Hindawi Publishing Corporation
EURASIP Journal on Wireless Communications and Networking
Volume 2008, Article ID 624542, 7 pages
doi:10.1155/2008/624542

Research Article

Space-Time Convolutional Codes over Finite Fields and Rings for Systems with Large Diversity Order

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Received 26 October 2007; Revised 18 March 2008; Accepted 6 May 2008

Recommended by Yonghui Li

We propose a convolutional encoder over the finite ring of integers modulo $p^k, \mathbb{Z}_{p^k}$, where p is a prime number and k is any positive integer, to generate a space-time convolutional code (STCC). Under this structure, we prove three properties related to the generator matrix of the convolutional code that can be used to simplify the code search procedure for STCCs over $\mathbb{Z}_{p^k}$. Some STCCs of large diversity order (≥4) designed under the trace criterion for $n = 2, 3$, and 4 transmit antennas are presented for various PSK signal constellations.

1. INTRODUCTION

Since the discovery of space-time trellis codes (STTCs) by Tarokh et al. [1], much research has been done in this area. Some authors [2–5] have concentrated their efforts to generate STTCs through an encoding structure wherein the inputs are binary symbols, the encoding operations are realised modulo 2^k, where k is any positive integer, and the 2^k-ary outputs are matched to a 2^k-ary signal constellation. Although this encoding structure facilitates the code search procedure, this search becomes prohibitively complex as the number of transmit antennas, states, or modulation size increases.

In order to simplify the design of STTCs, Abdool-Rassool et al. [6] have proven two theorems that allow one to significantly reduce the computational effort of the code search. In [7], utilising an alternative structure, the authors have considered STTCs generated by a convolutional encoder over the Galois field $GF(p) \equiv \mathbb{Z}_p$, p a prime, where the information symbols, the convolutional encoder tap gains, and the output symbols are elements of $\mathbb{Z}_p$, allowing for a spectral efficiency of $\log_2(p)$ b/s/Hz. These codes are referred to as space-time convolutional codes (STCCs). Using the structure proposed in [7], Hong and Chung [8] and Noronha-Neto and Uchôa-Filho [9] have presented some new STCCs over $GF(p)$ for two transmit antennas.

The design of good STTCs is based on the well-known rank and determinant [1] criteria or the trace [2, 3] criterion, depending on the system's diversity order. If the diversity order is greater or equal to 4, the trace criterion should be used in substitution of the determinant criterion while the rank criterion may be relaxed.

In this paper, utilising a nonsystematic feedfoward convolutional encoder over the finite ring of integers modulo p^k, $\mathbb{Z}_{p^k}$, and inspired by the results in [6], we prove three properties related to the generator matrix of the convolutional codes over $\mathbb{Z}_{p^k}$ that can simplify the code search procedure for STCCs over $\mathbb{Z}_{p^k}$. Essentially, the properties establish equivalences among STCCs so that many convolutional codes can be discarded in the code search without loosing anything. Herein we focus on systems with large diversity order, so only STCCs designed under the trace criterion are considered. By exploiting the structure of the convolutional encoder over $\mathbb{Z}_{p^k}$ and the simplifications coming from the properties, we obtain some good STCCs over finite fields ($k = 1$) and rings based on the trace criterion for 3,4,5,7,8, and 9-PSK modulations, $n = 2, 3, 4$ transmit antennas, and encoder memories 1, 2, and 3.

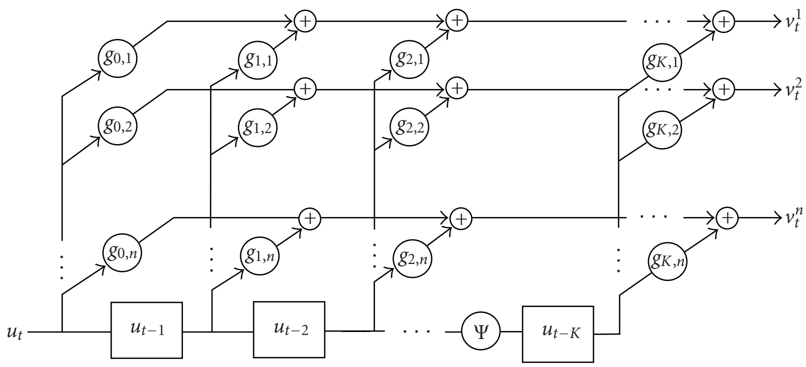

Figure 1: Rate $1/n$ convolutional encoder over $\mathbb{Z}_{p^k}$ with memory order K. The multiplier Ψ controls the number of encoder states.

