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Research Article

Z-Complementary Sets Based on Sequences with Periodic and Aperiodic Zero Correlation Zone

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Methods of constructing Z-complementary sequence sets are described. Different from those constructions using complementary sequences as a kernel, the proposed constructions are based on sequences with periodic and aperiodic zero-correlation zone (PAZCZ). These PAZCZ sequences are unitary sequences, not complementary sequences. By means of interleaving iteration and orthogonal matrix expansion of PAZCZ sequences, desirable bivalued zero cross-correlation zone (ZCCZ) characteristics can be obtained. Compared with Z-complementary sequence sets with single value ZCCZ which is generally equal to the length of an element sequence, the proposed Z-complementary sequence sets with bivalued ZCCZ have larger set size and then can support more users.

1. Introduction

Complementary sequences, first studied by Golay [1], have many useful properties for radar applications and spread-spectrum communications. Different from unitary sequences which work on a one-sequence-per-user basis, complementary sequences work on a one-flock-per-user basis [2]. It was shown in [3, 4] that orthogonal complementary (OC) sequences with ideal autocorrelation function (ACF) and cross-correlation function (CCF) can improve spectrum efficiency and bit error rate (BER) performance of code division multiple access (CDMA) systems.

Although OC sequences have ideal correlation properties, they cannot support the large number of users since the set size must not be larger than the flock size [2, 5]. To increase the set size of complementary sequence sets, the idea of zero-correlation zone (ZCZ) [6–8] or interference-free windows (IFWs) [9, 10] was exploited to the generation of complementary sequences, and generalized pairwise complementary (GPC) sequences [11] with the setwise uniform IFWs were constructed on the basis of complete complementary sequences [12] and generalized even shift orthogonal sequences [13]. Similar to the construction idea of GPC sequences, generalized pairwise Z-complementary (GPZ) sequences were generated in [14]. GPZ sequences are based on generalized even shift orthogonal-like sequences and include GPC sequences as a special case.

Compared with GPC and GPZ sequences which limit the flock size to be two, Z-complementary binary sequences [15] have fewer restrictions on the flock size. As a more general concept, Z-complementary sequences include the conventional complementary sequences as special cases. These sequences can efficiently resolve the limits on the lengths of conventional complementary sequences, and the theoretical bound given by [15] shows that Z-complementary sequences have much larger set size than conventional complementary sequences. In terms of the set of Z-complementary mates generated in [15], the zero cross-correlation zone (ZCCZ) is equal to the length of element sequences, and is usually larger than the zero autocorrelation zone (ZACZ) except that Z-complementary sequences become conventional complementary sequences.

Different from Z-complementary sequences with single value ZCCZ, intergroup complementary (IGC) sequences

[16] possess a bivalued ZCCZ. The IGC sequence set is divided into several sequence groups. CCFs of any two sequences from the same group are zero within an intragroup IFW, while CCFs of any two sequences from different groups are zero everywhere. According to the theoretical bound in [15], the set size of IGC sequences comes up to the maximum number. However, just like other complementary sequences, IGC sequences still use a complementary sequence set as a basic kernel, which limits the employment of unitary sequences in the construction of complementary sequences.

Actually, the idea of dividing a sequence set into multiple groups and hence obtaining bivalued ZCCZ first appears in [17] as mutually orthogonal sets of ZCZ sequences. Each of these sets is a ZCZ set, and any two sets are mutually orthogonal, which increases the number of available sequences for CDMA systems. Mutually orthogonal sets of ZCZ sequences can also be constructed in a new framework for constructing mutually orthogonal complementary sets and ZCZ sequences [18]. The framework involved characteristic matrices and mutually orthogonal Golay complementary sets (MOGCSs), and the required complete MOGCS in the framework was further studied on the basis of Reed-Muller codes in [19]. In comparison with MOGCS, GPC, Z-complementary sequences, IGC, and GPZ, it can be seen that mutually orthogonal sets of ZCZ sequences are unitary sequence sets, not complementary sequence sets.

This paper proposes methods to construct Z-complementary sets based on a set of sequences with PAZCZ which is a set of unitary sequences. By means of orthogonal matrix expansion and interleaving operation, two novel sets of Z-complementary sequences can be generated from a basic kernel of a PAZCZ sequence set. As an extension for Z-complementary sequences with single value ZCCZ in [15], the presented Z-complementary sequences have a bivalued ZCCZ like IGC sequences and can come up to the maximal set size as long as the kernel of PAZCZ sequences comes up to the maximal set size in terms of the theoretical bound on ZCZ sequences in [20].

The paper is organized as follows. In Section 2, we briefly introduce the definitions and notations used in the present paper. By using a PAZCZ sequence set as a basic kernel, Sections 3 and 4 present two construction schemes of Z-complementary sequences on the basis of orthogonal matrix expansion and interleaving operation, respectively. Some potential applications of the proposed sequence sets are described in Section 5, and we conclude the paper in Section 6.

2. Preliminary Considerations

Z-complementary sequences generally have larger set size than normal complementary sequences at the expense of correlation properties. Actually, a Z-complementary set can be considered to be a trade-off between correlation properties and set size. Although ideal correlation properties cannot be obtained, the CCFs and out-of-phase ACFs of Z-complementary sequences are zero within a region around the zero shift. Z-complementary sequences can be defined as follows.

