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## Research Article

# An Efficient Multibit Aggregation Scheme for Multihop Wireless Sensor Networks

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A single-hop wireless sensor network for distributed detection has been considered in the majority of the existing literature. However, a wireless sensor network for an event detection application with cheap and short range sensors is likely to be a multihop network. Here we consider a distributed detection problem in a multihop wireless sensor network with tree topology. We propose an optimum multibit decision fusion rule derived from the previously known optimum likelihood ratio for a single-hop network with star topology. Subsequently, we present an efficient multibit decision fusion rule for a multihop wireless sensor network with tree topology. Through numerical results, the proposed scheme is shown to achieve a significant improvement in detection accuracy over existing distributed detection schemes.

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#### 1. INTRODUCTION

Wireless sensor network (WSN) [1] is a network formed by a large number of sensor nodes deployed in the area of interest. These sensor nodes can sense, process, and communicate information among themselves to collaboratively perform a particular task. We consider the use of WSN for binary event detection [2–7]. The event of interest could be an intruder crossing the line of control, the rise of pollution level above some threshold, the detection of disasters like landslides [8, 9], fire [10], and volcanoes [11, 12]. The problem of binary event detection is a binary hypothesis testing problem in which the event hypothesis H takes two values  $\{0,1\}$  indicating the nonoccurrence and the occurrence of the event.

For event detection using WSN, we consider the deployment of a large number of inexpensive, albeit less precise, sensor nodes in an application area rather than having a few expensive and more precise sensor nodes. Each sensor node observes some real value corresponding to the phenomenon under consideration; for example, sensors may measure ambient temperature, moisture, humidity, strain, or any other parameter as required by the application. Sensor nodes may then use few (say q) bits to quantize the

information sensed by them. In star topology, nodes transmit the quantized information directly to the sink node, the only fusion center, which makes the final decision.

Distributed detection using multiple sensors with various network topologies has been considered in [4, 5]. A review on various decentralized detection schemes is available in [13]. In [3–5, 14–17], the observation of each sensor and the communication to its parent were considered to be one bit. In [3, 14, 16, 17], the authors considered the WSN with star topology having sensors with different probability of detection (*Pd*) and the probability of false alarm (*Pf*). The Chair-Varshney (CV) rule [14] gives an optimum way to fuse the 1-bit information received at fusion center from every sensor node in a star topology. This requires knowledge of the performance indices (*Pds* and *Pfs*) of all the sensors nodes.

WSN with star topology has limited sensing coverage area, because nodes deployed far from the sink may not be within its communication range. In [18], a wireless sensor network has been considered in a fading environment in the presence of relay nodes. The fusion center receives data from each sensor node in a multihop fashion via the relay nodes. Relay nodes themselves are not sensor nodes. Independent paths composed of series of independent Rayleigh fading

channels have been assumed from each sensor node to the fusion center.

In our work, we consider a tree topology where the sensor nodes lying far from the sink may transmit to the sink in multiple hops via the intermediate sensor nodes. In contrast to [18], the intermediate nodes act as sensors as well as relays. This significantly reduces the communication cost and makes the network more scalable in terms of coverage area. Further in tree topology, every node can be considered as a fusion center and can be used to do some processing on the received data before transmitting. Thus data aggregation in a tree topology is significantly different than that in a star topology.

For a WSN with tree topology, if the sensors detect binary value representing the occurrence of an event, the best scheme, as shown in [2], is when each intermediate node informs its parent about the actual number of its descendants observing "0" and the actual number of its descendants observing "1." Only one of these numbers is required to be transmitted if every node has a priori knowledge of the number of descendants of its each child (however, this may not be feasible in a dynamic network). Thus the sink node has the exact information about the total number of nodes in the entire network detecting "0" and the number of nodes detecting "1." Using this information, the sink node makes the final decision. In a large multihop network, this will require the intermediate nodes to transmit many bits to its parent. Note that even though the hypothesis as well as data sensed by each node is binary, the message transmitted by any intermediate sensor node is not binary in this case.

In this paper, for a tree topology, every sensor node quantizes its sensed data using q-bits. Each leaf node transmits its *q*-bit quantized information to its parent node. The intermediate node fuses the *q*-bit information sensed by itself with the information received from its children to make a summary of *l* bits for transmission to its parent. In general, l may be different for different nodes. But throughout this paper, we consider l to be the same for all the nodes and also the same as q. Henceforth, the q-bit summary information generated by each node or the q-bit quantized information of the leaf nodes will be called their decision. On receiving the decision information from all its children, the sink node makes the final 1-bit decision about the event. Since communication requires more power than computation [19], to save power, it is necessary to transmit as less number of bits as possible. However, the detection accuracy of the sink is expected to increase with the increase in q. Our numerical and simulation results in Section 5 will show this tradeoff.

We consider the problem of designing an efficient multibit (q-bit) decision fusion scheme, implemented at every node in a tree topology, so as to maximize the detection accuracy of the sink node. In [2], a 1-bit aggregation scheme based on the majority rule with a specific routing scheme was proposed for *tree topology*. Here a parent node takes the majority of its children's decision as its own decision. In [6], the weighted aggregation scheme (WAS) for WSN with tree topology has been proposed with q = 1. In

WAS, any non-leaf node weights the decision made by its each child with the minimum mean square error (MMSE) estimate of the number of descendants of that child deciding in the favor of event. In [6], it is assumed that every sensor node has knowledge of the number of descendants of its each child. In [7], a multibit generalization to WAS, *q*-bit weighted aggregation scheme (*q*-WAS), requiring *q* bits of transmission from any node to its parent was proposed. This provided a lifetime-accuracy tradeoff by varying *q*.

In this paper, we present a two-fold generalization of existing optimum 1-bit CV fusion rule: 1-bit to q-bits (q-CV) and star to tree topology. We first propose the optimum multibit fusion rule (q-CV) derived from likelihood ratio for a single-hop network with star topology. This enables the fusion center to optimally fuse q-bit information received from its each child to make either 1-bit decision or a q-bit summary as required. The multibit fusion scheme is then developed for WSN with tree topology which is a more generic topology as compared to star topology.

