measurements. Utilizing the bidirectional TOA and AOA measurements, taking the TOA measurements as the constraints of an optimization problem, a novel nonlinear programming (NLP) localization algorithm is proposed in [13] with the presence of the MB scattering path. Based on the linearization of TOA/AOD/AOA measurements with a first order Taylor series, a three-dimensional localization algorithm is proposed in [14] through utilizing the geometry relationship of the OB scattering paths. By assuming a ring of scatterer model with the OB scattering, a single MIMO BS (SMBS) algorithm with virtual BS based on TOA/AOD/AOA measurements is proposed in [15]. Based on the circular model of the OB scattering, a nonlinear constrained optimization localization algorithm is proposed in [16] by employing the geometry relationship among the location of BS, MS, and scatterers. The work in [17] utilizes the concept of virtual BS to identify the location of all virtual BSs, then the location of MS is determined by using only one OB scattering path and its corresponding virtual BS. Instead of obtaining the virtual BS, the work in [18] estimates the scattering point associated with the OB scattering path to determine the location of MS, and proposes the two-step elliptical Lagrange constrained optimization approach without any prior knowledge on the propagation environment. In [19], the authors analyze the performance of an mmWave localization approach that can utilize TOA/AOD/AOA measurements in an urban environment with both line of sight (LOS) and OB scattering, and proposes gradient-assisted particle filter method to accurately estimate the location of MS as well as nearby scatterers with radio-environmental mapping. After the identification of the LOS and OB scattering paths, a weighting localization algorithm is proposed to fuse the measured data of the LOS and OB scattering [20].

1.2 Methods and contributions of this work

All the above works assume that the network between MS and BSs is synchronized. That means the TOA measurements are only corrupted by the non-line of sight (NLOS) error and measured noise. However, the synchronization error is unavoidable for the TOA measurements, due to the unknown clock bias and drift [21–24]. Moreover, the conventional localization approaches assume that multiple BSs are involved in the localization process, which leads to the extra information exchange overhead and latency [25, 26]. Fortunately, as larger scale antenna arrays and wide bandwidth are adopted in the next generation wireless communication networks, the BSs can easily acquire higher multipath resolution in the angle and delay domains [27, 28]. These provide the possibility of localization with only the serving BS, which can significantly reduce the signaling overhead and latency of wireless network. To the best of our knowledge, hybrid localization that exploits the OB and MB scattering paths from one BS utilizing the TOA/AOA/AOD measurements with the synchronization error has not been studied in the literature. The main contributions of this paper are summarized as follows:

- In OB scattering environment, when the synchronization error is present, the localization accuracy of the related algorithms in [9–20] degrades significantly. In order to deal with the synchronization error, four linear least square (LLS), a quadratic programming (QP) and data fusion (DF) based localization algorithms are proposed in this paper. It is concluded that DF-based algorithms cannot improve the localization accuracy in comparison with LLS-based algorithms.
- 2. In OB scattering environment, the Cramer-Rao Lower Bound (CRLB) of our localization model is derived. Comparing it with the existing CRLB in [19], we find that the synchronization error greatly increases the CRLB of the estimated location of MS.
- 3. In hybrid OB and MB scattering environment, when the synchronization error is presented, due to the infeasibility of the existing MB identification methods, a novel double identification algorithm (DIA) is proposed to identify the MB scattering path.
- 4. Intensive simulations are performed to compare the performance of different algorithms. In OB scattering environment, it is shown that the LLS-based localization algorithms can effectively eliminate the effect of the synchronization error, and have higher localization accuracy than other algorithms. Moreover, in hybrid OB and MB scattering environment, the DIA can correctly identify the MB scattering path.

The rest of this paper is organized as follows. Section 2 introduces the localization model. In OB scattering environment, Sect. 3 shows our proposed localization algorithms and the CRLB in the presence of the synchronization error. Section 4 discusses the methods to detect and discard the MB scattering path. Simulation results are provided in Sect. 5 to demonstrate the performance of the proposed localization algorithms. Finally, Sect. 6 provides the concluding remarks.

2 System model

In dense multipath environments, only one BS is deployed, as shown in Fig. 1, the signal propagation of each path between the BS and MS is the OB or MB NLOS propagation. Due to an increased communication capacity, the next generation cellular communication system will likely adopt the mmWave and MIMO techniques. Thus, three important parameters of each propagation path such as TOA, AOD and AOA can be estimated when both BS and MS are equipped with antenna array [4, 5, 7, 8]. If all the propagation paths experience the OB scattering, the mathematical expressions are as follows [10–13, 20]

$$r_{i} = c \cdot (t_{i} - t_{0}) + n_{i} = r_{i}^{0} + c \cdot \Delta t + n_{i} \quad i = 1, ..., L$$

$$= \sqrt{(x - x_{i}')^{2} + (y - y_{i}')^{2}} + \sqrt{(x_{1} - x_{i}')^{2} + (y_{1} - y_{i}')^{2}} + \varepsilon + n_{i}$$

$$\alpha_{i} = \alpha_{i}^{0} + m_{i} = a \tan\left(\frac{y_{i}' - y}{x_{i}' - x}\right) + m_{i}$$

$$\beta_{i} = \beta_{i}^{0} + v_{i} = a \tan\left(\frac{y_{i}' - y_{1}}{x_{i}' - x_{1}}\right) + v_{i}$$

