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Research Article

Outage Performance Analysis of Cooperative Diversity with MRC and SC in Correlated Lognormal Channels

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The study of relaying systems has found renewed interest in the context of cooperative diversity for communication channels suffering from fading. This paper provides analytical expressions for the end-to-end SNR and outage probability of cooperative diversity in correlated lognormal channels, typically found in indoor and specific outdoor environments. The system under consideration utilizes decode-and-forward relaying and Selection Combining or Maximum Ratio Combining at the destination node. The provided expressions are used to evaluate the gains of cooperative diversity compared to noncooperation in correlated lognormal channels, taking into account the spectral and energy efficiency of the protocols and the half-duplex or full-duplex capability of the relay. Our analysis demonstrates that correlation and lognormal variances play a significant role on the performance gain of cooperative diversity against noncooperation.

1. Introduction

Cooperative diversity systems consist of multiple nodes that share resources in order to create multiple diversity channels and thereby improve system performance, typically in terms of availability, range, and throughput.

This paper considers cooperative diversity in lognormal fading channels. Lognormal channel models can be used to model indoor as well as outdoor propagation (see [1–3], [4, Section 4.2.1], [5] and references therein). Moreover, due to various propagation effects and geometrical parameters, the fading gains of differing propagations paths can be assumed to be correlated (see, e.g., [6–8] and references therein) thus following a multivariate lognormal distribution, as described in the next section.

A fundamental building block for cooperative diversity systems is the relaying channel [9], which has been studied in the context of fading channels in the recent years [10, 11]. To the authors' knowledge, related results on lognormal channels are very limited. Receive diversity combining with correlated lognormal branches has been studied in [12–15], while multihop communications over independent lognormal fading channels have been studied in [16]. Regarding cooperative diversity systems, the case of independent and identically distributed lognormal fading gains is studied in [17], where bounds on the pairwise error probability are provided. In [18], the impact of total power constraints is investigated through bounds of the outage probability and error probability of relaying with independent lognormal fading channel gains, utilizing Fenton-Wilkinson's method [19] for approximating the combiner output at the destination node. Finally, in [20] a distributed diversity system with amplify and forward [11] relays is studied, assuming independent channels.

This paper aims to cover a gap in the literature regarding the performance estimation of cooperative diversity systems in correlated lognormal channels. More specifically, this paper proposes exact integral expressions for the end-to-end outage probability of a cooperative diversity system where a decode-and-forward [11] relay node assists communication between a source and a destination nodes. The two formed diversity branches are combined coherently by the destination node using either Maximal Ratio Combining

(MRC) or Selection Combining (SC) [4]. In addition, some novel insights are obtained on the efficiency of cooperation by studying the impact of the multiple-access protocol on cooperation and comparing it to the performance of noncooperation. The choice of multiple-access protocol depends on the ability of the relay to perform half-duplex or full-duplex operation.

This paper is organized as follows. Section 2 presents the system model and correlated lognormal channel model including a description of the possible multiple-access protocols utilized by the cooperative diversity system. Section 3 then provides exact expressions for the outage probability of single-relay systems with MRC or SC at the destination. Section 4 establishes the energy and spectral efficiency of the cooperative protocols under consideration and proposes an appropriate direct link system for comparison purposes. Finally, Section 5 utilizes the proposed formulas and efficiency framework to numerically assess the impact of the various system parameters on the cooperative diversity system's performance.

2. System Model

2.1. Geometrical Parameters. The geometrical configuration of the considered cooperative wireless network is shown in Figure 1. The Source Node S communicates with the Destination Node D through two different routes. The first signal is directly transmitted by Node S to Node D and the second signal is transmitted by Node S to Node D through the Relay Node R (dual-hop transmission). These two signal paths form two diversity branches which are combined by the Node D using coherent combining [4] to form the final received signal.

In Figure 1, the length of each link j $(j = 1,2,3)$ is denoted as $L_j(m)$, while the links i, j $(i, j = 1,2,3,\ i \neq j)$ contain an angle $\phi_{ij}(\text{deg})$, where $\phi_{ij} = \phi_{ji}$.