Different definitions of network lifetime have been considered in literature [2, 6, 20]. In this paper, we consider three different measures of *network lifetime*. The simplest measure of network lifetime that we consider is where a network is considered to be alive till all the nodes in network are alive. Since in our schemes all the nodes transmit equal number of bits during data aggregation and we assume the transmission of equal power of each bit and each node, all the nodes are expected to have approximately the same node lifetime. We also consider another measure of network lifetime where a network is alive till 50% of the sensor nodes die or run out of power. Yet another measure of network lifetime we considered is where the network is considered alive as long as the detection accuracy is above a fixed threshold. The details of the network lifetime simulations are presented in Section 5.2.

The majority rule performs well when the tree is balanced [2] and all the sensor nodes have equal sensing accuracies because then the majority rule is equivalent to the optimum scheme. However, if the tree is not balanced, the performance of the majority rule will degrade from the optimum performance, [2, 20]. The majority rule is also not optimum when the different sensor nodes have different sensing accuracies. In either case, of an unbalanced tree or different sensing accuracies of the nodes, our scheme is expected to perform better than the majority rule.

In Section 2, we derive an optimum multibit fusion rule (q-CV) for a star topology. This is precursor to deriving a multibit q-CV fusion rule for tree topology in Section 3. Section 4 considers selection of the thresholds required in the proposed q-CV fusion rule. In Section 5, we present numerical and simulation results to compare the performance of the proposed fusion rule with the existing aggregation schemes in terms of detection accuracy, network lifetime, and perturbation analysis. It is seen that multibit q-CV fusion rule for tree topology has the highest accuracy (see Figure 9) among the known fusion rules. Finally, we conclude the paper in Section 6.

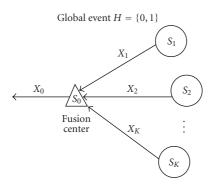


FIGURE 1: Star topology.

#### 2. OPTIMUM MULTIBIT DECISION FUSION RULE

In Section 2.1, we develop the optimum multibit fusion rule (q-CV) for single-hop WSN. In what follows, we first present the existing optimum 1-bit CV rule [14] for a star topology (see Figure 1).

The binary event, represented by binary random variable  $H \in \{0,1\}$ , is assumed to be observed by all sensors in an application area. Each sensor  $S_i$ , for i = 1, 2, ..., K, has 1-bit quantized information  $X_i \in \{0,1\}$  about the occurrence of the event with the probability of detection  $Pd_i = P(X_i = 1 \mid H = 1)$  and the probability of false alarm  $Pf_i = P(X_i = 1 \mid H = 0)$ .

In the existing optimum 1-bit CV rule [14], the fusion center  $S_0$  makes a 1-bit decision  $X_0$  to detect an event by thresholding

$$\Lambda_{\text{CV}} = \log \frac{P(H=1)}{P(H=0)} + \sum_{i=1}^{K} \left[ X_i \log \frac{Pd_i}{Pf_i} + (1 - X_i) \log \frac{1 - Pd_i}{1 - Pf_i} \right].$$
 (1)

The 1-bit decision  $X_0$  of  $S_0$  can be obtained using

$$X_0 = 1$$

$$\Lambda_{\text{CV}} \gtrsim T,$$

$$X_0 = 0$$
(2)

where T is the threshold which can be varied to achieve a tradeoff between the probability of detection  $Pd_0$  and the probability of false alarm  $Pf_0$  of the fusion center  $S_0$ .

## 2.1. Proposed optimum multibit fusion rule (q-CV)

The optimum multibit fusion rule (q-CV) for single-hop WSN (see Figure 1) is derived in this subsection. Here each node quantizes its observation using q-bits to make a q-bit quantized local decision about the occurrence of the binary event and transmits the same to the fusion center. The q-bit decision of a node has not only information of the binary event hypothesis as inferred by it but also has information of its confidence in deciding the event hypothesis.

In *q*-CV scheme, each node  $S_i$  (refer to Figure 1) transmits its *q*-bit decision  $X_i \in \{0, 1, ..., 2^q - 1\}$  to its parent

 $S_0$ . We represent  $X_i$  as  $(X_{i,1}, X_{i,2}, \dots, X_{i,q})$ , where  $X_{i,j} \in \{0, 1\}$  is the jth bit in  $X_i$ . Here  $X_{i,1}$  provides the binary event detection made by node  $S_i$ , while the remaining bits in  $X_i$  denote the confidence on the detection  $X_{i,1}$  made by node  $S_i$ .

If  $X_i$  is one bit, then probability of detection  $Pd_i = P(X_i = 1 \mid H = 1)$ , probability of false alarm  $Pf_i = P(X_i = 1 \mid H = 0)$ , probability of missed detection  $Pm_i = P(X_i = 0 \mid H = 1) = 1 - Pd_i$ , and probability of correct no event detection  $Pn_i = P(X_i = 0 \mid H = 0) = 1 - Pf_i$  are the performance indices of the decision  $X_i$ . These parameters should be known for optimum 1-bit fusion rule (CV rule). In the proposed q-CV ( $X_i$  is q-bits) rule, we require the knowledge of performance indices  $P_{mn}^i = P(X_i = m \mid H = n)$  for  $m \in \{0, 1, \dots, 2^q - 1\}$  and  $n \in \{0, 1\}$  about decision  $X_i$  of the sensor node  $S_i$ .