(1)

where *c* is the speed of light, t_i is the measured TOA of the *i*-th propagation path, t_0 is the time of signal transmission, Δt is the time synchronization error, (x_1, y_1) is the location of home BS, (x, y) is the location of MS, (x'_i, y'_i) is the location of scatterer from the MS to home BS in *i*-th OB scattering path, and *a* tan is the function of inverse tangent. α_i and α_i^0 are the measured and actual AOD of the *i*-th propagation path, respectively. β_i and β_i^0 are the measured and actual AOA of the *i*-th propagation path, respectively. *L* is the number of multipath. n_i , m_i and v_i are white Gaussian random variables with the



standard deviation σ_n , σ_α , and σ_β , respectively. When the propagation signal experiences the MB scattering path, as shown in Fig. 1, the number of scatterers is larger than one, the measured parameters will experience extra range and angle deviation.

If the measured noise and the synchronization error are ignored, only two OB scattering paths can decide the possible position of MS [9-13]. As shown in Fig. 2, the position of MS is the intersection point of line AB and line CD. If all the propagation paths experience the OB scattering path, the true range and angle parameters in (1) can be transformed into the linear form [13, 20, 29]

$$\left(\cos(\alpha_i^0) + \cos(\beta_i^0)\right) y - \left(\sin(\alpha_i^0) + \sin(\beta_i^0)\right) x = y_1 \left(\cos(\alpha_i^0) + \cos(\beta_i^0)\right) - x_1 \left(\sin(\alpha_i^0) + \sin(\beta_i^0)\right) - r_i^0 \sin(\alpha_i^0 - \beta_i^0), i = 1, \dots, L$$

$$(2)$$

3 Proposed algorithms with only OB scattering paths

Comparing with the NLOS error and measured noise, the synchronization error significantly affects the localization accuracy [21, 22]. In this section, four types of LLS-based algorithms, a quadratic programming (QP) algorithm and data fusion (DF)-based algorithms are presented to deal with the synchronization error.

3.1 LLS algorithm with a new variable

As the linear equations shown in (2), if we define the synchronization error as a new variable, the new linear equations can be easily obtained by ignoring the measured noise. Then putting measured parameters into it, we can obtain the following linear equations

$$Z = H \cdot X' \tag{3}$$



where $X' = [X^T, \varepsilon]^T$, $X = [x, y]^T$, *T* is the symbol of transpose,

$$Z = \begin{bmatrix} y_1(\cos(\alpha_1) + \cos(\beta_1)) - x_1(\sin(\alpha_1) + \sin(\beta_1)) - r_1\sin(\alpha_1 - \beta_1) \\ y_1(\cos(\alpha_2) + \cos(\beta_2)) - x_1(\sin(\alpha_2) + \sin(\beta_2)) - r_2\sin(\alpha_2 - \beta_2) \\ \vdots \\ y_1(\cos(\alpha_L) + \cos(\beta_L)) - x_1(\sin(\alpha_L) + \sin(\beta_L)) - r_L\sin(\alpha_L - \beta_L) \end{bmatrix}$$
$$H = \begin{bmatrix} -\sin(\alpha_1) - \sin(\beta_1) \cos(\alpha_1) + \cos(\beta_1) - \sin(\alpha_1 - \beta_1) \\ -\sin(\alpha_2) - \sin(\beta_2) \cos(\alpha_2) + \cos(\beta_2) - \sin(\alpha_2 - \beta_2) \\ \vdots \\ -\sin(\alpha_L) - \sin(\beta_L) \cos(\alpha_L) + \cos(\beta_L) - \sin(\alpha_L - \beta_L) \end{bmatrix}$$

Since the variables in (3) are independent, a new least square (LS) algorithm denoted as LLS is utilized to obtain the estimated location of MS and the synchronization error

$$\hat{X}' = (H^T H)^{-1} H^T Z \tag{4}$$

3.2 LLS algorithms with the synchronization error elimination

Instead of defining a new variable, we can obtain the linear equations by eliminating the synchronization error. Three different methods inspired from the literatures [30, 31] are introduced to eliminate the synchronization error, and construct the corresponding linear equations. Then, the LS algorithm is applied to obtain the estimated location of MS. The first method is called LLS-1, through selecting the first equation in (2) as the reference equation, we subtract it from the rest equations to eliminate the synchronization error. Then, the linear equations can be obtained through ignoring the measured noise and doing some mathematic manipulations

$$Z_1 = H_1 \cdot X \tag{5}$$

where

$$Z_{1} = \begin{bmatrix} c_{2} - c_{1} - r_{2} + r_{1} \\ c_{3} - c_{1} - r_{3} + r_{1} \\ \vdots \\ c_{L} - c_{1} - r_{L} + r_{1} \end{bmatrix}, \quad H_{1} = \begin{bmatrix} a_{2} - a_{1} \ b_{2} - b_{1} \\ a_{3} - a_{1} \ b_{3} - b_{1} \\ \vdots \\ a_{L} - a_{1} \ b_{L} - b_{1} \end{bmatrix}$$
$$a_{i} = \frac{-\sin(\alpha_{i}) - \sin(\beta_{i})}{\sin(\alpha_{i} - \beta_{i})}, \quad b_{i} = \frac{\cos(\alpha_{i}) + \cos(\beta_{i})}{\sin(\alpha_{i} - \beta_{i})},$$
$$c_{i} = \frac{y_{1}(\cos(\alpha_{i}) + \cos(\beta_{i})) - x_{1}(\sin(\alpha_{i}) + \sin(\beta_{i}))}{\sin(\alpha_{i} - \beta_{i})}.$$

