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Research Article

Rate-Optimized Power Allocation for DF-Relayed OFDM Transmission under Sum and Individual Power Constraints

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We consider an OFDM (orthogonal frequency division multiplexing) point-to-point transmission scheme which is enhanced by means of a relay. Symbols sent by the source during a first time slot may be (but are not necessarily) retransmitted by the relay during a second time slot. The relay is assumed to be of the DF (decode-and-forward) type. For each relayed carrier, the destination implements maximum ratio combining. Two protocols are considered. Assuming perfect CSI (channel state information), the paper investigates the power allocation problem so as to maximize the rate offered by the scheme for two types of power constraints. Both cases of sum power constraint and individual power constraints at the source and at the relay are addressed. The theoretical analysis is illustrated through numerical results for the two protocols and both types of constraints.

1. Introduction

In applications where it is difficult to locate several antennas on the same equipment, for size or cost issues, it has been proposed to mimic multiantenna configurations by means of cooperation among two or more terminals. Cooperation or relaying, also coined distributed MIMO, has gained a lot of interest recently. Cooperative diversity has been studied for instance in [1–3] (and references therein) for cellular networks.

In this paper we consider communication between a source and a destination, and the source is possibly assisted with a relay node. All the channels (source to destination, source to relay and relay to destination) are assumed to be frequency selective and in order to cope with that, OFDM modulation with proper cyclic extension is used. The relay operates in a DF mode. This mode is known to be suboptimum [4, 5]. Decode-and-forward is adopted here as a relaying strategy for its simplicity and its mathematical tractability. Two protocols (P1 and P2) are considered. Each protocol is made of two signaling periods, named time slots. The first time slot is identical for both protocols. During this period, on each carrier, the source broadcasts a symbol. This symbol (affected by the proper channel gain) is received by the destination and the relay. The relay may retransmit the same carrier-specific symbol to the destination during the second time slot. Whether the relay does it or not will be indicated by the optimization problem which is formulated and solved in this paper. The protocol P2 differs from the protocol P1 in that, in the latter, the source does not transmit during the second time slot, irrespective to whether the relay is active or not during the second time slot. For P2, on a per carrier basis, the source sends a new symbol if the relay is inactive. The reason for not having the source and the relay transmitting at the same time is to avoid the interference that would occur in this case, thus rendering the optimization problem somewhat tedious. Moreover in practice source and relay will have different carrier frequency offsets which is likely to require involved precorrection mechanisms. A scenario with interference will be investigated in the future.

For both protocols, whenever it is active, the relay uses the same carrier as the one used by the source. This is an a priori choice made here to make the optimization more tractable. It is however clear that carrier pairing between source and relay is a topic for possible further optimization of the scheme. At the destination, it is assumed that for the relayed carriers, the receiver performs maximum ratio combining of what is received from the source in the first time slot, and what is received from the relay in the second one, for each tone.

OFDM with relaying has already been investigated by some authors. In [6], the authors consider a general scenario in which users communicate by means of OFDMA (orthogonal frequency division multiple access). They propose a general framework to decide about the relaying strategy, and the allocation of power and bandwidth for the different users. The problem is solved by means of powerful optimization tools, for individual constraints on the power. In the current paper, we restrict ourselves to a single user scenario but we investigate more deeply the analytical solution and its understanding. We study power allocation to maximize the rate for both cases of sum power and individual power constraints. We also compare two different DF protocols and show the advantage of having the source also transmitting during the second time slot. In [7] the authors consider a setup which is similar to the one we address in this paper but with nonregenerative relays. In [8], the authors investigate OFDM transmission with DF relaying, and a rate maximizing power allocation for a global power constraint. They briefly investigate the power allocation for the protocol named P1 in the current paper, and a sum power constraint only. On the other hand they investigate optimized tone pairing. In [9], the authors consider OFDM with multiple decode and forward relays. They minimize the total transmission power by allocating bits and power to the individual subchannels. A selective relaying strategy is chosen. More recently, in [10] the authors also consider OFDM systems assisted by a single cooperative relay. The orthogonal half-duplex relay operates either in the selection detection-and-forward (SDF) mode or in the amplify-and-forward (AF) mode. The authors target the minimization of the transmit-power for a desired throughput and link performance. They investigate two distributed resource allocation strategies, namely flexible power ratio (FLPR) and fixed power ratio (FIPR).

The paper is organized as follows. The system under consideration is described in Section 2. The rate optimization for a sum power constraint is investigated in Section 3 for the two protocols. The cases of individual power constraints are dealt with in Section 4. Finally numerical results are discussed in Section 5.

2. System Description

We consider communication between a source and a destination, assisted with a relay node. All links are assumed to be frequency selective and this motivates the use of OFDM as a modulation technique. Assuming that the cyclic prefix is properly designed and that transmission over all links is synchronous, the scheme can be equivalently represented by a set of parallel subsystems corresponding to the different subchannels or frequencies used by the modulation and facing flat fading over each link. The block diagram associated with the system for one particular carrier (or tone) is depicted in Figure 1.

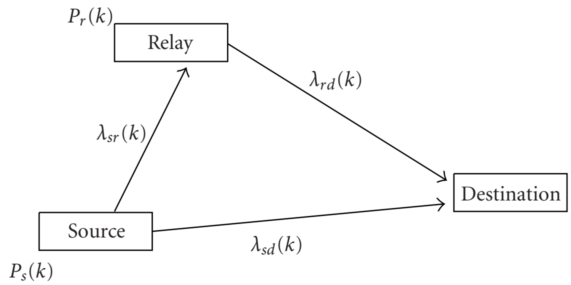

Figure 1: Structure of the system for carrier k.

During the first time slot, the source sends one modulated symbol on each carrier. During the second time slot, the relay selects some of the modulated symbols that it decodes, and retransmits them. For each relayed symbol, we constrain the relay to use the same carrier as that used by the source for the same symbol. Based on the two signalling intervals, the destination implements maximum ratio combining for the carriers with relaying. As explained earlier, we consider two protocols, called P1 and P2. In protocol P1, the carriers that are not relayed are simply not used in the second time slot (neither by the relay nor by the source). In protocol P2, a new carrier specific modulated symbol is sent by the source in the second time slot on each one of the carriers that are not used by the relay.

Let us denote by $A_s(k)$ (resp., $A_r(k)$) the amplitude of the symbol sent by the source (resp., the relay) on carrier k in the first (resp., second) time slot, and by $\lambda_{sd}(k)$ (resp., $\lambda_{rd}(k)$) the complex channel gain for tone k between source (resp., relay) and destination. The noise sample corrupting the transmission on tone k during the first time slot is $n_s(k)$, and $n_r(k)$ during the second period. These two noise samples are zero-mean circular Gaussian, white and uncorrelated with the same variance σ_n^2. Denoting by $s(k)$ the unit variance symbol transmitted over tone k, after proper maximum ratio combining at the destination, the decision variable obtained at the kth output of the FFT (Fast Fourier transform) is given by

$$\begin{aligned} r(k) = {} & A_s^2(k)\ |\lambda_{sd}(k)|^2\ s(k) + A_r^2(k)\ |\lambda_{rd}(k)|^2\ s(k) \\ & + A_s(k)\lambda_{sd}^*(k)\ n_s(k) + A_r(k)\lambda_{rd}^*(k)\ n_r(k). \end{aligned} \tag{1}$$

The associated signal to noise ratio is given by

$$\gamma(k) = \frac{P_s(k)|\lambda_{sd}(k)|^2 + P_r(k)|\lambda_{rd}(k)|^2}{\sigma_n^2}, \tag{2}$$

where we have used the following notations: $P_s(k) = A_s^2(k)$ and $P_r(k) = A_r^2(k)$.