For notational convenience, we denote P(H=0) and P(H=1) by  $P(H_0)$  and  $P(H_1)$ , respectively. Optimum log-likelihood ratio (LLR) test [14, 21, 22] for binary detection is given by

$$\Lambda = \log \frac{P(H_1 \mid \underline{X})}{P(H_0 \mid \underline{X})} \stackrel{X_0 = 1}{\underset{X_0 = 0}{\geq}} T, \tag{3}$$

where  $\underline{X} = [X_1, \dots, X_K];$ 

$$P(H_1 \mid \underline{X}) = \frac{P(H_1)P(\underline{X} \mid H_1)}{P(\underline{X})}$$

$$= \frac{P(H_1)P(X_1, \dots, X_K \mid H_1)}{P(\underline{X})}.$$
(4)

Since  $X_1, X_2, \dots, X_K$  are independent, we have

$$P(H_{1} \mid \underline{X}) = \frac{P(H_{1})}{P(\underline{X})} P(X_{1} \mid H_{1}) \cdots P(X_{K} \mid H_{1})$$

$$= \frac{P(H_{1})}{P(\underline{X})} \prod_{L_{o}} P_{01}^{i} \prod_{L_{1}} P_{11}^{i} \cdots \prod_{L_{1}(2^{q}-1)} P_{(2^{q}-1)1}^{i},$$
(5)

where  $L_i = \{i \mid X_i = j\}$  for i = 1, 2, ..., K. Similarly,

$$P(H_0 \mid \underline{X}) = \frac{P(H_0)}{P(\underline{X})} \prod_{L_0} P_{00}^i \prod_{L_1} P_{10}^i \cdots \prod_{L_{(p^q-1)}} P_{(2^q-1)0}^i.$$
 (6)

Using (3), we get

$$\Lambda = \log \frac{P(H_1)}{P(H_0)} + \sum_{L_0} \log \frac{P_{01}^i}{P_{00}^i} + \sum_{L_1} \log \frac{P_{11}^i}{P_{10}^i} + \cdots + \sum_{L_{(2^q-1)}} \log \frac{P_{(2^q-1)1}^i}{P_{(2^q-1)0}^i}.$$
(7)

Representing the likelihood ratio for q-CV by  $\Lambda_{CVq}$ , we get

$$\Lambda_{CVq} = \Lambda$$

$$= \log \frac{P(H_1)}{P(H_0)} + \sum_{i=1}^{K} \log \frac{P_{b_i 1}^i}{P_{b_i 0}^i},$$
(8)

where  $b_i \in \{0, 1, ..., 2^q - 1\}$  is the value taken by  $X_i$ . The fusion center  $S_0$  makes q-bit decision  $X_0$  by comparing  $\Lambda_{\text{CV}q}$ ,

from (8), with some thresholds  $T_0, T_1, \ldots, T_{(2^q-2)}$ ,

$$X_{0} = \begin{cases} 0, & \Lambda_{\text{CV}q} < T_{0}, \\ 1, & T_{0} \le \Lambda_{\text{CV}q} < T_{1}, \\ \vdots & \vdots \\ 2^{q} - 1 & \Lambda_{\text{CV}q} \ge T_{(2^{q} - 2)}, \end{cases}$$
(9)

The thresholds  $T_0, T_1, \ldots, T_{(2^q-2)}$  can be obtained according to the threshold selection criteria presented in Section 4. As far as binary event detection in a single-hop WSN is concerned, the *q*-bit decision of fusion center not only detects the event but also gives an extra information about the confidence of its decision. The presented multibit fusion rule gives an efficient way to increase the detection accuracy of the fusion center by simply increasing "q" that is number of bits transmitted by each node. This provides a tradeoff between detection accuracy and network lifetime which is shown in the results presented in Section 5.

## **2.2.** Special case of q = 2

In the proposed 2-CV scheme, for a star topology (see Figure 1) each node  $S_i$  transmits its 2-bit decision  $X_i$  to its parent  $S_0$ . Thus  $X_i \in \{00,01,10,11\}$  or  $X_i \in \{0,1,2,3\}$ . Here  $X_i = 3$  and  $X_i = 2$  represent decisions in the favor of the occurrence of the event with more and less confidence, respectively, while  $X_i = 0$  and  $X_i = 1$  represent decisions in the favor of the nonoccurrence of the event with more and less confidence, respectively. In 2-CV, the knowledge of the performance indices  $P_{mn}^i = P(X_i = m \mid H = n)$  for  $m \in \{0,1,2,3\}$  and  $n \in \{0,1\}$  about decision  $X_i$  of the sensor node  $S_i$  is required. That is, the fusion center  $S_0$  must know the following eight parameters for its each child  $S_i$  for  $i = 1,2,\ldots,K$ :

probability of strong detection =  $Pds_i = P_{31}^i$ , probability of weak detection =  $Pdw_i = P_{21}^i$ , probability of strong false alarm =  $Pfs_i = P_{30}^i$ , probability of weak false alarm =  $Pfw_i = P_{20}^i$ , probability of strong miss =  $Pms_i = P_{01}^i$ , probability of weak miss =  $Pmw_i = P_{11}^i$ , probability of strong no false alarm =  $Pnfs_i = P_{00}^i$ , probability of weak no false alarm =  $Pnfw_i = P_{10}^i$ .

The likelihood ratio for 2-CV, using (8), is given by

$$\Lambda_{\text{CV2}} = \log \frac{P(H_1)}{P(H_0)} + \sum_{i=1}^{K} \log \frac{P_{b_i 1}^i}{P_{b_i 0}^i}, \tag{10}$$

where  $b_i \in \{0, 1, 2, 3\}$  is the value taken by  $X_i$ . Using  $\Lambda_{CV2}$ , from (10), fusion center  $S_0$  can make a 2-bit decision

$$X_{0} = \begin{cases} 0, & \Lambda_{\text{CV2}} < T_{0}, \\ 1, & T_{0} \le \Lambda_{\text{CV2}} < T_{1}, \\ 2, & T_{1} \le \Lambda_{\text{CV2}} < T_{2}, \\ 3, & \Lambda_{\text{CV2}} \ge T_{2}, \end{cases}$$
(11)

where thresholds  $T_0$ ,  $T_1$ , and  $T_2$  can be selected as shown in Section 4.

