Hindawi Publishing Corporation
EURASIP Journal on Wireless Communications and Networking
Volume 2007, Article ID 17826, 11 pages
doi:10.1155/2007/17826

Research Article

Modeling of Call Dropping in Well-Established Cellular Networks

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Received 8 January 2007; Revised 6 July 2007; Accepted 11 October 2007

Recommended by Alagan Anpalagan

The increasing offer of advanced services in cellular networks forces operators to provide stringent QoS guarantees. This objective can be achieved by applying several optimization procedures. One of the most important indexes for QoS monitoring is the drop-call probability that, till now, has not deeply studied in the context of a well-established cellular network. To bridge this gap, starting from an accurate statistical analysis of real data, in this paper, an original analytical model of the call dropping phenomenon has been developed. Data analysis confirms that models already available in literature, considering handover failure as the main call dropping cause, give a minor contribution for service optimization in established networks. In fact, many other phenomena become more relevant in influencing the call dropping. The proposed model relates the drop-call probability with traffic parameters. Its effectiveness has been validated by experimental measures. Moreover, results show how each traffic parameter affects system performance.

1. INTRODUCTION

The drop-call probability is one of the most important quality of service indexes for monitoring performance of cellular networks. For this reason, mobile phone operators apply many optimization procedures on several service aspects for its reduction. As an example, they maximize service coverage area and network usage; or they try to minimize interference and congestion; or they exploit traffic balancing among different frequency layers (e.g., 900 and 1800 MHz in the European GSM standard).

There are several papers which study performance in cellular networks and, in particular, how the drop call probability is related to traffic parameters.

Paper [1] is a milestone in performance analysis of mobile radio systems. Drop call probability is analyzed with the classical assumption of exponential distribution for the call-holding time. In particular, it puts emphasis on handover and its effects on performance. Handover is considered the main cause for call dropping.

The other classic work [2] shows how drop call and blocking probabilities are affected by user mobility, considering different patterns for movements of mobile equipments. Again, handover is considered the cause of call dropping.

Authors of [3, 4] have studied the performance of a cellular network by considering more general distributions for the call and the channel holding times. They started from distributions described in the well-known papers [5, 6]. Analytical expressions for drop-call probability are obtained showing the effect of more realistic assumption on system behavior.

Influence of handover on mobile network performance is analyzed in depth in [7, 8], considering different patterns for user mobility. Also in [9], the relationship between handover failure and call dropping is analyzed.

In [10], handover and call dropping are studied considering a cellular mobile communication network with multiple cells and different classes of calls, that is, multiple types of service are assumed. Each class has different call-holding and cell-residence times.

Paper [11] estimates the drop-call probability considering a multimedia wireless network. An adaptive bandwidth allocation algorithm is exploited to improve system performance and to reduce, in particular, handover-blocking probability.

Whereas the previous cited papers assume wireless networks with an infinite number of users, [12] describes what happens when a finite user population is taken into account. In particular, the study considers also the presence of a hierarchical cellular structure.

The common denominator of all the previous works is assumptions about network characteristics. They implicitly consider that an appropriate radio planning has been carried out; therefore, propagation conditions are neglected. Moreover, they do not deal with mobile equipment failure and network equipment outages. Such assumptions lead to consider that calls are dropped only due to the failure of the handover procedure. That is, the connection of an active user changing cell several times is terminated only due to the lack of communication resources in the new cell. For this reason, researchers have focused their attention on developing analytical models which relate handovers with traffic characteristics.

Although the described models were very useful in the early phase of mobile network deployment, they are not very effective in a well-established cellular network. In such a system, network-performance optimization is carried out continuously by mobile phone operators. So that, in real mobile networks, the call dropping due to lack of communication resources is usually a rare event (i.e., blocking probability of new calls and handovers is negligible). In this paper, such a behavior has been confirmed by analyzing real telephone traffic data measured in the cellular network of Vodafone (Italy). In particular, we found that many phenomena become more relevant than handover in influencing the call dropping (e.g., propagation conditions, irregular user behavior, and so on). Hence, new analytical tools and models to study the call dropping phenomenon in a well-established network as a function of traffic parameters (e.g., call arrival rate, call duration, and so on) are needed. This could help operators in their work for optimizing network performance and, then, for increasing revenues.

The main objective of this paper is to find a new simple model to relate drop-call probability with traffic parameters in this well-established cellular network where handover failure becomes negligible. To the best of our knowledge, there are not similar models in literature which can effectively help operators in their analysis and predictions on this kind of networks. Thus, with respect to other related works, our main contribution is to bridge this gap.

To this aim, starting from real traffic data, we have identified call-dropping causes. Then, using well-known statistical tools, we have characterized call arrival and drop processes together with conversation and ringing durations. These results have driven us in developing the new analytical model.

The considered approach has been validated by comparing model results with real GSM data. Moreover, the impact of model parameters on performance has been studied.

Even if the proposed analysis has been validated only considering a GSM network, the developed approach is quite general. Indeed, following a similar procedure, model parameters can be easily derived from data obtained in other cellular systems (e.g., UMTS cellular networks). This means that the model can be fruitfully exploited for performance evaluation in different cellular networks.

The rest of the paper is organized as follows. Section 2 describes measured data. In Section 3, data are statistically analyzed. Then, in Section 4 the new analytical model is developed. Model validation and numerical results are reported in Section 5. Finally, conclusions are drawn in Section 6.

2. CHARACTERIZATION OF ESTABLISHED CELLULAR NETWORKS

As discussed before, the rationale of this work is related to the peculiar behavior of well established cellular networks. Herein, we characterize such a behavior by exploiting real measured data that have been collected in the GSM network of Vodafone (Italy). In particular, we have identified the main causes of call dropping. Moreover, using well-known statistical tools, call process has been studied.

We refer to a cellular network as well established if the number of customers is stable assuming that the system-planning phase has been completed. In this kind of network, during the years, many optimization procedures have been applied to several radio and propagation aspects (e.g., the maximization of network coverage area and the minimization of interference by a careful positioning of base transceiver stations and an accurate frequency-reuse planning). Moreover, the maximization of network usage, the minimization of congestion, and the traffic balancing among surrounding cells have been obtained as a result of the network management.

For our analysis, a total of about one million of calls has been monitored in Vodafone network during 2003−2004 years. All data come from the main metropolitan areas in the South of Italy. Traffic has been monitored during several days, typically one week.

