

Research Article

OFDM Link Performance Analysis under Various Receiver Impairments

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We present a methodology for OFDM link capacity and bit error rate calculation that jointly captures the aggregate effects of various real life receiver imperfections such as: carrier frequency offset, channel estimation error, outdated channel state information due to time selective channel properties and flat receiver I/Q imbalance. Since such an analytical analysis is still missing in literature, we intend to provide a numerical tool for realistic OFDM performance evaluation that takes into account mobile channel characteristics as well as multiple receiver antenna branches. In our main contribution, we derived the probability density function (PDF) of the received frequency domain signal with respect to the mentioned impairments and use this PDF to numerically calculate both bit error rate and OFDM link capacity. Finally, we illustrate which of the mentioned impairments has the most severe impact on OFDM system performance.

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1. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) is a widely applied technique for wireless communications, which enables simple one-tap equalization by cyclic prefix insertion. Conversely, the sensitivity of OFDM systems to various receiver impairments is higher than that of single-carrier systems. Furthermore, for OFDM system designers, it is often desirable to have easy to use numerical tools to predict the system performance under various receiver impairments. Within this article the term *performance* means both link capacity and uncoded bit-error rate (BER). Mostly, link level simulations are used to obtain reliable performance measures of a given system configuration. Unfortunately, simulations are highly time consumptive especially when the parameter space of the system under investigation is large. Therefore, the intention of this article is to introduce a stochastic/analytical method to predict the performance metrics of a given OFDM system configuration. To get realistic performance results, our approach takes into account a variety of receiver characteristics and impairments as well as mobile channel properties such as

- (i) residual carrier frequency offset (CFO) after synchronization;

- (ii) channel estimation errors;
- (iii) outdated channel state information due to time selective mobile channel properties;
- (iv) flat receiver I/Q imbalance in case of direct conversion receivers;
- (v) frequency selective mobile channel characteristics;
- (vi) multiple receiver branches to realize diversity combining methods such as maximum ratio combining (MRC).

In present OFDM standards, such as IEEE 802.11a/g or DVB-T, preamble (or pilots) are used to estimate and to compensate the CFO and channel impulse response. Unfortunately, after CFO estimation and compensation, the residual carrier frequency offset still destroys the orthogonality of the received OFDM signals and corrupts channel estimates, which worsen further the performance of OFDM systems during the equalization process. In the literature, the effects of carrier frequency offset on bit-error rate are mostly investigated under the assumption of perfect channel knowledge. The papers [5, 6] consider the effects of carrier frequency offset only (without channel estimation and equalization imperfections) and give exact analytical expressions in terms of SNR-loss and OFDM bit-error rate for the AWGN

channel. The authors of [8] extend the work of [5] toward frequency-selective fading channels and derive the correspondent bit-error rate for OFDM systems in case of CFO under the assumption of perfect channel knowledge.

Cheon and Hong [1] tried to analyze the joint effects of CFO and channel estimation error on uncoded bit-error rate for OFDM systems, but the used Gaussian channel estimation error model does not hold in real OFDM systems, especially when carrier frequency offset is large (see Section 5).

Additionally, receiver I/Q imbalance has been identified as one of the most serious concerns in the practical implementation of direct conversion receiver architectures (see, e.g., [12]). Direct conversion receiver designs are known to enable small and cheap OFDM terminals, highly suitable for consumer electronics. The authors of [11] investigated the effect of receiver I/Q imbalance on OFDM systems for frequency selective fading channels under the assumption of perfect channel knowledge and perfect receiver synchronization. Additionally, in order to cope with this impairment, the authors of [10] proposed a digital I/Q imbalance compensation method.

To our best knowledge, there is currently no literature available that describes a calculation method for OFDM BER and link capacity under the aggregate effect of all the mentioned impairments. Therefore, our intention is to describe the quantitative relationship between OFDM parameters, receiver impairments, and performance metrics such as bit-error rate and link capacity. Furthermore, we intend to provide a useful system engineering tool for the design and dimensioning of OFDM system parameters, pilot symbols, and receiver algorithms used for frequency synchronization, channel estimation, and I/Q imbalance compensation.

The structure of this article is as follows. After some general remarks on our proposed link capacity evaluation method in Section 2, we introduce our OFDM system model in section followed by a general probability density function analysis in Section 4. In Section 5, it will be explained how to model the correlation between channel estimates and received/impaired signals to derive uncoded bit-error rates of OFDM systems with carrier frequency offset and I/Q imbalance in Rayleigh frequency and time selective fading channels. It should be noted that the terms *bit-error rate* and *bit-error probability* are used with equal meaning. This is due to the fact that the bit-error rate converges toward bit-error probability with increasing observation time in a stationary environment. Finally, we introduce our link capacity calculation method in Section 6 and conclude in Section 7.

2. THE APPROACH

We choose link capacity, measured in bit/channel use, as an important performance metric for OFDM system designs. This information theoretic metric allows system designers to characterize the system behavior subject to real-life receiver impairments independently from any kind of channel coding and iterative detection methods. As explained in Section 6 and illustrated in Figure 1, the OFDM transceiver chain including channel and receiver properties can be characterized as effective channel between source and detector, often called

the modulation channel. The modulation channel is characterized by its conditional PDF $f_{Z|X}(z|x)$ that describes the statistical relationship between the discrete input symbols x and the continuously distributed decision variable z . Using any given complex M-QAM constellation alphabet X , the link capacity can be expressed as mutual information between source and sink that only depends on the input statistic of X and $f_{Z|X}(z|x)$. Since our performance analysis framework intends to describe the mutual information (and hence the link capacity under a given input statistic), we propose the following work flow.

- (1) We show how to derive $f_{Z|X}(z|x)$ under receiver impairments, given channel properties and OFDM system parameters.
- (2) We use the derived $f_{Z|X}(z|x)$ for uncoded BER calculation to verify its correctness by comparing the BER prediction results with those obtained from simulation.
- (3) We calculate the mutual information, that is, OFDM link capacity, using the verified statistic $f_{Z|X}(z|x)$.

3. OFDM SYSTEM MODEL

We consider an OFDM system with N -point FFT. The data is M-QAM modulated to different OFDM data subcarriers, then transformed to a time domain signal by IFFT operation and prepended by a cyclic prefix, which is chosen to be longer than the maximal channel impulse response (CIR) length L . The sampled discrete complex baseband signal for the l th subcarrier after the receiver FFT processing can be written as

$$Y_l = X_l H_l + W_l, \quad (1)$$

where X_l represents the transmitted complex QAM modulated symbol on subcarrier l , and W_l represents complex Gaussian noise. The coefficient H_l denotes the frequency domain channel transfer function on subcarrier l , which is the discrete Fourier transform (DFT) of the CIR $h(\tau)$ with maximal L taps

$$H_l = \sum_{\tau=0}^{L-1} h(\tau) e^{-j2\pi l\tau/N}. \quad (2)$$

In this paper, it is assumed that the residual carrier frequency offset (after frequency synchronization) is a given deterministic value. Furthermore, static (non-time-selective) channel characteristics are assumed during one OFDM symbol. The CFO-impaired complex baseband signal subcarrier l can be written as

$$Y_l = X_l H_l I(0) + \sum_{k=-N/2, k \neq l}^{N/2-1} X_k H_k I(k-l) + W_l. \quad (3)$$

