

Research Article

Application of Single-Mode Fiber-Coupled Receivers in Optical Satellite to High-Altitude Platform Communications

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In a free-space optical communication system employing fiber-optic components, the phasefront distortions induced by atmospheric turbulence limit the efficiency with which the laser beam is coupled into a single-mode fiber. We analyze different link scenarios including a geostationary (GEO) satellite, a high-altitude platform (HAP), and an optical ground station (OGS). Single-mode coupled optically preamplified receivers allow for efficient suppression of background noise and highly sensitive detection. While GEO-to-OGS communication suffers from atmospheric turbulence, we demonstrate that GEO-to-HAP communication allows for close to diffraction-limited performance when applying tip-tilt correction.

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1. INTRODUCTION

Free-space laser communications—with its ability to transmit information via a collimated laser beam at high data rates using compact, low-mass terminals, while avoiding interference problems and without exhausting the radio-frequency bandwidths—are a promising candidate to satisfy the ever increasing bandwidth demand associated with new communication services. While optical intersatellite links are already operable [1], laser communication from ground suffers from cloud coverage, harsh weather conditions, and atmospheric turbulence [2]. To find a remedy, current research concentrates on optical communications from or to high-altitude platforms (HAPs)—aircrafts which are situated well above the clouds at typical heights of 17 to 22 km—where the atmospheric impact on a laser beam is less severe than directly above ground [3, 4]. An optical link between satellite and HAP could serve as a broadband communication channel if data from several sensors or RF communication terminals onboard the HAP is to be transmitted to a satellite, or if the HAP works as a data relay station, receiving information from a satellite.

While existing satellite laser communication terminals use AlGaAs semiconductor lasers with wavelengths around

830 nm [1], recent research activities focus on communication wavelengths around $\lambda = 1550$ nm. At this wavelength, highly sensitive receivers incorporating Erbium-doped fiber amplifiers (EDFAs) for optical preamplification are available [5, 6]. When using such fiber-based technology, coupling into a single-mode fiber (SMF) is required and the reduction of the fiber coupling efficiency due to atmospheric turbulence is an important issue.

The Earth's atmosphere extends approximately 700 km above the surface and consists of several distinct layers [7]. Pronounced density is found within the lowest 20 km, still influencing satellite-to-HAP links. When a laser beam propagates through a turbulent medium like the atmosphere, one observes absorption, scattering, additional beam spreading and beam wander, scintillation, and phasefront distortions. The latter results in a loss of power when the light is coupled into a single-mode fiber and—in the worst case—may lead to a link failure. Knowledge of the efficiency with which random light (i.e., light that produces speckle patterns) can be coupled into optical fibers is therefore important, for example, to assess the feasibility of using single-mode coupled, optically preamplified receivers onboard of HAPs. Other effects of atmospheric turbulence than phasefront distortions are not addressed here as they influence all types of optical

TABLE 1: Typical parameter set for the satellite-to-HAP communication scenario.

Parameter	Symbol	Default value
HAP altitude	h_{HAP}	20 km
Satellite altitude	h_{SAT}	35786 km (GEO)
Telescope diameter	D	13.5 cm
Zenith angle	ζ	50°
Communication wavelength	λ	1550 nm

receivers in the same way (i.e., causing additional loss in power and intensity fluctuations).

The effect of aberrations and misalignments of optical components [8, 9], angular jitter [10], or atmospheric turbulence [11–13] on the single-mode fiber coupling efficiency has been analyzed in the field of optical communications, lidar, or astronomy. In most cases, a Gaussian approximation has been used for the fundamental fiber mode. We calculate the degradation of the mean power coupling efficiency into single-mode fiber coupled devices caused by atmospheric turbulence using the exact field of the fiber’s fundamental mode. We analyze different link scenarios, as GEO-to-HAP and GEO-to-OGS, and verify the applicability of fiber coupled receivers onboard HAPs due to the reduced impact of atmospheric turbulence at high altitudes. Additional tip-tilt correction even allows for close to diffraction limited performance. By comparing our calculations with analytical models for the Strehl ratio (a standard indicator of the performance of classical imaging systems), we show that the Strehl ratio is a good approximation for the coupling efficiency into a single-mode fiber.