In order to obtain numerically significant data, several cells have been considered. In particular, these cells were chosen as representative of the whole network taking into account cell extension, number of served subscribers in the area, and traffic load. Obviously, large datasets are needed to reduce errors in probability estimation from relative frequencies [13]. This is especially true when considering the call-dropping phenomenon which is a rare event in well-established networks. For this reason, both macro cells in an urban metropolitan environment and cell clusters in suburban areas were chosen. The macro cells are characterized by high traffic load and allow us to manage sufficiently large data samples. Whereas with suburban areas, we need to group together from 5 up to 7 neighboring cells to obtain significant data samples.

2.1. *Classification of drop call causes*

Data obtained from the network operator consist of several timestamps about the temporal evolution of the calls, such as the call start and end times. Besides, in the operator databases a *clear code* is associated to each call, that is, an alphanumerical label reporting the cause of call termination. By using these *clear codes*, calls are classified in *not dropped* and *dropped*, distinguishing causes of dropping. To exclude any influence of temporary or local phenomena, the analysis was repeated in different hours during the day for both single cells and cluster of cells belonging to several urban areas. Furthermore, data were collected for a period of about 2 years in

Table 1: Occurrence of call-dropping causes in a reference cell.

Drop Causes	Occurrence [%]
Electromagnetic causes	51.4
Irregular user behavior	36.9
Abnormal network response	7.6
Others	4.1

different network areas, allowing us to verify the absence of any seasonal or area-dependent phenomena.

As a typical example, the classification of drop-call causes for a single cell is reported in Table 1. It is straightforward to note that the call dropping is mainly due to electromagnetic causes (e.g., power attenuation, deep fading, and so on). A lot of calls are dropped due to irregular user behavior (e.g., mobile equipment failure, phones switched off after ringing, subscriber charging capacity exceeded during the call). Other causes are due to abnormal network response (e.g., radio and signaling protocols error).

We highlight that only few calls were blocked due to lack of resources (e.g., handover failure). As a consequence, the call-blocking probability (i.e., the probability that a call does not find an available communication channel) is negligible for any dataset. Usually, this result is obtained by network operators by means of traffic-balancing policies, which allow the sharing of overloaded traffic among neighboring cells.

A classification of drop causes similar to the one reported in Table 1 has been observed for both single cells or cluster of cells.

Therefore, the main conclusion of our analysis was that, in a well-established cellular network, it is not possible to find a prevailing cause for call dropping, but rather a mix of heterogeneous and independent causes. Mainly, the handover failure is almost a rare event in such environment thanks to the reliability and the effectiveness of the deployed handover control procedure. That is why this work does not deal with blocking and handover probabilities like other papers already known in literature. Yet, we need a new model to relate drop-call probability with traffic parameters.

2.2. *Analysis of stationarity*

To develop our novel model for the drop call probability, we started from the statistical characteristics of measured real data. First of all, the stationarities of two processes, the traffic entering into the cell and the call duration, were analyzed.

The traffic entering in the cell follows a nonstationary trend, since its profile strictly depends on the number of active users in the system and on their requests. For example, Figure 1 depicts the traffic load during the day for a cluster of seven neighboring cells. It is worthwhile noticing its typical "*M*" shape [14, 15]. That is, during the night there is a very low traffic load, while during the morning and the afternoon traffic load increases. Besides, two spikes are present in correspondence of the busiest hours related to business and commercial activities. In Figure 1, these two spikes are at 12:00 and 19:00, respectively.

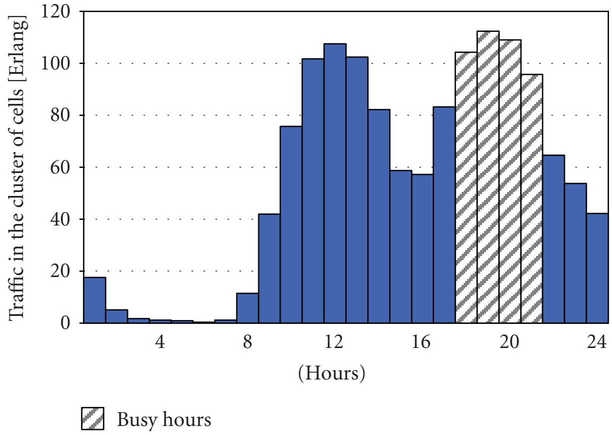

Figure 1: Daily traffic in a cluster of 7 neighboring cells.

To identify the size of the time window that satisfies the stationarity hypothesis for the traffic entering in the cell, we used the *run* and the *reverse arrangement tests* [16] which are hypothesis tests. They check for the presence of underlying trends or other variations in random data sequences.

To perform these stationarity tests, it has been assumed that the interarrival time between calls (i.e., the time between two successive call requests) is a random process $\{T_i\}_{i=1}^{n}$, where n is the total number of calls during one day. The stationarity of $\{T_i\}_{i=1}^{n}$ can be tested by the following steps.

(1) The trace of interarrival times $\{T_i\}_{i=1}^{n}$ is divided into m subtraces with equal time length (for simplicity multiples of one hour) obtaining m sequences $\{T_j^{(m)}\}_{j=1}^{N_m}$, where N_m is the number of samples of the mth subtrace.

(2) The mean value for each time interval is computed. The presence of underlying trends or variations in the sequence $\{T_j^{(m)}\}_{j=1}^{N_m}$ is tested, using both the *run test* and the *reverse arrangement test*.

(3) If in a subtrace there is an underlying trend on the considered time scale (i.e., the considered value of m), then the subtrace is not stationary with respect to the mean value.

(4) The size of the time window is decreased (i.e., the number, m, of subtraces is increased), repeating all the previous steps until obtaining positive response from both the tests, for all the subtraces.

We found that in all the cases, with a significance level of 0.05, data traces pass both the tests only when the size of the time window does not exceed four hours. Thus, we can analyze the traffic entering in the cell (and then the call arrival process) considering only a time window equal to or smaller than four hours. Given that the uncertainty of any statistical estimation decreases as the sample size increases (i.e., with larger sample, the influence of outliers is reduced), we chose an interval of four hours (i.e., the maximum window size which ensures stationarity) around the busiest day hour (i.e., the time interval with the maximum number of data samples).