The complex coefficients $I(K-l)$ represent the impact of the received signal at subcarrier k on the received signal at

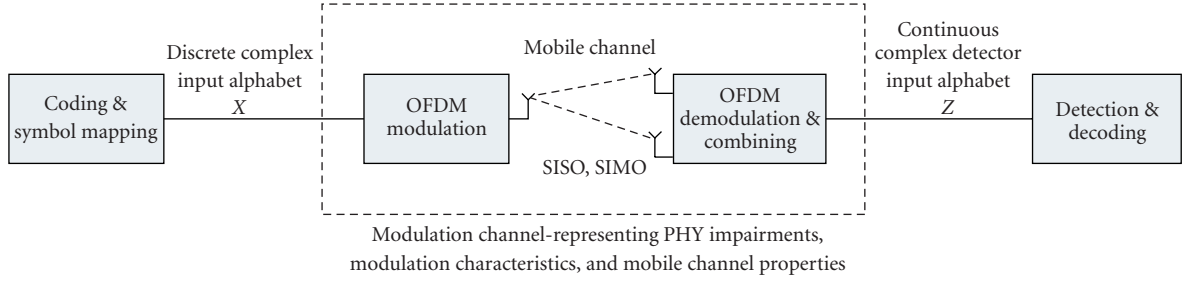


FIGURE 1: The modulation channel concept used for capacity evaluation.

subcarrier l due to the residual carrier frequency offset as defined in [5]

$$I(k-l) = e^{j\pi((k-l)+\Delta f)(1-1/N)} \frac{\sin(\pi((k-l)+\Delta f))}{N\sin(\pi((k-l)+\Delta f)/N)}, \quad (4)$$

where Δf is the residual carrier frequency offset normalized to the subcarrier spacing. In addition, later in this paper, the summation $\sum_{k=-N/2, k \neq l}^{N/2-1}$ will be abbreviated as $\sum_{k \neq l}$. In (3) we can see that residual CFO causes a phase rotation of the received signal ($I(0)$) and intercarrier interference (ICI). Furthermore, there is a time variant common phase shift for all subcarriers due to CFO as given in [8] that is not modeled here. This is due to the fact that this time variant common phase term is considered to be robustly estimated and compensated by continuous pilots that are inserted among the OFDM data symbols. I/Q imbalance of direct conversion OFDM receivers directly translates to a mutual interference between each pair of subcarriers located symmetrically with respect to the DC carrier [10]. Hence, the received signal Y_l at subcarrier l is interfered by the received signal Y_{-l} at subcarrier $-l$, and vice versa. Therefore, the undesirable leakage due to I/Q imbalance can be modeled by [10, 12]

$$\tilde{Y}_l = Y_l + K_l Y_{-l}^*, \quad (5)$$

where $(\cdot)^*$ represents the complex conjugation and K_l denotes a complex-valued weighting factor that is determined by the receiver phase and gain imbalance [10]. The image rejection capabilities of the receiver on subcarrier l can be expressed in terms of *image rejection ratio* (IRR) given by

$$\text{IRR}_l = \left| \frac{1}{K_l} \right|^2. \quad (6)$$

In this paper, we consider *flat* I/Q imbalance which simply means $\text{IRR}_l = \text{IRR}$ for all l . Subsequently, we consider preamble-based frequency domain least-square (FDLS) channel estimation to obtain the channel state information (\hat{H}_l) on subcarrier l :

$$\hat{H}_l = \frac{\tilde{Y}_{P,l}}{X_{P,l}} = \frac{I(0)H_l + \sum_{k \neq l} X_{P,k} H_k I(k-l) + W'_l}{X_{P,l}} + K_l \frac{\sum_m X_{P,m}^* H_m^* I^*(m+l) + W'_{-l}}{X_{P,l}}, \quad (7)$$

where $X_{P,l}$ and $\tilde{Y}_{P,l}$ denote the transmitted and received preamble symbol on subcarrier l . The Gaussian noise of the preamble part W'_l has the same variance as W_l of the data part ($\sigma_{W'_l}^2 = \sigma_{W_l}^2$). The channel estimate is used for frequency domain zero-forcing equalization before data detection

$$Z_l = \frac{\tilde{Y}_l}{\hat{H}_l}, \quad (8)$$

where Z_l is the decision variable that is feed into the detector/decoder stage. The power of preamble signals and the average power of transmitted data signals on all carriers are equivalent ($|X_P|^2 = \sigma_X^2$). In case of multiple (N_{Rx}) receiver branches, maximum ratio combining (MRC) is used at the receiver side. Therefore, the decision variable Z_l on subcarrier l is given by

$$Z_l = \frac{\sum_{\kappa=1}^{N_{Rx}} Y_{l,\kappa} \hat{H}_{l,\kappa}^*}{\sum_{\kappa=1}^{N_{Rx}} |\hat{H}_{l,\kappa}|^2}, \quad (9)$$

where κ denotes the receiver branch index. We assume that there is the same IRR and CFO on all branches, what is reasonable when considering one oscillator used for down-conversion in each branch. Furthermore, we assume uncorrelated channel coefficients among the branches,¹ that is,

$$E\{H_{l,\kappa_1} H_{l,\kappa_2}^*\} = 0 \quad \text{if } \kappa_1 \neq \kappa_2, \forall l. \quad (10)$$

3.1. Mobile channel characteristics

To obtain precise performance analysis results in case of subcarrier crosstalk induced by CFO and I/Q imbalance, it is desirable to use exact expressions of the subcarrier channel cross-correlation properties what is shown in more detail in Section 5. The cross-correlation properties between frequency domain channel coefficients are mainly determined by the power delay profile of the channel impulse response (CIR) and the CIR tap cross-correlation properties. Furthermore, the discrete nature of the sampled CIR is modeled as tapped delay line having L channel taps. Although our

¹ For sake of readability, we only include the antenna branch index κ if necessary.

analysis is not limited to a specific type of frequency selective channel, in our numerical examples, we consider mobile channels having an exponential power delay profile (PDP):

$$\sigma_\tau^2 = \frac{1}{C} e^{-D\tau/L}, \quad \tau = 0, 1, \dots, L-1, \quad (11)$$

where $\sigma_\tau^2 = E\{|h(\tau)|^2\}$ and the factor $C = \sum_{\tau=0}^{L-1} e^{-D\tau/L}$ is chosen to normalize the PDP as $\sum_{\tau=0}^{L-1} \sigma_\tau^2 = 1$, what leads to $\sigma_H^2 = E\{|H_l|^2\} = 1$, for all l . The channel taps $h(\tau)$ are assumed to be complex zero-mean Gaussian RV with uncorrelated real and imaginary parts. Hence, after DFT according to (2), the channel coefficients are zero-mean complex Gaussian random variables as well. Additionally, the CIR length L is assumed to be shorter than/equal to the cyclic prefix. The cross-correlation coefficient of the channel transfer function on subcarriers k and l in case of frequency selective fading is defined as

$$r_{k,l} = \frac{E\{H_k H_l^*\}}{\sigma_H^2} \neq 1, \quad \forall k \neq l, \quad (12)$$

where σ_H^2 is equivalent for all subcarriers. Assuming mutual uncorrelated channel taps of the CIR and applying (2), one gets

$$E\{H_k H_l^*\} = \sum_{\tau=0}^{L-1} \sigma_\tau^2 e^{-j2\pi(k-l)\tau/N}. \quad (13)$$

The cross-correlation property of the complex Gaussian channel coefficients can be formulated to be

$$H_k = r_{k,l} H_l + V_{k,l}, \quad (14)$$

where V_k is a complex zero-mean Gaussian with variance $\sigma_{V_{k,l}}^2 = \sigma_H^2(1 - |r_{k,l}|^2)$ and $E\{V_{k,l} H_l^*\} = 0$.