We should mention the important work of Carrasco and Pereira [10] that considers nonbinary space-time convolutional codes. There are significant differences between the present work and [10]. First, Carrasco and Pereira considered a systematic feedback convolutional encoder, which has approximately the same number of nonbinary coefficients as compared to the encoder in nonsystematic feedfoward form proposed in this paper. However, our structure gives rise to the three properties mentioned above for code equivalences, by which we can reduce the code search effort. Another important difference between our work and the work of Carrasco and Pereira is that in [10] they consider the determinant criterion regardless of the number of received antennas.

The remainder of this paper is organised as follows. In Section 2, we describe the proposed space-time coded system based on convolutional codes over $\mathbb{Z}_{p^k}$, and present the design criteria for obtaining good STCCs. In Section 3, we prove the three properties mentioned above and present guidelines for finding good STCCs over $\mathbb{Z}_{p^k}$. The new STCCs found with the code search are tabulated in Section 4. Also provided in that section is the frame error rate (FER) for some of the new STCCs obtained from computer simulations. Comparison results with existing STTCs are also given. Finally, in Section 5, we conclude the paper and make some final comments.

Throughout this paper, the conjugate, transpose, and hermitian (conjugate transpose) of a matrix/vector A are denoted by A^*, A^T, and A^H, respectively.

2. THE SPACE-TIME CONVOLUTIONALLY CODED SYSTEM AND DESIGN CRITERIA

We consider a space-time coded system employing n transmit antennas and m receive antennas. In the transmitter, at each discrete time t, a $\mathbb{Z}_{p^k}$-valued information symbol u_t is encoded by a rate $1/n$ convolutional encoder over $\mathbb{Z}_{p^k}$ with encoder memory K, shown in Figure 1. The encoder output at time t is a block of n coded symbols over $\mathbb{Z}_{p^k}$, $(v_t^1, v_t^2, \ldots, v_t^n)$, where

$$v_t^i \equiv \sum_{x=0}^{K} u_{t-x} g_{x,i} \pmod{p^k}, \qquad (1)$$

for $i = 1, \ldots, n$. The encoder tap gain associated with transmit antenna i and memory depth x is denoted by $g_{x,i}$. The coded symbols are mapped into a complex p^k-PSK signal constellation and transmitted simultaneously via the n transmit antennas. A complex codeword $\mathbf{c}$ of length l of the space-time code is a sequence of blocks

$$\mathbf{c} = \{(c_t^1, c_t^2, \ldots, c_t^n)\} = \{(e^{j(2\pi/p^k)v_t^1}, e^{j(2\pi/p^k)v_t^2}, \ldots, e^{j(2\pi/p^k)v_t^n})\} \qquad (2)$$

for $t = 1, 2, \ldots, l$, where c_t^i is the signal transmitted from the ith antenna at time t. The set of all codewords $\mathbf{c}$ is called the STCC, and is denoted by $\mathcal{C}$.

Note that in Figure 1 there is a multiplier Ψ between the $(K-1)$th and the Kth memory depths. This multiplier, a positive integer that divides p^k, has the purpose of controlling the number of encoder states. A similar structure has been adopted for the Gaussian channel by Massey and Mittelholzer in [11]. For $\Psi = 1$, the number of encoder states is p^{kK}. But for $\Psi > 1$ the number of encoder states is reduced due to the ring property that the product of two nonzero ring elements may be zero, which reduces the range of possible integer values that can be stored in the Kth encoder memory. We set the value of this multiplier to p^{k-z}, where $z = 1, 2, \ldots, k-1$, to obtain encoders with intermediate number of states between powers of p^k. The number of encoder states becomes p^{kK}/Ψ. For example, the encoders over $\mathbb{Z}_4$ with $(K = 1, \Psi = 1)$, $(K = 2, \Psi = 2)$, and $(K = 2, \Psi = 1)$ have 4, 8, and 16 states, respectively.

In the space-time coded system, the signal received by the jth antenna at time t, d_t^j, is given by

$$d_t^j = \sum_{i=1}^{n} \alpha_{i,j} c_t^i \sqrt{E_s} + \eta_t^j, \qquad (3)$$

where E_s is the average energy of the transmitted signal, η_t^j is a zero-mean complex white Gaussian noise with variance $N_0/2$ per dimension, and $\alpha_{i,j}$ denotes the flat fading coefficient of the channel from the ith transmit antenna to the jth receive antenna. Under the Rayleigh fading assumption, $\alpha_{i,j}$, for $1 \le i \le n$ and $1 \le j \le m$, are modelled as independent samples of a zero-mean complex Gaussian random process with variance 0.5 per dimension.