3. Rate Optimization for a Sum Power Constraint

We first investigate the case of a sum power constraint. The techniques used in this section will be useful in solving the problem with individual power constraints. It is well known [11, 12] that the optimization with individual power constraints can be solved by reformulating it properly into an equivalent problem with a sum power constraint. All channels gains are assumed to be perfectly known for the central device computing the power allocation. The overhead associated with channel updating is not discussed further in the current paper.

We investigate the two protocols separately.

3.1. Protocol P1. For protocol P1, the rate achieved by the system for a duration of 2 OFDM symbols is given by [13]:

$$R = \sum_{k\in S_s} \log\left(1 + \frac{P_s(k)|\lambda_{sd}(k)|^2}{\sigma_n^2}\right) + \sum_{k\in S_r} \min\left\{\log\left(1 + \frac{P_s(k)|\lambda_{sr}(k)|^2}{\sigma_n^2}\right), \log\left(1 + \frac{P_s(k)|\lambda_{sd}(k)|^2 + P_r(k)|\lambda_{rd}(k)|^2}{\sigma_n^2}\right)\right\}, \tag{3}$$

where S_s is the set of carriers (or tones) receiving power at the source only, and S_r the complementary set, that is the set of carriers receiving power at both source and relay. These sets are not known in advance and must be characterized in an optimal way. In [13] the signal to noise ratio without fading was assumed to be symmetric throughout the network. Here the model is more general and notations are introduced to possibly allow different transmit powers at the source and at the relay, not only for the same carrier but also for different carriers. For a relayed carrier, assuming a decode-and-forward mode, the rate is the minimum between the rate on link $s \to d$ and the rate on the link $s \to r$. The power allocation which maximizes (3) is first investigated for a sum power constraint

$$\sum_{k=1}^{N_t} [P_s(k) + P_r(k)] \le P_t, \tag{4}$$

where P_t is the total power budget available for the source and the relay together, and N_t is the total number of carriers. Below, the objective function will be worked out in order to find criteria enabling to decide about the set S_s or S_r to which each carrier has to be assigned.

The Lagrangian for the optimization of the rate, taking into account the total power constraint and the decode-and-forward constraints, is defined by

$$\begin{aligned}\mathcal{L}_1 = &\sum_k i_k \log\left(1 + \frac{P_s(k)|\lambda_{sd}(k)|^2}{\sigma_n^2}\right) \\ &+ \sum_k (1 - i_k) \log\left(1 + \frac{P_s(k)|\lambda_{sd}(k)|^2 + P_r(k)|\lambda_{rd}(k)|^2}{\sigma_n^2}\right) \\ &- \mu\left[\sum_k i_k\, P_s(k) + \sum_k (1 - i_k)\, [P_s(k) + P_r(k)] - P_t\right] \\ &- \sum_k \rho_k\, (1 - i_k)\, \Big[P_s(k)|\lambda_{sd}(k)|^2 + P_r(k)|\lambda_{rd}(k)|^2 \\ &\qquad - P_s(k)\, |\lambda_{sr}(k)|^2\Big],\end{aligned} \tag{5}$$

where μ is the Lagrange multiplier associated with the global power constraint and ρ_k is the Lagrange multiplier associated with the decodability (perfect decode and forward) constraint on carrier k. The i_k are indicators taking values 0 or 1 and whose optimization will provide the solution for the assignment to sets $S_s(i_k = 1)$ and $S_r(i_k = 0)$.

Let us first investigate whether the decodability constraints are active or not for relayed carriers. For relayed carrier q, $i_q = 0$. If a constraint is inactive, its associated Lagrange multiplier is zero [14]. Assuming this may be the case, setting the $\rho_q = 0$ and taking the derivative of the Lagrangian with respect to the powers for a relayed carrier leads to

$$\begin{aligned}\frac{\partial R}{\partial P_s(q)} &= \left(1 + \frac{P_s(q)\ |\lambda_{sd}(q)|^2 + P_r(q)\ |\lambda_{rd}(q)|^2}{\sigma_n^2}\right)^{-1} \times \frac{|\lambda_{sd}(q)|^2}{\sigma_n^2} = \mu, \\ \frac{\partial R}{\partial P_r(q)} &= \left(1 + \frac{P_s(q)\ |\lambda_{sd}(q)|^2 + P_r(q)\ |\lambda_{rd}(q)|^2}{\sigma_n^2}\right)^{-1} \times \frac{|\lambda_{rd}(q)|^2}{\sigma_n^2} = \mu.\end{aligned} \tag{6}$$

This shows that assuming that the constraint is not saturated, the equations lead to $|\lambda_{sd}(q)|^2 = |\lambda_{rd}(q)|^2$. This imposes a constraint on the current channel state, which is almost certain not to happen. Hence, except in very marginal cases, the decode-and-forward constraint has to be saturated. This means

$$\begin{aligned}P_s(k)\ |\lambda_{sr}(k)|^2 &= P_s(k)|\lambda_{sd}(k)|^2 + P_r(k)|\lambda_{rd}(k)|^2, \\ P_s(k) &= \frac{|\lambda_{\mathrm{rd}}(k)|^2\ P_r(k)}{|\lambda_{sr}(k)|^2 - |\lambda_{sd}(k)|^2} = \alpha(k)P_r(k),\end{aligned} \tag{7}$$

where the last line defines the coefficient $\alpha(k)$.

Hence for relayed carrier k, the total amount of power $P(k)$ allocated to that carrier will be given by $P(k) = P_s(k) + P_r(k) = (1 + \alpha(k))\ P_r(k) = P_s(k)(1 + \alpha(k))/\alpha(k)$. Therefore the Lagrangian can be written as:

$$\begin{aligned}\mathcal{L}_2 = &\sum_k i_k \log\left(1 + \frac{P(k)\ |\lambda_{sd}(k)|^2}{\sigma_n^2}\right) \\ &+ \sum_k (1 - i_k)\ \log\left(1 + \frac{P(k)\ |\lambda_{sr}(k)|^2}{\sigma_n^2}\frac{\alpha(k)}{1 + \alpha(k)}\right) \\ &- \mu\left[\sum_k i_k\ P(k) + \sum_k (1 - i_k)\ P(k) - P_t\right],\end{aligned} \tag{8}$$

where for $k \in S_s$, $P(k) = P_s(k)$ and $P_r(k) = 0$, while for $k \in S_r$, $P(k) = P_s(k) + P_r(k)$ with $P_s(k) = \alpha(k)\ P_r(k)$.

The solution for the carrier assignment can be found by taking the derivatives with respect to the indicators. We have that

$$\frac{\partial R}{\partial i_q} = \log\left(\frac{1+\left(P(q)\ |\lambda_{sd}(q)|^2/\sigma_n^2\right)}{1+\left(P(q)\,|\lambda_{sr}(q)|^2/\sigma_n^2\right)(\alpha(q)/(1+\alpha(q)))}\right), \quad \begin{cases} > 0, & i_q = 1, \\ < 0, & i_q = 0. \end{cases} \tag{9}$$

It appears that when

$$|\lambda_{sd}(q)|^2 \leq \frac{\alpha(q)}{1+\alpha(q)}\ |\lambda_{sr}(q)|^2 \tag{10}$$

the carrier should have $i_q = 0$ and be allocated to set S_r. By opposition, when

$$|\lambda_{sd}(q)|^2 \geq \frac{\alpha(q)}{1+\alpha(q)}\ |\lambda_{sr}(q)|^2 \tag{11}$$

the carrier should be allocated to set S_s.