2.2. Channel Models. The received Signal-to-Noise Ratio (SNR) of link j is given by (in linear scale)

$$\gamma_j = \frac{1}{N_0} P_{\mathrm{Tx}j} w_j, \tag{1}$$

where N_0 is the noise density in linear scale (assumed to be equal to 1 in this paper without loss of generality), $P_{\mathrm{Tx}j}$ is the transmitted power for link j, and w_j is the shadowing lognormal variable with parameters (in Neper):

$$\left(\mu_{w_j}, \sigma_j\right) = \left(-\ln\left(\mathrm{PL}_j\right), \sigma_j\right), \tag{2}$$

where PL_j is the path-loss of link j expressed in linear scale and σ_j is the variance of the shadowing parameter that depends on the specific propagation environment (values for various propagation scenarios are given in [7]).

The received SNR of link j is therefore a lognormal random variable with parameters (in Neper):

$$\left(\mu_j, \sigma_j\right) = \left(\left(\ln\left(P_{\mathrm{Tx}j}\right) + \mu_{w_j}\right), \sigma_j\right), \tag{3}$$

where μ_{w_j} is given by (2). Parameters expressed in Neper can also be expressed in dB, using $1\,\mathrm{Np} = \xi\,\mathrm{dB} = 10/\ln(10)\,\mathrm{dB}$. Moreover, the lognormal random variables γ_j $(j = 1,2,3)$ are assumed to be correlated and follow the trivariate lognormal distribution $f_{\gamma_1\gamma_2\gamma_3}(\gamma_1,\gamma_2,\gamma_3)$, as described in Section 3.

2.3. Diversity Techniques. The Relay retransmission takes place in the form of the Decode-and-Forward technique (Regenerative relay) [11], where the received signal is regenerated using the full receiver-transmitter processing chain containing the sequence of demodulation, channel decoding, encoding, and modulation.

The Destination Node D combines the direct-link signal with the signal from the dual-hop path. In this paper, two different combining techniques are examined: Selection Combining (SC) and Maximal Ratio Combining (MRC) [4]. In the SC technique, the diversity branch with the highest SNR is always selected. On the other hand, the MRC technique is a matched filter that weights the diversity branches by their respective complex fading gains and combines the result. In other words, if the MRC technique is adopted, Node D coherently adds the two received signals.

2.4. Medium Access Protocols. Access to the wireless medium of the participating nodes is facilitated through the use of either Time or Frequency Division Multiple Access (TDMA or FDMA) or a more bandwidth-efficient scheme such as Space Division Multiple Access (SDMA, smart antennas).

In the TDMA or FDMA scheme, termed as *protocol A* in this paper, two degrees of freedom (DOF) are utilized: in the first time/frequency slot the source node S broadcasts a signal to both the relay node R and the destination node D, while in the second time/frequency slot the relay R retransmits the received signal to the destination D. The protocol's double bandwidth usage will be modeled in Section 4.

In the SDMA scheme, termed as *protocol B* in this paper, only one DOF is used: S transmits and R receives while transmitting the previous transmission from S; the destination D simultaneously receives the transmission from S and the relayed transmission from R (interference is avoided through the use of spatial multiplexing such as smart or highly directive antennas). Protocol B can alternatively be viewed as a pipelining system, which explains the DOF superiority over protocol A. This protocol has the same spectral efficiency as a direct link system but is less efficient in terms of energy as it consumes more energy per DOF slot. In addition, protocol B has other potential advantages over protocol A, for example, it does not require any frame gap overheads, which are typically required in protocol A to enable error-resilient transmit-receive switching at the relay [21].

