

## Research Article

# Admission Control Threshold in Cellular Relay Networks with Power Adjustment

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In the cellular network with relays, the mobile station can benefit from both coverage extension and capacity enhancement. However, the operation complexity increases as the number of relays grows up. Furthermore, in the cellular network with cooperative relays, it is even more complex because of an increased dimension of signal-to-noise ratios (SNRs) formed in the cooperative wireless transmission links. In this paper, we propose a new method for admission capacity planning in a cellular network using a cooperative relaying mechanism called decode-and-forward. We mathematically formulate the dropping ratio using the randomness of “channel gain.” With this, we formulate an admission threshold planning problem as a simple optimization problem, where we maximize the accommodation capacity (in number of connections) subject to two types of constraints. (1) A constraint that the sum of the transmit powers of the source node and relay node is upper-bounded where both nodes can jointly adjust the transmit power. (2) A constraint that the dropping ratio is upper-bounded by a certain threshold value. The simplicity of the problem formulation facilitates its solution in real-time. We believe that the proposed planning method can provide an attractive guideline for dimensioning a cellular relay network with cooperative relays.

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## 1. Introduction

It is expected that both the operational complexity and the signaling burden are increased as the number of communication nodes increases in cellular networks. However, the large number of nodes distributed over the service area may act as a relay node for other nodes so that the transmit power and the achievable rate can be improved [1]. The use of relays is considered to be one of the most attractive strategies for the next generation wireless network [2]. Also, orthogonal frequency-division multiple access (OFDMA) is one of the most promising solutions to provide a high-performance physical layer in emerging cellular networks. OFDMA is based on OFDM and inherits immunity to intersymbol interference and frequency selective fading. Recently, adaptive resource management for multiuser OFDMA systems has attracted enormous research interest [3–7]. In [3], the authors studied how to minimize the total transmission power while satisfying a minimum rate constraint for each user. The problem was formulated as an

integer programming problem and a continuous-relaxation-based suboptimal solution method was studied. In [4], a class of computationally inexpensive methods for power allocation and subcarrier assignment were developed, which are shown to achieve comparable performance, but do not require intensive computation.

Specifically for data traffic, several studies have considered providing a *fair* opportunity for users to access a wireless system so that no user may dominate in resource occupancy while others starve. In [5], the authors proposed a fair scheduling scheme to minimize the total transmit power by allocating subcarriers to the users and then to determine the number of bits transmitted on each subcarrier. Also, they developed suboptimal solution algorithms by using the linear programming technique and the Hungarian method. A new scheme to fairly allocate subcarriers, rate, and power for multiuser OFDMA system was proposed [6], where a new generalized proportional fairness criterion, based on Nash bargaining solutions and coalitions, was used. The study in [6] is very different from the previous

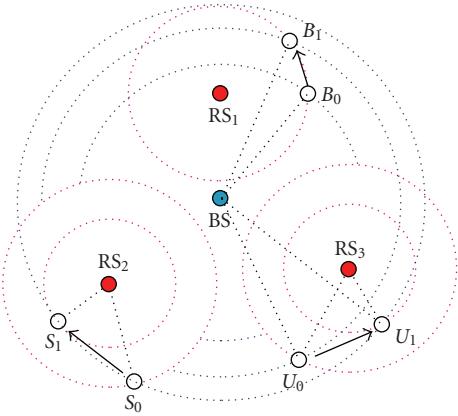


FIGURE 1: Three examples of channel gain change according to movement of mobile station (MS). Case 1 ( $B_0 \rightarrow B_1$ ): MS goes away from BS with RS-MS distance constant. The channel gain between RS and MS increases but it does not increase the achievable rate between MS and BS. Case 2 ( $S_0 \rightarrow S_1$ ): MS gets close to RS with BS-MS distance constant. The channel gain between BS and MS decreases and it decreases the achievable rate between BS and MS. Case 3 ( $U_0 \rightarrow U_1$ ): MS gets close to RS whereas it goes away from BS. The channel gain between RS and MS increases but the channel gain between BS and MS decreases, which finally decreases the achievable rate between BS and MS.

OFDMA scheduling studies in the sense that the resource allocation is performed with a game-theoretic decision rule. They proposed a very fast near-optimal algorithm using the Hungarian method. They showed by simulations that their fair-scheduling scheme provides a similar overall rate to that of the rate-maximizing scheme. In [7], they provided achievable rate formulations from the physical layer perspective and studied algorithms using the Lagrangian multiplier technique, where they showed that their algorithms can find the global optimum even in the case that the problems are nonconvex.