Let A be a flock of complex-valued sequences with N element sequences of length L, written as $A = \{A_i, 1 \le i \le N\}$ with each element sequence $A_i = (A_i(0), A_i(1), \ldots, A_i(L-1))$. Then $A = \{A_i, 1 \le i \le N\}$ is a Z-complementary sequence with flock size N if the sum $\Psi_{A,A}(\tau)$ of the aperiodic out-of-phase ACFs of N element sequences are zero within a region, namely,

$$\Psi_{A,A}(\tau) = \sum_{i=1}^{N} \psi_{A_i,A_i}(\tau) = \begin{cases} \sum_{i=1}^{N} E_{A_i}, & \tau = 0, \\ 0, & 1 \le |\tau| \le Z-1, \end{cases} \tag{1}$$

where $\psi_{A_i,A_i}(\tau)$ is the aperiodic ACF of element sequence A_i. The notations Z and E_{A_i} denote the length of ZCZ and the energy of element sequence A_i, respectively.

Let $B = \{B_i, 1 \le i \le N\}$ be another Z-complementary sequence with each element sequence $B_i = (B_i(0), B_i(1), \ldots, B_i(L-1))$. Then $B = \{B_i, 1 \le i \le N\}$ is referred to as a Z-complementary mate of $A = \{A_i, 1 \le i \le N\}$ if

$$\Psi_{A,B}(\tau) = \sum_{i=1}^{N} \psi_{A_i,B_i}(\tau) = 0, \quad |\tau| \le Z-1, \tag{2}$$

where $\Psi_{A,B}(\tau)$ represents the sum of the aperiodic CCFs of N element sequences of A and B. $\psi_{A_i,B_i}(\tau)$ denotes the aperiodic CCF of A_i and B_i, and is given by

$$\psi_{A_i,B_i}(\tau) = \begin{cases} \sum_{l=0}^{L-1-\tau} A_i(l) B_i^*(l+\tau), & 0 \le \tau \le L-1, \\ \sum_{l=0}^{L-1+\tau} A_i(l-\tau) B_i^*(l), & 1-L \le \tau < 0, \\ 0, & |\tau| \ge L, \end{cases} \tag{3}$$

where the symbol * denotes a complex conjugate. When $A_i = B_i$, the above definition becomes aperiodic ACF $\psi_{A_i,A_i}(\tau)$.

In addition to aperiodic correlation properties, periodic correlation properties are also important for system performance and should be analyzed as GPC, GPZ, and IGC sequences do. For two element sequences of A_i and B_i, their periodic CCF can be expressed as

$$\phi_{A_i,B_i}(\tau) = \sum_{l=0}^{L-1} A_i(l) B_i^*(l+\tau), \quad 0 \le \tau \le L-1, \tag{4}$$

where $l+\tau$ is performed modulo L. When $A_i = B_i$, the above definition becomes periodic ACF of A_i.

If any Z-complementary sequence in a set containing M Z-complementary sequences is a mate of other Z-complementary sequences in this set, then these Z-complementary sequences compose a Z-complementary set with set size M. In terms of [15], the set size M is bounded by

$$M \leq N\left\lfloor \frac{L}{Z} \right\rfloor, \tag{5}$$

where $\lfloor L/Z \rfloor$ denotes the largest integer smaller than or equal to L/Z.

From (5), a Z-complementary set generally has larger set size than a conventional complementary set even if the conventional complementary set is complete. However, for the fixed processing gain and the length of ZCZ, the set size of a Z-complementary set is smaller than that of mutually orthogonal ZCZ sequence sets in [17–19] and is usually comparable to the set size of each set of mutually orthogonal ZCZ sequence sets without regard to correlation properties.

When a Z-complementary set is divided into several sequence groups, the set will have the bivalued ZCCZ characteristics like IGC sequences. We assume that a Z-complementary set with set size M, flock size N, element sequence length L, and ZCZ length Z, contains G sequence groups, then the Z-complementary set can be denoted by $Z\text{-}CS(M, N, L, G, Z)$.

According to the definition of Z-complementary sequences, a Z-complementary set becomes a conventional complementary set when $Z = L$, and becomes a PAZCZ set when $N = 1$ in terms of the definition of PAZCZ sequences in [21, 22]. Then Z-complementary sequences include the conventional complementary sequences and PAZCZ sequences as special cases.

3. Sequence Construction Based on Orthogonal Matrix Expansion

A new scheme for the construction of Z-complementary set is presented in this section. The constructed sequences have good correlation properties and large set size. To show the performance of the generated sequences, a construction example is given.

3.1. The Construction Scheme. The proposed construction uses an arbitrary PAZCZ sequence set as a basic kernel. By means of orthogonal matrix expansion, the bivalued ZCCZ characteristics can be obtained.

