Hindawi Publishing Corporation
EURASIP Journal on Wireless Communications and Networking
Volume 2008, Article ID 325829, 11 pages
doi:10.1155/2008/325829

Research Article

Partial Transmit Sequences for Peak-to-Average Power Ratio Reduction in Multiantenna OFDM

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Received 30 April 2007; Accepted 17 September 2007

Recommended by Luc Vandendorpe

The major drawback of orthogonal frequency-division multiplexing (OFDM) is its high peak-to-average power ratio (PAR), which gets even more substantial if a transmitter with multiple antennas is considered. To overcome this problem, in this paper, the partial transmit sequences (PTS) method—well known for PAR reduction in single antenna systems—is studied for multiantenna OFDM. A directed approach, recently introduced for the competing selected mapping (SLM) method, proves to be very powerful and able to utilize the potential of multiantenna systems. To apply directed PTS, various variants for providing a sufficiently large number of alternative signal superpositions (the candidate transmit signals) are discussed. Moreover, affording the same complexity, it is shown that directed PTS offers better performance than SLM. Via numerical simulations, it is pointed out that due to its moderate complexity but very good performance, directed or iterated PTS using combined weighting and temporal shifting is a very attractive candidate for PAR reduction in future multiantenna OFDM schemes.

1. INTRODUCTION

Future wireless communication systems demand for higher and higher data rates. In order to cope with the peculiarities of the wireless channel, a combination of *orthogonal frequency-division multiplexing (OFDM)* and antenna arrays in transmitter and receiver is envisaged. Thereby, OFDM [1] is a very popular method for handling the temporal interferences (echoes) in the channel. Using multiantenna systems—hence creating a *multiple-input/multiple-output (MIMO) system*—it is possible to dramatically increase the channel capacity [2].

Since individual, independent signal components (the carriers) are superimposed in the OFDM transmitter, the transmit signal is almost Gaussian distributed and hence exhibits a very large *peak-to-average power ratio (PAR)*. This major drawback of OFDM significantly complicates implementation of the radio-frequency frontend. Using nonlinear power amplifiers, amplitude distortion and clipping of the signal is caused. This, in turn, generates out-of-band radiation which strictly has to be avoided.

In literature, numerous methods for reducing the PAR of single antenna OFDM systems are given (cf. [3]). Recently, first techniques for multiantenna systems were proposed. For PAR reduction, some degrees of freedom are introduced and (implicitly or explicitly) redundancy is added to each OFDM frame. The most important approaches (the list is not exhaustive) are *redundant signal representations*, that is, the design of multiple transmit signals which represent the same data, and from which the "best" representation is selected, in particular *selected mapping (SLM)* and *partial transmit sequences (PTS)* [4–8]; *(soft) clipping*, that is, the transmit signal (preferably the discrete-time symbols prior to pulse shaping) is passed through a nonlinear, memoryless device [9, 10]; *redundant coding techniques* (also combined with channel coding), that is, algebraic code constructions adopted to code over the frequency-domain symbols [11, 12]; *tone reservation*, that is, some carriers are omitted from data transmission and are selected via an algorithmic search (sometimes in an iterative way between frequency and time domain) [13, 14]; *(active) constellation expansion*, that is, the signal set is warped such that edge points are allowed to have (any) amplitude larger than the original one [15]; algorithms based on *lattice decoding*, that is, PAR reduction is formulated as a decoding problem and solved using "sphere decoders" [16–18].

In this paper, PAR reduction for MIMO OFDM is studied. In particular, the application of the concept of partial transmit sequences to the multiantenna setting is assessed. The recently presented approaches of MIMO selected mapping [7, 8, 19] are carried over to PTS; and new degrees of freedom (e.g., [20, 21]), only available using the concept of partial sequences, are utilized. It is evaluated which PTS variant offers the best tradeoff between PAR reduction and required arithmetic complexity.

Noteworthy, throughout this paper, a MIMO point-to-point scenario with receiver sided channel equalization is considered. Multiuser scenarios, where joint processing is not possible at both sides of the wireless link, are not taken into account.

The paper is organized as follows. In Section 2, the MIMO OFDM system model is established and the parameters for the numerical results are given. Section 3 reviews PTS for single antenna systems. The extensions of PTS to multiantenna systems are given in Section 4 together with numerical results to evaluate the performance of the various schemes. A comparison of PTS and SLM based on their computational complexity is performed in Section 5; Section 6 draws some conclusions.

2. SYSTEM MODEL

In this paper, vectors are designated by bold letters, whereas vectors in the frequency-domain are written as capital and in the time-domain as lower case letters; $\mathrm{E}\{\cdot\}$ is the expected value of a random variable and $\lceil\cdot\rceil$ denotes rounding to the nearest integer towards infinity.

Throughout this paper, we consider a MIMO point-to-point scenario with N_{T} transmit antennas. In order to equalize the temporal (intersymbol) interferences of the channel, an OFDM scheme is applied. The spatial (multiantenna) interferences in each subcarrier are eliminated through receiver-side equalization. As we are interested in the peak power at the power amplifier, it is sufficient to consider the transmitter.

As usual in OFDM, the information carrying symbols $A_{\mu,d}$ (drawn from a QAM alphabet with variance $\sigma_A^2 = E_{\forall\mu,\forall d}\{|A_{\mu,d}|^2\}$) of the μth transmit antenna are specified in frequency domain (carrier d) and are combined into the vector $\mathbf{A}_\mu = [A_{\mu,d}]$ of length D (number of subcarriers). This vector is transformed into the time-domain vector $\mathbf{a}_\mu$ (OFDM frame) via an inverse discrete Fourier transform (IDFT), written as $\mathbf{a}_\mu = \mathrm{IDFT}\{\mathbf{A}_\mu\}$, with components $a_{\mu,k} = (1/\sqrt{D})\sum_{d=0}^{D-1} A_{\mu,d}\cdot \mathrm{e}^{\mathrm{j}2\pi dk/D}$, $k = 0,\ldots,D-1$. Assuming statistically independence of the frequency-domain symbols $A_{\mu,d}$ and sufficiently large D, due to the central limit theorem, the resulting time-domain samples $a_{\mu,k}$ are approximately Gaussian distributed which leads to a high PAR. If multiple transmit antennas are present, we consider the worst-case peak power over all transmit antennas being crucial. Other criteria like the input power backoff, which is related to the harmonic mean of the PAR of each antenna [22] may also be taken into account. However, the harmonic mean is dominated by the worst-case PAR, which is hence a suited measure. As the IDFT is a unitary transformation, we define the PAR of one OFDM frame as

$$\mathrm{PAR} \stackrel{\mathrm{def}}{=} \max_{\substack{\mu=1,\ldots,N_{\mathrm{T}}\\ k=0,\ldots,D-1}} \frac{|a_{\mu,k}|^2}{\sigma_A^2}, \tag{1}$$

where the maximization is carried out over all time-domain samples within one OFDM frame and over all transmit antennas. As common in literature, we consider the PAR of the discrete time signal. Using oversampling, the results can readily be extended to control the PAR of the continuous-time signal. The performance measure for the different PAR reduction schemes is the *complementary cumulative distribution function* (ccdf) which gives the probability that the PAR exceeds a certain threshold PAR_0: $\Pr(\mathrm{PAR} > \mathrm{PAR}_0)$.

Assuming Gaussian time-domain samples $a_{\mu,k}$, the ccdf of MIMO OFDM is given by [7];

$$\Pr(\mathrm{PAR} > \mathrm{PAR}_0) = 1 - \left(1 - \mathrm{e}^{-\mathrm{PAR}_0}\right)^{N_{\mathrm{T}}D}. \tag{2}$$

This equation shows that for a fixed OFDM frame size the problem of high peak-power gets worse if the number of transmit antennas N_{T} is increased.