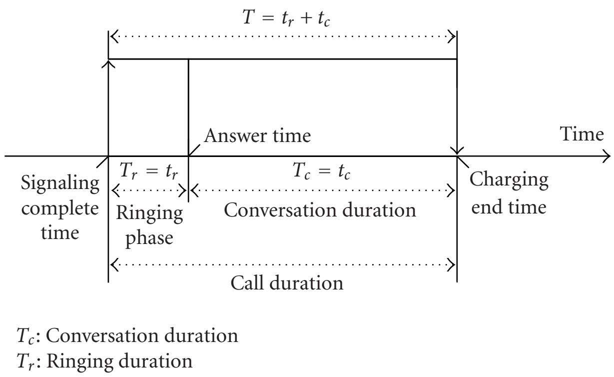

FIGURE 2: Time diagram to describe call behavior.

In Figure 1 the four hours around the busiest day hour are highlighted.

Concerning call duration, following a similar procedure (i.e., using run test and reverse arrangement tests), the stationarity was verified for any size of the time window. Specifically, we found that the mean call duration (evaluated in each hour) does not change appreciably during the day. Therefore, if we refer to the four hours around the busiest day hour, call duration is anyway a stationary process.

Given the aforesaid observations, unless otherwise specified, in the following the analysis will be referred to the four-hour time window around the busiest hour.

3. DATA ANALYSIS AND CHARACTERIZATION

To characterize the call dropping, we have analyzed the call arrival process and, specifically, the interarrival time between two successive new calls. Moreover, the interdeparture time between two successive dropped calls has been studied (i.e., the interval between call dropping instants); in the following, this time will be simply referred to as *interdeparture time*.

Likewise, to statistically characterize call duration, we have analyzed the durations of normally terminated calls (i.e., *not-dropped-calls* in operator database) and of dropped calls, distinguishing two phases: ringing and conversation (see Figure 2). The duration of the ringing phase is calculated as the difference between the *answer time* (i.e., the instant when the callee answers) and the *signaling complete time* (i.e., the instant when the communication setup finishes). The conversation duration is the difference between the *charging-end time* (i.e., the instant when the billing account stops) and the *answer time*. In our analysis, the *setup time* is not included in the evaluation of call duration because it does not depend on user behavior, but only on network characteristics.

The estimation of the mean, μ, and the variance, σ^2, of conversation duration (for both dropped and normally terminated calls) and of interarrival and interdeparture times were carried out. We used the following well-known convergent and not-polarized estimators [13]:

$$\widehat{\mu} = \frac{\sum_{i=1}^{n} x_i}{n}, \qquad \widehat{\sigma}^2 = \frac{\sum_{i=1}^{n} (x_i - \widehat{\mu})^2}{(n-1)}, \tag{1}$$

where $(x_1, x_2, \dots, x_n)$ is a sample vector of n elements.

Furthermore, the coefficient of variation, C, defined as the ratio between standard deviation and mean has been evaluated; this parameter is an index of data dispersion around the mean value. In Table 2, estimated statistical parameters (referred to 4 hours around the busy hour) are reported for five cells and two clusters of cells.

We observed that the conversation durations of normally terminated calls and dropped calls show a value of C greater than 1, whereas the interarrival and the interdeparture times have a coefficient of variation $C \simeq 1$. This behavior holds for both cells and cluster of cells. These results can suggest the choice of the pdf (probability density function) to represent each considered process. In particular, we made the hypothesis, validate by the following statistical analysis, that conversation duration of normally terminated calls and conversation duration of dropped calls have lognormal pdfs with different parameters [13]:

$$f_T(t) = \frac{1}{\varphi\sqrt{2\pi}t} e^{-(\ln t - \vartheta)^2/2\varphi^2}, \quad \varphi, \theta > 0, \ t \geq 0. \tag{2}$$

It is worthwhile to note that this result extends and generalizes the one reported in [17], where a lognormal function is used to fit only the channel-holding time in a single cell. Instead, the conversation duration, considered in this paper, is the sum of the channel-holding times in all the cells visited by the user during the same call.

For interarrival and interdeparture times we made the hypotheses of exponential pdfs, which are density functions with a coefficient of variation equal to one:

$$f_X(t) = \lambda e^{-\lambda t}, \quad \lambda > 0, \ t \geq 0. \tag{3}$$

It seems appropriate to mention that, although analysis of interarrival times has been reported in a previous scientific paper [17], the study of interdeparture time is a new result of this paper.

In the next sessions, the previous hypotheses about pdfs of conversation durations, interarrival time, and interdeparture time will be verified exploiting two suitable statistical methods.

3.1. *Analysis with probability plots*

In order to asses if a data set follows a given distribution, there are some useful graphical tools such as the *probability plot* [18].

The idea is to plot, together on the same graph, the cumulative distribution functions of the data sample and of a specific theoretical distribution, for the same quantile values. That is, on the axes there are the ordered values of the considered dataset and the theoretical distribution percentiles. For a given point on the probability plot, the quantile level is the same for both the variables on the axes. If the considered distribution really fits data, the points should lie approximately on a straight line.

Probability plots can be generated for several competing distributions to find which provides the best fit. Many aspects about distribution can be simultaneously tested and detected

Table 2: Estimated statistical parameters.

		Number of calls	$\widehat{\mu}\,[s]$	$\widehat{\sigma}\,[s]$	C
Cell 1	Conversation duration of normally terminated Calls	2339	121.74	205.65	1.69
	Conversation duration of dropped calls		96.01	172.09	1.79
	Interdeparture time		92.44	87.67	0.95
	Interarrival time		6.14	6.14	1.00
Cell 2	Conversation duration of normally terminated calls	2180	93.20	152.18	1.63
	Conversation duration of dropped calls		130.20	339.70	2.61
	Interdeparture time		67.72	78.23	1.16
	Interarrival time		6.60	6.54	0.99
Cell 3	Conversation duration of normally terminated calls	4555	100.97	134.89	1.34
	Conversation duration of dropped calls		92.86	159.35	1.72
	Interdeparture time		101.08	103.33	1.02
	Interarrival time		3.18	3.53	1.11
Cell 4	Conversation duration of normally terminated calls	2200	111.15	187.50	1.69
	Conversation duration of dropped calls		95.64	213.47	2.23
	Interdeparture time		85.01	94.28	1.11
	Interarrival time		6.54	7.01	1.07
Cell 5	Conversation duration of normally terminated calls	3586	108.35	198.13	1.83
	Conversation duration of dropped calls		97.27	174.25	1.79
	Interdeparture time		99.65	101.27	1.01
	Interarrival time		4.00	5.00	1.25
Cluster 1	Conversation duration of normally terminated calls	11748	100.41	212.21	2.10
	Conversation duration of dropped calls		94.92	199.69	2.11
	Interdeparture time		27.25	27.23	0.99
	Interarrival time		1.25	1.41	1.13
Cluster 2	Conversation duration of normally terminated calls	4448	107.70	208.94	1.94
	Conversation duration of dropped calls		91.42	161.67	1.77
	Interdeparture time		74.48	79.34	1.07
	Interarrival time		3.47	13.23	1.05

from this plot: shifts in location, shifts in scale, changes in symmetry, and the presence of outliers (see for details [18]).

