Research Article

Practical Approaches to Adaptive Resource Allocation in OFDM Systems

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Whenever a communication system operates in a time-frequency dispersive radio channel, the link adaptation provides a benefit in terms of any system performance metric by employing time, frequency, and, in case of multiple users, multiuser diversities. With respect to an orthogonal frequency division multiplexing (OFDM) system, link adaptation includes bit, power, and subcarrier allocations. While the well-known water-filling principle provides the optimal solution for both margin-maximization and rate-maximization problems, implementation complexity often makes difficult its application in practical systems. This paper presents a few suboptimal (low-complexity) adaptive loading algorithms for both single- and multiuser OFDM systems. We show that the single-user system performance can be improved by suitable power loading and an algorithm based on the incomplete channel state information is derived. At the same time, the power loading in a multiuser system only slightly affects performance while the initial subcarrier allocation has a rather big impact. A number of subcarrier allocation algorithms are discussed and the best one is derived on the basis of the order statistics theory.

1. INTRODUCTION

For a few last decades, the orthogonal frequency division multiplexing (OFDM) has gained a lot of practical and research interest because of the number of advantages that this technique exhibits compared with the single carrier modulation formats. These are primarily provisions of a high bitrate in a fading environment and relatively simple equalizer structure. In OFDM, a high bitrate is provided by frequency multiplexing where data is conveyed by a number of subcarriers. High spectral efficiency results from spectral overlapping of the data conveyed by the different subcarriers and its separation at the receiver is possible due to special assignment of frequency spacing between the subcarriers. OFDM is accepted as the standard in many current communication systems (e.g., [1–3]) and is considered as a strong candidate for next generation systems.

But when an OFDM system operates in a time-dispersive radio channel, the subcarriers with deep fading significantly deteriorate reliability of the data transmission, that is, enhance the error probability. A way to support reliable data transmission through spectrally shaped radio channels is to load each subcarrier according to the channel state information (CSI). Under a constrained transmit power, the well-known water-filling principle [4] gives the optimal solution of the problems of maximizing the bitrate under a constrained bit-error rate (BER) or BER minimizing under providing a given bitrate. The former problem is called rate maximization and the latter one is margin maximization [5]. Margin maximization is often formulated in the literature as the problem of power minimization under fixed bit and bit-error rates that is equivalent to the formulated above BER-minimizing problem.

Duality between margin maximization and rate maximization was proven in [6]. Particularly, this means that a loading algorithm providing optimization of one problem yields the optimal solution for another one. Therefore, without loss of generality we concentrate on the margin-maximization problem in this paper.

For last years, the problem of adaptive resource allocation in OFDM has been studied intensively and as a result a large number of algorithms have been developed on the

basis of Campello's conditions [5] providing implementation of the water-filling principle in practical systems. For example, a practical bit loading algorithm has been derived in [7], noniterative power-loading strategies have been suggested in [8, 9] and suboptimal water-filling algorithms with reduced computational complexities have been presented in [10–12]. Many algorithms however have not found wide application. The main reasons are still high computational complexity of implementation caused by the iterative structure of the algorithms and necessity to have a fresh (for mobile radio channels) and accurate CSI at the transmitter. The latter requirement results in a system overhead because of necessity to have a (fast) feedback channel for the CSI transmission. This fact and a sensitivity of the system performance to inaccuracies of the CSI make adaptive power loading actually unreasonable in mobile systems [13].

Recently, constant power suboptimal solutions have been derived in [14, 15]. Both algorithms employ equal power loading of a part of subcarriers. In [14], solely subcarriers with the power gain values exceeding some properly chosen power level are used for transmission, and in [15] the number of selected subcarriers is preliminary defined by the initial modulation format and is independent of specific power gain values of the subcarriers. The ordered subcarrier selection algorithm (OSSA) [15] results in a good BER performance close to optimal in a Rayleigh environment and its implementation complexity is very low because it is noniterative and employs a constant constellation size. Additionally, this method requires the CSI only in terms of "used-not used" subcarriers.

Solely, power loading is another suboptimal approach to the optimization problem. For example, in [16], we proposed a low-complexity technique that consists in (quasi-) inversion of subcarrier power gains. This technique provides a power gain at the expense of an additional overhead resulting from necessity to have information about subcarrier power gains at the transmitter. But because of the noniterative implementation procedure and a constant constellation size, this is still a low-complexity power-loading method. An interesting observation is that the combination of the algorithms [15, 16] improves the performances of the both algorithms and in some radio channels it results in a power-loading technique with the performance close to that of the much more complex optimal greedy algorithm [17–19].

In this paper, we propose a BER minimizing power-loading technique that employs only the CSI in terms of "strong-weak" subcarriers and does not require complete information about the power gain values. The technique is based on unequal power loading of the "strong" and "weak" subcarriers. As the method in [16], it is noniterative and uses a constant constellation size. We give the theoretical background and present simulation results that confirm efficiency of the proposed algorithm for both single- and multiuser cases. We prove that it provides a power gain in any time dispersive channel starting with some transmit signal-to-noise ratio (SNR). In a "truncated" radio channel derived in [15], the proposed method provides a power gain actually for all practical SNR values.