The second method is called LLS-2, the $L \times (L-1)/2$ linear equations are obtained by choosing two equations from (2) and subtracting each other. Thus, the following equations are employed for the estimated location of MS:

$$Z_2 = H_2 \cdot X \tag{6}$$

where

$$Z_{2} = \begin{bmatrix} c_{2} - c_{1} - r_{2} + r_{1} \\ c_{3} - c_{1} - r_{3} + r_{1} \\ \vdots \\ c_{L} - c_{L-1} - r_{L} + r_{L-1} \end{bmatrix}, \quad H_{2} = \begin{bmatrix} a_{2} - a_{1} & b_{2} - b_{1} \\ a_{3} - a_{1} & b_{3} - b_{1} \\ \vdots \\ a_{L} - a_{L-1} & b_{L} - b_{L-1} \end{bmatrix}.$$

The third method is called LLS-3, instead of obtaining the difference of the equations directly as the LLS-1 and LLS-2 methods, the average equation is obtained first, which is subtracted from all the equations, and will result in *L* new linear equations. The equations of the LLS-3 method can be expressed as

$$Z_3 = H_3 \cdot X \tag{7}$$

where

$$Z_{3} = \begin{bmatrix} c_{1} - f - r_{1} + g \\ c_{2} - f - r_{2} + g \\ \vdots \\ c_{L} - f - r_{L} + g \end{bmatrix}, \quad H_{3} = \begin{bmatrix} a_{1} - d & b_{1} - e \\ a_{2} - d & b_{2} - e \\ \vdots \\ a_{L} - d & b_{L} - e \end{bmatrix}$$

$$d = \frac{1}{L} \sum_{i=1}^{L} a_i, \ e = \frac{1}{L} \sum_{i=1}^{L} b_i, \ f = \frac{1}{L} \sum_{i=1}^{L} c_i, \ g = \frac{1}{L} \sum_{i=1}^{L} r_i.$$

3.3 QP algorithm

In the presence of the measured noise on the range and angle measurements, Eq. (3) will not hold in general. We resort to the optimization method, and define the residual error as the objective function

$$\min_{X'} ||Z - H \cdot X'||^2 \tag{8}$$

where $|| \cdot ||$ denotes the Euclidean norm.

As mentioned in [32], the NLOS error is always positive and assumed to be much larger than the range measurement noise. Therefore, we can relax the nonlinear constraints of X' into linear constraints as shown as follows:

$$x \le r_i - \varepsilon + x_1, -x \le r_i - \varepsilon - x_1$$

$$y \le r_i - \varepsilon + y_1, -y \le r_i - \varepsilon - y_1$$
(9)

Rewriting (9) in matrix form, we have

$$A_i X' \le B_i, i = 1, \cdots, L \tag{10}$$

where

$$A_{i} = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & -1 & 1 \end{bmatrix}, \quad B_{i} = \begin{bmatrix} r_{i} + x_{1} \\ r_{i} - x_{1} \\ r_{i} + y_{1} \\ r_{i} - y_{1} \end{bmatrix}$$

Based on the above objective function and linear inequalities, we can build a QP optimization problem, and it can be formulated as

$$\min_{X'} ||Z - H \cdot X'||^2$$
s.t. $A_i X' \leq B_{ij} i = 1, \dots, L$
(11)

The QP problem of (11) can be solved using the interior-point method [31].

3.4 Data fusion algorithm

From Eqs. (3) and (5)–(7), we know that only three OB scattering paths can obtain the estimated location of MS. If the number of multipath is larger than three, we can divide these measurements into different combinations. Each combination can obtain the intermediate estimate of MS. Finally, data fusion (DF) algorithm is utilized to fuse all the intermediate estimate. The key problem is how to measure the good or bad estimation of each combination. The simplest DF algorithm treated all the intermediate estimates as the same weight is to average all the estimated locations of MS. Thus, the algorithm is denoted as DF-LLS if the LLS is utilized to obtain the intermediate estimate of each combination, while it is denoted as DF-LLS-1 if the LLS-1 is utilized. In addition, the residual weighting algorithm [20] chooses the normalized residual as an indicator to fuse the intermediate estimate of each combination.