Most previous work on resource allocation in OFDMA systems, however, did not consider the connection-level performance which is limited by the fluctuations in performance, for example, signal-to-noise ratio (SNR), in the lower layer. Because of the random nature of user mobility, the average channel gain of a targeted group of users (referred simply as the average channel gain in the rest of the paper) in a cellular relay network changes over time, causing the average SNR of the user group to continuously change and fluctuate. Figure 1 presents an example where the maximum number of users have been accommodated in the best SNR case, which may cause a portion of them to be dropped if the SNR falls down from that point. Since the maximum achievable transmit rate is bounded by the SNR, ongoing connections may experience outage events and, furthermore, the dropping ratio increases for any given number of connections admitted in the system. Therefore, it is necessary to take the fluctuating nature of SNR into account when planning for the admission capacity threshold value.

In this paper, more specifically, we consider admission capacity planning for cellular networks with cooperative relays [1], considering the randomness of channel gains between three types of links formed in cooperative relaying as shown in Figure 1. In cooperative relaying through decode-and-forward, the achievable rate of the link between the source node and the destination node is characterized by the channel gains stochastic of three links: source-relay, source-destination, and relay-destination. This figure depicts three exemplary cases. In Case 1, where mobile station (MS) moves from point  $B_0$  to  $B_1$ , the distance between base station (BS) and MS gets longer with the distance between MS and relay station (RS) kept. The channel gain between RS and MS increases but the channel gain between BS and MS does not increase. Supposing that the current achievable rate between MS to BS (in the cooperatively formed link, but not in the direct transmission link) is upper bounded by the channel gain of BS-MS link, the movement from point  $B_0$  to point  $B_1$  will cause a reduction of the achievable rate in the cooperatively formed link between BS and MS. However, supposing that there is surplus power used in the transmitter (either of BS or of MS), the movement does not necessarily lead to a reduction in the achievable rate. In Case 2, where MS moves from point  $S_0$  to  $S_1$ , the MS-RS distance gets shorter with the MS-BS distance unchanged. In Case 3, where MS moves from point  $U_0$  to  $U_1$ , the MS-BS distance gets larger whereas the MS-RS distance gets shorter. For example, suppose that BS is transmitting some packets to MS. Also, suppose that the current transmit power vector is in equilibrium. Then BS needs to adjust the transmit power not to loose the current level of achievable rate. However, if BS adjust the transmit power but RS does not, RS may cause a certain level of power waste, also causing interference to other receivers to grow up.

In [8], Niyato and Hossain studied two call admission schemes in OFDMA networks. However, they did not consider the nonstationary nature of SNR in determining the threshold value for admission control. Also, the network model does not include relaying architectures. These two points are the major difference between their contributions and ours.

In [9], we considered a capacity planning problem in cooperative cellular relay network but no power adjustment was considered. In this paper, however, we propose a new method for admission capacity planning in OFMDA cellular networks with cooperative relays with power adjustment between source and relay nodes, which take into considerations of the random nature of the average channel gain. We derive the dropping ratio, and formulate an optimization problem to maximize the admission capacity subject to a dropping ratio constraint. The simplicity of the problem formulation enables the admission capacity planning problem to be solved in real-time.

There are extensive studies on subcarrier and power allocations in OFDM (see [3–7] and the literatures therein), where the authors assume that the SNR is not variable during the scheduling period. The results of these studies can be used in an adaptive manner in accordance with the frequent changes of SNR. Regardless of adaptations with respect to

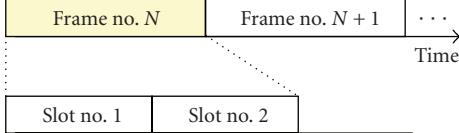


FIGURE 2: An example of frame structure (in the time domain) for decode-and-forward relaying. The first slot is used for source node to transmit whereas the second slot for relay node to relay the received data from the source node.

SNR variations, outage events of ongoing real-time connections are unavoidable in the cases that the instantaneous capacity with respect to the locations of users residing in a cell becomes lower than the minimum capacity required to serve those connections. A simple solution to improve the dropping ratio of ongoing connections is to apply a certain “bound” to the maximum number of connections. Because of simplicity of this type of solution, it is useful for practical applications. However, it is necessary to investigate how to find appropriate bounds for connection admission that take into account the particular characteristics of OFDM systems, which differentiates this problem from similar problems in the other wireless systems.

The main objective of this work is to find appropriate upper bounds of the number of connections that can be admitted in the system so that the dropping ratio is upper bounded by a certain threshold. More specifically, the objective is to maximize the admission capacity while keeping the dropping ratio upper bounded by a certain threshold value. In this paper, we call these upper bounds the “admission capacity.” We consider the case that the channel gain of user  $j$  using subcarrier  $i$ , denoted by  $G_{ij}$ , is a random variable that varies over time. In this case, the optimal subcarrier and power allocations will vary over time as they are completely dependent on the values the random variables  $G_{ij}$ ’s. We assume the perfect condition that optimum power and subcarrier allocations are made given the values of  $G_{ij}$ ’s. This assumption is necessary and widely adopted in the literature to enable an analytical evaluation of the achievable system capacity. For example, in capacity planning of CDMA systems with time-division duplex (TDD), it is commonly assumed to have perfect power control and resource allocation [10].