2. COMMUNICATION SCENARIO

Optical satellite-to-HAP connections allow establishing broadband (backhaul) links, which are required to globally receive from and distribute data to HAPs. The satellite is located at a height h_{SAT} above ground, which may range from 400 km for a low Earth orbiting satellite (LEO) to 35786 km for a geostationary satellite (GEO). The HAP altitude h_{HAP} gives the height above ground for the high-altitude platform. The telescope diameter D denotes the transmit telescope diameter in the uplink case and the receive telescope diameter in the downlink. Typically, the telescope diameters of optical antennas onboard of HAPs and satellites may vary between 0.05 m and 0.2 m. The zenith angle ζ is defined as the angle between the direction to zenith and the direction of the laser beam towards the satellite. Typical system parameter values used for the calculations in the following sections (assuming the HAP to be positioned in Central Europe) are given in Table 1.

Using a single-mode coupled optical preamplifier (cf., Figure 1) onboard the HAP allows for high sensitivity detection and is of advantage with respect to the suppression of background noise. Like in heterodyning, single-mode coupled receivers are less vulnerable to background light than,

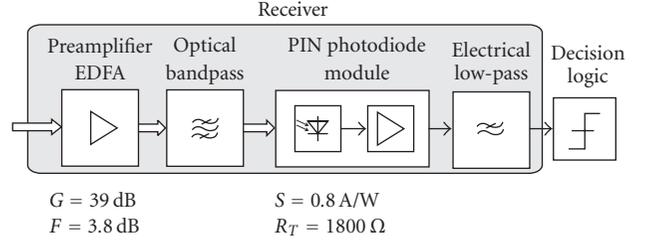


FIGURE 1: Setup of an optically preamplified receiver (G : preamplifier gain, F : preamplifier noise figure, S : photodiode responsivity, R_T : transimpedance).

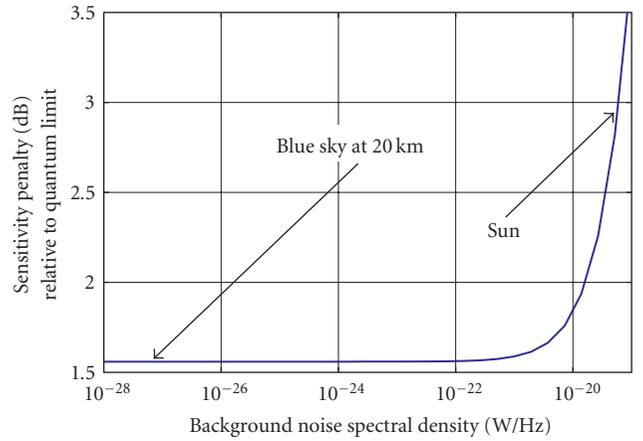


FIGURE 2: Calculated receiver sensitivity penalty of an optically preamplified receiver using RZ intensity modulation at a bit error probability of 10^{-9} as a function of background noise power spectral density.

for example, avalanche photodiode (APD) based receivers because the fiber provides an excellent spatial filter function and only one spatial mode (in two orthogonal directions of polarization) is detected [14].

Figure 2 shows the calculated receiver sensitivity penalty as a function of the background noise power spectral density. The sensitivity penalty is defined as the required number of received photons per bit at a bit error probability (BEP) of 10^{-9} relative to the quantum limit of 41 photons per bit [15, 16]. For the computation, we assumed return-to-zero (RZ) intensity modulation (with a duty cycle of 33%) using an external modulator with 20-dB extinction and no polarization control at the optically preamplified receiver. Beside an EDFA (gain $G = 39 \text{ dB}$ and noise figure $F = 3.8 \text{ dB}$), the receiver incorporates an optical fiber Bragg grating bandpass filter, a PIN-photodiode module with a sensitivity of $S = 0.8 \text{ A/W}$, a transimpedance amplifier with $R_T = 1800 \Omega$, and an electrical fifth-order Bessel lowpass filter (cf., Figure 1).