It should also be noted that deploying protocol B in a practical system faces several technological challenges. Protocol B is based on full-duplex relay operation, which is known to incur self-interference (feedback interference) caused by the relay's transmit antenna to the relay's receive

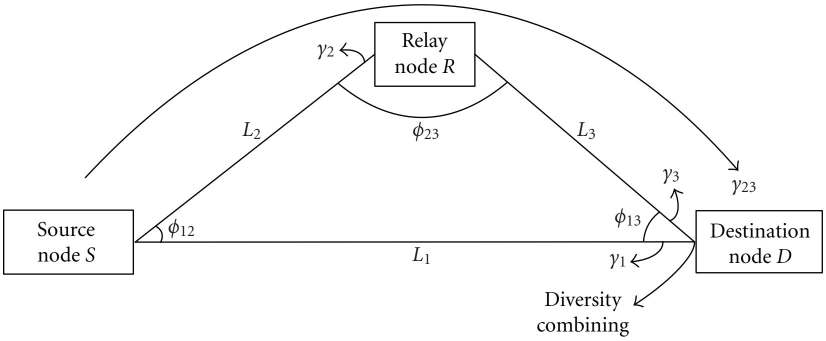

Figure 1: Geometrical configuration of the cooperative diversity network.

antenna. This problem can be solved by properly optimizing the antenna installation, ensuring sufficient isolation between the antennas, as well as by deploying interference cancellation signal processing techniques. In addition, the difficulty of antenna isolation depends on the propagation environment. For more information, the interested reader is referred to the techniques in [21–23] and references therein.

The efficiency of both protocols in terms of energy and spectral efficiency is studied in detail in Section 4.

3. Performance of Cooperative Diversity with SC and MRC

3.1. Total SNR. In the following analysis it is required that both the relay and the destination must perfectly decode the source signal. The total SNR, at the output of the MRC combiner, is given by [11]

$$\gamma_{\text{MRC}} = \min(\gamma_2, \gamma_1 + \gamma_3). \tag{4}$$

On the other hand, the total SNR given at the output of a Selection Combiner can be readily shown to be

$$\gamma_{\text{SC}} = \min(\gamma_2, \max(\gamma_1, \gamma_3)). \tag{5}$$

The first term in (4) and (5) corresponds to the event that the relay perfectly decodes the source signal. The second term in (4) and (5) is equal to the output at the combiner of MRC and SC, respectively [4] and corresponds to the event that the destination perfectly decodes the received signal.

3.2. Outage Probability. This section proposes exact integral expressions for the evaluation of the outage probability metric. The outage probability is defined as the fraction of time where the total SNR γ_C does not exceed a specified threshold γ_{th}:

$$P_{\text{out}} = P(\gamma_C < \gamma_{\text{th}}). \tag{6}$$

The threshold γ_{th} in (6) depends on the spectral efficiency of the medium access protocol under consideration (see Section 2.4) and will be analyzed in Section 4.

Using the trivariate lognormal distribution $f_{\gamma_1\gamma_2\gamma_3}(\gamma_1, \gamma_2, \gamma_3)$ and expressions (4) and (5), the outage probability of (6) can be calculated by

$$\begin{aligned} P_{\text{out}} = &\int_0^{\gamma_{\text{th}}} f_{\gamma_2}(\gamma_2)\, d\gamma_2 \\ &+ \int_0^{\gamma_{\text{th}}} d\gamma_1 \int_0^{\gamma_0} d\gamma_3 \int_{\gamma_{\text{th}}}^{\infty} d\gamma_2 f_{\gamma_1\gamma_2\gamma_3}(\gamma_1, \gamma_2, \gamma_3), \end{aligned} \tag{7}$$

where

$$\gamma_0 = \begin{cases} \gamma_{\text{th}}, & \text{for SC}, \\ \gamma_{\text{th}} - \gamma_1, & \text{for MRC}. \end{cases} \tag{8}$$

In expression (7), the first and the second term correspond to the first and the second term of (4) for the SC and of (5) for the MRC, respectively.

The trivariate lognormal distribution $f_{\gamma_1\gamma_2\gamma_3}(\gamma_1, \gamma_2, \gamma_3)$ can be derived by the trivariate normal distribution $f_{u_1u_2u_3}(u_1, u_2, u_3)$ [24, Section 7.2], using the transformations:

$$u_j = \frac{\left(\ln \gamma_j - \mu_j\right)}{\sigma_j} \quad (j = 1, 2, 3), \tag{9}$$

where μ_j, σ_j $(j = 1, 2, 3)$ are given by (3).