The numerical results from Sections 4.4 and 5 are based on a MIMO system with $N_{\mathrm{T}} = 2$, 4, or 8 transmit antennas. The OFDM block length (number of carriers) is always $D = 512$ and the symbol alphabet is chosen to a 4-QAM constellation.

3. REVIEW OF PARTIAL TRANSMIT SEQUENCES FOR SINGLE ANTENNA SYSTEMS

3.1. Original PTS (PTS-w)

The idea behind the original PTS scheme from [5, 23] is to divide the information carrying frequency-domain OFDM frame $\mathbf{A}$ into V pairwise disjoint parts $\overline{\mathbf{A}}_v$, the partial (transmit) sequences (the antenna index μ is suppressed in this section). Thereby, each symbol A_d is contained exactly in one part $\overline{\mathbf{A}}_v$; the remaining symbols of $\overline{\mathbf{A}}_v$ are set to zero. These partial sequences are transformed individually into time-domain vectors $\overline{\mathbf{a}}_v$, where the transformation length remains D. A weighted superposition of all V parts leads to the transmit signal

$$\mathbf{a}_{\mathrm{PTS-w}} = \sum_{v=1}^{V} b_v\cdot\overline{\mathbf{a}}_v. \tag{3}$$

For PAR reduction, the vector of weighting factors $\mathbf{b} = [b_1,\ldots,b_V]$ has to be optimized (weighted PTS, PTS-w). According to [23], b_v is preferably chosen from the set $\{\pm 1, \pm \mathrm{j}\}$; hence, only the *phase* is modified. This special choice of the weighting factors b_v guarantees that the frequency-domain symbols A_d are still taken from the original QAM constellation. Moreover, to avoid ambiguities and without any performance loss, the first weighting factor can be chosen to $b_1 = 1$.

This restriction of b_ν to a finite set leads to a discrete optimization problem with finite search space.

Besides a full search over all possible vectors $\mathbf{b}$, in literature a number of efficient decoding algorithms have been proposed [24–26]. For brevity, we refer to a straightforward search through a fixed set of vectors $\mathbf{b}$. Instead of searching over the maximum number $J_{\mathrm{b,max}} = 4^{V-1}$ of possible combinations of the weighting factors, a restriction of the search space to a given number of $J_{\mathrm{b}} \leq J_{\mathrm{b,max}}$ different, arbitrary chosen combinations (vectors $\mathbf{b}^{(\nu)}$, $\nu = 1, \ldots, J_{\mathrm{b}}$) is also possible. Thereby the complexity of the PAR reduction—given by the number $J = J_{\mathrm{b}}$ of superpositions (candidates) which have to be evaluated (calculating their PAR)—can be controlled. In addition, independent of the number of examined superpositions, V IDFTs have to be calculated to obtain the partial transmit sequences $\bar{\mathbf{a}}_\nu$.

In order to recover the transmitted signal correctly, for coherent reception the receiver must be aware of the actually used weighting vector $\mathbf{b}^{(\nu^*)}$. Thus, transmission of side information is necessary. Assuming a codebook of all J_{b} possible combinations $\mathbf{b}^{(\nu)}$; $\nu = 1, \ldots, J_{\mathrm{b}}$, is available jointly to transmitter and receiver, it is sufficient to transmit the index ν^* of the applied combination. This index can be represented by $\lceil \log_2(J_{\mathrm{b}}) \rceil$ bits.

3.2. Temporally shifted PTS (PTS-ts)

In [20] another variant to create alternative signal representations was presented. It is based on a cyclic shift of the time-domain partial sequences $\bar{\mathbf{a}}_\nu$ (temporally shifted PTS, PTS-ts). We define a function $\mathbf{y} \stackrel{\text{def}}{=} \mathrm{cycs}(\mathbf{x}, \delta)$ which cyclically shifts the vector $\mathbf{x}$ by δ elements to the left. The transmit signal is now given by

$$\mathbf{a}_{\text{PTS-ts}} = \sum_{\nu=1}^{V} \mathrm{cycs}(\bar{\mathbf{a}}_\nu, \delta_\nu). \tag{4}$$

According to [20] the number of positions to be shifted should be chosen to $\delta_\nu = \gamma \cdot D/4$, with $\gamma \in \{0, \ldots, 3\}$. This choice gives good results in PAR reduction and it does not affect the receiver side synchronization algorithm as, due to the shifting property of the DFT [27], all frequency-domain symbols of the partial sequences are weighted by $\{\pm 1, \pm \mathrm{j}\}$. As above, the symbol alphabet remains unchanged.

The different numbers δ_ν of positions to be shifted for all V signal parts are combined into the vector $\boldsymbol{\delta} = [\delta_1, \ldots, \delta_V]$. Again, the modification of the first partial sequence is fixed to $\delta_1 = 0$ in order to avoid ambiguities. The maximum number of combinations is given by $J_{\delta,\max} = 4^{V-1}$, and the search space can also be restricted to $J_\delta \leq J_{\delta,\max}$ combinations. Thus, the total number of superpositions is here given by $J = J_\delta$ and the number of redundant bits is $\lceil \log_2(J_\delta) \rceil$.

3.3. Weighted and temporally shifted PTS (PTS-wts)

As already published in [20], it is possible to combine the original (weighting) and temporally shifted PTS variants (weighted and temporally shifted PTS, PTS-wts). For a single antenna system this leads only to a slight better performance in PAR reduction (see numerical results [20, Figure 2]). When doing combined weighting and shifting, the transmit signal is calculated as

$$\mathbf{a}_{\text{PTS-wts}} = \sum_{\nu=1}^{V} \mathrm{cycs}(b_\nu \cdot \bar{\mathbf{a}}_\nu, \delta_\nu). \tag{5}$$

Now, optimization has to be carried out over weighting factors b_ν and shifts δ_ν, that is, over vector tuples $[\mathbf{b}, \boldsymbol{\delta}]$. Instead of searching over all $J_{\max} = J_{\mathrm{b}\delta,\max} \stackrel{\text{def}}{=} J_{\mathrm{b,max}} \cdot J_{\delta,\max} = 16^{V-1}$ possible combinations, restriction to $J = J_{\mathrm{b}\delta} \leq J_{\mathrm{b}\delta,\max}$ randomly selected weighting/shift vectors is again possible. Then, $\lceil \log_2(J_{\mathrm{b}\delta}) \rceil$ bits of side information have to be communicated.

Noteworthy, other operations than weighting and cyclically shifting can be introduced in order to increase the number of possible candidates. In [28], complex conjugation, frequency reversal, and circular shift in frequency domain are additionally used. Since only marginal improvements are achieved, in this paper we concentrate on combined weighting and temporal shifting.

4. PARTIAL TRANSMIT SEQUENCES FOR MIMO OFDM

4.1. Ordinary, simplified, and directed PTS

In [7], Baek et al. presented a generalization of the *selected mapping* techniques to a MIMO point-to-point scenario, namely, ordinary SLM (oSLM) and simplified SLM (sSLM). Using SLM, U alternative signal representations are generated by multiplying the frequency-domain vector $\mathbf{A}$ elementwise with a phase vector $\mathbf{P}$ [4]. These alternative OFDM frames are transformed into time domain and the best one, that is the one exhibiting the lowest PAR, is chosen for transmission.