When optimizing the optical and electrical filter bandwidths, the calculated receiver sensitivity is only 1.6 dB above the quantum limit in the absence of background noise. In the case of blue sky as background source, that is, with the receiver onboard the HAP, no degradation of receiver

performance is found when compared to the case where there is no background light at all. Even when directly looking into the Sun (with a background noise power spectral density of $4.1 \cdot 10^{-20}$ W/Hz) only an additional deterioration of 1.4 dB has to be expected.

In a HAP-to-satellite uplink, the turbulent eddies forming the atmosphere are right in front of the transmitter and comparatively large relative to the optical beam (cf., Figure 3(a)). The resulting large beam spreading causes the phasefront disturbance to be negligible within the small receiving aperture at the satellite [7]. In the downlink, the turbulent eddies are relatively small when compared to the beam diameter, thus leading to noticeable phasefront distortions (cf., Figure 3(b)).

3. SINGLE-MODE FIBER COUPLING EFFICIENCY

We model the coupling as shown in Figure 4. The optical input field E —which is collected by the receive telescope of diameter D —is focussed by a thin diffraction limited lens to the bare end of a single-mode fiber, whose field distribution of the fundamental mode F as a function of the radial coordinate r is given by the Bessel function of the first kind and of zero order within the core and by the modified Bessel function of the second kind and of zero order in the cladding [17].

The power coupling efficiency η is defined as the ratio of the average power carried by the fiber mode and the average power available in the focal plane. It is possible—and often more convenient—to calculate the coupling efficiency in the aperture plane \mathcal{A} , that is, just in front of the coupling lens (cf., Figure 4). The mean coupling efficiency is given by

$$\langle \eta \rangle = \left\langle \left| \iint_{\mathcal{A}} F(r)E(r)\exp(j\phi(r))dA \right|^2 \right\rangle, \quad (1)$$

where the angled brackets represent the ensemble-average operator, $E(r)$ is the incident optical field within plane \mathcal{A} normalized to a power of unity within the input aperture, and $F(r)$ is the deterministic fiber mode backpropagated into the aperture plane by means of an inverse spatial Fourier transform. A phase function $\Phi(r)$ covers any deviations from an ideal plane wavefront of the input field E . The coupling efficiency depends also on the lens-to-fiber coupling geometry, which can be taken into account via an additional coupling design parameter $\chi = \rho/r_S$, with ρ being the core radius of the fiber and r_S being the Airy radius. The parameter χ takes into account the properties of the incoming beam and of the coupling optics, while the normalized frequency $V = 2\pi\rho/\lambda(n_{co}^2 - n_{cl}^2)^{1/2}$, with core and cladding refractive indices, n_{co} and n_{cl} , determine the characteristics (e.g., the numerical aperture) of the optical fiber. Without loss of generality, we assume for the calculations to follow the fiber to be operated at single-mode cutoff ($V = 2.405$) and the lens-to-fiber geometry to be optimized for maximum coupling efficiency ($\chi = 0.5345$). In the case of a perfect phasefront, $\Phi(r) = \text{const.}$, and when approximating the fiber’s eigenmode by a Gaussian field distribution a maximum coupling efficiency of $\eta_{\text{max}} = 81.5\%$ can be achieved [9]. When using

