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Research Article

Low-Complexity Estimation of CFO and Frequency Independent I/Q Mismatch for OFDM Systems

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CFO and I/Q mismatch could cause significant performance degradation to OFDM systems. Their estimation and compensation are generally difficult as they are entangled in the received signal. In this paper, we propose some low-complexity estimation and compensation schemes in the receiver, which are robust to various CFO and I/Q mismatch values although the performance is slightly degraded for very small CFO. These schemes consist of three steps: forming a cosine estimator free of I/Q mismatch interference, estimating I/Q mismatch using the estimated cosine value, and forming a sine estimator using samples after I/Q mismatch compensation. These estimators are based on the perception that an estimate of cosine serves much better as the basis for I/Q mismatch estimation than the estimate of CFO derived from the cosine function. Simulation results show that the proposed schemes can improve system performance significantly, and they are robust to CFO and I/Q mismatch.

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1. Introduction

Orthogonal Frequency Division Multiplexing (OFDM) becomes the foundation technique for broadband wireless communications because of its various advantages including high spectrum efficiency, low complexity equalization and great flexibility in resource optimization. However, one well-known disadvantage of OFDM is its high sensitivity to carrier frequency offset (CFO) [1]. CFO refers to the frequency difference between the local oscillators in the transmitter and receiver. CFO causes intercarrier interference (ICI) and could deteriorate the system performance seriously. CFO itself is not difficult to estimate and compensate, using either training-based or blind estimation schemes [2, 3]. However, when some distortions, in particular, I/Q mismatch, are entangled with CFO, the performance of conventional CFO estimator will degrade significantly [4].

I/Q mismatch is caused by the imbalance between the components of the Inphase (I-) and Quadrature (Q-) branches in I/Q modulated systems. I/Q mismatch includes gain and phase mismatches. Gain mismatch is caused by the gain difference of amplifiers or filters in I- and Q- branches. Phase mismatch is caused by the nonideal $\pi/2$ rotation in local oscillators and the phase difference between analogue filters in I- and Q- branches. In a practical receiver with analog I/Q separation, I/Q mismatch always exists and contributes as interference in general CFO estimation. On the other hand, without the knowledge of CFO, a training-based estimator cannot estimate I/Q mismatch accurately. CFO estimation in the presence of I/Q mismatch is not trivial, and has been investigated in, for example, [5–14]. Each of these schemes partially solves the CFO estimation problem in the presence of I/Q mismatch, with respective drawbacks. In [5, 6], initial CFO is estimated in the presence of errors caused by I/Q imbalance. Then, based on the CFO estimates, [5] proposes an iterative I/Q mismatch estimation approach, which requires five iterations to obtain the gain parameter. In [6], a simple time domain I/Q mismatch estimation method is proposed, but the performance degrades significantly when CFO is small. [6] also proposes a frequency domain estimator which improves performance when CFO is small, however, it is sensitive to transmitter side mismatch. In [7], an iterative scheme is proposed, requiring special training symbols which contain many zeros to suppress the I/Q

mismatch effect in the receiver. In [8], a searching-based CFO estimator is developed. The high computational complexity, however, may prevent it from practical applications. In [12] iterative estimators are proposed, and they have relatively high complexity. In [13], a frequency domain adaptive I/Q mismatch compensation scheme is proposed, however, it requires perfect CFO knowledge. In [14], perfect CFO knowledge is required either in the training based RLS method or in forming the per-tone-equalizer. In [9, 10], CFO estimators based on three identical training symbols are proposed. However, [9] only uses a cosine function of the CFO to estimate the CFO parameter. The scheme is thus very sensitive to noise when CFO and/or I/Q mismatch is small, and has a phase ambiguity problem with positive and negative phases. Improvement to [9] is made in [10], using two groups of three identical training symbols. Although this estimator is robust to both transmitter and receiver I/Q mismatch, the special long training symbols designed for CFO estimation increase system overhead and are incompatible with current standards. In [11], a complete CFO and I/Q mismatch estimation and compensation scheme is proposed based on the CFO estimator in [9]. However, I/Q mismatch parameters are estimated based on the CFO estimates, which is sensitive to noise, particularly when CFO is small.

In our early work [15], we independently developed a CFO estimation scheme partially similar to the approach in [10]. Different to [10], our scheme only requires one group of three identical training symbols by forming an approximated estimator for the CFO. The scheme works well for various I/Q mismatch values when the CFO is not too small (say, 10% of the normalized CFO), and the performance otherwise degrades. In this paper, we propose some novel estimation schemes which are robust to any values of both transmitter and receiver I/Q mismatch, and have better accuracy of the I/Q mismatch estimation for small CFO. The schemes use a group of at least three identical training symbols, which are generally present in the preamble of current systems, for example, WLAN and WiMAX systems. They serially estimate I/Q mismatch and CFO with low complexity, without incurring iterative process. The schemes mainly consist of three steps. Firstly, a cosine function of the CFO, which is free of I/Q mismatch interference, is formed using a group of three identical training symbols. Secondly, based on the estimated value of the cosine function instead of the CFO estimate, the I/Q mismatch parameters are estimated. Thirdly, the I/Q mismatch is compensated using the estimates, and a sine function of the CFO is formed based on the compensated signal. Combining the results of cosine and sine functions, CFO can then be estimated accurately. The use of cosine value instead of the CFO estimate for I/Q mismatch estimation is from the insight that the cosine value is much more robust to noise than the CFO estimate. The rest of the paper is organized as follows. Section 2 formulates the problem of CFO and I/Q mismatch estimation in OFDM systems. In Section 3, the proposed CFO and I/Q mismatch estimation schemes are developed. Simulation results are presented in Section 4. Section 5 concludes the paper.

2. Problem Formulation and System Structure

An OFDM system model with CFO and I/Q mismatch estimation and compensation is shown in Figure 1. Let transmitter's gain mismatch be η and phase mismatch be γ. Denoting the baseband signal as $s(t) = s_I(t) + js_Q(t)$, the analog signal radiated from the transmitter antenna (denoted as RF signal hereafter) can be represented as

$$\check{s}(t) = (1+\eta)s_I(t)\cos(\omega_c t + \gamma) - (1-\eta)s_Q(t)\sin(\omega_c t - \gamma), \tag{1}$$

where ω_c is the carrier frequency. The received RF signal $\check{r}(t)$ becomes

$$\check{r}(t) = \check{s}(t) \otimes h(t) + \xi(t), \tag{2}$$

where $h(t)$ is the channel impulse response, $\xi(t)$ is additive white Gaussian Noise (AWGN), and $\otimes$ denotes the linear convolution. The signal is down-converted to baseband by an oscillator with imbalanced inphase input $(1+\varepsilon)\cos(\omega_c t - \omega_d t - \theta)$ and quadrature input $(1-\varepsilon)\sin(\omega_c t - \omega_d t + \theta)$, where ε and θ represent gain and phase mismatch in the receiver, respectively, and ω_d is the frequency offset between the transmitter and receiver oscillators. The received signal is then filtered by a Low Pass Filter (LPF). The filtered signal is sampled at a sampling rate $f_s = 1/T_s$, where T_s is the sampling period. The sampled baseband signal, consisted of signals in I- and Q- branches, can be represented as

