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Research Article

CDMA Transmission with Complex OFDM/OQAM

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We propose an alternative to the well-known multicarrier code-division multiple access (MC-CDMA) technique for downlink transmission by replacing the conventional cyclic-prefix orthogonal frequency division multiplexing (OFDM) modulation by an advanced filterbank-based multicarrier system (OFDM/OQAM). Indeed, on one hand, MC-CDMA has already proved its ability to fight against frequency-selective channels thanks to the use of the OFDM modulation and its high flexibility in multiple access thanks to the CDMA component. On the other hand, OFDM/OQAM modulation confers a theoretically optimal spectral efficiency as it operates without guard interval. However, its orthogonality is limited to the real field. In this paper, we propose an orthogonally multiplex quadrature amplitude modulation (OQAM-) CDMA combination that permits a perfect reconstruction of the complex symbols transmitted over a distortion-free channel. The validity and efficiency of our theoretical scheme are illustrated by means of a comparison, using realistic channel models, with conventional MC-CDMA and also with an OQAM-CDMA combination conveying real symbols.

1. INTRODUCTION

Multicarrier code-division multiple access (MC-CDMA) systems have been initially proposed in [1, 2]. This technique constitutes a popular way to combine CDMA and orthogonal frequency division multiplexing (OFDM) with cyclic prefix (CP). Nowadays, MC-CDMA is considered as one of the possible candidates for the downlink of B3G communication systems. Indeed, on one hand, this technique proposes a good way to fight against frequency-selective channels thanks to the OFDM modulation and, on the other hand, it has a high flexibility in the multiple access scheme thanks to the CDMA component. However, the insertion of the CP leads to spectral efficiency loss since this "redundant" symbol part does not carry useful data information. In addition, the conventional OFDM modulation is based on a rectangular windowing in the time domain which leads to a poor ($\text{sinc}(x)$) behavior in the frequency domain. Thus, CP-OFDM gives rise to 2 drawbacks: loss of spectral efficiency and sensitivity to frequency dispersion (e.g., Doppler spread). Both of them can be counteracted using a variant of OFDM introduced in [3, 4] known as orthogonally multiplex quadrature amplitude modulation (OQAM) [5] or more recently as OFDM/OQAM [6], where OQAM then stands for Offset QAM. Here for concision, we will call it the OQAM modulation.

OQAM has many common features with OFDM. Indeed, in OQAM, the basic principle is also to divide the total transmission bandwidth into a large number of uniform subbands. As for OFDM systems, the transmitter and receiver implementations can also benefit of fast Fourier transform (FFT) algorithms. However, instead of a single FFT or inverse fast Fourier transform (IFFT), a uniform filter bank is used. So, one can get a better frequency separation between subchannels, reducing the intercarrier interference (ICI) in the presence of frequency shifts. It is also of interest to examine if these attractive features can also be efficiently exploited when OQAM is used in combination with spread spectrum techniques and also if this combination leads to some new advantages.

If a CDMA spreading is applied to OQAM in the frequency domain, leading to OQAM-CDMA, we get a transmission scheme similar to MC-CDMA, both being of a particular interest in a multiuser downlink transmission context. It is shown in [7] that, not surprisingly, we can keep the inherent advantage of OQAM over CP-OFDM of a better spectral efficiency. Furthermore, as for OQAM, the orthogonality only holds in the real field, that is, for the transmission of real symbols, it is suggested in [7], instead of simply discarding them, to use the imaginary parts of the demodulated and despread signals for resynchronization. In [8] it is also shown, with a wavelet-based OFDM-CDMA system, that a pulse-shaped CDMA multicarrier system can also bring improvements with respect to the multiuser interference. In [7, 8], the data symbols transmitted over each subcarrier are real-valued. In this paper, we show that for OQAM-CDMA, a transmission of complex-valued data symbols, keeping the same symbol rate, is possible if the spreading codes are appropriately selected.

The mathematical foundations of the OQAM scheme with spread spectrum are presented in Section 2. Then, in the following sections, we analyze for a distortion-free channel the OQAM-CDMA scheme considering Walsh-Hadamard (W-H) codes. An analysis of the imaginary component, in the single user case, is provided in Section 3. In Section 4, we present a construction rule about the W-H spreading code selection that in the multiuser case leads to a perfect cancellation of the imaginary interference created by the transmission of complex-valued data with OQAM. Section 5 provides a global analysis of the main features of the complex version of OQAM-CDMA with respect to the real version and to MC-CDMA. Finally, in Section 6, some comparisons in terms of bit error rate (BER) and regarding to the systems load are carried out, using realistic channel models, between the real and complex version of OQAM-CDMA and also with MC-CDMA.

2. PROBLEM STATEMENT

We can write the baseband equivalent of a continuous-time multicarrier OQAM signal as follows [6]:

$$s(t) = \sum_{m=0}^{M-1} \sum_{n\in\mathbf{Z}} a_{m,n} \underbrace{g(t - n\tau_0) e^{j2\pi m F_0 t} \nu_{m,n}}_{g_{m,n}(t)} \quad (1)$$

with $M = 2N$ an even number of subcarriers, $F_0 = 1/T_0 = 1/2\tau_0$ the subcarrier spacing, g the pulse shape, and $\nu_{m,n}$ an additional phase term. Here, as in [9], we set $\nu_{m,n} = j^{m+n}(-1)^{mn}$. The prototype filter g is real-valued and we also assume that its length is a multiple of M such that $L = bM = 2bN$, with b an integer. The transmitted data symbols $a_{m,n}$ are real-valued. They are obtained from a 2^{2K}-QAM constellation, taking the real and imaginary parts of these complex-valued symbols of duration $T_0 = 2\tau_0$, where τ_0 denotes the time offset between the two parts [5, 6, 9, 10].

Assuming a distortion-free channel, the perfect reconstruction of the real data symbols is obtained owing to the following real orthogonality condition:

$$\Re\{\langle g_{m,n} \mid g_{p,q}\rangle\} = \Re\left\{\int g_{m,n}(t) g_{p,q}^{*}(t)dt\right\} = \delta_{m,p}\delta_{n,q}, \quad (2)$$

where $\delta_{m,p} = 1$ if $m = p$, and $\delta_{m,p} = 0$ if $m \neq p$. To express the complex inner product, it may be convenient to use the ambiguity function A_g of the prototype function g. Defining it as follows:

$$A_g(n,m) = \int_{-\infty}^{\infty} g(u - n\tau_0) g(u) e^{2j\pi m F_0 u} du \quad (3)$$

and taking into account the limited duration of g with the indicating function $I_{|n-n_0|<2b}$, equal to 1 if $|n - n_0| < 2b$ and 0 elsewhere, it can be easily shown that

$$\langle g_{m,n}, g_{p,n_0}\rangle = \delta_{m-p,n-n_0} + j\gamma_{m,n}^{(p,n_0)} I_{|n-n_0|<2b}, \quad (4)$$

where $\gamma_{m,n}^{(p,n_0)}$ is given by

$$\gamma_{m,n}^{(p,n_0)} = \Im\{(-1)^{m(n+n_0)} j^{m+n-p-n_0} A_g(n - n_0, m - p)\}. \quad (5)$$

The block diagram illustrating the OQAM transmission scheme is depicted in Figure 1. Compared to conventional CP-OFDM, real-data symbols are transmitted via an OQAM modulator involving an IFFT operation followed by a filtering operation polyphase with the polyphase components of g [9, 10]. At the receiver side, the dual operations are carried out; and thanks to the real orthogonality demodulation, followed by one-tap equalization, the data symbols are recovered. Different kinds of prototype functions can be implemented as the isotropic orthogonal transform algorithm (IOTA) prototype [6] or some other prototypes directly optimized in discrete time using the time-frequency localization (TFL) criterion [11].

Let us now present the CDMA component of the proposed transmission scheme. We denote by N_c the length of the CDMA code used and assume that $N_0 = M/N_c$ is an integer number. Let us denote by $\underline{c}_u = [c_{0,u} \;\; \cdots \;\; c_{N_c-1,u}]^t$ the code used by the uth user. Then, for a user u_0 at a given time n_0, N_0 different data are transmitted denoted by $d_{u_0,n_0,0}, d_{u_0,n_0,1}, \ldots, d_{u_0,n_0,N_0-1}$. Then by *spreading* with the $\underline{c}_u$ codes, we get the real symbol a_{m_0,n_0} transmitted at frequency m_0 and time n_0 by

$$a_{m_0,n_0} = \sum_{u=0}^{U-1} c_{m_0/N_c,u} d_{u,n_0,\lfloor m_0/N_c\rfloor}, \quad (6)$$

where U is the number of users, / the modulo operator, and $\lfloor\cdot\rfloor$ the floor operator. From the a_{m_0,n_0} term, the reconstruction of $d_{u,n_0,p}$ (for $p \in [0, N_0 - 1]$) is insured thanks to the orthogonality of the code, that is, $\underline{c}_{u1}^T\underline{c}_{u2} = \delta_{u1,u2}$ (see [12] for more details). Therefore, the *despreading* operator leads to

$$d_{u,n_0,p} = \sum_{m=0}^{N_c-1} c_{m,u} a_{pN_C+m,n_0}. \quad (7)$$

In [7], it is shown that, thanks to the real orthogonality of the OQAM modulation, the transmission of these spread real data ($d_{u,n_0,p}$) can be insured at a symbol rate which is more than twice the one used for transmitting complex MC-CDMA data as no CP is inserted. Figure 2 depicts the real OQAM-CDMA transmission scheme where after the *despreading* operation, only the real part of the symbol is kept whereas the imaginary component is not detected.