To verify our hypothesis about pdf of the conversation time, we can consider the probability plot for the logarithm of conversation duration versus the normal standard distribution. In fact, as well known, the normal and lognormal distributions are closely related, that is, if X is lognormally distributed with parameters θ and φ, then $\log(X)$ is normally distributed with the same parameters [13]. For example, with reference to the normally terminated calls in a cell monitored for 4 hours, Figure 3 reports the probability plot for the logarithm of conversation duration versus normal standard distribution. A similar result holds also for the conversation duration of dropped calls. Figure 4 shows the probability plot for the interarrival time versus the exponential distribution.

From both figures, it can be noticed that data lie on a straight line, confirming our hypotheses about pdfs. We highlight that also the probability plots for the interdeparture time between dropped calls, which have not been reported for lack of space, show similar agreement.

Regarding the ringing time, T_r, measures have shown that there are many values close to zero, a lot of values around 5 seconds, and few larger values. So that, it does not follow any known distribution. By using again the probability plots (not reported for lack of space), it has been verified that a suitable pdf for fitting ringing time data was a weighted mixture of exponential and lognormal pdfs:

$$f_{T_r}(t) = \alpha\lambda e^{-\lambda t} + \frac{(1-\alpha)}{\varphi\sqrt{2\pi t}} e^{-(1/2)((\log(t)-\theta)/\varphi)^2}; \quad t \geq 0,\ \alpha \in [0,1], \tag{4}$$

where α is a weight coefficient.

3.2. The χ^2-goodness-of-fit-test results

The probability plot remains a qualitative graphical test. To confirm our assumption, we need to deploy also a hypothesis test such as the χ^2-goodness-of-fit test (χ^2-test) [19]. Such a test requires the estimation, from the sample data, of parameters for each distribution under testing.

We use the well-known maximum likelihood method [13]. Let X be a random variable with its pdf dependent on the parameter γ and let

$$f(X,\gamma) = f(x_1,\gamma)\cdot f(x_2,\gamma)\cdots f(x_n,\gamma) \tag{5}$$

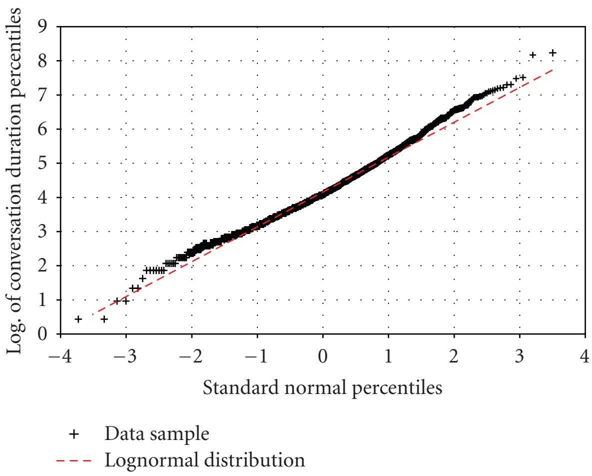

FIGURE 3: Probability plot for the logarithm of conversation duration (for normally terminated calls) versus normal standard distribution.

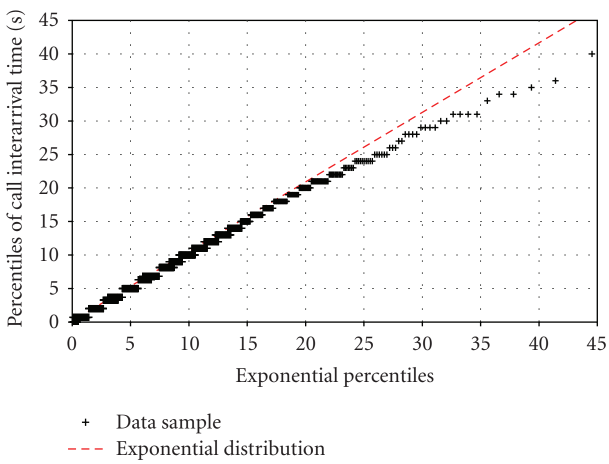

FIGURE 4: Probability plot of calls interarrival time versus exponential distribution.

be the joint density function of n samples x_i of X. This density, considered as a function of γ, is called the *likelihood function* of X.

The value γ^* of γ that maximizes $f(X,\gamma)$ is the maximum likelihood estimation of γ. The logarithm of $f(X,\gamma)$,

$$L(X,\gamma) = \ln f(X,\gamma) = \sum_{i=l}^{n} \ln f(x_i,\gamma), \tag{6}$$

is the *log-likelihood* function of X.

From the monotonicity of logarithm, it follows that γ^* also maximizes the function $L(x,\gamma)$ and is the solution of the equation

$$\frac{\partial L(X,\gamma)}{\partial \gamma} = \sum_{i=1}^{n} \frac{1}{f(x_i,\gamma)} \frac{\partial f(x_i,\gamma)}{\partial \gamma} = 0. \tag{7}$$

As shown in [13], the maximum likelihood estimator is asymptotically normal, unbiased, with minimum variance.

For our purpose, the maximum likelihood estimators for the parameters of the exponential and the lognormal pdfs can be easily obtained solving (7) applied to (2) and (3). The estimators are, respectively (see [13, 17]),

$$\begin{gathered}\hat{\lambda} = n \Big/ \sum_{i=1}^{n} t_i, \\ \hat{\vartheta} = \frac{1}{n}\sum_{i=1}^{n} \ln(t_i), \qquad \hat{\varphi} = \frac{1}{n}\sum_{i=1}^{n} \ln(t_i)^2 - \hat{\vartheta}^2,\end{gathered} \tag{8}$$

where t_i are the time samples.

Unfortunately, it is not possible to obtain a closed form expression for the four estimators of the parameters in (4), since from (7) we obtain a nonlinear equation system. Nevertheless, such a system can be solved by numerical methods. Specifically, as described in [20, 21], a subspace trust region method based on the interior-reflective Newton method has been implemented.