Investigating (3) it should be clear that when one has $|\lambda_{sr}(q)|^2 \leq |\lambda_{sd}(q)|^2$, because of the min, the rate obtained by allocating the carrier to the set S_s will always be higher than the rate obained if the carrier were allocated to S_r. It is worth noting that, if $|\lambda_{sr}(q)|^2 \geq |\lambda_{sd}(q)|^2$, the inequality between $(\alpha(q)/(1+\alpha(q)))\ |\lambda_{sr}(q)|^2$ and $|\lambda_{sd}(q)|^2$ is equivalent to the inequality between $|\lambda_{rd}(q)|^2$ and $|\lambda_{sd}(q)|^2$. As a matter of fact, with the definition of $\alpha(q)$,

$$\frac{\alpha(q)}{1+\alpha(q)}\ |\lambda_{sr}(q)|^2 = \frac{|\lambda_{sr}(q)|^2\ |\lambda_{rd}(q)|^2}{|\lambda_{sr}(q)|^2 - |\lambda_{sd}(q)|^2 + |\lambda_{rd}(q)|^2}. \tag{12}$$

Then,

$$\begin{aligned} &\frac{|\lambda_{sr}(q)|^2\ |\lambda_{rd}(q)|^2}{|\lambda_{sr}(q)|^2 - |\lambda_{sd}(q)|^2 + |\lambda_{rd}(q)|^2} \\ &\quad \geq |\lambda_{\mathrm{sd}}(q)|^2, \\ &|\lambda_{sr}(q)|^2\ |\lambda_{rd}(q)|^2 - |\lambda_{sr}(q)|^2\ |\lambda_{sd}(q)|^2 \\ &\quad \geq \left(|\lambda_{rd}(q)|^2 - |\lambda_{sd}(q)|^2\right)\ |\lambda_{sd}(q)|^2, \\ &|\lambda_{sr}(q)|^2\ \left(|\lambda_{rd}(q)|^2 - |\lambda_{sd}(q)|^2\right) \\ &\quad \geq |\lambda_{sd}(q)|^2\left(|\lambda_{rd}(q)|^2 - |\lambda_{sd}(q)|^2\right). \end{aligned} \tag{13}$$

The above shows that

$$|\lambda_{sd}(q)|^2 \leq \frac{\alpha(q)}{1+\alpha(q)}\ |\lambda_{sr}(q)|^2 \Longleftrightarrow |\lambda_{sd}(q)|^2 \leq |\lambda_{rd}(q)|^2. \tag{14}$$

This means that when $|\lambda_{sr}(q)|^2 \geq |\lambda_{sd}(q)|^2$, the allocation to S_s or to S_r of the carrier may be based on either comparisons in (14) because they are equivalent. And in short, to be relayed, a carrier should fulfil the following two conditions simultaneously: $|\lambda_{sr}(q)|^2 \geq |\lambda_{sd}(q)|^2$ and $|\lambda_{rd}(q)|^2 \geq |\lambda_{sd}(q)|^2$.

Now that the assignment is known, the Karush-Kuhn-Tucker (KKT) optimality conditions are such that, at the optimum, for $k \in S_s$,

$$\frac{\partial R}{\partial P(k)} = \left[P(k) + \frac{\sigma_n^2}{|\lambda_{sd}(k)|^2}\right]^{-1} = \mu \tag{15}$$

for the carriers to be served, and for carrier k such that

$$\frac{\partial R}{\partial P(k)} < \mu \tag{16}$$

the power should be set to $P(k) = 0$. For carriers $k \in S_r$ and to be served with power,

$$\frac{\partial R}{\partial P(q)} = \left[P(k) + \frac{\sigma_n^2}{|\lambda_{sr}(k)|^2}\frac{1+\alpha(k)}{\alpha(k)}\right]^{-1} = \mu \tag{17}$$

while if

$$\frac{\partial R}{\partial P(q)} < \mu \tag{18}$$

we should set $P(q) = 0$.

All these derivations basically also show that, after the assignment step, our constrained optimization problem can actually be solved thanks to the seminal waterfilling algorithm, applied to a water container built either from $\sigma_n^2/|\lambda_{sd}(k)|^2$ or from $(\sigma_n^2/|\lambda_{sr}(k)|^2)((1+\alpha(k))/\alpha(k))$. The latter values actually show that the constraint related to the DF operating mode of the relay leads to particular values to be used for the container. More specifically, for the set S_r, these values are modified values with respect to the $|\lambda_{\mathrm{sr}}(k)|^2$.

3.2. Protocol P2. In this case, the rate achieved by the system over a duration of 2 OFDM symbols is given by [13]:

$$\begin{aligned} R = {} & 2\sum_{k\in S_s}\log\left(1+\frac{P_s(k)}{2}\ \frac{|\lambda_{sd}(k)|^2}{\sigma_n^2}\right) \\ & + \sum_{k\in S_r}\min\left\{\log\left(1+\frac{P_s(k)|\lambda_{sr}(k)|^2}{\sigma_n^2}\right),\right. \\ & \left.\qquad \log\left(1+\frac{P_s(k)|\lambda_{sd}(k)|^2+P_r(k)|\lambda_{rd}(k)|^2}{\sigma_n^2}\right)\right\}, \end{aligned} \tag{19}$$

where S_s is the set of carriers (or tones) receiving power at the source only, and S_r is the complementary set, that is, carriers receiving power at both the source and the relay. We also denote by $P_s(k)$ the power allocated to a carrier at the source. If this carrier is not relayed, each protocol instant uses $P_s(k)/2$.

Analysis of this objective function shows that the DF constraint is also saturated on all carriers using the relay, like for protocol P1. Hence for a relayed carrier with an allocated power $P(q)$ the rate evolves as

$$R_r(q) = \log\left(1 + \frac{P(q)\ |\lambda_{sr}(k)|^2}{\sigma_n^2}\frac{\alpha(q)}{1+\alpha(q)}\right). \tag{20}$$

For a nonrelayed carrier q, and a total allocated power $P(q)$ (over the two instants), the rate evolves as

$$R_s(q) = \log\left[\left(1 + \frac{P(q)\ |\lambda_{sd}(q)|^2}{2\ \sigma_n^2}\right)^2\right]. \tag{21}$$

When $|\lambda_{sd}(q)|^2 > |\lambda_{sr}(q)|^2(\alpha(q)/(1+\alpha(q)))$ we have that $R_s(q) > R_r(q)$ for any value of $P(q)$. On the contrary, when $|\lambda_{sd}(q)|^2 < |\lambda_{sr}(q)|^2(\alpha(q)/(1+\alpha(q)))$ we have $R_s(q) < R_r(q)$. However this is only valid for $P(q) \leq \lambda_t$ where

$$\lambda_t = 4\sigma_n^2\ \frac{|\lambda_{sr}(q)|^2(\alpha(q)/(1+\alpha(q))) - |\lambda_{sd}(q)|^2}{|\lambda_{sd}(q)|^4}. \tag{22}$$

If $P(q) \geq \lambda_t$, even when $|\lambda_{sd}(q)|^2 < |\lambda_{sr}(q)|^2(\alpha(q)/(1+\alpha(q)))$, the power is better used by allocating the carrier to set S_s. Let us define the following Lagrangian, with a Lagrange multiplier μ associated with the global power constraint, and taking into account the saturation of the DF constraints:

$$\mathcal{L}_3 = R - \mu\left[\sum_{k\in S_s} P_s(k) + \sum_{k\in S_r} P(k) - P_t\right] \tag{23}$$

with

$$R = 2\sum_{k\in S_s}\log\left(1 + \frac{P_s(k)}{2}\ \frac{|\lambda_{sd}(k)|^2}{\sigma_n^2}\right) + \sum_{k\in S_r}\log\left(1 + \frac{P(k)\ |\lambda_{sr}(k)|^2}{\sigma_n^2}\frac{\alpha(k)}{1+\alpha(k)}\right). \tag{24}$$