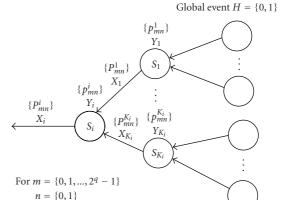


FIGURE 2: A local view of a tree topology showing an intermediate node  $S_i$  with its  $K_i$  children  $S_1, S_2, \ldots, S_{Ki}$ . For any node  $S_i$ ,  $Y_i$  represents its q-bit observation while  $X_i$  corresponds to its q-bit decision.

## 3. MULTIBIT DECISION FUSION RULE FOR TREE TOPOLOGY

The optimum multibit (q-bit) fusion rule (q-CV) for star topology had been derived in Section 2, and is given by (8) and (9). In this section, we develop the multibit decision fusion rule (q-CV) for tree topology.

#### 3.1. Proposed q-CV fusion rule for tree topology

A local view of a tree topology is shown in Figure 2, in which  $S_i$  is any intermediate node having  $K_i$  children  $S_1, S_2, \ldots, S_{K_i}$ . In a tree topology, any non-leaf node has the following function: it senses the data (observation) and fuses the data sensed by itself with the data received from its children to form a summary, and it forwards the summary to its parent (data transmission), while a leaf node observes the data and transmits it to its parent.

Here every node  $S_i$  quantizes its sensed data using q bits to obtain its q-bit quantized observation  $Y_i \in \{0, 1, ..., 2^q - 1\}$ 1}. We represent  $Y_i$  as  $(Y_{i,1}, Y_{i,2}, ..., Y_{i,q})$ , where  $Y_{i,j} \in \{0, 1\}$ is the jth bit in  $Y_i$ . The performance indices associated with  $Y_i$  are  $p_{mn}^i = P(Y_i = m \mid H = n)$  for  $m \in \{0, 1, ..., 2^q - 1\}$ and  $n \in \{0,1\}$ . Further, each node (say  $S_i$ ) acts as a fusion center to make its q-bit decision  $X_i \in \{0, 1, ..., 2^q - 1\}$ using the proposed q-bit optimum fusion rule (q-CV). Recall that *q*-bit decision  $X_i = (X_{i,1}, X_{i,2}, \dots, X_{i,q})$  of node  $S_i$ , where  $X_{i,j} \in \{0,1\}$  is the jth bit in  $X_i$ . The performance indices associated with  $X_i$  are  $P_{mn}^i = P(X_i = m \mid H = n)$  for  $m \in \{0, 1, \dots, 2^q - 1\}$  and  $n \in \{0, 1\}$ . To make local optimum q-bit decision  $X_i$ , node  $S_i$  uses the proposed optimum q-bit fusion rule (q-CV). The likelihood ratio for q-CV fusion rule for star topology has been given by (8). In a tree topology, the likelihood ratio for q-CV fusion rule at any node  $S_i$  having  $K_i$ children takes the form

$$\Lambda_{\text{CV}q} = \log \frac{P(H_1)}{P(H_0)} + \log \frac{p_{a_11}^i}{p_{a_i0}^i} + \sum_{j=1}^{K_i} \log \frac{P_{b_j1}^j}{P_{b_i0}^j}, \tag{12}$$

where  $a_i \in \{0, 1, ..., 2^q - 1\}$  is the value taken by  $Y_i$ , and  $b_j \in \{0, 1, ..., 2^q - 1\}$  is the value taken by  $X_j$ . The fusion center  $S_i$  makes its q-bit decision  $X_i$  by comparing  $\Lambda_{CVq}$  with the thresholds  $T_0, T_1, ..., T_{(2^q-2)}$  using (9).

Thus each node  $S_i$  requires the knowledge of  $p_{mn}^i$  and  $P_{mn}^J$  of its each child  $S_j$ , for  $j = 1, ..., K_i$ ,  $m \in \{0, 1, ..., 2^q - 1\}$ , and  $n \in \{0, 1\}$ . At a first glance, it looks difficult to have the information about the performance indices associated with the decision made by every sensor node practically available. However, as will be seen latter, for implementing q-CV for tree structure, we can compute the performance indices for every node's decision, in a hierarchical manner, starting from the leaf nodes to the sink node.

## Computation of performance indices

Here we show how the performance indices  $P_{mn}^i$ , for any node  $S_i$ , can be computed when the performance indices,  $p_{mn}^i$  of its observation and the performance indices  $P_{mn}^j$  of the decision made by its each child  $S_j$ ,  $j = 1, 2, ..., K_i$ , are known.

For any  $0 \le m \le 2^q - 1$ , let  $\mathcal{X}_m$  be the set of values  $\underline{X} = [Y_i, X_1, X_2, \dots, X_{K_i}] \in \{0, 1, \dots, 2^q - 1\}^{K_i + 1}$  which gives  $X_i = m$ . Then for any  $n \in \{0, 1\}$ , we have

$$P_{mn}^{i} = P(X_{i} = m \mid H = n)$$

$$= \sum_{(a_{i},b_{1},...,b_{K_{i}}) \in \mathcal{X}_{m}} (P(Y_{i} = a_{i} \mid H = n) P(X_{1} = b_{1} \mid H = n))$$

$$\cdot \cdot \cdot P(X_{K_{i}} = b_{K_{i}} \mid H = n))$$

$$= \sum_{(a_{i},b_{1},...,b_{K_{i}}) \in \mathcal{X}_{m}} \left( p_{a_{i}n}^{i} \prod_{j=1}^{K_{i}} P_{b_{j}n}^{j} \right).$$
(13)

During the *initial setup*, the performance indices associated with the decision made by each node are computed starting from leaf nodes to the sink node. Further, each node transmits its performance indices to its parent as soon as they are computed. In a static network, there is no change in the topology and the performance indices of the observation made by the nodes. This leads to fixed performance indices of the decisions made by all of the nodes. Thus for a *static topology*, the computation and transmission of the performance indices happen only once, that is, during the initial setup.

