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Research Article

Scalable Ad Hoc Networks for Arbitrary-Cast: Practical Broadcast-Relay Transmission Strategy Leveraging Physical-Layer Network Coding

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The capacity of wireless ad hoc networks is constrained by the interference of concurrent transmissions among nodes. Instead of only trying to avoid the interference, physical-layer network coding (PNC) is a new approach that embraces the interference initiatively. We employ a network form of interference cancellation, with the PNC approach, and propose the multihop, broadcast-relay transmission strategy in linear, rectangular, and hexagonal networks. The theoretical analysis shows that it gains the transmission efficiency by the factors of 2.5 for the rectangular networks and 2 for the hexagonal networks. We also propose a practical signal recovery algorithm in the physical layer to deal with the influence of multipath fading channels and time synchronization errors, as well as to use media access control (MAC) protocols that support the simultaneous receptions. This transmission strategy obtains the same efficiency from one-to-one communication to one-to-many. By our approach, the number of the users/terminals of the network has better scalability, and the overall network throughput is improved.

1. INTRODUCTION

In wireless communication, a node may broadcast information through the electromagnetic (EM) waves to all of its neighboring nodes. At the same time, a node may receive several signals simultaneously sent from its neighbors. Due to the additive nature of the EM waves, information cannot be recovered from these scrambled signals correctly without appropriate protocols. This is a problem called multiple access interference (MAI). A similar problem is illustrated in the pioneering work of Gupta and Kumar [1], from which it can be concluded that the capacity of wireless ad hoc networks is constrained by the mutual interference of concurrent transmissions among nodes (i.e., the MAI problem). When the number of nodes in a distributed ad hoc network gets larger, information is transmitted through a "multihop" method from the source node to the sink nodes. As a result, the opportunity of such a problem is additionally increased. Hence, many researchers attempt to find new approaches to boost the network capacity. Network coding (NC) [2] is a new method in information theory which allows nodes to combine several input packets into one or several output packets, instead of simply forwarding them. Combining multiple information flows into one flow has the potential to save the system resources, promote the network capacity, and bring better robustness. Li et al. [3] proved that linear combination is sufficient for multicast, while Koetter and Medard [4] gave an algebraic approach to network coding. Furthermore, Wu et al. [5] utilized the broadcast characteristic of wireless communication for network coding and Fragouli et al. [6] addressed how NC can be used in practice. Their transmission scheduling scheme assumes that signals are received separately and then taken into a linear operation. Due to the mutual interference of concurrent transmissions as stated above, however, in a wireless network, NC does not change the key problem, which is the MAI problem that constrains the network capacity. Moreover, Liu et al. [7] recently obtained the result that NC cannot increase the order of the network throughput for multipair unicast case when nodes are half-duplex in wireless networks (similar result is obtained by Li and Li [8], in which it is shown that NC has no throughput gain for unicast and broadcast case to wired networks, and can only provide at most twice the throughput with no NC in an undirected graph). Therefore, using NC alone does not demonstrate all of the potentials of a wireless network.

In contrast to the investigations ([9], etc.) conducted on how to avoid or reduce the MAI problem (e.g., the RTS/CTS strategy [10], in 802.11), physical-layer network coding (PNC) [11], which makes use of the additive nature of simultaneously received signals (regardless of whether it is within the plan or there is a collision), is a new effective approach to solve the MAI problem and attain more transmission throughput. Despite the authors [11] assumption that all the transmission and reception are ideally synchronized and without any interference, which is hard to implement, they advanced the innovation of turning the MAI into an extra throughput gain and opened up a new research area because of its new implementation and design requirements for the physical, MAC, and network layers of ad hoc wireless stations. Some related works are as follows.

Zhang et al. [12] showed that synchronization is not an issue in the three-node-network case, which gives us an inspired positive result that supports PNC in practice. However, they only analyzed the BPSK modulation. They did not consider the channel influence which will badly change the shape of the signals. Moreover, in their discussion, time synchronization errors not only decrease the desired signal power, but also introduce an intersymbol interference (ISI). They restricted the time synchronization error Δt_s by $\Delta t_s < T_s/2$, where T_s is the sampling interval, so that the signal-over-interference-and-noise ratio (SINR) decreases slightly. However, this assumption is the subject of debate, that is, if the time synchronization error becomes larger, which would probably occur in distributed wireless ad hoc networks because nonneighboring nodes do not know the status of each other accurately, then the performance of this simultaneous reception will deteriorate severely.

Traditional signal recovery methods are not suitable for PNC because the receiver nodes will get the mix of more than one signal simultaneously, so more recent investigations turned to the new signal recovery algorithm. Fan et al. [13] introduced a unidirectional transmission strategy by using PNC, where their signal recovery method are based on signal correlation. They analyzed the AWGN channel and obtained a precise signal recovery. However, wireless environments are more complex than the AWGN channel. If the wireless channel, such as a multipath fading channel, changes the shape of the mixed signals, their correlation analysis method will not work. In addition, if the information that the two mixed signals in [13] take is the same, it will embarrass the correlation analysis as well.

Since the nodes in PNC scheme will receive more than one signal simultaneously, the technique of cochannel interference cancellation is also concerned by this paper. In contrast to the traditional physical-layer interference cancellation (such as [14, 15], etc.), which cancels the cochannel interference by equalizer to increase the throughput of one communication channel between two nodes, in this paper, we employ a network form of interference cancellation with network coding approach to increase the throughput of the whole network. In our approach, we leverage the results on PNC, whose key insight is to *embrace the interference*, and we propose a new transmission strategy from the physical layer up to the network layer by making the nodes send or receive concurrently without avoiding the mutual interference. Our contributions are as follows.

In the physical layer, we extend the framework of PNC to the orthogonal frequency-division multiplexing (OFDM) setup. By extending the idea to OFDM, we resolve the problems of PNC in practice to decode in the air, such as the time synchronization problem with $\Delta t_s > T_s/2$ and the influence of multipath fading channel, because OFDM system works well over fading channels and is less sensitive to time synchronization errors than conventional systems. To compensate for the distorted signals, we use frequency-domain, phase-shifting orthogonal pilots to do the channel estimation. As a result, the influence of the channel's interferences and the signals' synchronization errors can be estimated simultaneously and released together. This physical-layer analysis is presented in Section 3.

As stated above, our transmission strategy takes advantage of PNC, and it deals with the simultaneous reception by decoding the mixed signals as well as canceling out the information in successive packets. To the best of our knowledge, the previous discussions of PNC transmission strategy are restricted to the three-node-network or linear unidirectional case, and the transmission models are for the unicast or bidirectional information exchange in a three-node-network. In order to extend our transmission strategy to a general wireless ad hoc network, taking physical-layer techniques alone is not sufficient. Hence, in this paper, the signal identification and access control protocols in the MAC layer to support the simultaneous transmissions are also considered. In comparison to the restriction of the previous works of PNC, we propose the broadcast-relay transmission strategy in linear, rectangular, and hexagonal networks, respectively, for any arbitrary-cast case, including unicast, multicast, and broadcast. This transmission strategy extends the concept and the application of PNC, and it is introduced in Section 4.

Furthermore, in traditional transmission strategies, the transmission efficiency may decrease when the number of sink nodes (users/terminals) increases, because the opportunity of the MAI problem among multiple transmission paths increases at the same time. Hence, the average throughput of each unicast pair is ordinally decreased from one-to-one communication to one-to-many in the same network. In contrast, our transmission strategy has the same transmission efficiency when the number of transmission paths increases, that is to say, if the network topologies (such as linear/rectangular/hexagonal networks) are kept, regardless of the number of the users/terminals, our transmission strategy is scalable for the unicast, multicast, and broadcast cases! Aside from this, the performance of the multiple signal reception and recovery techniques in our transmission strategy can meet the performance of conventional OFDM systems for single-signal reception. The approach conjectured in [16] to gain the highest attainable capacity that combines multiple packet reception (MPR) and NC together

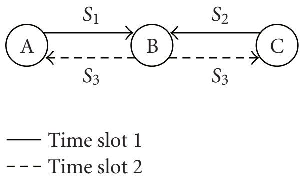


FIGURE 1: Physical-layer network coding.

may therefore become truly practical by our transmission strategy. This result is addressed in Section 5.

2. MODEL AND PRELIMINARIES

2.1. Physical model

Our basic physical-layer network coding transmission model is shown in Figure 1. In time slot 1, node A and node C transmit modulated signals S_1 and S_2, which take the information a and b, respectively. Different from the straightforward network coding scheme, at this time node B operates on the mixed signals $S_1 \dotplus S_2$, and in time slot 2 broadcasts a remapping result, such as $a \oplus b$, in the analog signal S_3 to node A and node C (node B gets this remapping result by decoding the interfered signals and re-encoding a new packet [11]). Then, the receiver nodes A and C decode S_3 by using their own knowledge of a and b to get the new information b and a, respectively. Specifically, it is supposed that nodes will send and receive signals in the two time slots, respectively. In the sending time slot, nodes broadcast signals to all their neighbors; and in the receiving time slot, nodes receive signals from all their neighbors, simultaneously. A node is a sending node if it works in its sending time slot, and is a receiving node if it works in its receiving time slot. Besides, if a node does not send or receive any signal, we call it in idle status. In this paper, different from the conventional operation "+", functor "$\dotplus$" in $S_1 \dotplus S_2$ represents the addition of two signals' EM waves. In particular, in Section 4, the addition of the two signals that take the information x_1 and x_2 are represented as $x_1 \dotplus x_2$.

In [11], it is required that nodes are able to decode from the simultaneously received signals. Therefore, the arriving time of signals S_1 and S_2 should be precisely synchronized, and the shape, including the amplitude and the phase of the signals in each sampling time, cannot be changed. However, in the wireless environment, the channels of node A to B and node C to B may be different multipath fading channels. In addition, there may be a delay between the arriving time of the signals S_1 and S_2. In this situation, OFDM technology has an advantage because it is designed for anti-multipath interference. Moreover, by inserting cyclic prefix (CP) for the guard interval, the OFDM system becomes less sensitive to the time offset than conventional systems because time synchronization errors do not violate the orthogonality of transmitted waveforms, which differ from the case discussed in [12]. In this paper, as we will consider the influence of the signals and the channels on applying PNC and mainly use OFDM for the case to discuss the physical-layer technique, we describe the signals of the physical layer in both time-domain and frequency-domain, for the OFDM technology converts them between the two domains.