In current OFDM systems such as 802.11a/n or 802.16, there is a typical OFDM block structure. An OFDM block consists of a set of preamble symbols used for acquisition, synchronization, and channel estimation, followed by a set of serially concatenated OFDM data symbols. User mobility gives rise to a considerable variation of the mobile channel during one OFDM block (*fast fading*) what causes *outdated channel information* in certain OFDM symbols if there is no appropriate channel tracking. To be precise, during the time period λ between channel estimation and OFDM symbol reception, the channel changes in a way that the estimated channel information used for equalization does not fit the actual channel anymore. If there is no channel tracking at the receiver side, our aim is to incorporate the effect of outdated channel information into the performance analysis framework. Therefore, we have to define the autocorrelation properties of channel coefficients H_l . The autocorrelation coefficient of subcarrier l is defined as follows:

$$r_H(l, \lambda) = \frac{E\{H_l(t) H_l^*(t + \lambda)\}}{\sigma_H^2}. \quad (15)$$

Applying (2) we get

$$\begin{aligned} & E\{H_l(t) H_l^*(t + \lambda)\} \\ &= E\left\{ \sum_{\tau=0}^{L-1} \sum_{\nu=0}^{L-1} h(\tau, t) h^*(\nu, t + \lambda) e^{-2\pi l((\tau-\nu)/N)} \right\}. \end{aligned} \quad (16)$$

When assuming uncorrelated channel taps, it follows

$$E\{H_l(t) H_l^*(t + \lambda)\} = \sum_{\tau=0}^{L-1} r_h(\tau, \lambda) \sigma_\tau^2. \quad (17)$$

For sake of simplicity, it is assumed that all channel taps have the same autocorrelation coefficient, that is, $r_h(\tau, \lambda) = r_h(\lambda)$, for all $0 \leq \tau \leq L-1$. Substituting the relation $\sum_{\tau=0}^{L-1} \sigma_\tau^2 = \sigma_H^2$ and (16) into (15), we obtain

$$r_H(l, \lambda) = r_h(\lambda). \quad (18)$$

For the numerical BER and link capacity evaluations done in Section 5.2 and 6.2, the time selectivity of the complex Gaussian channel taps was modeled as follows:

$$h(\tau, t + \lambda) = r_h(\tau, \lambda) h(\tau, t) + v_{\tau, \lambda}, \quad (19)$$

with

$$E\{|h(\tau, t)|^2\} = E\{|h(\tau, t + \lambda)|^2\} = \sigma_\tau^2, \quad (20)$$

where $v_{\tau, \lambda}$ is a complex Gaussian RV with variance $\sigma_{v_{\tau, \lambda}}^2 = \sigma_\tau^2(1 - |r_h(\tau, \lambda)|^2)$ and $E\{h(\tau, t) v_{\tau, \lambda}^*\} = 0$. For sake of simplicity, it is assumed that the channel is stationary during one OFDM symbol but changes from symbol to symbol in the above defined manner. In our analysis, we intentionally avoid any assumptions on concrete fast-fading models in order to obtain fundamental results. Anyway, one of the commonly used statistical descriptions of fast channel variations is the Jakes' model [7], where the channel autocorrelation coefficient $r_h(\tau)$ is given by

$$r_h(\tau) = J_0(2\pi f_{D, \max} \tau), \quad (21)$$

and $f_{D, \max}$ denotes the maximum Doppler frequency that is determined by the mobile velocity and carrier frequency of the system. It should be noted that $r_h(\tau)$ is real due to uncorrelated i.i.d. real and imaginary parts of the CIR taps.

4. PROBABILITY DENSITY FUNCTION ANALYSIS

The author of [9] suggested a correlation model regarding channel estimation for single-carrier systems and derived the correspondent symbol error-rate and bit-error rate of QAM-modulated signals transmitted in flat Rayleigh and Ricean channels. In this section, a short review of the contribution of [9] will be given in order to further extend these results to OFDM systems for time and frequency selective fading channels with CFO, I/Q imbalance, and channel estimation error. The single-carrier transmission model without carrier frequency offset for flat Rayleigh fading channels can be written as

$$y = hx + w, \quad (22)$$

where y , h , x , and w denote the complex baseband representation of the received signal, the channel coefficient, the transmitted data symbol, and the additive Gaussian noise

with variance σ_w^2 , respectively. In [9], the channel estimate \hat{h} is assumed to be biased and used for zero forcing equalization as follows:

$$z = \frac{y}{\hat{h}} \quad \text{with} \quad \hat{h} = \alpha h + \nu, \quad (23)$$

where α denotes the deterministic multiplicative bias of the channel estimates and ν represents zero-mean complex Gaussian noise with variance σ_ν^2 . The channel coefficient h and Gaussian noise ν are assumed to be uncorrelated. Hence, the case of perfect channel knowledge can be easily modeled by $\alpha = 1$ and $\sigma_\nu^2 = 0$.

In [9], the joint PDF of the decision variable $z = z_r + jz_i$ in case of transmit symbol x is derived in cartesian coordinates and can be written as

$$f_{Z|X}(z|x) = \frac{a^2(x)}{\pi(|z - b(x)|^2 + a^2(x))} \Big|_{z=z_r+jz_i}. \quad (24)$$

The PDF mainly depends on the complex parameter $b(x)$, given by [4, 9]

$$\begin{aligned} b(x) &= \Re\{b\} + j\Im\{b\} = b_r(x) + jb_i(x) \\ &= x \left(\frac{\alpha^* r_h(\lambda) \sigma_h^2}{|\alpha|^2 \sigma_h^2 + \sigma_\nu^2} \right) \end{aligned} \quad (25)$$

and the real parameter $a(x)$ that can be written according [4, 9] as

$$a^2(x) = |x|^2 \frac{|\alpha|^2 \sigma_h^4 (1 - r_h^2(\lambda)) + \sigma_\nu^2 \sigma_h^2}{(|\alpha|^2 \sigma_h^2 + \sigma_\nu^2)^2} + \frac{\sigma_w^2}{|\alpha|^2 \sigma_h^2 + \sigma_\nu^2}. \quad (26)$$

Additionally, the closed form integral of (24) with $z = z_r + jz_i$ is given by [9] to be

$$\begin{aligned} F_{Z|X}(z|x) &= \frac{(z_i - b_i(x)) \arctan\left(\frac{(z_r - b_r(x))}{\sqrt{a^2(x) + (z_i - b_i(x))^2}}\right)}{2\pi\sqrt{a^2(x) + (z_i - b_i(x))^2}} \\ &+ \frac{(z_r - b_r(x)) \arctan\left(\frac{(z_i - b_i(x))}{\sqrt{a^2(x) + (z_r - b_r(x))^2}}\right)}{2\pi\sqrt{a^2(x) + (z_r - b_r(x))^2}}. \end{aligned} \quad (27)$$