Now, we can apply the χ^2-test to check our hypotheses about pdfs following the algorithm introduced by Fisher [19]. Using the significance level $\alpha = 0.01$, the tests gave positive results in all the trials. As in [17], also in this work it was necessary to filter data samples which showed an anomalous relative frequency. But, whereas in [17] the 26% of the sample data were rejected, in our analysis never more than 5% of data have been discharged.

The obtained results show that both conversation durations of normally terminated calls and dropped calls are lognormal distributed. Moreover, our statistical analysis confirms the exponential hypothesis both for interarrival time between two successive new calls and for the interdeparture time between two successive dropped calls. Finally, χ^2-test confirms that ringing time has the pdf reported in (4). Even if some of this results are similar to previous ones [17], we highlight that, to the best of our knowledge, the analyses of interdeparture time, of conversation duration for dropped calls, and of ringing time do not appear in any previous scientific papers.

As an example, in Figure 5 the measured data and the fitted lognormal pdf for the conversation duration of normal terminated calls are reported. In Figure 6, the same information is reported, but referring to the dropped calls. In Figures 7 and 8 the interdeparture time between dropped calls and the interarrival time between calls are fitted by exponential pdfs. Finally, in Figure 9 the ringing duration pdf of a cluster of 7 cells monitored for 4 hours is fitted by a mixture of exponential and lognormal pdfs. We point out that the conclusions about the characterization of call durations, interarrival time between calls, and interdeparture time between dropped calls hold both for single cells and for cell clusters.

4. ANALYTICAL MODEL

In this section, starting from the results of data analysis, a new analytical model to predict the drop-call probability as a function of traffic parameters has been developed.

As verified in the previous section, the interarrival times for new calls and interdeparture time for dropped calls have an exponential distribution. With the additional hypotheses

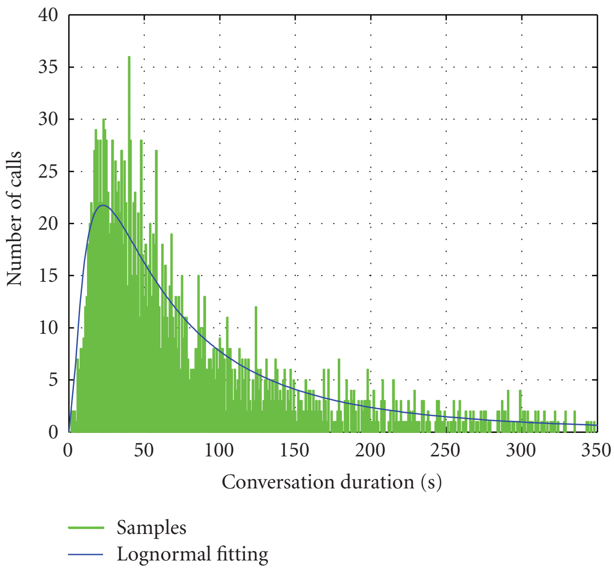

Figure 5: Fitting of conversation duration of normally terminated calls with a lognormal pdf (cell 1 observed for 4 hours).

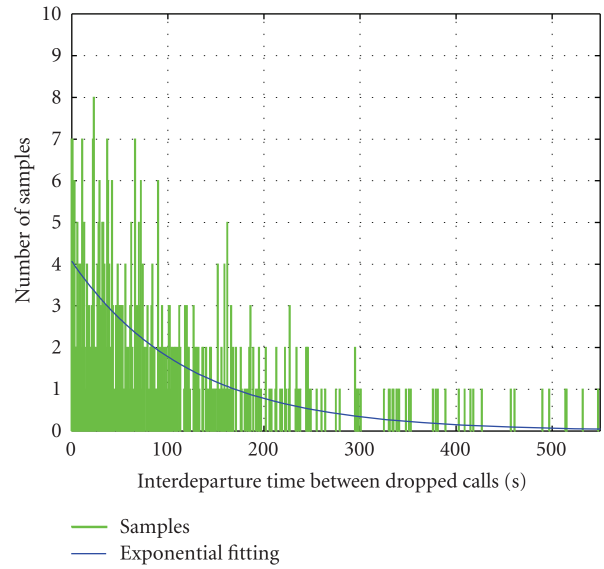

Figure 7: Fitting of interdeparture time between dropped calls with an exponential pdf (cell 1 observed for 24 hours).

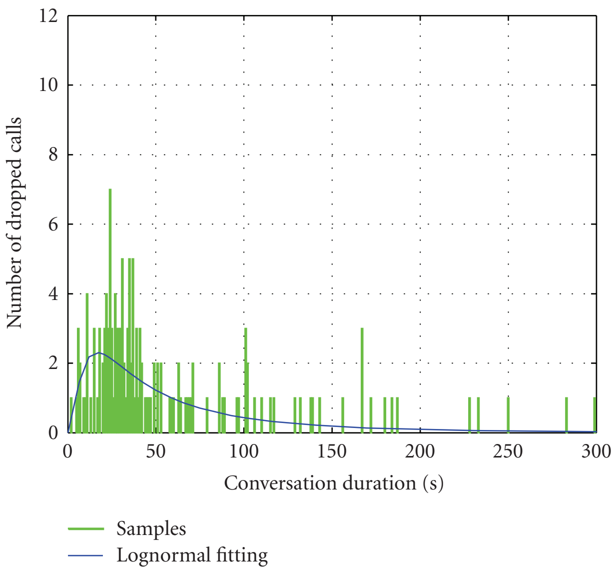

Figure 6: Fitting of conversation duration of dropped calls with a lognormal pdf (cell 1 observed for 4 hours).

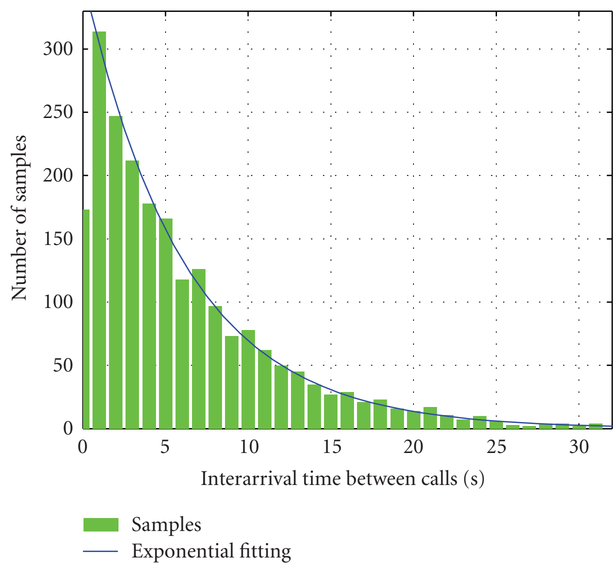

Figure 8: Fitting of interarrival time between calls with an exponential pdf (cell 1 observed for 4 hours).

of independence for both arrival events and dropping events, we can state that these processes can be considered Poissonian. This result extends the one reported in [14] in which, starting from measures, the classical Poisson hypothesis is verified only for call arrivals.