$$y(n) = y_I(n) + jy_Q(n), \tag{3}$$

where

$$\begin{aligned} y_I(n) &= \xi_I(n) + \frac{(1+\varepsilon)}{2}\cos(\omega_d n + \theta) \\ &\quad \times \left[(1+\eta)r_I(n)\cos\gamma - (1-\eta)r_Q(n)\sin\gamma\right] \\ &\quad - \frac{(1+\varepsilon)}{2}\sin(\omega_d n + \theta) \\ &\quad \times \left[-(1+\eta)r_I(n)\sin\gamma + (1-\eta)r_Q(n)\cos\gamma\right], \\ y_Q(n) &= \xi_Q(n) + \frac{(1-\varepsilon)}{2}\cos(\omega_d n - \theta) \\ &\quad \times \left[-(1+\eta)r_I(n)\sin\gamma + (1-\eta)r_Q(n)\cos\gamma\right] \\ &\quad + \frac{(1-\varepsilon)}{2}\sin(\omega_d n - \gamma) \\ &\quad \times \left[(1+\eta)r_I(n)\cos\gamma - (1-\eta)r_Q(n)\sin\gamma\right]. \end{aligned} \tag{4}$$

The $r_I(n)$ and $r_Q(n)$ in (4) are the sampled real and imaginary outputs of the convolution between $s(t)$ and the baseband

channel impulse response, respectively, $\xi_I(n)$ and $\xi_Q(n)$ are the noise in I- and Q- branches, respectively. Define

$$x_I(n) \triangleq \frac{1}{2}[(1+\eta)r_I(n)\cos\gamma - (1-\eta)r_Q(n)\sin\gamma],$$
$$x_Q(n) \triangleq \frac{1}{2}[-(1+\eta)r_I(n)\sin\gamma + (1-\eta)r_Q(n)\cos\gamma]. \tag{5}$$

Equation (4) can be rewritten as

$$\begin{aligned} y_I(n) &= g_1[x_I(n)\cos(\phi_n+\theta) - x_Q(n)\sin(\phi_n+\theta)] + \xi_I(n) \\ &= g_1\{\cos(\theta)[x_I(n)\cos\phi_n - x_Q(n)\sin\phi_n] \\ &\quad - \sin(\theta)[x_I(n)\sin\phi_n + x_Q(n)\cos\phi_n]\} + \xi_I(n), \\ y_Q(n) &= g_2[x_I(n)\sin(\phi_n-\theta) + x_Q(n)\cos(\phi_n-\theta)] + \xi_Q(n) \\ &= g_2\{\cos(\theta)[x_I(n)\sin\phi_n + x_Q(n)\cos\phi_n] \\ &\quad - \sin(\theta)[x_I(n)\cos\phi_n - x_Q(n)\sin\phi_n]\} + \xi_Q(n), \end{aligned} \tag{6}$$

where

$$\phi_n = \omega_d n, \qquad g_1 = 1+\varepsilon, \qquad g_2 = 1-\varepsilon. \tag{7}$$

Equation (6) shows that the transmitter side and the receiver side I/Q mismatch impacts can be decoupled and the transmitter side I/Q mismatch is only contained in $x_I(n)$ and $x_Q(n)$. If the channel is static during CFO estimation, periodically transmitted training symbols lead to periodical $x_I(n)$ and $x_Q(n)$ at the receiver. In the CFO and I/Q mismatch estimation algorithms to be presented, only the periodicity of the baseband signal is required and exploited, and the detailed information of $x_I(n)$ and $x_Q(n)$ is not required. After the CFO and receiver side I/Q mismatch are compensated, the transmitter-side I/Q mismatch can be estimated via joint estimation of channel and I/Q mismatch proposed in [6] or by a least square estimator. In the following, we propose some CFO and I/Q mismatch joint estimators, which only require the periodicity of training sequences instead of the actual signal values.

The complex signal in (3) can also be written as

$$y(n) = \alpha x(n)e^{j\omega_d n} + \beta x^*(n)e^{-j\omega_d n} + \xi(n), \tag{8}$$

where

$$\begin{aligned} x(n) &= x_I(n) + jx_Q(n), \\ \alpha &= \cos(\theta) + j\varepsilon\sin(\theta), \\ \beta &= \varepsilon\cos(\theta) - j\sin(\theta), \end{aligned} \tag{9}$$

and the superscript "*" denotes the conjugate.

According to (8), the received signal becomes the sum of the scaled original signal and the interference from its own conjugation. It is clear that CFO is always entangled with I/Q mismatch. Even when CFO is known, without the information of I/Q mismatch, the second part in (8) cannot be eliminated, so CFO cannot be compensated correctly. Thus it is a natural task to estimate CFO and I/Q mismatch jointly.

3. CFO and I/Q Mismatch Estimation

Referring to Figure 1, the proposed scheme consists of three steps, including forming a cosine estimator for CFO which is free of I/Q mismatch interference, estimating I/Q mismatch using the estimated cosine value, and forming a sine estimator for CFO by removing I/Q mismatch in the received signal using the estimated I/Q mismatch parameters. The CFO is then estimated by combining the sine and cosine estimator. In the process, both CFO and I/Q mismatch are estimated in the presence of minimum interference from each other, introduced by the residual estimation error due to the noise.

3.1. Cosine Estimator Free of I/Q Mismatch Interference. Denote the number of samples in each training symbol as L_p, and let $\phi = \omega_d L_p$. From (6), in I- branch, we have

$$\begin{aligned} &y_I\left(n+2L_p\right) + y_I(n) \\ &= 2g_1\cos\phi[x_I(n)\cos(\phi_n+\theta+\phi) - x_Q(n)\sin(\phi_n+\theta+\phi)] \\ &\quad + \xi_I(n) + \xi_I\left(n+2L_p\right) \\ &= 2\cos\phi\left[y_I\left(n+L_p\right) - \xi_I\left(n+L_p\right)\right] + \xi_I(n) + \xi_I\left(n+2L_p\right), \end{aligned} \tag{10}$$

where the sum and difference formulas of sine and cosine functions are used.