We consider an OFDMA cellular relay network with cooperative relaying called *decode-and-forward* [1]. A cell has a total of  $C$  subcarriers and each user has a transmission power limit of  $\bar{p}$ . In a single link (without cooperative diversity scheme), the throughput of user  $j$  using subcarrier  $i$ , denoted by  $R_{ij}$ , is given by

$$R_{ij} = W \log_2(1 + a \cdot G_{ij} p_{ij}), \quad (1)$$

where  $W$  is the bandwidth of a subcarrier,  $a \approx -1.5/(\sigma^2 \cdot \log(5 \cdot \text{BER}))$  ( $\text{BER}$  denotes desired bit-error rate),  $G_{ij}$  denotes the channel gain of user  $j$  at subcarrier  $i$ ,  $\sigma^2$  is the thermal noise power, and  $p_{ij}$  denotes the power allocated to user  $j$  at subcarrier  $i$  [6]. Each connection has the minimum rate requirement  $\phi$  such that an outage event occurs if the

assigned rate is smaller than the minimum required transmit rate  $\phi$ .

In cellular networks, the user nodes are normally mobile, which implies that the channel gains  $G_{ij}$ ’s can be considered as random variables. The allocation of subcarrier and power is dependent upon the instantaneous values of the random variables. In such situations, we propose an alternative to approximate the total rate of connections when  $y$  connections are ongoing as follows [11]:

$$\begin{aligned} R(y; \bar{p}_s, \bar{p}_r) &\approx \left( C \cdot \frac{W}{2} \right) \\ &\cdot \min \left\{ \log_2 \left( 1 + \frac{ay}{C} \cdot \bar{G}_{s,r} \bar{p}_s \right), \right. \\ &\quad \left. \log_2 \left( 1 + \frac{ay}{C} \cdot \{\bar{G}_{s,d} \bar{p}_s + \bar{G}_{r,d} \bar{p}_r\} \right) \right\}, \end{aligned} \quad (2)$$

where  $W$  is the bandwidth of a subcarrier,  $1/2$  is because of two-slot frame structure for cooperative relaying (as in Figure 2),  $\bar{G}_{(\cdot,\cdot)} = (1/yC) \sum_{i=1}^C \sum_{j=1}^y G_{ij}$  in the associated link  $(\cdot, \cdot)$ , and  $(y/C) \cdot \bar{p}_{(\cdot)}$  is the average power allocated to a subcarrier at the associated node  $(\cdot)$ . By letting  $\alpha \triangleq CW/2$  and  $\beta(y) \triangleq (a/C)y$ , we can rewrite (2) as

$$\begin{aligned} R(y; \bar{p}_s, \bar{p}_r) &\approx \alpha \cdot \min \{ \log_2(1 + \beta(y) \cdot \bar{G}_{s,r} \bar{p}_s), \\ &\quad \log_2(1 + \beta(y) \cdot (\bar{G}_{s,d} \bar{p}_s + \bar{G}_{r,d} \bar{p}_r)) \}. \end{aligned} \quad (3)$$

There are practical reasons to use  $\bar{G}$  instead of the individual random variables  $G_{ij}$ ’s. First, the variances of  $G_{ij}$ ’s with respect to indices  $i$  and  $j$  are small in the case of group-mobility users because the users are located at the nearly same position with respect to the base station. Second, the mean value  $\bar{G}$  is an unbiased estimator that provides sufficient statistical information on the targeted population. The probability density function (pdf) of random variable  $\bar{G}$  is denoted by  $f_G(\cdot)$ . In the case of a system filled with individual mobility users, the approximation used in (3) may not be sufficiently accurate because the channel gains and allocated powers of individual mobility users are quite different, which is beyond the scope of this work. In the case of group-mobility users, however, because of the first reason, the approximation is much more accurate.

## 2. Dropping Ratio Formulation

In this section, we derive the dropping ratio  $D(y; \bar{p}_s, \bar{p}_r)$  when there are  $y$  connections are ongoing. The *dropping ratio* is defined as the average fraction of the total number of connections suffering from outages:

$$D(y; \bar{p}_s, \bar{p}_r) = \Pr(R(y; \bar{p}_s, \bar{p}_r) < y \cdot \phi). \quad (4)$$

By letting  $\rho(y) = (y \cdot \phi/\alpha)$ , we have

$$\begin{aligned} D(y; \bar{p}_s, \bar{p}_r) &= \Pr \left( \min \{1 + \beta(y) \cdot \bar{G}_{s,r} \bar{p}_s, \right. \\ &\quad \left. 1 + \beta(y) \cdot (\bar{G}_{s,d} \bar{p}_s + \bar{G}_{r,d} \bar{p}_r)\} < 2^{\rho(y)} \right). \end{aligned} \quad (5)$$