In practice, to achieve independent fading the antennas must be physically separated by a distance in the order of a few wavelengths. For the quasistatic, flat-fading channel, it is assumed that the fading coefficients remain constant during a frame and change independently from one frame to another.

Also, we assume that the receiver perfectly knows the channel state information and that the Viterbi algorithm with the Euclidean metric is used in the decoder. Under these conditions, and for high signal-to-noise ratio (SNR), Tarokh et al. [1] have shown that the pairwise error probability is upperbounded by

$$P(\mathbf{c} \longrightarrow \mathbf{e}) \leq \left(\prod_{i=1}^{r}\lambda_i\right)^{-m}\left(\frac{E_s}{4N_0}\right)^{-rm}, \quad (4)$$

where r is the rank of the difference matrix of complex codewords (arranged as a matrix):

$$B(\mathbf{c},\mathbf{e}) \triangleq \begin{pmatrix} e_1^1 - c_1^1 & e_2^1 - c_2^1 & \cdots & e_l^1 - c_l^1 \\ e_1^2 - c_1^2 & e_2^2 - c_2^2 & \cdots & e_l^2 - c_l^2 \\ \vdots & \vdots & \ddots & \vdots \\ e_1^n - c_1^n & e_2^n - c_2^n & \cdots & e_l^n - c_l^n \end{pmatrix}, \quad (5)$$

and λ_i, for $i = 1,\dots,r$, are the nonzero eigenvalues of $A(\mathbf{c},\mathbf{e}) \triangleq B(\mathbf{c},\mathbf{e})B(\mathbf{c},\mathbf{e})^H$. To minimise $P(\mathbf{c}\rightarrow\mathbf{e})$ in (4), we should maximise the minimum rank r of the matrix $B(\mathbf{c},\mathbf{e})$ over all pairs of distinct complex codewords (rank criterion), and maximise the minimum geometric mean (η_{det}) of the nonzero eigenvalues of the matrix $A(\mathbf{c},\mathbf{e})$ over all pairs of distinct complex codewords with minimum rank (determinant criterion).

As shown by Chen et al. [2, 3], the rank and the determinant criteria should be adopted whenever $rm < 4$. If $rm \geq 4$, they have shown that the pairwise error probability is upperbounded by

$$P(\mathbf{c} \longrightarrow \mathbf{e}) \leq \frac{1}{4}\exp\left(-m\frac{E_s}{4N_0}\sum_{i=1}^{n}\sum_{j=1}^{l}|e_i^j - c_i^j|^2\right), \quad (6)$$

which indicates that to minimise $P(\mathbf{c}\rightarrow\mathbf{e})$ we should maximise the minimum squared Euclidean distance over all pairs of distinct complex codewords (trace criterion). It should be noted that the squared Euclidean distance between $\mathbf{c}$ and $\mathbf{e}$ is equal to the trace of $A(\mathbf{c},\mathbf{e})$, denoted by η_{tr}. In this paper, we consider only systems with $rm \geq 4$, but r needs not to be equal to n.

3. GUIDELINES FOR FINDING GOOD SPACE-TIME CONVOLUTIONAL CODES OVER $\mathbb{Z}_{p^K}$

In this section, we prove three properties that can be used to reduce the code search procedure for STCCs over $\mathbb{Z}_{p^k}$. But first, let us denote G as the $n(K+1)$ scalar generator matrix of the rate $1/n$ convolutional encoder over $\mathbb{Z}_{p^k}$ of Figure 1, which is defined in this paper as

$$G \triangleq \begin{bmatrix} g_{0,1} & g_{1,1} & \cdots & g_{K,1} \\ g_{0,2} & g_{1,2} & \cdots & g_{K,2} \\ \vdots & \vdots & \ddots & \vdots \\ g_{0,n} & g_{1,n} & \cdots & g_{K,n} \end{bmatrix}. \quad (7)$$

The first property is based on a result in [6, Section 3.2] for STCCs generated by an encoder with binary input and 2^k-ary tap gains. Herein, this result is extended to the case of a convolutional encoder over $\mathbb{Z}_{p^k}$.