Then $\cos\phi$ can be estimated by

$$\widehat{\cos}\phi_n = \frac{y_I\left(n+2L_p\right) + y_I(n)}{2y_I\left(n+L_p\right)}. \tag{11}$$

To reduce the noise effect, final estimate needs to be averaged over a number of samples. The general approach is to use a maximal ratio combining (MRC). Denote the number of total samples in the training sequence as N_L. For I-branch, the estimate of $\cos\phi$ based on MRC is given by

$$\widehat{\cos}\phi = \frac{\sum_{n=1}^{N_L-2L_p}\left\{y_I\left(n+L_p\right)\left[y_I\left(n+2L_p\right) + y_I(n)\right]\right\}}{2\sum_{n=1}^{N_L}\left|y_I\left(n+L_p\right)\right|^2}. \tag{12}$$

The formulation of (12) is similar to [10], where the estimator is derived based on mixed signals from I/Q branches. As an alternative to the MRC approach we propose a lower complexity combiner. For I-branch, the estimator is given by

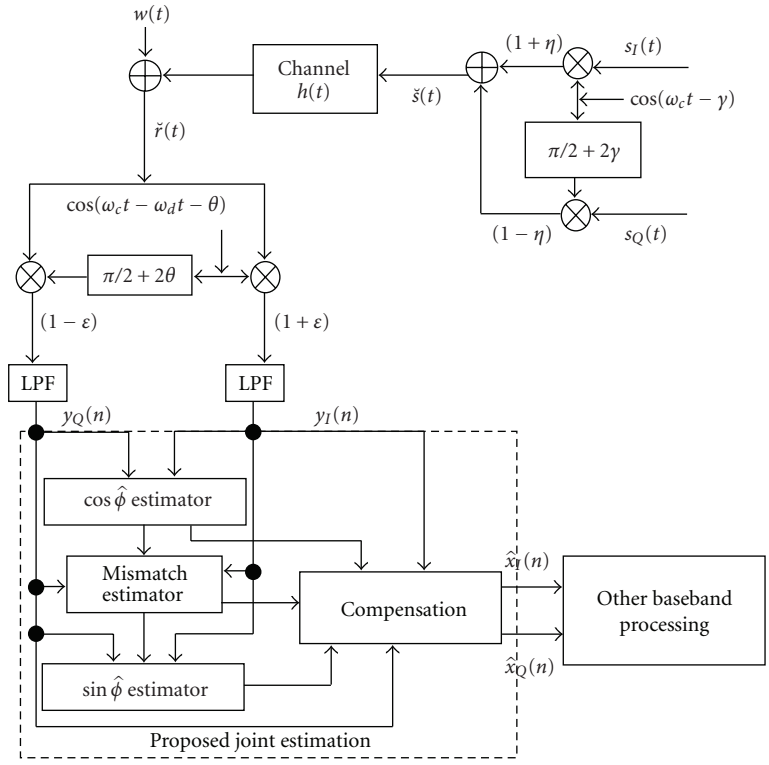


Figure 1: Block diagram of an OFDM system showing CFO and I/Q mismatch, and the proposed estimators.

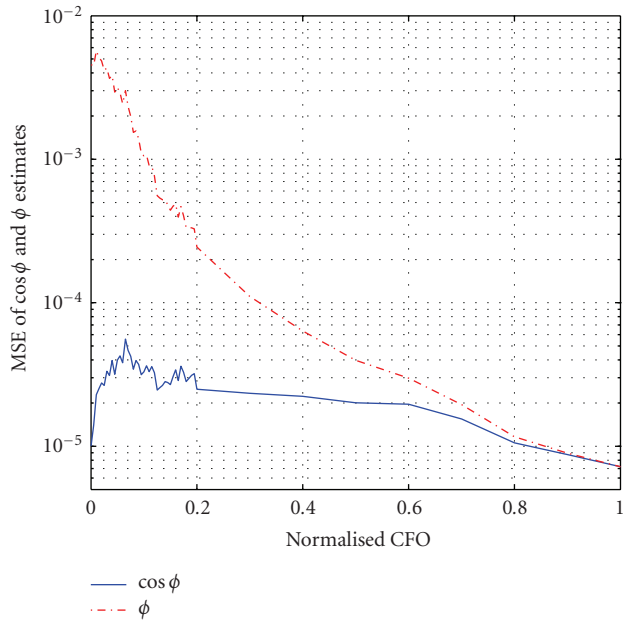


Figure 2: Mean square error of the estimates for $\cos\phi$ and ϕ versus normalized CFO, where SNR = 22 dB.

$$\widehat{\cos\phi} = \frac{\sum_{n=1}^{N_L-2L_p}\left\{\operatorname{sign}\left(y_I\left(n+L_p\right)\right)\left[y_I\left(n+2L_p\right)+y_I(n)\right]\right\}}{2\sum_{n=1}^{N_L}\left|y_I\left(n+L_p\right)\right|}, \tag{13}$$

where $\operatorname{sign}(x) = x/|x|$ for real $x \neq 0$ and $\operatorname{sign}(0) = 0$. The combiner is similar to an equal gain combiner (EGC), with the function $\operatorname{sign}(x)$ ensuring samples to be combined in a constructive way. This combiner, which will be called as EGC hereafter, only requires one division, plus $2(N_L - 2L_p)$ additions.

The EGC estimator even promises better performance than MRC when the number of training symbols is large and the CFO is small. The reason is that the MRC is the best one only when (1) signal and noise are independent and (2) noise samples are uncorrelated. However, when more than three training symbols are used in averaging, each noise samples could appear several times in combining. These repeated noise samples are scaled by $-\cos\phi$, and in EGC, some of the items have opposite phases and a noise cancellation effect can be achieved when $\cos\phi$ approaches 1. Thus the total noise can be partially cancelled due to the noise correlation in the EGC estimator when $\cos\phi$ is approaching 1.

For Q-branch, we can form a similar estimator. By combining I- and Q- branches, the final cosine estimator using EGC is given by (14)

$$\widehat{\cos\phi} = \frac{\sum_{n=1}^{N_L-2L_p}\left\{\operatorname{sign}\left(y_I\left(n+L_p\right)\right)\left[y_I\left(n+2L_p\right)+y_I(n)\right]+\operatorname{sign}\left(y_Q\left(n+L_p\right)\right)\left[y_Q\left(n+2L_p\right)+y_Q(n)\right]\right\}}{2\sum_{n=1}^{N_L-2L_p}\left(\left|y_I\left(n+L_p\right)\right|+\left|y_Q\left(n+L_p\right)\right|\right)}. \tag{14}$$

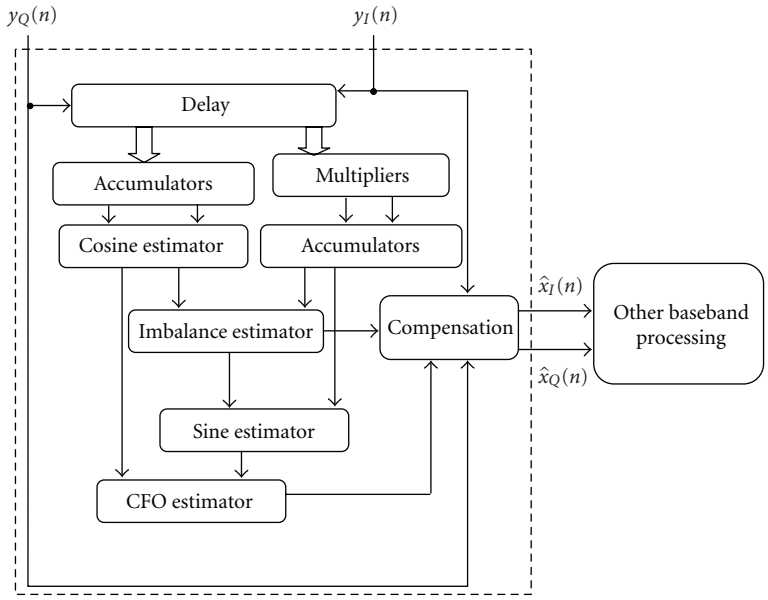


Figure 3: Implementation structure of the proposed estimators.