OFDM has been recognized not only as an efficient modulation format but also as an effective way of supporting a multiple access (e.g., [1, 20]). The orthogonal frequency division multiple access (OFDMA) principle employs assignment of orthogonal subcarrier sets to a number of system users. A lot of research activities have been focused on adaptive resource allocation for OFDMA and a large number of techniques have been presented (see, e.g., [21–23]). In [21], the authors present a heuristic algorithm based on constructive initial subcarrier assignment with further iterations improving the system power efficiency. Another computationally efficient suboptimal algorithm employing fast initial subcarrier allocation and further iterative refinement is given in [22]. In [23], both the optimal loading algorithm providing different bitrate services with different target bit-error rates (that is however NP-hard) and its reduced complexity version are derived.

Most of the proposed algorithms are based on the water-filling principle. It is worth mentioning that in a multiuser environment, the water-filling principle does not provide fairness between users in terms of the bitrates because it always "encourages" "stronger" subcarriers by giving them more power at the expense of the "weaker" ones.

In this paper, we study the margin maximization problem for an OFDMA system with a constant and equal bitrate for each user. A way to simplify implementation of adaptive resource allocation in OFDMA is a disjoint subcarrier, power, and bit allocation. Herein, we further simplify the optimization problem and restrict adaptive resource allocation by only disjoint subcarrier selection and power assignment.

We consider a number of subcarrier selection algorithms and compare their performances for different channel statistics. For the Rayleigh environment we prove that when using subcarrier assignment with iterations over users, starting iteration from the "worst" user (i.e., with the smallest average power gain) achieves better performance than the other user orderings. The performance is similar to the initial constructive allocation from [21] when it is combined with the OSSA [15].

The observation that the OSSA releases a part of the subcarriers and thus has a potential for increasing the multiuser diversity in multiple access has resulted in an extension of the algorithm to OFDMA [24]. A low-complexity implementation of the OSSA in OFDMA includes initial subcarrier allocation to users and next employing the OSSA for each user. Only one adaptive initial subcarrier allocation algorithm was presented and analyzed in [24] and it was shown that the application of the OSSA provides a significant power gain while the procedure of implementation is noniterative.

In this paper, we study combinations of the OSSA with different initial subcarrier allocation schemes. We show that the algorithm given in [24] is not the best one and on the basis of the order statistics theory we propose a technique that provides a better performance.

The paper is organized as follows. In Section 2, we briefly describe the OFDM-OFDMA concepts and formulate the optimization problem. Section 3 presents the proposed power loading algorithm and subcarrier allocation schemes.

In Section 4, the simulation results are given and Section 5 summarizes and concludes the contents.

2. SYSTEM DESCRIPTION

2.1. OFDM-OFDMA basics

In an OFDM system with N subcarriers, the input information data is mapped onto M-QAM constellation and in such a way, a sequence of the N-dimensional input data vectors is formed. The samples of an OFDM symbol are obtained by the application of the N-point inverse Fourier transform to an input data vector and next a cyclical extension of the symbol with the last N_G samples, that is, the so-called guard interval is added at the beginning of each symbol.

The power efficiency η of the system is defined by the relative length of the information part of the symbol with respect to its total length:

$$\eta = \frac{N}{N + N_G}. \tag{1}$$

In an OFDMA system, the N subcarriers are shared between the K users and a set of maximum L subcarriers is allocated to each user.

2.2. M-QAM OFDM BER-performance

Since we consider the margin maximization problem, an analytical expression for the BER is of interest.

In case of Gray coding, the BER on the ith subcarrier with the power gain $|H_i|^2 = x_i$ is as [25]

$$\text{BER}_i \cong a_M \text{erfc}\left(\sqrt{b_M x_i}\right), \tag{2}$$

where erfc$(\cdot)$ is the complementary error function, $a_M = (\sqrt{M}-1)/\sqrt{M}\log_2\sqrt{M}$, $b_M = (E_b^i/N_o)(3\log_2(M)\eta/2(M-1))$, and E_b^i/N_o defines the transmit SNR of the ith subcarrier.

Averaging (2) through the subcarriers and channel statistics results in the expression for average BER of the system:

$$\text{BER}_{\text{aver}} = \frac{1}{N} E\left\{\sum_{i=1}^{N} \text{BER}_i\right\}, \tag{3}$$

where E means the expectation.

2.3. Optimization problem

The margin maximization problem can be formulated as follows.

(i) For a single-user OFDM,

$$\min \text{BER}_{\text{aver}} \tag{4}$$

subject to

$$p_T \leq p_{\max}, \tag{5}$$

where BER_{aver} is defined by (3) and p_T denotes the transmit power.

(ii) For OFDMA,

$$\min \text{BER}_{\text{aver}} = \frac{1}{K} E\left\{\sum_{k=1}^{K} \sum_{\pi_k \in \pi} P_{\pi_k} \cdot P_k\right\} \tag{6}$$

subject to

$$R_k = R, \tag{7a}$$

$$p_{T_k} \leq p_{\max} \text{ (for the uplink)} \quad \text{or}$$

$$\sum_{k=1}^{K} p_{T_k} \leq p_M \text{ (for the downlink)}, \tag{7b}$$

where R_k and p_{T_k} denote the bitrate and transmit power of each user, respectively, π_k is the set of $L' \leq L$ subcarriers allocated to the kth user and π is the set of all possible permutations. In (6), P_{π_k} denotes the probability of assignment of the set π_k to the kth user and P_k is the conditional user's error probability assuming that the set π_k is allocated to the user:

$$P_k = 1/L' \sum_{i \in \pi_k} P_{er/H_{ik}}, \tag{8}$$

where $P_{er/H_{ik}}$ is the error probability conditioned to the specific subcarrier (characterized by the gain H_{ik}) allocation.