In this way, we can model all the causes of call dropping as a single phenomenon which follows the Poisson statistic.

Let λ_t be the total traffic entering in the generic cell. Since in a well-established cellular network the call-blocking probability is almost negligible (i.e., the system can be considered as nonblocking), λ_t is also the total traffic accepted in the cell. The drop call probability, P_d, is equal to the fraction of this traffic dropped due to the phenomena described in Section 2.

To evaluate P_d, let us consider, for sake of simplicity, the probability that a call is normally terminated, P_{nt}, related to P_d by the following expression:

$$P_d = 1 - P_{\text{nt}}. \tag{9}$$

A call request is served by a generic channel, randomly selected, and the call will finish, if correctly terminated, after a duration time, T (see Figure 2). From the results reported

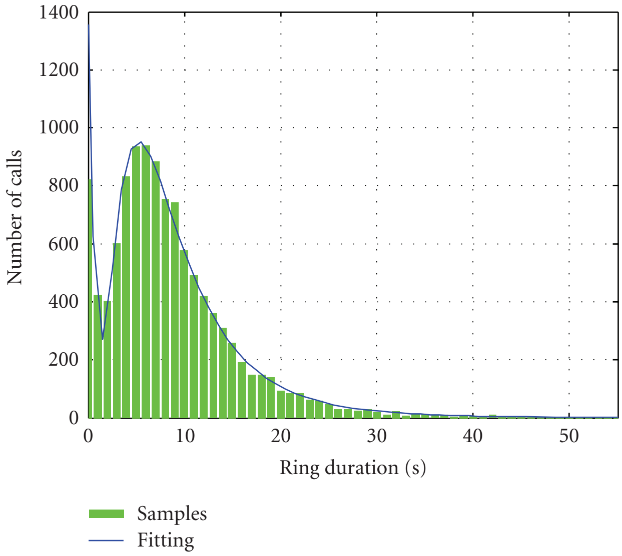

FIGURE 9: Fitting of ringing duration with a mix of exponential and lognormal pdf (cluster 1 observed for 4 hours).

in the previous section, we can state that the call duration, T, is the sum of the two random variables T_r and T_c which model the ringing and conversation times, respectively. The random variable (r.v.) T_r models the ringing duration with a pdf $f_{T_r}(t)$, according to (4). The r.v. T_c models the conversation duration with a lognormal pdf $f_{T_c}(t)$, according to (2). Assuming that T_r and T_c are independent, the pdf $f_T(t)$ of the call duration for the normally terminated calls can be obtained as the following convolution between pdfs [13]:

$$f_T(t) = f_{T_r}(t) * f_{T_c}(t) = \int_0^t f_{T_c}(t-\tau)\cdot f_{T_r}(\tau)d\tau. \quad (10)$$

The probability $P_{\text{nd}}(1)$ that a call, among k active ones, is not involved by a single drop event (i.e., call is not dropped), during the duration time $T = t$, is $(k-1)/k$. Obviously, given that drop events are assumed to be independent, if there are n drop events, this probability becomes

$$P_{\text{nd}}(n) = \left(\frac{k-1}{k}\right)^n. \quad (11)$$

On the other hand, as said before, dropping events constitute a Poisson process; let ν_d be its intensity. Hence, if Y is the r.v. which counts the number of drops, the probability that there are n drops in the interval $T = t$ is [13]

$$P(Y = n) = \frac{(\nu_d t)^n}{n!}e^{-\nu_d t}, \quad n \geq 0. \quad (12)$$

By using (11) and (12), the probability that a call with duration $T = t$ is normally terminated, in the presence of k contemporary calls and n drop events, is equal to the probability that drop events do not affect the considered call:

$$P_{\text{nt}}(T = t, k, n) = P_{\text{nd}}(n)\cdot P(Y = n) = \left(\frac{k-1}{k}\right)^n \frac{(\nu_d t)^n}{n!}e^{-\nu_d t}. \quad (13)$$

By applying the total probability theorem to the number of drop events, the probability that a call with duration $T = t$ is normally terminated, in the presence of k contemporary calls (i.e., the call is not dropped), can be estimated as

$$\begin{aligned} P_{\text{nt}}(T = t, k) &= \sum_{n=0}^{\infty} P_{\text{nt}}(T = t, k, n) \\ &= \sum_{n=0}^{\infty}\left(\frac{k-1}{k}\right)^n \frac{(\nu_d t)^n}{n!}e^{-\nu_d t} \\ &= e^{-\nu_d t}\sum_{n=0}^{\infty}\frac{1}{n!}\left[\frac{(k-1)\nu_d t}{k}\right]^n \\ &= e^{-\nu_d t}\cdot e^{((k-1)/k)\,\nu_d t} = e^{-\nu_d t/k}. \end{aligned} \quad (14)$$

Using again the total probability theorem, summing over all the possible numbers of contemporary active calls, the probability that a call is normally terminated with duration t is

$$P_{\text{nt}}(T = t) = \sum_{k=1}^{\infty} P_{\text{nt}}(T = t, k)\cdot P_a(k), \quad (15)$$

where $P_a(k)$ is the probability that there are k active users (i.e., k calls in progress).

As experimentally verified (see Section 2), in well-established cellular networks operating in normal conditions, the call dropping is not due to unavailability of communication channels (i.e., the blocking and handover probabilities are negligible). Thus, we can model the system as a queue with infinite number of servers, which is a nonblocking queue. Considering as service time the call duration, we have to consider a queue with a general service time distribution. This means that, by using the queuing theory notation [22], the system can be modeled as an M/G/∞ queue. Therefore, the probability $P_a(k)$ that there are k active users is given by [22]

$$P_a(k) = c_N\cdot\frac{\rho^k}{k!}, \quad k \geq 1, \quad (16)$$

where ρ is the utilization factor, given by the product between the total traffic λ_t and the mean service time $E[T]$; c_N is a normalization coefficient which considers that there is at least one ongoing call.