We now propose to consider the transmission of complex data, denoted by $d_{n,u,p}^{(c)}$, using U well-chosen Walsh-Hadamard codes. In order to establish the theoretical features of this complex OQAM-CDMA scheme, we suppose that the transmission channel is free of any type of distortion. Also

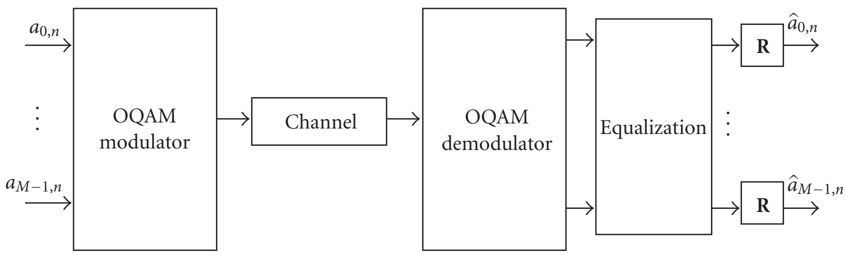

FIGURE 1: Conventional OQAM transmission scheme.

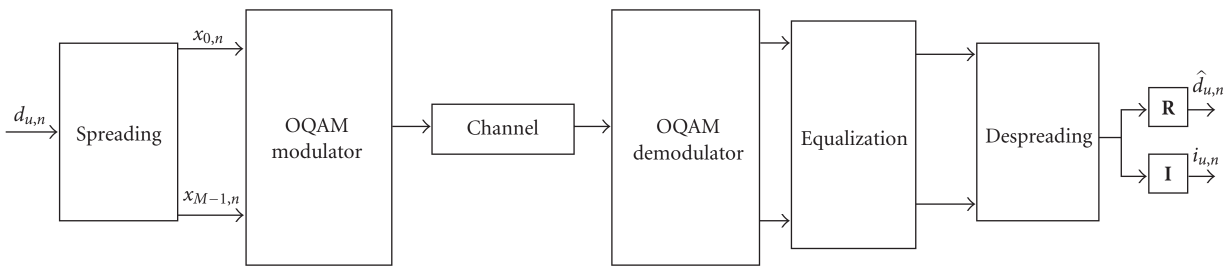

FIGURE 2: Real OQAM-CDMA transmission scheme.

for simplicity reasons, we assume a maximum frequency diversity, $M = 2N = N_c$. Then we can denote by $d_{n,u}^{(c)}$ the transmitted complex data and by $a_{m,n,u}^{(c)} = c_{m,u} d_{n,u}^{(c)}$ the complex symbol transmitted at time $n\tau_0$ over the carrier m and for the code u. As usual, the length of the W-H codes are supposed to be a power of 2, $M = 2N = 2^q$ with q an integer.

The corresponding transmission scheme is depicted in Figure 3. This complex OQAM-CDMA transmission case has similarities with the MC-CDMA one. However, the modulation and demodulation operations include a specific mapping and demapping in relation to the time offset of OQAM and also a pulse shaping. Furthermore, the subsets of W-H codes have to be appropriately selected (see Sections 3 and 4). The baseband equivalent of the transmitted signal can be written as

$$s(t) = \sum_{n\in\mathbf{Z}} \sum_{m=0}^{2N-1} x_{m,n} g_{m,n}(t) \quad \text{with } x_{m,n} = \sum_{u=0}^{U-1} a_{m,n,u}^{(c)}. \tag{8}$$

As the channel is distortion-free, the received signal is $y(t) = s(t)$ and the demodulated symbols are obtained as follows:

$$y_{m_0,n_0}^{(c)} = \langle y, g_{m_0,n_0} \rangle. \tag{9}$$

Then, the despreading operation gives us the despread data for any code, for example, for u_0, we get

$$z_{n_0,u_0}^{(c)} = \sum_{p=0}^{2N-1} c_{p,u_0} y_{p,n_0}^{(c)} = \sum_{p=0}^{2N-1} c_{p,u_0} \sum_{n\in\mathbf{Z}} \sum_{m=0}^{2N-1} x_{m,n} \langle g_{m,n}, g_{p,n_0} \rangle. \tag{10}$$

Replacing $x_{m,n}$ and $\langle g_{m,n}, g_{p,n_0} \rangle$ by their expression given in (8) and (4), respectively, we get

$$z_{n_0,u_0}^{(c)} = \sum_{p=0}^{2N-1} c_{p,u_0} \sum_{n=-2b+1}^{2b-1} \sum_{m=0}^{2N-1} \times \sum_{u=0}^{U-1} c_{m,u} d_{n+n_0,u}^{(c)} \left(\delta_{m-p,n-n_0} + j\gamma_{m,n+n_0}^{(p,n_0)} \right). \tag{11}$$

Then, splitting the summation over n in two parts, with n equal or not to 0, (11) can be rewritten as

$$z_{n_0,u_0}^{(c)} = \sum_{u=0}^{U-1} d_{n_0,u}^{(c)} \sum_{p=0}^{2N-1} c_{p,u_0} c_{p,u} + j \left(\sum_{u=0}^{U-1} \sum_{\substack{n=-2b+1 \\ n\neq 0}}^{2b-1} d_{n+n_0,u}^{(c)} \left(\sum_{p=0}^{2N-1} \sum_{m=0}^{2N-1} c_{p,u_0} c_{m,u} \gamma_{m,n+n_0}^{(p,n_0)} \right) \right). \tag{12}$$

The W-H codes being orthogonal, that is,

$$\sum_{p=0}^{2N-1} c_{p,u_0} c_{p,u} = \begin{cases} 1 & \text{if } u = u_0, \\ 0 & \text{if } u \neq u_0, \end{cases} \tag{13}$$

we finally obtain:

$$z_{n_0,u_0}^{(c)} = d_{n_0,u_0}^{(c)} + j \left(\sum_{u=0}^{U-1} \sum_{\substack{n=-2b+1 \\ n\neq 0}}^{2b-1} d_{n+n_0,u}^{(c)} \left(\sum_{p=0}^{2N-1} \sum_{m=0}^{2N-1} c_{p,u_0} c_{m,u} \gamma_{m,n+n_0}^{(p,n_0)} \right) \right). \tag{14}$$

The aim now is to show that, when $U \leq M/2$, for an appropriate choice of the U codes we can get $z_{n_0,u_0}^{(c)} = d_{n_0,u_0}^{(c)}$. Let us first examine the single user case.

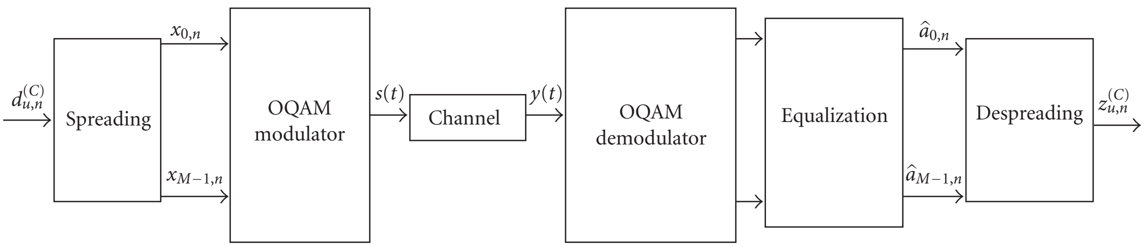

Figure 3: Complex OQAM-CDMA transmission scheme.

3. SINGLE USER CASE $U = 1$

As the channel is assumed to be distortion-free if there is only one single user, the demodulated and despread signal is the one obtained in (14) setting, for one user u_0, $U = 1$. Then by splitting the summations over m and p in two parts, one for $m = p$ and the other one for $m \neq p$, we get

$$z_{n_0,u_0}^{(c)} = d_{n_0,u_0}^{(c)} + j\left(\sum_{n\neq 0} d_{n+n_0,u_0}^{(c)}\left(s_1(n) + s_2(n)\right)\right), \tag{15}$$

with

$$\begin{aligned} s_1(n) &= \sum_{p=0}^{2N-1} c_{p,u_0}c_{p,u_0}\Im\{(-1)^{pn}j^n A_g(n,0)\},\\ s_2(n) &= \sum_{p=0}^{2N-1}\sum_{m=0}^{p-1} c_{p,u_0}c_{m,u_0}\Big[\Im\Big\{(-1)^{mn}j^{m+n-p}A_g(n,m-p)\Big\}\\ &\qquad + \Im\Big\{(-1)^{pn}j^{p+n-m}A_g(n,p-m)\Big\}\Big]. \end{aligned} \tag{16}$$

For W-H codes, we have, $\forall n : c_{p,u_0}c_{p,u_0} = 1/2N$ and the prototype filter g being real-valued, $A_g(n,0)$, see (3), is also real-valued. Then, it is straightforward to show that for every n, $s_1(n) = 0$.