Construction 1. Given an arbitrary PAZCZ sequence set S with set size K and sequences length L, let S possess the ZCZ of one-sided length Z for periodic and aperiodic correlation functions. Then S can be denoted by $Z\text{-}(K, L, Z)$ and be arranged in matrix form as follows:

$$S = \begin{bmatrix} S_1 \\ S_2 \\ \vdots \\ S_K \end{bmatrix} = \begin{bmatrix} S_1(0) & S_1(1) & \cdots & S_1(L-1) \\ S_2(0) & S_2(1) & \cdots & S_2(L-1) \\ \vdots & \vdots & \ddots & \vdots \\ S_K(0) & S_K(1) & \cdots & S_K(L-1) \end{bmatrix}. \tag{6}$$

Let H be a $U \times V$ orthogonal matrix given by

$$H = \begin{bmatrix} h_{1,1} & h_{1,2} & \cdots & h_{1,V} \\ h_{2,1} & h_{2,2} & \cdots & h_{2,V} \\ \vdots & \vdots & \ddots & \vdots \\ h_{U,1} & h_{U,2} & \cdots & h_{U,V} \end{bmatrix}, \tag{7}$$

where any two distinct rows are orthogonal each other, and $U \leq V$ (generally $U = V$). Each entry is a complex number with unity amplitude. Then

$$C = H \otimes S = \begin{bmatrix} h_{1,1}S & h_{1,2}S & \cdots & h_{1,V}S \\ h_{2,1}S & h_{2,2}S & \cdots & h_{2,V}S \\ \vdots & \vdots & \ddots & \vdots \\ h_{U,1}S & h_{U,2}S & \cdots & h_{U,V}S \end{bmatrix}, \tag{8}$$

is a Z-complementary set $Z\text{-}CS(UK, V, L, U, Z)$, where the notation $\otimes$ denotes Kronecker product.

The set C is divided into U sequence groups. In the matrix of (8), each row

$$\begin{bmatrix} h_{u,1}S & h_{u,2}S & \cdots & h_{u,V}S \end{bmatrix} = \begin{bmatrix} h_{u,1}S_1 & h_{u,2}S_1 & \cdots & h_{u,V}S_1 \\ h_{u,1}S_2 & h_{u,2}S_2 & \cdots & h_{u,V}S_2 \\ \vdots & \vdots & \ddots & \vdots \\ h_{u,1}S_K & h_{u,2}S_K & \cdots & h_{u,V}S_K \end{bmatrix}, \quad (1 \leq u \leq U) \tag{9}$$

becomes a group, and each group consists of K sequences with each sequence containing V element sequences of length L. Then the constructed Z-complementary set C consists of UK Z-complementary sequences, and the $((u-1)K+k)$th Z-complementary sequence which is also referred to as the kth Z-complementary sequence in the uth group can be denoted by $C_{u,k} = \{h_{u,1}S_k, \ldots, h_{u,v}S_k, \ldots, h_{u,V}S_k\}$, where $1 \leq u \leq U$, $1 \leq v \leq V$, $1 \leq k \leq K$, and $h_{u,v}S_k = (h_{u,v}S_k(0), h_{u,v}S_k(1), \ldots, h_{u,v}S_k(L-1))$ denotes the vth element sequence of $C_{u,k}$.

When $U = V = 1$, the above Z-complementary set becomes the PAZCZ sequence set $Z\text{-}(K, L, Z)$. Therefore, $Z\text{-}(K, L, Z)$ is a special case of the constructed Z-complementary set and can be also denoted by $Z\text{-}CS(K, 1, L, 1, Z)$.

3.2. Correlation Properties of the Constructed Set. The aperiodic and periodic correlation properties of the designed set C in Construction 1 satisfy the following theorem.

Theorem 1. *The generated Z-complementary set C in Construction 1 has bivalued ZCCZ. The ZCCZ of any two sequences in a group is equal to Z, and the ZCCZ of any two sequences from different groups is equal to L, that is, sequences from different groups are completely complementary. The ZACZ of the set C is Z.*

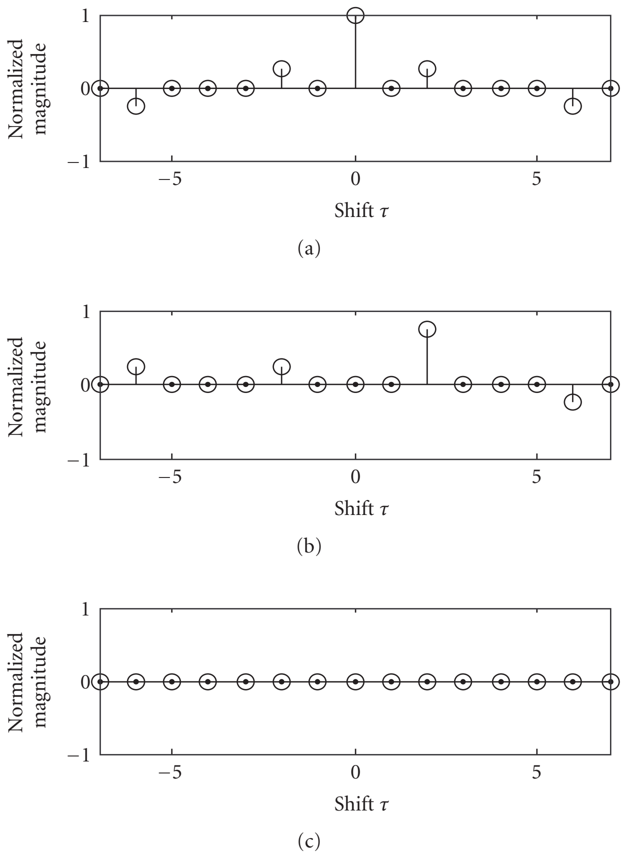

Figure 1: The distribution of aperiodic ACF and CCF values of Z-complementary set $Z\text{-}CS(8,2,8,2,2)$. (a) ACF of Z-complementary sequence $C_{1,1}$; (b) intragroup CCF of $C_{1,1}$ and $C_{1,2}$; (c) intergroup CCF.