The corresponding CFO estimate is given by

$$\hat{\omega}_d = \frac{\arccos(\widehat{\cos}\phi)}{L_p}. \tag{15}$$

There are two problems with this estimator though it is robust to I/Q mismatch. One is the phase ambiguity problem as the range for ϕ in the estimator needs to be limited to $[0, \pi]$. The other is, when ϕ is small, the estimation error of ϕ increases rapidly even with $\cos\phi$ varying slightly. This is because the gradient of $\cos\phi$ is large in this case. The effect can be observed from Figure 2, where the variances of the estimation errors for $\widehat{\cos}\phi$ and $\hat{\phi}$ obtained from $\cos\phi$ are plotted against the normalized CFO. The results are obtained by using the general CFO estimation scheme in (14) in an IEEE802.11a system without introducing I/Q mismatch.

To eliminate the phase ambiguity and reduce the estimation error for small ϕ, a complementary sine estimator is generally needed. Such a sine estimator free of I/Q mismatch cannot be constructed directly. In [10], a sine estimator is proposed based on special training symbols, which are created by taking the original training sequences and superimposing an artificial CFO to generate point-wise 90-degree phase rotation. In [15], we introduce an approximated sine estimator, which can work without changing the training symbols for the cosine estimator. However the estimator in [15] sees interference from I/Q mismatch, particularly when the mismatch is large. It is thus natural to consider the approach of forming a sine estimator free of I/Q mismatch after estimating and compensating it.

3.2. Estimation of I/Q Mismatch Parameters. As can be seen from Figure 2, when ϕ is small, the estimate of $\cos\phi$ is much more robust to noise than ϕ. Next we develop an algorithm to estimate the I/Q mismatch parameters based on the estimate of $\cos\phi$ instead of ϕ. This approach can estimate mismatch parameters more accurately, particularly when ϕ is small.

From (8), the I/Q mismatch can be compensated as

$$\begin{aligned} x(n)e^{j\omega_d n} &= \frac{\alpha^* y(n) - \beta y^*(n)}{|\alpha|^2 - |\beta|^2} \\ &= \frac{\alpha^*}{|\alpha|^2 - |\beta|^2}\left[\frac{y(n) - \beta}{\alpha^* y^*(n)}\right]. \end{aligned} \tag{16}$$

Since I/Q mismatch is generally fixed during one transmission, $\alpha^*/(|\alpha|^2 - |\beta|^2)$ is a fixed constant, and it will not contribute to the CFO estimation and can be absorbed in channel coefficients for I/Q mismatch compensation. Thus we only need to know β/α^* to compensate the I/Q mismatch for the moment. The value of β/α^* can be computed via $\mu \triangleq \alpha\beta/(|\alpha|^2 + |\beta|^2)$, which can be estimated from $\cos\phi$. The formulation of estimating μ from $\cos\phi$ is shown in the appendix, and the result is given by

$$\hat{\mu} = \frac{\sum_{n=1}^{N_L - L_p}\left[y^2(n) + y^2\left(n + L_p\right) - 2y(n)y\left(n + L_p\right)\cos\phi\right]}{2\sum_{n=1}^{N_L - L_p}\left[\left|y(n + L_p)\right|^2 + |y(n)|^2 - 2\mathcal{R}\left(y\left(n + L_p\right)y^*(n)\right)\cos\phi\right]}, \tag{17}$$

where $\mathcal{R}(x)$ denotes the real part of x.

From the estimate $\hat{\mu}$, β/α^* can be computed by finding its phase and magnitude separately. The phase of β/α^* is obtained by

$$\angle\left(\frac{\beta}{\alpha^*}\right) = \angle\left(|\alpha|^2\frac{\beta}{\alpha^*}\right) = \angle(\alpha\beta) = \angle\mu. \tag{18}$$

To find the magnitude of β/α^*, we use

$$\left|\frac{1}{\mu}\right| = \frac{|\alpha|^2 + |\beta|^2}{|\alpha\beta|} = \left|\frac{\alpha^*}{\beta}\right| + \left|\frac{\beta}{\alpha^*}\right|. \tag{19}$$

Solving the equation, we get

$$\left|\frac{\beta}{\alpha^*}\right| = \frac{1-\sqrt{1-4|\mu|^2}}{2|\mu|}. \tag{20}$$

Note that we have dropped another solution which is impractically large. Since $|\mu| < |\beta/\alpha|$ and in general systems the I/Q mismatch is not very large, we have $|\mu| \ll 1$. Applying Taylor series to (20), the amplitude of μ can be approximated by

$$\left|\frac{\beta}{\alpha^*}\right| \approx |\mu| - \frac{1}{4}|\mu|^3. \tag{21}$$

Thus the estimate of β/α^* can be calculated as

$$\frac{\beta}{\alpha^*} = e^{j\angle\mu}\left(|\mu| - \frac{1}{4}|\mu|^3\right), \quad \text{with } \mu \text{ obtained from (17).} \tag{22}$$

As pointed out in the appendix, the estimation accuracy of μ becomes low when CFO is small and $\sin\phi$ is approaching zero. This is the common drawback of general I/Q mismatch estimation schemes based on the periodicity of the training sequence. To improve the performance of the proposed schemes, further processing can be applied. For example, a threshold can be set to initiate a frequency domain least square estimator or a joint estimator for I/Q mismatch and channel response [6] when the estimated CFO from $\cos\phi$ is smaller than the threshold. This threshold can be set as 0.1 according to our simulation results. The detailed discussion is beyond the scope of this paper.

3.3. CFO Estimation after I/Q Mismatch Compensation.

3.3.1. Autocorrelation-Based CFO Estimation. When I/Q mismatch parameters are known, a general approach is to compensate the signal in time domain, and then apply conventional autocorrelation-based CFO estimation given in [3]. With estimated β/α^* given in (22), I/Q mismatch can be compensated via (16), generating samples

$$\hat{x}_c(n) \triangleq x(n)e^{j\omega_d n}. \tag{23}$$

An autocorrelation-based CFO estimator can then be applied to the compensated samples, generating CFO estimates

$$\widehat{\omega_d} = \frac{1}{L_p}\angle\left(\sum_{n=1}^{N_L+L_p}\hat{x}_c(n)\hat{x}_c^*\left(n+L_p\right)\right). \tag{24}$$

The performance of this estimator depends on the accuracy of the estimated I/Q mismatch parameters.