In the transmitters of OFDM systems, the serial data $\{S_k\}$ in the frequency-domain is transformed into parallel data in order to perform the inverse fast Fourier transform (IFFT), and then the result is reverted into serial data $\{S_n\}$. Thus, we have

$$S_n = \frac{1}{N}\sum_{k=0}^{N-1} S_k e^{j2\pi nk/N}, \quad n, k = 0, 1, \ldots, N-1. \tag{1}$$

Denoting the data after CP (with length G) is inserted by $\{x_n\}$, the structure of one frame of the sequence $\{x_n\}$ from x_0 to x_{N+G-1} is $S_{N-G}, \ldots, S_{N-1}, S_0, \ldots, S_{N-1}$.

An equivalent expression is

$$x_n = \begin{cases} S_{N-G+n} & n \in [0, G-1], \\ S_{n-G} & n \in [G, N+G-1]. \end{cases} \tag{2}$$

If $x(t)$ is the analog signal which has passed through the D/A module, the signal after D/A can be expressed as

$$x(t) = \sum_{n=0}^{N+G-1} x_n p(t - nT_s), \tag{3}$$

where $p(t)$ is the pulse waveform, T_s is the time interval of sampling signal, and $T = NT_s$ is the time length of an OFDM frame.

For a common OFDM system (e.g., the system of IEEE 802.11, in which we can apply our transmission strategy), the length of CP is 1/4 of the length of an OFDM frame, that is, $T_{\mathrm{CP}} = GT_s = (1/4)T = (1/4)NT_s$, where the value of T_s depends on the transmission speed (the bandwidth). As stated in Section 1, in this paper we consider the delay between the arriving time of the signals S_1 and S_2 as $\Delta t_s > T_s/2$ and $\Delta t_s \sim nT_s$, $n < G$.

The receivers in OFDM systems will do the reverse transformation of the signals.

2.2. Network model

Since we will discuss the transmission strategy in linear, rectangular, and hexagonal networks for the unicast, multicast, and broadcast cases, for preliminaries, we present some basic definitions first.

Definition 1 (distance). The *distance* between two nodes is the minimum number of hops between these two nodes in an ad hoc network.

Definition 2 (distance-n-network). The *distance-n-network* is an ad hoc network with one source in which all the *distances* between the source node and the other nodes are not larger than n.

Definition 3 (full distance-n rectangular network). The *full distance-n rectangular network* is a rectangular *distance-n-network* that contains $2n(n+1)+1$ nodes. All the possible nodes with distance-n to the source in the rectangular network topology exist in this network.

Definition 4 (full distance-n hexagonal network). The *full distance-n hexagonal network* is a hexagonal *distance-n-network* that contains $3n(n+1)/2+1$ nodes. All the possible nodes with distance-n to the source in the hexagonal network topology exist in this network.

Definition 5 (transmission path). All the nodes that take part in one transmission from the source node to a sink node constitute the *transmission path*. The combination of all the transmission paths in a multicast case excludes the nodes which are always idle during one transmission and connects the source node and all the sink nodes in the network together; the combination of all the transmission paths in a broadcast case includes all the nodes in the network.

In this paper, the *broadcast case* in a distance-n-network means that there is one source node in a distance-n-network, and all the other nodes are the sink nodes, where the farmost sink node is distance-n away from the source node; the *multicast case* in a distance-n-network means that there is one source node and several sink nodes in a distance-n-network, where the farmost sink node is distance-n away from the source node; the *unicast case* means that there is one source node and one sink node which is distance-n away from the source node; and the *arbitrary-cast case* contains these three cases above.

The rest of the paper is organized as follows. The physical-layer techniques will be introduced in Section 3, where we will show the combination of PNC and OFDM, with the channel estimation methods. A unicast transmission strategy in a linear network with the consideration of channel influence and time synchronization error, which is a basic component of general multihop transmission strategy, will be presented as well. In Section 4, we will propose the broadcast-relay transmission strategy in rectangular and hexagonal networks for any arbitrary-cast with the MAC-layer protocols that support the simultaneous transmission. Some simulation results and discussions of performance and the trade-off between the transmission efficiency gain and the cost will be shown in Section 5. Finally, we conclude this paper in Section 6.

3. PHYSICAL LAYER: THE COMBINATION OF PNC AND OFDM

Zhang et al. [12] showed that synchronization is not an issue on applying PNC. However, as stated in Section 1, there still exist two problems in the physical layer:

(1) the influence of the signals' time offset: here we do not restrict the time offset (time synchronization error) by $\Delta t_s < T_s/2$, but let $\Delta t_s \sim nT_s$, where T_s is the sampling interval and n is an integer less than the length of CP;

(2) the influence of the channels: the shape of the signal will be badly changed by the channel, in particular, if in the fading channel, different, multiple copies of the signals arrive continuously, then the scrambled signals cannot be recognized without accurate compensations.

Deriving the influence of these two issues in the OFDM system, we first draw the conclusion in a former way in this theorem.

Theorem 1. *In the OFDM system, if two data sequences* $\{S_{1k}\}$ *and* $\{S_{2k}\}$ *are transmitted through two different multipath fading channels and are performed through simultaneous reception by one node with the time synchronization error* $\Delta t_s \sim nT_s$, *where* T_s *is the sampling interval and n is an integer less than the length of* CP, *the received mixed signals (without noise) can be expressed as*

$$R_k = S_{1k}H_1'(k) + S_{2k}H_2'(k), \tag{4}$$

where $H_1'(k)$ *and* $H_2'(k)$ *are the functions of the frequency-domain index k. The power of the noise does not change as well.*

In the following two subsections, we will prove this theorem.

3.1. Analysis for the time synchronization error

Consider the basic network unit shown in Figure 1. In time slot 1, node B receives two overlapped signals with length-N data and length-G CP, whose structure is shown in Figure 2. Let $\hat{T}_s$, $\hat{f}_c$, $\hat{\phi}$ be the estimations for the time interval T_s of the sampling signal, the carrier frequency f_c, and the carrier-phase ϕ, respectively. Since the synchronization errors of these three parameters are proven to be not an issue in [12] and we only have interest in the influence of the time synchronization error, here we suppose $\hat{T}_s = T_s$, $\hat{f}_c = f_{c1} = f_{c2}$, and $\hat{\phi} = \phi_1 = \phi_2$, and denote the time interval between the two signals by Δt_s. Thus, the received mixed signals are

$$\begin{aligned} y(t) &= \left[x_1(t)e^{(j2\pi f_{c1}t+\phi_1)} + x_2(t-\Delta t_s)e^{(j2\pi f_{c2}(t-\Delta t_s)+\phi_2)} + \eta(t)\right] \\ &\quad \cdot e^{-j(2\pi \hat{f}_c t+\hat{\phi})} \\ &= x_1(t) + x_2(t-\Delta t_s)e^{-j(2\pi \hat{f}_c \Delta t_s)} + \eta', \end{aligned} \tag{5}$$

where $\eta(t)$ is AWGN on simultaneous reception with $N_0/2$ as its double-sided noise power spectral density, and $\eta' = \eta(t)e^{-j(2\pi \hat{f}_c t+\hat{\phi})}$. It is more appropriate to use one noise term $\eta(t)$ than several noise terms $\eta_1(t)$, $\eta_2(t)$,... to represent the noise in the simultaneous reception for the following reasons. A receiver's noise is not only caused by the channel of the transmission, but also from the interferences caused by other nodes as well as the receiver itself. It seems that the receiver node receives two mixed signals from two different channels, however, the receiver node is in fact just to receive one signal which is interfered by the other signal. Therefore, in (5) and the following parts of this paper, we use one noise term $\eta(t)$ to represent all kinds of noise in the receiver system.

The segments of the received signals including data and CP will be taken by an FFT window (as shown in Figure 2, the parts of the signals within the window will be sampled and utilized, and the parts outside the window will be dropped). We denote the FFT window offsets by Δt_{f1} and Δt_{f2}, where $\Delta t_s = \Delta t_{f2} - \Delta t_{f1}$. Then, the received sequence $\{r_n\}$ $(n = 0, 1, \ldots, N-1)$ is given by

$$r_n = \left[x_1(t) + x_2(t)e^{-j(2\pi \hat{f}_c \Delta t_s)}\right]\Big|_{t=(n+G)\hat{T}_s - \Delta t_{f1}} + \eta'. \tag{6}$$

Consequently, FFT(N) will begin at the sampling position of $\Delta n_{f1} = \Delta t_{f1}/\hat{T}_s$ and $\Delta n_{f2} = \Delta t_{f2}/\hat{T}_s$ before the data segment for each signal, respectively. Because CP is inserted before the data, we have $r_{-1} = r_{N-1}, \ldots, r_{-G} = r_{N-G}$. Therefore, the received sequence after FFT is given by

$$R_k = \sum_{n=0}^{N-1} r\left((n - \Delta n_{f1}) \bmod N\right) e^{-j2\pi nk/N}) + \eta'', \tag{7}$$

where $\eta'' = \eta' e^{-j2\pi nk/N}$. Let $m = n - \Delta n_f$, then

$$\begin{aligned}
R_k &= \sum_{m=-\Delta n_{f1}}^{N-1-\Delta n_f} r_m\, e^{-j2\pi(m+\Delta n_{f1})k/N} + \eta'' \\
&= \sum_{m=-\Delta n_{f1}}^{-1} r_m\, e^{-j2\pi(m+\Delta n_{f1})k/N} + \sum_{m=0}^{N-1} r_m\, e^{-j2\pi(m+\Delta n_{f1})k/N} \\
&\quad - \sum_{m=N-\Delta n_{f1}}^{N-1} r_m\, e^{-j2\pi(m+\Delta n_{f1})k/N} + \eta''.
\end{aligned} \tag{8}$$