Applying the normalization condition, the coefficient c_N is evaluated as

$$c_N = \frac{1}{e^\rho - 1}. \quad (17)$$

Note that exploiting the utilization factor ρ, we can also evaluate the mean number of active users $E[N]$:

$$E[N] = \sum_{k=1}^{\infty} k\cdot c_N\frac{\rho^k}{k!} = \frac{e^\rho}{e^\rho - 1}\rho. \quad (18)$$

Using (17) in (16), we obtain

$$P_a(k) = \frac{1}{e^\rho - 1}\cdot\frac{\rho^k}{k!}, \quad k \geq 1. \quad (19)$$

Substituting (19) and (14) in (15), we have

$$P_{\mathrm{nt}}(T=t)=\sum_{k=1}^{\infty} e^{-\nu_d t/k}\cdot\frac{1}{e^{\rho}-1}\cdot\frac{\rho^k}{k!}. \tag{20}$$

Now, it is straightforward to evaluate the probability of a normally terminated call, P_{nt}, simply considering every possible call duration:

$$\begin{aligned} P_{\mathrm{nt}} &= \int_0^{\infty} P_{\mathrm{nt}}(T=t) f_T(t)dt \\ &= \frac{1}{e^{\rho}-1}\sum_{k=1}^{\infty}\frac{\rho^k}{k!}\int_0^{\infty} f_T(t)e^{-\nu_d t/k}dt, \end{aligned} \tag{21}$$

where $f_T(t)$ is the pdf defined by (10).

Finally, from (9), it results that the drop-call probability is

$$P_d = 1-\frac{1}{e^{\rho}-1}\sum_{k=1}^{\infty}\frac{\rho^k}{k!}\int_0^{\infty} f_T(t)e^{-\nu_d t/k}dt. \tag{22}$$

It is worth noticing that (22) depends on the drop-call rate ν_d, the pdf $f_T(t)$ of the call duration of normally terminated calls, and the utilization factor ρ (which in turn depends on the traffic λ_t).

Equation (22) can be exploited to study the effect of traffic parameters on drop-call probability, but it can be also applied to predict such a probability starting from real data. In the latter case, equation parameters should be obtained from measured data following the same analysis described in Section 3.

The development of our model did not require any assumption on a particular technology. Thus, the model can be exploited to predict the drop-call probability in different cellular networks (e.g., PCS, UMTS). In fact, we need only measured datasets to find the pdfs that best fit ringing time, conversation duration, interarrival time, and interdeparture time. Then, we can characterize (10) and find the drop-call probability in this kind of systems by applying (22).

5. NUMERICAL RESULTS

The developed model has been validated by using the real data analyzed in Section 3. Moreover, it has been exploited to study the effect of its parameter on network performance (i.e., we evaluated the model sensitivity to its parameters).

For the validation, in each considered cell, the drop-call probability and its confidence interval [13] (with confidence level $1-\alpha=0.95$) have been estimated directly from measured data. This is to establish the acceptance region for results from our model. Then, the drop call probability has been analytically estimated just applying (22). Parameters of this equation have been obtained by the data analysis reported in Section 3. Results coming out from the analytical model can be considered acceptable if they fall in the confidence interval of the measured drop-call probability.

In Table 3, results of validation are reported for the same cells and cluster of cells considered in Table 2 (i.e., the datasets for which we have explicitly reported numerical results of statistical analysis). They show that, in every case, the analytical results fall in the confidence interval of measured drop-call probability. This result has been confirmed for all the sets of measured data, thus validating our model.

Table 3: Drop-call probability results.

	(By measures) P_d[%]	(By model) P_d [%]	Confidence interval [%]
Cell 1	6.79	6.52	[5.84; 7.88]
Cell 2	7.29	7.47	[6.27; 8.46]
Cell 3	3.07	3.12	[2.61; 3.61]
Cell 4	6.72	6.74	[5.75; 7.84]
Cell 5	4.04	4.00	[3.44; 4.74]
Cluster 1	4.61	4.29	[4.13; 5.14]
Cluster 2	4.68	4.34	[4.08; 5.37]

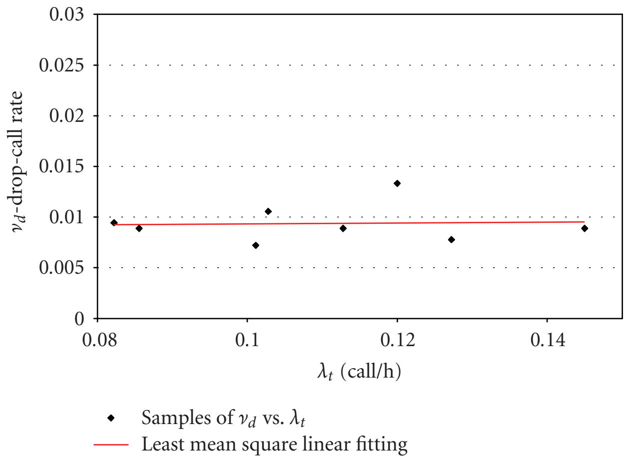

Figure 10: Total entering traffic in a cell, λ_t, versus the drop-call rate, ν_d.

A better agreement between real data and model results could be achieved by using larger data sample [13]. In fact, as the dataset gets larger, the confidence interval gets smaller. Hence, the estimation of the input parameters (i.e., ν_d, λ_t, and so on) for the analytical model gets more accurate. It is evident from the comparison of Tables 2 and 3 that the narrowest confidence intervals (i.e., the better estimations for our model) correspond to the largest datasets (i.e., *Cell 3* and *Cluster 1*).

The model can be also exploited to evaluate network performance as a function of traffic parameters. For example, it allows us to asses the sensitivity of the drop call probability to call duration distribution, to the offered traffic load, and so on. To this aim, first the correlation between ν_d and λ_t has been studied from data. We found that a linear dependence between these two parameters exists, that is,

$$\nu_d = m\lambda_t + b, \tag{23}$$

where m and b could be obtained with a least square regression technique [13].

Figure 10 shows that relatively large variations of λ_t produce only small changes for ν_d.