Using the transformations in (9) and after employing Bayes' theorem [24], the outage probability of (7) can be expressed as

$$\begin{aligned} P_{\text{out}} = 1 - &\operatorname{erfc}\frac{(u_{2,0}/\sqrt{2})}{2} \\ &+ \frac{1}{2}\int_{-\infty}^{u_{1,0}} du_1 \int_{-\infty}^{u_{3,0}} f_{u_1u_3}(u_1, u_3) \operatorname{erfc}\left(\frac{u_{2,0} - \mu_{2/1,3}}{\sqrt{2}\sigma_{2/1,3}}\right) du_3, \end{aligned} \tag{10}$$

where

$$\begin{aligned} u_{3,0} &= \frac{(\ln \gamma_0 - \mu_3)}{\sigma_3}, \\ u_{j,0} &= \frac{\left(\ln \gamma_{\text{th}} - \mu_j\right)}{\sigma_j} \quad (j = 1, 2), \end{aligned} \tag{11}$$

and γ_0 is given by (8) for the SC and MRC cases.

The joint normal probability density function of the random variables u_1, u_3 is denoted as $f_{u_1u_3}(u_1, u_3)$ [24].

The parameters $\mu_{2/1,3}, \sigma_{2/1,3}$ [24, Chapter 7] are given by

$$\mu_{2/1,3} = \frac{\rho_{12} - \rho_{13}\rho_{23}}{1 - \rho_{13}{}^2}u_1 + \frac{\rho_{23} - \rho_{12}\rho_{13}}{1 - \rho_{13}{}^2}u_3,$$
$$\sigma_{2/1,3} = \sqrt{\frac{1 - \rho_{12}{}^2 - \rho_{13}{}^2 - \rho_{23}{}^2 + 2\rho_{12}\rho_{13}\rho_{23}}{1 - \rho_{13}{}^2}}. \tag{12}$$

In expression (12), the correlation coefficient between the lognormal variables (γ_i, γ_j), where $(i, j = 1, 2, 3,\ i \neq j)$, is denoted as ρ_{ij}, where $\rho_{ij} = \rho_{ji}$.

The next section investigates both the spectral and energy efficiency of the two medium access protocols, specified in Section 2.

4. Protocol Efficiency of Cooperative Diversity

During the evaluation of cooperative diversity systems, the question of whether they offer any benefit compared to direct link transmission often occurs. This section provides a suitable framework for providing fair comparisons between protocol A, protocol B and direct link systems, in terms of two important system resources, namely the spectral efficiency and the total transmission energy. It should be noted that the presented framework is more general than [11] because it covers both the half- and full-duplex relay cases.

In other words, the performance of the three systems is compared when each system consumes the same transmission energy in total and offers the same data rate (spectral efficiency). The results illustrate the optimal system operation regions where cooperation is of benefit, taking into account both energy and rate constraints.

4.1. Energy Efficiency of Protocols A and B. The two protocols A and B consume the same energy for symbol transmission. The *energy per transmitted symbol* of protocols A and B is given by

$$E_{\text{protocol(A)}} = E_{\text{protocol(B)}} = \sum_{i=1}^{2} p_{\text{total}(t=i)}T, \tag{13}$$

where $p_{\text{total}(t=i)}$ is the total system power transmitted at multiple-access slot $t = i$, and T is the duration of each multiple-access slot.

A typical direct link system, consisting of source S and destination D without a relay, utilizes less transmission energy per symbol compared to the cooperative system under consideration (since the latter deploys two transmissions per symbol). For example, if the transmit power of the relay R is equal to the source S, then the cooperative system has a total transmission energy (per symbol) which is double the energy of a reference direct link system consisting of nodes S and D. In other words, the additional performance offered by cooperation comes at the expense of increased transmit power consumption. This fact was considered in [18] in order to compare cooperation with equal transmit powers to a reference direct link system.