In case of N_{Rx} receiver branches, maximum ratio combining (MRC) is used for decision variable computation what can be formulated as

$$z = \frac{\sum_{\kappa=1}^{N_{Rx}} y_\kappa \hat{h}_\kappa^*}{\sum_{\kappa=1}^{N_{Rx}} |\hat{h}_\kappa|^2}, \quad (28)$$

where κ represents the antenna branch index, and the κ th channel estimate can be written according to the SISO case as

$$\hat{h}_\kappa = \alpha_\kappa h_\kappa + \nu_\kappa. \quad (29)$$

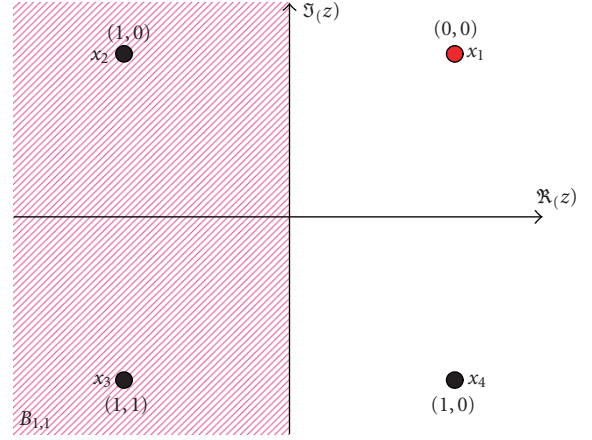


FIGURE 2: The QPSK constellation diagram, showing the decision region for one bit position of symbol x_1 .

Since it is quite reasonable to assume that the same channel estimation scheme is used in each receive antenna branch, we have $\alpha_\kappa = \alpha$, for all κ .

The authors of [9] also derived the PDF of z in case of transmit symbol x and N_{Rx} receiver branches that is given by

$$f_{Z|X, N_{Rx}}(z|x, N_{Rx}) = \frac{N_{Rx} (a^2(x))^{N_{Rx}}}{\pi(|z - b(x)|^2 + a^2(x))^{N_{Rx}+1}}. \quad (30)$$

It is easy to observe that the PDF (30) for the MRC case takes the SISO form of (24) in case of $N_{Rx} = 1$. Additionally, the closed form integral $F_{Z|X, N_{Rx}}(z|x, N_{Rx})$ of $f_{Z|X, N_{Rx}}(z|x, N_{Rx})$ can be found in [9] that also takes the SISO form (27) in case of $N_{Rx} = 1$. To enhance readability and to simplify our notation, we omit the receiver branch number N_{Rx} in the conditional PDF and its closed form integral, that is, in the following we write $f_{Z|X}(z|x)$ instead of $f_{Z|X, N_{Rx}}(z|x, N_{Rx})$.

Finally, the result of (27) can be used to calculate the bit-error rate of a given M-QAM constellation. In an M-QAM constellation there are $M \log_2(M)$ different possible bit positions with respect to the M-QAM constellation. The probability of an erroneous bit with respect to the m th QAM transmit symbol x_m can be calculated by using the closed form integral (27) and an appropriate decision region $B_{m,\nu}$ for the ν th bit position (see Figure 2) that takes into account the bit mapping of the QAM constellation. In the paper, we always use Gray mapping in our numerical results, but it is worth mentioning that the described method can be used for arbitrary bit mappings as well.

As already stated, we propose to use bit-error rate prediction to verify the correctness of the derived probability density function $f_{Z|X}(z|x)$ that is later used to determine the OFDM link capacity of a given transceiver configuration. Therefore, the bit-error probability $P_b(x_m)$ takes the form

$$P_b(x_m) = \frac{1}{\log_2(M)} = \sum_{\nu=1}^{\log_2(M)} \frac{1}{\log_2(M)} \left[[F_{Z|X}(z|x_m)] \right]_{B_{m,\nu}}, \quad (31)$$

where $[[F_{Z|X}(z|x_m)]]_{B_{m,\nu}}$ denotes the 2-dimensional evaluation of the closed form integral $F_{Z|X}(z|x_m)$ subject to the

decision region $B_{m,\nu}$. Finally, the bit-error probability can be obtained by averaging over all possible constellation points, when assuming equal probable M-QAM symbols as follows:

$$P_b = \frac{1}{M} \sum_{m=1}^M P_b(x_m). \quad (32)$$

5. OFDM BIT-ERROR RATE ANALYSIS

In this section, the derivation of the bit-error rate of OFDM systems with carrier frequency offset, I/Q imbalance, and channel estimation error in Rayleigh frequency and time-selective fading channels will be given. The central idea of our BER derivation is to map the OFDM system model of Section 3 to the statistics given in Section 4. To be precise, we have to map the OFDM system model to the parameters α , a^2 (26) and b^2 (25) as explained below.

5.1. Mathematical derivation

Firstly, we can rewrite the channel estimates of subcarrier l in (7) with respect to the frequency selective fading characteristic given in (14) to be

$$\hat{H}_l = I(0)H_l \left(1 + \frac{\sum_{k \neq l} r_{k,l} X_{P,k} I(k-l)}{I(0)X_{P,l}} + e \frac{K_l \sum_m r_{m,l}^* X_{P,m}^* I^*(m+l)}{I(0)X_{P,l}} \right) + \tilde{\nu}_l, \quad (33)$$

where e denotes the term $e^{-j2\phi_l}$. This comes due to the fact that the complex Gaussian channel coefficient can be written as $H_l = |H_l|e^{j\phi_l}$. Hence, we have $H_l^*/H_l = e^{-j2\phi_l} = e$, where ϕ_l is an equally distributed RV in the interval $[-\pi : \pi]$. From (33) we obtain an (23)-like expression as follows:

$$\hat{H}_l = \tilde{\alpha}_l \tilde{H}_l + \tilde{\nu}_l, \quad (34)$$

by defining *effective channel* $\tilde{H}_l = I(0)H_l$ and *effective bias* $\tilde{\alpha}_l$ as

$$\tilde{\alpha}_l = 1 + \frac{\sum_{k \neq l} r_{k,l} X_{P,k} I(k-l) + e K_l \sum_m r_{m,l}^* X_{P,m}^* I^*(m+l)}{I(0)X_{P,l}}, \quad (35)$$

where $\tilde{\alpha}_l$ is a stochastic quantity with given subcarrier index l , a set of deterministic preamble symbols $X_{P,k}$, a fixed pre-determined frequency offset, a given IRR constant K_l and RV $e = e^{j2\phi_l}$. It should be noted that the stochastic part of α_l is negligible in case of moderate I/Q imbalance (IRR ≥ 30 dB) and moderate CFO. Hence, we have that

$$e K_l \sum_m r_{m,l}^* X_{P,m}^* I^*(m+l) \approx 0, \quad (36)$$

and α_l can be well modeled to be a deterministic quantity. This is due to the fact that the pilot symbols $X_{P,k}$ as well as the CFO are given deterministic values and the channel cross-correlation coefficients $r_{k,l}$ can be calculated using (12) and (13).