Let us now look at

$$\begin{aligned} s_2(n) &= \sum_{p=0}^{2N-1}\sum_{m=0}^{p-1} c_{p,u_0}c_{m,u_0}\Big[\Im\Big\{(-1)^{mn}j^{m+n-p}A_g(n,m-p)\Big\}\\ &\qquad + \Im\Big\{(-1)^{pn}j^{p+n-m}A_g(n,p-m)\Big\}\Big]. \end{aligned} \tag{17}$$

With g being a real function, then $A_g(n,m) = A_g^*(n,-m)$, thus the imaginary terms in (17) are such that

$$\begin{aligned} S_I &= \Im\Big\{(-1)^{mn}j^{m+n-p}A_g(n,m-p)\Big\}\\ &\quad + \Im\Big\{(-1)^{pn}j^{p+n-m}A_g(n,p-m)\Big\}\\ &= \Im\Big\{(-1)^{mn}j^{m+n-p}A_g(n,m-p)\\ &\quad + (-1)^{pn}j^{p+n-m}A_g^*(n,m-p)\Big\}. \end{aligned} \tag{18}$$

It can be easily seen that for n even, $S_I = 0$, while for n odd, the result depends upon the parity of m and p being given by

$$S_I = \begin{cases} -2(-1)^{mn}j^{n+1}\Re\{j^{m-p}A_g(n,m-p)\} & \text{if } m \text{ and } p \text{ have the same parity}\\ 0 & \text{otherwise.} \end{cases} \tag{19}$$

Then, the computation of (15) can be restricted to the terms obtained for odd values of n with p and m being of identical parity. After some computations, setting $v = m - p$, it can be shown that

$$\begin{aligned} z_{n_0,u_0}^{(c)} &= d_{n_0,u_0}^{(c)} + 2\left(\sum_{\substack{n=-b\\ n\neq 0}}^{b-1} d_{n_0+2n+1,u_0}^{(c)} j^{2n+1}\sum_{v=0}^{N}\Re\right.\\ &\qquad \left.\times\left\{A_g(2n+1,2v)\sum_{k=0}^{2N-1-2v}(-1)^k c_{k+2v,u_0}c_{k,u_0}\right\}\right). \end{aligned} \tag{20}$$

With a first property of the W-H codes shown in Appendix A:

$$\sum_{m=0}^{k}(-1)^k c_{k+2v,u_0}c_{k,u_0} = 0 \quad \text{for } v = 0,\ldots,N; \quad k = 0,\ldots,2N-1-2v, \tag{21}$$

(20) becomes

$$\forall n_0, u_0, \quad z_{n_0,u_0}^{(c)} = d_{n_0,u_0}^{(c)}. \tag{22}$$

This last equality is the result of a straightforward derivation of the demodulated and despread signal. It leads us to a property that a priori could not be easily intuitively apprehended. Nevertheless, we can attempt to justify it a posteriori. Let us notice, firstly, that if instead of complex data, we transmit real data over a distortion-free channel, thanks to the real orthogonality of the OQAM modulation scheme, we exactly recover these real data by taking the real part in (20). Again using (20), it can then be seen that to cancel the imaginary part, the interference, the condition (21) on the W-H CDMA codes is essential. Therefore, in the one user case, using the *system linearity*, we can transmit complex data and recover them perfectly at the receiver.

4. MULTIUSER CASE WITH $U \leq M/2$

In order to generalize the relation (22) to a multiuser case, we propose in this section a selection mode for the subsets of W-H codes. Then, the generalization can be carried out step by step, considering firstly a two-user OQAM-CDMA system and secondly a U-user system with $U \leq M/2$.

4.1. Selection of the U codes

For a Walsh-Hadamard matrice of size $M = 2N = 2^n$, there are two subsets of column indices, S_1^n and S_2^n, with cardinal equal to $M/2$ making a partition of all the index set. We propose a recurrent rule of construction for these two subsets that can guarantee the absence of interference between users.

For $n_0 = 1$, each subset is initialized setting $S_1^1 = \{0\}$ and $S_2^1 = \{1\}$.

Let us now assume that, for a given integer $n = n_0$, the two subsets contain the following list of indices:

$$S_1^{n_0} = \{i_{1,1}, i_{1,2}, i_{1,3}, \dots, i_{1,2^{n_0-1}}\},$$
$$S_2^{n_0} = \{i_{2,1}, i_{2,2}, i_{2,3}, \dots, i_{2,2^{n_0-1}}\}. \tag{23}$$

These subsets are afterwards used to build new subsets of identical size such that

$$\overline{S}_1^{n_0} = \{i_{2,1} + 2^{n_0}, i_{2,2} + 2^{n_0}, i_{2,3} + 2^{n_0}, \dots, i_{2,2^{n_0-1}} + 2^{n_0}\},$$
$$\overline{S}_2^{n_0} = \{i_{1,1} + 2^{n_0}, i_{1,2} + 2^{n_0}, i_{1,3} + 2^{n_0}, \dots, i_{1,2^{n_0-1}} + 2^{n_0}\}. \tag{24}$$

Then, we get the subsets of higher size, $n = n_0 + 1$, as follows:

$$S_1^{n_0+1} = S_1^{n_0} \cup \overline{S}_1^{n_0}, \qquad S_2^{n_0+1} = S_2^{n_0} \cup \overline{S}_2^{n_0}. \tag{25}$$

4.2. Case of two users in the same subset ($U = 2$)

In the second step of our proof, we want to show now that, again for W-H codes such that $M = 2N = 2^n$, if two users u_0 and u_1 take their codes into the same subset, for example, all in S_1^n or all in S_2^n, there in no interference between these 2 users, $z_{n,u_0}^{(c)} = d_{n,u_0}^{(c)}$ and $z_{n,u_1}^{(c)} = d_{n,u_1}^{(c)}$.

Let us show at first that for u_0 and $u_1 \in S_1^n$ (resp., S_2^n), $z_{n_0,u_0}^{(c)} = d_{n_0,u_0}^{(c)}$. Indeed, setting $U = 2$ in (14), for two given users u_0 and $u_1 \in S_1^n$ (resp., S_2^n), we get

$$\begin{aligned} z_{n_0,u_0}^{(c)} &= d_{n_0,u_0}^{(c)} + j\left(\sum_{\substack{n=-2b+1\\ n\neq 0}}^{2b-1} d_{n+n_0,u_0}^{(c)}\left(\sum_{p=0}^{2N-1}\sum_{m=0}^{2N-1} c_{p,u_0} c_{m,u_0} \gamma_{m,n+n_0}^{(p,n_0)}\right)\right) \\ &\quad + j\left(\sum_{\substack{n=-2b+1\\ n\neq 0}}^{2b-1} d_{n+n_0,u_1}^{(c)}\left(\sum_{p=0}^{2N-1}\sum_{m=0}^{2N-1} c_{p,u_0} c_{m,u_1} \gamma_{m,n+n_0}^{(p,n_0)}\right)\right). \end{aligned} \tag{26}$$

As it has been shown for one user that $z_{n_0,u_0}^{(c)} = d_{n_0,u_0}^{(c)}$, based on (20), we can deduce that at the right-hand side the second term is zero.

Then, by splitting again the summation over m in two parts, one for $m = p$ and the second one for $m \neq p$, we get

$$z_{n_0,u_0}^{(c)} = d_{n_0,u_0}^{(c)} + j\left(\sum_{\substack{n=-2b+1\\ n\neq 0}}^{2b-1} d_{n+n_0,u_1}^{(c)}\left(w(n) + T_g(n,u_0,u_1)\right)\right), \tag{27}$$

with $w(n)$ containing the terms obtained for $p = m$ and $T_g(n,u_0,u_1)$, the ones for $m \neq p$ leading to

$$\begin{aligned} w(n) &= \sum_{p=0}^{2N-1} c_{p,u_0} c_{p,u_1} \Im\{(-1)^{pn} j^n A_g(n,0)\}, \\ T_g(n,u_0,u_1) &= \sum_{p=0}^{2N-1}\sum_{m=0}^{p-1} \big(c_{p,u_0} c_{m,u_1} \Im\{(-1)^{mn} j^{m+n-p} A_g(n,m-p)\} \\ &\qquad + c_{p,u_1} c_{m,u_0} \Im\{(-1)^{pn} j^{p+n-m} A_g(n,p-m)\}\big), \end{aligned} \tag{28}$$

respectively.

