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Research Article

Coordinated Transmission of Interference Mitigation and Power Allocation in Two-User Two-Hop MIMO Relay Systems

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This paper considers coordinated transmission for interference mitigation and power allocation in a correlated two-user two-hop multi-input multioutput (MIMO) relay system. The proposed transmission scheme utilizes statistical channel state information (CSI) (e.g., transmit correlation) to minimize the cochannel interference (CCI) caused by the relay. To this end, it is shown that the CCI can be represented in terms of the eigenvalues and the angle difference between the eigenvectors of the transmit correlation matrix of the intended and CCI channel, and that the condition minimizing the CCI can be characterized by the correlation amplitude and the phase difference between the transmit correlation coefficients of these channels. Then, a coordinated user-scheduling strategy is designed with the use of eigen-beamforming to minimize the CCI in an average sense. The transmit power of the base station and relay is optimized under separate power constraint. Analytic and numerical results show that the proposed scheme can maximize the achievable sum rate when the principal eigenvectors of the transmit correlation matrix of the intended and the CCI channel are orthogonal to each other, yielding a sum rate performance comparable to that of the minimum mean-square error-based coordinated beamforming which uses instantaneous CSI.

1. Introduction

The use of wireless relays with multiple antennas, so-called multi-input multioutput (MIMO) relay, has received a great attention due to its potential for significant improvement of link capacity and cell coverage in cellular networks [1–8]. Previous works mainly focused on the capacity bound of point-to-point MIMO relay channels from the information-theoretic aspects [9, 10]. Recently, research focus has moved into point-to-multipoint MIMO relay channels, so-called multiuser MIMO relay channel [11]. When relay users and direct-link users coexist in multiuser MIMO relay channels, it is of an important concern to develop a MIMO relay transmission strategy that mitigates cochannel interference (CCI) caused by the relay [4]. However, the capacity region of the multiuser MIMO relay channel is still an open issue in interference-limited environments [12–14]. It is a complicated design issue to determine how to simultaneously schedule relay users and direct-link users, and how to co-optimize the transmit beamforming and the power of MIMO relays without major CCI effect [15].

In a multiuser MIMO cellular system, recent works have shown that the CCI caused by adjacent base stations (BSs) can be mitigated with the use of coordinated beamforming (CBF) [15–17]. They derived a closed-form expression for the minimum mean square error (MMSE) and zero-forcing (ZF)-based CBF [15] in terms of maximizing the signal-to-interference plus noise ratio (SINR) [18, 19]. However, they did not consider the user scheduling together and may require a large feedback signaling overhead and computational complexity due to the use of instantaneous channel state information (CSI) at every frame [20]. Moreover, it can suffer from so-called channel mismatch problem due to the time delay for the exchange of instantaneous CSI via a backbone network among the BSs [21, 22]. As a consequence, previous works for multiuser MIMO cellular systems may not directly be applied to multiuser MIMO relay systems.

The problem associated with the use of instantaneous CSI can be alleviated with the use of statistical characteristics (e.g., correlation information) of MIMO channel [23–27]. Measurement-based researches show that the MIMO channel is often correlated in real environments [26, 27]. It is shown that the channel correlation is associated with the scattering characteristics, antenna spacing, Doppler spread, and angle of departure (AoD) or arrival (AoA) [27]. In spite of efforts on the capacity of correlated single/multiuser MIMO channels [28–33], the capacity of correlated multiuser MIMO relay channels remains unknown. This motivates the design of an interference-mitigation strategy with the use of channel correlation information in a multiuser MIMO relay system.

Along with the interference mitigation, it is also of an interesting topic to determine how to allocate the transmit power of the relay since the capacity of MIMO relay channel is determined by the minimum capacity of multihops [1–4]. It was shown that the minimum capacity can be improved by adaptively allocating the transmit power according to the channel condition of multihops [34–36]. However, it may need to consider the effect of CCI in a multiuser MIMO relay system [4, 11]. Nevertheless, to authors' best knowledge, few works have considered combined use of CCI mitigation and power allocation in a multiuser MIMO relay system.

In this paper, we consider coordinated transmission for the CCI mitigation and power allocation in a correlated two-user two-hop MIMO relay system, where one is served through a relay and the other is served directly from the BS. (We consider a simple scenario of two hops, which is most attractive in practice because the system complexity and transmission latency are strongly related to the number of hops [4].) The proposed coordinated transmission scheme utilizes the transmit correlation to minimize the CCI in an average sense. To this end, it is shown that the CCI can be expressed in terms of the eigenvalues and the angle difference between the eigenvectors of the transmit correlation matrix of the intended and the CCI channel, and that the condition minimizing the CCI can be characterized by the correlation amplitude and the phase difference between the transmit correlation coefficients of these channels. Using the statistics of the CCI, a coordinated user-scheduling criterion is designed with the use of eigen-beamforming to minimize the CCI in an average sense. The transmit power is optimized for rate balancing between the two hops, yielding less interference while maximizing the minimum rate of the two hops. It is also shown that the proposed scheme can maximize the achievable sum rate when the principal eigenvectors of the transmit correlation matrix of the intended and the interfered user are orthogonal to each other, and that the maximum sum rate approaches to that of the MMSE-CBF while requiring less complexity and feedback signaling overhead.