The topology of a sensor network may change because of the death of the nodes which occurs due to exhaustion of the battery power or hardware failure of the nodes. The change in topology due to the elimination of a single node in a tree topology may result in changes in the performance indices of several nodes. Further, the performance indices of the observation made by the nodes may also vary with time, which in turn results in a variation in the performance indices of all their successors. Thus as nodes die, either the performance indices should be computed and transmitted periodically throughout the network, or the algorithm has to run with the original, but incorrect, values of performance indices. While the first option adds periodic transmission

overhead and thus reduces the network lifetime, the second option compromises the accuracy.

#### Data transfer in a session

After the *initial setup*, a *q*-bit decision is made by each node, in a hierarchical manner, using (12) and (9) which requires precomputed values of the performance indices and the *q*-bit decisions received from its children. The decision made by a node is further transmitted to its parent. This process starts from leaf nodes and is continued till the sink node makes the final decision about an event. This process of the decision making and the data transfer by all the nodes will be termed as a *session* in a tree topology.

#### Performance evaluation

To evaluate the performance of the algorithm, we now compute the performance indices  $P_{mn}^{\rm sink}$  of the decision made by the sink node using (13). Let the final q-bit decision made by sink node be  $X_{\rm sink}$ . We represent  $X_{\rm sink}$  as  $(X_{\rm sink,1},X_{\rm sink,2},\ldots,X_{\rm sink,q})$ , where  $X_{\rm sink}$ ,  $j \in \{0,1\}$  is the jth bit in  $X_{\rm sink}$ . The most significant bit  $X_{\rm sink,1}$  of  $X_{\rm sink}$  indicates the detection decision made by the sink node. Thus  $PD = P(X_{\rm sink,1} = 1 \mid H = 1)$  is the probability that the sink node makes a decision in favor of the event given that the event has actually occurred (H = 1), while  $PF = P(X_{\rm sink,1} = 1 \mid H = 0)$  is the probability that the sink node makes a decision in favor of the event when the event has actually not occurred (H = 0). These system level probability of detection PD and probability of false alarm PF can be computed using the performance indices  $P_{mn}^{\rm sink}$  of the sink node using

$$PD = \sum_{j=2^{(q-1)}}^{2^{q}-1} P_{j1}^{\text{sink}},$$

$$PF = \sum_{j=2^{(q-1)}}^{2^{q}-1} P_{j0}^{\text{sink}}.$$
(14)

We define the accuracy of the sink's decision,

Accuracy = 
$$0.5PD + 0.5(1 - PF)$$
, (15)

as another performance evaluating parameter. The equal weightage to maximize PD and minimize PF suggests its suitability for the applications where false alarms are equally intolerable as is the loss of probability of detection. In general, these weights should be chosen based on the relative significance of PD and PF in the particular application.

The multibit (q-CV) fusion rule presented for the tree topology locally optimizes the decision made by intermediate nodes. However, it may not be the globally optimum fusion rule for a tree topology.

#### Precision of sensors

For performance comparison with the algorithms presented in [2, 6], in numerical results (see Section 5), we consider

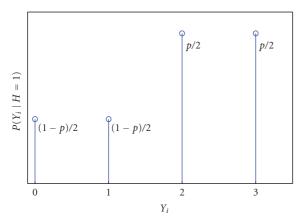


FIGURE 3: Probability mass function  $P(Y_i \mid H = 1)$  of 2-bit observation  $Y_i$  made by sensor node  $S_i$ . Here p is the precision of sensor and in general p > 0.5. For the above plot, an example value of precision p = 0.7 has been considered without the loss of generality.

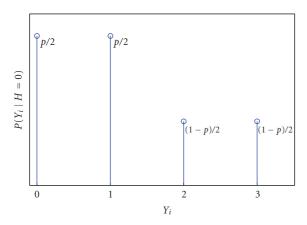


FIGURE 4: Probability mass function  $P(Y_i \mid H = 0)$  of 2-bit observation  $Y_i$  made by sensor node  $S_i$ . Here p is the precision of sensor and in general p > 0.5. For the above plot, an example value of precision p = 0.7 has been considered.

all sensors forming a tree topology to be *equally* precise with known precision p. Here the precision of a sensor indicates its probability of correct detection for both the possibilities of the binary hypothesis. Thus, the precision  $p = P(Y_{i,1} = 1 \mid H = 1) = P(Y_{i,1} = 0 \mid H = 0)$  for any sensor  $S_i$ . At sensor  $S_i$ , its q-bit quantized observation  $Y_i$  follows the distributions  $P(Y_i \mid H = 1)$  (see Figure 3), and  $P(Y_i \mid H = 0)$  (see Figure 4). For q = 2, we hence assume the following for each sensor  $S_i$ :

$$P(Y_{i} = 3 \mid H = 1) = P(Y_{i} = 2 \mid H = 1) = \frac{p}{2},$$

$$P(Y_{i} = 0 \mid H = 1) = P(Y_{i} = 1 \mid H = 1) = \frac{1 - p}{2},$$

$$P(Y_{i} = 3 \mid H = 0) = P(Y_{i} = 2 \mid H = 0) = \frac{1 - p}{2},$$

$$P(Y_{i} = 1 \mid H = 0) = P(Y_{i} = 0 \mid H = 0) = \frac{p}{2}.$$
(16)

When all the nodes in a tree topology have the same precision, then all the leaf nodes will have the same performance indices for their decisions. This is because the decision made by any leaf node is the same as its observation. However, the performance indices associated with the decision made by the non-leaf nodes may be different because of an *unbalanced* tree. This is because in a tree topology each node's decision is influenced by the decision made by its descendants and the topology formed by those descendants.