It is straightforward to apply the same technique to PTS, hence we call these schemes ordinary PTS (oPTS) and simplified PTS (sPTS). Both methods are just a simple application of single antenna PTS (all three variants from Section 3 can be applied, of course) at all N_{T} antennas of the transmitter. A block diagram of these PAR reduction schemes is depicted in Figure 1.

Ordinary PTS is the straightforward application of single antenna PTS to each transmit antenna. Thus $N_{\mathrm{T}} V$ computations of the IDFT and the assessment of $J = J_{\mathrm{b}/\delta/\mathrm{b}\delta}$ superpositions per antenna are necessary in this case. The number of side information bits increases to $N_{\mathrm{T}} \lceil \log_2(J_{\mathrm{b}/\delta/\mathrm{b}\delta}) \rceil$.

Simplified PTS optimizes the PAR by applying the same weighting or shifting to all transmit antennas. This PTS variant performs worse, as less possible combinations of weighting factors $\mathbf{b}$ or shifting positions $\boldsymbol{\delta}$ are available. Nevertheless, the computational effort compared to oPTS remains $N_{\mathrm{T}} V$ evaluations of the IDFT and $J = J_{\mathrm{b}/\delta/\mathrm{b}\delta}$ superpositions. The only advantage of this technique compared to oPTS is the reduced amount of side information which is the same as for single antenna PTS, namely, $\lceil \log_2(J_{\mathrm{b}/\delta/\mathrm{b}\delta}) \rceil$ bits.

In [8] a "directed" approach to SLM (dSLM) has been proposed which utilizes the potential of multiple transmit antennas. The dSLM algorithm does not consider the

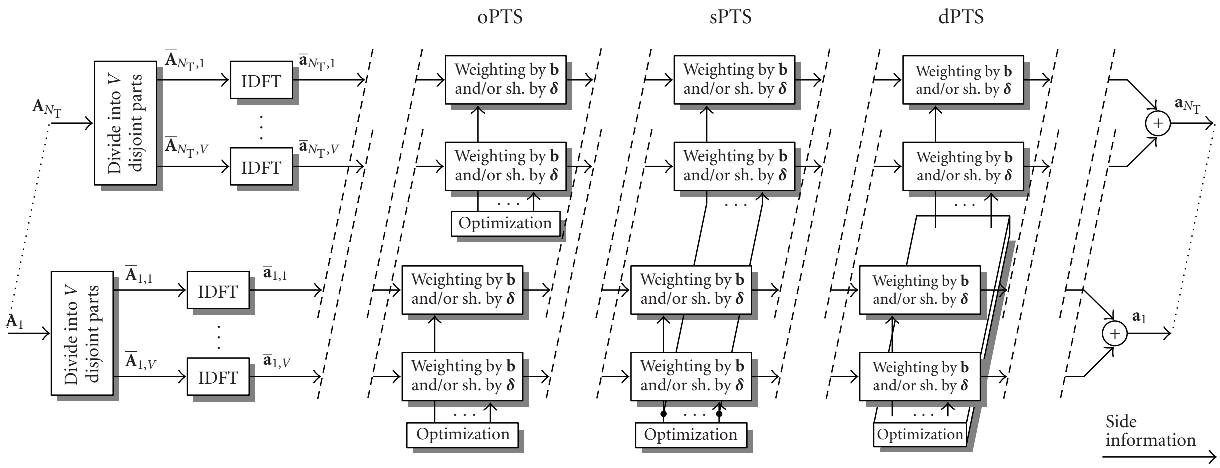

FIGURE 1: Block diagram of ordinary, simplified, and directed PTS.

antennas separately, and hence equally, but concentrates on the antenna exhibiting the highest PAR. Thereby, significant gains compared to a single antenna system (comparable to a diversity gain) are achieved.

It is natural to apply this directed approach to partial transmit sequences. Consequently, we denote this approach by dPTS. The idea of this technique is to increase the number of possible alternative signal representations (by increasing the combinations of the weighting factors J_{b} or numbers of positions to be shifted J_δ), but to keep the complexity (i.e., the amount of IDFT computations V and superpositions J) the same compared to ordinary or simplified PTS. As in dSLM, not all possible candidates are evaluated for each transmit antenna, but this method always considers that antenna which currently exhibits the highest PAR and tries to reduce it.

A pseudocode description of the dPTS algorithm is given in Algorithm 1. First, the partial sequences of all antennas are determined, and the PAR of each transmit antenna is set to infinity. In each iteration of the for-loop (lines 02 to 08), the antenna with the highest PAR is considered and another signal representation is tested. Here, line 04A corresponds to the weighting PTS variant (Section 3.1), 04B to the shifting variant (Section 3.2), and 04C to combined weighting and shifting (Section 3.3). As all PAR_μ are initialized with infinity the loop determines in its first N_{T} cycles the PAR of all N_{T} transmit antennas. The remaining budget of $N_{\mathrm{T}}(J-1)$ superpositions is successively spent on that antenna exhibiting the worst PAR.

The number of alternative signal representations (achieved through weighting or shifting), which should be evaluated in Algorithm 1, must be restricted to $J = J_{\mathrm{b}/\delta/\mathrm{b}\delta} \le (J_{\mathrm{b}/\delta/\mathrm{b}\delta,\max} - 1)/N_{\mathrm{T}} + 1$. If in each cycle of the for-loop (line 02 to 08, Algorithm 1) always one certain antenna exhibits the currently worst PAR $N_{\mathrm{T}}(J-1)+1$ candidates are assessed. This number, of course, has to be smaller than the maximum possible number of candidates for each antenna.

Compared to oPTS/sPTS the average number of superpositions is given by $J = J_{\mathrm{b}/\delta/\mathrm{b}\delta}$ and the number of side information bits is $N_{\mathrm{T}}\lceil \log_2(N_{\mathrm{T}}(J_{\mathrm{b}/\delta/\mathrm{b}\delta}-1)+1)\rceil$.

4.2. Spatially permuted PTS

All above PTS approaches optimize (individually or jointly) the way the partial sequences are superimposed. However, in case of PTS there is an additional way to exploit the presence of multiple transmit antennas by permuting the partial sequences between the antennas. We call this variant spatially permuted PTS (PTS-sp). A similar scheme was already described in [21] which uses cyclic shifting of the partial sequences between the antennas. This cyclic shifting is just a special case of the more general permutation described here.

We introduce the bijective permutation function $y \stackrel{\mathrm{def}}{=} \mathrm{perm}(x)$ of the set $x, y \in \{1, \ldots, N_{\mathrm{T}}\}$ into itself. Instead of using weighting factors for generating the different signal representations we apply different permutations of the partial sequences between the antennas. The time-domain transmit signal of the μth antenna is now given by

$$\mathbf{a}_{\mu,\mathrm{PTS\text{-}sp}} = \sum_{\nu=1}^{V} \bar{\mathbf{a}}_{\mathrm{perm}_\nu(\mu),\nu}, \tag{6}$$

where $\mathrm{perm}_\nu(\mu)$ is the permutation function applied to the νth partial transmit sequence. To avoid ambiguities the permutation function of the first partial sequence is chosen to $\mathrm{perm}_1(\mu) = \mu$.