Because

$$\begin{aligned}
\sum_{m=N-\Delta n_{f1}}^{N-1} r_m\, e^{-j2\pi(m+\Delta n_{f1})k/N} &= \sum_{m=-\Delta n_{f1}}^{-1} r_m\, e^{-j2\pi(m+\Delta n_{f1}+N)k/N} \\
&= \sum_{m=-\Delta n_{f1}}^{-1} r_m\, e^{-j2\pi(m+\Delta n_{f1})k/N},
\end{aligned} \tag{9}$$

we have

$$\begin{aligned}
R_k &= e^{-j2\pi\Delta n_{f1}k/N} \sum_{m=0}^{N-1} r_m\, e^{-j2\pi mk/N} + \eta'' \\
&= A_{c1} S_{1k} e^{-j2\pi k\Delta n_{f1}/N} + A_{c2} e^{-j(2\pi \hat{f}_c \Delta t_s)} S_{2k} e^{-j2\pi k\Delta n_{f2}/N} + \eta'' \\
&= A'_{c1} S_{1k} e^{-j2\pi k\Delta n_{f1}/N} + A'_{c2} S_{2k} e^{-j2\pi k\Delta n_{f2}/N} + \eta'',
\end{aligned} \tag{10}$$

where $\Delta t_{f2} = \Delta t_{f1} + \Delta t_s$, A_{c1}, A_{c1}, are the amplitude coefficients of the two signals, respectively, and $A'_{c1} = A_{c1}$, $A'_{c2} = A_{c2} e^{-j(2\pi \hat{f}_c \Delta t_s)}$.

Indeed, the time synchronization error does not violate orthogonality of the symbols, and the power of the noise is not changed. For the time offsets, many methods such as ([17], etc.) are useful to deal with the phase rotation. Therefore, the influence of time-domain synchronization error can be estimated and compensated if the time offset $\Delta t_s \sim nT_s$ is less than the length of CP. Moreover, we will propose a channel estimation and signal recovery method in Section 3.3 in order to compensate for the infection of multipath fading channels and time synchronization errors, simultaneously, which do not need to do the time-offset estimation independently.

3.2. Analysis for the multipath fading channel

The OFDM system is designed for anti-multipath interference. After adding cyclic prefix extensions to each frame, the linear convolution becomes e quivalent to a circular convolution, which will greatly help us deal with the multipath interference.

To analyze the influence of the channel, we suppose that the channels of node A to B and node C to B in Figure 1 are different multipath fading channels. Without synchronization error, if the spread time of multipath signals is less than the time length of CP, then in time slot 1, the received signals of node B are given by

$$r_n = \sum_{l=1}^{2}\left(\sum_{i=1}^{P_l} \frac{m_{l,i}}{N} \sum_{k=0}^{N-1} x_l(k) e^{j2\pi k(n-\theta_{l,i})}\right), \tag{11}$$

where $l = 1$ is for receiving the signal S_1 and $l = 2$ is for S_2. The number of paths of signal l is denoted by P_l, with $m_{l,i}$, $\theta_{l,i}$ being the amplitude and phase coefficients of each path of the two signals, respectively.

After FFT, we have

$$R_k = \mathrm{FFT}\{r_n\} = S_{1k} H_1(k) + S_{2k} H_2(k), \tag{12}$$

where,

$$\begin{aligned}
H_1(k) &= \sum_{i=1}^{P_1} m_{1,i}\, e^{-j2\pi k\theta_{1,i}}, \\
H_2(k) &= \sum_{i'=1}^{P_2} m_{2,i}\, e^{-j2\pi k\theta_{2,i}}.
\end{aligned} \tag{13}$$

Now, we involve the time synchronization problem and the noise. If there exist time offsets, after FFT, a phase rotation will be added to the signal as stated in the last subsection. Denoting $H'_1(k) = H_1(k) A'_{c1} e^{-j2\pi k\Delta n_{f1}/N}$, $H'_2(k) = H_2(k) A'_{c2} e^{-j2\pi k\Delta n_{f2}/N}$, we get $R_k = S_{1k} H'_1(k) + S_{2k} H'_2(k) + \eta''$, where $H'_1(k)$, $H'_2(k)$ contain the phase rotation caused by the time-domain offsets of applying PNC. Therefore, Theorem 1 has been proven.

As a result, if we get the estimation of $H'_1(k)$, $H'_2(k)$ and get enough (at least two) independent linear combinations of the two signals, we can compensate for the influence of the multipath fading channels and the synchronization error together and recover the data. This signal recovery can be put in any kind of nodes. For example, in Figure 1, node B does not need to recover the two signals and only the end nodes (A and C) will do the signal recovery. However, in our broadcast-relay transmission strategy which will be shown in Section 3.4 and the next section, all the nodes including the relay nodes and the end nodes will recover the unknown data by the information in successive packets.

Moreover, because the deduction above does not lie on the number of the signals, we have the following.

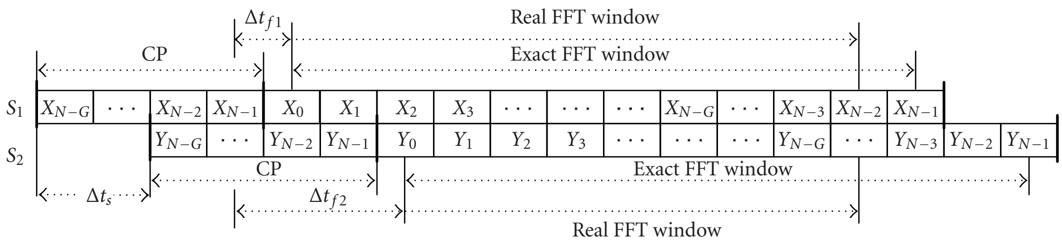


Figure 2: Structure of the mixed signals.

Corollary 1. *In the* OFDM *system, if serial data* $\{S_{1k}\}, \{S_{2k}\}, \ldots, \{S_{Lk}\}$ *are transmitted through L different multipath fading channels and are performed through simultaneous reception by one node with the maximum time synchronization error* $\Delta t_{s(\max)} \sim nT_s$*, where* T_s *is the sampling interval and n is an integer less than the length of CP, the received mixed signals can be expressed as*

$$R_k = \sum_{l=1}^{L} S_{lk} H_l'(k), \tag{14}$$

where each $H_l'(k)$ *is the function of the frequency-domain index k.*

Proof. For the influence of time synchronization error, each signal S_{lk} is transformed into $A_{cl}' S_{lk} e^{-j2\pi k \Delta n_{fl}/N}$, and for the influence of multipath channel, it is $S_{lk} H_l(k)$, so finally each signal will be transformed into $S_{lk} H_l'(k) = S_{lk} H_l(k) A_{cl}' e^{-j2\pi k \Delta n_{fl}/N}$, respectively. The addition of all the signals are $R_k = \sum_{l=1}^{L} S_{lk} H_l'(k)$. □

3.3. Channel estimation algorithm

Channel estimation may help us recover the signals. However, by conventional methods, we cannot simultaneously get $H_1'(k)$ and $H_2'(k)$, respectively, that is why we choose orthogonal pilot sequences for channel estimation. An nth element of a length-P *Chu* sequence, which has constant amplitude in the frequency-domain as pilot, is given by [18]

$$c_n = \begin{cases} e^{j\pi q n^2/P}, & P = \text{even}, \\ e^{j\pi q n(n+1)/P}, & P = \text{odd}, \end{cases} \tag{15}$$

where q is relatively prime to P. For the two signals reception scheme, we suspend a length-$N/2$ *Chu* sequence behind itself to form a length-N pilot sequence of one node. Consequently, in the frequency-domain, it equals to 0 on even subcarriers (as shown in Figure 3(a)). Furthermore, we use a shifting sequence as another node's pilot, which is 0 on odd subcarriers in the frequency-domain.

The received signals of the mixed pilot frames after the multipath channel are $R_k = P_{1k} H_1(k) + P_{2k} H_2(k)$. As stated above, P_{1k} and P_{2k} are orthogonal *Chu* sequences in the frequency-domain. Therefore, after removing CP, $\widehat{H}_1(k) = R_k/P_{1k}$ is the estimated value of $H_1(k)$ for even k, and $\widehat{H}_2(k) = R_k/P_{2k}$ is for $H_2(k)$ on odd k. If there exists a time offset between the two signals, we have $\widehat{H}_1(k) = R_k/P_{1k} = H_1'(k) = H_1(k) A_{c1}' e^{-j2\pi k \Delta n_{f1}/N} (k = \text{even})$, and $\widehat{H}_2(k) = R_k/P_{2k} = H_2'(k) = H_2(k) A_{c2}' e^{-j2\pi k \Delta n_{f2}/N} (k = \text{odd})$, respectively, which is the estimation in the frequency-domain while in the time-domain it is a circular shifting of the original channel (as shown in Figure 3(b)). To generate the other half of index k of the estimation, we can do the interpolation (as shown in Figure 3(c)) by

$$\begin{aligned} \widehat{H}_1(k)\big|_{k=0,\ldots,N-1} &= F_N W_{N/2} F_N^{-1} \widehat{H}_1(k)\big|_{k=\text{even}}, \\ \widehat{H}_2(k)\big|_{k=0,\ldots,N-1} &= F_N W_{N/2} F_N^{-1} \widehat{H}_2(k)\big|_{k=\text{odd}}, \end{aligned} \tag{16}$$

where F_N is a normalized DFT-N matrix and $W_{N/2}$ is a length-$N/2$ rectangular windowing vector. In particular, the algorithm can be further described as follows:

(1) *obtain an initial channel estimate,*
(2) *convert the channel estimate into the time domain,*
(3) *convert the first length-N/2 sequence of this time-domain signal back into the frequency domain.*

We insert several of this kind of pilot frames into the data transmission dispersively and let the time interval of two pilot frames be less than the channel's coherence time. As a result, the channel can be recognized as a time invariable channel during the interval between two pilot frames. By the method stated above, all the k-index $H_1'(k)$ and $H_2'(k)$ can be estimated with both of the influences of the channel and the time offset, and we do not need to estimate the time offset of simultaneous reception independently. Moreover, this method can be easily extended to generate orthogonal sequences in group of three, four, or M pilots, that is, the ith pilot in groups of M pilots may have nonzero values in the $(nM + i)$th subcarrier and are 0 in the other subcarriers, where $n = 0, 1, 2, \ldots$.