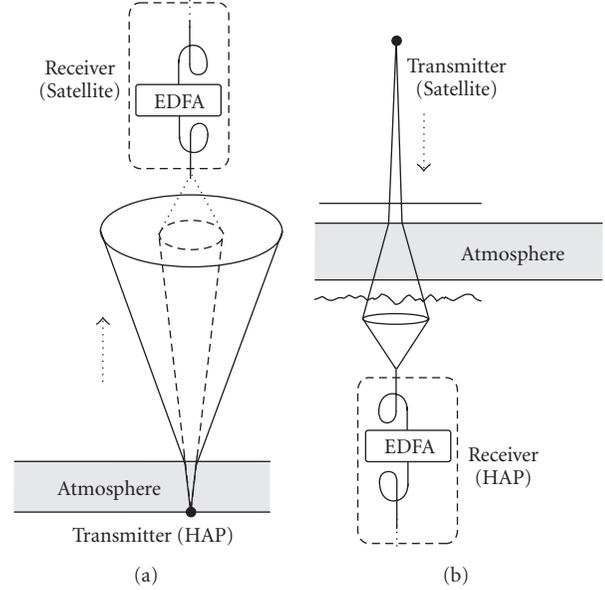


FIGURE 3: (a) Uplink: Negligible phasefront disturbance, large beam spreading. (b) Downlink: Negligible beam spread, large phasefront disturbance over receiving aperture.

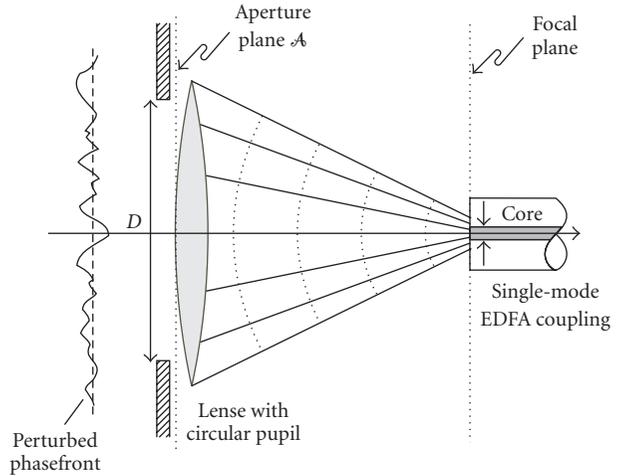


FIGURE 4: Coupling geometry: A thin lens focuses the incident field E , which is assumed to have constant amplitude but a statistically disturbed phase, onto the bare end of a single-mode fiber.

the exact field of the fiber’s fundamental mode the maximum coupling efficiency amounts to $\eta_{\text{max}} = 78.6\%$, corresponding to a minimum power coupling loss of 1.05 dB.

4. TURBULENCE-INDUCED PHASEFRONT DISTORTIONS

To evaluate the influence of a phasefront disturbance on the fiber coupling efficiency, the phase function $\Phi(r)$ is modeled as a normal distribution with zero mean ($\mu_\phi = 0$) and standard deviation σ_ϕ . Using (1), the resulting coupling efficiency

is calculated by means of a Monte Carlo simulation. Based on the Zernike representation of the Kolmogorov spectrum of turbulence, σ_ϕ can be derived depending on the receiving aperture diameter D and the Fried parameter r_0 as [7, 18, 19]

$$\sigma_\phi = \sqrt{1.0299(D/r_0)^{5/3}}. \quad (2)$$

The Fried parameter can be interpreted as the diameter of an aperture over which there is approximately 1 rad of rms phase distortion σ_ϕ^2 . In accordance with [7], we calculate the Fried parameter as

$$r_0 = \left[0.423k^2 \sec(\zeta) \int_{h_{\text{HAP}}}^{h_{\text{SAT}}} C_n^2(h) dh \right]^{-3/5}, \quad (3)$$

using the optical wave number $k = 2\pi/\lambda$ in rad/m and the structure parameter $C_n^2(h)$ in $\text{m}^{-2/3}$ at height h . The structure parameter is a measure of turbulence strength and represents the total amount of energy contained in the stochastic field of the refractive index fluctuations. It can be approximated according to the Hufnagle-Valley model [20],

$$C_n^2(h) = 5.94 \cdot 10^{-3} \text{ m}^{-8/3} \text{ s}^2 \left(\frac{\nu_{\text{RMS}}}{27} \right)^2 \left(\frac{h}{10^5 \text{ m}} \right)^{10} \exp\left\{ \frac{-h}{1000 \text{ m}} \right\} \\ + 2.7 \cdot 10^{-16} \exp\left\{ \frac{-h}{1500 \text{ m}} \right\} + C_n^2(0) \exp\left\{ \frac{-h}{100 \text{ m}} \right\}. \quad (4)$$