3.3.2. Sine Estimator. The estimator given by (24) depends on the estimation of I/Q mismatch, and estimation error of I/Q mismatch affects both the cos and sin parts of the CFO estimate. Alternatively, we can form a complementary sine estimator to exploit the cosine estimator developed in Section 3.1 which is free of I/Q mismatch. With estimated I/Q mismatch parameters, a sine estimator can be formed as follows.

It is easy to verify that

$$\begin{aligned}\mathcal{R}\left(\hat{x}_c\left(n+2L_p\right) - \hat{x}_c(n)\right) &= -2\mathcal{I}\left(\hat{x}_c\left(n+L_p\right)\right)\sin\phi\\ \mathcal{I}\left(\hat{x}_c\left(n+2L_p\right) - \hat{x}_c(n)\right) &= 2\mathcal{R}\left(\hat{x}_c\left(n+L_p\right)\right)\sin\phi,\end{aligned} \tag{25}$$

where $\mathcal{I}(x)$ denotes the imaginary part of x. Then $\sin\phi$ can be estimated as

$$\widehat{\sin\phi} = \frac{\mathcal{R}\left(x_c\left(n+2L_p\right) - x_c(n)\right)}{-2\mathcal{I}\left(x_c\left(n+L_p\right)\right)}, \tag{26}$$

or

$$\widehat{\sin\phi} = \frac{\mathcal{I}\left(x_c\left(n+2L_p\right) - x_c(n)\right)}{2\mathcal{R}\left(x_c\left(n+L_p\right)\right)}. \tag{27}$$

Combining them together and incorporating MRC over a group of samples, the final estimate of $\sin\phi$ is given as follows:

$$\widehat{\sin\phi} = \frac{\sum_{n=1}^{N_L-L_p}\left[\mathcal{R}\left(\hat{x}_c(n) - \hat{x}_c\left(n+2L_p\right)\right)\mathcal{I}\left(\hat{x}_c\left(n+L_p\right)\right) + \mathcal{I}\left(\hat{x}_c\left(n+2L_p\right) - \hat{x}_c(n)\right)\mathcal{R}\left(\hat{x}_c\left(n+L_p\right)\right)\right]}{\sum_{n=1}^{N_L-L_p} 2\left|\hat{x}_c(n+L_p)\right|^2} \tag{28}$$

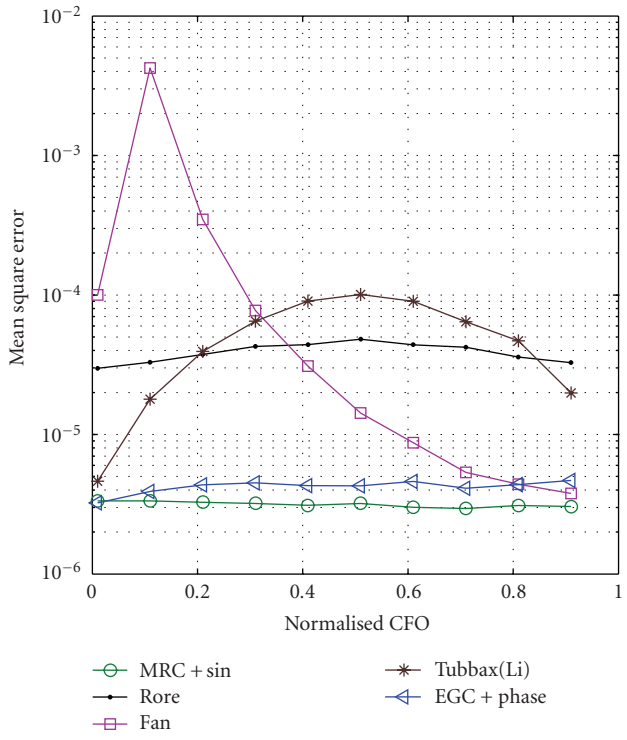


Figure 4: MSE of CFO estimates at SNR = 22 dB in the presence of 2 dB gain mismatch and 5° phase mismatch.

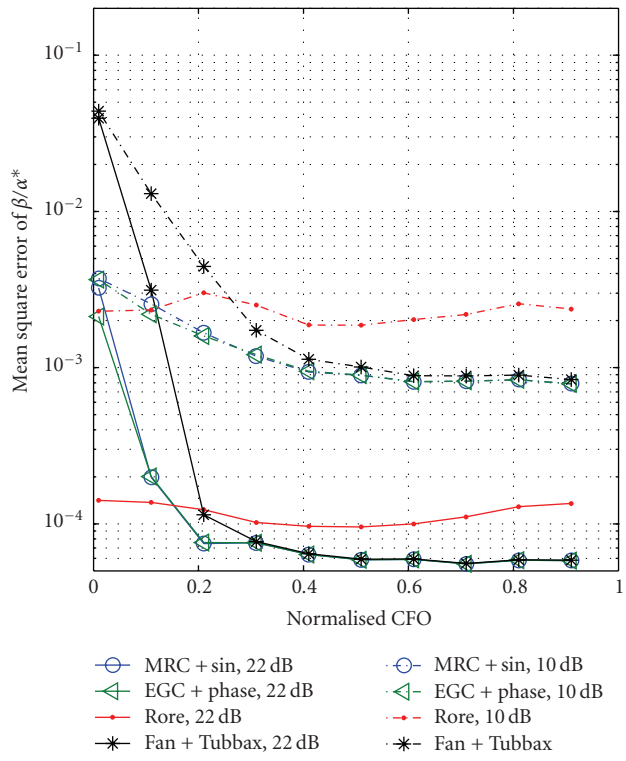


Figure 5: MSE of β/α^* at SNR = 10 dB and 22 dB in the presence of 1 dB gain mismatch and 1° phase mismatch.

Combining $\widehat{\sin}\,\phi$ and $\widehat{\cos}\,\phi$, the CFO ω_d is given by

$$\widehat{\omega_d} = \frac{1}{L_p}\arctan\left(\frac{\widehat{\sin\phi}}{\widehat{\cos\phi}}\right). \tag{29}$$

The complexity of this scheme is approximately half of the autocorrelation-based one as only $\sin\phi$ needs to be estimated. Its performance, however, could be better than the latter because of the use of the interference-free cosine estimator.