In order to evaluate the exact performance gain of cooperation compared to noncooperation, an *energy-effective direct link system* should be considered, taking into account the total system transmit energy used for cooperation. The energy-effective direct link system is a direct link system where the transmitted energy (per symbol) of the source S is the same as the total energy of the cooperative system, that is, the sum of the transmit powers of the source S and the relay R. In other words, the effective direct link system has an increased transmission energy $E_{\text{direct(eff)}} = T \cdot p_{\text{Tx(eff)}}$ (per transmitted symbol), equal to the total energy of either cooperative diversity protocols given by (13). The transmission power of the energy-effective direct link system is therefore given by

$$p_{\text{Tx(eff)}} = p_{\text{Tx1}} + p_{\text{Tx2}}, \tag{14}$$

where $p_{\text{Tx}j}$ is the transmitted power for link j of the cooperative diversity system.

Using (3) on expression (14), the energy effective direct link system under consideration can be viewed as a direct link system with a larger lognormal mean. This is possible by incorporating the transmit power of the energy effective direct link system into the direct-link lognormal parameters (in Neper):

$$(\mu_{\text{eff}}, \sigma_{\text{eff}}) = (\mu_{w_{\text{eff}}} + \ln(\exp\{\mu_1 - \mu_{w_1}\} + \exp\{\mu_3 - \mu_{w_3}\}), \sigma_1), \tag{15}$$

where $\mu_{w_{\text{eff}}} = \mu_{w_1}$. It should be noted that in (15), the energy-effective direct link system has a lognormal variance σ_{eff} equivalent to the variance σ_1 of the channel between the Source-Destination link of Figure 1, thus experiencing an identical propagation scenario.

4.2. Spectral Efficiency of Protocols A and B. As described in Section 2, multiple access protocol A occupies two multiple-access slots. This inefficiency in terms of bandwidth usage can be viewed in terms of the probability that the capacity of the combiner output $C_{C(A)}$ for protocol A does not exceed a specified threshold R:

$$P_{\text{out}(A)} = P(C_{C(A)} < R) \tag{16}$$

or equivalently:

$$P_{\text{out}(A)} = P\left(\frac{1}{2}\log_2(1 + \gamma_C) < \log_2(1 + \gamma_{\text{th}\cdot\text{norm}})\right), \tag{17}$$

where γ_C is given by (4) or (5) according to the chosen diversity combining technique, and $\gamma_{\text{th}\cdot\text{norm}}$ is the rate-normalized SNR threshold. The rate-normalized SNR threshold is the SNR corresponding to spectral efficiency threshold R and is a common information-theoretic tool to facilitate system performance assessment based on a single performance metric, for example, outage probability.

Simplifying (17), the rate-normalized outage probability for protocol A is equivalent to

$$P_{\text{out}(A)} = P(\gamma_C < \gamma_{\text{th}\cdot\text{norm}}{}^2 + 2\gamma_{\text{th}\cdot\text{norm}}). \tag{18}$$

The outage probability of protocols A and B can be expressed using (10), combined with a modified threshold which contains the rate-normalized SNR threshold $\gamma_{\text{th}\cdot\text{norm}}$:

$$\gamma_{\text{th}} = \begin{cases} \gamma_{\text{th}\cdot\text{norm}}{}^{2} + 2\gamma_{\text{th}\cdot\text{norm}}, & \text{for protocol A,} \\ \gamma_{\text{th}\cdot\text{norm}}, & \text{for protocol B.} \end{cases} \tag{19}$$

Given a reference outage probability $P_{\text{out(ref)}}$, the rate-normalized SNR threshold $\gamma_{\text{th}\cdot\text{norm}}$ can be expressed as

$$\gamma_{\text{th}\cdot\text{norm}}\big|_{P_{\text{out}}=P_{\text{out(ref)}}} = \begin{cases} \sqrt{1+\gamma_{\text{th}}\big|_{P_{\text{out}}=P_{\text{out(ref)}}}} - 1, & \text{protocol A,} \\ \gamma_{\text{th}}\big|_{P_{\text{out}}=P_{\text{out(ref)}}}, & \text{protocol B.} \end{cases} \tag{20}$$

5. Numerical Results and Discussion

In this section numerical results for the performance of the SC and MRC cooperative diversity systems are presented, using numerical evaluations of the proposed expression (10).