Property 1. Consider an STCC $\mathcal{C}$ over $\mathbb{Z}_{p^k}$ generated by a generator matrix G with coefficients $g_{x,i}$, for $x = 0,1,\dots,K$ and $i = 1,2,\dots,n$. Let $\widetilde{\mathcal{C}}$ be the STCC over $\mathbb{Z}_{p^k}$ generated by the generator matrix $\widetilde{G}$ with coefficients $\widetilde{g}_{x,i} = p^k - g_{x,i}$, for $x = 0,1,\dots,K$ and $i = 1,2,\dots,n$. Then, every pair of codewords $\mathbf{c},\mathbf{e} \in \mathcal{C}$ is associated with a pair $\widetilde{\mathbf{c}},\widetilde{\mathbf{e}} \in \widetilde{\mathcal{C}}$ such that $A(\mathbf{c},\mathbf{e}) = B(\mathbf{c},\mathbf{e})B(\mathbf{c},\mathbf{e})^H$ and $A(\widetilde{\mathbf{c}},\widetilde{\mathbf{e}}) = B(\widetilde{\mathbf{c}},\widetilde{\mathbf{e}})B(\widetilde{\mathbf{c}},\widetilde{\mathbf{e}})^H$ have the same rank, determinant, and trace. Therefore, the two STCCs $\mathcal{C}$ and $\widetilde{\mathcal{C}}$ are entirely equivalent.

Proof. Consider that the output of the encoder shown in Figure 1 is as given in (1). Changing the encoder coefficient to $p^k - g_{x,i}$ yields the following output:

$$\begin{aligned} \widetilde{v}_t^i &\equiv \sum_{x=0}^{K} u_{t-x}(p^k - g_{x,i}) \pmod{p^k} \\ &\equiv \sum_{x=0}^{K}(u_{t-x}p^k) - (u_{t-x}g_{x,i}) \pmod{p^k} \\ &\equiv \sum_{x=0}^{K} - u_{t-x}g_{x,i} \pmod{p^k} \equiv -v_t^i \pmod{p^k} \\ &\equiv p^k - v_t^i \pmod{p^k}. \end{aligned} \quad (8)$$

Each element of $B(\mathbf{c},\mathbf{e})$ is a difference of complex numbers of the form

$$b_{i,j} = \exp\left(\frac{j2\pi v}{p^k}\right) - \exp\left(\frac{j2\pi w}{p^k}\right).$$

The associated element of $B(\widetilde{\mathbf{c}},\widetilde{\mathbf{e}})$ is

$$\begin{aligned} \widetilde{b}_{i,j} &= \exp\left(\frac{j2\pi(p^k - v)}{p^k}\right) - \exp\left(\frac{j2\pi(p^k - w)}{p^k}\right) \\ &= \exp\left(\frac{-j2\pi v}{p^k}\right) - \exp\left(\frac{-j2\pi w}{p^k}\right) \\ &= b_{i,j}{}^*. \end{aligned} \quad (9)$$

From (9), we can conclude that

$$\begin{aligned} A(\widetilde{\mathbf{c}},\widetilde{\mathbf{e}}) &= B(\widetilde{\mathbf{c}},\widetilde{\mathbf{e}})B(\widetilde{\mathbf{c}},\widetilde{\mathbf{e}})^H \\ &= B(\mathbf{c},\mathbf{e})^*(B(\mathbf{c},\mathbf{e})^*)^H \\ &= B(\mathbf{c},\mathbf{e})^*B(\mathbf{c},\mathbf{e})^T \\ &= (B(\mathbf{c},\mathbf{e})B(\mathbf{c},\mathbf{e})^H)^* \\ &= A(\mathbf{c},\mathbf{e})^*. \end{aligned} \quad (10)$$

Table 1: New good STCCs over finite fields based on the trace criterion.

p^k	n	ϑ	**G**	rank	η_{tr}	η_{det}
3	3	3	[1 1 ; 1 2 ; 2 1]	2	18	—
	3	9	[1 1 1 ; 1 1 2 ; 1 2 1]	3	27	3
	3	27	[1 0 1 2 ; 1 1 1 1 ; 1 1 2 1]	3	33	4.32
	4	3	[1 1 ; 1 1 ; 1 1 ; 1 2]	2	24	—
	4	9	[0 2 1 ; 1 1 1 ; 1 2 1 ; 2 2 1]	3	33	—
	4	27	[2 1 2 2 ; 2 0 2 1 ; 1 1 2 2 ; 2 2 2 1]	4	45	3
5	3	5	[1 1 ; 1 2 ; 2 2]	2	15	—
	3	25	[1 1 1 ; 1 3 2 ; 2 3 1]	3	21.38	1
	4	5	[1 2 ; 1 2 ; 2 1 ; 2 1]	2	20	—
7	3	7	[2 4 ; 3 5 ; 6 1]	2	14	—
	4	7	[1 1 ; 1 2 ; 2 3 ; 3 3]	2	17.19	—

Since $A(\mathbf{c},\mathbf{e})$ is Hermitian, then $A(\mathbf{c},\mathbf{e})$ and $A(\mathbf{c},\mathbf{e})^*$ have the same rank, determinant, and trace. □

Note that by this property it is possible to reduce by approximately one half the number of STCCs to be tested without any sacrifice in terms of finding the best code.