In this paper, our multipath model is corresponding to the factors of the environment variety (such as the nodes' moving speed), and our channel estimation result holds only for slow fading channels. In fast fading channels, by contraries, although we are able to compensate for the phase rotation caused by frequency shift, we cannot use one OFDM frame for the pilot to estimate the fast-changing channel.

3.4. Transmission strategy for multihop unicast

Consider a unidirectional transmission session, for example, a distance-4 linear wireless ad hoc network (as shown in

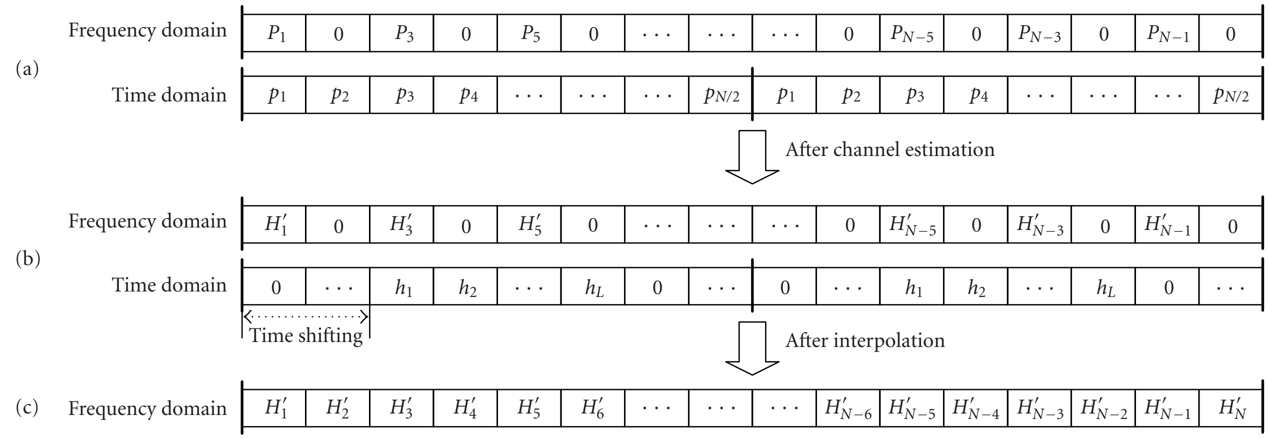


Figure 3: Channel estimation algorithm.

Figure 4), where node A intends to transmit three frames a, b and c to sink E. In time slot 3, while C is forwarding a to D, A cannot send the next frame b to B. Otherwise, the signals of a and b will collide and cause the MAI problem. In contrast, through the strategy shown in Figure 5, which utilizes the receiving and decoding function of PNC and embraces the interference, in time slot 3, nodes A and C transmit their own data, respectively, then node B will receive a mixed signal of a and b. Because node B has received the information of a before time slot 3, the data of b can be recovered from the mixed signal, which is to be forwarded to the next relay C, and so on.

Zhang et al. [13] observed the number of time slots in these two transmission strategies. From their result, we can get that by the strategy leveraging PNC, as shown in Figure 5, the transmission efficiency gain is $G_n = (n + 3b - 3)/(n + 2b - 2)$ with $G_n \to 1.5$ as $b \to \infty$, where n is the distance between the source node and the farmost sink node; and b is the number of blocks of data. At this point, the strategy in Figure 5 makes use of the broadcast nature of wireless networks and the transmission efficiency is increased.

For this transmission strategy, as shown in time slot 3 of Figure 5, the mixed signals that node B receives are given by

$$R_k = S_{1k}H'_1(k) + S_{2k}H'_2(k) + \eta'', \qquad (17)$$

where η'' is the noise, S_{2k} is the signal that takes the information a received from node C, S_{1k} is the signal that takes b received from node A, and $H'_1(k)$, $H'_2(k)$ denote the characteristics of channel $A \to B$ and $C \to B$, respectively, which also contain the phase rotation caused by the time offsets of simultaneous reception. At this time, if the nodes (including the relay nodes and the end node) get the estimation of $H'_1(k)$, $H'_2(k)$ by the channel estimation method stated in the last subsection, they can compensate for the influence of the multipath fading channel and the synchronization error together. As S_{2k} is already known by node B, the unknown signal S_{1k} can be recovered by subtracting S_{2k}.

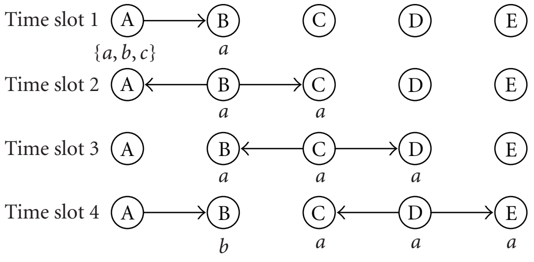


Figure 4: The traditional wireless multihop unicast strategy with three messages in a distance-4 linear network.

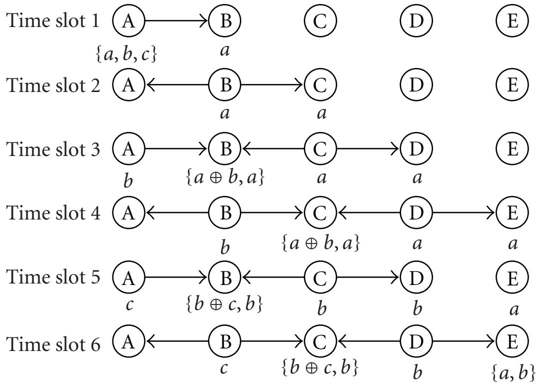


Figure 5: An example of a unicast strategy leveraging PNC with three messages in a distance-4 linear network.

Thus, we can get the benefit of PNC, which promotes the throughput of the network and solves the MAI problem, and use the OFDM technology to cope with the synchronization problem and multipath interference. Moreover, the error will not diffuse while the information is relayed, because no matter whether a reception of one signal is correct or not, it will become useful information to recover other signals. The performance of this signal recovery method will be shown in Section 5.

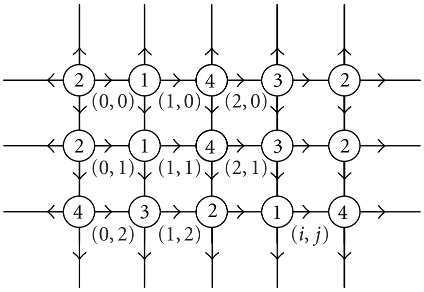


Figure 6: Rectangular grid network.

4. MAC-LAYER FOR ASSISTANCE: TRANSMISSION STRATEGY FOR WIRELESS AD HOC NETWORKS

In order to extend the transmission strategy leveraging PNC from a unidirectional line to networks, we consider a broadcast-relay approach with the physical-layer techniques stated in the last section, which utilizes the broadcast nature of wireless nodes and the additive nature of electromagnetic waves. In ad hoc networks, because nodes can receive all their neighbors' signals, which come from several directions, some protocols in the MAC layer are needed for assistance. In this section, we will first propose the transmission strategy in two representative topologies of ad hoc networks and then introduce the MAC layer protocols.

4.1. Transmission strategy in rectangular grid network

Consider the example of the transmission process from a source node to several receiver nodes in random unknown locations on a rectangular grid network, as shown in Figure 6. Each node can send/receive signals to/from its neighboring nodes through the wireless links, and the sending and receiving behaviors are in two time slots, respectively. All the links are bidirectional in Figure 6, and the arrows on the links represent the directions of the data flow, where we suppose the source node is at coordinate $(0,0)$ and one of the sink nodes is at coordinate (i,j). The number on each node represents the pilot it uses, which we will explain in Section 4.3. An example of our broadcast-relay transmission strategy is as follows.

(i) In time slot 1, the source node $(0,0)$ broadcasts the source information x_1 to its neighbors, and all of its neighbors such as node $(0,1)$ and node $(1,0)$ receive the signal and get the information.

(ii) In time slot 2, node $(0,1)$ and node $(1,0)$ broadcast information x_1 to all their neighbors. At this time, node $(1,1)$ will receive two mixed signals from node $(0,1)$ and node $(1,0)$ simultaneously and get the information x_1 from them.

(iii) In time slot 3, node $(0,0)$ broadcasts the next information x_2 to node $(0,1)$ and node $(1,0)$. At the same time, node $(2,0)$, node $(1,1)$, and node $(0,2)$ broadcast the information x_1 to all their neighbors. Thus, this time node $(0,1)$ and node $(1,0)$ will receive x_1 from node $(2,0)$, node $(1,1)$, and node $(0,2)$, as well as x_2 from node $(0,0)$. They decode the mixed signals and get the new information x_2.

(iv) In time slots 4, 5, 6 and so on, repeat the process of time slots 1–3.

Thus, in time slot s, the nodes in the network can be divided into three sets by their behavior: sending, receiving, and idle. We have

(1) node (i,j) will be idle if $|i|+|j|>s$ otherwise it will be a sending or receiving node,

(2) if $|i|+|j|\leqslant s$, and $2|(|i|+|j|+s)$, node (i,j) will be a receiving node otherwise, it will be a sending node, (We consider node $(0,0)$ as a receiving node while it is not sending information).

(3) for the sending nodes, each of them will send information to their four neighbors,

(4) for receiving node (i,j), if $|i|+|j|<s$, it will receive four additive signals simultaneously, from its four neighbors, respectively; if $|i|+|j|=s$, it will receive either one signal (if $i\cdot j=0$) or two additive signals (if $i\neq 0$ and $j\neq 0$).