Proof. We first consider aperiodic correlation properties. Let $C_{u_1,k_1} = \{h_{u_1,1}S_{k_1}, h_{u_1,2}S_{k_1}, \ldots, h_{u_1,V}S_{k_1}\}$ and $C_{u_2,k_2} = \{h_{u_2,1}S_{k_2}, h_{u_2,2}S_{k_2}, \ldots, h_{u_2,V}S_{k_2}\}$ be any two Z-complementary sequences of the constructed Z-complementary set C, where $1 \le u_1,\ u_2 \le U$ and $1 \le k_1,\ k_2 \le K$. Then the aperiodic correlation function of C_{u_1,k_1} and C_{u_2,k_2} is given by

$$\begin{aligned}\Psi_{C_{u_1,k_1},C_{u_2,k_2}}(\tau) &= \sum_{v=1}^{V} \psi_{h_{u_1,v}S_{k_1},h_{u_2,v}S_{k_2}}(\tau)\\ &= \psi_{S_{k_1},S_{k_2}}(\tau) \cdot \sum_{v=1}^{V} h_{u_1,v}\, h^*_{u_2,v}.\end{aligned} \tag{10}$$

For (10), when $u_1 = u_2$ and $k_1 = k_2$, $\Psi_{C_{u_1,k_1},C_{u_1,k_1}}(\tau)$ becomes ACF of $\{h_{u_1,1}S_{k_1}, h_{u_1,2}S_{k_1}, \ldots, h_{u_1,V}S_{k_1}\}$ and is equal to $V \cdot \psi_{S_{k_1},S_{k_1}}(\tau)$. Then the ZACZ of set C is the same as one of the PAZCZ set S and is Z.

When $u_1 = u_2$ and $k_1 \ne k_2$, (10) represents CCF of two sequences from the same group, namely, intragroup CCF. We have $\Psi_{C_{u_1,k_1},C_{u_1,k_2}}(\tau) = V \cdot \psi_{S_{k_1},S_{k_2}}(\tau)$. Then the intragroup ZCCZ of set C is the same as the ZCCZ of the PAZCZ set S and is Z.

When $u_1 \ne u_2$, (10) denotes CCF of two sequences from different groups, namely, intergroup CCF. Due to the orthogonality of matrix H, we have $\sum_{v=1}^{V} h_{u_1,v}\, h^*_{u_2,v} = 0$, and then $\Psi_{C_{u_1,k_1},C_{u_2,k_2}}(\tau) = 0$.

According to the above analysis, the aperiodic correlation properties of the constructed sequences satisfy Theorem 1. Similar to aperiodic correlation properties, periodic correlation properties can be easily analyzed, and the corresponding proof is omitted. □

In terms of Theorem 1, it is obvious that the constructed Z-complementary sequences have better correlation properties than unitary sequences, such as m-sequence and gold sequences. In addition, for the same intragroup ZCCZ, the Z-complementary set with bivalued ZCCZ has larger intergroup ZCCZ than mutually orthogonal ZCZ sequence sets, since the generated Z-complementary set has ideal intergroup cross-correlation properties while a sequence from one set of mutually orthogonal ZCZ sequence sets is usually nonorthogonal to a sequence from another set for nonzero shifts. Compared with the idea of large ZCZ set with smaller subsets of larger ZCZ than the original set [18, 19], the intergroup ZCCZ of the constructed Z-complementary set with bivalued ZCCZ is usually larger than its intragroup ZCCZ.

3.3. Set Size of the Constructed Set. We assume that the PAZCZ sequence set $Z\text{-}(K, L, Z)$ has the maximum set size in terms of the theoretical bound in [20], namely, $K = L/Z$. Then it is obvious that the constructed Z-complementary set $Z\text{-}CS(UK, V, L, U, Z)$ has the maximum set size in terms of (5) as long as $U = V$, in which case the set size of $Z\text{-}CS(UK, V, L, U, Z)$ is K times large as that of conventional complete complementary set like complete MOGCS.

3.4. An Example. We use a ternary PAZCZ sequence set $Z\text{-}(4, 8, 2)$ in [22] as a basic kernel which is given by

$$S = \begin{bmatrix} S_1\\ S_2\\ S_3\\ S_4 \end{bmatrix} = \begin{bmatrix} 0 & + & 0 & + & 0 & + & 0 & -\\ 0 & + & 0 & + & 0 & - & 0 & +\\ 0 & + & 0 & - & 0 & + & 0 & +\\ 0 & + & 0 & - & 0 & - & 0 & - \end{bmatrix}. \tag{11}$$

Let H be a 2×2 Walsh-Hadamard matrix and be expressed as $H = [\begin{smallmatrix} + & + \\ + & - \end{smallmatrix}]$.