3.4. CFO and I/Q Mismatch Compensation. With all the parameters estimated, CFO and I/Q mismatch can be compensated in time domain by

$$\hat{x}(n) = \frac{\alpha^*}{|\alpha|^2 - |\beta|^2} e^{-j\widehat{\omega_d} n}\left(y(n) - \frac{\hat{\beta}}{\hat{\alpha}^*} y^*(n)\right), \tag{30}$$

where we note that the term $\alpha^*/(|\alpha|^2 - |\beta|^2)$ will be absorbed in the channel coefficients and does not need to be known and compensated here.

3.5. Implementation Issues. Although the proposed schemes are divided into three steps, they can be implemented in a parallel manner. Thus very little memory is required in the hardware and the processing delay is very small. As can be seen from (12), (13), (17), (24), and (28), all the sums can be implemented in parallel because the CFO and I/Q imbalance parameters in these equations are fixed and independent of the received signal samples. Parameters can be estimated based on the final sums.

Figure 3 shows the implementation structure of the proposed algorithms. The input signals at the I and Q branches are first passed through register banks to generate the delayed signal $y(n + L_p)$ and $y(n + 2L_p)$. Then they are added up in the first accumulator bank consisting of 4 accumulators to get the summations required for the cosine estimator. In the meantime, products between $y_I(n)$, $y_I(n + L_p)$, $y_I(n + 2L_p)$, $y_Q(n)$, $y_Q(n + L_p)$, and $y_Q(n + 2L_p)$ are generated by the multipliers. Then these products are summed up in other accumulator banks consisting of 9 accumulators. After all the training symbols are received, during the reception of the guarding intervals of the next OFDM block, estimation and compensation of the CFO and I/Q imbalance can be processed. The cosine estimator provides an estimate of $\cos\phi$ based on the summation. Then the mismatch estimator calculates the term β/α^* using the estimated $\cos\phi$ and the results of the accumulators. After obtaining β/α^*, the sine estimator calculates $\sin\phi$ using the estimated mismatch parameters and the outputs of the accumulators. Compensation is then performed using the estimates of CFO and β/α^* for the following OFDM blocks. When the guarding interval is long enough, in most cases over 16 samples, estimation can be completed within this interval and will not cause delay in processing data symbols.

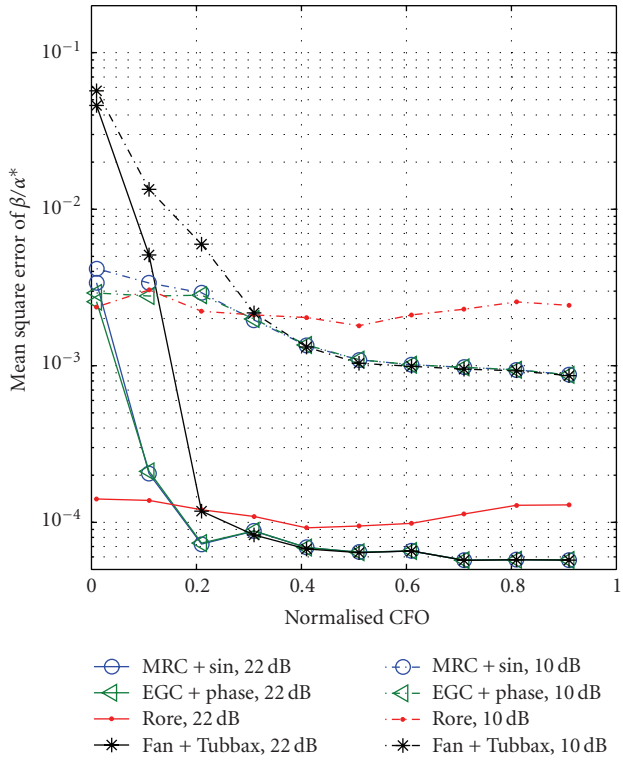


FIGURE 6: MSE of β/α^* at SNR = 10 dB and 22 dB in the presence of 2 dB gain mismatch and 5° phase mismatch.

4. Simulation Results

The proposed schemes can be used in any OFDM systems with more than three periodical training symbols in the preamble. In our simulation, the 54Mbps option in the IEEE802.11a standard is followed, and 64QAM modulation and 3/4 convolutional coding are used. We use ETSI Multipath A [16] in the simulation. Both mean square error (MSE) of estimates and bit error rate (BER) are used to evaluate the performance of the proposed systems. Each result presented was averaged over 5000 packets, each with 1024 bytes. Basically, at least 3 periodical training sequence are required for the proposed estimator, however, we assume 7 short training symbols are available for CFO and I/Q mismatch estimation which is the minimum requirement of the methods in [10].

In the simulation, our schemes are mainly compared with three known approaches: the two CFO estimators robust to I/Q mismatch proposed in [10, 11] and the time domain I/Q mismatch estimator proposed in [6, 10]. To ensure the compared schemes to have similar complexity, the frequency domain I/Q mismatch estimator for small CFO in [6] is not used for comparison. When simulating the scheme in [10], half of the training sequences are revised accordingly so that the total number of training symbols are the same for all schemes, and we assume the starting of the second group of training sequences are perfectly identified. Although in Section 3.3 we mentioned that incorporating a frequency domain I/Q mismatch estimator can improve the performance for small CFO (<0.1), it is not used in our simulation to ensure fair comparisons with other methods. Notations for different schemes are as follows:

(i) *MRC*+sin: MRC-based cosine and sine estimators using MRC cosine estimator and (28);

(ii) *EGC+Phase*: EGC-based cosine estimator (14) to generate I/Q mismatch estimate, and autocorrelation estimator (24) for CFO estimation;

(iii) *Tubbax(Li)*: Joint CFO and time domain I/Q mismatch estimator in [6] which applies conventional autocorrelation-based CFO estimator in [3];

(iv) *Fan/Fan+Tubbax*: CFO estimator in [11] plus the time domain I/Q mismatch estimator in [6] (combination of the two schemes generates much better performance than the single one in [11].);

(v) *Rore*: CFO estimator in [10] using special training sequence.

We first examined the MSE of CFO estimates in the presence of I/Q mismatch (2 dB for gain mismatch and 5° for phase mismatch). All the estimators designed for CFO estimation in the presence of I/Q mismatch, showed robustness to I/Q mismatch in the experiments. Figure 4 shows the MSE of the CFO estimates for relatively large mismatch, where the signal to noise power ratio (SNR) is 22 dB. From the figure, we can see that the proposed schemes are robust to any CFO and achieve great improvement over the "Fan" and "Rore" schemes, particularly at smaller CFO. As mentioned in the introduction, the "Fan" scheme sees significant performance degradation at smaller CFO. The "Tubbax(Li)" scheme, which is the only one that ignores I/Q mismatch in CFO estimation, suffers from large performance degradation, particularly in the middle range of CFO values.