3.5 Cramer-Rao lower bound (CRLB)

CRLB is the performance bound in terms of the minimum achievable variance provided by any unbiased estimators. The CRLB of our localization model can be easily obtained as the same method in [30]. Given the unknown parameters $\theta = [X^T, \varepsilon, x'_1 \cdots, x'_L, y'_1 \cdots, y'_L]^T$, the joint probability density function of the range and angle measurements is as follows:

$$p(r_1, \alpha_1, \beta_1 \cdots r_L, \alpha_L, \beta_L | \theta) = \prod_{i=1}^L \frac{1}{\sqrt{(2\pi)^3 (\sigma_n^2 \sigma_\alpha^2 \sigma_\beta^2)^3}} e^{-\frac{(r_i - r_i^0 - \varepsilon)^2}{2\sigma_n^2} - \frac{(\alpha_i - \alpha_i^0)^2}{2\sigma_\alpha^2} - \frac{(\beta_i - \beta_i^0)^2}{2\sigma_\beta^2}}$$
(12)

The Fisher information matrix (FIM) with 2L + 3 unknown parameters can be defined as

$$I(\theta) = \begin{bmatrix} I_{xx} & I_{xy} & \cdots & I_{xy'_{L}} \\ I_{xy} & I_{yy} & \cdots & I_{yy'_{L}} \\ \vdots & \vdots & \ddots & \vdots \\ I_{xy'_{L}} & I_{yy'_{L}} & \cdots & I_{y'_{L}y'_{L}} \end{bmatrix}_{(2L+3)\times(2L+3)}$$
(13)

where $[I(\theta)]_{ij} = -E[\frac{\partial^2 \ln p(r_1,\alpha_1,\beta_1\cdots r_L,\alpha_L,\beta_L|\theta)}{\partial \theta_i \partial \theta_j}]$, the expressions of them can be found in Appendix A.

The CRLB of variables in θ is the (i, i) entry of $I^{-1}(\theta)$, i = 1, ..., L. If root mean square error (RMSE) is used as the performance criterion. The CRLB about the estimated location of MS in terms of RMSE is given as

$$CRLB = \sqrt{[I^{-1}(\theta)]_{11} + [I^{-1}(\theta)]_{22}}$$
(14)

4 MB identification methods with hybrid OB and MB scattering paths

In dense multipath environment, there will be paths that are subject to the MB scattering. The experiments in [33] show that the MB scattering path will have large attenuation and much lower signal strength on mmWave frequencies in comparison with the OB scattering path. As the mmWave technique is used for the next generation cellular networks, our localization model can be extended to abundant OB scattering paths with few MB scattering path. This section firstly presents the existing methods to detect and discard MB path with hybrid OB and MB scattering paths, then proposes a novel method to identify the MB path. After the MB path is discarded, the remaining paths are utilized to estimate the location of MS by utilizing the proposed algorithms discussed in Sect. 3.

4.1 Statistical proximity test

A two-step detection scheme with the statistical proximity test (SPT) is used to detect and discard the MB scattering path [11]. The first step of the test is to find the centroid of line of possible MS location for all the paths, which is defined as

$$C = \frac{1}{3L} \sum_{\substack{j=1 \ m=1}}^{L} \sum_{\substack{m=1 \ w_j > 0.1}}^{3} Z_m^j$$
(15)

where $Z_m^j = (x_m^j, y_m^j)$ is the point along each path, their mathematical expressions are shown as

$$x_{1}^{j} = x_{1} + r_{j} \times \cos(\beta_{j}), y_{1}^{j} = y_{1} + r_{j} \times \sin(\beta_{j})$$

$$x_{3}^{j} = x_{1} - r_{j} \times \cos(\alpha_{j}), y_{3}^{j} = y_{1} - r_{j} \times \sin(\alpha_{j})$$

$$x_{2}^{j} = \frac{x_{1}^{j} + x_{3}^{j}}{2}, y_{2}^{j} = \frac{y_{1}^{j} + y_{3}^{j}}{2}$$
(16)

The normalized weighting factor w_i in (15) is defined as

$$w_{j} = \frac{\frac{1}{L} \sum_{i=1}^{L} r_{i}/r_{j}}{\sum_{j=1}^{L} w_{j}}$$
(17)

It ranges between 0 and 1, a 10% weighting decision is stringent enough to provide a good calculation of the centroid C [11]. The second step is to calculate the normalized Euclidean distance between the midpoint of each path and the estimated centroid, which is formulated as

$$\delta_j = \frac{||Z_2' - C||}{\sum\limits_{j=1}^L \delta_j}$$
(18)

where δ_j is the normalized Euclidean distance between the *j*-th path of the BS and centroid. It ranges between 0 and 1, and 20 percent of the maximum limit is set to be sufficient to provide the correct rejection [11]. That means, the *j*-th path is decided as the MB path if $\delta_j > 0.2$, otherwise, it is the OB path.

The above SPT method has high ratio to reject the MB path with the experiment and simulation scenarios in [11]. However, if the experiment or simulation scenario is changed, it cannot work well as before. As shown in Fig. 3, when the Euclidean distance between the midpoint of the OB path and the centroid is almost the same as or larger than the Euclidean distance between the midpoint of MB path and the centroid, this method cannot correctly identify the MB path or mistakenly discard the OB path. In addition, it is difficult to decide whether the thresholds 0.1 and 0.2 are suitable to compute the centroid C and discard the MB path in any experiments or simulation scenarios, respectively.