The rest of this paper is organized as follows. Section 2 describes a correlated two-user two-hop MIMO relay system in consideration. In Section 3, previous works are briefly discussed for ease of description. Section 4 proposes a coordinated transmission strategy for the CCI mitigation and power allocation, and analyzes its performance in terms of the achievable sum rate. Section 5 verifies the analytic results by computer simulation. Finally, conclusions are given in Section 6.

Notation. Throughout this paper, lower- and uppercase boldface are used to denote a column vector $\mathbf{a}$ and matrix $\mathbf{A}$, respectively; $\mathbf{A}^T$ and $\mathbf{A}^*$, respectively, indicate the transpose and conjugate transpose of $\mathbf{A}$; $\|\mathbf{a}\|$ denotes the Euclidean norm of $\mathbf{a}$; $\mathrm{tr}(\mathbf{A})$ and $\det(\mathbf{A})$, respectively, denote the trace and the determinant of $\mathbf{A}$; $\mathbf{I}_M$ is an $(M \times M)$ identity matrix; $E\{\cdot\}$ stands for the expectation operator.

2. System Model

Consider the downlink of a two-user two-hop MIMO relay system with the use of half-duplex decode and forward (DF) protocol as shown in Figure 1, where the BS transmits the signal to the relay during the first time slot, and the relay decodes/re-encodes and transmits it to user i during the second time slot. We refer this link to the relay link. Simultaneously, the BS transmits the signal to user k during the second time slot through the frequency band allocated to user i, which is referred to the access link. We assume that only a single data stream is transmitted to users. We also assume that the BS and the relay, respectively, transmit the signal using M_1 and M_2 antennas with own amplifiers [35], and that each user has a single receive antenna (primarily for the simplicity of description).

Let $\mathbf{H}_1^{(1)} = \begin{bmatrix} \mathbf{h}_1^{(1)} & \cdots & \mathbf{h}_{M_2}^{(1)} \end{bmatrix}$ be an $(M_1 \times M_2)$ channel matrix from the BS to the relay and $\mathbf{h}_i^{(2)}$ be an $(M_2 \times 1)$ channel vector from the relay to user i, where the superscript (n) indicates the time slot index. Then, during the first time slot, the received signal at the relay can be represented as

$$\mathbf{y}_1^{(1)} = \sqrt{P_{\mathrm{BS}}\Gamma_1^{(1)}}\mathbf{H}_1^{(1)*}\mathbf{x}_1^{(1)} + \mathbf{n}_1^{(1)}, \tag{1}$$

where P_{BS} is the transmit power of the BS, $\Gamma_1^{(1)}$ denotes the large-scale fading coefficient of the first hop, $\mathbf{x}_1^{(1)} = \mathbf{w}_1^{(1)} s_1^{(1)}$, and $\mathbf{n}_1^{(1)}$ is an $(M_2 \times 1)$ additive white Gaussian noise (AWGN) vector with covariance matrix $\sigma_1^2 \mathbf{I}_{M_2}$. Here, $\mathbf{w}_1^{(1)}$ and $s_1^{(1)}$ denote an $(M_1 \times 1)$ transmit beamforming vector with unit norm and the transmit data, respectively. During the second time slot, the received signal of user i and k can be, respectively, represented as

$$\begin{aligned} y_i^{(2)} &= \sqrt{P_{\mathrm{RS}}\Gamma_i^{(2)}}\mathbf{h}_i^{(2)*}\mathbf{x}_i^{(2)} + n_i^{(2)}, \\ y_k^{(2)} &= \sqrt{P_{\mathrm{BS}}\Gamma_k^{(2)}}\mathbf{h}_k^{(2)*}\mathbf{x}_k^{(2)} + \sqrt{P_{\mathrm{RS}}\Gamma_{k,\mathrm{CCI}}^{(2)}}\mathbf{h}_{k,\mathrm{CCI}}^{(2)*}\mathbf{x}_i^{(2)} + n_k^{(2)}, \end{aligned} \tag{2}$$

where P_{RS} is the transmit power of the relay, $\mathbf{h}_{k,\mathrm{CCI}}^{(2)}$ denotes an $(M_2 \times 1)$ CCI channel vector from the relay to user k, and $n_i^{(2)}$ and $n_k^{(2)}$ denote zero-mean AWGN with variance σ_i^2 and σ_k^2, respectively.