Figures 5 and 6 show the MSE of the mismatch estimates for different CFO values in both small and large I/Q mismatch cases for different SNRs (10 dB and 22 dB). To be consistent with the metric used in [6], the MSE of $\hat{\beta}/\hat{\alpha}^*$ is used. From Figure 4 we know that the "Tubbax" or "Li" CFO estimator is not robust to I/Q mismatch, and it is not simulated for I/Q mismatch estimation. From the figures, we can see that when CFO is small, the proposed schemes largely outperform the "Fan+Tubbax" schemes, particularly for SNR = 10 dB. This mainly contributes to the high stability of cosine function to noise at small CFO.

We further test the BER performance of different schemes. First, performance is examined for some specific CFO values with random I/Q mismatch values in Figure 7. The I/Q mismatch is set to be uniform distributed random variables in the range of $[0, 1]$ dB for gain and $[0, 4°]$ for phase. The SNR is 22 dB. The figures show that the proposed schemes outperform the "Tubbax" and "Fan+Tubbax" methods, especially at smaller CFO. The "Rore" method can achieve better performance over other when CFO is small, the proposed methods, "Fan+Tubbax" and "Tubbax" outperform "Rore" method. This coincides with the observation for CFO and I/Q mismatch estimation from Figures 4, 5, and 6.

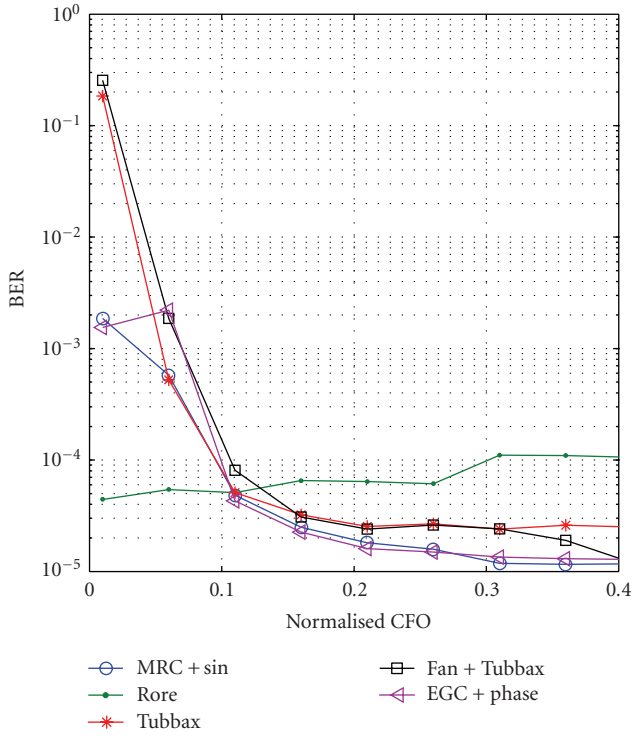


Figure 7: BER at SNR = 22 dB versus normalized CFO in the presence of small random mismatch which is uniformly distributed in $[0, 1]$ dB for gain mismatch and $[0, 4^\circ]$ for phase mismatch.

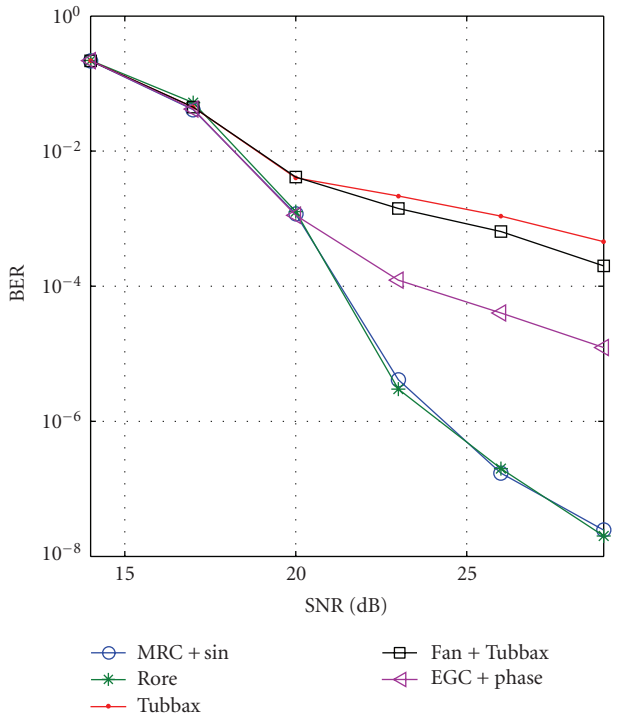


Figure 8: BER at different SNRs in coded system with random CFO in $[0, 1]$ and random I/Q mismatch which is uniformly distributed in $[0, 1]$ dB for gain mismatch and $[0, 4^\circ]$ for phase mismatch.

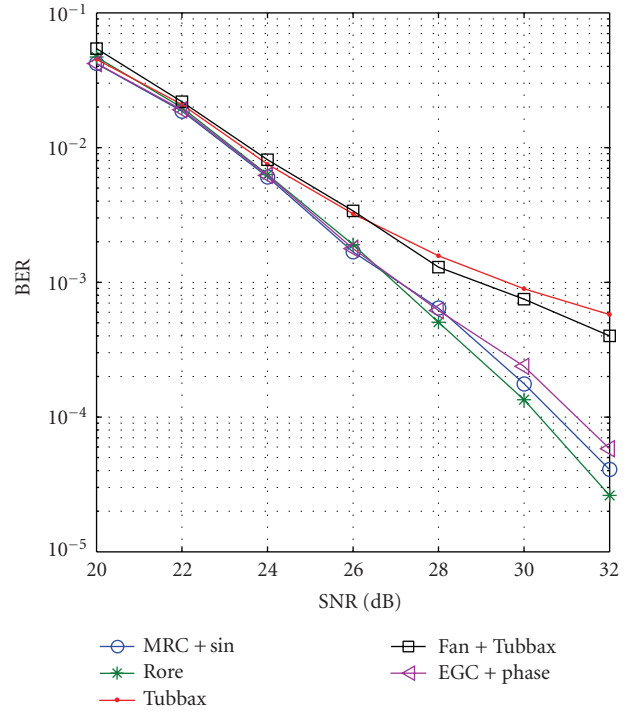


Figure 9: BER at different SNRs in uncoded system with random CFO in $[0, 1]$ and random I/Q mismatch which is uniformly distributed in $[0, 1]$ dB for gain mismatch and $[0, 4^\circ]$ for phase mismatch.

System performance versus SNR for coded and uncoded systems is shown in Figures 8 and 9 respectively, where CFO is uniformly distributed over $[0, 1]$ and the gain and phase I/Q mismatch are uniform distributed over $[0, 1]$ dB and $[0, 4^\circ]$ respectively. It is shown that the proposed method "MRC+Sin" can achieve similar performance as the "Rore" method, and the proposed "EGC+Phase" scheme experiences performance degradation in high SNR cases. The proposed methods outperform the "Tubbax" and "Fan+Tubbax" methods in all cases. Comparing Figures 8 and 9, we can see that without coding, AWGN has more impact on system performance and the BER difference between different methods is much smaller than that in the coded cases.