Equating to 0 the derivatives of this Lagrangian with respect to the power, we get for $k \in S_s$,

$$P_s(k) = 2\left[\frac{1}{\mu} - \frac{\sigma_n^2}{|\lambda_{sd}(k)|^2}\right]_+, \tag{25}$$

where $[\cdot]_+$ stands for max $[0, .]$. Similarly, for $k \in S_r$,

$$P(k) = \left[\frac{1}{\mu} - \frac{\sigma_n^2}{|\lambda_{sr}(k)|^2}\frac{1+\alpha(k)}{\alpha(k)}\right]_+. \tag{26}$$

Again the derivations show that the constrained optimization problem can be solved using the waterfilling algorithm, applied to a water container built either from $\sigma_n^2/|\lambda_{sd}(k)|^2$ or from $(\sigma_n^2/|\lambda_{sr}(k)|^2)((1+\alpha(k))/\alpha(k))$. It is also important to note that for the nonrelayed carriers two identical values have to be used for the water container, corresponding to the two protocol instants. At the end of the waterfilling one checks if any of the relayed carriers receives an amount of power larger than the threshold given by (22). If this happens, the relayed carrier fulfilling this condition and for which the rate increase is the largest one is moved from the set S_r to the set S_s. The waterfilling is applied again. This procedure is iterated till none of the relayed carrier receives an amount of power larger than its associated threshold. In the sequel this procedure will be named the reallocation step.

4. Rate Maximization for Individual Power Constraints

This section is devoted to the power allocation which maximizes the rates under individual power constraints on the source and the relay respectively:

$$\sum_{k=1}^{N_t} P_s(k) \leq P_s, \tag{27}$$

$$\sum_{k=1}^{N_t} P_r(k) \leq P_r. \tag{28}$$

First, note that for the optimum power allocation with individual power constraints, it might happen that constraint (28) is inactive for certain values of channel parameters, but constraint (4) will always be active. In other words, at the optimum, the full available power will always be used at the source, while some of the power available at the relay may not be used. This can be explained using simple intuitive arguments. Assume a solution is found such that P_s is not fully used. The rate can be further increased by allocating the remaining source power to a carrier in set S_s or in set S_r. For the relay power, things may be different. For instance, it may even happen that all carriers are allocated to the set S_s in which case the relay does not transmit at all. One way to take this particular case into account is to perform a first optimization (called first step hereafter), trying to allocate the source power in an optimum way, not considering the constraint on the relay power. After this allocation process of the source power, one has to check whether the relay power is sufficient or not. If it is sufficient, then the optimum solution corresponds to this particular situation in which the full relay power is not used. If not, it can now be assumed that the relay power constraint is satisfied with equality at the optimum, and the full iterative method explained below should be used. Let us first describe the first step.

4.1. First Step. Again, we analyze the two protocols separately.

4.1.1. Protocol P1. The problem in this case is still to maximize (3) where it is now assumed that the constraint on P_r may not be active. This means that there is enough relay power such that for a relayed carrier k, $P_r(k)$ can always be made large enough to have

$$P_s(k)\ |\lambda_{sd}(k)|^2 + P_r(k)\ |\lambda_{rd}(k)|^2 \geq P_s(k)\ |\lambda_{sr}(k)|^2. \tag{29}$$

As discussed above, the constraint on the source power being saturated the associated Lagrange multiplier μ_s may be different from 0. Here we investigate a solution for the case where the relay power is not saturated and the related

Lagrange multiplier is then 0. The corresponding Lagrange function can be written as:

$$\mathcal{L}_4 = \sum_{k\in S_s} \log\left(1 + \frac{P_s(k)\ |\lambda_{sd}(k)|^2}{\sigma_n^2}\right) + \sum_{k\in S_r} \log\left(1 + \frac{P_s(k)\ |\lambda_{sr}(k)|^2}{\sigma_n^2}\right) - \mu_s\left[\sum_{k\in S_s} P_s(k) - P_s\right]. \tag{30}$$

In agreement with the indicator variables used above, when $|\lambda_{sd}(q)|^2 \geq |\lambda_{sr}(q)|^2$ carrier q should be allocated to set S_s. In the reverse case, it should be allocated to set S_r. Once the assignment is known, taking the derivative with respect to $P_s(q)$ with $q \in S_s$ and equating it to 0, it comes

$$\frac{\partial R}{\partial P_s(q)} = D_q(P_s) = \left[P_s(q) + \frac{\sigma_n^2}{|\lambda_{sd}(q)|^2}\right]^{-1} = \mu_s. \tag{31}$$

For a carrier q in the other set, S_r, we get

$$\frac{\partial R}{\partial P_s(q)} = D'_q(P_s) = \left[P_s(q) + \frac{\sigma_n^2}{|\lambda_{sr}(q)|^2}\right]^{-1} = \mu_s. \tag{32}$$

Hence the problem can be solved by means of a water-filling procedure, where the container is built from values $\sigma_n^2/|\lambda_{sd}(q)|^2$ in set S_s, and values $\sigma_n^2/|\lambda_{sr}(q)|^2$ in set S_r. With such an allocation procedure, the minimum power required at the relay is given by $\sum_{S_r} P_r(q)$ where $P_r(q) = P_s(q)/\alpha(q)$. If this value is below the power available at the relay, the problem is solved. This would correspond to a situation where the relay is located far away from the source, and, in a sense, not very useful for the protocol used here. Otherwise one has to investigate the situation where both power constraints are active (saturated), which is of most interest.

4.1.2. Protocol P2. The corresponding Lagrange function can be written:

$$\mathcal{L}_5 = 2\sum_{k\in S_s} \log\left(1 + \frac{P_s(k)}{2}\ \frac{|\lambda_{sd}(k)|^2}{\sigma_n^2}\right) + \sum_{k\in S_r} \log\left(1 + \frac{P_s(k)\ |\lambda_{sr}(k)|^2}{\sigma_n^2}\right) - \mu_s\left[\sum_{k\in S_s} P_s(k) - P_s\right]. \tag{33}$$

Taking the derivative with respect to $P_s(q)$ with $q \in S_s$ and equating it to 0, it comes

$$\frac{\partial R}{\partial P_s(q)} = D_q(P_s) = \left[\frac{P_s(q)}{2} + \frac{\sigma_n^2}{|\lambda_{sd}(q)|^2}\right]^{-1} = \mu_s. \tag{34}$$

For a carrier q in the other set, S_r,

$$\frac{\partial R}{\partial P_s(q)} = D'_q(P_s) = \left[P_s(q) + \frac{\sigma_n^2}{|\lambda_{sr}(q)|^2}\right]^{-1} = \mu_s. \tag{35}$$

So the conclusions are similar to those drawn for protocol P1. The problem can again be solved by means of a waterfilling procedure, where the container is built from the values $\sigma_n^2/|\lambda_{sd}(q)|^2$, and the values $\sigma_n^2/|\lambda_{sr}(q)|^2$ in set S_r. However it has to be noted that for the values related to set S_s those values have to be used twice because of the two time slots. Besides that, the reallocation procedure has to be implemented: it has to be checked whether any of the carrier allocated to set S_r receives an amount of power above a certain threshold. If this happens, carriers have to be moved from set S_r to set S_s, and the waterfilling has to be applied till this no longer happens, as explained above. The value to be used for the threshold is similar to (22), where $|\lambda_{sr}(q)|^2$ has to be used instead of $|\lambda_{sr}(q)|^2\alpha(q)/(1 + \alpha(q))$.