Let  $A = 1 + \beta(y) \cdot \bar{G}_{sr} \bar{p}_s$  and  $B = 1 + \beta(y) \cdot (\bar{G}_{sd} \bar{p}_s + \bar{G}_{rd} \bar{p}_r)$ . Under the assumption that random variables  $\bar{G}_{sr}$ ,  $\bar{G}_{sd}$ , and  $\bar{G}_{rd}$  are mutually independent, we can rewrite the above expression as

$$\begin{aligned} D(y; \bar{p}_s, \bar{p}_r) &= \Pr(\min(A, B) < 2^{\rho(y)}) \\ &= 1 - \Pr(\min(A, B) \geq 2^{\rho(y)}) \\ &= 1 - \Pr(A \geq 2^{\rho(y)}) \cdot \Pr(B \geq 2^{\rho(y)}), \end{aligned} \quad (6)$$

and, therefore,

$$\begin{aligned} D(y; \bar{p}_s, \bar{p}_r) &= 1 - \{1 - \Pr(1 + \beta(y) \cdot \bar{G}_{sr} \bar{p}_s < 2^{\rho(y)})\} \\ &\quad \cdot \{1 - \Pr(1 + \beta(y) \cdot (\bar{G}_{sd} \bar{p}_s + \bar{G}_{rd} \bar{p}_r) < 2^{\rho(y)})\} \\ &= 1 - \left\{1 - \Pr\left(\bar{G}_{sr} < \frac{2^{\rho(y)} - 1}{\beta(y) \cdot \bar{p}_s}\right)\right\} \\ &\quad \cdot \left\{1 - \Pr\left(\bar{G}_{sd} \bar{p}_s + \bar{G}_{rd} \bar{p}_r < \frac{2^{\rho(y)} - 1}{\beta(y)}\right)\right\}. \end{aligned} \quad (7)$$

We can rewrite this as follows:

$$\begin{aligned} D(y; \bar{p}_s, \bar{p}_r) &= \Pr\left(\bar{G}_{sr} < \frac{2^{\rho(y)} - 1}{\beta(y) \cdot \bar{p}_s}\right) \\ &\quad + \int_0^{((2^{\rho(y)} - 1)/\beta(y)\bar{p}_r)} F_{\bar{G}_{sd}}\left(\frac{2^{\rho(y)} - 1}{\beta(y)\bar{p}_s} - \frac{\bar{p}_r}{\bar{p}_s} \cdot x\right) \cdot f_{\bar{G}_{rd}}(x) dx \\ &\quad - \Pr\left(\bar{G}_{sr} < \frac{2^{\rho(y)} - 1}{\beta(y) \cdot \bar{p}_s}\right) \\ &\quad \cdot \int_0^{((2^{\rho(y)} - 1)/\beta(y)\bar{p}_r)} F_{\bar{G}_{sd}}\left(\frac{2^{\rho(y)} - 1}{\beta(y)\bar{p}_s} - \frac{\bar{p}_r}{\bar{p}_s} \cdot x\right) \cdot f_{\bar{G}_{rd}}(x) dx \\ &= F_{\bar{G}_{sr}}\left(\frac{2^{\rho(y)} - 1}{\beta(y) \cdot \bar{p}_s}\right) \\ &\quad + \int_0^{((2^{\rho(y)} - 1)/\beta(y)\bar{p}_r)} F_{\bar{G}_{sd}}\left(\frac{2^{\rho(y)} - 1}{\beta(y)\bar{p}_s} - \frac{\bar{p}_r}{\bar{p}_s} \cdot x\right) \\ &\quad \cdot f_{\bar{G}_{rd}}(x) dx - F_{\bar{G}_{sr}}\left(\frac{2^{\rho(y)} - 1}{\beta(y) \cdot \bar{p}_s}\right) \\ &\quad \cdot \int_0^{((2^{\rho(y)} - 1)/\beta(y)\bar{p}_r)} F_{\bar{G}_{sd}}\left(\frac{2^{\rho(y)} - 1}{\beta(y)\bar{p}_s} - \frac{\bar{p}_r}{\bar{p}_s} \cdot x\right) \cdot f_{\bar{G}_{rd}}(x) dx. \end{aligned} \quad (8)$$

### 3. Minimization of Dropping Ratio

We find the maximum  $y$  that satisfies a dropping ratio constraint by solving the following simple problem  $(P)$ .

**3.1. Problem Formulation.** We have the following:

$$(P) \quad \begin{aligned} &\text{maximize } y \\ &\text{subject to } D(y; \bar{p}_s, \bar{p}_r) \leq \gamma_0 \end{aligned} \quad (9)$$

$$w \cdot \bar{p}_s + (1 - w) \cdot \bar{p}_r \leq \bar{p}, \quad (10)$$

where  $p_s, p_r$  are nonnegative real numbers,  $y$  is nonnegative integer, and  $w, \bar{p}$  are given values.