For each partial sequence there exist $N_{\mathrm{T}}!$ possible permutations. As $\mathrm{perm}_1(\mu)$ is fixed there are in total $J_{\mathrm{p},\max} = N_{\mathrm{T}}!^{V-1}$ possibilities for creating representations of the transmit signal. In general it is too complex to consider all possibilities for finding the best solution. Hence, we again limit the number of different signal representations by choosing $J_{\mathrm{p}} \le J_{\mathrm{p},\max}$ arbitrary, distinct sets of permutation functions. The average number of superpositions is now given by $J = J_{\mathrm{p}}$. Compared to the variants discussed above (Section 4.1), here the number of superpositions J can be increased extremely.

given:

$V, J, [\mathbf{b}^{(1)}, \ldots, \mathbf{b}^{(N_\mathrm{T}(J-1)+1)}]$ or $[\mathbf{ffi}^{(1)}, \ldots, \mathbf{ffi}^{(N_\mathrm{T}(J-1)+1)}]$ or $[[\mathbf{b}^{(1)}, \mathbf{ffi}^{(1)}], \ldots, [\mathbf{b}^{(N_\mathrm{T}(J-1)+1)}, \mathbf{ffi}^{(N_\mathrm{T}(J-1)+1)}]]$

generate V disjoint parts $\overline{\mathbf{A}}_{\mu,1}, \ldots, \overline{\mathbf{A}}_{\mu,V}$ of $\mathbf{A}_\mu$, $\mu = 1, \ldots, N_\mathrm{T}$

$\bar{\mathbf{a}}_{\mu,\nu} := \mathrm{IDFT}\{\overline{\mathbf{A}}_{\mu,\nu}\}$, $\nu = 1, \ldots, V$ and $\mu = 1, \ldots, N_\mathrm{T}$

function $[\mathbf{a}_1, \ldots, \mathbf{a}_{N_\mathrm{T}}] = \mathrm{dPTS}([\bar{\mathbf{a}}_{1,1}, \ldots, \bar{\mathbf{a}}_{1,V}, \ldots, \bar{\mathbf{a}}_{N_\mathrm{T},1}, \ldots, \bar{\mathbf{a}}_{N_\mathrm{T},V}])$

01 $\mathrm{PAR}_\mu := \infty, \mu = 1, \ldots, N_\mathrm{T}$

02 **for** $\nu = 1, \ldots, N_\mathrm{T} J$

03 $\mu_{\max} := \mathrm{argmax}_{\forall \mu = 1,\ldots,N_\mathrm{T}} \mathrm{PAR}_\mu$

04A $\mathbf{a}_\mathrm{new} := \sum_{\nu=1}^{V} b_\nu^{(\nu)} \cdot \bar{\mathbf{a}}_{\mu_{\max},\nu}$, calc. $\mathrm{PAR}_\mathrm{new}$

04B $\mathbf{a}_\mathrm{new} := \sum_{\nu=1}^{V} \mathrm{cycs}(\bar{\mathbf{a}}_{\mu_{\max},\nu}, \delta_\nu^{(\nu)})$, calc. $\mathrm{PAR}_\mathrm{new}$

04C $\mathbf{a}_\mathrm{new} := \sum_{\nu=1}^{V} \mathrm{cycs}(b_\nu^{(\nu)} \cdot \mathbf{a}_{\nu,\mu_{\max}}, \delta_\nu^{(\nu)})$, calc. $\mathrm{PAR}_\mathrm{new}$

05 **if** $(\mathrm{PAR}_\mathrm{new} < \mathrm{PAR}_{\mu_{\max}})$

06 $\mathbf{a}_{\mu_{\max}} := \mathbf{a}_\mathrm{new}, \mathrm{PAR}_{\mu_{\max}} := \mathrm{PAR}_\mathrm{new}$

07 **endif**

08 **endfor**

Algorithm 1: Pseudocode description of the dPTS algorithm.

As already mentioned, a cyclic shift [21] between the antennas is just a special case of the present permutation. Using cyclic shifting, there are only N_T^{V-1} possibilities to create alternative signal representations.

In order to inform the receiver about the permutation of the partial sequences it is necessary to transmit $\lceil \log_2(J_\mathrm{p}) \rceil$ bits of side information.

4.3. Hybrid PTS variant: spatially permuted and weighted/temporally shifted PTS

In order to increase performance of PTS, the number J of tested signal superpositions may be increased. This number, however, is limited by the maximum number of possible combinations of the weighting factors J_b or positions to be shifted J_δ. This limitation is especially important in dPTS, since here the maximum possible number has to be much higher (factor N_T) than the average number of assessed combinations. In order to provide more signal combinations, the different PTS variants may be combined.

As already shown for the single antenna case, the combined weighting and temporal shifting variant may be applied leading to a maximum of $J_{\mathrm{b},\max} \cdot J_{\delta,\max}$ possible candidates.

Another way to increase the number $J_{\max}$ of maximum possible superpositions is to combine weighted/temporally shifted PTS (PTS-wts) with spatially permuted PTS (PTS-sp). As above, to avoid a full search, a straightforward strategy would be to search over a given number of J randomly selected combinations of weights b_ν, shifts δ_ν, and permutations $\mathrm{perm}_\nu(\mu)$, that is, vector triples $[\mathbf{b}, \boldsymbol{\delta}, [\mathrm{perm}_1(\mu), \ldots, \mathrm{perm}_V(\mu)]]$; ambiguities should be removed. We denote this approach as spatially permuted and weighted/temporally shifted PTS (PTS-spwts). Since each new vector influences all antennas simultaneously and the search is now done jointly over the antennas, no "directed" approach is possible in this case.

Another strategy is to separate the search over the permutations and the weights/shifts. A promising procedure is to perform dPTS with respect to the weights/temporal shifts (dPTS-wts) and repeat this optimization with different spatial permutations (PTS-sp). Using J_p (randomly selected) permutations and (on the average) $J_{\mathrm{b}\delta}$ combinations of weights/shifts, the total number of average candidates per antenna is given by $J = J_\mathrm{p} \cdot J_{\mathrm{b}\delta}$. In Algorithm 2, a pseudocode description of this *iterated spatially permuted and weighted/temporally shifted PTS (iPTS-spwts)* is given. Main advantage of this variant is its dramatically increased number of maximum possible candidates, allowing for much higher numbers of (average) candidates than the pure (weighting, shifting, or permuting) variants. In turn, better performance can be achieved at the price of additional complexity. The redundancy of iPTS is given by the sum of the redundancies of dPTS and PTS-sp. Hence, in total $N_\mathrm{T}\lceil \log_2(N_\mathrm{T}(J_{\mathrm{b}/\delta/\mathrm{b}\delta} - 1) + 1) \rceil + \lceil \log_2(J_\mathrm{p}) \rceil$ bits of side information have to be transmitted.

4.4. Numerical results

To evaluate the performance of the different PAR reduction techniques numerical simulations were conducted. The performance measure is the ccdf which gives the probability that the PAR of an OFDM frame exceeds a certain threshold PAR_0. As usual, transmission of side information is not considered in the following.

In the top of Figure 2, we compare the ccdf in case of no PAR reduction with that of ordinary, simplified, and directed PTS. All these schemes base on the original weighting (phase) variant. The plot shows the behavior for a different number of transmit antennas ($N_\mathrm{T} = 2, 4, 8$) for $J = 8$ superpositions per antenna. Each OFDM frame is divided into $V = 4$ partial sequences (adjacent carriers are combined into the partial sequences, i.e., block partitioning is used). As reference the results for a single antenna system are also given (gray dotted for no PAR reduction and gray solid for PTS with $J = 8$).