Now, we present the second property, which is also an extension of a result in [6, Theorem 2] to the ring $\mathbb{Z}_{p^k}$.

Property 2. Consider an STTC $\mathcal{C}$ over $\mathbb{Z}_{p^k}$ generated by a generator matrix G. Any STCC over $\mathbb{Z}_{p^k}$ generated by a generator matrix whose rows correspond to a permutation of the rows of G is entirely equivalent to $\mathcal{C}$.

Proof. A permutation of the rows of G implies a permutation of the encoder outputs in Figure 1, and also induces the same permutation of the rows of $B(\mathbf{c},\mathbf{e})$. It is easy to show that the rank, determinant, and trace of the corresponding matrix $A(\mathbf{c},\mathbf{e})$ are not affected by any permutation of the rows of $B(\mathbf{c},\mathbf{e})$. □

Observe that with Property 2 it is possible to obtain a reduction in the code search space by a factor of approximately $n!$. In this paper, we utilised Properties 1 and 2 to reduce the code search effort under the trace criterion, but they can also be applied to the rank and the determinant criteria. The last property, presented next, applies to the trace criterion only.

Property 3. Consider an STCC over $\mathbb{Z}_{p^k}$ generated by a matrix G with coefficients $g_{x,i}$, for $x = 0,1,\ldots,K$ and $i = 1,2,\ldots,n$. Changing the coefficients $g_{x,i}$ of χ rows of G to $p^k - g_{x,i}$, where $1 \le \chi \le n$, does not affect the trace of the matrix $A(\mathbf{c},\mathbf{e})$ for any pair of STCC codewords $\mathbf{c}$ and $\mathbf{e}$.

Proof. Consider a rate $R = 1/n$ convolutional encoder over $\mathbb{Z}_{p^k}$ with scalar generator matrix G. As proved in Property 1, if the coefficients $g_{x,i}$, where $x = 0,1,\ldots,K$, of the ith row of the matrix G are changed to their corresponding complements modulo p^k, that is, $p^k - g_{x,i}$, where $x = 0,1,\ldots,K$, then the ith row of the matrix $B(\mathbf{c},\mathbf{e})$ changes to its complex conjugate. Since $A = BB^H$, then the ith diagonal element $a_{i,i}$ of the matrix A is the sum of the squared modulus of the elements of the ith row of B. Since $|b_{i,j}|^2 = |b_{i,j}{}^*|^2$, and the trace of a matrix is the sum of its diagonal elements, Property 3 is proved. □

By utilising Property 3, it is possible to reduce the code search space by a factor of 2^n. Note that when all rows of G are changed to their corresponding complements modulo p^k, that is, when $\chi = n$, this property becomes Property 1.

It is worth mentioning that the structure of convolutional encoders over $\mathbb{Z}_{p^k}$, adopted in this paper, offers a reduced search space as compared to the structure based on binary inputs. For our structure, the number of possible codes is $p^{kn(K+1)}$, while for the structure with binary input (standard structure) this number is $p^{k^2 n(K+1)}$. This reduction is possible because the structure over $\mathbb{Z}_{p^k}$ yields a smaller number of coefficients. Of course, since we consider a smaller search space, it is possible that in some cases the standard structure will produce better codes. On the other hand, the code search based on the standard structure becomes prohibitive as the number of transmit antennas, states, or constellation size increases, and quite often only partial (nonexhaustive) search results are presented (see, e.g., [12]). The STCCs presented herein have, in many cases, the same performance parameters of the STCCs found with the standard structure for the same number of antennas and the same complexity. For the cases where the STCC is over GF(p), that is, $k = 1$, the structure utilised in this paper becomes the only option.

We should also mention that a computer program routine to discard those equivalent codes, according to the three properties, can be easily prepared. So the cut in the search effort is quite significant.