$A_g(n,0)$ being real-valued, it is obvious that $w(n) = 0$ for n even. For n odd, it is shown in Appendix B that

$$\sum_{p=0}^{2N-1} (-1)^p c_{p,u_0}^{(n)} c_{p,u_1}^{(n)} = 0 \quad \text{for } u_0, u_1 \in S_1^n \text{ (resp., } S_2^n\text{)}, \tag{29}$$

which leads again to $w(n) = 0$. Thus for every n, $w(n) = 0$.

The expression of T_g can be rewritten introducing a new variable $u = p - m$ and using the fact that $A_g(n,-u) = A_g^*(n,u)$, thus we obtain

$$\begin{aligned} T_g(n,u_0,u_1) &= \sum_{u=1}^{2N-1} \Im\Bigg\{\sum_{m=0}^{2N-1-u} (-1)^{mn} j^{n-u} c_{m+u,u_0} c_{m,u_1} A_g^*(n,u) \\ &\qquad + \sum_{m=0}^{2N-1-u} (-1)^{mn+un} c_{m+u,u_1} c_{m,u_0} j^{n+u} A_g(n,u)\Bigg\}. \end{aligned} \tag{30}$$

For n even ($n = 2k$), we get

$$\begin{aligned} T_g(2k,u_0,u_1) &= \sum_{u=1}^{2N-1} (-1)^k \Im\Bigg\{\sum_{m=0}^{2N-1-u} c_{m+u,u_0} c_{m,u_1} j^{-u} A_g^*(n,u) \\ &\qquad + \sum_{m=0}^{2N-1-u} c_{m+u,u_1} c_{m,u_0} j^{u} A_g(n,u)\Bigg\}. \end{aligned} \tag{31}$$

In Appendix C, it is shown that for $s > 0$, and for any W-H matrix of order n, that is, a size $M = 2^n$, the corresponding codes $c_{m,u_0}^{(n)}$ are such that

$$\sum_{m=0}^{2^n-1-s} c_{m,u_0}^{(n)} c_{m+s,u_1}^{(n)} = \sum_{m=0}^{2^n-1-s} c_{m,u_1}^{(n)} c_{m+s,u_0}^{(n)}, \quad \text{for } u_0, u_1 \in S_1^n \text{ (resp., } S_2^n\text{)}. \tag{32}$$

Then as $T_g(2k, u_0, u_1)$ is the imaginary part of the sum of two conjugate quantities, we have

$$T_g(2k, u_0, u_1) = 0. \tag{33}$$

The same lines of arguments can be applied to show that if n is odd ($n = 2k + 1$), we get $T_g(2k + 1, u_0, u_1) = 0$.

The computation for n odd uses the following properties of Walsh-Hadamard codes:

$$\begin{aligned}
&\sum_{m=0}^{2^n-1-2s} (-1)^m c_{m,u_0}^{(n)} c_{m+2s,u_1}^{(n)} \\
&\quad = -\sum_{m=0}^{2^n-1-2s} (-1)^m c_{m,u_1}^{(n)} c_{m+2s,u_0}^{(n)} \quad \text{for } u_0, u_1 \in S_1^n \text{ (resp., } S_2^n\text{)}, \\
&\sum_{m=0}^{2^n-2-2s} (-1)^m c_{m,u_0}^{(n)} c_{m+2s+1,u_1}^{(n)} \\
&\quad = \sum_{m=0}^{2^n-2-2s} (-1)^m c_{m,u_1}^{(n)} c_{m+2s+1,u_0}^{(n)} \quad \text{for } u_0, u_1 \in S_1^n \text{ (resp., } S_2^n\text{)}.
\end{aligned} \tag{34}$$

The proof of these properties, not reported here to avoid another lengthy mathematical derivation, is quite similar to the one used to get by recurrence the result presented in Appendix B.

Finally, as

$$T_g(n, u_0, u_1) = 0 \quad \text{for } u_0, u_1 \in S_1^n \text{ (resp., } S_2^n\text{)}, \tag{35}$$

we get

$$\forall n_0, u_0, \quad z_{n_0,u_0}^{(c)} = d_{n_0,u_0}^{(c)}. \tag{36}$$

As in the one-user case, this last equality is the result of a straightforward derivation of the demodulated and despread signal and it could not be so easily intuitively apprehended. However in this case, based on our previous study of OQAM-CDMA systems for real-data transmission [7], it was clear that to cancel the imaginary part, some specific conditions on W-H codes were required. Indeed looking at [7, Figures 4 and 5], it is clear that whatever the orthogonal pulse shape $g(t)$ being used, the imaginary part is zero only for some pairs of codes. What we show here is that these pairs of W-H codes can be grouped in two subsets, forming a partition of the set of all codes (see Section 4.1), where they satisfy the essential relations (29), (32), (34). So using again *the system linearity*, we can transmit complex data and recover them perfectly at the receiver.

4.3. *Case of* U *users in the same subset* ($U \leq M/2$)

Now let us consider the case $U \leq M/2$ where the U codes are all chosen either in S_1^n or in S_2^n. Setting $U \leq M/2$ in (14) for U given users $\in S_1^n$ (resp., S_2^n), we get

$$z_{n_0,u_0}^{(c)} = d_{n_0,u_0}^{(c)} + j \sum_{u=0}^{U-1} X(u_0, u), \tag{37}$$

where

$$X(u_0, u) = \sum_{\substack{n=-2b+1 \\ n \neq 0}}^{2b-1} d_{n+n_0,u}^{(c)} \sum_{p=0}^{2N-1} \sum_{m=0}^{2N-1} c_{p,u_0} c_{m,u} \gamma_{m,n+n_0}^{(p,n_0)}. \tag{38}$$

It has been shown for one user, with $u = u_0$ (see (15) in Section 3), and afterwards for 2 users, with u_0 and $u_1 \in S_1^n$ (resp., S_2^n) (see (26) in Section 4.2), that $X(u_0, u) = 0$. Therefore, if the U codes are all chosen in $\in S_1^n$ (resp., S_2^n), we get

$$\forall n_0, u_0, \quad z_{n_0,u_0}^{(c)} = d_{n_0,u_0}^{(c)}. \tag{39}$$

So, in this last and more general case the result can be a posteriori justified using the same lines of arguments we developed previously for the one- and two-user case.

5. ANALYSIS OF COMPLEX OQAM-CDMA

In MC-CDMA, and CP taking apart, the transmitted data are complex and the full load is obtained when using all the codes of the W-H matrix ($U = M$). When considering the full diversity, that is, one spread symbol transmitted over all modulated carriers, the maximum spectral efficiency (full load) is obtained for M complex data symbols transmitted at every T_0 symbol duration.

In OQAM-CDMA with real-data symbol transmission, the full load is again obtained when $U = M$. Therefore, we obtain the maximum spectral efficiency when M real-data symbols are transmitted at every $T_0/2$ symbol duration, which is equivalent to the transmission of M complex-data symbols at T_0.

In the proposed OQAM-CDMA scheme with complex data symbol transmission, the system guarantees a complex orthogonality up to a number of users $U = M/2$ which corresponds to the maximum load. As for the complex OQAM-CDMA system, $M/2$ complex-data symbols are transmitted at every $T_0/2$ symbol duration, this scenario is equivalent to the one where M complex-data symbols are transmitted every T_0 duration.

So, these 3 scenarios lead to the same spectral efficiency, without taking into account the CP, but consider different number of spreading codes to reach this spectral efficiency. Since the number of spreading codes used directly impacts on the multiple access interference (MAI) [13, 14], a first analysis of the different systems shows that using less spreading codes may lead to better performance results. Indeed, if U increases, the MAI term also increases. As an illustration of the reduction of the MAI, we can notice that when there is only one user in the OQAM-CDMA complex transmission scheme, that is, no MAI, the same spectral efficiency is obtained either in MC-CDMA or in OQAM-CDMA real-transmission schemes, with the use of 2 W-H spreading codes, with a nonzero MAI term. So, the OQAM-CDMA with complex symbol transmission should outperform the two other systems as it uses twice less spreading codes to achieve the same spectral efficiency. Some simulation results will also confirm this analysis in the following section.

6. SYSTEM PARAMETERS AND SIMULATION RESULTS

This section gives the main parameters used in simulations and provides an evaluation of the 3-transmission schemes: MC-CDMA, OQAM-CDMA with real symbols transmission, and the new proposed OQAM-CDMA with complex-symbol transmission. This evaluation leads to a fair comparison between the 3 systems either in terms of BER or in percentage of load.

6.1. System parameters

The static propagation channel is modelled by a 3-tap delay profile having the following characteristics.

(i) Delay (μs): 0 0.2527 0.32.
(ii) Powers (in dB): −0 − 3 − 2.2204.

The other main parameters of the uncoded system are the following.