Compared to the situation with no PAR reduction, all reduction schemes are able to reduce the peak power significantly. (The values of PAR_0 at a clipping probability of

given:

N_T, V, $J_{\mathrm{b}/\delta/\mathrm{b}\delta}$, J_p, $[\mathrm{perm}_{1,1}(x),\ldots,\mathrm{perm}_{1,V}(x),\ldots,\mathrm{perm}_{J_\mathrm{p},1}(x),\ldots,\mathrm{perm}_{J_\mathrm{p},V}(x)]$ and

$[\mathbf{b}^{(1)},\ldots,\mathbf{b}^{(N_\mathrm{T}(J_\mathrm{b}-1)+1)}]$ or $[\boldsymbol{\delta}^{(1)},\ldots,\boldsymbol{\delta}^{(N_\mathrm{T}(J_\delta-1)+1)}]$ or $[[\mathbf{b}^{(1)},\boldsymbol{\delta}^{(1)}],\ldots,[\mathbf{b}^{(N_\mathrm{T}(J_{\mathrm{b}\delta}-1)+1)},\boldsymbol{\delta}^{(N_\mathrm{T}(J_{\mathrm{b}\delta}-1)+1)}]]$

generate V disjoint parts $\overline{\mathbf{A}}_{\mu,1},\ldots,\overline{\mathbf{A}}_{\mu,V}$ of $\mathbf{A}_\mu$, $\mu=1,\ldots,N_\mathrm{T}$

$\overline{\mathbf{a}}_{\mu,\nu} := \mathrm{IDFT}\{\overline{\mathbf{A}}_{\mu,\nu}\}$, $\nu=1,\ldots,V$ and $\mu=1,\ldots,N_\mathrm{T}$

function $[\mathbf{a}_1,\ldots,\mathbf{a}_{N_\mathrm{T}}] = \mathrm{iPTS}([\overline{\mathbf{a}}_{1,1},\ldots,\overline{\mathbf{a}}_{1,V},\ldots,\overline{\mathbf{a}}_{N_\mathrm{T},1},\ldots,\overline{\mathbf{a}}_{N_\mathrm{T},V}])$

$\mathrm{PAR}_\mathrm{max} := \infty$

for $\nu=1,\ldots,J_\mathrm{p}$

$\widetilde{\mathbf{a}}_{\mu,\nu} := \overline{\mathbf{a}}_{\mathrm{perm}_{\nu,\nu}(\mu),\nu}$, $\mu=1,\ldots,N_\mathrm{T}$, $\nu=1,\ldots,V$

$[\mathbf{a}_{\mathrm{new},1},\ldots,\mathbf{a}_{\mathrm{new},N_\mathrm{T}}] = \mathrm{dPTS}([\widetilde{\mathbf{a}}_{1,1},\ldots,\widetilde{\mathbf{a}}_{1,V},\ldots,\widetilde{\mathbf{a}}_{N_\mathrm{T},1},\ldots,\widetilde{\mathbf{a}}_{N_\mathrm{T},V}])$

calc. PAR_μ of $\mathbf{a}_{\mathrm{new},\mu}$, $\mu=1,\ldots,N_\mathrm{T}$, $\mathrm{PAR}_\mathrm{new} := \max_{\forall\mu=1,\ldots,N_\mathrm{T}} \mathrm{PAR}_\mu$

if $(\mathrm{PAR}_\mathrm{new} < \mathrm{PAR}_\mathrm{max})$

$\mathrm{PAR}_\mathrm{max} := \mathrm{PAR}_\mathrm{new}$

$[\mathbf{a}_1,\ldots,\mathbf{a}_{N_\mathrm{T}}] = [\mathbf{a}_{\mathrm{new},1},\ldots,\mathbf{a}_{\mathrm{new},N_\mathrm{T}}]$

endif

endfor

Algorithm 2: Pseudocode description of iterated PTS.

10^{-5} are approximately 12.6 dB, 12.8 dB, and 13 dB for $N_\mathrm{T} = 2, 4, 8$.) Evidently, sPTS performs worse than oPTS as less combinations of the weighting factors are utilized. For high values of PAR_0 the difference between sPTS and oPTS gets smaller. Both reduction schemes perform worse than PTS in the single antenna case and for an increasing number of transmit antennas N_T the results get even worse. This reflects the fact that simplified and ordinary PTS are just a simple application of single antenna PTS to a multiantenna transmitter. In contrast to that, the "directed" approach from Section 4.1 is able to exploit the multiple transmit antennas; dPTS always outperforms single antenna PTS and the performance gets even better for increasing N_T.

The above results are in perfect agreement with the ones of sSLM, oSLM, and dSLM [8, 19]. In [19] it has been shown that the ccdf of dSLM exhibits a steeper decay if the number of transmit antennas is increased, whereas the slope of oSLM remains constant. The same effect can be observed here, too, where oPTS has always the same decay independent of the number of transmit antennas. In case of dPTS the ccdf curves get steeper.

The middle plots of Figure 2 show performance results of the different PTS schemes based on temporal shifting and weighting/temporal shifting of the partial sequences. Basically, the results are equal to that described above. But the performance of the temporal shifting variant is better than that for the original (phase) variant, and combined weighting/temporal shifting performs best. This effect, although not fully understood yet, has already been observed in [20]. Hence, in the following, we concentrate on the weighting/temporal shifting variant.

A hint, why cyclic shifting offers better results, can be obtained when considering small DFT sizes and small numbers of partial sequences and allowed phases/shifts. For example, for $D = 4$, $V = 2$ (carrier $d = 0$ and 1 are combined and 2 with 3), BPSK signaling and 2 phases/shifts ($+1$, -1/no shift, shifting by 2 positions), assessment of all $2^4 = 16$ OFDM frames reveals that in case of weighting a maximum PAR of 3 dB occurs. In case of shifting, the worst case PAR is 0 dB. One particular example is given by $\overline{\mathbf{A}}_1 = [1, 1, 0, 0]$ and $\overline{\mathbf{A}}_2 = [0, 0, 1, 1]$. Since $\overline{\mathbf{a}}_1 = [0.5, 0.25{+}0.25\mathrm{j}, 0, 0.25{-}0.25\mathrm{j}]$ and $\overline{\mathbf{a}}_2 = [0.5, -0.25-0.25\mathrm{j}, 0, -0.25+0.25\mathrm{j}]$, the best weighted combination is $\overline{\mathbf{a}}_1 - \overline{\mathbf{a}}_2 = [0, 0.5{+}0.5\mathrm{j}, 0, 0.5{-}0.5\mathrm{j}]$ with a PAR of 3 dB. In case of shifting $\overline{\mathbf{a}}_1 + \mathrm{cycs}(\overline{\mathbf{a}}_2, 2) = [0.5, 0.5\mathrm{j}, 0.5, -0.5\mathrm{j}]$ with a PAR of 0 dB. Similar results are possible for larger D and V and QPSK.

Numerical results of the PTS-sp scheme are compared in the bottom plot of Figure 2. In the considered range of PAR_0 this variant of MIMO PTS performs worse than single antenna PTS-ts. Up to a PAR_0 value of about 9.5 dB the PAR reduction performance gets worse for an increasing number of transmit antennas. Due to the different slopes of the curves, there is an intersection point where this behavior reverses. It may be stated that PTS-sp is also able to exploit the multiple transmit antennas in order to reduce the PAR. However, the performance of this scheme is relatively bad compared to the other PTS variants.

Next, we turn to the hybrid PTS variants. In Figure 3, different variants of dPTS are compared. As above, each OFDM frame ($D = 512$) is divided into $V = 4$ partial sequences (block partitioning). $J = 4$, 8, and 16 (randomly selected) superpositions are assessed, respectively. For each value of J and in the region of clipping probabilities greater than 10^{-5}, PTS-spwts performs worst, followed by dPTS-w and dPTS-ts (temporally shifting is slightly better), and dPTS-wts performs best. Hence, the directed approach (Algorithm 1) proves again to be most powerful, and the increased number of freedoms due to combined weighting and shifting can be utilized gainfully. Interestingly, the PTS-spwts variant (where no directed approach is possible) shows a slightly steeper decay than the other one. This variant seems to be able to use the multiple antennas in the same way as dSLM (achieving some form of "diversity gain"). However, only for very low clipping probabilities, an advantage can be gained.