4.2. Second Step. A second step is needed unless the power used at the relay by the procedure described in the first step is below the available relay power. Two Lagrange multipliers, μ_s ad μ_r, now have to be used for the power contraints. One element in the direction of the solution lies in the observation [12] that the rate only depends on the products of powers and (possibly modified) channel gains. Hence allocating power P to a carrier with gain $|\lambda|^2$ provides the same rate as allocating power μP to a carrier with gain $|\lambda|^2/\mu$. Let us assume for the moment that the optimum μ_s and μ_r are known. The allocation rules proposed above to define the sets S_s and S_r should be revisited with gains modified as: $|\lambda_{sd}^{\mu}|^2 = |\lambda_{sd}|^2/\mu_s$; $|\lambda_{sr}^{\mu}|^2 = |\lambda_{sr}|^2/\mu_s$ and $|\lambda_{rd}^{\mu}|^2 = |\lambda_{rd}|^2/\mu_r$. The equivalent powers under consideration are now $P_s^{\mu}(q) = \mu_s\ P_s(q)$ and $P_r^{\mu}(q) = \mu_r\ P_r(q)$.

4.2.1. Protocol P1. Let us define the following Lagrangian:

$$\begin{aligned}\mathcal{L}_1^{\mu} = {} & \sum_k i_k \log\left(1 + \frac{P_s^{\mu}(k)\ \left|\lambda_{sd}^{\mu}(k)\right|^2}{\sigma_n^2}\right) \\ & + \sum_k (1 - i_k) \log\left(1 + \frac{P_s^{\mu}(k)\left|\lambda_{sd}^{\mu}(k)\right|^2 + P_r^{\mu}(k)\left|\lambda_{rd}^{\mu}(k)\right|^2}{\sigma_n^2}\right) \\ & - \left[\sum_k P_s^{\mu}(k) - \mu_s P_s\right] - \left[\sum_k (1 - i_k)\ P_r^{\mu}(k) - \mu_r P_r\right] \\ & - \sum_{k\in S_r} \rho_k (1 - i_k)\left[P_s^{\mu}(k)\left|\lambda_{sd}^{\mu}(k)\right|^2 + P_r^{\mu}(k)\left|\lambda_{rd}^{\mu}(k)\right|^2 \right. \\ & \left. - P_s^{\mu}(k)\ \left|\lambda_{sr}^{\mu}(k)\right|^2\right].\end{aligned} \tag{36}$$

It is interesting to compare this Lagrangian with the one given by (5). Actually they both have the same structure. The first difference is that (5) is based on P's and λ's while (36) is based on P^μ's and λ^μ's. Assuming that μ_s and μ_r are known, and thanks to the use of the modified gains and powers, the individual power constraints give rise to a single sum power constraint. The associated Lagrange multiplier now has to be equal to 1.

Based on these observations, it turns out that for fixed μ_s and μ_r all the results derived in Section 3 apply to our problem with individual power constraints, and to the powers and the gains that have been properly normalized. In particular it can be concluded that for the carriers using the relay, the decode-and-forward constraint will be saturated, leading to $P_r^\mu(q) = P_s^\mu(q)/\alpha^\mu(q)$. Hence $P_r^\mu(q)$ and $P_s^\mu(q)$ should be allocated simultaneously leading to a total power denoted by $P^\mu(q) = P_r^\mu(q) + P_s^\mu(q) = (1+\alpha^\mu(q))P_r^\mu(q) = P_s^\mu(q)(1+\alpha^\mu(q))/\alpha^\mu(q)$ where

$$\alpha^\mu(q) = \frac{\left|\lambda_{rd}^\mu(q)\right|^2}{\left|\lambda_{sr}^\mu(q)\right|^2 - \left|\lambda_{sd}^\mu(q)\right|^2} = \frac{\mu_s}{\mu_r}\alpha(q). \tag{37}$$

Considering that $P^\mu(q) = P_s^\mu(q)(1+\alpha^\mu(q))/\alpha^\mu(q)$, we also have

$$\begin{aligned} P_s^\mu(q)\ \left|\lambda_{sr}^\mu(q)\right|^2 &= P^\mu(q)\ \left|\lambda_{sr}^\mu(q)\right|^2 \frac{\alpha^\mu(q)}{1+\alpha^\mu(q)} \\ &= P^\mu(k)\ \left|\lambda_{sr}^\mu(k)\right|^2 \frac{\mu_s\ \alpha(k)}{\mu_r + \alpha(k)\mu_s} \\ &= P^\mu(q)\ \left|\lambda_{sr}^\mu(q)\right|^2 \frac{\alpha(q)}{\mu_r + \mu_s\alpha(q)}. \end{aligned} \tag{38}$$

Therefore, omitting the indicators, the Lagrangian can be rewritten as

$$\begin{aligned} \mathcal{L}_2^\mu = &\sum_{k\in S_s} \log\left(1 + \frac{P_s^\mu(k)\ \left|\lambda_{sd}^\mu(k)\right|^2}{\sigma_n^2}\right) \\ &+ \sum_{k\in S_r} \log\left(1 + P^\mu(k)\frac{\left|\lambda_{sr}^\mu(k)\right|^2}{\sigma_n^2}\frac{\mu_s\ \alpha(k)}{\mu_r+\alpha(k)\mu_s}\right) \\ &- \left[\sum_k P_s^\mu(k) + \sum_k P^\mu(k) - \mu_s P_s - \mu_r P_r\right]. \end{aligned} \tag{39}$$

Carrier q should be placed in set S_s if

$$\left|\lambda_{sd}^\mu(q)\right|^2 \geq \left|\lambda_{sr}^\mu(q)\right|^2 \frac{\alpha^\mu(q)}{1+\alpha^\mu(q)} = \left|\lambda_{sr}(q)\right|^2 \frac{\alpha(q)}{(\mu_r + \alpha(q)\ \mu_s)}. \tag{40}$$

Based on the above, and relations (14) to be adapted with λ^μ and α^μ it turns out that the selection rule when $|\lambda_{sd}^\mu(q)|^2 \leq |\lambda_{sr}^\mu(q)|^2$ amounts to choosing S_s when $|\lambda_{sd}^\mu(q)|^2 \geq |\lambda_{rd}^\mu(q)|^2$ or when

$$\frac{\left|\lambda_{sd}(q)\right|^2}{\left|\lambda_{rd}(q)\right|^2} \geq \frac{\mu_s}{\mu_r} \tag{41}$$

and vice-versa. Therefore, the allocation procedure of the carriers turns out to be equivalent to that in the sum power case, with properly modified channel gains.

There is however one important exception to this rule which is related to the particular case where the equality $|\lambda_{sd}(q)| = |\lambda_{rd}(q)|$ holds. It has been assumed previously that this particular case needs not being investigated as it is very unlikely to happen. This applies for the sum power constraint. However, in the case of individual power constraints, the procedure is now working with the modified values $\lambda^\mu(q)$ which are no longer given but depend on the Lagrange parameters μ_s and μ_r. It may happen (and has been encountered for some of the channels randomly generated) that the optimal values of these Lagrange parameters are such that the equality is exactly met on some carriers (usually at most one). This particular situation needs a few additional developments and adjustments which have been presented in [15] and will not be repeated here.