(i) Carrier frequency: f_c = 1000 MHz.
(ii) FFT size = 32.
(iii) Sampling frequency = 10 MHz.
(iv) Symbol duration, τ_0 (T_0): 1.6 μs (3.2 μs).
(v) Cyclic prefix = 0.5 μs for MC-CDMA.
(vi) Walsh-Hadamard spreading codes of length 32.
(vii) One-tap MMSE (minimum mean squared error) equalization.
(viii) For OFDM/OQAM, either the IOTA prototype filtering of length 128, b = 4, or TFL prototype of length 32, b = 1, is implemented.

When considering the MC-CDMA technique, the performance results are given by taking into account the loss in power ($10\log_{10}(T_0/(T_0+\mathrm{CP}) = 0.63$ dB) induces by the cyclic prefix insertion. For the OQAM-CDMA with complex symbol transmission, the W-H codes are issued from the first subset S_1^5. In MC-CDMA and OQAM-CDMA with real symbol transmission, when S_1^5 is not sufficient to achieve the targeted spectral efficiency (use of more than $M/2$ codes), then the spreading codes from the S_2^5 subset are selected.

6.2. Simulation results

Figure 4 shows the performance results obtained at 1/16 of the maximum system spectral efficiency. To achieve this spectral efficiency, the MC-CDMA and real OQAM-CDMA techniques use 2 W-H spreading codes, whereas the complex OQAM-CDMA system uses only one W-H code. It is shown that the OQAM-CDMA with real symbols outperforms the MC-CDMA technique of the gain induced by the absence of the CP insertion. When comparing both OQAM-CDMA schemes, we can note that the complex-symbol transmission system provides around a 2 dB gain at BER= 10^{-2} compared to the real-symbol transmission. This gain shows that using only one W-H code (complex-symbol transmission) instead of 2 (real-symbol transmission) allows to reduce the MAI term and so to obtain better performance.

Figure 5 shows the performance results obtained at the maximum system spectral efficiency. To achieve this spectral efficiency, the MC-CDMA and the real-OQAM-CDMA techniques use the 32 W-H spreading codes, whereas the complex OQAM-CDMA system uses 16 W-H codes corresponding to the whole S_1^5 subset. It is shown that both OQAM-CDMA systems have the same performance results and outperform the MC-CDMA system as no CP is required. In that case, we note that the MAI term does not provide any gain in favour of the OQAM-CDMA with complex symbol transmission.

In Figure 5, it can also be noted that the OQAM and OFDM curves merge around E_b/N_0 = 13 dB. Indeed, in this context, where we assume a perfect channel knowledge and a one-tap MMSE equalization, the OQAM system, which has

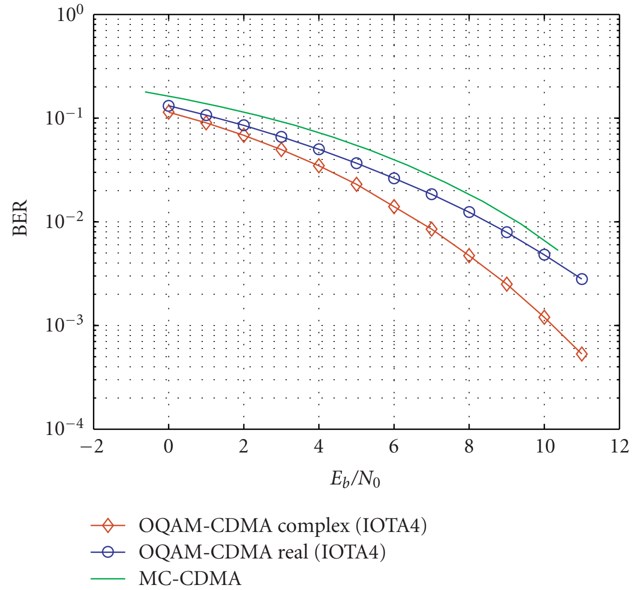

Figure 4: BER performance results of the 3-transmission schemes at 1/16 of the maximum spectral efficiency.

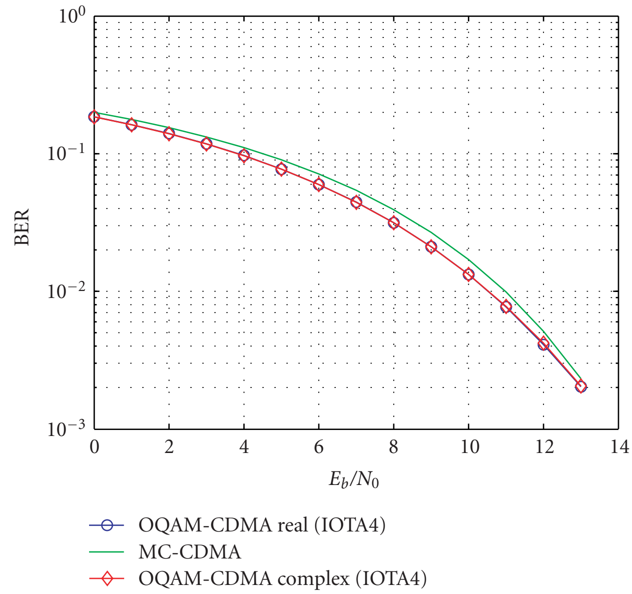

Figure 5: BER performance results of the 3-transmission schemes at the maximum spectral efficiency.

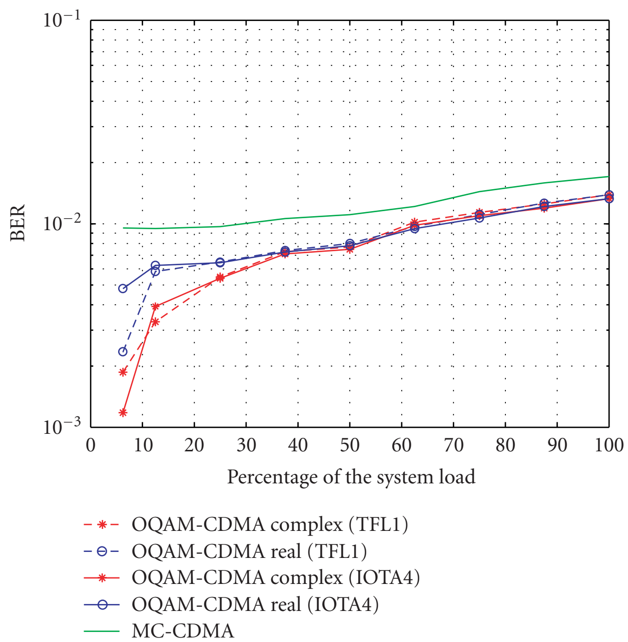

Figure 6: BER performance result regarding to the system's load for the 3-transmission schemes.

no guard interval, suffers from ISI in the presence of a time dispersive channel. So for E_b/N_0 beyond 13 dB the curves cross. This phenomenon, named intrinsic interference, is explained in details in [15, 16]. As also shown in [17], if the delay spread is not too long, less than 1/8 of τ_0, a one tap equalization may be enough. For larger delay spread, as is the case here (being 20% of τ_0), a more complex equalization procedure should be used.

To quantify the impact of the MAI term, we have plotted in Figure 6, the performance with regard to the system load at fixed E_b/N_0 (a fixed E_b/N_0 ratio leads to a lower BER in OQAM systems compared to MC-CDMA since the CP is taken into account). This E_b/N_0 is the same for the 3 systems and is equal to 10 dB that corresponds approximatively to a BER= 10^{-2} at full load. For OQAM-CDMA systems, we have either considered the IOTA prototype function or the TFL one. Figure 6 shows that OQAM-CDMA systems give better performance results than the MC-CDMA whatever the load. This gain is always provided by the no-CP insertion. When comparing OQAM-CDMA systems, Figure 6 shows that until 35% of the load, the OQAM-CDMA with complex symbol transmission outperforms the OQAM-CDMA with real-symbol transmission. These results illustrate the impact of the MAI term on the performance results, showing the advantage of using the OQAM-CDMA complex-symbol transmission. Now, if we compare the results with regard to the prototype function, we can comment that the TFL prototype provides better performance results than the IOTA one in real-symbol transmission and for 2 W-H codes. For the complex OQAM-CDMA transmission, both prototypes have almost the same performance. Note also that, for complexity implementation, TFL prototype is more suitable than the IOTA one since its length is 4 times less.

7. CONCLUSION

In this paper, we have proposed an OQAM-CDMA system with complex-data symbol transmission, which allows a reduction of the MAI term while keeping the same spectral efficiency as in MC-CDMA (CP taking apart) or OQAM-CDMA with real-data symbol transmission. We have proved that the transmission of complex symbols in OQAM-CDMA requires a judicious selection of the W-H spreading codes to guarantee the complex orthogonality, that is, in theory limited to the real field in OQAM. The performance results obtained in the considered system have shown that OQAM-CDMA with complex symbol transmission outperforms the MC-CDMA technique whatever the system load thanks to the no-CP insertion and to the lower number of spreading codes used. Compared to OQAM-CDMA with real-symbol transmission, owing to the reduction of the MAI term, our proposed technique gives better performance results up to 35% of the system load. The choice of the prototype function in OQAM-CDMA has no major impact on the performance results in our studied system. However, the TFL prototype function has an advantage with regard to the implementation.