Suppose that all the nodes have caches and have the ability of decoding:

(a) cache: all the nodes are able to cache the information they have received in the last time slot; no more caches are needed;

(b) decoding: if the nodes receive two additive signals that take the information $x_1\dotplus x_1$, they can decode them and get the information x_1; if the nodes receive four additive signals such as $x_2\dotplus x_2\dotplus x_1\dotplus x_1$ or $x_2\dotplus x_1\dotplus x_1\dotplus x_1$, they can get the information x_2 by the cache of x_1;

(c) the sending nodes only send the information $x_1, x_2, \ldots$ that have been decoded by themselves.

Based on the assumptions above, in time slot s, the information that node (i,j) receives is

(i) x_1, if $|i|+|j|=s$, and $ij=0$; (Decoding is not needed here, and the node can send the information to its four neighbors in the next time slot.)

(ii) $x_1\dotplus x_1$, if $|i|+|j|=s$, and $i\neq 0,\ j\neq 0$; (At this time, the node gets x_s by decoding from the mixed signals and sends the result to its four neighbors in the next time slot.)

(iii) $x_{[(s-|i|-|j|)/2]}\dotplus x_{[(s-|i|-|j|)/2]}\dotplus x_{[(s-|i|-|j|)/2]}\dotplus x_{[(s-|i|-|j|)/2]+1}$, if $|i|+|j|<s$, and $ij=0$; (At this time, the node gets $x_{[(s-|i|-|j|)/2]+1}$ by the cache of $x_{[(s-|i|-|j|)/2]}$, and sends $x_{[(s-|i|-|j|)/2]+1}$ to its four neighbors in the next time slot.)

(iv) $x_{[(s-|i|-|j|)/2]}\dotplus x_{[(s-|i|-|j|)/2]}\dotplus x_{[(s-|i|-|j|)/2]+1}\dotplus x_{[(s-|i|-|j|)/2]+1}$, if $|i|+|j|<s$, and $i\neq 0,\ j\neq 0$. (At this time, the node gets $x_{[(s-|i|-|j|)/2]+1}$ by the cache of $x_{[(s-|i|-|j|)/2]}$, and sends $x_{[(s-|i|-|j|)/2]+1}$ to its four neighbors in the next time slot.)

Thus, by this strategy, the information from a source node (such as node $(0,0)$) can be transmitted to any group

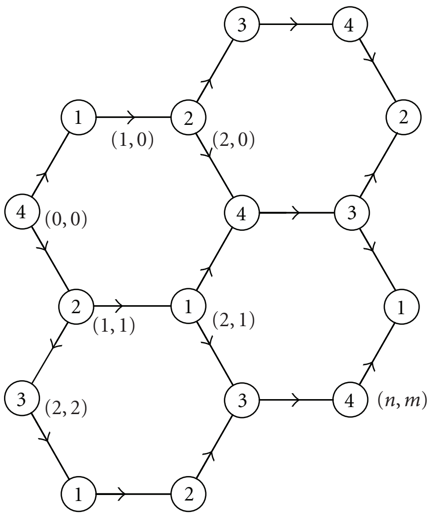


FIGURE 7: Hexagonal network.

of the sink nodes (such as node (i, j)) by the broadcasting relays.

4.2. *Transmission strategy in hexagonal network*

Consider another network topology in common use namely, hexagonal cells, for the same problem of the last subsection, as shown in Figure 7. Based on the same assumption, each node can send/receive signals to/from its neighboring nodes through the wireless links, wherein the sending and receiving behaviors are in two time slots, respectively. All the links are bidirectional, and the arrows in the links represent the directions of the data flow, where we denote every node by a reference coordinate such as (n, m). n is the minimum number of hops from the source node to node (n, m), and m is the sequence number of the nodes which are minimum n-hops away from the source node. Hence, the source node is at coordinate $(0, 0)$, and one of the receiver nodes is at (n, m), which means this node is the mth node that receives the source information by minimum n-hops. In addition, the number on each node represents the pilot it uses, which will also be explained in Section 4.3.

By the same transmission strategy of the rectangular networks, in time slot s, the nodes in the network can be divided into three sets by their behavior: sending, receiving, and idle. We have

(1) node (n, m) will be idle if $n > s$, otherwise it will be sending or receiving node;
(2) if $n \leqslant s$, and $2 \mid (n + s)$, node (n, m) will be a receiving node otherwise, it will be a sending node; (We consider node $(0, 0)$ as a receiving node while it is not sending information.)
(3) for the sending nodes, each of them will send information to its three neighbors;
(4) for receiving node (n, m), if $n < s$, it will receive three additive signals simultaneously from its three neighbors, respectively; if $n = s$, it will receive either one signal (if there is only one disjoint minimum ℕ hops path from the source node to this node) or two additive signals (if there are two disjoint minimum n hops paths from the source node to this node).

Suppose that all the nodes have caches and have the ability of decoding;

(a) cache: all the nodes are able to cache the information they have received in the last time slot; no more caches are needed;
(b) decoding: if the nodes receive two additive signals that take the information $x_1 \dotplus x_1$, they can decode them and get the information x_1; if the nodes receive three additive signals such as $x_2 \dotplus x_2 \dotplus x_1$ or $x_2 \dotplus x_1 \dotplus x_1$, they can get the information x_2 by the cache of x_1;
(c) the sending nodes only send the information $x_1, x_2, \ldots$ that have been decoded by themselves.

Based on the assumptions above, in time slot s, the information that node (n, m) receives is

(i) x_1, if $n = s$, and there is only one disjoint minimum n hops path from the source node to this node; (Decoding is not needed here, and the node can send the information to its three neighbors in the next time slot.)
(ii) $x_1 \dotplus x_1$, if $n = s$, and there are two disjoint minimum n hops paths from the source node to this node; (At this time, the node gets x_s by decoding from the mixed signals, and sends the result to its three neighbors in the next time slot.)
(iii) $x_{[(s-n)/2]} \dotplus x_{[(s-n)/2]} \dotplus x_{[(s-n)/2]+1}$, if $n < s$, and there is only one disjoint minimum n hops path from the source node to this node; (At this time, the node gets $x_{[(s-n)/2]+1}$ by the cache of $x_{[(s-n)/2]}$, and sends $x_{[(s-n)/2]+1}$ to its three neighbors in the next time slot.)
(iv) $x_{[(s-n)/2]} \dotplus x_{[(s-n)/2]+1} \dotplus x_{[(s-n)/2]+1}$, if $n < s$, and there are two disjoint minimum ℕ hops paths from the source node to this node. (At this time, the node gets $x_{[(s-n)/2]+1}$ by the cache of $x_{[(s-n)/2]}$, and sends $x_{[(s-n)/2]+1}$ to its three neighbors in the next time slot.)

Thus, by the strategy introduced above, the information from a source node (such as node $(0, 0)$) can be transmitted to any group of the sink nodes (such as node (n, m)) by the broadcast-relays. The transmission efficiency gain of this strategy in rectangular and hexagonal network will be shown in Theorems 2 and 3 in Section 5, respectively.

4.3. *Crosslayer design of the physical layer and MAC Layer*

In the transmission strategy stated in the last two subsections, the key requirement is that nodes should be able to get the new information they need by decoding from the mixed signals, such as getting x_1 from $x_1 \dotplus x_1$ or getting x_2 from $x_2 \dotplus x_1 \dotplus x_1 \dotplus x_1$ or $x_2 \dotplus x_2 \dotplus x_1 \dotplus x_1$ by the cache of x_1 in the rectangular network, as well as getting x_1 from $x_1 \dotplus x_1$, or getting x_2 from $x_2 \dotplus x_1 \dotplus x_1$ or $x_2 \dotplus x_2 \dotplus x_1$ by the cache of x_1 in the hexagonal network. Denoting the signals that take the information x_1 or x_2 in the frequency-domain by S_{1k}, S_{2k}, respectively, by Theorem 1 and Corollary 1, the mixed signals could be expressed as one of the following equations:

$$\begin{aligned} R_k &= S_{1k}H_1'(k) + S_{1k}H_2'(k) + \eta'', \\ R_k &= S_{2k}H_1'(k) + S_{1k}H_2'(k) + S_{1k}H_3'(k) + S_{1k}H_4'(k) + \eta'', \\ R_k &= S_{2k}H_1'(k) + S_{2k}H_2'(k) + S_{1k}H_3'(k) + S_{1k}H_4'(k) + \eta'', \end{aligned} \tag{18}$$

in the rectangular network, and

$$\begin{aligned} R_k &= S_{1k}H_1'(k) + S_{1k}H_2'(k) + \eta'', \\ R_k &= S_{2k}H_1'(k) + S_{1k}H_2'(k) + S_{1k}H_3'(k) + \eta'', \\ R_k &= S_{2k}H_1'(k) + S_{2k}H_2'(k) + S_{1k}H_3'(k) + \eta'', \end{aligned} \tag{19}$$

in the hexagonal network.

If the neighbors of each node transmit orthogonal pilot frames before sending data, by the estimation algorithm introduced in Section 3, we can get all the channel parameters $H'(k)$ in (18) and (19) including the influences of multipath interference and time synchronization error. In ad hoc networks, making all the pilots of every node's neighbors orthogonal is a dyeing problem. For example, the distribution of nodes with orthogonal pilots in the rectangular network is shown in Figure 6, where the number on each node represents the pilot it uses. As a result, four different pilots are used in all. If we still use four different pilots for the solution of hexagonal network as shown in Figure 7 (because the length of OFDM frames is always in the form of 2^n), then every node within distance 2 may have different pilots, which is a stronger solution for such a dyeing problem. Thus, we may extend the by-twos orthogonal pilots introduced in Section 3 to the orthogonal pilots in groups of four, (e.g., four neighbors of node $(1,1)$ in Figure 6 should transmit four different orthogonal pilots, such as node $(1,0)$ uses Pilot number 1, node $(0,1)$ uses Pilot number 2, node $(1,2)$ uses Pilot number 3 and node $(2,1)$ uses Pilot number 4), and then, as a result of the simultaneous reception, the sequence of each pilot will be at a different position in the mixed pilot frame as shown in the pilot segment of Figure 8, where the sequence of Pilot number i is on the position $4n+i$ of the mixed pilot frame, and $n = 0, 1, 2, \ldots$. If there is no signal coming from the node that uses Pilot number i, it will be 0 on the position $4k+i$ in the mixed pilot frame.