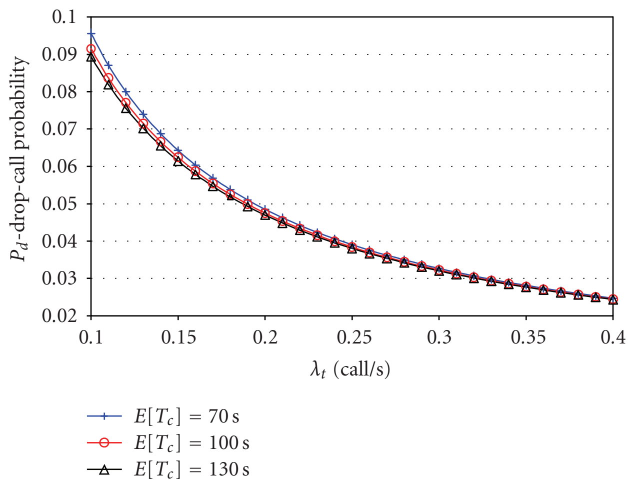

FIGURE 11: Drop-call probability versus traffic λ_t with several mean conversation durations.

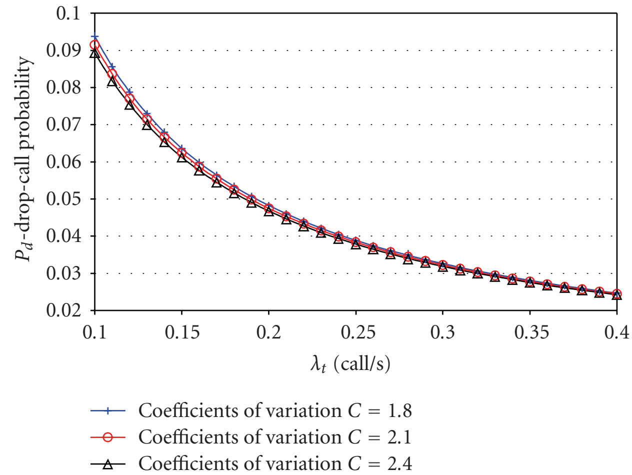

FIGURE 12: Drop-call probability versus traffic λ_t with several coefficients of variation C.

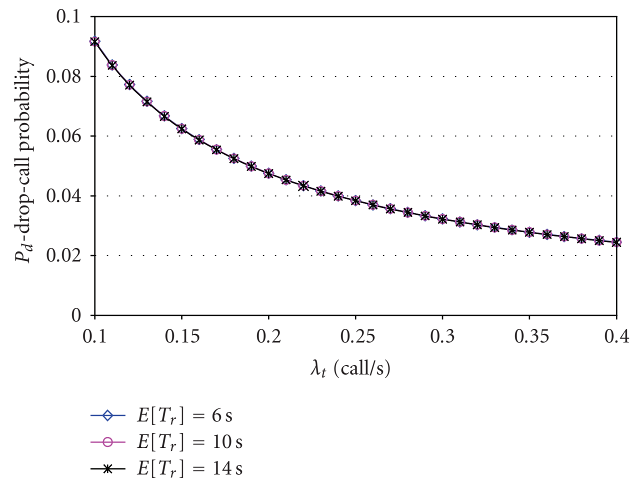

FIGURE 13: Drop-call probability versus λ_T with several mean ringing durations.

Hence, in (22) the effect of the drop call rate ν_d can be studied by considering only the effect of the call arrival rate λ_t. At the same time, the other parameter of the model (i.e., the utilization factor ρ) is defined as the product between the mean call duration $E[T]$ and the call arrival rate λ_t. Therefore, we can simply analyze the impact on model results of the call-arrival rate and of the call duration.

In Figure 11, the drop-call probability obtained by the model is reported as a function of the total traffic entering in the cell, λ_t (measured in calls per second [call/s]). The graphs are reported for several values of the mean conversation duration $E[T_c]$ (from 70 seconds to 130 seconds) with a fixed coefficient of variation, C, equal to 1.3, near to the typical value observed in measured data (see Table 2). The mean ringing duration is equal to 10 seconds. The drop call rate ν_d was varied accordingly with (23).

System performance improves as the traffic entering in the cell increases. Since there is a linear dependence between λ_t and ν_d, increasing the traffic load, the number of dropped calls remains quite constant. For this reason, the drop-call rate decreases.

Furthermore, drop-call probability remains quite constant when mean call duration increases. Only for small values of λ_t, that is, for a low traffic load, there are appreciable differences.

In Figure 12, the drop-call probability is reported as a function of the total traffic entering in the cell, λ_t, with several values for the coefficient of variation. The mean conversation duration is assumed equal to 100 seconds, near to the typical value observed in the measured data (see Table 2). The other system parameters have the same values used for obtaining Figure 11.

The more interesting result coming out from this figure is the effect of coefficient of variation on drop-call probability, particularly at low traffic load. This probability decreases as coefficient of variation increases; that is, fixing mean conversation duration, values more dispersed around this mean reduce drop-call probability. Similar results on other system performance parameters are reported in literature [2, 23]. Such a behavior can partially explain the performance improvement of some well-established mobile networks. In fact, in these networks the presence at the same time of many different services leads to a larger differentiation of call durations; consequently, values are more dispersed around the mean and the drop-call probability gets smaller.

Finally, Figure 13 reports the sensitivity of the proposed model as a function of λ_T, for several values of the mean ringing duration. The mean call duration is equal to 100 seconds. The other model parameters are the same previously used. It is worth noting that ringing duration variation does not affect the drop-call probability. In fact, the curves for the different $E[T_r]$ values are practically indistinguishable.

6. CONCLUSIONS

In this paper, starting from the statistical analysis of data measured in a large real well-established cellular network, a new model to study the call-dropping phenomenon has been developed.

We started from the verification that handover failure, considered prevailing in the classical cellular performance models, has become negligible in this kind of networks. With both planning optimization and fine tuning of network parameters, several secondary phenomena (e.g., irregular user behaviors, abnormal network response, power attenuation, and so on) become significant. This requires new modeling of the call dropping process.

Using statistical tools on measured data from a real network, we have characterized dropped calls and call durations (distinguishing between ringing and conversation phases). Results of this data analysis have driven the development of a new analytical model which relates drop-call probability to the drop-call rate, the pdf of the call duration, and the traffic load.

The proposed model has been validated comparing its results with the ones obtained by measures, in a wide range of traffic load conditions for both cells and cluster of neighboring cells. Moreover, the impact of its parameters on drop-call probability has been studied.

The developed model can be easily extended to different cellular networks simply characterizing the distribution of the call duration.

ACKNOWLEDGMENTS

The authors would like to thank Dr. Massimo Siviero from Vodafone, Italy, for his helpful contribution in this work; in particular, in the phase of measure collection. A preliminary version of this paper was presented at IEEE VTC'05 Spring Conference.

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