Aiming at low complexity of implementation, we restrict ourselves by identical constellation sizes for each user. This restriction combined with (7a) results in the equal number of subcarriers allocated to each user.

3. PROPOSED ALGORITHMS

3.1. Power loading based on incomplete CSI

In this section, we derive an algorithm of unequal power loading of "strong" and "weak" subcarriers.

Let all the subcarriers of a user be ordered according to their power-gain values, that is, $x_N \geq x_{N-1} \geq \cdots x_1$. As in [15, 16] we assume identical M-QAM modulation of each subcarrier that considerably facilitates the transceiver implementation. Let the total transmit power per symbol be

$$P = N\log_2 M \cdot E_b, \tag{9}$$

that is, E_b is the power per bit under equal power loading of all subcarriers.

The following lemma is valid.

Lemma 1. *In any frequency-selective channel, the power-loading algorithm*

$$E_b^i = \begin{cases} \dfrac{2kE_b}{k+1} & \text{if } N/2 < i \leq N, \\ \dfrac{2E_b}{k+1} & \text{if } 1 \leq i \leq N/2, \end{cases} \quad 0 < k < 1 \tag{10}$$

always improves the average BER-performance (through the subcarriers and channel statistics) starting with some transmit SNR value.

The proof is given in the appendix.

The procedure (10) assigns more power to the "weak" half of subcarriers and actually is a simplified algorithm of equalization of the received SNR. This method is opposite to the optimal water filling. Generally, such power loading may result in ineffective use of the transmit power since it is primarily used for compensation of deep fading and according to the lemma a power gain is observed only for high enough transmit SNR values. But below we show that in some radio channels, the proposed method provides a power gain actually for all practical transmit SNR values for both single- and multiuser scenarios.

3.2. BER-performance analysis of the power-loading algorithm

We assume that the channel power gains are identically and independently distributed with the probability density function (pdf) $f(x)$ and cumulative distribution function $F(x)$. Then the probability density function $f_i(x)$ of the ith order statistic is [26]

$$f_i(x_i) = \frac{N! F^{i-1}(x_i)\left[1 - F(x_i)\right]^{N-i} f(x_i)}{(i-1)!(N-i)!}. \qquad (11)$$

For example, for uncorrelated Rayleigh fading with a normalized expectation $E(x) = 1$,

$$f(x) = \exp(-x), \qquad F(x) = 1 - \exp(-x). \qquad (12)$$

Using (11) we obtain that the BER_{aver} under power loading defined by (10) is

$$\begin{aligned}
\text{BER}_{\text{aver}} = \frac{a_M}{N} \sum_{i=1}^{N/2} \Bigg\{ & \int_0^\infty \text{erfc}\left(\sqrt{b_M \frac{2kx}{k+1}}\right) f_{i+N/2}(x)dx \\
& + \int_0^\infty \text{erfc}\left(\sqrt{b_M \frac{2 \cdot x}{k+1}}\right) f_i(x)dx \Bigg\}.
\end{aligned} \qquad (13)$$

Since calculation of the BER_{aver} in (3) involves the expectation operation, the value of k minimizing (3) is essentially defined by the channel statistics and can be found for example numerically. We test the application of the power-loading algorithm (10) in a "truncated" radio channel [15] because the technique given herein is of low complexity and provides performance close to optimal. In this case, the required CSI at the transmitter is expressed in terms of "used strong-used weak-not used" subcarriers. It turned out that in a single-user case, for both Rayleigh and Nakagami (with different scale parameters) independent fading, the optimal k value is practically independent of N and E_b/N_o and is $k_{\text{opt}} \cong 0.53$.

The graphs of the BER_{aver} versus k for a Rayleigh uncorrelated channel and the total number of subcarriers $N = 192$ and $N = 96$ are shown in Figure 1. The curves in Figure 1 are given for the case of applying the OSSA for the transmit SNR values 5, 10, 15, and 17 dB.

3.3. Subcarrier allocation algorithms for OFDMA

We propose and analyze subcarrier assignment with iteration over users based on the user average power-gain values:

$$M_k = 1/N \sum_{i=1}^N x_{ki}, \qquad (14)$$

where $x_{ki} = |H_{ki}|^2$ is a subcarrier power gain.

We order the users according to their average power-gain values defined by (14) in such a way that

$$M_1 \le M_2 \cdots \le M_K. \qquad (15)$$

Then at least two algorithms of subcarrier assignment based on (15) can be proposed.

Algorithm "W" (starting with the "worst" user). Each user orders subcarriers according to the individual power-gain values, that is, puts them in such a way that

$$x_{k1} \le x_{k2} \cdots \le x_{kN}. \qquad (16)$$

Then the stronger subcarriers are assigned sequentially to each user starting from the worst one (i.e., in the ascending order in (15)). If a selected subcarrier of the user k has been allocated to another user, the next ordered vacated subcarrier of the user k is assigned to it.

Algorithm "B" (starting with the "best" user). The algorithm is similar to the previous one with the difference that the iteration over users is performed in the reverse order, that is, in the descending order in (15).

Then the following lemma is valid.

Lemma 2. *For an OFDMA system with equal users' bitrates operating in a Rayleigh channel, the initial subcarrier allocation according to Algorithm "W" always provides a better BER-performance defined by* (6) *compared with Algorithm "B" under other equal conditions.*

The proof is given in the appendix.