These physical-layer orthogonal pilots will help us estimate the channel parameters $H'(k)$. If all of the $H'(k)$ are known, the receiver nodes can get the new information because there is only one unknown variable in (18) and (19) if the former information is cached. However, it is not sufficient for distinguishing which kind of mixed signals it is that the nodes receive, so the receiver nodes have trouble deciding which kind of equations to use in (18) and (19) for decoding.

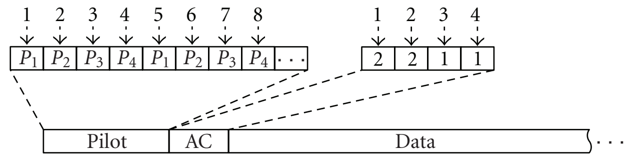


FIGURE 8: Pilot frame and access control header.

Therefore, we insert an access control header after the pilot frame, as shown in Figure 8, the access control (AC) segment. The AC segment of the node which uses pilot number 1 is in the position 1. As an analogy of this, positions 1, 2, 3, and 4 of this segment represent different neighbors of the receiver node, which are orthogonal in the frequency-domain. The content in position i of the AC segment represents the original source where the information comes from in the last time slot. For example, node $(1,1)$ in Figure 6 receives the mixed AC segment as $(2,2,1,1)$ in positions 1, 2, 3, and 4, respectively. That means, the information received from the first neighbor of node $(1,1)$ (which is actually node $(1,0)$, with pilot no. 1) comes from the node with pilot number 2 (which is actually node $(0,0)$) in the last time slot. Similarly, node $(1,1)$'s other neighbor node $(0,1)$'s information comes from node $(0,0)$ (which uses the pilot no. 2) as well. For the other neighbors, the original source of node $(1,2)$ and node $(2,1)$ is node $(1,1)$ in the last time slot, which is the receiver node itself, with pilot number 1. (If there is no signal coming from the node that uses pilot i, it will be 0 on the position i.) Thus, the receiving node will not only get which neighbors send signals to it, but will also know the original source of information in the last time slot. It can thereby decide to use which equation in (18) and (19) for decoding. If one of the original source in the last time slot is the receiver node itself, for example, the signal $S_{1k}H_3'(k)$ in the mixed signals $S_{2k}H_1'(k) + S_{2k}H_2'(k) + S_{1k}H_3'(k)$ in (19), this signal must be the receiver node's cached signal (this is kept by the solution of the orthogonal pilot dyeing problem), so it should be subtracted from the mixed signals. If not, the signal will contain new information which should be merged for decoding (merge the sum of $S_{2k}H_1'(k) + S_{2k}H_2'(k)$ as $S_{2k}(H_1'(k) + H_2'(k))$ and then get S_{2k}).

By the support of the access control header in the MAC-layer, the transmission problem of the rectangular and hexagonal networks can be resolved. This transmission strategy makes use of the additive nature of EM waves to boost the network throughput. Besides, the different type of the content in the AC segment lies in the number of orthogonal pilots. For example, there are $\binom{4}{1} + \binom{4}{2} = 10$ kinds of values in the AC segment of the transmission strategy shown in Figures 6 and 7 to represent the pilots of the original source, which are for the pilots of 1, 2, 3, 4, 1 and 2, 1 and 3, 1 and 4, 2 and 3, 2 and 4, and 3 and 4, respectively.

5. PERFORMANCE ANALYSIS

In the last two sections, our transmission strategy is based on the decoding techniques. In practice, the problem of decoding is to subtract the cached information from the mixed signals and get the new information, which depends on the physical-layer techniques of the OFDM system and the channel estimation algorithm. Compared to the conventional OFDM system which is for single signal reception, our system will deal with the issue of multiple signals

Table 1: Parameters used in simulation.

Parameters	Values
Channel coding	1/2 convolutional codes
Modulation	QPSK
Number of FFT points	64, 128, 256, 512, 1024, 2048, 4096, 8192
Guard interval	1/4 length of the FFT points
Multipath model	2 path independent

simultaneous transmission. Thus, in this section, to evaluate our works we will first present simulation results in the physical layer which shows the performance of our signal recovery techniques against the conventional OFDM system, as well as the accuracy of our channel estimation algorithm. Afterwards, we turn to the performance of our transmission strategy, where we will discuss the transmission efficiency of our strategy in linear, rectangular, and hexagonal networks, whose performance is scalable for any arbitrary-cast case, including unicast, multicast, and broadcast. The transmission efficiency gain is a trade-off from the complexity of the transmitters and receivers. So finally, we will analyze the implementation times of the channel estimation/decoding and the bandwidth consumption to ensure synchronization, which is the cost of the efficiency gain.

5.1. *The performance of the decoding techniques*

To validate Theorem 1 and our signal recovery algorithm, here we present sample simulation results of the physical-layer techniques. As stated in Section 3, if the time interval of two pilot frames is less than the channel's coherence time, the channel can be recognized as a time-invariable channel because the old channel estimation result will be dropped if a new pilot frame appears. Therefore, we assume that the channel is invariable during such a time interval. Table 1 shows the simulation parameters. In the representative situation of two signals' simultaneous reception, we will demonstrate that the channel estimation error is much smaller than the demodulation error, as well as the performance of our method in processing multiple signal reception can meet the performance of the conventional OFDM system in processing a single signal.

Figures 9 and 10 present the mean square error (MSE) of our channel estimation algorithm stated in Section 3. From Figure 9, the performance improves while the symbol length N (the pilot sequence's length) increases. Figure 10 shows the performance with some typical N under the different noise power.

Figure 11 presents the BER performance for the signal recovery of our transmission strategy, which meets the performance of conventional OFDM systems in the same situation. That is mainly because the order of the channel estimation error is much smaller than the order of the error of demodulating unknown signals. Besides, from (5), we only use one noise $\eta(t)$ to represent the noise along with the received signal that is interfered, where the assumption of the noise is the same as conventional OFDM systems. From Theorem 1, the power of the noise does not change in the simultaneous reception. Hence, there is no loss due to noise accumulation in Figure 11.

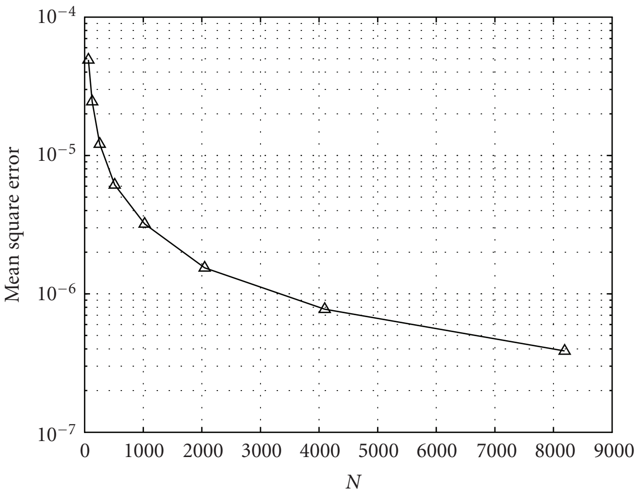


Figure 9: Channel estimation MSE against FFT point N.

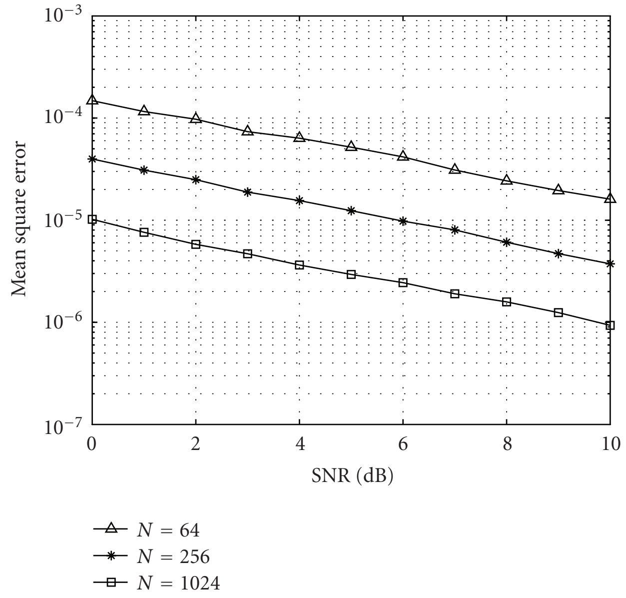


Figure 10: Channel estimation MSE against SNR.

Denoting the length of the multipath spreading by T_{delay}, and the length of CP by T_{CP}, the timing offset ΔT_s of the two mixed signals (as shown in Figure 2) should satisfy $\Delta t_s < T_{\text{CP}} - T_{\text{delay}}$ in order to maintain the orthogonality of transmitted waveforms. Figure 12 shows that if this condition is satisfied, the estimate algorithm works steadily. If not, the orthogonality of subcarriers is violated and the BER performance deteriorates gradually.