4.2 Kmeans clustering

In dense multipath environment, the propagation path is the OB or MB scattering. Different to SPT method, classification technique denoted as *K* means clustering [34] can be used to detect and discard the MB path. From our localization model, we know that each path is characterized by three parameters denoted as $P(i) = (r_i, \alpha_i, \beta_i), i = 1, ..., L$. *K* means clustering aims to partition the *L* path samples into sets $S = \{S_1, S_2\}$, where S_1 is the subset of OB paths, S_2 is the subset of MB path. And it proceeds in three steps:

(1) Initial Step

Compute the squared Euclidean distance between two arbitrary paths and return the index of the two paths whose squared Euclidean distance is the largest.



$$D(i,j) = ||P(i) - P(j)||^2, i, j = 1, \dots, L, i \neq j$$

$$[i^0, j^0] = \arg \max_{i,j} (D(i,j))$$
(19)

The initial means about sets *S* are set as $\mu_1^0 = P(i^0)$ and $\mu_2^0 = P(j^0)$.

- (2) Assignment Step
 - Assign each sample P(i), i = 1, ..., L to the cluster S_i , $1 \le i \le 2$ whose mean has the smallest squared Euclidean distance, this is intuitively the nearest mean. It can be mathematically expressed as

$$S_k^{(t)} = \{P(i) : \left\| P(i) - \mu_k^{(t)} \right\|^2 \le \left\| P(i) - \mu_j^{(t)} \right\|^2 \forall j, 1 \le j \le 2\}, k = 1, \cdots, L$$
(20)

where *t* is the number of iterations.

(3) Update step

When all the samples are assigned, the new means can be calculated to the centroids of the samples in the new cluster

$$\mu_i^{(t+1)} = \frac{1}{|S_i^{(t)}|} \sum_{P(j) \in S_i^{(t)}} P(j), i = 1, 2$$
(21)

The algorithm goes back and forth between step 2 and step 3, and it has converged until no further change in the cluster assignments. In the end, this algorithm outputs the subset of S_1 and S_2 .

4.3 Double identification algorithm

In general, when the signal experiences the MB scattering path, it travels more distance than the one that experiences OB scattering path. Therefore, the significant difference between the OB path and MB path is the range measurement. Moreover, as shown in Fig. 3, the midpoints of the OB paths almost lie in one zone, whereas the midpoint of the MB path lies in the other zone. This motivates us to adopt the simple classification method to identify the MB path. Based on the above two considerations, a novel double identification algorithm (DIA) is proposed, and it proceeds as three steps:

1. Range Identification

Through calculating the average of the measured distance and dividing it by the measured distance of each path, we obtain a new value about each path. If this value is less than 1, it is the MB, otherwise, it is the OB.

2. Centroid Identification

From (16), we can calculate the centroid of each path. The initial classification is achieved through selecting the centroid corresponding to the minimum measurement distance as the OB, and the centroid to the maximum measurement distance as the MB. The Euclidean distances are calculated between the remaining centroid and the initial classification, respectively. Comparing this Euclidean distances with each other to determine whether it belongs to the OB or the MB, and then updating the

centroid of the OB and the MB. Repeating this process until all the centroid has been allocated, the sets of the OB and MB are obtained.

3. Incorporation

The final set of the MB is obtained by performing the set intersection of the two MB sets that obtained from step1 and step 2, and the rest is the OB set.

5 Simulation results and discussion

This section presents simulation results to evaluate the performance of the proposed localization algorithms in dense multipath environment. Figure 4 shows the distribution of the MS and BS with the OB and MB NLOS propagation paths, as well as the locations of MS, BS, and scatterers. As the same methods in [9-13], we can produce the TOA/AOD/AOA measurement data from (1), then the estimated location of MS is obtained with our proposed methods. The performance criteria are the Root Mean Square Error (RMSE) of the MS location estimate, which can be calculated as

RMSE =
$$\sqrt{\frac{1}{M} \sum_{i=1}^{M} (\hat{x}_i - x)^2 + (\hat{y}_i - y)^2}$$
 (22)

where *M* is the number of Monte-Carlo simulations, and (\hat{x}_i, \hat{y}_i) is the estimated location of MS in the *i*-th Monte-Carlo simulation. It is doubt that a different distribution of scatterer may lead to different results. However, we change the coordinates of the scatterer, a little change happens in the figures, but the same conclusion can be obtained.

5.1 Performance comparison with OB scattering paths

The simulation scenario is shown in Fig. 4, five OB scattering paths are considered. The LLS-based algorithms, QP algorithm and DF-based algorithms shown in Sect. 3 are compared. Moreover, in order to illustrate the effect of the synchronization error on CRLB, we denote the CRLB of our localization model as CRLB-S, whereas the CRLB in [19] is denoted as CRLB-NS. The effect of the standard deviation of range and angle measurements, i.e., σ_n , σ_α , and σ_β , as well as the synchronization error Δt , on the RMSE of the proposed localization algorithms are examined. The angle measurements are assumed to have the same standard deviation $\sigma_\alpha = \sigma_\beta$, and all the simulation results are obtained through 5000 independent runs. Figure 5 shows the effect of the synchronization error Δt on the localization accuracy when $\sigma_\alpha = \sigma_\beta = 1^0$ and $\sigma_n = 5$ m. As the Δt