The parametric methods, like the CV rule, are known to result in a better performance than the majority rule even in the star topology when the nodes have different performance indices [14]. The majority rule is also known to perform suboptimally in a tree topology when the tree is unbalanced [2, 20]. Since the tree topology obtained by randomly deploying nodes in practice is rarely balanced, this motivates the use of the proposed q-CV fusion rule in a tree topology. Thus the proposed scheme will also work well for the tree topology when nodes have different precision. However, their performance indices are required to be known in any case to implement q-CV fusion rule.

## 4. THRESHOLD SELECTION

The present section considers the selection of the thresholds,  $T_0, T_1, \ldots, T_{2^q-2}$ , that are required in the proposed q-CV fusion rule given by (8) and (9). We first discuss the threshold selection for q = 2 and later generalize it for any value of q. In our numerical results (see Section 5), the threshold selection is done as described here.

The (q-1) most significant bits in q-CV are the same as the decision in (q-1)-CV rule. Thus, from [4], we know that the optimum threshold T for CV rule and the optimum threshold  $T_1$  for the most significant bit of decision using 2-CV are given by

$$T_1 = T = \log \frac{P(H_0)}{P(H_1)} + \log \frac{(C_{10} - C_{00})}{(C_{01} - C_{11})},$$
 (17)

where  $C_{ij}$ , for  $i = \{0, 1\}$  and  $j = \{0, 1\}$  is the cost of deciding H = i given that H = j has occurred. For example, if we consider P(H = 1) = P(H = 0) = 0.5,  $C_{11} = C_{00} = 0$  (no cost for correct detection), and  $C_{10} = C_{01}$  (equal cost for false alarm and miss detection), the optimum thresholds are  $T_1 = T = 0$ .

Figures 5 and 6 show the simulated probability mass function (PMF) of  $\Lambda_{\rm CV}$  (1) for 1-bit CV rule and that of  $\Lambda_{\rm CV2}$  (10) for 2-bit 2-CV rule, respectively. Here we consider the star topology with K=10 nodes. The binary hypothesis H is assumed to be equiprobable. For 1-CV, each node  $S_i$  transmits its 1-bit decision with  $Pd_i \in [0.8,1]$  and  $Pf_i \in [0,0.2]$  to the fusion center; while for 2-CV,  $S_i$  transmits its 2-bit decision with  $Pds_i = Pdw_i = (Pd_i/2) \in [0.4,0.5]$  and  $Pfs_i = Pfw_i = (Pf_i/2) \in [0,0.1]$  to the fusion center. PMF of  $\Lambda_{\rm CV2}$  shows a better separation of plots on both sides of the threshold " $T_1$ " as compared to the separation of plots on both sides of the threshold " $T_1$ " in the PMF of  $\Lambda_{\rm CV}$ . Thus 2-CV is expected to achieve better accuracy as compared to CV rule.

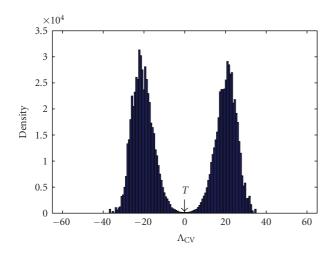


FIGURE 5: Simulated probability mass function (PMF) of  $\Lambda_{CV}$  considering P(H=0) = P(H=1).

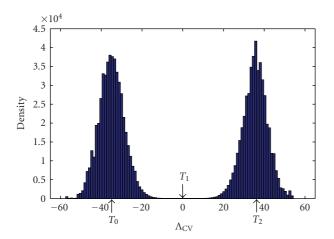


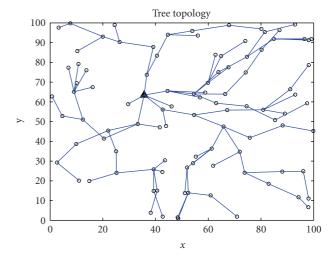
FIGURE 6: Simulated probability mass function (PMF) of  $\Lambda_{CV2}$  considering P(H=0)=P(H=1).

From (17), for 2-CV, it is not clear how the thresholds  $T_0$  and  $T_2$  should be chosen to result in best global performance. We suggest selecting the thresholds  $T_0$  and  $T_2$  as the medians of the conditional PMF of  $\Lambda_{\rm CV2}$  given  $\Lambda_{\rm CV2} < T_1$  and the median of conditional PMF of  $\Lambda_{\rm CV2}$  given  $\Lambda_{\rm CV2} > T_1$ , respectively. This selection of thresholds maximizes the information carried by the second bit in  $X_0$  about the likelihood ratio  $\Lambda_{\rm CV2}$ . Under this selection criterion, the thresholds  $T_0$  and  $T_2$  satisfy  $P(\Lambda_{\rm CV2} < T_0) = P(T_0 \le \Lambda_{\rm CV2} < T_1)$  and  $P(T_1 \le \Lambda_{\rm CV2} < T_2) = P(\Lambda_{\rm CV2} \ge T_2)$ , respectively.

Similarly, for q-CV, the threshold  $T_{2^{(q-1)}-1}$  for the most significant bit is chosen, as in (17), using

$$T_{2^{(q-1)}-1} = \log \frac{P(H_0)}{P(H_1)} + \log \frac{(C_{10} - C_{00})}{(C_{01} - C_{11})}.$$
 (18)

The remaining thresholds,  $T_0, T_1, \ldots, T_{2^q-2}$  (except  $T_{2^{(q-1)}-1}$ ), are chosen by dividing the conditional PMF of  $\Lambda_{CVq}$  given the



- Node
- ▲ Sink node

FIGURE 7: The spanning tree as a result of Bellman-Ford routing algorithm on 100 nodes deployed with uniform distribution.

most significant bit is 0 or 1 as appropriate, in equal parts. Thus the thresholds are selected so that the probabilities  $P(\Lambda_{\text{CV}q} < T_0)$  and  $P(T_i \le \Lambda_{\text{CV}q} < T_{i+1})$ , for  $i = 0, 1, \ldots, 2^{(q-1)} - 2$ , are equal, and the probabilities  $P(T_i \le \Lambda_{\text{CV}q} < T_{i+1})$ , for  $i = 2^{(q-1)} - 1, 2^{(q-1)}, \ldots, 2^q - 3$  and  $P(\Lambda_{\text{CV}q} \ge T_{2^q-2})$  are equal.