5. Conclusions

In this paper, some low-complexity joint CFO and I/Q mismatch estimators are proposed. The estimators are formed based on the observation that a cosine estimator of the CFO, which is free of I/Q mismatch, serves much better as the basis for I/Q mismatch estimation than an initial estimate of CFO. The proposed schemes are robust to any values of CFO and I/Q mismatch, and can improve the accuracy of CFO and I/Q mismatch estimates significantly. The proposed schemes are applicable to systems with conventional training symbols and have low complexity, and they are very promising for broadband systems where I/Q mismatch could deteriorate system performance significantly.

Appendix

We derive the estimation of $\mu = \alpha\beta/(|\alpha|^2 + |\beta|^2)$ from $\cos\phi$ here.

According to (8), the received signals at $n + L_p$ and n are given by

$$\begin{aligned} y(n) &= \alpha x(n)e^{j\phi_n} + \beta x^*(n)e^{-j\phi_n} + \xi(n), \\ y\left(n+L_p\right) &= \alpha x\left(n+L_p\right)e^{j(\phi_n+\phi)} + \beta x^*\left(n+L_p\right)e^{-j(\phi_n+\phi)} \\ &\quad + \xi\left(n+L_p\right). \end{aligned} \tag{A.1}$$

Since the signal is periodic, we have $x(n + L_p) = x(n)$. Let $z(n) \triangleq y(n + L_p) - y(n)\cos\phi$. The energy of $z(n)$ is

$$\begin{aligned} |z(n)|^2 &= \left[y\left(n+L_p\right) - y(n)\cos\phi\right]\left[y\left(n+L_p\right) - y(n)\cos\phi\right]^* \\ &= \left|y\left(n+L_p\right)\right|^2 + |y(n)|^2\cos^2\phi \\ &\quad - 2\mathcal{R}\left(y\left(n+L_p\right)y^*(n)\right). \end{aligned} \tag{A.2}$$

Adding $|y(n)\sin\phi|^2$ to both sides of (A.2), we get

$$\begin{aligned} &|y(n)\sin\phi|^2 + |z(n)|^2 \\ &= \left|y(n+L_p)\right|^2 + |y(n)|^2 - 2\mathcal{R}\left(y\left(n+L_p\right)y^*(n)\right)\cos\phi. \end{aligned} \tag{A.3}$$

On the other hand, replacing $y(n)$ with (8), we have

$$\begin{aligned} z(n) &= y\left(n+L_p\right) - y(n)\cos\phi \\ &= \frac{\alpha}{2}e^{j\phi_n}x(n)\left[2e^{j\phi} - \left(e^{j\phi} + e^{-j\phi}\right)\right] \\ &\quad + \frac{\beta}{2}e^{-j\phi_n}x^*(n)\left[2e^{-j\phi} - \left(e^{j\phi} + e^{-j\phi}\right)\right] \\ &\quad + \xi\left(n+L_p\right) - \xi(n)\cos\phi \\ &= j\sin\phi\left[\alpha x(n)e^{j\phi_n} - \beta x^*(n)e^{-j\phi_n}\right] \\ &\quad + \xi\left(n+L_p\right) - \xi(n)\cos\phi. \end{aligned} \tag{A.4}$$

Using the results in (A.4) and (8), we compute

$$\begin{aligned} &|y(n)\sin\phi|^2 + |z(n)|^2 \\ &\quad = 2\left(|\alpha|^2 + |\beta|^2\right)|x(n)|^2\sin^2\phi + W_1(n), \end{aligned} \tag{A.5}$$

where $W_1(n)$ is the noise term. Combining (A.5) and (A.3), we get

$$\begin{aligned} &\left|y(n+L_p)\right|^2 + |y(n)|^2 - 2\mathcal{R}\left(y\left(n+L_p\right)y^*(n)\right) \\ &\quad = 2\left(|\alpha|^2 + |\beta|^2\right)|x(n)|^2\sin^2\phi + W_1(n), \end{aligned} \tag{A.6}$$

which gives us the denominator of μ. An averaging is performed on (A.6).

We continue to find the numerator of μ. Similar to the computation of $|z(n)|^2 + |y(n)\sin\phi|^2$, we compute the difference between $z^2(n)$ and $(j\sin\phi)^2y^2(n)$, giving

$$\begin{aligned} &z^2(n) - (j\sin\phi)^2y^2(n) \\ &\quad = \left[y(n+L_p) - y(n)\cos\phi\right]^2 + y^2(n)\sin^2\phi \\ &\quad = y^2(n) + y^2\left(n+L_p\right) - 2y(n)y\left(n+L_p\right)\cos\phi, \\ &z^2(n) - (j\sin\phi)^2y^2(n) \\ &\quad = (j\sin\phi)^2\left\{[\alpha x(n) - \beta x^*(n)]^2 - [\alpha x(n) + \beta x^*(n)]^2\right\} \\ &\quad = 4\alpha\beta|x(n)|^2\sin^2\phi + W_2(n). \end{aligned} \tag{A.7}$$

where $W_2(n)$ is the noise term. Combining the results in (A.7), we have

$$\begin{aligned} &y^2(n) + y^2\left(n+L_p\right) - 2y(n)y\left(n+L_p\right)\cos\phi \\ &\quad = 4\alpha\beta|x(n)|^2\sin^2\phi + W_2(n), \end{aligned} \tag{A.8}$$

which gives the numerator of μ.

Synthesizing the results in (A.6) and (A.8), the estimate of μ can be calculated as

$$\begin{aligned} \hat{\mu} &= \frac{y^2(n) + y^2\left(n+L_p\right) - 2y(n)y\left(n+L_p\right)\cos\phi}{2\left[\left|y(n+L_p)\right|^2 + |y(n)|^2 - 2\mathcal{R}\left(y\left(n+L_p\right)y^*(n)\right)\cos\phi\right]} \\ &= \frac{4\alpha\beta|x(n)|^2\sin^2\phi + W_2(n)}{4\left(|\alpha|^2 + |\beta|^2\right)|x(n)|^2\sin^2\phi + 2W_1(n)} \\ &= \frac{\alpha\beta + \left(W_2(n)/4|x(n)|^2\sin^2\phi\right)}{|\alpha|^2 + |\beta|^2 + \left(W_1(n)/2|x(n)|^2\sin^2\phi\right)}. \end{aligned} \tag{A.9}$$

Equation (A.9) shows that the noise effect in the estimates of μ is inversely proportional to $\sin\phi$. So the smaller the $\sin\phi$ is, the larger the noise effects in the $\hat{\mu}$ are. When ω_d is small and $\sin\phi$ is approaching zero, the estimation accuracy of μ degrades. Averaging over a group of samples and replacing $\cos\phi$ with its estimate establishes the final estimate as shown in (17).

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