As a final consideration, we should note that although in this paper we utilise only PSK constellations, quadrature amplitude modulation (QAM) constellations could also be used. However, Properties 1 and 3 would not hold for QAM, and the search space reduction provided by these properties would be lost. On the other hand, Property 2 could still be used without any modification if QAM signal constellations were adopted. It is well known that QAM has better Euclidean distance properties than PSK. So, using

TABLE 2: New good STCCs over finite rings based on the trace criterion.

p^k	n	ϑ	**G**	rank	η_{tr}	η_{det}
4	2	4	[1 1 ; 1 2]	2	10	2
	2	8	[1 1 0 ; 2 1 1]	2	12	3.46
	2	16	[1 1 2 ; 2 1 3]	2	16	3.46
	2	64	[1 0 1 2 ; 1 1 2 1]	2	18	5.29
	3	4	[1 1 ; 1 1 ; 1 2]	2	16	—
	3	8	[3 3 0 ; 1 0 1 ; 1 3 1]	2	18	—
	3	16	[1 1 1 ; 1 2 2 ; 2 1 3]	2	24	—
	3	64*	[2 2 3 3 ; 1 2 1 3 ; 1 1 3 2]	3	32	2.88
	4	4	[1 1 ; 1 1; 1 2 ; 1 2]	2	20	—
	4	8	[1 0 1 ; 1 1 0 ; 1 1 1 ; 1 3 1]	2	26	—
	4	16	[1 1 1 ; 1 1 2 ; 1 2 2 ; 2 1 3]	3	32	—
	4	64*	[1 3 2 3 ; 1 2 1 1 ; 2 2 1 2 ; 3 3 1 0]	4	40	2
8	2	8	[1 2 ; 4 3]	2	7.17	1.41
	2	16	[2 1 0 ; 3 0 1]	2	8	2
	2	64*	[5 1 6 ; 1 1 3]	2	10.58	1.17
	3	8	[1 1 ; 2 2 ; 3 4]	2	12	—
	4	8	[1 1 ; 1 2 ; 2 3 ; 3 4]	2	16.52	—
9	3	9	[1 3 ; 6 4 ; 7 2]	2	12	—

TABLE 3: Comparison of STCCs found with different encoder structures.

p^k	n	ϑ	η_{tr} [12]	η_{det} [12]	η_{tr}	η_{det}
4	2	4	10	2	10	2
	2	8	12	2.82	12	3.46
	2	16	16	2.82	16	3.46
	2	64	18	4	18	5.29
	3	4	16	—	16	—
	3	8	20	—	18	—
	3	16	24	—	24	—
	3	64*	28	—	32	2.88
	4	4	20	—	20	—
	4	8	26	—	26	—
	4	16	32	—	32	—
	4	64*	38	—	40	2
8	2	8	7.17	1.41	7.17	1.41
	2	16	8	0.82	8	2
	3	8	12	—	12	—
	4	8	16.58	—	16.58	—

the encoding structure proposed in this paper, it is possible that STCCs for QAM constellation better than STCCs for PSK constellation of the same size exist. However, since the demonstration of algebraic properties to reduce the code search effort constitutes an important part of this paper, QAM will not be considered herein.

4. CODE SEARCH AND SIMULATION RESULTS

In this section, we present some new STCCs generated by a rate $1/n$ convolutional encoder over $\mathbb{Z}_{p^k}$, and show their performance on the quasistatic flat Rayleigh fading channel. Since we are considering large diversity order, the code search was based only on the trace criterion. Tables 1 and 2 show the search results for STCCs over finite fields and rings, respectively, with various p^k-PSK modulations, number of states (ϑ), and number of transmit antennas (n). In these tables, the STCCs marked with $*$ are the result of a partial search. All other codes are optimal for the structure of Figure 1. In [12], we can find STCCs for the 4 and 8-PSK modulations. For the same number of states and number of transmit antennas, those codes in most cases have the same

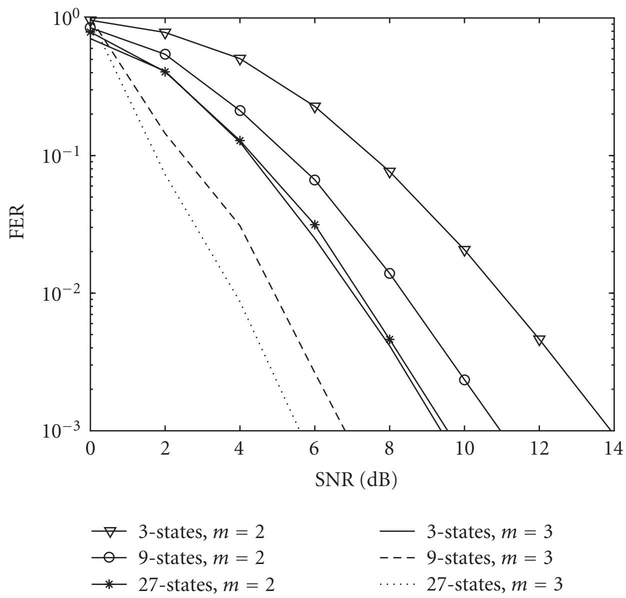

Figure 2: FER versus SNR for the STCCs over $\mathbb{Z}_3$ for 3-PSK based on the trace criterion with $n = 3$, $m = 2, 3$, and 3, 9, and 27 states.