The final expression (10) is easily calculated numerically and converges very fast due to the monotonically decreasing nature of the integrand functions. More specifically, replacing the infinite limit $-\infty$ with an appropriate negative number results in the desired numerical precision for the outage probability. The value of this number can be chosen to offer sufficient accuracy for any specific system parameters. Nevertheless, a value of -10 has been found to be sufficient in all the following numerical results and has also been proposed in similar expressions of multiple-branch receive diversity in [15].

The results are compared to the performance of the energy effective direct link system defined in Section 4, given by $P_{\text{out(eff)}} = P(\gamma_{\text{eff}} < \gamma_{\text{th}\cdot\text{norm(eff)}})$ or

$$P_{\text{out(eff)}} = 1 - \frac{1}{2}\operatorname{erfc}\left(\frac{\ln(\gamma_{\text{th}\cdot\text{norm(eff)}}) - \mu_{\text{eff}}}{\sigma_{\text{eff}}\sqrt{2}}\right) \tag{21}$$

with lognormal parameters $(\mu_{\text{eff}}, \sigma_{\text{eff}})$ given by expression (15), and $\gamma_{\text{th}\cdot\text{norm(eff)}} = \gamma_{\text{th}\cdot\text{norm}}$.

Unless otherwise noted, the values for the system parameters are assumed to be $\mu_j = 0\,\text{dB}$ and $\mu_{w_j} = 0\,\text{dB}$ for all j. In addition, in Figures 2 and 3, a model for shadowing correlation [7, 8] is used

$$\rho_{ij} = \left(\frac{\phi_T}{\phi_{ij}}\right)^{\alpha}\sqrt{\frac{L_i}{L_j}} \quad (i, j = 1, 2, 3,\ i \neq j), \tag{22}$$

where $L_j \geq L_i$, $\phi_T = 2\sin^{-1}(L_c/2L_i)$, $L_c = 20\,\text{m}$ is the decorrelation distance, and $\alpha = 0.3$ is a parameterization exponent that depends on geometrical parameters such as the size and heights of the terrain and clutter, and the antenna heights of relative nodes.

In Figure 2, the outage probability of the SC and MRC cooperative systems for both protocols A and B is plotted against the rate-normalized SNR threshold $\gamma_{\text{th}\cdot\text{norm}}$. The values of the system parameters are $\sigma_1 = \sigma_3 = 8\,\text{dB}$, $\sigma_2 = 3.4\,\text{dB}$, $L_2 = L_3 = 2000\,\text{m}$ and $\phi_{23} = 30^\circ$. The variances correspond to a Source and Relay situated above the rooftop (ART) and the Destination place below the rooftop (BRT) [7].

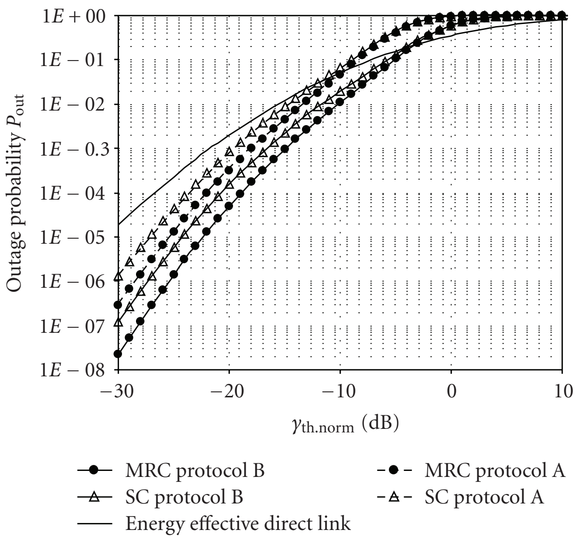

Figure 2: Outage probability versus rate-normalized threshold of the cooperative diversity system using MRC and SC under both multiple access protocols, compared to the reference direct link channel.