4. SIMULATION RESULTS

Figure 2 presents the simulation results for the scheme where the proposed power-loading algorithm based on the incomplete CSI is combined with the OSSA. The graphs are shown for a single user with 256 subcarriers in an uncorrelated Rayleigh channel. The number of subcarriers was chosen according to the WiMAX standard [1]. Here the value of $k = 0.53$ was used. Other graphs in the figure show BER for the ordered selection with equal power loading ($k = 1$) [15], ordered selection with subcarrier power gain inversion (labelled as inversion), described in [16], and optimal greedy algorithm [17–19]. The simulation results for the systems with the above power-loading algorithms but operating in correlated Rayleigh fading are shown in Figure 3. The channel model used for the simulations was the reduced typical urban channel [27].

It is seen that for both channels the algorithm (10) provides the BER-performance close to that under inversion of
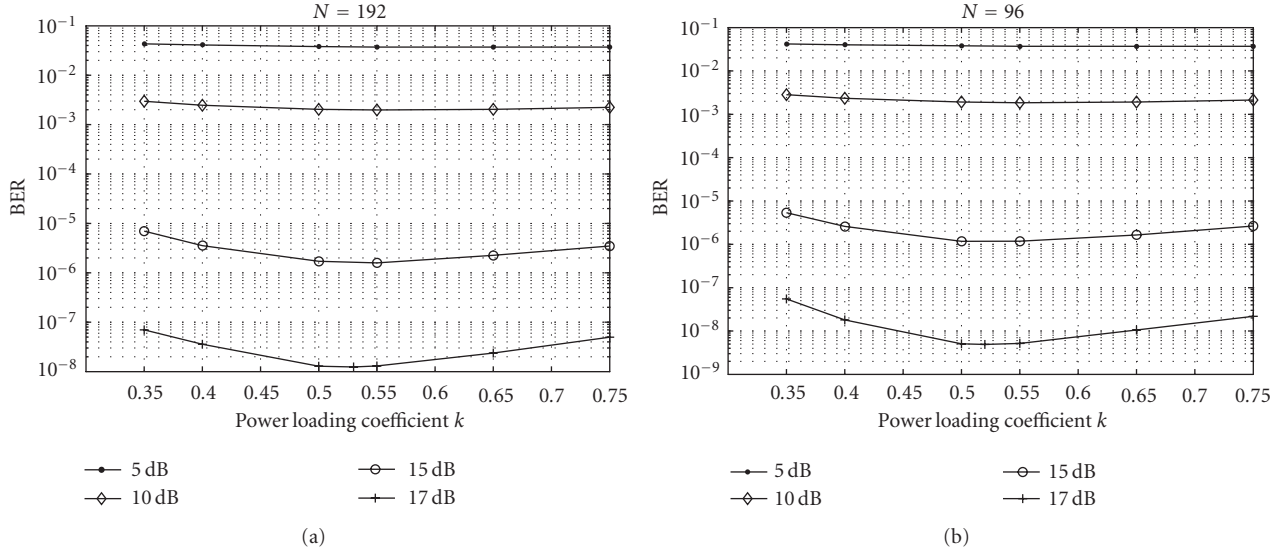
FIGURE 1: BER versus k in uncorrelated Rayleigh fading for different numbers of subcarriers.
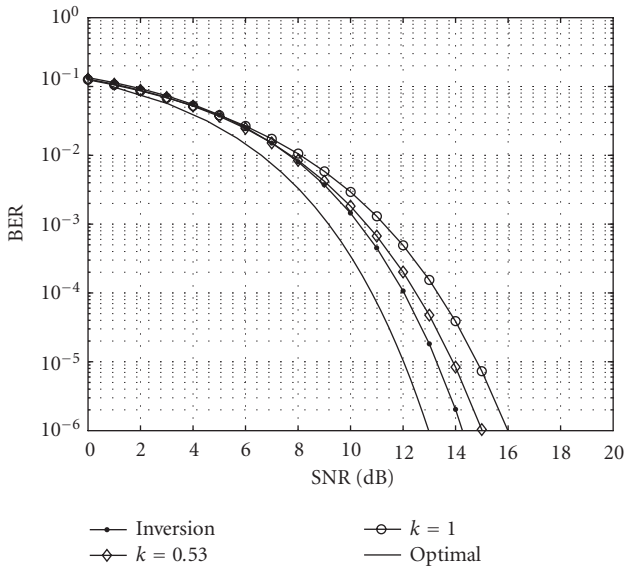

FIGURE 2: BER-performance of a few power loading schemes combined with OSSA in uncorrelated Rayleigh fading, single-user case.
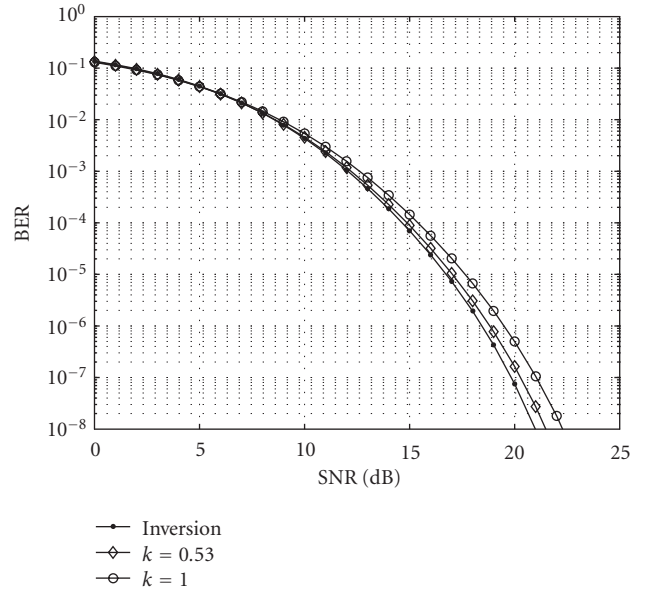
FIGURE 3: BER-performance of a few power-loading schemes combined with OSSA in correlated Rayleigh fading, single-user case.

the subcarrier power gains. The observed difference results from incomplete CSI in case of application of (10).