The structure constant on ground, $C_n^2(0)$, ranges from $10^{-17} \text{ m}^{-2/3}$ (during night and weak turbulence conditions) to $10^{-13} \text{ m}^{-2/3}$ (during day and strong turbulence conditions). The required rms wind speed ν_{RMS} is calculated using the Bufton wind model [7, 20]:

$$\nu_{\text{RMS}} = \left[\frac{1}{15 \cdot 10^3} \int_{5 \cdot 10^3}^{20 \cdot 10^3} \left\{ \nu_{\text{wind}} + \nu_T \exp\left[-\left(\frac{h-h_T}{d_T} \right)^2 \right] \right\}^2 dh \right]^{1/2}. \quad (5)$$

The quantity ν_{wind} is the ground wind speed, ν_T is the wind speed at the tropopause, h_T is the height of the tropopause, and d_T is its thickness. For our calculations, we assume $\nu_{\text{wind}} = 3 \text{ m/s}$, $\nu_T = 30 \text{ m/s}$, $h_T = 9.4 \text{ km}$, $d_T = 4.8 \text{ km}$, and a structure constant on ground of $C_n^2(0) = 1.7 \cdot 10^{-14} \text{ m}^{-2/3}$, which are typical values for clear sky and weak turbulence conditions.

5. COMPARISON OF LINK SCENARIOS

A comparison between the single-mode fiber coupling efficiency in a satellite-to-ground communication link and a satellite-to-HAP scenario shows the feasibility of using fiber coupled receivers onboard HAPs due to the reduced impact of atmospheric turbulence at high altitudes. Figure 5 gives the coupling efficiency as a function of the rms phasefront perturbation, RMS_λ , expressed in fractions of the wavelength λ . For the calculations (using the methods as described in Sections 3 and 4), we assumed a wavelength of 1550 nm and a standard (telecommunications) single-mode fiber with a

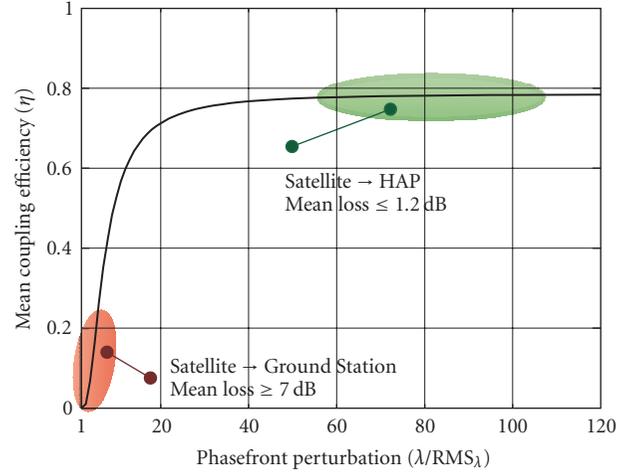


FIGURE 5: Mean coupling efficiency $\langle \eta \rangle$ for a standard single-mode fiber as a function of the rms phasefront perturbation RMS_λ given in fractions of the wavelength λ .

core diameter of $10 \mu\text{m}$ a core refractive index of 1.46, and a core/cladding refractive indices difference of 0.3%. As expected, the characteristic shown in Figure 5 is rather flat for small perturbations, but drops dramatically to very small values for large disturbances. For typical satellite-to-HAP communication links (i.e., with $h_{\text{HAP}} > 17 \text{ km}$ and $D < 20 \text{ cm}$), the mean coupling loss is always less than 1.2 dB, while for communication links to a ground station it significantly increases to values larger than 7 dB.