When $\mathbf{H}_1^{(1)}$ experiences spatially correlated Rayleigh fading, it can be represented as [37]

$$\mathbf{H}_1^{(1)} = \mathbf{R}_1^{(1)/2}\widetilde{\mathbf{H}}_1^{(1)}\mathbf{G}_1^{(1)/2}, \tag{3}$$

where $\widetilde{\mathbf{H}}_1^{(1)}$ denotes an uncorrelated channel matrix whose elements are independent and identically distributed (i.i.d.)

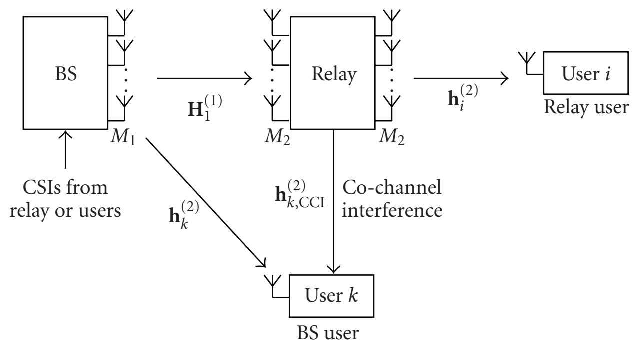


Figure 1: Modeling of a two-user two-hop MIMO relay system.

zero-mean complex Gaussian random variables with unit variance; $\mathbf{R}_1^{(1)/2}$ and $\mathbf{G}_1^{(1)/2}$, respectively, denote the square root of the transmit and receive correlation matrix (i.e., $\mathbf{R}_1^{(1)} = \mathbf{R}_1^{(1)/2}\mathbf{R}_1^{(1)/2*}$ and $\mathbf{G}_1^{(1)} = \mathbf{G}_1^{(1)/2}\mathbf{G}_1^{(1)/2*}$) defined by [38] (to derive the statistical characteristics of the CCI and analyze its geometrical meaning in following sections, we consider the exponential decayed correlation model, which is physically reasonable in the sense that the correlation decreases as the distance between antennas increases [24, 25])

$$\mathbf{R}_1^{(1)} = \begin{bmatrix} 1 & \rho_1^{(1)} & \cdots & \rho_1^{(1)^{M_1-1}} \\ \rho_1^{(1)*} & 1 & \cdots & \rho_1^{(1)^{M_1-2}} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_1^{(1)*^{M_1-1}} & \rho_1^{(1)*^{M_1-2}} & \cdots & 1 \end{bmatrix},$$
$$\mathbf{G}_1^{(1)} = \begin{bmatrix} 1 & \varphi_1^{(1)} & \cdots & \varphi_1^{(1)^{M_2-1}} \\ \varphi_1^{(1)*} & 1 & \cdots & \varphi_1^{(1)^{M_2-2}} \\ \vdots & \vdots & \ddots & \vdots \\ \varphi_1^{(1)*^{M_2-1}} & \varphi_1^{(1)*^{M_2-2}} & \cdots & 1 \end{bmatrix}, \quad (4)$$

where $\rho_1^{(1)}(= \alpha_1^{(1)} e^{j\theta_1^{(1)}})$ and $\varphi_1^{(1)}(= \beta_1^{(1)} e^{j\varpi_1^{(1)}})$ are the complex-valued transmit and receive correlation coefficient, respectively. Here, $\alpha_1^{(1)}, \beta_1^{(1)}$ $(0 \leq \alpha_1^{(1)}, \beta_1^{(1)} \leq 1)$ and $\theta_1^{(1)}, \varpi_1^{(1)} (-\pi \leq \theta_1^{(1)}, \varpi_1^{(1)} \leq \pi)$ denote those amplitude and phase, respectively. Similarly, $\mathbf{h}_i^{(2)}$ can be represented as

$$\mathbf{h}_i^{(2)} = \mathbf{R}_i^{(2)/2}\tilde{\mathbf{h}}_i^{(2)} = \begin{bmatrix} 1 & \rho_i^{(2)} & \cdots & \rho_i^{(2)^{M_2-1}} \\ \rho_i^{(2)*} & 1 & \cdots & \rho_i^{(2)^{M_2-2}} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_i^{(2)*^{M_2-1}} & \rho_i^{(2)*^{M_2-2}} & \cdots & 1 \end{bmatrix}^{1/2} \tilde{\mathbf{h}}_i^{(2)}, \quad (5)$$

where $\tilde{\mathbf{h}}_i^{(2)}$ denotes an uncorrelated channel vector whose elements are i.i.d. zero-mean complex Gaussian random variables with unit variance and $\rho_i^{(2)}(= \alpha_i^{(2)} e^{j\theta_i^{(2)}})$. Here, $\alpha_i^{(2)}$ $(0 \leq \alpha_i^{(2)} \leq 1)$ and $\theta_i^{(2)}$ $(-\pi \leq \theta_i^{(2)} \leq \pi)$. Since $\mathbf{R}_i^{(2)}$ is a positive semidefinite Hermitian matrix, it can be decomposed as [39]