The simulation results for multiuser systems with the above power-loading algorithms operating in Rayleigh uncorrelated and correlated fadings are shown in Figure 4 and Table 1, respectively. For the former case $k_{\mathrm{opt}} \cong 0.75$ and for the latter $k_{\mathrm{opt}} \cong 0.9$.

It is seen that proposed power loading improves the BER-performance in all considered cases. This is more evident for the single-user case where in both uncorrelated and correlated Rayleigh channels the performance of the proposed method is close to that of inversion. However, for the consid-

TABLE 1: $\mathrm{Log}_{10}(\mathrm{BER})$ versus SNR of OFDMA in correlated Rayleigh fading; 8 users sharing 256 subcarriers.

SNR, dB	10	12	14	16	18	20
Invers.	−2.935	−3.662	−4.494	−5.422	−6.454	−7.611
$k = 0.9$	−2.931	−3.653	−4.482	−5.406	−6.434	−7.594
$k = 1$	−2.922	−3.641	−4.466	−5.386	−6.413	−7.576

ered multiuser cases, the proposed method is still beneficial although the provided power gain is small in the considered correlated Rayleigh channel.
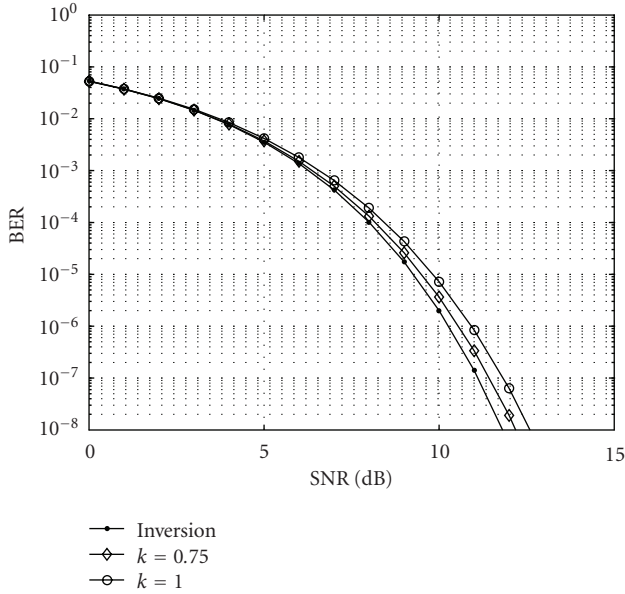
FIGURE 4: BER-performance of OFDMA with different power-loading algorithms combined with OSSA in uncorrelated Rayleigh fading; 8 users sharing 256 subcarriers.
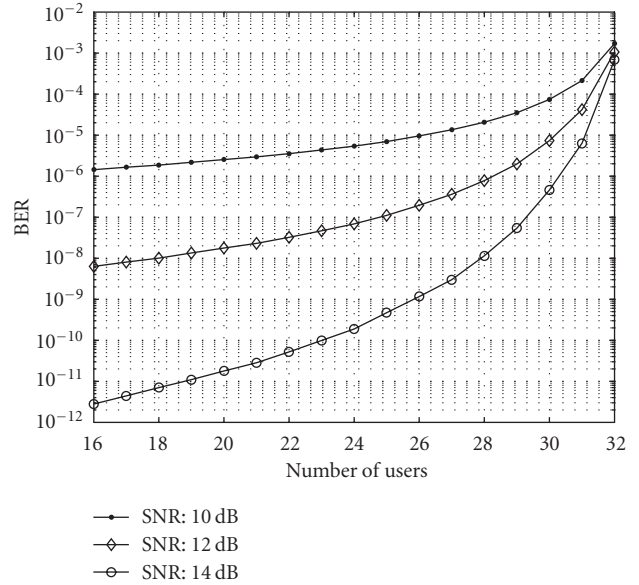

FIGURE 6: BER performance versus number of users in uncorrelated Rayleigh fading; OSSA is applied.
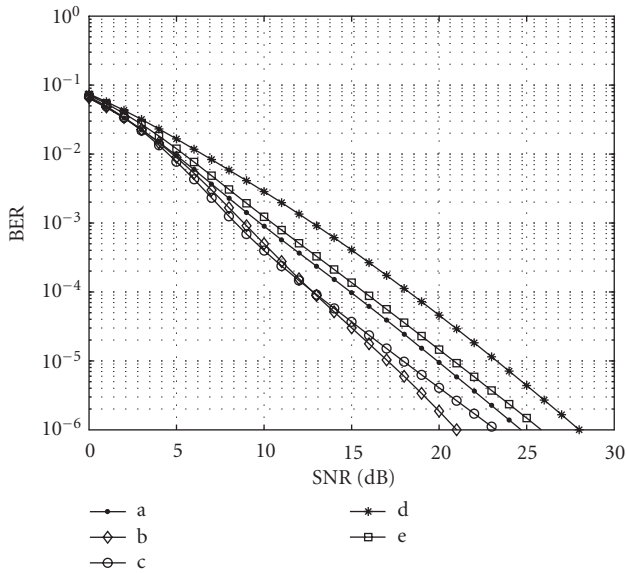

FIGURE 5: BER performance of a few subcarrier allocation schemes in correlated Rayleigh fading with equal power loading.