5.2. *Transmission efficiency*

Figure 13(a) shows the traditional transmission schedule of broadcast-relay in the rectangular network, while

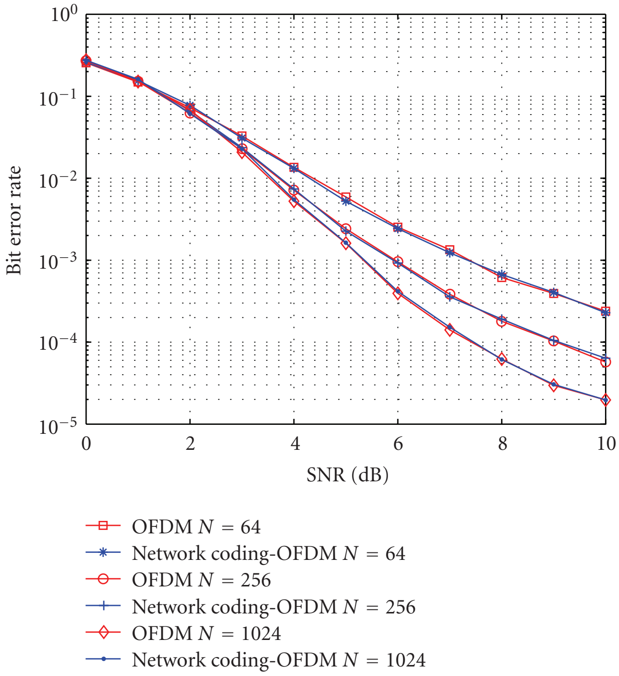


FIGURE 11: BER performance for signal recovery.

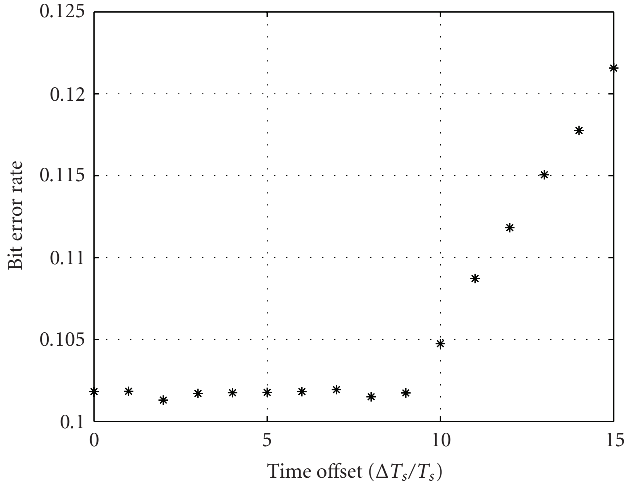


FIGURE 12: BER for signal recovery versus time-domain offset (without convolutional codes).

Figure 13(b) shows the schedule of our transmission strategy, where the number on each node represents the time slot that the node broadcasts signals to all its neighbors. Intuitively, they are optimal schedules to avoid the MAI problem by using the least number of time slots for *broadcast case*. In the following, we will give the transmission efficiency gain of our strategy for broadcast case in a distance-n-network.

Theorem 2. *Let the source node be at coordinate* $(0, 0)$ *in a full distance-n rectangular network. Let* $\mathbb{T}$ *denote the minimum number of time slots needed to spread b blocks of data from the source to all the other nodes by using the broadcast-relay strategy. Then,*

$$T = \begin{cases} 2n + 5b - 6, & \text{Traditional strategy,} \\ n + 2b - 2, & \text{Our strategy.} \end{cases} \tag{20}$$

As a result, the transmission efficiency gain is $G_n = (2n + 5b - 6)/(n + 2b - 2)$ *with* $G_n \to 2.5$ *as* $b \to \infty$.

Proof. Consider a full distance-n rectangular network in which the source node is at coordinate $(0, 0)$. In our transmission strategy shown in Figure 13(b), transmitting one message from the source to all the farmost sink nodes needs n time slots. Then, the sink nodes can receive the next message after 2 time slots. Thus, the total number of time slots for transmitting b blocks of data should be $n + 2(b - 1)$. In contrast to the traditional strategy of the optimum transmission schedule shown in Figure 13(a), 2 time slots are needed to spread the information of distance 1, because we should arrange the nodes to send the information by turns in order to avoid the MAI problem. Thus, to spread one message from the source to all the nodes in a full distance-n rectangular network needs $2n - 1$ time slots. In addition, the sink nodes will receive the next message after 5 time slots. Therefore, the total number of time slots for transmitting b blocks of data should be $2n - 1 + 5(b - 1)$. The transmission efficiency gain is then $G_n = (2n + 5b - 6)/(n + 2b - 2)$ with $G_n \to 2.5$ as $b \to \infty$. □

For the transmission strategies in the hexagonal network as shown in Figures 13(c) and 13(d), the number on each node represents the time slot that the node broadcasts signals to all its neighbors. Similar to the rectangular topology, we have

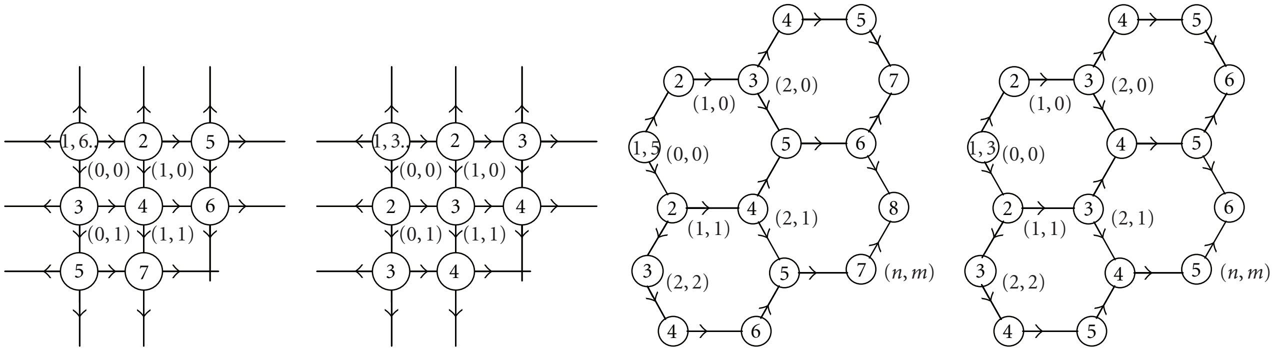
Theorem 3. *Let the source node be at coordinate* $(0, 0)$ *in a full distance-n hexagonal network. Let* $\mathbb{T}$ *denote the minimum number of time slots needed to spread b blocks of data from the source to all the other nodes by using the broadcast-relay strategy. Then,*

$$T = \begin{cases} \dfrac{3n}{2} + 4b - 5, & \text{Traditiona strategy, } (2|n), \\ \dfrac{3(n-1)}{2} + 4b - 3, & \text{Traditiona strategy, } (2\nmid n), \\ n + 2b - 2, & \text{Our strategy.} \end{cases} \tag{21}$$

As a result, the transmission efficiency gain is $G_n = ((3n/2) + 4b - 5)/(n + 2b - 2)$, $(2|n)$, *or* $G_n = ((3(n - 1)/2) + 4b - 3)/(n + 2b - 2)$, $(2 \nmid n)$ *with* $G_n \to 2$ *as* $b \to \infty$.

Proof. The proof of this theorem is similar to Theorem 2. □

If some nodes are not sink nodes in a full distance-n rectangular or hexagonal network, the broadcast problem will be degraded to a multicast problem. Moreover, if there is only one sink node in the network, it is the unicast problem



(a) Traditional transmission schedule in rectangular network

(b) Our transmission schedule leveraging PNC in rectangular network

(c) Traditional transmission schedule in hexagonal network

(d) Our transmission schedule leveraging PNC in hexagonal network

Figure 13: Time slots distribution in transmission schedule.

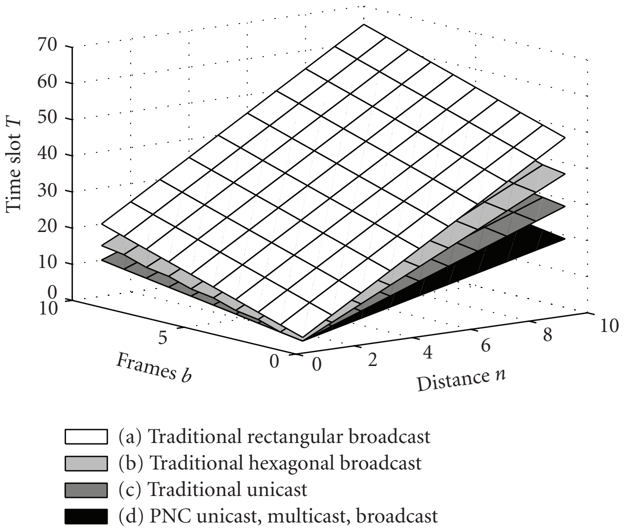


Figure 14: The comparison of transmission efficiency.

which has been discussed in Section 3. In the unicast case of transmitting b blocks of data to the distance-n sink node, it needs $n + 2b - 2$ time slots for our transmission strategy and $n + 3b - 3$ time slots for the traditional strategy. Therefore, for an arbitrary-cast case, including unicast, multicast, and broadcast, in a full distance-n rectangular or hexagonal network, in which the farmost sink node is distance-n away from the source node, our transmission strategy always uses $n + 2b - 2$ time slots to finish the transmission of b blocks of data. In contrast, the traditional strategy needs $2n + 5b - 6$ time slots in the rectangular network, $3n/2 + 4b - 5$ or $3(n - 1)/2 + 4b - 3$ time slots in the hexagonal network for broadcast case, and $n + 3b - 3$ for unicast case. Its transmission efficiency for the multicast case is between the broadcast and the unicast case because if the number of sink nodes decreases in a full distance-n rectangular or hexagonal network, some nodes will not be in the transmission paths and the MAI problem can be partly reduced, thus saving the time slots. A comparison of transmission efficiency is shown in Figure 14, where the two horizontal axes are for the network distance n and the number of frames b, respectively, and the vertical axis is for the needed time slots T. Surface (d) represents the transmission efficiency of our proposed strategy for any *arbitrary-cast*, including unicast, multicast, and broadcast case. Thus, if n and b are fixed, then the transmission efficiency does not decrease as the number of sink nodes increases, thereby providing a scalable transmission with the number of users/terminals. Surfaces (a) and (b) are for the traditional strategy of *broadcast case* in rectangular and hexagonal networks, respectively, and Surface (c) shows the *unicast case* of traditional strategy. For the *multicast case* of traditional strategy, as stated above, the transmission efficiency is between Surfaces (a) and (c) in the rectangular network, or between Surfaces (b) and (c) in the hexagonal network. As a result, when $b \to \infty$, the transmission efficiency gain of our strategy for any arbitrary-cast is from 1.5 to 2.5 in the rectangular network, and is from 1.5 to 2 in the hexagonal network.