The performance of the iterated hybrid PTS variants is compared in Figure 4. For reference, oPTS-ts (worst PAR

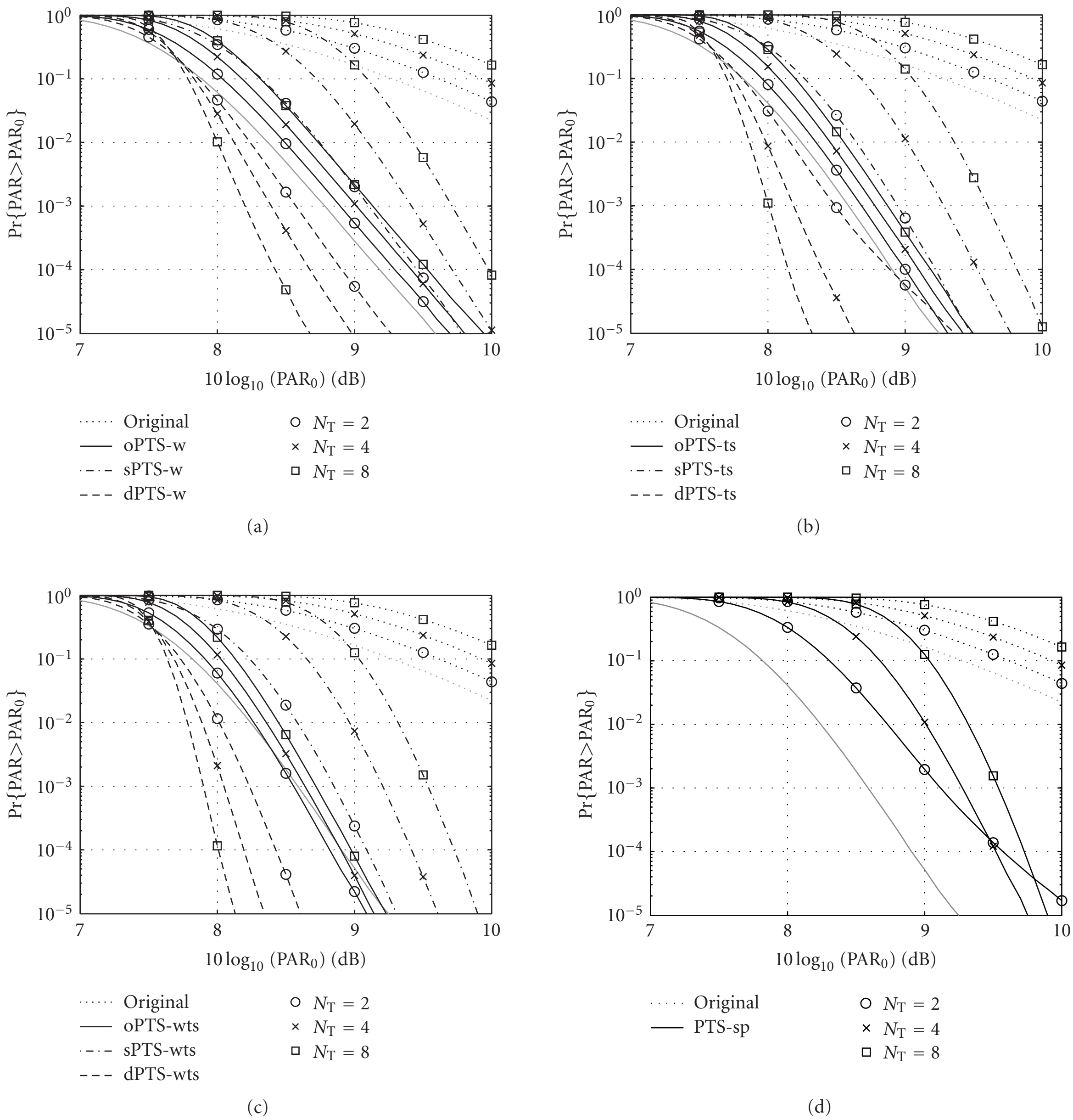

FIGURE 2: Comparison of the ccdf of original (a), temporal shifted (b), weighted and temporally shifted (c), and permuted (d) PTS. MIMO systems with $N_T = 2$ (∘), $N_T = 4$ (×), and $N_T = 8$ (□) transmit antennas. Average number of superpositions $J = 8$, and number of partial sequences $V = 4$. The required number of bits of side information reads oPTS 6, 12, 24; sPTS 6, 12, 24; dPTS 8, 20, 48; PTS-sp 3, 3, 3 (for $N_T = 2, 4, 8$). As reference the single antenna case is plotted in gray with no PAR reduction (dotted) and PTS (solid).

reduction), dPTS-wts (best performance), and PTS-spwts are shown as well. The iterated PTS approaches either use $J_p = 2$ or 4 (randomly selected) spatial permutations. Since the (average) number of superpositions per antenna is fixed, the number of weighting/shifting vectors is equal to $J_{b\delta} = 4$ or 2 ($J = 8$) and $J_{b\delta} = 16$ or 8 ($J = 32$). Unfortunately, these approaches are not able to reach the performance of pure directed PTS with weighting and temporal shifting of the partial sequences. However, choosing J large, the maximum number of possible combinations J_{max} will not be sufficient to perform dPTS solely using weighting and temporal shifting (dPTS-wts is only possible up to $J = 1024$). Here, the (iterated) hybrid variants, which offer a much larger number of maximum possible candidates, are the best choice. Again, only for very low clipping probabilities, PTS-spwts will outperform the other variants.

In summary it can be stated that the directed approach using combined weighting and temporal shifting is the most powerful approach to PTS for multiantenna OFDM schemes if the number J of assessed superpositions is small. For large J, iterated PTS with spatial permutation and temporal shifting is an interesting alternative.

5. COMPARISON WITH SELECTED MAPPING

Besides PTS, *selected mapping* (SLM) is another popular PAR reduction method. The fundamental idea of PTS and SLM is very similar: several alternative signal representations are calculated from the initial information carrying OFDM frame. The one exhibiting the lowest PAR is selected for transmission. The number, U, of alternative signal representations directly corresponds to PAR reduction performance. In this section, we compare the performance of PTS and SLM and point out their differences with respect to computational complexity. (According to [29], we concentrate on complex multiplications as complexity measure. In addition to that, the number of complex additions is considered, too.)

In principle, the complexity analysis holds for every PTS and SLM approach (ordinary, simplified, or directed). Since directed PTS/SLM performs best, subsequently we will concentrate on this approach.

In case of PTS, the computational effort per transmit antenna consists of the IDFTs (always assumed to be implemented as fast Fourier transform (FFT) [27]) of the V partial sequences, the J superpositions of all partial sequences, and the calculation of the PARs (metric) for selection. The complexity of PTS, normalized per transmit antenna, is then given as

$$c_{\text{PTS}} = V \cdot c_{\text{FFT}} + J \cdot (c_{\text{sp}} + c_{\text{met}}). \tag{7}$$

According to [27], the complexity of the FFT algorithm sums up to $(D/2)\cdot\log_2(D)$ complex multiplications and $D\log_2(D)$ complex additions. Counting each complex addition as two real additions and each complex multiplication as four real multiplications and two real additions, the numbers of real-valued operations account to

$$c_{\text{FFT}} = \begin{cases} 2D\log_2(D) & \text{mult.,} \\ 3D\log_2(D) & \text{add.} \end{cases} \tag{8}$$

To calculate the superpositions of the partial sequences no multiplications are necessary but the number of additions are given by

$$c_{\text{sp}} = \begin{cases} 0 & \text{mult.,} \\ 2D(V-1) & \text{add.} \end{cases} \tag{9}$$

Weighting of the partial sequences does not contribute to complexity, as multiplication by $\{\pm 1, \pm \mathrm{j}\}$ does only result in a change of sign or in an exchange of real and imaginary parts. Temporal shifting or spatial permutation of the partial sequences does also not require any arithmetic operation.