## 5. NUMERICAL AND SIMULATION RESULTS

In the current setup, 100 nodes are deployed with uniform distribution in a square area of 100 square units. We use the Bellman-Ford routing algorithm to obtain the spanning tree, as in [2, 6, 7], considering the sink node as the final data aggregation node. The range of communication for every node is fixed based on the maximum transmission power. This is assumed to be equal for all nodes. For a given node, the nodes which fall within its communication range become its neighbors. Among its neighbors, each node selects its parent in order to minimize the sum of the link costs associated with the path chosen to reach the sink node. Each link (i, j) is associated with the cost  $C_{ij} =$  $I_i/B_i$  [2], where  $I_i$  is the total number of nodes capable of transmitting to the node  $S_i$ , and  $B_i$  is the remaining battery power of the node  $S_i$ . As suggested in [2], the selection of this cost balances the tree to some extent, which helps the *counting* rule (*majority* rule) to perform reasonably well. However, WAS and the proposed q-CV fusion rule are independent of the link cost used for tree formation since they perform equally well for unbalanced trees. Figure 7 shows an example of the resulting spanning

The threshold in 1-bit CV rule can be either fixed for all the nodes or variable for different nodes in a tree topology. We thus categorize CV rule for tree topology as follows.

- (1) CV with fixed threshold (CV-FT). Here the threshold *T* at every node is considered to be the same and can be fixed to a value which results in the best possible accuracy of the sink node.
- (2) CV with variable threshold (CV-VT). Here every sensor node  $S_i$  throughout the tree uses a different threshold  $T_i$ . We do the local optimization at each node by selecting a threshold which maximizes the local accuracy of the decision made by that particular node. We believe that this may also have an impact on the detection accuracy of the sink node. However, even CV-VT may not provide the global optimal solution.

For numerical and simulation results, we consider a binary event hypothesis equiprobable in 0 and 1. All 100 nodes deployed in the area are assumed to observe the event with the same precision p. The precision of a sensor is defined in Section 3. In our numerical results, we vary p from 0.55 to 0.95, in steps of 0.05. In particular, we compare the proposed 2-bit CV rule (2-CV) and 1-bit CV rule (CV-FT and CV-VT) with the existing weighted aggregation scheme (WAS) and the *counting* rule in terms of numerically computed *accuracy* and system level PD and PF.

## 5.1. Detection performance

The numerically computed *PD/PF* and *accuracy* plots are shown in Figures 8 and 9, respectively. The results presented here are averaged over ten different random deployments, where in each deployment a tree topology is formed by Bellman-Ford routing algorithm.

Accuracy plots in Figure 9 show that the proposed *q*-CV results in the highest accuracy among all the other schemes. We further observe that even with the use of sensors with precision as less as 65%, the accuracy obtained by 2-CV is close to 100%. It can be inferred from Figure 9 that among all 1-bit aggregation schemes, both CV-FT and CV-VT show significant improvement in accuracy for a tree topology compared to the existing 1-bit aggregation schemes. The *PD*, *PF* plots in Figure 8 show that 2-CV results in system level probability of detection almost equal to "1" and the probability of false alarm equal to "0" when the nodes have precision more than 65%.

The results further demonstrate that increasing the number of bits transmitted with q-CV rule results in a significant gain in accuracy of the system. This gain in accuracy for the multibit scheme (q-CV) comes at the cost of network lifetime since it requires the transmission of almost q times the number of bits required for 1-bit aggregation schemes in the data aggregation sessions. The overhead due to routing is approximately the same for all the aggregation schemes. However, in disaster detection applications, we find the application of the more accurate multibit scheme preferable, since the cost incurred in the loss of life and property is of great importance.

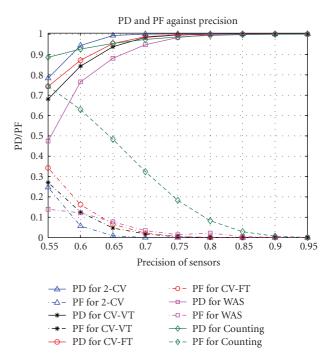


FIGURE 8: Numerical plots of system level probability of detection (PD), probability of false alarm (PF) with respect to precision of sensors p for various aggregation schemes.

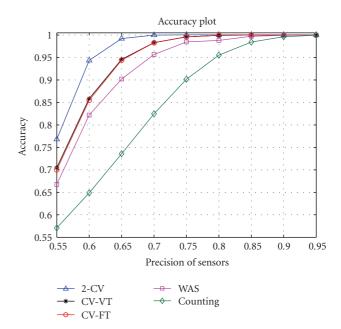


FIGURE 9: Numerical plots of detection *accuracy* with respect to precision of sensors *p* for various aggregation schemes.

## 5.2. Network lifetime-accuracy tradeoff

Each node in a network is considered to be alive till it has enough battery power for its operation. In simulations, we do not consider factors like hardware failure to be responsible

for the death of nodes. Network lifetime has been defined in literature [2, 6, 20] in various ways. They are as follows.

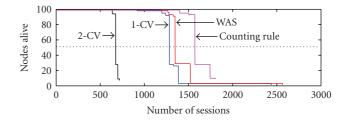
- (1) *NL1*. In [6], we considered network lifetime to be the duration for which all the nodes in a network are alive. Thus network lifetime *NL1* is considered to be the time till the death of the first node in a network.
- (2) *NL2*. As considered in [2], *NL2* refers to the duration for which more than 50% of the nodes in a network are alive.
- (3) *NL3*. In [20], the network lifetime has been defined as the duration for which the probability of error delivered by a network is below a certain error threshold. This ensures the delivery of a desired level of performance by a network during the course of its entire lifetime. Here we consider the network lifetime *NL3* to be the duration for which the detection accuracy is above some accuracy threshold.