The role of problem  $(P)$  is to find the maximum  $y$  that satisfies a dropping ratio constraint. In other words, it is to maximize  $y$  subject to a constraint that the dropping ratio is less than or equal to  $\gamma_0$ .

### 3.2. Solution Method of $(P)$ .

**Proposition 1.** *The dropping probability  $D(y; \bar{p}_s, \bar{p}_r)$  is a strictly increasing function of  $y$ .*

*Proof.* Let

$$h(y) \triangleq \frac{2^{\rho(y)} - 1}{\beta(y) \bar{p}_s},$$

$$\begin{aligned} H(y) &\triangleq \int_0^{((2^{\rho(y)} - 1)/\beta(y)\bar{p}_r)} F_{\bar{G}_{sd}}\left(\frac{2^{\rho(y)} - 1}{\beta(y)\bar{p}_s} - \frac{\bar{p}_r}{\bar{p}_s} \cdot x\right) \cdot f_{\bar{G}_{rd}}(x) dx \\ &= \int_0^{\infty} F_{\bar{G}_{sd}}\left(\frac{2^{\rho(y)} - 1}{\beta(y)\bar{p}_s} - \frac{\bar{p}_r}{\bar{p}_s} \cdot x\right) \cdot f_{\bar{G}_{rd}}(x) dx. \end{aligned} \quad (11)$$

Then

$$\begin{aligned} \frac{dD(y; \bar{p}_s, \bar{p}_r)}{dy} &= \frac{d}{dy} \left\{ F_{\bar{G}_{sr}}\left(\frac{2^{\rho(y)} - 1}{\beta(y) \cdot \bar{p}_s}\right) \right\} + \frac{d}{dy} H(y) \\ &\quad - \frac{d}{dy} \left\{ F_{\bar{G}_{sr}}\left(\frac{2^{\rho(y)} - 1}{\beta(y) \cdot \bar{p}_s}\right) \right\} \cdot H(y) \\ &\quad - F_{\bar{G}_{sr}}\left(\frac{2^{\rho(y)} - 1}{\beta(y) \cdot \bar{p}_s}\right) \cdot \frac{d}{dy} H(y) \\ &= f_{\bar{G}_{sr}}\left(\frac{2^{\rho(y)} - 1}{\beta(y) \cdot \bar{p}_s}\right) \frac{dh(y)}{dy} \\ &\quad + \int_0^{\infty} f_{\bar{G}_{sd}}\left(\frac{2^{\rho(y)} - 1}{\beta(y)\bar{p}_s} - \frac{\bar{p}_r}{\bar{p}_s} \cdot x\right) \frac{dh(y)}{dy} \cdot f_{\bar{G}_{rd}}(x) dx \\ &\quad - f_{\bar{G}_{sr}}\left(\frac{2^{\rho(y)} - 1}{\beta(y) \cdot \bar{p}_s}\right) \frac{dh(y)}{dy} \cdot H(y) - F_{\bar{G}_{sr}}\left(\frac{2^{\rho(y)} - 1}{\beta(y) \cdot \bar{p}_s}\right) \\ &\quad \cdot \int_0^{\infty} f_{\bar{G}_{sd}}\left(\frac{2^{\rho(y)} - 1}{\beta(y)\bar{p}_s} - \frac{\bar{p}_r}{\bar{p}_s} \cdot x\right) \frac{dh(y)}{dy} \cdot f_{\bar{G}_{rd}}(x) dx. \end{aligned} \quad (12)$$

This can be rewritten as

$$\begin{aligned}
& \frac{dD(y; \bar{p}_s, \bar{p}_r)}{dy} \\
&= f_{\bar{G}_{sr}} \left( \frac{2^{\rho(y)} - 1}{\beta(y) \cdot \bar{p}_s} \right) \frac{dh(y)}{dy} \cdot \overbrace{\{1 - H(y)\}}^{>0} \\
&\quad + \overbrace{\left\{ 1 - F_{\bar{G}_{sr}} \left( \frac{2^{\rho(y)} - 1}{\beta(y) \cdot \bar{p}_s} \right) \right\}}^{>0} \\
&\quad \cdot \int_0^\infty f_{\bar{G}_{sd}} \left( \frac{2^{\rho(y)} - 1}{\beta(y) \bar{p}_s} - \frac{\bar{p}_r}{\bar{p}_s} \cdot x \right) \frac{dh(y)}{dy} \cdot f_{\bar{G}_{rd}}(x) dx. \tag{13}
\end{aligned}$$