The noise part $\tilde{\nu}_l$ of the channel estimate can be written as

$$\tilde{\nu}_l = W'_l + K_l W'_{-l} + \frac{\sum_{k \neq l} X_{P,k} V_{k,l} I(k-l)}{X_{P,l}} + K_l \frac{\sum_m X_{P,m}^* V_{m,l}^* I^*(m+l)}{X_{P,l}}. \quad (37)$$

For $\sigma_{\tilde{\nu}_l}^2$, which represents the additive Gaussian noise variance of the channel estimates, we obtain

$$\begin{aligned} \sigma_{\tilde{\nu}_l}^2 = & \sum_{k \neq l} \sum_{n \neq l} X_{P,k} X_{P,n}^* I(k-l) I^*(n-l) (r_{k,n} - r_{k,l} r_{n,l}^* \sigma_H^2) \\ & + |K_l|^2 \sum_k \sum_n X_{P,k}^* X_{P,n} I^*(k+l) I(n+l) \\ & \times (r_{k,n}^* - r_{k,l}^* r_{n,l} \sigma_H^2) + \sigma_W^2 (1 + |K_l|^2). \end{aligned} \quad (38)$$

Applying the same method as above for (3) and (5), the same definition of effective channel \tilde{H}_l can be used to get a (22)-like expression as follows:

$$Y_l = \tilde{H}_l \left(X_l + \frac{\sum_{k \neq l} r_{k,l} X_k I(k-l)}{I(0)} + e \frac{K_l \sum_m r_{m,l}^* X_m^* I^*(m+l)}{I(0)} \right) + \tilde{W}_l = \tilde{H} \tilde{X}_l + \tilde{W}_l. \quad (39)$$

Given (39), the *effective symbol* \tilde{X}_l can be defined that is no longer a deterministic value but a stochastic quantity due to i.i.d. data symbols on subcarriers $k \neq l$:

$$\tilde{X}_l = X_l + \underbrace{\frac{\sum_{k \neq l} r_{k,l} X_k I(k-l) + e K_l \sum_m r_{m,l}^* X_m^* I^*(m+l)}{I(0)}}_{\text{stochastic part of the effective transmit symbol}}. \quad (40)$$

Assuming a certain transmit symbol X_l and assuming randomly transmitted data symbols X_k with $k \neq l$, we can decompose the effective symbol \tilde{X}_l as follows:

$$\tilde{X}_l = X_l + J_l, \quad (41)$$

which shows the stochastic nature of \tilde{X}_l due to the random interference part J_l due to ICI and I/Q imbalance. Applying the central limit theorem, we assume that the interference J_l term is a complex zero-mean Gaussian random variable $J_l = p + jq$. The mutual uncorrelated real and imaginary parts p and q have the same variance for all constellation points

$$\tilde{\sigma}_{J_l}^2 = \frac{\sum_{k \neq l} |I(k-l)|^2 |r_{k,l}|^2 + |K_l|^2 \sum_m |I(m+l)|^2 |r_{m,l}|^2}{2|I(0)|^2}. \quad (42)$$

According to (25) and (26), we calculate the parameters $b_l = b_{l,r} + jb_{l,i}$ and a_l^2 for M-QAM effective data symbols \tilde{X}_l on subcarrier l in frequency and time selective fading channels:

$$b_l(\tilde{X}_l) = \tilde{X}_l \left(\frac{\tilde{\alpha}_l^* r_h(\lambda) \sigma_{\tilde{H}}^2}{|\tilde{\alpha}_l|^2 \sigma_{\tilde{H}}^2 + \sigma_{\tilde{V}}^2} \right),$$

$$a_l^2(\tilde{X}_l) = |\tilde{X}_l|^2 \frac{|\tilde{\alpha}_l|^2 \sigma_{\tilde{H}}^4 (1 - r_h^2(\lambda)) + \sigma_{\tilde{V}}^2 \sigma_{\tilde{H}}^2}{(\sigma_{\tilde{H}}^2)^2} + \frac{\sigma_{\tilde{W}_l}^2}{\sigma_{\tilde{H}}^2}, \quad (43)$$

where $\sigma_{\tilde{H}}^2 = |I(0)|^2 \sigma_H^2$ and $\sigma_{\tilde{H}_l}^2 = |\tilde{\alpha}_l|^2 |I(0)|^2 \sigma_H^2 + \sigma_{\tilde{V}}^2$. From (43) one can observe that the parameter $\sigma_{\tilde{W}_l}^2$ has to be calculated exactly to obtain reliable results. The term \tilde{W}_l represents the *effective noise* of the received signal that consists of AWGN parts W_l , W_{-l} , and ICI parts, respectively. If we substitute (3) and (14) into (5), we get

$$\tilde{W}_l = W_l + K_l W_{-l} + \sum_{k \neq l} X_k V_{k,l} I(k-l) + K_l \sum_m X_m^* V_{m,l}^* I^*(m+l). \quad (44)$$

For an exact expression of $\sigma_{\tilde{W}_l}^2$, we take (44), $\sigma_{V_{k,l}}^2 = \sigma_H^2 (1 - |r_{k,l}|^2)$ together with the assumptions of mutually uncorrelated data symbols and obtain

$$\sigma_{\tilde{W}_l}^2 = \sigma_W^2 (1 + |K_l|^2) + \sigma_H^2 \sum_{k \neq l} |I(k-l)|^2 (1 - |r_{k,l}|^2) + |K_l|^2 \sigma_H^2 \sum_{m \neq l} |I(m+l)|^2 (1 - |r_{m,l}|^2). \quad (45)$$

As an example, for one QPSK constellation point with index $m = 1$ on subcarrier l , $X_{1,l} = (1/\sqrt{2})(1, 1) = (1/\sqrt{2})(1+j)$, we need to recalculate $b_l(\tilde{X}_{1,l})$ and parameter $a_l^2(\tilde{X}_{1,l})$ separately for each effective symbol realization

$$\tilde{X}_{1,l} = X_{1,l} + p + jq = \frac{1}{\sqrt{2}}(1+j) + p + jq \quad (46)$$

to use the closed form integral and (31) for BER calculation. Subsequently, the bit-error rate on subcarrier l for the m th constellation point can be expressed using (31) by the following double integral involving the Gaussian PDFs of p and q :

$$\bar{P}_b(X_{m,l}) = \iint_{-\infty}^{\infty} \frac{P_b(X_{m,l} + p + jq)}{2\pi\tilde{\sigma}_{j_l}^2} e^{-(p^2+q^2)/2\tilde{\sigma}_{j_l}^2} dp dq. \quad (47)$$

Finally, to obtain the general bit-error rate, we have to average (47) over all N_C data subcarriers with index l and M-QAM constellation points with index m as follows:

$$\tilde{P}_b = \frac{1}{MN_C} \sum_{l=-N_C/2}^{N_C/2-1} \sum_{m=1}^M \bar{P}_b(X_{m,l}). \quad (48)$$

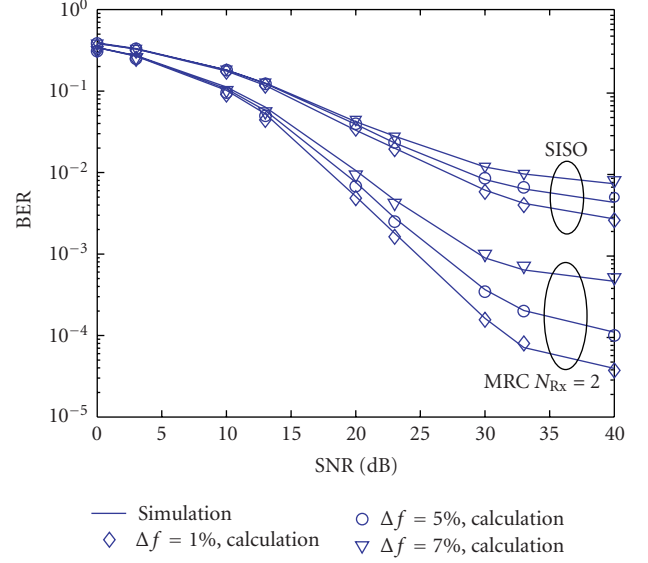


FIGURE 3: The comparison of simulated and calculated uncoded BER versus SNR for 16-QAM OFDM under residual CFO in non-time-selective channel environment and IRR = 30 dB.