The mean value of the coupling efficiency into a SMF as a function of the zenith angle according to (1) is shown in Figure 6(a) for the following three scenarios: (i) a GEO-to-HAP link with the HAP at $h_{\text{HAP}} = 20 \text{ km}$ and a receive telescope diameter of $D = 0.135 \text{ m}$, (ii) a GEO-to-HAP link with $D = 0.135 \text{ m}$ and $h_{\text{HAP}} = 2400 \text{ m}$, and (iii) a GEO-to-ground link with a telescope diameter $D = 1 \text{ m}$ at an optical ground station.

At high altitudes, $\langle \eta \rangle$ is roughly invariant with zenith angle and very close to the maximum possible coupling efficiency of 78.6%. With decreasing height also the coupling efficiency decreases due to increasing turbulence leading to noticeable phasefront perturbations especially at zenith angles $\zeta > 50^\circ$. The phasefront perturbation over a large aperture like at the OGS is more severe than at a HAP, leading to a poor coupling efficiency which, in practice, excludes the use of a single-mode coupled receiver within a ground station.

If the spatial period of the phasefront perturbation is clearly smaller than the pupil diameter, the normalized mean power coupling efficiency is sometimes estimated by the Strehl ratio SR [13], which is defined as the ratio of peak power in the center of the image plane compared to that of an equivalent unaberrated (diffraction-limited) system. It is useful to have an approximate predictor for the coupling efficiency based on the SR which is also a standard indicator of the performance of classical and adaptive (astronomical) imaging systems. Under general conditions, not limited to