Here,

$$\begin{aligned}
\frac{dh(y)}{dy} &= \frac{\rho'(y) 2^{\rho(y)} \ln 2 \cdot \beta(y) - (2^{\rho(y)} - 1) \cdot \beta'(y)}{(\beta(y))^2} \cdot \frac{1}{\bar{p}_s} \\
&= \frac{2^{\rho(y)} \cdot \{\rho(y) \cdot \ln 2 - 1\} + 1}{\beta(y) \cdot y} \cdot \frac{1}{\bar{p}_s} (\because \beta(y) = \beta'(y) \cdot y). \tag{14}
\end{aligned}$$

Let

$$g(y) \triangleq 2^{\rho(y)} \cdot \{\rho(y) \cdot \ln 2 - 1\} + 1. \tag{15}$$

Then we have the following:

$$\begin{aligned}
g(0) &= 0, \\
g(y) &> g(0), \quad \forall y > 0 \ (\because \rho'(y) > 0). \tag{16}
\end{aligned}$$

Thus,  $dh(y)/dy > 0$ . This yields  $dD(y; \bar{p}_s, \bar{p}_r)/dy > 0$ . This completes the proof.  $\square$

**Proposition 2.** For a given value of  $\bar{p}_s$ , a feasible solution  $y^*$  is the global optimal solution if and only if

$$y^* = \lfloor \sup \{y : D(y; \bar{p}_s, \bar{p}_r) \leq y_0\} \rfloor. \tag{17}$$

In other words, the following solution:

$$y^* = \lfloor D^{-1}(y_0) \rfloor \tag{18}$$

is the unique global optimum. Since  $dD/dy > 0$ ,  $D$  is invertible.

*Proof.* Suppose that there is a feasible solution  $y_0$  better than  $y^*$ : that is,  $y_0 \geq y^* + 1$  and  $D(y_0) \leq y_0$ . Since  $D(y; \bar{p}_s, \bar{p}_r)$  is strictly increasing,  $D(y^* + k) > y_0$  for all  $k \geq 1$ , which yields the two inequalities under this assumption cannot hold at the same time. Therefore, there are no solutions better than  $y^*$ .  $\square$

From these two Propositions, we have the following result.

**Proposition 3.** A feasible solution  $y^*$  is the global optimal solution if and only if

$$y^* = \lfloor \sup \{y : D(y; \bar{p}_s, \bar{p}_r) \leq y_0, \forall \bar{p}_s\} \rfloor. \tag{19}$$

TABLE 1: Parameters used in experiments.

Item	Value	Description
$\bar{p}$	50	Avg. transmit power (mW)
$\sigma^2$	$1e - 11$	Thermal noise level (W)
$C$	128	No. of subcarriers
BER	$1e - 5$	Desired bit-error rate
$W$	25000	Bandwidth of subcarrier (Hz)
$\phi$	100	Min. required rate per connection (Kbps)
$\bar{G}$	$\sim \mathcal{N}(100, 5)$	
$w$	0.5	

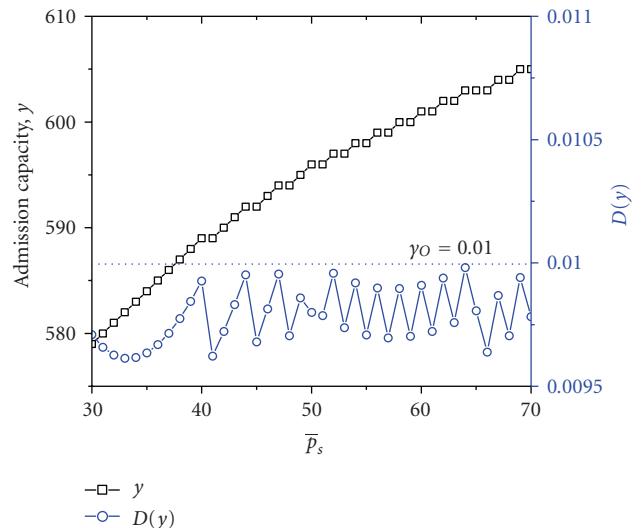


FIGURE 3: Relation between the admission capacity and the transmit power of the source node when transmitting its own traffic  $y_0 = 0.01$ .

In other words, the following solution:

$$y^* = \lfloor D^{-1}(y_0) \rfloor, \quad \forall \bar{p}_s \tag{20}$$

is the unique global optimum.

**Proposition 4.** The constraint (10) is binding at the optimum.

*Proof.* The accommodation capacity  $y$  is an increasing function of  $\bar{p}_s$  and  $\bar{p}_r$ . However, in decode-and-forward cooperation, the capacity is not a strictly increasing function of them because there may exist a portion of wastage in either side, which cannot necessarily contribute to the increase of the achievable rate. However, if there is a waste in one side, either source node or relay node, there is a binding in the other side. Therefore, the former can reduce a certain portion of the transmit power and the latter can increase a certain portion of the transmit power such that the constraint (10) is not violated. As far as there is a waste in one side, the other is binding; this means all power vector  $(\bar{p}_s, \bar{p}_r)$  that has a waste has room to improve the capacity  $y$ , that is, it is not optimal. Therefore, even if  $y$  is not a strictly increasing function of  $\bar{p}_s$  and  $\bar{p}_r$ , the power vector  $(\bar{p}_s, \bar{p}_r)$  is not optimal if the equality does not hold.  $\square$