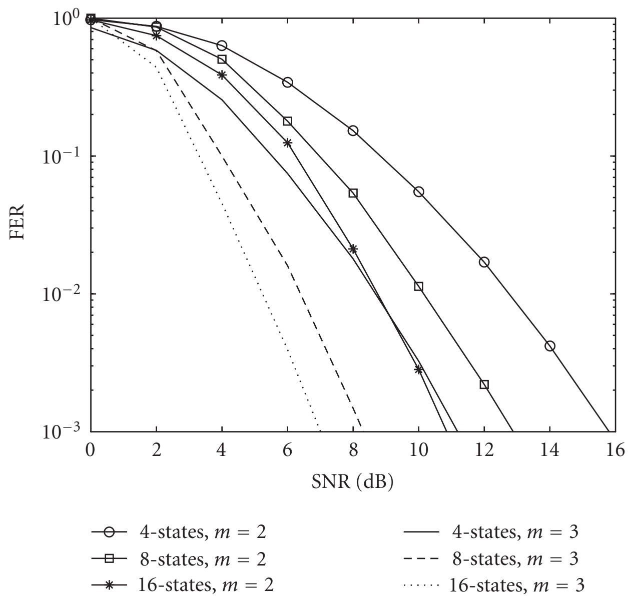

Figure 3: FER versus SNR for STCCs over $\mathbb{Z}_4$ for 4-PSK based on the trace criterion with $n = 3$, $m = 2, 3$, and 4, 8, and 16 states.

trace as the STCCs presented in Table 2. Table 3 compares STCCs for the 4 and 8-PSK modulations found with different structures. It can be seen that with the proposed structure we obtained an STCC with improved trace in two cases, a worse trace in one case, and the same trace in all other cases.

All the new STCCs in Table 1 and the STCC for 9-PSK in Table 2 have no corresponding competitors in the literature. The STCCs over GF(p) with two transmit antennas based on the trace criterion can be found in [9].

In Figures 2 and 3, we show the FER versus SNR (in decibels) curves for the STCCs over GF(3) and $\mathbb{Z}_4$, respectively, where we can observe the performance of the codes for different numbers of states and receive antennas. In Figure 4, we show the performance comparison of the STCCs for 4-PSK found with the encoder structure over $\mathbb{Z}_{p^k}$ and with the standard structure. For $n = 3$ transmit antennas, $m = 2$ receive antennas, and for 4, 8, and 64 states, we can observe that the performances of these codes are very similar, although the codes have been generated by different encoder structures and have different traces in the cases of 8 and 64 states. In all simulations presented in this section, we considered a frame length $l = 130$ symbols.

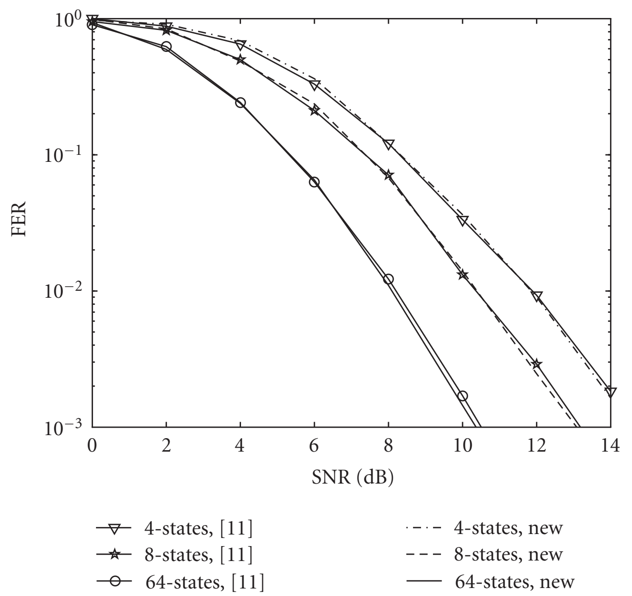

Figure 4: Performance comparison of STCCs for 4-PSK obtained with different encoder structures for $n = 3$, $m = 2$, and 4, 8, and 64 states.