In future work, we will investigate the potential utilization of more than $M/2$ codes in OQAM-CDMA system transmitting complex-data symbols in order to increase the system spectral efficiency and exceed the theoretical MC-CDMA and OQAM-CDMA transmitting real-data symbol systems.

APPENDICES

A. A FIRST PROPERTY OF W-H CODES

We denote by $A^{(n)} = [\underline{c}_1^{(n)}, \underline{c}_2^{(n)}, \dots, \underline{c}_{M-1}^{(n)}]$ the W-H matrix of order n of size $M \times M$ with $M = 2^n$, and with a kth column given by $\underline{c}_k^{(n)} = [c_{0,k}^{(n)}, c_{1,k}^{(n)}, \dots, c_{M-1,k}^{(n)}]^T$ for $k = 0, 1, \dots, M-1$.

In this appendix, we show that for any positive integer n and any code of index k, that is, for the kth column of $A^{(n)}$, we have

$$\sum_{m=0}^{2^n-1-2p} (-1)^m c_{m+2p,k}^{(n)} c_{m,k}^{(n)} = 0 \quad \text{for } p = 0, 1, \dots, 2^{n-1}, \quad m = 0, 1, \dots, 2^n - 1 - 2p. \tag{A.1}$$

The proof is carried out in 2 steps.

Step 1. We first show by recurrence that

$$c_{2m,k}^{(n)} c_{2p,k}^{(n)} = c_{2m+1,k}^{(n)} c_{2p+1,k}^{(n)} \quad \text{for } n = 1, 2, \dots, \infty; \quad m = 0, 1, \dots, 2^{(n-1)} - 1;\ p = 0, 1, \dots, 2^{(n-1)} - 1. \tag{A.2}$$

Case $n = 1$

The W-H matrix being such that

$$A^{(1)} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}, \tag{A.3}$$

it can be easily checked that $c_{0,0}^{(1)}c_{0,0}^{(1)} = c_{1,0}^{(1)}c_{1,0}^{(1)}$ and $c_{0,1}^{(1)}c_{0,1}^{(1)} = c_{1,1}^{(1)}c_{1,1}^{(1)}$, which shows that the property is true for $n = 1$.

Case $n = n_0$

We assume that the property is true for $n = n_0$,

$$c_{2m,k}^{(n_0)}c_{2p,k}^{(n_0)} = c_{2m+1,k}^{(n_0)}c_{2p+1,k}^{(n_0)}, \quad \text{for } p = 0,1,\dots,2^{(n_0-1)}-1; \quad m = 0,1,\dots,2^{(n_0-1)}-1-2p. \tag{A.4}$$

Case $n = n_0 + 1$

Let us show that the property is therefore true for $n = n_0 + 1$. As

$$A^{(n_0+1)} = \begin{pmatrix} A^{(n_0)} & A^{(n_0)} \\ A^{(n_0)} & -A^{(n_0)} \end{pmatrix}, \tag{A.5}$$

we also have

$$\underline{c}_k^{(n+1)} = \begin{cases} \left(\underline{c}_k^{(n)}, \underline{c}_k^{(n)}\right)^T & \text{if } k < 2^{n_0}, \\ \left(\underline{c}_{k-2^n}^{(n)}, -\underline{c}_{k-2^n}^{(n)}\right)^T & \text{if } k > 2^{n_0}-1. \end{cases} \tag{A.6}$$

Let us now only consider the case $(k > 2^{n_0} - 1)$ knowing that the computation principle is similar for the case $(k < 2^{n_0}-1)$. For $m = 0,1,\dots,2^{n_0}-1$ and $p = 0,1,\dots,2^{n_0}-1$, we have the following set of equalities:

$$\begin{gathered} c_{2m,k}^{(n_0+1)} = c_{2m,k-2^n}^{(n_0)}; \qquad c_{2p,k}^{(n_0+1)} = c_{2p,k-2^n}^{(n_0)} \\ c_{2m+1,k}^{(n_0+1)} = c_{2m+1,k-2^n}^{(n_0)}; \qquad c_{2p+1,k}^{(n_0+1)} = c_{2p,k-2^n}^{(n_0)} \\ \text{if } 2m < 2^{n_0}-1,\ 2p < 2^{n_0}-1, \\ c_{2m,k}^{(n_0+1)} = c_{2m,k-2^n}^{(n_0)}; \qquad c_{2p,k}^{(n_0+1)} = -c_{2p,k-2^n}^{(n_0)} \\ c_{2m+1,k}^{(n_0+1)} = c_{2m+1,k-2^n}^{(n_0)}; \qquad c_{2p+1,k}^{(n_0+1)} = -c_{2p,k-2^n}^{(n_0)} \\ \text{if } 2m < 2^{n_0}-1,\ 2p > 2^{n_0}-1, \\ c_{2m,k}^{(n_0+1)} = -c_{2m,k-2^n}^{(n_0)}; \qquad c_{2p,k}^{(n_0+1)} = -c_{2p,k-2^n}^{(n_0)} \\ c_{2m+1,k}^{(n_0+1)} = -c_{2m+1,k-2^n}^{(n_0)}; \qquad c_{2p+1,k}^{(n_0+1)} = -c_{2p,k-2^n}^{(n_0)} \\ \text{if } 2m > 2^{n_0}-1,\ 2p > 2^{n_0}-1. \end{gathered} \tag{A.7}$$

As the case $2m > 2^{n_0} - 1$ and $2p < 2^{n_0} - 1$ can be directly derived, exchanging m and p, from the one where $2m < 2^{n_0} - 1$ and $2p > 2^{n_0} - 1$, we always get

$$\begin{aligned} c_{2m,k}^{(n_0+1)}c_{2p,k}^{(n_0+1)} &= c_{2m,k-2^n}^{(n_0)}c_{2p,k-2^n}^{(n_0)}, \\ c_{2m+1,k}^{(n_0+1)}c_{2p+1,k}^{(n_0+1)} &= c_{2m+1,k-2^n}^{(n_0)}c_{2p+1,k-2^n}^{(n_0)}. \end{aligned} \tag{A.8}$$

Then using the recurrence assumption for $n = n_0$ given in (A.4), we get

$$c_{2m,k}^{(n)}c_{2p,k}^{(n)} = c_{2m+1,k}^{(n)}c_{2p+1,k}^{(n)} \quad \text{for } n = 1,2,\dots,\infty, \quad m = 0,1,\dots,2^{(n-1)}-1,\ p = 0,1,\dots,2^{(n-1)}-1. \tag{A.9}$$

Step 2. Let us simply notice that for a given m and p, we have

$$\sum_{m=0}^{2^n-1-2p} (-1)^m c_{m+2p,k}^{(n)} c_{m,k}^{(n)} = \sum_{u=0}^{2^{n-1}-1-p} \left(c_{2u+2p,k}^{(n)}c_{2u,k}^{(n)} - c_{2p+2u+1,k}^{(n)}c_{2u+1,k}^{(n)}\right). \tag{A.10}$$

Then, with the result proved in Step 1, we conclude that

$$\sum_{m=0}^{2^n-1-2p} c_{m+2p,k}^{(n)} c_{m,k}^{(n)} (-1)^m = 0. \tag{A.11}$$

B. A SECOND PROPERTY OF W-H CODES

Keeping the same notations as in Appendix A and using the definitions of the subsets S_1^n, S_2^n, $\overline{S}_1^{n_0}$, and $\overline{S}_2^{n_0}$ presented in Section 4.1, we can also show that

$$\sum_{p=0}^{2^n-1} (-1)^p c_{p,u_0}^{(n)} c_{p,u_1}^{(n)} = 0 \quad \text{for } u_0, u_1 \in S_1^n \text{ (resp., } S_2^n\text{)}. \tag{B.1}$$

The proof is established by recurrence on n, starting by $n = 2$.

Case $n = 2$

The W-H matrix is given by

$$A^{(2)} = \frac{1}{\sqrt{4}} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}. \tag{B.2}$$

It can be easily checked, testing the different possible cases when $u_0, u_1 \in S_1^2 = \{0, 3\}$: $u_0 = u_1 = 0$ or 3, $u_0 = 0$ and $u_1 = 3$, similarly for $u_0, u_1 \in S_2^2 = \{1, 2\}$ that the property (B.1) is true for $n = 2$.