5.2. Bit-Error rate performance: numerical results

In this section, the derived analytical expressions for bit-error rate are compared with appropriate simulation results for both SISO (single-input single-output) OFDM transmission as well as SIMO (single-input multiple-output) OFDM using MRC and two receiver antenna branches. Furthermore, we consider an IEEE 802.11a-like OFDM system [3] with 64-point FFT. The data is 16-QAM modulated to the data subcarriers, then transformed to the time domain by IFFT operation and finally prepended by a 16-tap long cyclic prefix. The data is randomly generated and one OFDM pilot symbol was used for channel estimation. The used BPSK pilot data in the frequency domain is given by

$$X_{P,l} = (-1)^l \quad \text{for subcarrier index } l = [-26 : 1 : 26], l \neq 0. \quad (49)$$

The data and pilot symbols are modulated on 52 data carriers. The DC carrier as well as the carriers at the spectral edges are not modulated and are often called “*virtual carriers*.” For simulation and numerical BER analysis, we use an 8 taps exponential PDP frequency selective Rayleigh fading channel with $D = 7$ (see Section 3). Furthermore, we choose statistical independent channel realizations for the two antenna branches in case of SIMO OFDM transmission.

The double integral of (47) is evaluated numerically using Matlab built-in integration functions having a numerical tolerance of 10^{-8} and upper/lower integration bounds of ± 10 .

Figure 3 illustrates the calculated and simulated 16-QAM BER versus SNR (σ_X^2/σ_W^2) with given carrier frequency offset Δf (in % subcarrier spacing) and IRR = 30 dB under non-time variant mobile channel conditions.

Figure 4 illustrates the calculated and simulated 16-QAM BER versus SNR (σ_X^2/σ_W^2) with given carrier frequency offset

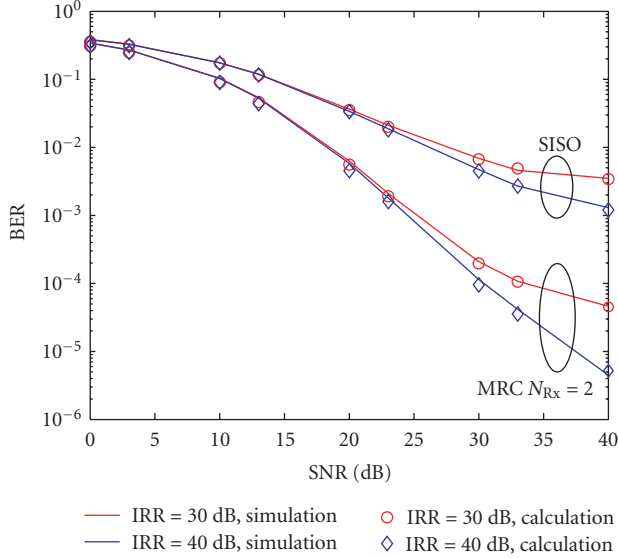


FIGURE 4: The comparison of simulated and calculated uncoded BER versus SNR for 16-QAM OFDM with residual CFO of 3% under non-time selective channel conditions under IRR = 30 dB/40 dB.

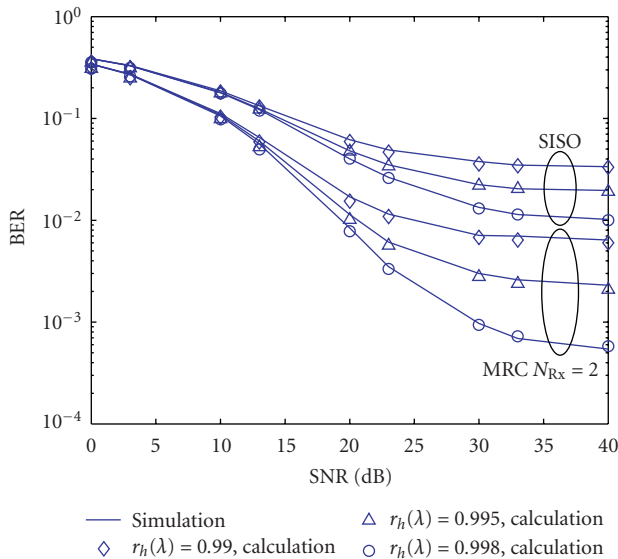


FIGURE 5: The comparison of simulated and calculated uncoded BER versus SNR for 16-QAM OFDM with residual CFO of 3% and IRR = 30 dB under time selective channel conditions.

Δf (in % subcarrier spacing) and IRR = 30 dB under non-time variant mobile channel conditions.

In Figure 5, we use a fixed Δf of 3% to investigate 16-QAM BER versus SNR for time variant mobile channel properties, characterized by the channel tap autocorrelation coefficients $r_h(\lambda)$.

The results illustrate that our analysis can approximate the simulative performance very accurately if the channel power delay profile, the image rejection ratio of the direct conversion receiver, and carrier frequency offset are known.

6. CAPACITY ANALYSIS OF IMPAIRED OFDM LINKS

To perform OFDM link capacity analysis, it seems mandatory to review the main principles and basic equations of how to calculate average mutual information between source and sink of a modulation channel. An excellent overview of this topic can be found in [7] that is summarized in the following. In an OFDM system, we have a number of parallel channels, that is, data subcarriers. Hence we propose to calculate the mutual information for each of the parallel data carriers independently and to finally average the link capacity among the data carriers.

Let us consider real input and output alphabets X and Z . Both alphabets can be characterized in terms of information content carried by the elements of each alphabet what leads to the concept of information entropy $H(X)$ and $H(Z)$. The entropy of the discrete alphabet X having elements X_m with appropriate probability $P(X_m)$ is given by

$$H(X) = -\sum_m P(X_m) \log_2(P(X_m)). \quad (50)$$

Conversely, Z is assumed to be a real continuously distributed RV having realizations z . As a result, Z can be characterized by its differential entropy as

$$H(Z) = -\int_z f_Z(z) \log_2(f_Z(z)) dz, \quad (51)$$

where $f_Z(z)$ denotes the PDF of Z . Finally, the mutual information $I(X; Z)$ of X and Z can be formulated as [7]

$$I(X; Z) = \sum_m P(X_m) \int_z f_{Z|X}(z|X_m) \times \log_2 \left(\frac{f_{Z|X}(z|X_m)}{\sum_n f_{Z|X}(z|X_n) P(X_n)} \right) dz. \quad (52)$$