The performance estimates of different subcarrier allocation schemes for OFDMA discussed in the Section 3.3 are shown in Figures 5–8.

Performance of algorithms "W" and "B" in terms of average BER was evaluated and compared with performance of several other algorithms for subcarrier allocation. The algorithms were simulated in correlated and uncorrelated Rayleigh channels with different power-loading techniques.

Figure 5 shows the simulation results for the correlated Rayleigh channel with equal power loading for all the subcarriers and 4-QAM modulation. As the channel model, we use the reduced typical urban channel [27]. The algorithms shown there are the following: (a) allocation by sorted channel gains with iteration over subcarriers similar to the initial constructive allocation in [21] with the difference that we use a randomly permuted user order, (b) allocation by channel gains normalized by the user's mean gain and iteration over subcarriers, this normalization enhances fairness and decreases the mean BER, (c) algorithm "W," (d) algorithm "B", (e) allocation with iteration over users with the randomly selected user order, this allocation was used in [24].

As we can see from the figure, the best performance with equal power loading is shown by Algorithm "b" and "W" with "B" having the worst performance which validates the lemma's assertion.

Further, we combine the above-mentioned algorithms with the OSSA. There are a few reasons for employing the OSSA in OFDMA. Firstly, for a fixed number of the system users, releasing a part of the subcarriers results in a more effective use of multiuser diversity that improves the system performance [24]. Secondly, the system capacity can be enhanced by the allocation of the released subcarriers to extra users. Clearly, increasing the number of the system users deteriorates the error probability. However, the tradeoff between the number of the users and BER performance can be included into OFDMA design considerations. For example, in Figure 6 we show the simulation results for the BER-performance of an OFDMA system with 256 subcarriers when the number of users, with 8 subcarriers allocated to each of them, is increased from 16 to 32. The subcarrier allocation algorithm employed is that with

iterations over users with the randomly selected user order [24].

Effects of different subcarrier allocation algorithms on the BER-performance are shown in Figures 7-8. The simulation results for the uncorrelated and correlated Rayleigh channels with the same algorithms as in Figure 5 but additionally employing the OSSA are presented in Figures 7 and 8, respectively. There we can see that the best performance is shown by the algorithms "a" and "W" with the worst performance again shown by "B" which validates the lemma also in these environments. We observe that for all the cases, the algorithm "W" achieves good performance and that performance of the randomly permuted user ordering, algorithm "e", lies in the middle between "W" and "B".

5. CONCLUSIONS

In this paper, we consider practical approaches to the problem of optimal resource allocation in OFDM-based systems. We study both single- and multiuser systems and show that the single-user system performance can be improved by a suitable power loading and an algorithm based on the incomplete channel state information is derived. We also show that in a multiuser system the power loading only slightly affects performance while the initial subcarrier allocation has a rather big impact. A number of the subcarrier allocation algorithms are discussed.

When deriving the algorithm of power loading, we assume that only incomplete CSI in terms of the "strong" and "weak" subcarriers is available at the transmitter. Under such assumptions, we propose a technique of unequal power loading of the "strong" and "weak" groups. We give the theoretical background and simulation results that confirm efficiency of the proposed algorithm.

Actually, the proposed algorithm distributes the available transmit power by giving more power to the "weak" group and less to the "strong" one. Clearly, the technique approximately (i.e., only on the basis of incomplete CSI) equalizes the transmit SNR and thus it is an opposite one to the optimal water-filling procedure.

We prove that the algorithm is efficient in any time-dispersive channel starting with some transmit SNR value. It is interesting that in a truncated radio channel suggested in [15], the proposed technique gives a power gain actually for all practical transmit SNR values. In fact, the combination with [15] renders a new power and subcarrier selection algorithm for OFDM that achieves performance close to that of the optimal (but rather complex in implementation) algorithm, and therefore can be regarded as a simplified water-filling technique.

Such features of the presented algorithm as the noniterative structure, a constant constellation size, and a low overhead allow to refer it to a group of low-complexity techniques that make it attractive for practical implementation in OFDM-based transmission systems.

For OFDMA, we study performance of subcarrier allocation algorithms with iterations over users contrasted to the more conventional approach of iteration over sub-
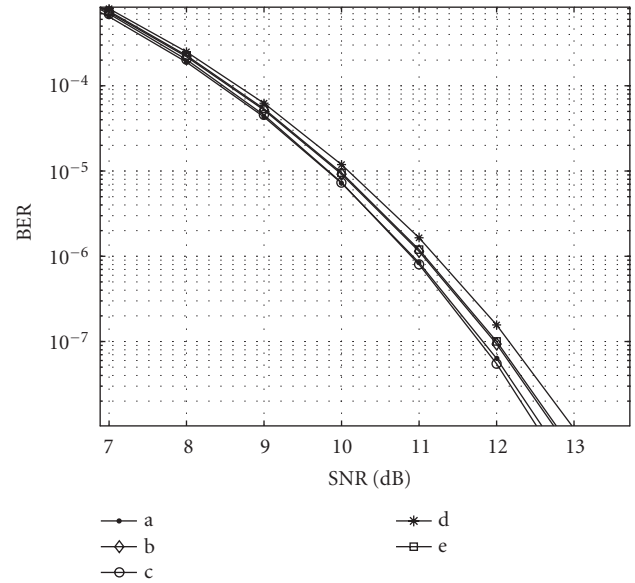