gets larger, the RMSE of the proposed algorithms remain unchanged. It is demonstrated that the proposed algorithms can deal with the effect of the synchronization error. Figure 6 depicts the effect of $\sigma_{\alpha} = \sigma_{\beta}$ on RMSE when $\sigma_n = 5$ m and $\Delta t = 1$ µs. The RMSE of our proposed algorithms slightly increases as the increase of $\sigma_{\alpha} = \sigma_{\beta}$. Figure 7 shows the effect of σ_n when $\sigma_{\alpha} = \sigma_{\beta} = 1^0$ and $\Delta t = 1$ µs. The same conclusion can be obtained as Fig. 6, the RMSE of all the proposed algorithms increase as σ_n gets larger. From Figs. 5, 6 and 7, we can get some observed results. The performance of the LLS is the same as the QP, while the LLS-1, LLS-2, and LLS-3 also have the same performance. Due to the low computational complexity, we select LLS and LLS-1 as the comparison. The LLS-1 has the highest localization accuracy, followed by LLS, and then DF-based





algorithms. It is demonstrated that DF-based algorithm cannot improve the localization accuracy for our localization scenario. The reason is that the combination of DF-based algorithm may have low localization accuracy to degrade the overall performance. Figures 5, 6 and 7 also show the CRLB performance against different parameters, i.e., σ_n , σ_α , σ_β , and Δt . It clearly shows that the CRLB increases greatly with the presence of the synchronization error. The derived CRLB increases greatly as the increase of σ_n , σ_α , and σ_β , whereas it stays the same as the increase of Δt . The effect of the number of multipath on RMSE for the different localization algorithms is shown in Fig. 8. It is clearly indicated that the proposed algorithms have the relatively high localization accuracy when there are five OB scattering paths. As the number of OB path increases, there is few performance improvements, whereas the localization accuracy is declined greatly as it decreases. This can explain why the DF-based algorithms cannot improve the localization accuracy. Moreover, when there is only four OB paths, the QP has comparatively higher localization accuracy in comparison with other algorithms. That indicates the QP is suitable for the scenario with a small number of multipath.

5.2 Performance comparison with hybrid OB and MB scattering paths

This section first compares the MB identification performance of three algorithms shown in Sect. 4, and then presents the RMSE comparison of different algorithms. The simulation scenario is shown in Fig. 4, five OB and one MB scattering paths are considered.

5.2.1 The identification performance comparison

To demonstrate the capability of our proposed DIA algorithm, two other algorithms denoted as SPT and Kmeans are chosen to be compared. Since the OB path may be identified as the MB path, two indicators called "Exact" and "with Extra Path" are used to measure the identification performance. "Exact" represents only MB path is identified and no OB path is mistakenly identified as MB path, whereas "with Extra Path" represents the MB path is identified together with some OB paths are identified as the MB path. Figures 9, 10 and 11 show the effect of the synchronization error, angle deviation and range deviation on the identification ratio of MB path. Some observed results are shown that the synchronization error and angle deviation do not affect the identification performance of MB path, whereas the identification ratio of MB path decreases as the increase in range deviation. The SPT is almost impossible to correctly identify the MB path, especially when the synchronization error is large. The proposed DIA "with Extra path" can identify the MB path with one hundred percent, but "Exact" DIA demonstrates







that some OB paths are mistakenly judged as the MB path. In addition, comparing with the SPT and the Kmeans, the DIA has the highest identification ratio of MB path.

5.2.2 The RMSE performance comparison

According to the conclusions in Sect. 5.1, two algorithms LLS and LLS-1 are selected to combine with the MB identification algorithms. The effect of σ_n , σ_α , σ_β , and Δt on the RMSE are examined with hybrid OB and MB scattering paths. Figure 12 depicts the RMSE of different algorithms about the different Δt , it shows that the DIA-based





algorithms have the best localization accuracy, due to the correct identification of MB path. Because of the lowest ratio of MB identification, the performance of the Kmeans-based algorithms are the worst. These are consistent with the previous simulation results. When the Δt is very small, the SPT-based algorithms have comparatively higher localization accuracy. However, the localization accuracy significantly increases as it increases. Figures 13 and 14 show the effect of angle deviation and range deviation on different algorithms, respectively. The same conclusion can be obtained that the DIA-based algorithms perform the best, followed by SPT-based, then the Kmeans-based. As the angle deviation and range deviation get larger, the



RMSEs of both DIA-based and SPT-based algorithms increase slightly. However, the Kmeans-based algorithms remain unchanged.

6 Conclusion

In this paper, the synchronization error was introduced into the hybrid TOA/AOD/AOA measurement model, four LLS algorithms, one QP algorithm and DF-based algorithms with one BS were proposed in OB scattering environment. Moreover, the DIA was proposed to identify the MB path in hybrid OB and MB scattering environment. Simulation results demonstrated: (1) In OB scattering environment, the LLS had the same localization accuracy as the QP, while the LLS-1, LLS-2, and LLS-3 also had the same localization accuracy. Moreover, DF-based algorithms could not improve the localization accuracy. Further, the LLS-1 had the highest localization accuracy, followed by LLS, and then DF-based algorithms. (2) In hybrid OB and MB scattering environment, the DIA could correctly identify the MB path in comparison with the SPT and Kmeans algorithms, and the RMSE comparison further proved the validity of our proposed algorithm. (3) The standard deviation of range and angle measurements could affect the performance of our proposed algorithms, whereas the synchronization error had little effect about them. However, due to the limitation of experimental conditions, our conclusions were based on the simulation results. In the next step, we will focus on the experimental data to verify these conclusions.