For obtaining the PAR (metric), the quotient of infinity norm (peak power) and Euclidean norm (average power) of the considered OFDM frame has to be calculated. Assuming 4-QAM per carrier, average power is constant for each candidate, as neither phase modification nor shifting or permutation changes this quantity. Hence, only peak power has to be evaluated, which requires $2D$ real multiplication and D real additions (calculation of the squared magnitudes of the time-

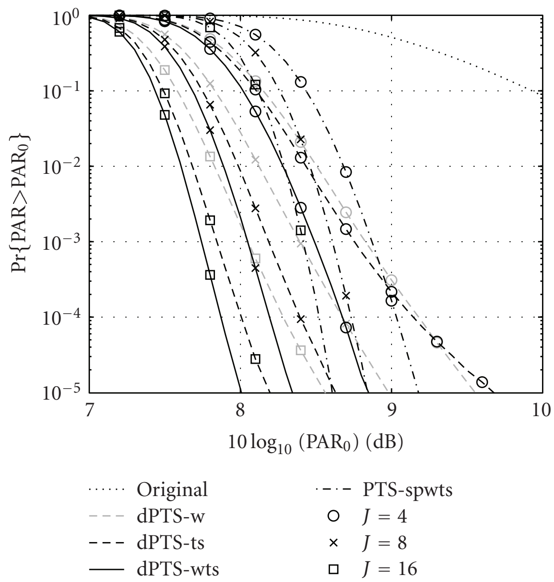

Figure 3: Comparison of the ccdf of PTS variants. Dashed: directed PTS with weighting (PTS-w) and temporal (cyclic) shifting (PTS-ts) of the partial sequences; dash-dotted: PTS with spatial permutation and weighting/temporal shifting (PTS-spwts); solid: dPTS with weighting and temporal shifting (PTS-wts). $N_{\text{T}} = 4$ transmit antennas; $V = 4$ partial transmit sequences (per antenna); average number of superpositions $J = 4, 8, 16$. Required number of side information bits: dPTS 13, 29, 61; PTS-spwts 8, 12, 16 (for $J = 4, 8, 16$).

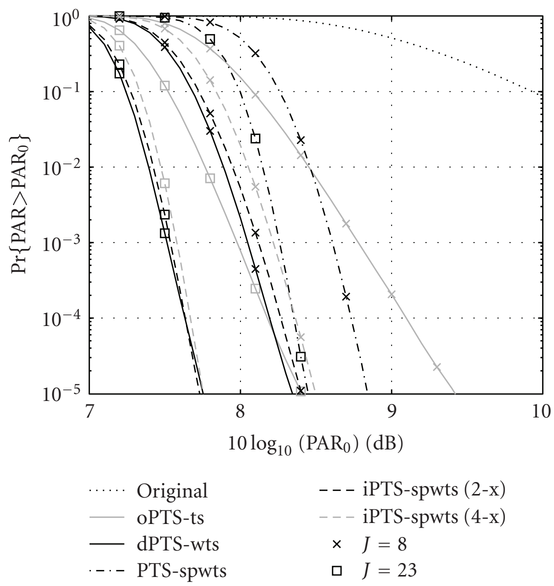

Figure 4: Comparison of the ccdf of PTS variants. Solid: oPTS and dPTS with temporal (cyclic) shifting (o/dPTS-ts); dash-dotted: PTS with spatial permutation and weighting/temporal shifting (PTS-spwts); dashed: iterated PTS with spatial permutation and weighting/temporal shifting (iPTS-spwts); $N_{\text{T}} = 4$ transmit antennas; $V = 4$ partial transmit sequences (per antenna); average number of superpositions $J = 8, 32$. Required number of side information bits: oPTS 12, 20; dPTS 20, 28; PTS-spwts 12, 32; iPTS (2-x) 17, 25; iPTS (4-x) 14, 26 (for $J = 8, 32$).

domain samples); no arithmetic operations are required for finding the largest value. Hence, the complexity is

$$c_{\text{met}} = \begin{cases} 2D & \text{mult.,} \\ D & \text{add.} \end{cases} \quad (10)$$

If, for example, for larger constellations, average power is also of importance, spending $D - 1$ real additions this quantity may immediately be obtained from the squared magnitudes. Via one additional division, PAR may then be calculated.

The computational effort of SLM consists of U calls of the FFT algorithm. As the resulting signals are the alternative signal representations only the metric calculations have to be done. In this case the complexity per antenna is given by

$$c_{\text{SLM}} = U \cdot (c_{\text{FFT}} + c_{\text{met}}). \quad (11)$$

The top row of Figure 5 compares dPTS (the phase, temporal shifting, and combined weighting/temporal shifting variants) using $V = 4$ partial sequences and $J = 16$ superpositions with dSLM [8] using $U = 4$ alternative signal representations. The computational complexity due to the FFTs is the same in both cases. As dPTS takes more different signal representations into account ($J > U$) and each candidate contributes to complexity, computational effort is higher than that of dSLM. However, due to the increased number of candidates, dPTS (especially the combined weighting/temporal shifting variant) performs much better than dSLM.

For a fair comparison of both methods, the number U of alternative signal representations in dSLM should be chosen such that the number of multiplications is (almost) the same in dPTS and dSLM. Using again $V = 4$ and $J = 16$ in dPTS, $U = 6$ candidates may be studied in dSLM (cf. middle row of Figure 5). In the relevant range of PAR_0, dPTS still outperforms dSLM slightly. However, as the slope of the dSLM curve is slightly larger than that of dPTS, an intersection below clipping probabilities 10^{-5} will occur.

Following [19], the directed approach gives a ccdf according to

$$\Pr\{\text{PAR} > \text{PAR}_0\} = \left(1 - \left(1 - \mathrm{e}^{-\text{PAR}_0/\Delta}\right)^D\right)^{N_{\text{T}}C}, \quad (12)$$

where C gives the slope of the curve and Δ represents a horizontal shift. (Due to the central limit theorem, the partial sequences $\bar{\mathbf{a}}_{\mu,\nu}$ are almost Gaussian distributed. However, since the partial sequences are not superimposed in a controlled way, the samples of the actual transmit sequence are no longer Gaussian. Hence, contrary to the SLM cases [4, 19, 30] it is not easily possible to derive an exact analytical expression for the ccdf of PTS. Nevertheless, Gaussian samples are assumed in deriving the ccdf and the approximation from [19] is used.) Based on a large number of simulations, we conjecture that given the number V of partial sequences and number J of candidates, for PTS the slope may well be approximated by

$$C = \frac{V}{2} \cdot \sqrt{\frac{J}{2}}. \quad (13)$$

For reference, this theoretical curve for choosing (on average) among $C = 5.66$ independent candidates ($V = 4$, $J = 16$) is included, as well (gray). Interestingly, dPTS-wts exhibits this theoretical performance (here, Δ is slightly larger than one; the theoretical curve is plotted for $\Delta = 1$).