For a carrier belonging to the set S_s, the rate gain and optimality conditions are given by

$$\frac{\partial R}{\partial P_s^\mu(q)} = \left[P_s^\mu(q) + \frac{\sigma_n^2}{\left|\lambda_{sd}^\mu(q)\right|^2}\right]^{-1} = 1. \tag{42}$$

This leads to

$$P_s^\mu(q) = \left[1 - \frac{\sigma_n^2\ \mu_s}{\left|\lambda_{sd}(q)\right|^2}\right]_+. \tag{43}$$

For a carrier belonging to the set S_r, the gain and optimality conditions are given by

$$\frac{\partial R}{\partial P^\mu(q)} = \left[P^\mu(q) + \frac{\sigma_n^2}{\left|\lambda_{sr}(q)\right|^2}\frac{\mu_s\ \alpha(q)+\mu_r}{\alpha(q)}\right]^{-1} = 1. \tag{44}$$

The corresponding power allocation is given by

$$P^\mu(q) = \left[1 - \frac{\sigma_n^2}{\left|\lambda_{sr}(q)\right|^2}\frac{\mu_r + \alpha(q)\mu_s}{\alpha(q)}\right]_+. \tag{45}$$

So far, we have assumed that μ_r and μ_s were known. In fact there is a single pair (μ_s, μ_r) for which the two power constraints are simultaneously fulfilled. To find this pair, the following algorithm is proposed. The idea is to scan all possible assignments to sets S_s and S_r. For carriers such that $|\lambda_{sd}(q)|^2 \geq |\lambda_{sr}(q)|^2$, as discussed above, the carrier will be assigned to set S_s. For the other carriers, with $|\lambda_{sd}(q)|^2 \leq |\lambda_{sr}(q)|^2$, relaying may be considered. Equation (41) says that the assignment of a carrier candidate for relaying depends on the ratio $|\lambda_{rd}(q)|^2/|\lambda_{sd}(q)|^2$. By sorting the carriers candidates for relaying by decreasing order of the ratios $|\lambda_{rd}(q)|^2/|\lambda_{sd}(q)|^2$, all possible assignments can be considered. As a matter of fact, if a single carrier gets relayed it will be the first one in the sorted set. If two get relayed, it will be the first two, and so forth. Therefore, by considering all possible sets of first carriers in this sorted set, all possible assignments can be investigated. We have as many situations to consider as we have carriers being candidates to be relayed. For each situation, the assignment to sets S_s and S_r is fixed. For a fixed assignment, the optimization problem to be solved is convex. The corresponding dual problem is also convex. The dual problem can be solved by taking the derivatives of the dual objective with respect to μ_s and μ_r, and equating these derivatives to zero. The values of μ_s^* and μ_r^* solving these equations can be entered in the primal problem, and the optimum power values can be obtained. The problem is that the equations to find the optimum μ_s and μ_r are nonlinear. They can be solved for instance in an iterative manner.

These derivatives with respect to μ_s and μ_r correspond to the two power constraints that have to be fulfilled. Hence any classical method known to find the roots of a function (here the derivatives with respect to μ_s and μ_r) can be used. A typical method used is the so-called "subgradient method" where the correction to the Lagrange variables μ_s and μ_r at step i is made proportionally to the error on the constraints. Here we try to improve this classical method by using a Newton-Raphson algorithm where the first derivative of the objective function (here the objectives are the constraints) is also used. A Newton-Raphson approach is known to have quadratic convergence, and to always converge for a convex objective function. At iteration i, the power prices μ_r and μ_s are updated according to

$$\begin{bmatrix} \mu_s^{i+1} \\ \mu_r^{i+1} \end{bmatrix} = \begin{bmatrix} \mu_s^{i} \\ \mu_r^{i} \end{bmatrix} - \lambda \begin{bmatrix} \dfrac{\partial \sum_q P_s(q)}{\partial \mu_s} & \dfrac{\partial \sum_q P_s(q)}{\partial \mu_r} \\ \dfrac{\partial \sum_q P_r(q)}{\partial \mu_s} & \dfrac{\partial \sum_q P_r(q)}{\partial \mu_r} \end{bmatrix}^{-1} \times \begin{bmatrix} \sum\limits_q P_s(q) - P_s \\ \sum\limits_q P_r(q) - P_r \end{bmatrix}. \tag{46}$$

This Newton-Raphson procedure is thus to be repeated for each one of the possible assignments.

4.2.2. Protocol P2. Adapting the results of the previous subsection leads to the following Lagrangian with the modified gains and powers:

$$\begin{aligned} \mathcal{L}_3^{\mu} = {} & 2 \sum_{k \in S_s} \log\left(1 + \frac{P_s^{\mu}(k)}{2} \frac{\left|\lambda_{sd}^{\mu}(k)\right|^2}{\sigma_n^2}\right) \\ & + \sum_{k \in S_r} \log\left(1 + P^{\mu}(k) \frac{\left|\lambda_{sr}^{\mu}(k)\right|^2}{\sigma_n^2} \frac{\mu_s\, \alpha(k)}{\mu_r + \alpha(k)\mu_s}\right) \\ & - \left[\sum_k P_s^{\mu}(k) + \sum_k P^{\mu}(k) - \mu_s P_s - \mu_r P_r\right]. \end{aligned} \tag{47}$$

For a carrier belonging to the set S_s, the rate gain and optimality conditions are given by

$$\frac{\partial R}{\partial P_s^{\mu}(q)} = \left[\frac{P_s^{\mu}(q)}{2} + \frac{\sigma_n^2}{\left|\lambda_{sd}^{\mu}(q)\right|^2}\right]^{-1} = 1 \tag{48}$$

which leads to

$$P_s^{\mu}(q) = 2\left[1 - \frac{\sigma_n^2\, \mu_s}{\left|\lambda_{sd}(q)\right|^2}\right]_+. \tag{49}$$

For a carrier belonging to the set S_r, the gain and optimality conditions are given by

$$\frac{\partial R}{\partial P^{\mu}(q)} = \left[P^{\mu}(q) + \frac{\sigma_n^2}{\left|\lambda_{sr}(q)\right|^2} \frac{\mu_s\, \alpha(q) + \mu_r}{\alpha(q)}\right]^{-1} = 1. \tag{50}$$

The corresponding power allocation is given by

$$P^{\mu}(q) = \left[1 - \frac{\sigma_n^2}{\left|\lambda_{sr}(q)\right|^2} \frac{\mu_r + \alpha(q)\mu_s}{\alpha(q)}\right]_+. \tag{51}$$

Equations (49) and (51) also show that the powers are given by a waterfilling procedure with a common water level 1 or a common power constraint, and containers defined by these equations. The problem is again equivalent to the sum power case and the procedure defined for the maximisation problem in Section 3.2 can be reused. The $|\lambda_{sd}(q)|^2$ have to be replaced by $|\lambda_{sd}(q)|^2/\mu_s$, and the $|\lambda_{sr}(q)|^2\alpha(q)/(1+\alpha(q))$ by $|\lambda_{sr}(q)|^2\alpha(q)/(\mu_r + \alpha(q)\mu_s)$. The comments about the allocation of the carrier to set S_s or S_r are the same as in the case of protocol P1. Recall also that the reallocation step has to be implemented. The Newton-Raphson procedure for the updating of μ_s and μ_r is similar to that used for protocol P1.

5. Results

In order to illustrate the theoretical analysis, numerical results are provided and discussed. The number of carriers is set to $N_t = 128$. Channel impulse responses (CIR) of length 32 are generated. The taps are randomly generated from independent zero mean unit variance circular complex gaussian distributions. Hence the power delay profile is flat. All taps have a unit variance for all links. From these CIRs, FFT are computed to provide the corresponding λ_{xy} ($x \in \{s, r\}, y \in \{r, d\}$). We set $\sigma_n^2 = 1$.