According to Construction 1, we can generate a Z-complementary set $Z\text{-}CS(8, 2, 8, 2, 2)$ as follows:

$$C = \begin{bmatrix} C_{1,1}\\ C_{1,2}\\ C_{1,3}\\ C_{1,4}\\ C_{2,1}\\ C_{2,2}\\ C_{2,3}\\ C_{2,4} \end{bmatrix} = \begin{bmatrix} 0+0+0+0-, & 0+0+0+0-\\ 0+0+0-0+, & 0+0+0-0+\\ 0+0-0+0+, & 0+0-0+0+\\ 0+0-0-0-, & 0+0-0-0-\\ 0+0+0+0-, & 0-0-0-0+\\ 0+0+0-0+, & 0-0-0+0-\\ 0+0-0+0+, & 0-0+0-0-\\ 0+0-0-0-, & 0-0+0+0+ \end{bmatrix}. \tag{12}$$

The constructed Z-complementary set C has maximum set size in terms of (5). Figure 1 shows aperiodic correlation properties of the constructed Z-complementary set $Z\text{-}CS(8,2,8,2,2)$. It is obvious that the generated set has bivalued ZCCZ characteristics. In this example, the intragroup ZCCZ is 2, and the intergroup ZCCZ is 8.

4. Sequence Construction Based on Interleaving Iteration

According to Construction 1, the orthogonal matrix expansion method assures that the generated Z-complementary sequences from different groups are completely complementary. However, this method cannot increase the ZACZ and intragroup ZCCZ which are the same as the ZCZ of a basic PAZCZ set kernel. In order to obtain larger ZCZ than one of a basic PAZCZ set kernel, the interleaving method is used to construct Z-complementary sets in this section.

4.1. The Iterative Construction. The interleaving operation is a significant expansion method and has wide applications to constructions of conventional complementary sequences [5, 23]. In [15], the bit-interleaved operation can be used to lengthen the element sequences of a single Z-complementary sequence, and then a new Z-complementary sequence can be obtained from a shorter Z-complementary sequence. Compared with the interleaving method for a single Z-complementary sequence in [15], we generate a novel Z-complementary set by interleaving an arbitrary PAZCZ set kernel which is a unitary sequence set. Different from the results with single value ZCCZ in [5, 15, 23], the constructed Z-complementary set with multiple sequence groups has bivalued ZCCZ characteristics while its ZACZ and intragroup ZCCZ can be increased by the iterative procedure of the interleaving operation.

Construction 2. Let S be an arbitrary PAZCZ sequence set $Z\text{-}(K,L,Z)$ which is denoted by

$$S = \begin{bmatrix} S_1 \\ S_2 \\ \vdots \\ S_K \end{bmatrix} = \begin{bmatrix} S_1(0) & S_1(1) & \cdots & S_1(L-1) \\ S_2(0) & S_2(1) & \cdots & S_2(L-1) \\ \vdots & \vdots & \ddots & \vdots \\ S_K(0) & S_K(1) & \cdots & S_K(L-1) \end{bmatrix}. \quad (13)$$

Given the iterative origin $\Delta^{(0)} = S$, then the set $\Delta^{(n)}$ of the nth iteration ($n \geq 1$) is obtained by

$$\Delta^{(n)} = \begin{bmatrix} \Delta^{(n-1)} \odot \Delta^{(n-1)} & \left(-\Delta^{(n-1)}\right) \odot \Delta^{(n-1)} \\ \left(-\Delta^{(n-1)}\right) \odot \Delta^{(n-1)} & \Delta^{(n-1)} \odot \Delta^{(n-1)} \end{bmatrix}, \quad (14)$$

where $-\Delta$ represents the matrix whose ijth entry is the ijth negation of Δ, and $\Delta_1 \odot \Delta_2$ denotes the bit-interleaved operation between Δ_1 and Δ_2. Then the set $\Delta^{(n)}$ in matrix form is a Z-complementary set $Z\text{-}CS(2^nK, 2^n, 2^nL, 2^n, 2^nZ)$.

Each row of the set $\Delta^{(n)}$ in matrix form is a Z-complementary sequence. After n iterations, there are 2^nK Z-complementary sequences in total. These sequences are divided into 2^n groups with each group containing K Z-complementary sequences. The sequences from the $((r-1)K+1)$th row to the rKth row in $\Delta^{(n)}$ compose the rth group, where $1 \leq r \leq 2^n$. The sth element sequence of the kth Z-complementary sequence in the rth group for $\Delta^{(n)}$ can be denoted by $\Delta^{(n)}_{r,k,s}$, where $1 \leq r,\ s \leq 2^n$ and $1 \leq k \leq K$.

In comparison with the interleaving operation in [15], the proposed method in Construction 2 can generate multiple sequence groups and obtain larger set size, which guarantees that the CDMA systems using such Z-complementary sets can accommodate more users.

4.2. Correlation Properties of the Constructed Set

Theorem 2. *The generated Z-complementary set C in Construction 2 has bivalued ZCCZ. Intergroup ZCCZ is 2^nL, that is, sequences from different groups are completely complementary. Both of ZACZ and intragroup ZCCZ are equal to 2^nZ.*

Proof. We first consider aperiodic correlation properties. Let $\Delta^{(1)}_{r_1,k_1}$ and $\Delta^{(1)}_{r_2,k_2}$ denote the k_1th Z-complementary sequence in the r_1th group and the k_2th Z-complementary sequence in the r_2th group for $\Delta^{(1)}$, respectively, where $1 \leq r_1,\ r_2 \leq 2$ and $1 \leq k_1,\ k_2 \leq K$.