It can be seen from (52) that $I(X; Z)$ requires knowledge of a-priori probabilities $P(X_m)$ and conditional PDFs $f_{Z|X}(z|X_m)$ only. Mostly we have that $P(X_m) = 1/M$ in case of M -ary constellations. Since the above defined mutual information calculation scheme assumes one-dimensional output variables and z is a two-dimensional complex RV of real part z_r and imaginary part z_i , we have to solve a double integral to obtain the corresponding mutual information as follows:

$$I(X; Z) = \sum_m P(X_m) \int_{z_r} \int_{z_i} f_{Z|X}(z_r + jz_i | X_m) \times \log_2 \left(\frac{f_{Z|X}(z_r + jz_i | X_m)}{\sum_n f_{Z|X}(z_r + jz_i | X_n) P(X_n)} \right) dz_r dz_i. \quad (53)$$

6.1. Mutual information under carrier crosstalk

Recalling the two-dimensional conditional SISO PDF $f_{Z|X}(z_r + jz_i | X_m)$ on subcarrier l as given in Section 4, we have that

$$f_{Z|X}(z_r + jz_i | X_m) = \frac{a_l^2(X_m)}{\pi(|z_r + jz_i - b_l(X_m)|^2 + a_l^2(X_m))}, \quad (54)$$

where $a_l^2(X)$ and $b_l(X)$ contain the entire OFDM link impairment information (channel estimation error, I/Q imbalance, CFO, outdated channel information, and channel power delay profile). According to Section 4, the complex-valued transmit symbol is stochastic by nature due to CFO and I/Q imbalance carrier crosstalk and can be expressed as $X_m + J = X_m + p + jq$, where m represents the constellation point index while p and q represent the effects of I/Q imbalance and residual CFO. Both, p and q can be modeled as i.i.d. zero-mean Gaussian RV as done in Section 4. Additionally, both parameters $a_l^2(X)$ and $b_l(X)$ are subcarrier-dependent. As a result (54) has to be reformulated for subcarrier l as

$$f_{Z|X,P,Q}(z_r + jz_i | X_m + p + jq) = \frac{a_l^2(X_m + p + jq)}{\pi(|z_r + jz_i - b_l(X_m + p + jq)|^2 + a_l^2(X_m + p + jq))^2}. \tag{55}$$

Hence, the calculation of p/q -independent conditional marginal PDFs can be done via numerical double integration as

$$f_{Z|X}(z_r + jz_i | X_m) = \iint_{-\infty}^{\infty} f_{Z|X,P,Q}(z_r + jz_i | X_m + p + jq) \times f_Q(q) f_P(p) dp dq. \tag{56}$$

According to Section 5, we have the Gaussian distribution for each p and q :

$$f_P(p) = f_Q(q) = \frac{1}{\sqrt{2\pi\sigma_j^2}} e^{-(p,q)^2/2\sigma_j^2}, \tag{57}$$

where σ_j^2 is given in (42). In case of MRC multiantenna reception, we have to proceed in the same manner.

6.2. OFDM link capacity: numerical examples

The quantitative relationship between receiver impairments, OFDM system parameters and link capacity is an essential piece of information for the dimensioning of I/Q imbalance compensation algorithms as well as frequency synchronization methods. Moreover, the effects of time-selective mobile channels on link capacity can be used to design scattered pilot structures for channel estimation and tracking as done in [2]. Generally, link capacity indicates the maximum data rate that can be achieved with strong channel coding under a given input constellation and a specified receiver architecture.

The numerical examples of average mutual information are chosen such that we illustrate the effects of channel estimation error, outdated channel state information (CSI), residual CFO, and flat receiver I/Q imbalance on the link capacity of SISO and SIMO OFDM links. Therefore, we choose the same IEEE 802.11a-like OFDM system parameters as introduced in Section 5.2, assume an 8 taps exponential PDP mobile channel and the use of 16-QAM modulation on each data carrier. Again, statistical independent channel realizations for the N_{RX} antenna branches in case of SIMO OFDM

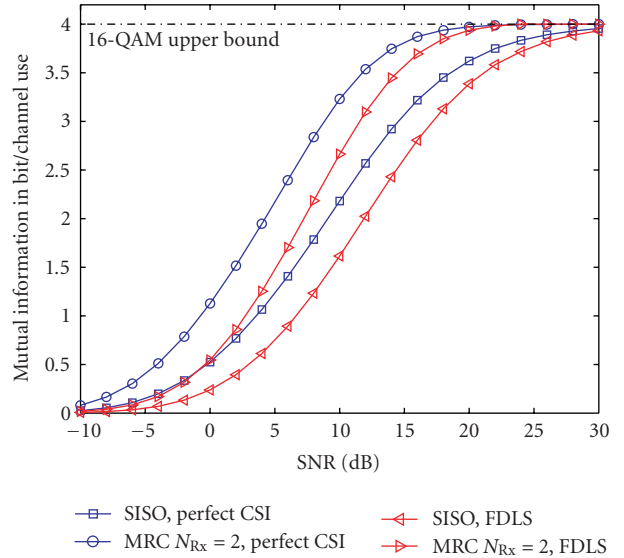


FIGURE 6: The mutual information, averaged over all data carriers, comparison between perfect channel-state information and real FDLS channel estimation for SISO and SIMO OFDM, CFO = 0%, no I/Q imbalance, static Rayleigh fading channel.

transmission are assumed. The mutual information (measured in Bit/Channel Use) is averaged among the data carriers and plotted over SNR (σ_X^2/σ_W^2).

In Figure 6, we illustrate the effect of real-life frequency domain least-square (FDLS) channel estimation on the link capacity of SISO and SIMO OFDM, respectively, assuming no I/Q imbalance, a perfect frequency synchronization (CFO = 0%) and static (non-time-selective) channel properties. As reference, we plotted the case of perfect channel state information that can easily be modelled by $\alpha_l = 1$ and $\sigma_{y_l}^2 = 0$.

In Figure 7, we show the aggregate effect of I/Q imbalance and FDLS channel estimation under static-channel conditions and perfect frequency synchronization. It is easy to see that I/Q imbalance has only little effect on the averaged mutual information performance, what is especially the case at realistic image rejection ratios above 30 dB. Interestingly, a worst case IRR of 20 dB heavily impacts the SISO performance but causes only a small performance loss in case of receiver diversity combining.

Figure 8 depicts the effect of CFO on averaged link capacity under real FDLS channel estimation and no I/Q imbalance under static-channel conditions. It can be shown that a moderate CFO of 3% causes only a negligible degradation of SISO and SIMO OFDM link capacity. The worst case performance in case of CFO = 10% is plotted to illustrate the lower sensitivity of the SIMO link compared to the SISO link. Nevertheless, we have to state that in case of realistic frequency synchronization techniques, it is highly improbable to have a residual CFO larger than 3% at moderate SNR (> 10 dB). This fact is also mentioned in [4] where the authors derived the PDF of the residual CFO in case of real frequency synchronization under Rayleigh fading channels and given SNR.