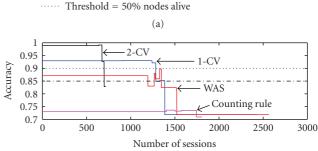
In a simulation setup, we consider a deployment of 100 nodes, each with an initial battery power of 300 000 units in a square area. A fixed transmission power of 300 units and reception power of 100 units is considered for the simulation of network lifetime. Bellman-Ford routing algorithm along with computation and communication of the parameters necessary for the fusion rule is done during the initial setup. For instance, in the existing weighted aggregation scheme [6], the update of the total number of descendants of each node is provided by each child node to its parent. In the proposed *q*-CV fusion rule, the performance indices of the decision made by each node are communicated to its parents. However, in the *Counting* rule parameter, communication is not required.

Each time a node dies due to the loss of battery power, the initial setup is required to be re-executed to obtain a new tree topology with the remaining nodes. The plot of the number of nodes alive with respect to number of sessions of operation for various data fusion schemes is shown in Figure 10(a).

A threshold at 50% of the nodes alive is shown in Figure 10(a) which defines the network lifetime NL2 for various fusion rules in terms of the number of sessions. The NL2 for various aggregation schemes is shown in the second row of Table 1. It can be observed that 2-CV rule has approximately half the network lifetime compared to 1-CV and WAS schemes. Whereas, the *counting* rule has slightly better network lifetime compared to the 1-CV and WAS because of low overhead in the initial setup. The first row in Table 1 represents the network lifetime NL1, which is the time till the death of the first node in the network. This also shows the network lifetime of the *counting* rule to be the highest while the network lifetime of the 2-CV fusion rule to be the lowest.

As the nodes die in the network, the Bellman-Ford routing algorithm is re-executed to obtain the new tree topology. With loss of more nodes in a network, the detection accuracy is expected to decrease. Thus for any change in topology due to death of nodes, the accuracy delivered by the network is numerically recomputed. We consider all sensors to be precise with precision p=0.65 for numerical results of





····· Accuracy threshold = 0.9 ·-·- Accuracy threshold = 0.85 (b)

FIGURE 10: (a) Simulation results for network lifetime with respect to number of sessions for various aggregation schemes. (b) Numerical results of accuracy with respect to number of sessions for various aggregation schemes.

TABLE 1: Network lifetime in the number of sessions for various aggregation schemes.

Network lifetime	Fusion rule			
	Counting	WAS	1-CV	2-CV
NL1	1400	1193	916	636
NL2	1571	1347	1283	674
NL3 with accuracy threshold 0.85	0	1194	1283	698
NL3 with accuracy threshold 0.9	0	0	1283	698

accuracy plots shown in Figure 10(b). Accuracy thresholds of 0.9 and 0.85 are shown in Figure 10 which defines two different sets of network lifetimes NL3. The selection of high-accuracy thresholds (0.9 and 0.85) is to guarantee the superior performance of the schemes when the precision of the individual nodes is as low as 0.65.

The last two rows of Table 1 show the network lifetime NL3 for various schemes by considering an accuracy thresholds of 0.85 and 0.9, respectively. These results indicate that the lifetime of the proposed 1-CV is the highest among all 1-bit aggregation schemes. Figure 10(b) also shows the significant improvement in accuracy for 1-CV as compared to the WAS and the *counting* rule as was seen in Figure 9. Further, this gain in accuracy comes without any loss in the network lifetime.

The network lifetime for 2-CV is approximately half of the 1-CV because it requires double the number of transmission for each node in each aggregation session. As seen from Figure 10(b), 2-CV however has a significant

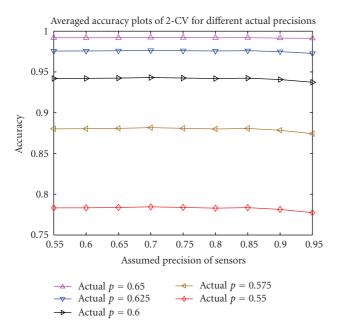


FIGURE 11: Perturbation analysis: numerically computed accuracy plot with respect to assumed precision of sensors. Different plots are for different values of actual precision *p*.

improvement in detection accuracy as compared to 1-CV fusion rule.

## 5.3. Perturbation analysis

The proposed *q*-CV fusion rule requires knowledge of the performance indices of the observation made by each node. Results of the proposed q-CV presented in Figures 8, 9, and 10 assumed correct knowledge of the precision p of all the sensors. In this subsection, we study the performance sensitivity of q-CV to incorrect knowledge of precision p. It should be noted here that the majority-based aggregation scheme does not need the knowledge of p. Figure 11 shows the numerically computed accuracy of the proposed q-CV fusion rule for various values of actual precision p with respect to an assumed (incorrect) precision used for decision making. A negligible variation in accuracy is observed even when the precision assumed for decision making was significantly different from the actual precision of the nodes. It can be inferred that the proposed q-CV fusion rule is robust to inaccuracy in the knowledge of the performance indices of nodes. One possible explanation for this robustness is that with an incorrect value of the precision p, the calculated performance indices for all the sensors have similar perturbation from their true value. Since the decision in a node depends only on the relative value of the performance indices of its children, the decision is robust against error in the assumed value of *p*.

## 6. CONCLUSION

The problem of distributed data fusion in wireless sensor networks with tree topology was considered in the context of binary event detection. An efficient multibit (q-bit) decision fusion rule (q-CV) for tree topology had been proposed in the current work. This scheme achieves a significant gain

in detection accuracy for q > 1. However, with increasing q, the network lifetime decreases due to more transmission per aggregation session. This significant improvement in accuracy at the cost of network lifetime with increasing q offers a good tradeoff between accuracy and network lifetime.

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