According to (14), when $r_1 = r_2$, the aperiodic correlation function of $\Delta^{(1)}_{r_1,k_1}$ and $\Delta^{(1)}_{r_1,k_2}$ can be given by

$$\begin{aligned} \Psi_{\Delta^{(1)}_{r_1,k_1},\Delta^{(1)}_{r_1,k_2}}\left(\tau^{(1)}\right) &= \psi_{S_{k_1}\odot S_{k_1},S_{k_2}\odot S_{k_2}}\left(\tau^{(1)}\right) \\ &\quad + \psi_{\left(-S_{k_1}\right)\odot S_{k_1},\left(-S_{k_2}\right)\odot S_{k_2}}\left(\tau^{(1)}\right), \end{aligned} \quad (15)$$

where $\tau^{(m)}$ denotes the chip index of the correlation function of $\Delta^{(m)}$. Equation (15) is the ACF of $\Delta^{(1)}$ when $k_1 = k_2$ and is the intragroup CCF of $\Delta^{(1)}$ when $k_1 \neq k_2$.

For (15), when $\tau^{(1)}$ is an even number and is denoted by $\tau^{(1)} = 2\tau^{(0)}$, where $-(L-1) \leq \tau^{(0)} \leq L-1$, we have

$$\begin{aligned} \Psi_{\Delta^{(1)}_{r_1,k_1},\Delta^{(1)}_{r_1,k_2}}\left(2\tau^{(0)}\right) &= \left[2\psi_{S_{k_1},S_{k_2}}\left(\tau^{(0)}\right)\right] \\ &\quad + \left[\psi_{-S_{k_1},-S_{k_2}}\left(\tau^{(0)}\right) + \psi_{S_{k_1},S_{k_2}}\left(\tau^{(0)}\right)\right] \\ &= 4\psi_{S_{k_1},S_{k_2}}\left(\tau^{(0)}\right). \end{aligned} \quad (16)$$

When $\tau^{(1)}$ is an odd number and is denoted by $\tau^{(1)} = 2\tau^{(0)}+1$, where $-(L-1) \leq \tau^{(0)} \leq L-1$, we have

$$\begin{aligned} &\Psi_{\Delta^{(1)}_{r_1,k_1},\Delta^{(1)}_{r_1,k_2}}\left(2\tau^{(0)}+1\right) \\ &\quad = \left[\psi_{S_{k_1},S_{k_2}}\left(\tau^{(0)}\right) + \psi_{S_{k_1},S_{k_2}}\left(\tau^{(0)}+1\right)\right] \\ &\qquad + \left[\psi_{-S_{k_1},S_{k_2}}\left(\tau^{(0)}\right) + \psi_{S_{k_1},-S_{k_2}}\left(\tau^{(0)}+1\right)\right] \\ &\quad = \left[\psi_{S_{k_1},S_{k_2}}\left(\tau^{(0)}\right) + \psi_{S_{k_1},S_{k_2}}\left(\tau^{(0)}+1\right)\right] \\ &\qquad + \left[-\psi_{S_{k_1},S_{k_2}}\left(\tau^{(0)}\right) - \psi_{S_{k_1},S_{k_2}}\left(\tau^{(0)}+1\right)\right] \\ &\quad = 0. \end{aligned} \quad (17)$$

When $r_1 \neq r_2$, the correlation function of $\Delta^{(1)}_{r_1,k_1}$ and $\Delta^{(1)}_{r_2,k_2}$ becomes the intergroup CCF of $\Delta^{(1)}$ and can be given by

$$\Psi_{\Delta^{(1)}_{r_1,k_1},\Delta^{(1)}_{r_2,k_2}}\left(\tau^{(1)}\right) = \psi_{S_{k_1}\odot S_{k_1},(-S_{k_2})\odot S_{k_2}}\left(\tau^{(1)}\right) + \psi_{(-S_{k_1})\odot S_{k_1},S_{k_2}\odot S_{k_2}}\left(\tau^{(1)}\right). \quad (18)$$

Similar to the analysis of (15), we calculate (18) in terms of $\tau^{(1)} = 2\tau^{(0)}$ and $\tau^{(1)} = 2\tau^{(0)} + 1$. When $\tau^{(1)} = 2\tau^{(0)}$, we have

$$\begin{aligned}
\Psi_{\Delta^{(1)}_{r_1,k_1},\Delta^{(1)}_{r_2,k_2}}\left(2\tau^{(0)}\right)
&= \left[\psi_{S_{k_1},-S_{k_2}}\left(\tau^{(0)}\right) + \psi_{S_{k_1},S_{k_2}}\left(\tau^{(0)}\right)\right] \\
&\quad + \left[\psi_{-S_{k_1},S_{k_2}}\left(\tau^{(0)}\right) + \psi_{S_{k_1},S_{k_2}}\left(\tau^{(0)}\right)\right] \\
&= \left[-\psi_{S_{k_1},S_{k_2}}\left(\tau^{(0)}\right) + \psi_{S_{k_1},S_{k_2}}\left(\tau^{(0)}\right)\right] \\
&\quad + \left[-\psi_{S_{k_1},S_{k_2}}\left(\tau^{(0)}\right) + \psi_{S_{k_1},S_{k_2}}\left(\tau^{(0)}\right)\right] \\
&= 0.
\end{aligned} \quad (19)$$