Case $n = n_0$

We now assume the property (B.1) is true for $n = n_0$. Otherwise, for the sets $S_1^{n_0}$ and $S_2^{n_0}$, see (23) and $u_0, u_1, \dots$, we have

$$\sum_{p=0}^{2^{n_0}-1} (-1)^p c_{p,u_0}^{(n_0)} c_{p,u_1}^{(n_0)} = 0. \tag{B.3}$$

Case $n = n_0 + 1$

Let us show that (B.3) also holds true for $n = n_0 + 1$.

For $n = n_0 + 1$, (B.3) can be rewritten as

$$\sum_{p=0}^{2^{n_0+1}-1} (-1)^p c_{p,u_0}^{(n_0+1)} c_{p,u_1}^{(n_0+1)} = \sum_{p=0}^{2^{n_0}-1} (-1)^p c_{p,u_0}^{(n_0+1)} c_{p,u_1}^{(n_0+1)} + \sum_{p=0}^{2^{n_0}-1} (-1)^p c_{p+2^{n_0},u_0}^{(n_0+1)} c_{p+2^{n_0},u_1}^{(n_0+1)}, \tag{B.4}$$

where the second summation results from a substitution of p by $p-2^{n_0}$.

Let us consider the 3 possible cases

(1) $u_0,u_1\in S_1^{n_0}$ (resp., $\in S_2^{n_0}$). Then for $p<2^{n_0}$, we get

$$c_{p+2^{n_0},u_0}^{(n_0+1)}=c_{p,u_0}^{(n_0)};\qquad c_{p,u_0}^{(n_0+1)}=c_{p,u_0}^{(n_0)};$$
$$c_{p+2^{n_0},u_1}^{(n_0+1)}=c_{p,u_1}^{(n_0)};\qquad c_{p,u_1}^{(n_0+1)}=c_{p,u_1}^{(n_0)}. \tag{B.5}$$

Based on (B.4) and (B.3), we get

$$\sum_{p=0}^{2^{n_0+1}-1}(-1)^p c_{p,u_0}^{(n_0)}c_{p,u_1}^{(n_0)}=2\sum_{p=0}^{2^{n_0}-1}(-1)^p c_{p,u_0}^{(n_0)}c_{p,u_1}^{(n_0)}=0. \tag{B.6}$$

(2) $u_0,u_1\in \overline{S}_1^{n_0}$ (resp., $\overline{S}_2^{n_0}$).

For $p<2^{n_0}$, based on the same principles of computation, we now obtain

$$c_{p+2_0^n,u_0}^{(n_0+1)}=-c_{p,u_0-2_0^n}^{(n_0)};\qquad c_{p,u_0}^{(n_0+1)}=c_{p,u_0-2_0^n}^{(n_0)};$$
$$c_{p+2_0^n,u_1}^{(n_0+1)}=-c_{p,u_1-2_0^n}^{(n_0)};\qquad c_{p,u_1}^{(n_0+1)}=c_{p,u_1-2_0^n}^{(n_0)}. \tag{B.7}$$

Then using relation (B.4) and noting that by the substitution $v_0=u_0-2^{n_0}$ and $v_1=u_1-2^{n_0}$ $v_0,v_1\in S_2^{n_0}$ (resp., $\in S_1^{n_0}$), then taking the recurrence relation (B.3) into account, we get

$$\sum_{p=0}^{2^{n_0+1}-1}(-1)^p c_{p,u_0}^{(n_0+1)}c_{p,u_1}^{(n_0+1)}=2\sum_{p=0}^{2^{n_0}-1}(-1)^p c_{p,v_0}^{(n_0)}c_{p,v_1}^{(n_0)}=0. \tag{B.8}$$

(3) $u_0\in S_1^{n_0}$ (resp., $S_2^{n_0}$) and $u_1\in\overline{S}_1^{n_0}$ (resp., $\overline{S}_2^{n_0}$)

For $p<2^{n_0}$, the recurrence relation between the columns of Hadamard matrices of successive order leads to

$$c_{p+2^{n_0},u_0}^{(n_0+1)}=c_{p,u_0}^{(n_0)};\qquad c_{p,u_0}^{(n_0+1)}=c_{p,u_0}^{(n_0)};$$
$$c_{p+2^{n_0},u_1}^{(n_0+1)}=-c_{p,u_1-2^{n_0}}^{(n_0)};\qquad c_{p,u_1}^{(n_0+1)}=c_{p,u_1-2^{n_0}}^{(n_0)}. \tag{B.9}$$

Then, we find

$$\sum_{p=0}^{2^{n_0+1}-1}(-1)^p c_{p,u_0}^{(n_0+1)}c_{p,u_1}^{(n_0+1)}=\sum_{p=0}^{2^{n_0}-1}(-1)^p c_{p,u_0}^{(n_0)}c_{p,u_1-2^{n_0}}^{(n_0)}-\sum_{p=0}^{2^{n_0}-1}(-1)^p c_{p,u_0}^{(n_0)}c_{p,u_1-2^{n_0}}^{(n_0)}=0. \tag{B.10}$$

To conclude, for any integer n, there are two subsets, S_1^n and S_2^n, which give a partition of all the index set such that for $u_0,u_1\in S_1^n$ (resp., S_2^n) the property (B.1) is satisfied.

C. THIRD PROPERTY OF W-H CODES

Using again the notations introduced in Appendix A and in Section 4, we are going to show that for $0<s<2^n-1$, the W-H codes satisfy the following properties:

$$\sum_{m=0}^{2^n-1-s}c_{m,u_0}^{(n)}c_{m+s,u_1}^{(n)}=\sum_{m=0}^{2^n-1-s}c_{m,u_1}^{(n)}c_{m+s,u_0}^{(n)}\quad\text{for } u_0,u_1\in S_1^n(\text{resp., } S_2^n), \tag{C.1}$$

$$\sum_{m=0}^{2^n-1-s}c_{m,u_0}^{(n)}c_{m+s,u_1}^{(n)}=-\sum_{m=0}^{2^n-1-s}c_{m,u_1}^{(n)}c_{m+s,u_0}^{(n)}\quad\text{for } u_0\in S_1^n,\ u_1\in S_2^n. \tag{C.2}$$

As in the previous appendices, we use a recurrence on n.

Case $n=1$

For the W-H matrix of order 1

$$A^{(1)}=\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}, \tag{C.3}$$

it can be checked with the partition $S_1^1=\{0\}$ and $S_2^1=\{1\}$ that for $s=1$, the relations (C.1) are (C.2) are true.

Case $n=n_0$

Let us now assume that these relations are also true for $n=n_0$, that is, for $0<s<2^{n_0}-1$, we have

$$\sum_{m=0}^{2^{n_0}-1-s}c_{m,u_0}^{(n_0)}c_{m+s,u_1}^{(n_0)}=\sum_{m=0}^{2_0^n-1-s}c_{m,u_1}^{(n_0)}c_{m+s,u_0}^{(n_0)}\quad\text{for } u_0,u_1\in S_1^{n_0}\ (\text{resp., } S_2^{n_0}), \tag{C.4}$$

$$\sum_{m=0}^{2^{n_0}-1-s}c_{m,u_0}^{(n_0)}c_{m+s,u_1}^{(n_0)}=-\sum_{m=0}^{2_0^n-1-s}c_{m,u_1}^{(n_0)}c_{m+s,u_0}^{(n_0)}\quad\text{for } u_0\in S_1^{n_0},\ u_1\in S_2^{n_0}. \tag{C.5}$$

Case $n=n_0+1$

To show that these properties are also true for $n=n_0+1$, the seven different cases have to be considered

(1) $u_0,u_1\in S_1^{n_0+1}$ (resp., $S_2^{n_0+1}$) and $u_0,u_1\in S_1^{n_0}$ (resp., $S_2^{n_0}$).

(2) $u_0,u_1\in S_1^{n_0+1}$ (resp., $S_2^{n_0+1}$) and $u_0,u_1\notin S_1^{n_0}$ (resp., $S_2^{n_0}$).

(3) $u_0,u_1\in S_1^{n_0+1}$ (resp., $\in S_1^{n_0+1}$) and $u_1\in S_1^{n_0}$ (resp., $S_2^{n_0}$) and $u_0\notin S_1^{n_0}$ (resp., $S_2^{n_0}$).

(4) $u_0\in S_1^{n_0+1}$, $u_1\in S_2^{n_0+1}$ and $u_0\in S_1^{n_0}$, and $u_1\in S_2^{n_0}$.

(5) $u_0 \in S_1^{n_0+1}$, $u_1 \in S_2^{n_0+1}$ and $u_0 \in S_1^{n_0}$, and $u_1 \notin S_2^{n_0}$.
(6) $u_0 \in S_1^{n_0+1}$, $u_1 \in S_2^{n_0+1}$ and $u_0 \notin S_1^{n_0}$, and $u_1 \notin S_2^{n_0}$.
(7) $u_0 \in S_1^{n_0+1}$, $u_1 \in S_2^{n_0+1}$ and $u_0 \notin S_1^{n_0}$, and $u_1 \in S_2^{n_0}$.