5. CONCLUSION AND FINAL COMMENTS

In this paper, we have considered space-time convolutional codes over finite fields and rings for the quasistatic, flat Rayleigh fading channel. Based on this encoding structure, we proved three properties that can be used to simplify the code search based on the trace criterion. Good STCCs for $n = 2, 3, 4$ transmit antennas and various p^k-PSK constellations were presented. The resulting spectral efficiencies, namely, $\log_2(p^k)$ b/s/Hz, can serve a wide range of multimedia applications.

As the STCCs presented herein are designed by the trace criterion, they do not achieve the optimal diversity-multiplexing gain (DM-G) tradeoff [13, 14] for system with more than one receive antenna. Therefore, it is possible that STTCs constructed to achieve the optimum DM-G tradeoff perform better than the codes in this paper, under the same spectral efficiency.

ACKNOWLEDGMENTS

This work was supported by CEFET/SC under a research grant and by the Brazilian National Council for Scientific and Technological Development (CNPq) under Grants no. 484391/2006-2 and 303938/2007-2.

REFERENCES

[1] V. Tarokh, N. Seshadri, and A. R. Calderbank, "Space-time codes for high data rate wireless communication: performance criterion and code construction," *IEEE Transactions on Information Theory*, vol. 44, no. 2, pp. 744–765, 1998.

[2] Z. Chen, J. Yuan, and B. Vucetic, "Improved space-time time trellis coded modulation scheme on slow Rayleigh fading channels," *Electronics Letters*, vol. 37, no. 7, pp. 440–441, 2001.

[3] Z. Chen, B. Vucetic, J. Yuan, and K. L. Lo, "Space-time trellis codes for 4-PSK with three and four transmit antennas in quasi-static flat fading channels," *IEEE Communications Letters*, vol. 6, no. 2, pp. 67–69, 2002.

[4] S. Baro, G. Bauch, and A. Hansmann, "Improved codes for space-time trellis-coded modulation," *IEEE Communications Letters*, vol. 4, no. 1, pp. 20–22, 2000.

[5] R. S. Blum, "Some analytical tools for the design of space-time convolutional codes," *IEEE Transactions on Communications*, vol. 50, no. 10, pp. 1593–1599, 2002.

[6] B. Abdool-Rassool, M. R. Nakhai, F. Heliot, L. Revelly, and H. Aghvami, "Search for space-time trellis codes: novel codes for Rayleigh fading channels," *IEE Proceedings: Communications*, vol. 151, no. 1, pp. 25–31, 2004.

[7] M. de Noronha-Neto, R. D. Souza, and B. F. Ucha-Filho, "Space-time convolutional codes over GF(p) achieving full 2-level diversity," in *Proceedings of the IEEE Wireless Communications and Networking Conference (WCNC '03)*, vol. 1, pp. 408–413, New Orleans, La, USA, March 2003.

[8] S. K. Hong and J.-M. Chung, "Prime valued space-time convolutionsl Z_w code achieving full 2-level diversity," *Electronics Letters*, vol. 40, no. 4, pp. 253–254, 2004.

[9] M. de Noronha-Neto and B. F. Uchoa-Filho, "Space-time convolutional codes over GF(p) for two transmit antennas," *IEEE Transactions on Communications*, vol. 56, no. 3, pp. 356–358, 2008.

[10] R. A. Carrasco and A. Pereira, "Space-time ring TCM codes for QAM over fading channels," *IEE Proceedings: Communications*, vol. 151, no. 4, pp. 316–321, 2004.

[11] J. L. Massey and T. Mittelholzer, "Convolutional codes over rings," in *Proceedings of the 4th Joint Swedish-Soviet International Workshop on Information Theory*, pp. 14–18, Gotland, Sweeden, August-September 1989.

[12] B. Vucetic and J. Yuan, *Space-Time Coding*, John Wiley & Sons, New York, NY, USA, 2003.

[13] L. Zheng and D. N. C. Tse, "Diversity and multiplexing: a fundamental tradeoff in multiple-antenna channels," *IEEE Transactions on Information Theory*, vol. 49, no. 5, pp. 1073–1096, 2003.

[14] R. Vaze and B. S. Rajan, "On space-time trellis codes achieving optimal diversity multiplexing tradeoff," *IEEE Transactions on Information Theory*, vol. 52, no. 11, pp. 5060–5067, 2006.