It can be seen that when either protocol A or protocol B is used, both the MRC and SC techniques outperform the energy effective direct link for normalized SNR thresholds $\gamma_{\text{th}\cdot\text{norm}}$ of practical interest (i.e., for thresholds smaller than 0.1 and 0.01 for protocol B and protocol A, resp.). Therefore the use of cooperative diversity with either protocol increases the system performance. Finally, Figure 2 shows that the relative performance advantage of MRC compared to SC is approximately 2 dB for outage probabilities of interest regardless of the utilized multiple-access protocol.

A useful metric for assessing the performance of various system configurations is the gain of the rate-normalized SNR threshold defined for a reference outage probability $P_{\text{out(ref)}}$, given by

$$\begin{aligned} G_{\gamma_{\text{th}\cdot\text{norm}}}[\text{dB}] = {} & \gamma_{\text{th}\cdot\text{norm}}\big|_{P_{\text{out}}=P_{\text{out(ref)}}}[\text{dB}] \\ & - \gamma_{\text{th}\cdot\text{norm(eff)}}\big|_{P_{\text{out}}=P_{\text{out(ref)}}}[\text{dB}], \end{aligned} \tag{23}$$

that is, for a given $P_{\text{out(ref)}}$, the difference between the rate-normalized threshold $\gamma_{\text{th}\cdot\text{norm}}[\text{dB}]$ of the cooperative system (given in linear form by (20)), and $\gamma_{\text{th}\cdot\text{norm(eff)}}[\text{dB}]$ is the rate-normalized threshold of the energy effective direct link system.

In Figure 3, the gain of the rate-normalized SNR threshold $G_{\gamma_{\text{th}\cdot\text{norm}}}$ in dB, as defined by (23), is depicted for MRC using protocol A and for destination D positions varying both horizontally and vertically when the source S is at position 0 and the relay R is fixed at 1 km distance. Figure 3 can be viewed as a performance map showcasing the benefit of relaying for varying destination D positions against the noncooperative case, taking into account the spectral and

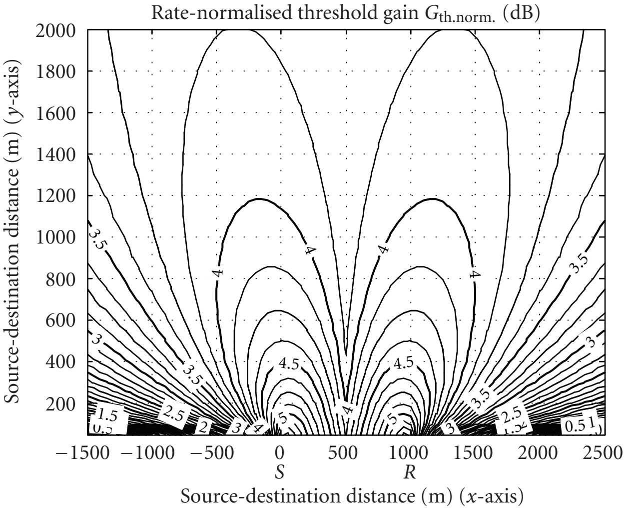

FIGURE 3: Variation of SNR gain with destination position for MRC cooperative diversity system deploying protocol A.

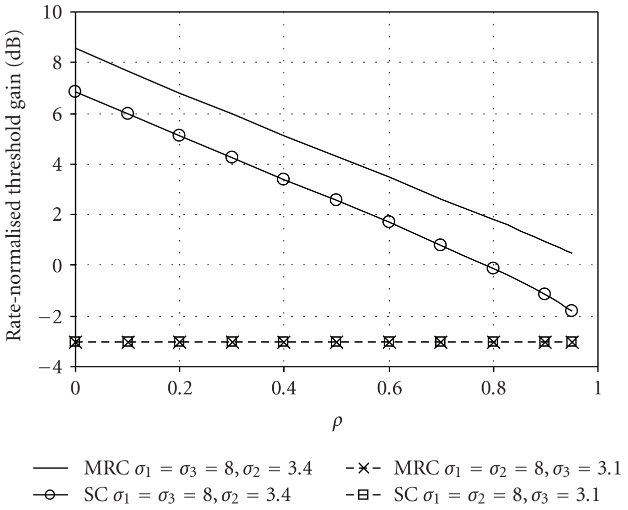

FIGURE 4: Impact of correlation on the SNR threshold gain of MRC and SC for various lognormal variances.

power efficiencies of the cooperative diversity protocol. Parameters assumed for this figure are $P_{\text{out(ref)}} = 10^{-4}$ and $(\sigma_1, \sigma_2, \sigma_3) = (8, 3.4, 8)$ dB according to [7].