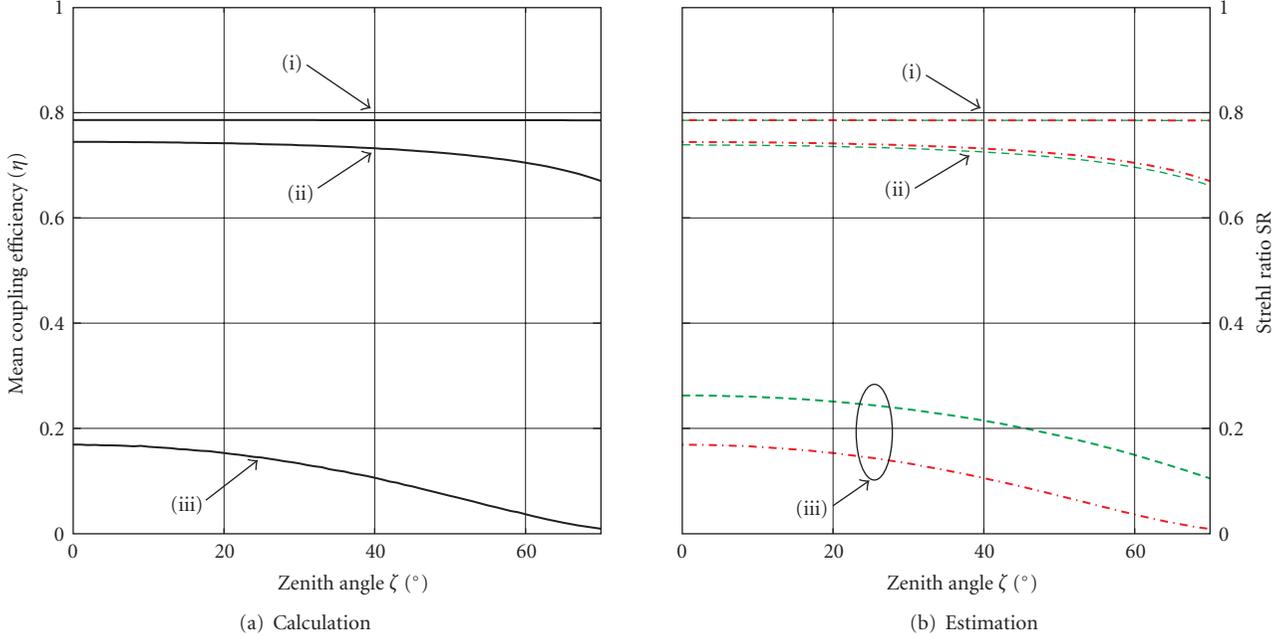


FIGURE 6: (a) Calculated coupling efficiency (1) and (b) estimation of coupling efficiency using the Strehl ratio (dashed line: (6), dashed-dotted line: (7)) versus zenith angle for three different link scenarios: (i) GEO-to-HAP with $h_{\text{HAP}} = 20$ km and $D = 0.135$ m, (ii) GEO-to-HAP with $h_{\text{HAP}} = 2400$ m and $D = 0.135$ m, and (iii) GEO-to-ground with $h_{\text{ground}} = 2400$ m and $D = 1$ m at the OGS.

weak fluctuations, the Strehl ratio for an untracked beam can be approximated by the expression [7]

$$\text{SR}_1 = \left[1 + \left(\frac{D}{r_0} \right)^{5/3} \right]^{-6/5}. \quad (6)$$

Another widely used empirical formula is the Marechal approximation [13, 19],

$$\text{SR}_2 = \exp(-\sigma_\phi^2), \quad (7)$$

where σ_ϕ^2 is the variance of the phase aberration across the receiving aperture D . An estimation of the coupling efficiency using these approximations is given in Figure 6(b), showing good agreement with the computationally more intensive calculation of Figure 6(a) in the communication scenarios where HAPs are involved, that is, at high altitudes and when using small telescope diameters.

The phase variance averaged over a circular aperture of diameter D as given in (2) drops to

$$\sigma_{\phi, \text{track}} = \sqrt{0.134(D/r_0)^{5/3}}, \quad (8)$$

if the tip and tilt components of the atmospheric perturbations are removed by means of active target tracking [18, 19]. Thus applying tip-tilt correction reduces the phase variance by a factor of $1.0299/0.134 = 7.7$, independent of the diameter of the telescope. Figure 7 compares the coupling loss into an SMF for an untracked beam and for a tip-tilt corrected system in a GEO-to-HAP downlink (at different turbulence conditions and using the parameters as given in Table 1). The

coupling loss is normalized to the maximum mean coupling efficiency of 78.6%. When tracking is performed by means of a tiltable mirror, the coupling loss is noticeably reduced compared to the untracked case. The advantage of using large wavelengths and HAP altitudes of more than 15 km, leading to negligible coupling loss, is also shown in Figure 7.

Figures 6 and 7 illustrate that using a telescope with D less than a few times r_0 as it is the case in a typical satellite-to-HAP communication scenario, will, after tip-tilt correction, lead to a residual phase variance which is small compared to unity and therefore results in close-to-diffraction-limited performance.

6. SUMMARY

We have analyzed the effect of atmospheric turbulence on a free-space optical communication system employing fiber-optic components for different link scenarios including a geostationary satellite, a high-altitude platform, and an optical ground station. We show that single-mode coupled optically preamplified receivers allow for efficient suppression of background noise and highly sensitive detection. While the phase perturbations in a downlink from GEO-to-OGS prevent the use of optically preamplified receivers because of the low achievable coupling efficiency, the perturbations are small in typical GEO-to-HAP links, thus allowing for efficient single-mode fiber coupling. Active tip-tilt correction reduces phasefront perturbations and allows for achieving close to diffraction limited performance. We showed that the Strehl ratio is a good approximation for the coupling efficiency into a single-mode coupled receiver.