5.3. The trade-off between the transmission efficiency gain and the cost

The transmission efficiency gain discussed in the last subsection is a trade-off from the complexity of the transmitters and receivers, so here we observe the implementation times of the channel estimation/decoding and the bandwidth consumption to ensure synchronization, which are the cost of the efficiency gain.

Theorem 4. *In the broadcast-relay transmission strategy, the average decoding times of each node in the transmission path of a linear, rectangular, or hexagonal distance-n-network after time slot* s_0, $s_0 > n$ *is* $(1/2)(n + 2b - 2)$ *for transmitting b blocks of data.*

Proof. In our broadcast-relay transmission strategy, each node in the transmission path should do the decoding while it is the receiving node. After time slot s_0, $s_0 > n$, all the nodes in the transmission paths take part in the transmission, and each node is a receiving node or a sending node in

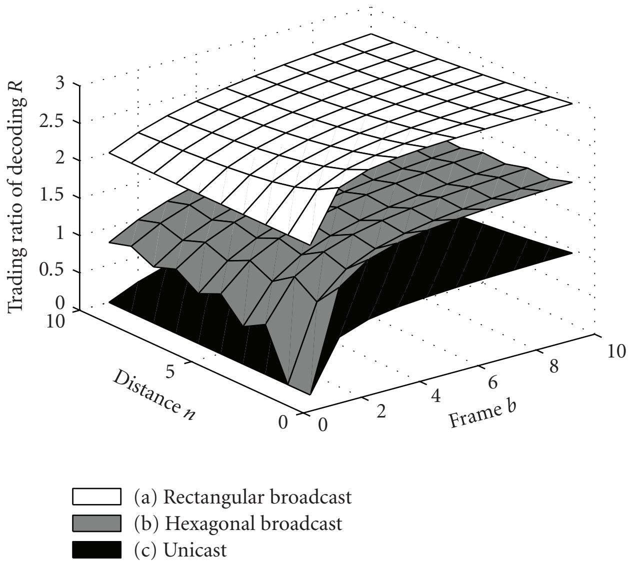


Figure 15: Trading ratio of decoding.

alternate time slot. Because the total number of time slots for transmitting b blocks of data is $n+2(b-1)$ for any arbitrary-cast, and the decoding is only taken by the receiving node, the decoding times of each node in average is $(1/2)(n+2b-2)$. □

To evaluate the trade-off between the transmission efficiency gain and the implement times of the decoding, we give the definition of the *trading ratio of decoding.*

Definition 6 (Trading ratio of decoding). The expression of the trading ratio of decoding R is $R = T_{\text{save}}/D$, where T_{save} is the number of time slots that the broadcast-relay strategy has saved against the traditional strategy, and D is the implementation times of the decoding.

Figure 15 shows the distribution of the trading ratio of decoding R against the network distance n and the transmission block number b for the broadcast case of rectangular and hexagonal networks and the unicast case (Surfaces (a), (b), and (c), resp.). For the multicast case, the distribution of R is between Surfaces (a) and (c) in the rectangular network; and it is between Surfaces (b) and (c) in the hexagonal network.

If we suppose that channel estimation is taken before performing decoding every time, after time slot s_0, $s_0 > n$, the implementation times of the channel estimation depends on the degree of the nodes in the networks, which is $(1/2)(n+2b-2)\cdot 4$ in the rectangular network, $(1/2)(n+2b-2)\cdot 3$ in the hexagonal network, and $(1/2)(n+2b-2)\cdot 2$ in the linear network. If the wireless channel's coherent time gets larger, this implementation times can be reduced. Under the extreme condition, if the wireless channel is invariable, every node in the transmission path of a distance-n-network only needs to take $(1/2)n\cdot 4$, $(1/2)n\cdot 3$, and $(1/2)n\cdot 2$ times of channel estimation in average for the rectangular, hexagonal, and linear network, respectively.

Furthermore, in our transmission strategy leveraging PNC, all the nodes could use the same group of N subcarriers as conventional OFDM systems, which makes the most of the system bandwidth. The only penalty of the bandwidth is paid by the channel estimation frame (the pilot) and the access control header which are to ensure synchronization and obtain the channel parameters. Hence, the time interval of two pilot frames should be less than the channel's coherence time, and it will consume $T_{\text{pilot}} + T_{\text{AC}}/T_{\text{CO}}$ of the whole bandwidth of the system to ensure synchronization and estimate the channels, where T_{pilot} is the time length of the pilot, T_{AC} is the time length of the access control header, and T_{CO} is the channel's coherence time.

6. CONCLUSION

In this paper, we have discussed the broadcast-relay transmission strategies in linear, rectangular, and hexagonal networks. It is shown that in applying PNC in practice, the signal's time synchronization error can be released by CP in the OFDM system, which can also be estimated and compensated together with the multipath fading channel's influence by orthogonal pilots. By our broadcast-relay strategy, which performs mixed-signal reception in the physical layer and with the help of the access control protocol that distinguishes the original source of the signals of the simultaneous transmission, the transmission efficiency is no longer limited by the number of sink nodes (users/terminals) and the MAI problem, but by the complexity of the nodes. As Garcia-Luna-Aceves et al. [16] offered the conjecture that the combination of MPR and NC constitutes the best approach for capacity gain, we believe that the transmission strategies which leverage PNC and support the simultaneous reception may have great development prospects to render scalable ad hoc networks.

REFERENCES

[1] P. Gupta and P. R. Kumar, "The capacity of wireless networks," *IEEE Transactions on Information Theory*, vol. 46, no. 2, pp. 388–404, 2000.

[2] R. Ahlswede, N. Cai, S.-Y. R. Li, and R. W. Yeung, "Network information flow," *IEEE Transactions on Information Theory*, vol. 46, no. 4, pp. 1204–1216, 2000.

[3] S.-Y. R. Li, R. W. Yeung, and N. Cai, "Linear network coding," *IEEE Transactions on Information Theory*, vol. 49, no. 2, pp. 371–381, 2003.

[4] R. Koetter and M. Medard, "An algebraic approach to network coding," *IEEE/ACM Transactions on Networking*, vol. 11, no. 5, pp. 782–795, 2003.

[5] Y. Wu, P. A. Chou, and S.-Y. Kung, "Information exchange in wireless networks with network coding and physical-layer broadcast," in *Proceedings of the 39th Annual Conference on Information Sciences and Systems (CISS '05)*, Baltimore, Md, USA, March 2005.

[6] C. Fragouli, J.-Y. L. Boudec, and J. Widmer, "Network coding: an instant primer," *ACM SIGCOMM Computer Communication Review*, vol. 36, no. 1, pp. 63–68, 2006.

[7] J. Liu, D. Goeckel, and D. Towsley, "Bounds on the gain of network coding and broadcasting in wireless networks," in *Proceedings of the 26th IEEE International Conference on Computer Communications (INFOCOM '07)*, pp. 724–732, Anchorage, Alaska, USA, May 2007.

[8] Z. Li and B. Li, "Network coding in undirected networks," in *Proceedings of the 38th Annual Conference on Information Sciences and Systems (CISS '04)*, Princeton, NJ, USA, March 2004.

[9] Y. Zhou and S. M. Nettles, "Balancing the hidden and exposed node problems with power control in CSMA/CA-based wireless networks," in *Proceedings of IEEE Wireless Communications and Networking Conference (WCNC '05)*, vol. 2, pp. 683–688, New Orleans, La, USA, March 2005.

[10] C.-K. Toh, *Ad Hoc Mobile Wireless Networks: Protocols and Systems*, Prentice Hall, Upper Saddle River, NJ, USA, 2002.

[11] S. Zhang, S.-C. Liew, and P. P. Lam, "Hot topic: physical-layer network coding," in *Proceedings of the 12th Annual International Conference on Mobile Computing and Networking (MobiCom '06)*, pp. 358–365, Los Angeles, Calif, USA, September 2006.

[12] S. Zhang, S.-C. Liew, and P. P. Lam, "On the synchronization of physical-layer network coding," in *Proceedings of IEEE Information Theory Workshop (ITW '06)*, pp. 404–408, Chengdu, China, October 2006.

[13] P. Fan, K. B. Letaief, J. Zhang, and K. Cai, "A network coding unicast strategy for wireless multi-hop networks," in *Proceedings of IEEE Wireless Communications and Networking Conference (WCNC '07)*, pp. 4221–4226, Kowloon, Hong Kong, March 2007.

[14] K. Fukawa and H. Suzuki, "Blind interference canceling equalizer for mobile radio communications," *IEICE Transactions on Communications*, vol. E77-B, no. 5, pp. 580–588, 1994.

[15] B. C. W. Lo and K. B. Letaief, "Adaptive equalization and interference cancellation for wireless communication systems," *IEEE Transactions on Communications*, vol. 47, no. 4, pp. 538–545, 1999.

[16] J. Garcia-Luna-Aceves, H. R. Sadjadpour, and Z. Wang, "Challenges: towards truly scalable ad hoc networks," in *Proceedings of the 13th Annual ACM International Conference on Mobile Computing and Networking (MobiCom '07)*, pp. 207–214, Montreal, Quebec, Canada, September 2007.

[17] Y. Sato, "A method of self-recovering equalization for multilevel amplitude-modulation systems," *IEEE Transactions on Communications*, vol. 23, no. 6, pp. 679–682, 1975.

[18] J. Siew, J. Coon, R. J. Piechocki, et al., "A channel estimation alogorithm for MIMO-SCFDE," *IEEE Communications Letters*, vol. 8, no. 9, pp. 555–557, 2004.