Appendix A

Suppose the range and angle measurements are mutually independent, the joint probability density function of the range and angle measurements is shown in (12). We first perform two order differential calculation about the variables of θ , and then compute its expected value. Thus, the elements of $I(\theta)$ can be derived as follow:

$$\begin{split} I_{xx} &= \sum_{i=1}^{L} \left\{ \frac{1}{\sigma_n^2} \left(\frac{\partial r_i^0}{\partial x} \right)^2 + \frac{1}{\sigma_\alpha^2} \left(\frac{\partial \alpha_i^0}{\partial x} \right)^2 \right\}, \\ I_{xy} &= \sum_{i=1}^{L} \left\{ \frac{1}{\sigma_n^2} \frac{\partial r_i^0}{\partial y} \frac{\partial r_i^0}{\partial x} + \frac{1}{\sigma_\alpha^2} \frac{\partial \alpha_i^0}{\partial y} \frac{\partial \alpha_i^0}{\partial x} \right\}, \quad I_{x\varepsilon} = \sum_{i=1}^{L} \frac{1}{\sigma_n^2} \frac{\partial r_i^0}{\partial x} \\ I_{xx_i'} &= \frac{1}{\sigma_n^2} \frac{\partial r_i^0}{\partial x_i'} \frac{\partial r_i^0}{\partial x} + \frac{1}{\sigma_\alpha^2} \frac{\partial \alpha_i^0}{\partial x_i'} \frac{\partial \alpha_i^0}{\partial x}, \\ I_{xy_i'} &= \frac{1}{\sigma_n^2} \frac{\partial r_i^0}{\partial y_i'} \frac{\partial r_i^0}{\partial x} + \frac{1}{\sigma_\alpha^2} \frac{\partial \alpha_i^0}{\partial y_i'} \frac{\partial \alpha_i^0}{\partial x}, \\ I_{yy} &= \sum_{i=1}^{L} \left\{ \frac{1}{\sigma_n^2} \left(\frac{\partial r_i^0}{\partial y} \right)^2 + \frac{1}{\sigma_\alpha^2} \left(\frac{\partial \alpha_i^0}{\partial y_i'} \right)^2 \right\}, \quad I_{y\varepsilon} = \sum_{i=1}^{L} \frac{1}{\sigma_n^2} \frac{\partial r_i^0}{\partial y}, \\ I_{yx_i'} &= \frac{1}{\sigma_n^2} \frac{\partial r_i^0}{\partial x_i'} \frac{\partial r_i^0}{\partial y} + \frac{1}{\sigma_\alpha^2} \frac{\partial \alpha_i^0}{\partial x_i'} \frac{\partial \alpha_i^0}{\partial y}, \\ I_{yx_i'} &= \frac{1}{\sigma_n^2} \frac{\partial r_i^0}{\partial y_i'} \frac{\partial r_i^0}{\partial y} + \frac{1}{\sigma_\alpha^2} \frac{\partial \alpha_i^0}{\partial y_i'} \frac{\partial \alpha_i^0}{\partial y}, \\ I_{yy_i'} &= \frac{1}{\sigma_n^2} \frac{\partial r_i^0}{\partial y_i'} \frac{\partial r_i^0}{\partial y} + \frac{1}{\sigma_\alpha^2} \frac{\partial \alpha_i^0}{\partial y_i'} \frac{\partial \alpha_i^0}{\partial y}, \quad I_{\varepsilon\varepsilon} &= \frac{L}{\sigma_n^2}, \quad I_{\varepsilon x_i'} &= \frac{1}{\sigma_n^2} \frac{\partial r_i^0}{\partial x_i'}, \quad I_{\varepsilon y_i'} &= \frac{1}{\sigma_n^2} \frac{\partial r_i^0}{\partial y_i'}, \\ I_{x_i'x_j'} &= \left\{ \frac{1}{\sigma_n^2} \left(\frac{\partial r_i^0}{\partial x_i'} \right)^2 + \frac{1}{\sigma_\alpha^2} \left(\frac{\partial \alpha_i^0}{\partial x_i'} \right)^2 + \frac{1}{\sigma_\beta^2} \left(\frac{\partial \beta_i^0}{\partial x_i'} \right)^2, \quad i = j \\ I_{x_i'x_j'} &= \left\{ \frac{1}{\sigma_n^2} \left(\frac{\partial r_i^0}{\partial y_i'} \right)^2 + \frac{1}{\sigma_\alpha^2} \left(\frac{\partial \alpha_i^0}{\partial x_i'} \right)^2 + \frac{1}{\sigma_\beta^2} \left(\frac{\partial \beta_i^0}{\partial y_i'} \right)^2, \quad i = j \\ I_{x_j'y_j'} &= \left\{ \frac{1}{\sigma_n^2} \left(\frac{\partial r_i^0}{\partial y_i'} \right)^2 + \frac{1}{\sigma_\alpha^2} \left(\frac{\partial \alpha_i^0}{\partial x_i'} \frac{\partial r_i^0}{\partial y_i'} + \frac{1}{\sigma_\alpha^2} \frac{\partial \alpha_i^0}{\partial \alpha_i^2} \frac{\partial \alpha_i^0}{\partial x_i'} \frac{\partial \beta_i^0}{\partial y_i'} \right\}, \quad i = j \\ I_{x_j'y_j'} &= \left\{ \frac{1}{\sigma_\alpha^2} \left(\frac{\partial r_i^0}{\partial y_i'} \right)^2 + \frac{1}{\sigma_\alpha^2} \left(\frac{\partial \alpha_i^0}{\partial \alpha_i'} \frac{\partial \alpha_i^0}{\partial y_i'} + \frac{1}{\sigma_\beta^2} \frac{\partial \beta_i^0}{\partial \alpha_i'} \frac{\partial \beta_i^0}{\partial y_i'} + \frac{1}{\sigma_\beta^2} \frac{\partial \beta_i^0}{\partial \alpha_i'} \frac{\partial \beta_i^0}{\partial y_i'} \right\}, \quad i = j \\ \end{array} \right\}$$