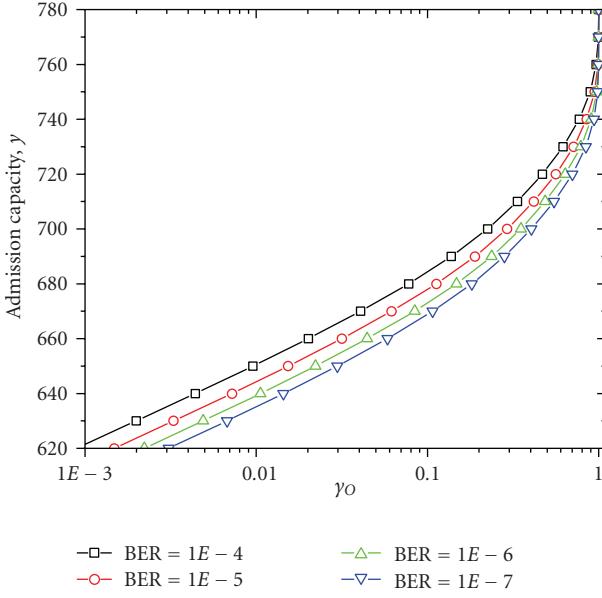


FIGURE 4: The maximum number of connections  $y$  versus  $\gamma_0$  with respect to BER ( $\bar{p}_s = \bar{p}_r = 50$  mW,  $\sigma^2 = 10^{-11}$ ,  $\phi = 100$  Kbps).

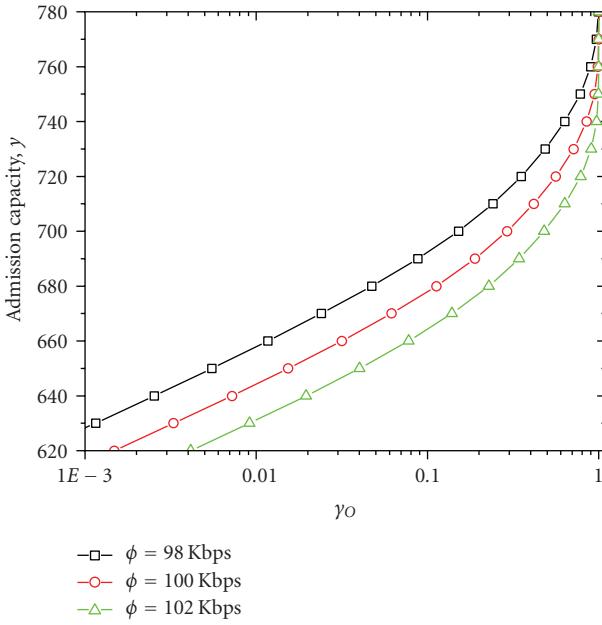


FIGURE 5: The maximum number of connections  $y$  versus  $\gamma_0$  with respect to  $\phi$  ( $\bar{p}_s = \bar{p}_r = 50$  mW,  $\sigma^2 = 10^{-11}$ , BER =  $1e-5$ ).

Using Proposition 4, we may eliminate one of variables  $\bar{p}_s, \bar{p}_r$  by setting the equality of (10). With this, we can simply solve the problem.

**3.3. Experimental Results.** We examine the three proposed methods for various pdfs of the average channel gain  $\bar{G}$  and for various values of BER,  $\phi$ ,  $\sigma^2$ , and  $\bar{p}$ . In our simulation setups, the transmission power is  $\bar{p} = 50$  mW, the thermal noise power is  $\sigma^2 = 10^{-11}$  W, the number of subcarriers is  $C =$

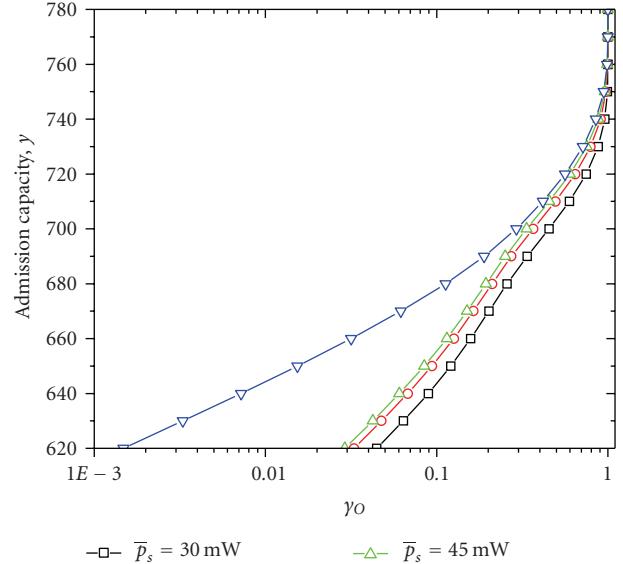


FIGURE 6: The maximum number of connections  $y$  versus  $\gamma_0$  with respect to  $\bar{p}_s$  ( $\bar{p}_r = 50$  mW,  $\sigma^2 = 10^{-11}$ ,  $\phi = 100$  Kbps, BER =  $1e-5$ ).