As the proofs are quite similar for these different cases in this appendix, as a matter of example, we only provide the proofs for the first 2 cases.

(1) $u_0, u_1 \in S_1^{n_0+1}$ (resp., $S_2^{n_0+1}$) and $u_0, u_1 \in S_1^{n_0}$ (resp., $S_2^{n_0}$). We split the interval of analysis for $0 < s < 2^{n_0+1} - 1$ into two subcases.

Subcase $0 < s < 2^{n_0} - 1$

Let S_L denote the left-hand side in relation (C.1), it can be split into three terms:

$$S_L = \sum_{m=0}^{2^{n_0}-1-s} c_{m,u_0}^{(n_0+1)} c_{m+s,u_1}^{(n_0+1)} + \sum_{m=2^{n_0}-s}^{2^{n_0}-1} c_{m,u_0}^{(n_0+1)} c_{m+s,u_1}^{(n_0+1)} + \sum_{m=2^{n_0}}^{2^{n_0+1}-1-s} c_{m,u_0}^{(n_0+1)} c_{m+s,u_1}^{(n_0+1)}. \tag{C.6}$$

Making a substitution by $m \rightarrow m - 2^{n_0} + s$ and $m \rightarrow m - 2^{n_0}$ in the second and third summations, respectively, then using in the first and third summations the relation of recurrence between the columns of W-H matrices of consecutive order, we get

$$\begin{aligned} S_L &= \sum_{m=0}^{2^{n_0+1}-1-s} c_{m,u_1}^{(n_0+1)} c_{m+s,u_0}^{(n_0+1)} \\ &= 2 \sum_{m=0}^{2^{n_0}-1-s} c_{m,u_1}^{(n_0)} c_{m+s,u_0}^{(n_0)} + \sum_{m=0}^{2^{n_0}-\hat{s}-1} c_{m,u_0}^{(n_0)} c_{m+\hat{s},u_1}^{(n_0)} \end{aligned} \tag{C.7}$$

with $\hat{s} = 2^{n_0} - s$.

Then using the recurrence hypothesis (C.4), we have

$$\sum_{m=0}^{2^{n_0+1}-1-s} c_{m,u_1}^{(n_0+1)} c_{m+s,u_0}^{(n_0+1)} = \sum_{m=0}^{2^{n_0+1}-1-s} c_{m,u_0}^{(n_0+1)} c_{m+s,u_1}^{(n_0+1)} \tag{C.8}$$

Subcase $2^{n_0} < s$

Then we can write

$$\begin{aligned} S_L &= \sum_{m=0}^{2^{n_0+1}-1-s} c_{m,u_0}^{(n_0)} c_{m+s-2^{n_0},u_1}^{(n_0)} \\ &= \sum_{m=0}^{2^{n_0+1}-2^{n_0}-1-(s-2^{n_0})} c_{m,u_0}^{(n_0)} c_{m+s-2^{n_0},u_1}^{(n_0)} \\ &= \sum_{m=0}^{2^{n_0}-1-\hat{s}} c_{m,u_0}^{(n_0)} c_{m+\hat{s},u_1}^{(n_0)} \end{aligned} \tag{C.9}$$

with again $\hat{s} = s - 2^{n_0}$. From the recurrence hypothesis (C.4), we have

$$\sum_{m=0}^{2^{n_0+1}-1-s} c_{m,u_1}^{(n_0+1)} c_{m+s,u_0}^{(n_0+1)} = \sum_{m=0}^{2^{n_0+1}-1-s} c_{m,u_0}^{(n_0+1)} c_{m+s,u_1}^{(n_0+1)}. \tag{C.10}$$

(2) $u_0, u_1 \in S_1^{n_0+1}$ (resp., $S_2^{n_0+1}$) and $u_0, u_1 \notin S_1^{n_0}$ (resp., $S_2^{n_0}$).

To prove the properties for $0 < s < 2^{n_0+1} - 1$, we proceed as in the first case, splitting the analysis into two 2 intervals for s.

Case $0 < s < 2^{n_0} - 1$

S_L, the left-hand side in relation (C.1), can again be split into three terms:

$$S_L = \sum_{m=0}^{2^{n_0}-1-s} c_{m,u_0}^{(n_0+1)} c_{m+s,u_1}^{(n_0+1)} + \sum_{m=2^{n_0}-s}^{2^{n_0}-1} c_{m,u_0}^{(n_0+1)} c_{m+s,u_1}^{(n_0+1)} + \sum_{m=2^{n_0}}^{2^{n_0+1}-1-s} c_{m,u_0}^{(n_0+1)} c_{m+s,u_1}^{(n_0+1)}. \tag{C.11}$$

Making a substitution by $m \rightarrow m - 2^{n_0} + s$ and $m \rightarrow m - 2^{n_0}$ in the second and third summations, respectively, then using, in the first and third summations, the relation of recurrence between the columns of W-H matrices of consecutive order, we get

$$\begin{aligned} S_L &= \sum_{m=0}^{2^{n_0+1}-1-s} c_{m,u_0}^{(n_0+1)} c_{m+s,u_1}^{(n_0+1)} \\ &= 2 \sum_{m=0}^{2^{n_0}-1-s} c_{m,v_0}^{(n_0)} c_{m+s,v_1}^{(n_0)} - \sum_{m=0}^{2^{n_0}-\hat{s}-1} c_{m,v_1}^{(n_0)} c_{m+\hat{s},v_0}^{(n_0)} \end{aligned} \tag{C.12}$$

with $\hat{s} = 2^{n_0} - s$, $v_1 = u_1 - 2^n$, $v_0 = u_0 - 2^n$, and $v_0, v_1 \in S_2^{n_0}$ (resp., $S_1^{n_0}$). In the same manner, we get

$$\begin{aligned} &\sum_{m=0}^{2^{n_0+1}-1-s} c_{m,u_1}^{(n_0+1)} c_{m+s,u_0}^{(n_0+1)} \\ &\quad = 2 \sum_{m=0}^{2^{n_0}-1-s} c_{m,v_1}^{(n_0)} c_{m+s,v_0}^{(n_0)} - \sum_{m=0}^{2^{n_0}-\hat{s}-1} c_{m,v_0}^{(n_0)} c_{m+\hat{s},v_1}^{(n_0)}. \end{aligned} \tag{C.13}$$

From the recurrence hypothesis (C.4), we have

$$\sum_{m=0}^{2^{n_0+1}-1-s} c_{m,u_1}^{(n_0+1)} c_{m+s,u_0}^{(n_0+1)} = \sum_{m=0}^{2^{n_0+1}-1-s} c_{m,u_0}^{(n_0+1)} c_{m+s,u_1}^{(n_0+1)} \tag{C.14}$$

Case $2^{n_0} < s$

$$\begin{aligned} S_L &= \sum_{m=0}^{2^{n_0+1}-1-s} c_{m,u_0}^{(n_0+1)} c_{m+s,u_1}^{(n_0+1)} \\ &= -\sum_{m=0}^{2^{n_0+1}-1-s} c_{m,u_0-2^{n_0}}^{(n_0)} c_{m+s-2^{n_0},u_1-2^{n_0}}^{(n_0)} \\ &= -\sum_{m=0}^{2^{n_0+1}-2^{n_0}-1-(s-2^{n_0})} c_{m,v_0}^{(n_0)} c_{m+s-2^{n_0},v_1}^{(n_0)} \\ &= -\sum_{m=0}^{2^{n_0}-1-\hat{s}} c_{m,v_0}^{(n_0)} c_{m+\hat{s},v_1}^{(n_0)} \end{aligned} \tag{C.15}$$

with $\hat{s} = 2^{n_0} - s$, $v_1 = u_1 - 2^n$, $v_0 = u_0 - 2^n$ and $v_0, v_1 \in S_2^{n_0}$ (resp., $S_1^{n_0}$). In the same manner, we get

$$\sum_{m=0}^{2^{n_0+1}-1-s} c_{m,u_1}^{(n_0+1)} c_{m+s,u_0}^{(n_0+1)} = -\sum_{m=0}^{2^{n_0}-1-\hat{s}} c_{m,v_1}^{(n_0)} c_{m+\hat{s},v_0}^{(n_0)}. \tag{C.16}$$

From the recurrence hypothesis (C.4), we have

$$\sum_{m=0}^{2^{n_0+1}-1-s} c_{m,u_1}^{(n_0+1)} c_{m+s,u_0}^{(n_0+1)} = \sum_{m=0}^{2^{n_0+1}-1-s} c_{m,u_0}^{(n_0+1)} c_{m+s,u_1}^{(n_0+1)}. \tag{C.17}$$

To conclude, for any integer n, there are two subsets, S_1^n and S_2^n, which give a partition of all the index set such that for $u_0, u_1 \in S_1^n$ (resp., S_2^n), the property (C.1) is satisfied and for $u_0 \in S_1^n$ (resp., S_2^n) and $u_1 \in S_2^n$ (resp., S_1^n), the property (C.2) is satisfied.

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