On the bottom of Figure 5, PTS and SLM are compared for an increased complexity. Only the combined phase/temporal shifted variant of dPTS is able to provide (on the average) $J = 64$ candidates. A comparable complexity in SLM is achieved for $U = 10$. Here, the gap between dSLM and dPTS (which achieves a performance of $C = 11.3$ independent candidates) is even larger. In summary, it can be stated that based on the same complexity, dPTS shows better performance than dSLM. The complexity is not primarily invested in calculating IDFTs as in SLM, but in metric calculations, hence PTS is able to assess a larger number of candidates, which in turn leads to the gain over SLM.

Looking only at the slope of the curve ((13), neglecting the horizontal shift), for given total complexity according to (7), an optimal exchange between V and J (and hence number of IDFTs and number of metric calculations) can be calculated. Straightforward optimization gives $J_{\text{opt}} = c_{\text{PTS}}/3(c_{\text{sp}} + c_{\text{met}})$ and $V_{\text{opt}} = 2c_{\text{PTS}}/3c_{\text{FFT}}$. For a total complexity of $c_{\text{PTS}} = 10^5$ and $c_{\text{sp}} + c_{\text{met}} = 1024$, $c_{\text{FFT}} = 9 \cdot 1024$ (multiplications), $J \approx 32$, $V \approx 8$, and $C = 14.47$ results, which shows a slight improvement over the above choice $J = 64$, $V = 4$. The simulation result together with the theoretical curve are also shown in Figure 5. For very low clipping probabilities this set of parameters indeed will provide slightly better performance.

6. CONCLUSIONS

In this paper, the application of partial transmit sequences for peak-to-average power ratio reduction in multiantenna point-to-point OFDM has been studied. In particular, the approaches (ordinary, simplified, and directed), recently introduced for selected mapping, have been transfered to PTS. Unfortunately, the PAR problem in OFDM gets worse where more transmit antennas are present; oPTS and sPTS also suffer from this problem. In contrast, the directed approach to PTS (dPTS) is able to utilize the multiple antennas, that is, employing more transmit antennas, better PAR reduction performance can be achieved. As in SLM, a sort of diversity gain can be achieved with respect to the ccdf of PAR.

One problem in dPTS is that this approach has to keep ready a higher number of alternative signal representations (increased by the number of antennas compared to oPTS/sPTS). Hence, performance is limited by the maximum number of candidates which can be generated. The presented solution is to combine different variants, in particular the original weighting with temporal shifting [20] and/or with spatial shifting/permutation [21]. For a very large number of desired candidates, iterated directed PTS has been introduced.

Spending the same complexity in PTS and SLM, it has been shown that PTS offers better performance, as this method is able to assess more candidates with a lower number of IDFTs.

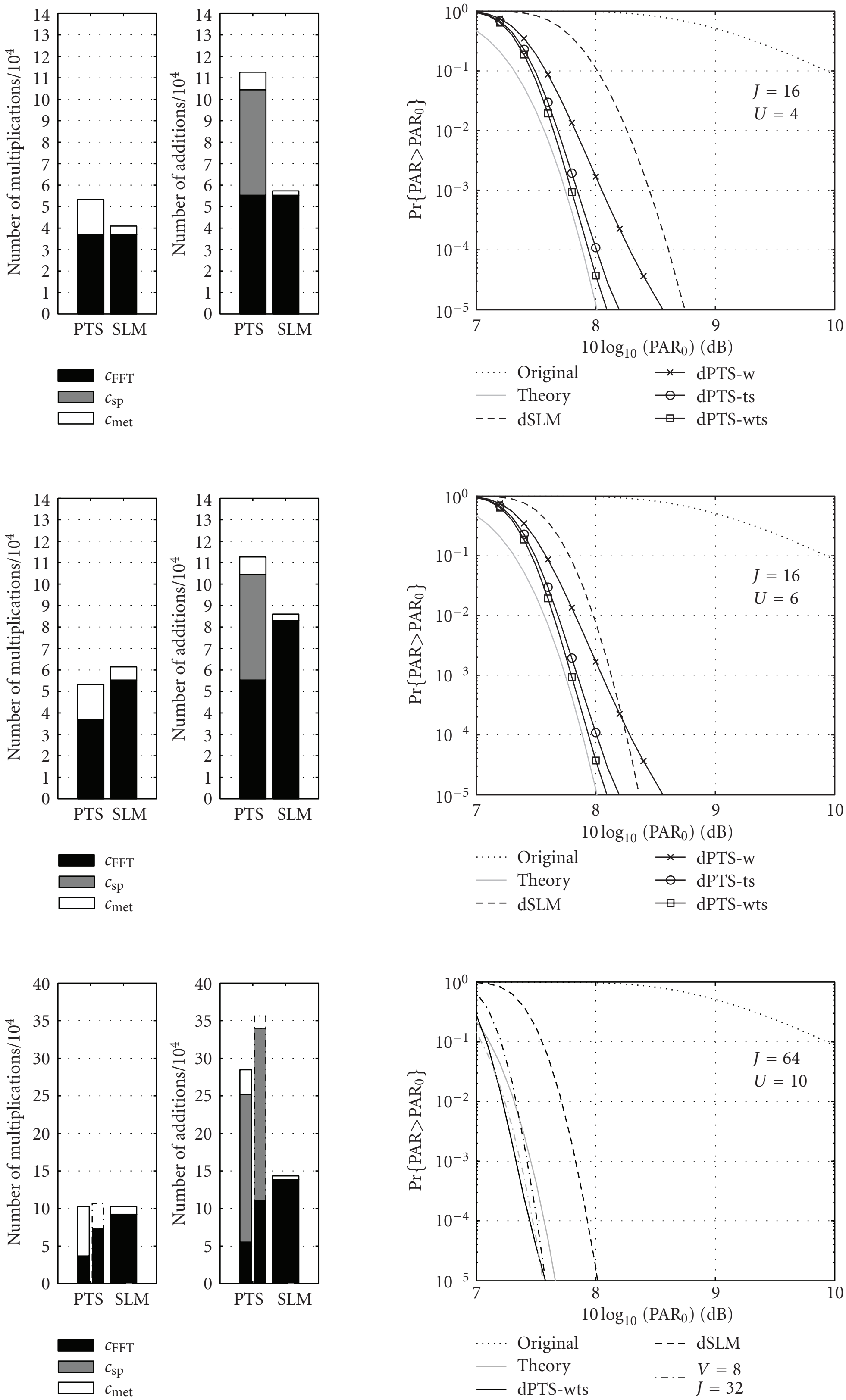

Figure 5: Comparison of dPTS (weighting, temporal shifting, and combined weighting/temporal shifting) and dSLM with respect to computational complexity (left; real multiplications and additions) and ccdf (right). $N_\mathrm{T} = 4$, $V = 4$; Top: $J = 16$, $U = 4$ (same number of IDFTs), required number of side information bits dPTS 24, dSLM 16; Middle: $J = 16$, $U = 6$ (approximately same complexity), required number of side information bits: dPTS 24, dSLM 20; Bottom: $J = 64$, $U = 10$ (approximately same complexity, here no pure weighting or temporal shifting variant is possible since $J_{\max}$ is to small), required number of side information bits: dPTS 32, dSLM 24; dash-dotted: $J = 32$, $V = 8$.

In summary it can be stated that due to its very good performance directed PTS using combined weighting and temporal shifting is a very attractive candidate for PAR reduction in future multiantenna OFDM schemes.

ACKNOWLEDGMENT

This work was supported by Deutsche Forschungsgemeinschaft (DFG) within the framework TakeOFDM under Grant FI 982/1-1.

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