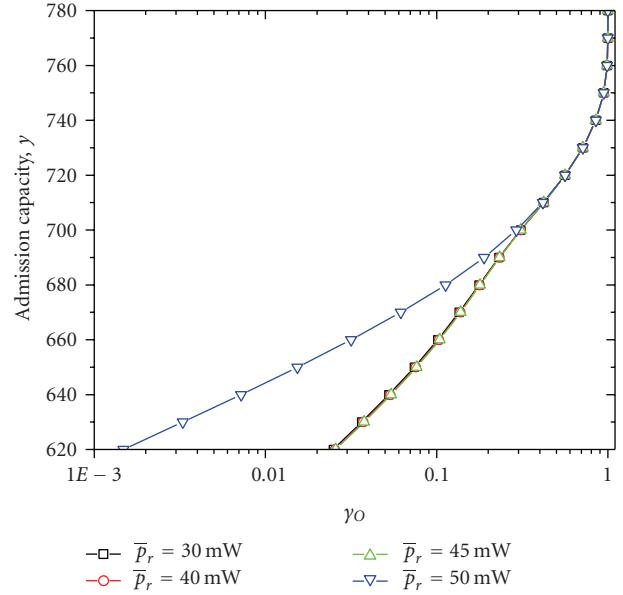


FIGURE 7: The maximum number of connections  $y$  versus  $\gamma_0$  with respect to  $\bar{p}_r$  ( $\bar{p}_s = 50$  mW,  $\sigma^2 = 10^{-11}$ ,  $\phi = 100$  Kbps, BER =  $1e-5$ ).

128 over a 3.2-MHz band, BER =  $10^{-5}$ , and the minimum rate requirement is  $\phi = 100$  kbps, which are used as default values.

Figure 3 shows the relation between the admission capacity and the transmit power of the source node when the source node spends 50% of its whole activity duration for transmitting its own traffic and the other 50% for relaying others' traffic. It is observed that the admission capacity increases as the portion of used power at the source node becomes greater than that of the relay node.

For different requirements of bit-error rates, Figure 4 shows the maximum number of connections that can be accommodated in a cell with a target dropping ratio of  $\gamma_0$ . It is observed that the admission capacity decreases as the bit-error rate requirement becomes stringent. For the given setup of this numerical experiment, it is observed for  $\gamma_0 = 0.01$  that the admission capacity increases at a rough rate of 8% per 10-fold increase in the targeted bit-error rate.

For different values of required data rates, Figure 5 shows the maximum number of connections that can be accommodated in a cell with a target dropping ratio of  $\gamma_0$ . We test how much increase or decrease in the admission capacity we may have if there is 2% of decrease and increase. For  $\gamma_0 = 0.01$ , the admission capacity at  $\phi = 100$  Kbps is 644. If there is 2% decrease in required data rate, the admission capacity increases to 659 (2.3% of increase) whereas if there is 2% increase in required data rate, the admission capacity decreases to 631 (2.0% of decrease).

For different levels of transmit power for the source node and relay node, Figures 6 and 7 show the maximum number of connections that can be accommodated in a cell with a target dropping ratio of  $\gamma_0$ . It is commonly observed, for any  $\gamma_0$  less than a certain value (e.g., 0.1), that an increase in transmit power greater than a certain value (e.g., 45 mW) results in a remarkable increase in admission capacity. Also, it is observed, by comparison of these figures, that adjustment in transmit power of relay nodes does not have good impact on the increase of admission capacity when the transmit power level is small (e.g., less than 45 mW in this experimental setup).

## 4. Conclusions

Since the admission capacity, defined as the upper bound of the number of connections that a base station can accommodate, fluctuates in accordance with the signal-to-noise ratio in cellular networks with node cooperation diversity, it is highly probable that a portion of ongoing connections may be dropped prior to their normal completion because of outage events. In this paper, we have developed a challenging method for admission capacity planning in an OFDMA-based cellular relay system with cooperative diversity scheme called *decode-and-forward*. Taking into account of the fluctuations of the average channel gains in the multihop cellular network, we have derived dropping ratio at the connection level. Based on the metric, we have formulated a problem to optimize admission capacity under given conditions. Because of the simplicity of its formulation, each problem can be solved in real-time. We believe that the proposed capacity planning method can be effectively applied in the design and dimensioning of OFDMA cellular networks with cooperative relays.

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