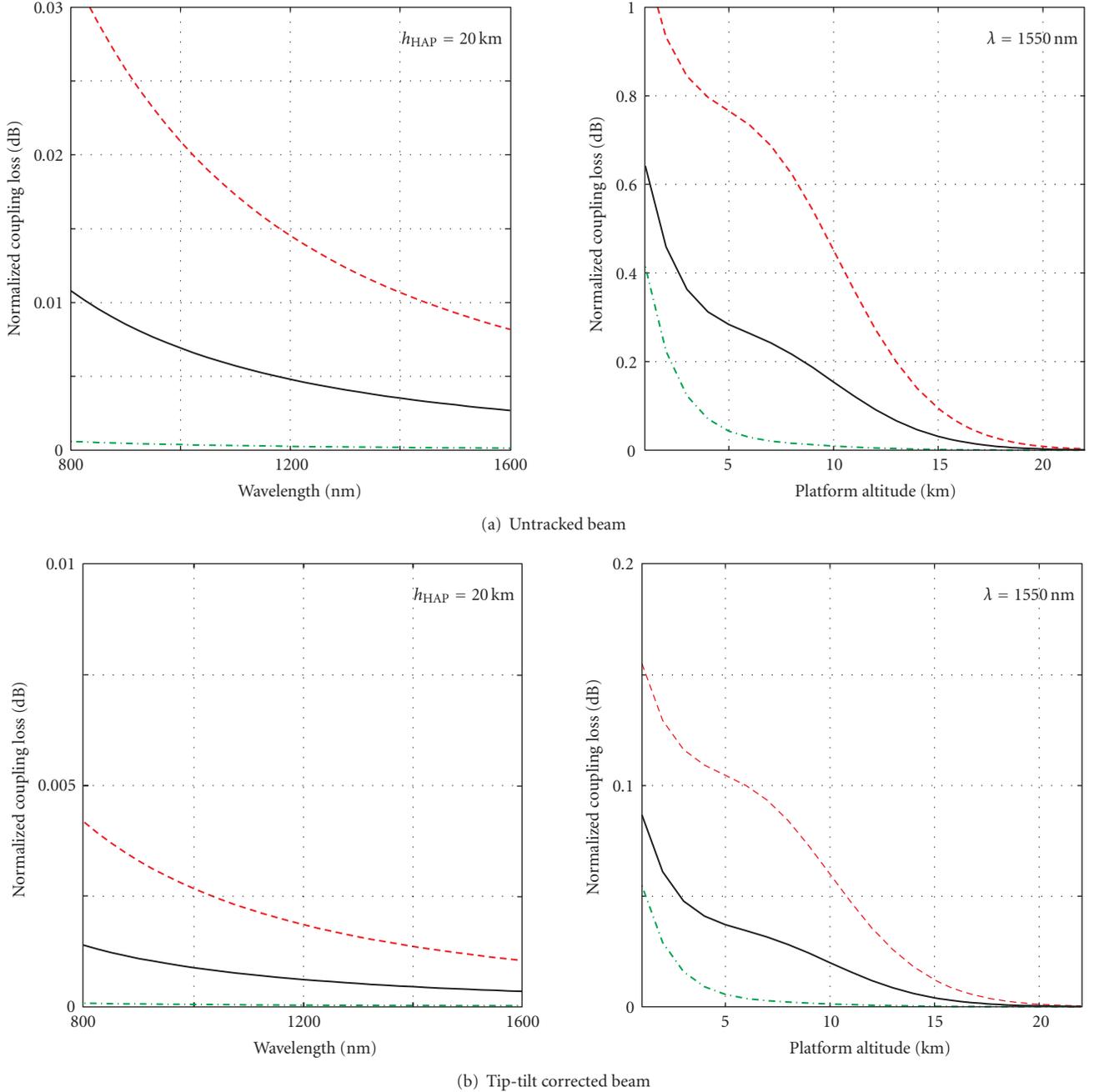


FIGURE 7: Normalized coupling loss into a standard single-mode fiber for a GEO-to-HAP downlink versus wavelength λ , and platform altitude h_{HAP} in the case of (a) an untracked beam and (b) a tip-tilt corrected beam. (dashed-dotted line: $v_{\text{wind}} = 0$ m/s, $C_n^2(0) = 10^{-17} \text{ m}^{-2/3}$, solid line: $v_{\text{wind}} = 3$ m/s, $C_n^2(0) = 1.7 \cdot 10^{-14} \text{ m}^{-2/3}$, dashed line: $v_{\text{wind}} = 20$ m/s, $C_n^2(0) = 10^{-13} \text{ m}^{-2/3}$).

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