For illustrative purposes, results are first presented for one particular channel realization. The power is set to $P_t = 200$ for the sum power constraint, and to $P_s = 100$ and $P_r = 100$ for the case of individual power constraints. Figure 2 shows the gains $|\lambda_{sr}(k)|^2$ (solid curve), $|\lambda_{sd}(k)|^2$ (dash-dotted), $|\lambda_{rd}(k)|^2$ (dashed) in dBW of the channels. Figure 3 shows, for protocol P1 and the sum power constraint, the result about the power allocation ($\circ$) and the possible additional split whenever relevant among source power (solid line) and relay power (dashed). The $\times$s indicate whether the relay is active ($\times$ at the top of the figure) or not ($\times$ in 0). In this case, the power used by the source is 136 and that by the relay is 64. The total rate obtained here is 275.45 bits per a duration of 2 OFDM symbols. If preferred, this rate N_b (bits) per 2 OFDM symbols may readily be converted to a spectral efficiency by computing $N_b/2N_t(1+\beta)$ (bits/sec/Hz) where β is the roll-off factor. Figure 4 reports the power allocation for protocol P2 with a sum power

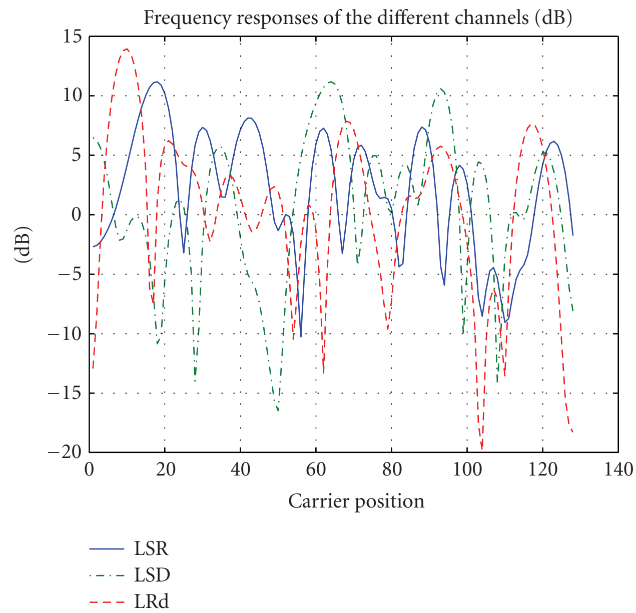

FIGURE 2: Gains $|\lambda_{sr}(k)|^2$, $|\lambda_{sd}(k)|^2$, $|\lambda_{rd}(k)|^2$ in dBW.

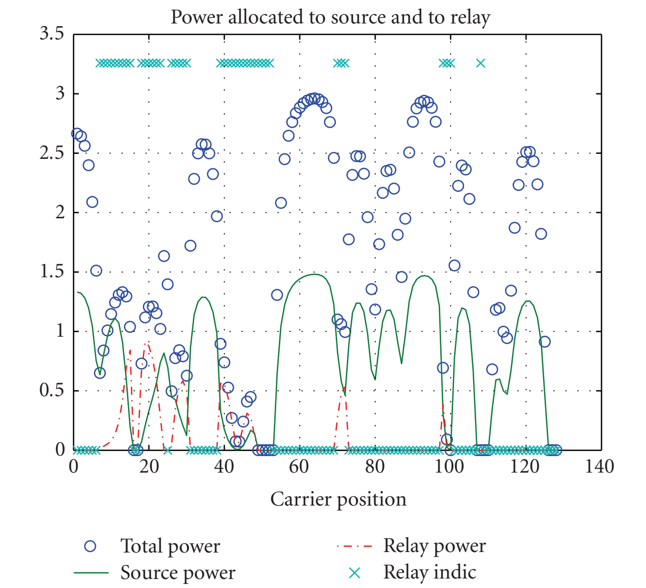

FIGURE 4: Final power allocation to source and relay in the sum power case for protocol P2.

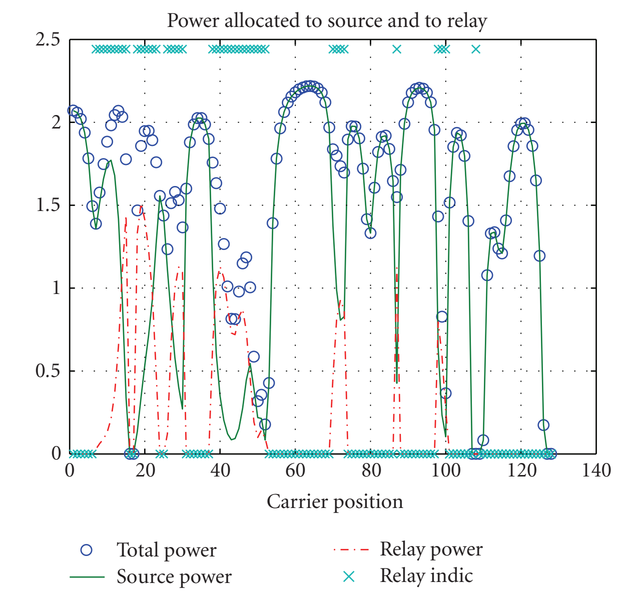

FIGURE 3: Final power allocation to source and relay in the sum power case and protocol P1.

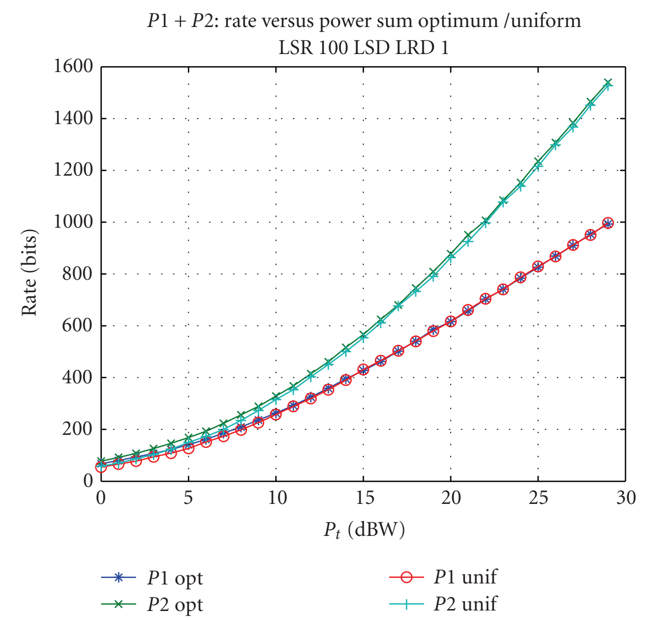

FIGURE 5: Rate versus P_t (dBW) for the two protocols and uniform and optimized power allocation for the sum power constraint. Taps of the $|\lambda_{sr}(k)|^2$ have a variance 20 dBs above those associated with the $|\lambda_{sd}(k)|^2$ and the $|\lambda_{rd}(k)|$.

constraint. Recall that for a nonrelayed carrier the amount of source power shown has to be used twice: once per time slot. The rate achieved for the particular channel realization under consideration here is 377.45 bits for a duration of 2 OFDM symbols. It is also interesting to mention that in this case, the power allocated to the source for the channel realization under consideration is 186.8 and to the relay, the remainder meaning 13.2. Compared to protocol P1, the gain is noticeable and is clearly due to the better exploitation of the second time slot.

With protocol P1 and individual power constraints, the bit rate achieved is 239.74 bits for a duration of 2 OFDM symbols. Compared to the same protocol with the sum power constraint, the observed rate loss is due to the values chosen here for the individual power constraints (100-100) which are rather different from the values devoted to the two categories of carriers by the sum power case (136-64). For individual power constraints and protocol P2, the total rate is 318 bits per 2 OFDM symbols duration. The loss incurred compared to the sum power case can be explained

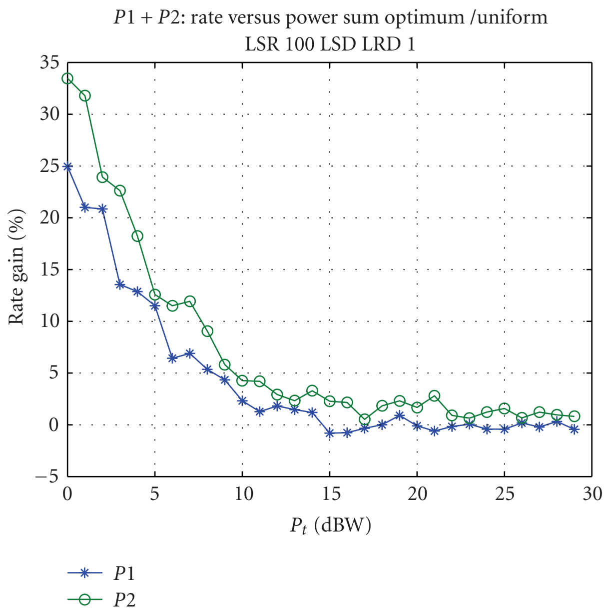

FIGURE 6: Rate gain with the optimized power allocation compared to the uniform one, versus P_t (dBW) for the two protocols and the sum power constraint. Taps of the $|\lambda_{sr}(k)|^2$ have a variance 20 dBs above those associated with the $|\lambda_{sd}(k)|^2$ and the $|\lambda_{rd}(k)|^2$.