FIGURE 7: BER performance of subcarrier allocation algorithms "a"–"e" supplemented by OSSA for uncorrelated Rayleigh fading.
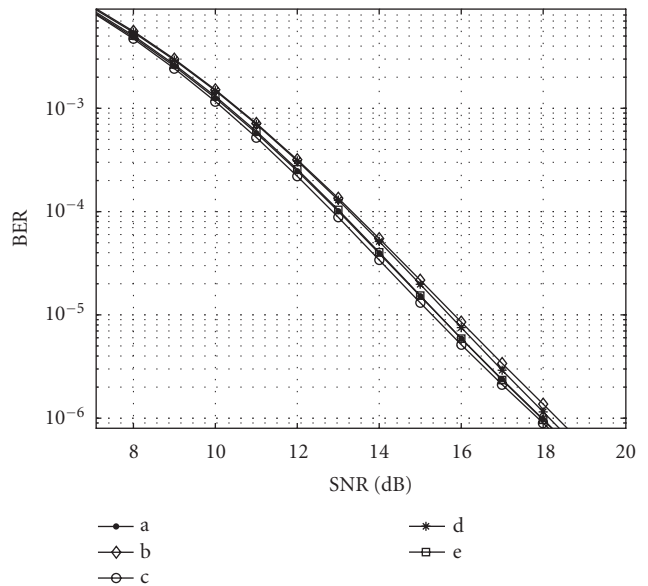

FIGURE 8: BER performance of subcarrier allocation algorithms "a"–"e" supplemented by OSSA for correlated Rayleigh fading.

carriers. We show that the performance of this scheme is defined by user ordering. Particularly, we prove that the algorithm based on the iteration starting from the worst user (with the smallest average power gain) outperforms other orderings. The analytical proof is validated by the simulation results that also show that the suggested algorithm achieves good performance with different power-loading techniques, while performance of algorithms with iteration over subcarriers depends on the chosen power loading.

APPENDIX

A.1. Proof of Lemma 1

The difference between the average BER for equal (non-adaptive) power loading BER^{eq} and the proposed algorithm $\text{BER}^{\text{adapt}}$ is expressed as

$$\text{BER}^{\text{eq}} - \text{BER}^{\text{adapt}}$$

$$= 1/N \times a_M E \left\{ \sum_{i=1}^{N/2} \left(\text{erfc}\left(\sqrt{b_M x_i}\right) - \text{erfc}\left(\sqrt{\frac{b_M x_i 2}{k+1}}\right) \right) \right.$$
$$\left. - \sum_{i=N/2+1}^{N} \left(\text{erfc}\left(\sqrt{\frac{b_M x_i \cdot 2k}{k+1}}\right) - \text{erfc}\left(\sqrt{b_M x_i}\right) \right) \right\}.$$
$$(A.1)$$

The following inequalities are valid for $0 < k < 1$,

$$\frac{2x_i}{k+1} > x_i, \qquad \frac{2kx_i}{k+1} < x_i. \qquad (A.2)$$

The derivative of the erfc-function

$$\frac{d}{dx}\text{erfc}\left(\sqrt{bx}\right) = -\sqrt{\frac{b}{\pi x}} \exp(-bx) \qquad (A.3)$$

and thus the function

$$F(x) = \frac{\text{erfc}\left(\sqrt{bx}\right)}{\exp(-bx)} \qquad (A.4)$$

is strictly decreasing for $x > 0$.

Therefore, we obtain that for $x_2 > x_1$,

$$\frac{\text{erfc}\left(\sqrt{bx_1}\right)}{\text{erfc}\left(\sqrt{bx_2}\right)} > \frac{\exp(-bx_1)}{\exp(-bx_2)}. \qquad (A.5)$$

For example, from (A.5) we have that

$$\text{erfc}\left(\sqrt{bx_1}\right) > C \times \text{erfc}\left(\sqrt{bx_2}\right) \qquad (A.6)$$

if

$$(x_2 - x_1) \geq \frac{\ln C}{b}, \qquad (A.7)$$

where C is a positive constant.

We compare components of the first and second sums at the right-hand side (RHS) of (A.1) elementwise. We obtain from (A.6) that each component of the first sum at the RHS of (A.1) satisfies to an inequality:

$$\text{erfc}\left(\sqrt{b_M x_i}\right) - \text{erfc}\left(\sqrt{\frac{b_M x_i \cdot 2}{k+1}}\right)$$
$$> (C-1) \times \text{erfc}\left(\sqrt{\frac{b_M x_i \cdot 2}{k+1}}\right) \qquad (A.8)$$

if

$$\frac{x_i \cdot 2}{k+1} - x_i > \frac{\ln C}{b_M}. \qquad (A.9)$$

Clearly, validity of (A.8)-(A.9) can be provided by a proper assignment of b_M. Moreover, a value of $b_{M\max}$ can be assigned such that (A.8)-(A.9) hold for each x_i ($1 \leq i \leq N/2$). At the same time, we have for each component of the second sum at the RHS of (A.1) that