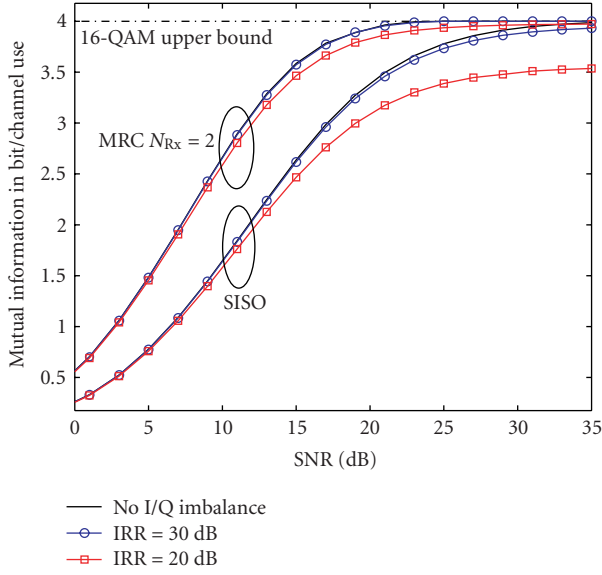


FIGURE 7: The mutual information averaged over all data carriers under the aggregate effect of I/Q imbalance and FDLS channel estimation for SISO OFDM, 16-QAM, CFO = 0%, static 8 taps exponential PDP Rayleigh fading channel.

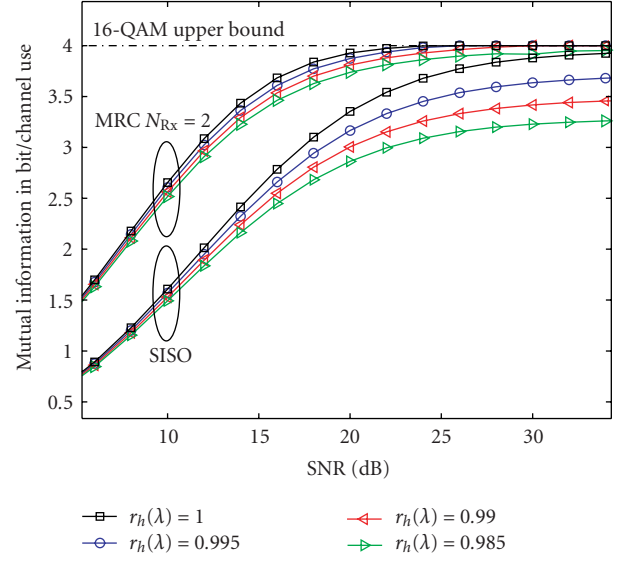


FIGURE 9: The mutual information averaged over all data carriers under time-selective channel properties and FDLS channel estimation for SISO OFDM, 16-QAM, CFO = 0%, IRR = 30 dB, time variant 8 taps exponential PDP Rayleigh fading channel.

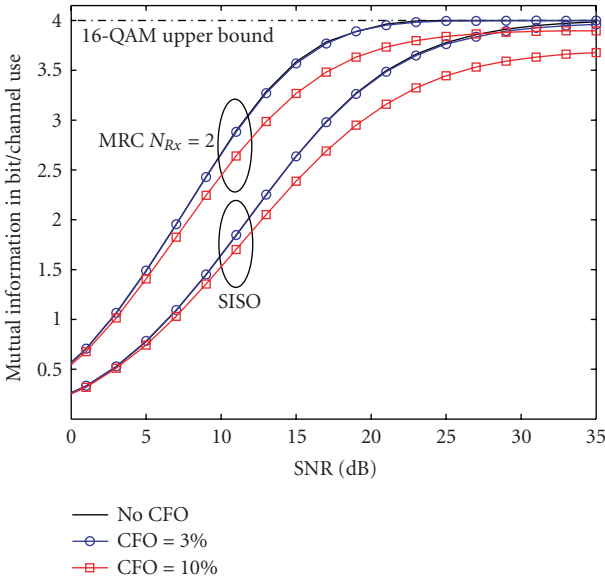


FIGURE 8: The mutual information averaged over all data carriers under CFO, FDLS channel estimation is assumed, 16-QAM modulation on all subcarriers, no I/Q imbalance, time variant 8 taps exponential PDP Rayleigh fading channel.

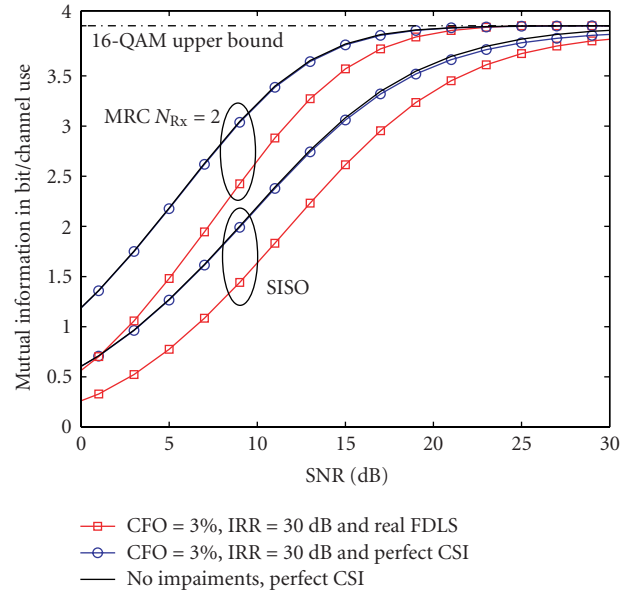


FIGURE 10: The mutual information averaged over all data carriers, comparing the effect of receiver impairments in case of perfect CSI and real FDLS channel estimation, 16-QAM modulation on all subcarriers, static 8 taps exponential PDP Rayleigh fading channel.

Figure 9 depicts the effect of outdated channel-state information quantified by appropriate channel autocorrelation coefficients $r_h(\lambda)$, FDLS channel estimation and I/Q imbalance under 8 taps exponential PDP Rayleigh fading channel conditions and perfect frequency synchronization. Again, the performance loss in case of diversity combining is smaller than the loss that we have in case of conventional SISO receiver designs. Moreover, we have to state that even in case

of very small deviations of $r_h(\lambda)$ from the ideal static case $r_h(\lambda) = 1$, the effect of outdated channel-state information causes much larger performance losses than realistic CFO and I/Q imbalance.

Finally, we want to highlight the fact that in case of moderate receiver impairments the performance loss mainly comes due to channel-estimation errors. This important observation is illustrated in Figure 10 where we plotted

averaged mutual information versus SNR under CFO = 3% and IRR = 30 dB assuming static channel properties. As reference we use a plot without any I/Q imbalance, CFO, or channel estimation error. Interestingly the impairment plots in case of perfect CSI are almost equivalent to the reference curves but we observe a severe performance degradation in case of real FDLS channel estimation.

7. CONCLUSIONS

In this paper, we show how to analytically evaluate the uncoded bit-error rate as well as link capacity of OFDM systems subject to carrier frequency offset, channel estimation error, outdated channel state information, and flat receiver I/Q imbalance in Rayleigh frequency and time-selective mobile fading channels. The probability density function of the frequency domain received signal subject to the mentioned impairments is derived. Furthermore, this PDF is verified by means of bit-error rate calculation. We show that our approach can be used to exactly evaluate uncoded bit-error rates when a priori knowledge of the mobile channel power delay profile, the image rejection ratio and receiver CFO is used. Furthermore, we show how to use the derived PDF to calculate OFDM link capacity under the aggregate effects of receiver impairments and mobile channel characteristics. Finally, we highlight the fact that channel uncertainty induced by channel estimation errors as well as outdated channel state information have much severer impact on OFDM capacity than CFO or I/Q imbalance.

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