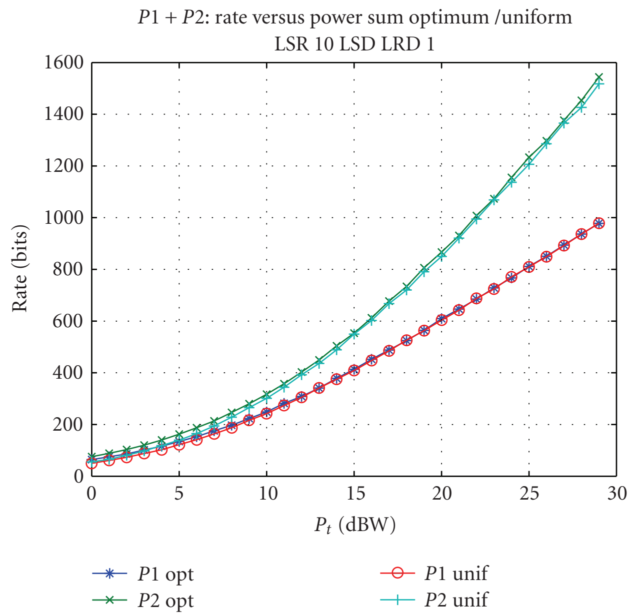

FIGURE 7: Rate versus P_t (dBW) for the two protocols and uniform and optimized power allocation for the sum power constraint. Taps of the $|\lambda_{sr}(k)|^2$ have a variance 10 dBs above those associated with the $|\lambda_{sd}(k)|^2$ and the $|\lambda_{rd}(k)|^2$.

in a manner identical to that discussed for protocol P1. And again the advantage of this protocol compared to P1 is visible.

Systematic results have also been produced for the two protocols, the sum power case, and different values of P_t. For each value of P_t the results reported are obtained by averaging over 250 channel realizations. The CIRs associated with the $|\lambda_{sr}(k)|^2$, have a variance of 20 dBs above those associated with the $|\lambda_{sd}(k)|^2$ and the $|\lambda_{rd}(k)|^2$. The results obtained with the optimized power allocation are contrasted against uniform power allocation. For protocol P1 with uniform power allocation, the carrier allocation to sets S_s and S_r is performed as in the optimized case. The power available is uniformly divided between the N_t carriers. For the carriers to be relayed, the per carrier power is further split between source and relay according to the ratio associated with the saturation of the decodability constraint (7). For protocol P2, the allocation of the carrier to set S_s or S_r is based on the comparison of $|\lambda_{sd}(q)|^2$ with $|\lambda_{sr}(q)|^2(\alpha(q)/(1+\alpha(q)))$. If N_s carriers are allocated to set S_s and $N_t - N_s$ to set S_r the total power is divided by $2N_s + N_t - N_s = N_t + N_s$ in order to take into account the use of the two time slots for the carriers in S_s. At this point the reallocation step is implemented and some carriers may be moved from S_r to S_s. For the carriers remaining in set S_r the power is further split among source and relay according to the ratio associated with the saturation of the decodability constraint (7). Figure 5 reports the rate obtained with the two protocols, and for each protocol, with the optimized and the uniform power allocation. In order to have a better understanding of the gain associated with the optimized power allocation with respect to the uniform one, the rate gain in % between uniform power allocation and optimized allocation is also reported in Figure 6. The rate results (Figure 5) clearly show the higher efficiency of protocol P2 compared to P1. This is due to the better use of the second time slot for the nonrelayed carriers. For high values of P_t and protocol P2, all carriers will be allocated to set S_s (because of the reallocation step). Because each carrier

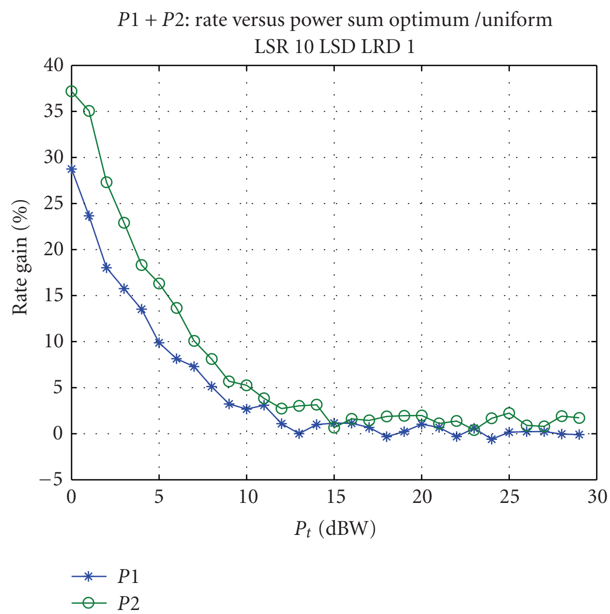

FIGURE 8: Rate gain with the optimized power allocation compared to the uniform one, versus P_t (dBW) for the two protocols and the sum power constraint. Taps of the $|\lambda_{sr}(k)|^2$ have a variance 10 dBs above those associated with the $|\lambda_{sd}(k)|^2$ and the $|\lambda_{rd}(k)|^2$.

is used over the two time slots, the rate grows with a slope $2N_t$ for P2 whereas the slope is only N_t with P1. The rate gain results (Figure 6) show how the rate gain evolves with P_t. Clearly and as expected, the benefit of the optimized power allocation decreases with P_t. For high values of P_t the optimized power allocation tends to become a uniform one.

Figures 7 and 8 report similar results for the case where the CIRs associated with the $|\lambda_{sr}(k)|^2$, have a variance of 10 dBs (instead of 20 dBs) above those associated with the $|\lambda_{sd}(k)|^2$ and the $|\lambda_{rd}(k)|^2$. These results lead to similar conclusions.

6. Conclusion

In this paper we considered an OFDM point to point link enhanced by means of a relay. When a symbol is received by the relay on a certain tone, it may be relayed to the destination on the same tone. We have investigated the problem of power allocation to the source and to the relay in order to maximize the rate of the whole transmission for a global power constraint and for individual power constraints at the source and at the relay. Two protocols have been considered; the second one makes a better use of the second time slot whenever the relay is inactive. It is assumed that the destination implements MRC between what is received from the source and what is received from the relay, for each tone. The DF operating mode of the relay puts an additional constraint on the design. The carrier classification (whether a carrier has to be relayed or not) has first been investigated for the sum power case. The power allocation problem has been shown to be of the waterfilling type with a specific construction of the container. It has also been shown how the problem for individual constraints could be recast into an equivalent waterfilling problem by using the technique of equivalent powers and equivalent channels. It has been proposed to find iteratively the two Lagrange multipliers in this second case by means of a Newton-Raphson method implemented for each possible carrier assignment. Numerical results have been provided to illustrate the schemes and have shown the advantage of protocol P2 over protocol P1.

Future work will be devoted to the cases of multiple relays, nonperfect channel state information and a refinement of the power allocation across the two signaling intervals. Moreover, coding will also be included in the transmission scheme and taken into account. Besides these topics, the (peak to average power ratio) PAPR might also be a problem to be considered. PAPR issues are well known with OFDM transmission and are likely to be impacted by power allocation.

Acknowledgments

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