When $\tau^{(1)} = 2\tau^{(0)} + 1$, we have

$$\begin{aligned}
\Psi_{\Delta^{(1)}_{r_1,k_1},\Delta^{(1)}_{r_2,k_2}}\left(2\tau^{(0)} + 1\right)
&= \left[\psi_{S_{k_1},-S_{k_2}}\left(\tau^{(0)}\right) + \psi_{S_{k_1},S_{k_2}}\left(\tau^{(0)} + 1\right)\right] \\
&\quad + \left[\psi_{S_{k_1},S_{k_2}}\left(\tau^{(0)}\right) + \psi_{-S_{k_1},S_{k_2}}\left(\tau^{(0)} + 1\right)\right] \\
&= \left[-\psi_{S_{k_1},S_{k_2}}\left(\tau^{(0)}\right) + \psi_{S_{k_1},S_{k_2}}\left(\tau^{(0)} + 1\right)\right] \\
&\quad + \left[\psi_{S_{k_1},S_{k_2}}\left(\tau^{(0)}\right) - \psi_{S_{k_1},S_{k_2}}\left(\tau^{(0)} + 1\right)\right] \\
&= 0.
\end{aligned} \quad (20)$$

From (19)-(20), we can obtain that intergroup CCF (where $r_1 \neq r_2$) of $\Delta^{(1)}$ is zero. From (16)-(17), the odd-shift ACF (where $r_1 = r_2$ and $k_1 = k_2$) and intragroup CCF (where $r_1 = r_2$ and $k_1 \neq k_2$) of $\Delta^{(1)}$ are zero, while the even-shift ACF and intragroup CCF of $\Delta^{(1)}$ are determined by the ACF and CCF of $\Delta^{(0)} = S$, respectively. Thus, at the nth iteration, we have

$$\Psi_{\Delta^{(n)}_{r_1,k_1},\Delta^{(n)}_{r_2,k_2}}\left(\tau^{(n)}\right) = \begin{cases} 4^n \psi_{S_{k_1},S_{k_2}}\left(\tau^{(0)}\right), & \tau^{(n)} = 2^n \tau^{(0)},\ r_1 = r_2, \\ 0, & \text{otherwise}, \end{cases} \quad (21)$$

where $1 \leq r_1,\ r_2 \leq 2^n$ and $1 \leq k_1,\ k_2 \leq K$.

Since $\psi_{S_{k_1},S_{k_2}}(\tau^{(0)}) = 0, |\tau^{(0)}| \leq Z - 1$ (where $k_1 \neq k_2$) and $\psi_{S_{k_1},S_{k_1}}(\tau^{(0)}) = 0,\ 1 \leq |\tau^{(0)}| \leq Z - 1$, we have

$$\Psi_{\Delta^{(n)}_{r_1,k_1},\Delta^{(n)}_{r_1,k_2}}\left(\tau^{(n)}\right) = 0, \quad \left|\tau^{(n)}\right| \leq 2^n Z - 1,\ k_1 \neq k_2, \quad (22)$$

$$\Psi_{\Delta^{(n)}_{r_1,k_1},\Delta^{(n)}_{r_1,k_1}}\left(\tau^{(n)}\right) = 0, \quad 1 \leq \left|\tau^{(n)}\right| \leq 2^n Z - 1. \quad (23)$$

From (19), (20), (22), and (23), we have proved that the aperiodic correlation properties of the constructed sequences in Construction 2 satisfy Theorem 2. For periodic correlation properties, similar analysis can be easily obtained, and the corresponding proof is omitted. □

4.3. Set Size of the Constructed Set. Similar to Construction 1, when the PAZCZ sequence set Z-(K, L, Z) has the maximum set size (namely, $K = L/Z$), the constructed Z-complementary set Z-$CS(2^n K, 2^n, 2^n L, 2^n, 2^n Z)$ in Construction 2 has the maximum set size in terms of (5).

4.4. An Example. The ternary PAZCZ sequence set Z-$(4, 8, 2)$ in the example of Construction 1 is still used as a basic kernel. According to Construction 2, the Z-complementary set Z-$CS(8, 2, 16, 2, 4)$ can be generated when $n = 1$ in (14). The constructed set comes up to the theoretical bound in (5) and can be given by

$$\Delta^{(1)} = \begin{bmatrix} \Delta^{(1)}_{1,1} \\ \Delta^{(1)}_{1,2} \\ \Delta^{(1)}_{1,3} \\ \Delta^{(1)}_{1,4} \\ \Delta^{(1)}_{2,1} \\ \Delta^{(1)}_{2,2} \\ \Delta^{(1)}_{2,3} \\ \Delta^{(1)}_{2,4} \end{bmatrix} = \begin{bmatrix} 00++00++00++00--, & 00-+00-+00-+00+- \\ 00++00++00--00++, & 00-+00-+00+-00-+ \\ 00++00--00++00++, & 00-+00+-00-+00-+ \\ 00++00--00--00--, & 00-+00+-00+-00+- \\ 00-+00-+00-+00+-, & 00++00++00++00-- \\ 00-+00-+00+-00-+, & 00++00++00--00++ \\ 00-+00+-00-+00-+, & 00++00--00++00++ \\ 00-+00+-00+-00+-, & 00++00--00--00-- \end{bmatrix}. \quad (24)$$