Figure 3 can be viewed as a snapshot of the impact of the mechanisms of the correlation model (as given by (22)) on performance. It can be seen that there exist two optimal performance regions for the Destination, one found near the Source and one near the Relay. The symmetry of the graph around the middle of the *S-R* link can be explained by the fact that the SNRs of the direct link (γ_1) and the second hop (γ_3) have the same impact on expressions (4) and (5). It can also be observed that correlation (given by (22)) can severely impact the relative gain of cooperative diversity, which is evident by the low relative performance regions on the *x*-axis far from the *S-R* region.

Finally, Figure 4 depicts the SNR threshold gain $G_{\gamma_{\text{th-norm}}}$ against correlation for protocol B with equal correlations $\rho_{12} = \rho_{23} = \rho_{13} = \rho$. This assumption refers to a system with equilateral triangle geometry (i.e., $L_1 = L_2 = L_3 = L$) where L varies from L_c to $+\infty$ corresponding to correlations ρ from 1 to 0, respectively, according to (22).

Two sets of lognormal variances are investigated, $(\sigma_1, \sigma_2, \sigma_3) = (8, 3.4, 8)$ dB as in the previous figures and $(\sigma_1, \sigma_2, \sigma_3) = (8, 8, 3.1)$ dB, the second set corresponding to the Source at ART and both Relay and Destination at BRT [7].

Regarding the first set of variances, it can be observed that for both SC and MRC the correlation has a linear degradation effect to the performance gain. More specifically, the SC technique is inferior to an energy effective direct link for large values of correlation. On the other hand, the MRC technique outperforms noncooperative systems throughout the whole range of correlation.

Results using the second set of variances indicate that both MRC and SC fail to provide any performance gains over noncooperation. In addition, both techniques offer the same performance.

A direct comparison between the two sets of variances clearly indicates that the quality of the Source-Relay link is detrimental to the performance of the cooperative diversity system. This result shows agreement with the results in [11] for independent Rayleigh channels. More specifically, the cooperative system outperforms noncooperation only for values of σ_2 significantly smaller than σ_1. Considering the above sets of lognormal variances and the details of the physical model [7], it can be concluded that a Relay station should be placed ART, in order to guarantee good SNR with the Source γ_2 (line of sight condition) and offer significant performance gains over noncooperation.

Future work would involve the extension of our analysis to cover different cooperative diversity techniques [11, 25], further enriching our understanding of cooperative diversity in correlated lognormal channels and the significance of the channel model parameters on its performance. Another interesting task would be to generalize the presented framework to more complex system architectures, investigating the gains and efficiency of using multiple relays.

6. Conclusions

This paper presented the study of the relaying channel for the decode-and-forward case and both SC and MRC at the destination. Exact analytical expressions for the probability of outage performance have been derived which enable the evaluation of cooperative diversity for various lognormal propagation model parameters such as varying correlation and variances. Our analysis was complemented by the inclusion of the cooperative multiple-access protocol efficiency such as the spectral efficiency (expressed as the rate-normalized SNR) and the energy efficiency, compared to noncooperative direct link systems. Moreover, direct numerical evaluations of our formulas indicate that the efficiency of the cooperative protocol can have a significant impact on the performance of cooperative diversity.

Regarding lognormal parameters, it has been found that in efficient cooperative diversity systems, the lognormal variance of the Source-Relay link must always be smaller than the variance of the Source-Destination link. Therefore, it has been verified that the Source-Relay link is the most critical part of a cooperative system with regenerative relays, regardless of the multiple-access protocol. Finally, it has been found that in the operation regions where cooperation outperforms noncooperation, the relative performance gains strongly depend on correlation.

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