where

$$\begin{split} s_{i} &= \sqrt{(x - x_{i}')^{2} + (y - y_{i}')^{2}} \\ \frac{\partial r_{i}^{0}}{\partial x} &= \frac{x - x_{i}'}{s_{i}}, \ \frac{\partial r_{i}^{0}}{\partial y} &= \frac{y - y_{i}'}{s_{i}}, \ \frac{\partial r_{i}^{0}}{\partial x_{i}'} &= \frac{x_{i}' - x}{s_{i}} + \frac{x_{i}' - x_{1}}{r_{i}^{0} - s_{i}}, \ \frac{\partial r_{i}^{0}}{\partial y_{i}'} &= \frac{y_{i}' - y}{s_{i}} + \frac{y_{i}' - y_{1}}{r_{i}^{0} - s_{i}} \\ \frac{\partial \alpha_{i}^{0}}{\partial x} &= \frac{y_{i}' - y}{s_{i}^{2}}, \ \frac{\partial \alpha_{i}^{0}}{\partial y} &= \frac{x - x_{i}'}{s_{i}^{2}}, \ \frac{\partial \alpha_{i}^{0}}{\partial x_{i}'} &= \frac{y - y_{i}'}{s_{i}^{2}}, \ \frac{\partial \alpha_{i}^{0}}{\partial y_{i}'} &= \frac{x_{i}' - x}{s_{i}^{2}} \\ \frac{\partial \beta_{i}^{0}}{\partial x_{i}'} &= \frac{y_{1} - y_{i}'}{(r_{i}^{0} - s_{i})^{2}}, \ \frac{\partial \beta_{i}^{0}}{\partial y_{i}'} &= \frac{x_{i}' - x_{1}}{(r_{i}^{0} - s_{i})^{2}} \end{split}$$

Abbreviations

MS: Mobile station; OB: One-bound; TOA: Time of arrival; AOD: Angle of departure; AOA: Angle of arrival; BS: Base stations; LLS: Linear least square; CRLB: Cramer-Rao lower bound; RMSE: Root mean-square error; MB: Multiple-bound; DIA: Double identification algorithm; QP: Quadratic programming; LBS: Location-based services; MIMO: Massive multiple-inputmultiple-output; LS: Least square; LOS: Line of sight; NLOS: Non-line of sight; DF: Data fusion; FIM: Fisher information matrix; SPT: Statistical proximity test.

Acknowledgements

We appreciate Dr. Shengjun Zhang for his contributions to this paper. We also respect his decision that he would like not to be an author of this paper. In addition, we thank the editor and anonymous reviewers for their helpful comments and valuable suggestions.

Authors' contributions

All authors took part in the discussions of the work described herein. All simulations are carried out by the first author, and improvements are integrated after discussions among the authors. Paper has been written collaboratively by the authors. Dr. Miao Zhang made great contributions to the revised manuscript. He worked on mathematical formulation, simulations and the response to the reviewers, as well as the writing. All authors read and approved the final manuscript.

Funding

This research work was supported in part by the Science and Technology Research Program of Chongqing Municipal Education Commission of China under grant number KJQN202000703, in part by the Key research and development projects of Sichuan-Chongqing cooperation in Sichuan Province under grant number 2020YFQ0057, in part by the 2018 Chongqing Technology Innovation and Application Demonstration Major Theme Special Project under grant number cstc2018jszx- cyztzxX0034, in part by the Natural Science Foundation of China under Grant 62101080, in part by the Science Foundation of China under Grant 62101080, in part by the Science Foundation of China under Grant 62101080, in part by the Research Start Up Funding of Chongqing Jiaotong University under Grant 2020020007 and XJ2021000501.

Availability of data and materials

The datasets used and/or analyzed during the current study are available upon request to the corresponding author.

Declarations

Competing interests

The authors declare that they have no competing interests.

Received: 15 June 2021 Accepted: 19 December 2021 Published online: 04 January 2022

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