Different from Z-complementary sets on the basis of the interleaving method in [15], the constructed set $\Delta^{(1)}$ is divided into two groups, namely, $\{\Delta^{(1)}_{1,1}, \ldots, \Delta^{(1)}_{1,4}\}$ and $\{\Delta^{(1)}_{2,1}, \ldots, \Delta^{(1)}_{2,4}\}$. As a result, the set size is increased from 4 to 8 after iterating once. To compare with the designed Z-complementary set in Construction 1, we give Figure 2 which shows aperiodic correlation properties of Z-$CS(8, 2, 16, 2, 4)$. From Figure 2, we can see that Construction 2 provides bivalued ZCCZ characteristics like Construction 1. However, the generated Z-complementary set in terms of Construction 2 has larger ZACZ and intragroup ZCCZ. At the first iteration, both of the ZACZ and intragroup ZCCZ are 4, and this value is as two times large as the ZCZ of Z-$(4, 8, 2)$.

5. Applications

Compared with PAZCZ sequence sets, the proposed Z-complementary sets in this paper have bivalued ZCCZ

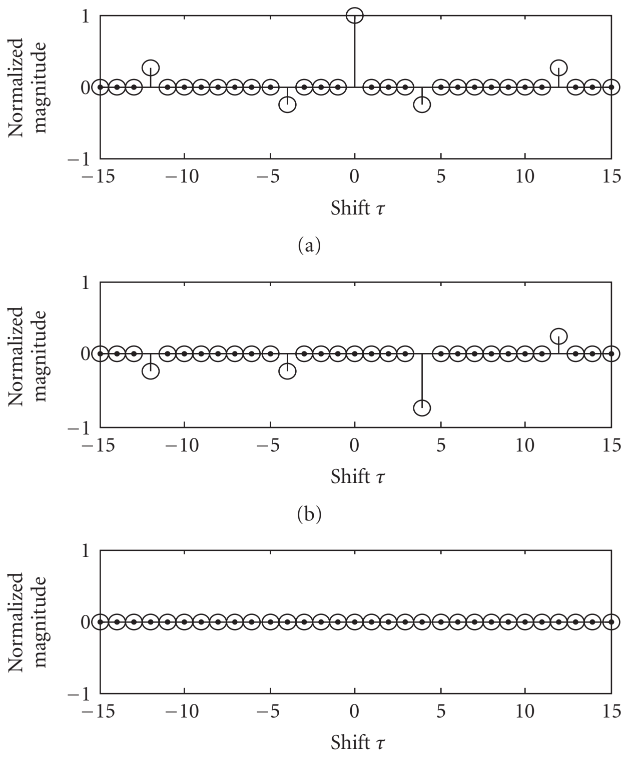

Figure 2: The distribution of aperiodic ACF and CCF values of Z-complementary set $Z\text{-}CS(8, 2, 16, 2, 4)$. (a) ACF of Z-complementary sequence $\Delta_{1,3}^{(1)}$; (b) intragroup CCF of $\Delta_{1,3}^{(1)}$ and $\Delta_{1,4}^{(1)}$; (c) intergroup CCF.

characteristics. Then better correlation properties can be obtained, and wider applications can be considered. The constructed sequences can be used in IGC-CDMA systems [16] since IGC sequences also possess bivalued ZCCZ characteristics. In addition, when the flock sizes of Constructions 1 and 2 are fixed to be two, we can apply the generated Z-complementary sets to those systems exploiting GPC [11] and GPZ [14] sequences. By choosing a suitable PAZCZ set kernel and controlling construction parameters in Constructions 1 and 2, the designed sequences can be adapted to various requirements of the above systems.

It should be noted that sequences with ZCZ just for periodic correlation function, which are usually called ZCZ sequences, can be used as a basic kernel when some CDMA systems only need good periodic correlation properties. Compared PAZCZ sequences, ZCZ sequences have received more attention. Then one can have a wider choice to generate Z-complementary sets just with periodic ZCZ in terms of Constructions 1 and 2.

Since correlation functions of complementary sequences are based on a flock of element sequences jointly instead of a single sequence, these element sequences should be transmitted via different channels which are usually different carriers like the cases in [3, 4, 16]. Therefore, complementary sequences, such as IGC sequences and Z-complementary sequences, are more sensitive to frequency selective fading channel than mutually orthogonal ZCZ sequence sets which are unitary sequence sets. It actually becomes the main drawback in applications of complementary sequences to multicarrier systems.

6. Conclusions

In this paper, two novel kinds of Z-complementary sets on the basis of a PAZCZ set are generated. As a basic kernel, the PAZCZ set affects significantly the ZCZ length and set size of the designed Z-complementary set, and it can be expanded by means of orthogonal matrix and the interleaving iteration. The proposed expansions of a PAZCZ set provide efficient and flexible ways to construct Z-complementary sets with bivalued ZCCZ characteristics.

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