$$\text{erfc}\left(\sqrt{\frac{b_M x_i \cdot 2k}{k+1}}\right) - \text{erfc}\left(\sqrt{b_M x_i}\right) < \text{erfc}\left(\sqrt{\frac{b_M x_i \cdot 2k}{k+1}}\right) \qquad (A.10)$$

and thus

$$\text{erfc}\left(\sqrt{b_{M\max} x_i}\right) - \text{erfc}\left(\sqrt{\frac{b_{M\max} x_i \cdot 2}{k+1}}\right)$$
$$- \text{erfc}\left(\sqrt{\frac{b_{M\max} x_{i+N/2} \cdot 2k}{k+1}}\right) + \text{erfc}\left(\sqrt{b_{M\max} x_{i+N/2}}\right) > (C-1)$$
$$\times \text{erfc}\left(\sqrt{\frac{b_{M\max} x_i \cdot 2}{k+1}}\right) - \text{erfc}\left(\sqrt{\frac{b_{M\max} x_{i+N/2} 2k}{k+1}}\right). \qquad (A.11)$$

It follows from (A.11) that the left-hand side of (A.11) is positive if such is the RHS of (A.11). We observe that owing to ordering $x_{i+N/2} > x_i$ and thus if $k \times x_{i+N/2} > x_i$ the RHS of (A.11) is positive for $C \geq 2$. But even if $k \times x_{i+N/2} < x_i$, the RHS of (A.11) can be made positive by proper assignment of the constant C that in turn can be provided by a large value of $b_{M\max}$ (see (A.6)-(A.7)).

Thus starting from some value of b_0, the inequality (A.11) holds for $b_{M\max} > b_0$. This means that the RHS of (A.1) is positive that in turn proves that the proposed power-loading procedure starting with some transmit SNR value improves the average BER performance.

A.2. Proof of Lemma 2

Let the matrix of the channel power gains be $\mathbf{X} = \{x_{ki}\}$ with the elements ordered according to (15) and (16).

We consider two algorithms of the initial subcarrier allocation that differ only by the first step. These steps of Algorithm I and Algorithm II are those of Algorithm "W" and Algorithm "B," respectively. Then the error probability for Algorithms I (II) will be $\text{BER}_{I(II)}$:

$$\text{BER}_I = \frac{1}{K} E\left\{ P_{\text{err}/x_{1N},\dots x_{1N-L'+1}} + \sum_{i=2}^{K} \sum_{\pi_i \in \pi} P_{\pi_i} \cdot P_{I/\pi_i} \right\}, \qquad (A.12)$$

$$\text{BER}_{II} = \frac{1}{K} E\left\{ P_{\text{err}/x_{KN},\dots x_{KN-L'+1}} + \sum_{i=1}^{K-1} \sum_{\pi_i \in \pi} P_{\pi_i} \cdot P_{II/\pi_i} \right\}, \qquad (A.13)$$

where $P_{\text{err}/x_{1N},\dots x_{1N-L'+1}}$ and $P_{\text{err}/x_{KN},\dots x_{KN-L'+1}}$ are the error probabilities conditioned to that the best subcarriers are allocated to the worst and best user, respectively. The second components in the brackets in (A.7)-(A.8) express the error probabilities for the rest of the users (see (6)).

The difference between the second components of the sums at the RHS of (A.12) and (A.13) is that the set $\pi_{L'}$ is

assigned to the Kth user in (A.12) while in (A.13) it is allocated to the 1st user, that is,

$$
\mathrm{BER}_{II} - \mathrm{BER}_{I} = \frac{1}{K}\Bigg[- E\big\{P_{\mathrm{err}/x_{1N},\dots,x_{1N-L'+1}} - P_{\mathrm{err}/x_{KN},\dots,x_{KN-L'+1}}\big\} \\
+ E\bigg\{ \sum_{\pi_{L'} \in \pi} (P_{1/\pi_{L'}} - P_{K/\pi_{L'}})P_{\pi_{L'}} \bigg\}\Bigg],
$$

$$(A.14)$$

where $P_{\pi_{L'}}$ is the probability that a specific subcarrier set $\pi_{L'}$ is assigned to the 1st (Kth) user.

The function $\mathrm{erfc}(\sqrt{bx})$ that defines the error probability in (A.14) (see (2)) is a rapidly decreasing function with the rapidly decreasing derivative defined by (A.3). It follows from (A.3) that the erfc-function decreases faster than the exponential function that in turn means that the difference between two values of the erfc-function in the area of large arguments is smaller than that in the area of small arguments if the difference of the small arguments is not smaller than the natural logarithm of the difference of the large and small arguments.

We observe that the first component of the sum at the RHS of (A.14) is just defined by the difference of erfc-function values in the area of large values of the argument while the second component is defined by that in the area of smaller argument values. We recall that the power-gain values of each user for Rayleigh fading are subject to the exponential distribution and such are $(x_{1i}-x_{1i-1})$ and $(x_{Ki}-x_{Ki-1})$ [26].

The expectations $E\{x_{ji} - x_{ji-1}\}$, $j = 1,\dots,K$ decrease linearly as i decreases [26] and thus due to (A.3) the RHS of (A.14) is positive.

Likewise we can prove that each next step of the initial subcarrier allocation of Algorithm "W" provides a power gain compared with that of Algorithm "B".

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