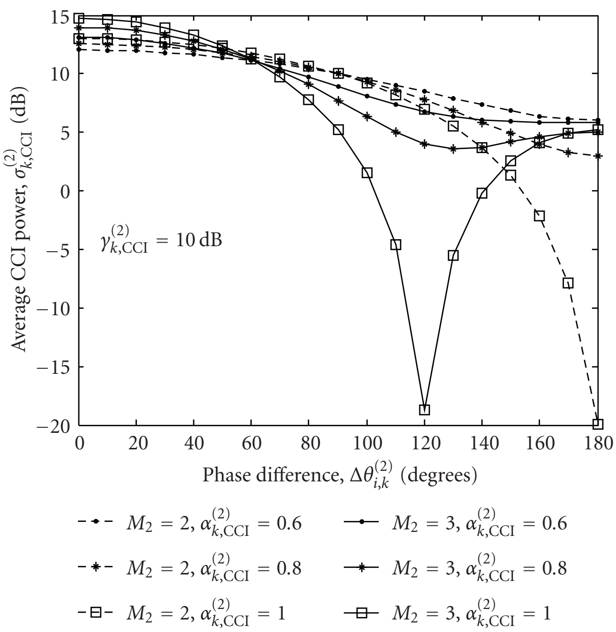


Figure 2: Average CCI power according to $\Delta\theta_{i,k}^{(2)}$.

$$\mathbf{R}_i^{(2)} = \mathbf{U}_i^{(2)}\mathbf{\Lambda}_i^{(2)}\mathbf{U}_i^{(2)*}, \quad (6)$$

where $\mathbf{U}_i^{(2)} = \begin{bmatrix} \mathbf{u}_{i,1}^{(2)} & \cdots & \mathbf{u}_{i,M_2}^{(2)} \end{bmatrix}$ is an $(M_2 \times M_2)$ unitary matrix whose columns are the normalized eigenvectors of $\mathbf{R}_i^{(2)}$, and $\mathbf{\Lambda}_i^{(2)}$ is an $(M_2 \times M_2)$ diagonal matrix whose diagonal elements are $\{\lambda_{i,1}^{(2)}, \ldots, \lambda_{i,M_2}^{(2)}\}$, where $\lambda_{i,1}^{(2)} \geq \cdots \geq \lambda_{i,M_2}^{(2)} \geq 0$. We define $\mathbf{u}_{i,\max}^{(2)}$ by the principal eigenvector corresponding to the largest eigenvalue $\lambda_{i,1}^{(2)}$ of $\mathbf{R}_i^{(2)}$ (i.e., $\mathbf{u}_{i,1}^{(2)} = \mathbf{u}_{i,\max}^{(2)}$).

3. Previous Works

In this section, we briefly review relevant results which motivate the design of interference mitigation scheme for ease of description.

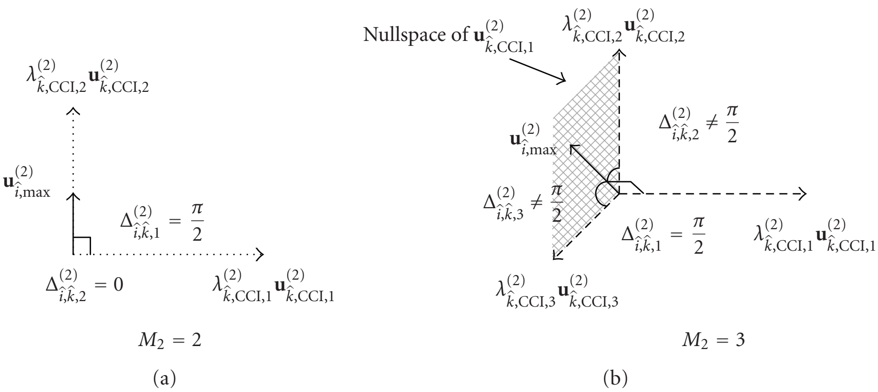


Figure 3: Design concept of the coordinated eigen-beamforming with geometrical interpretation.

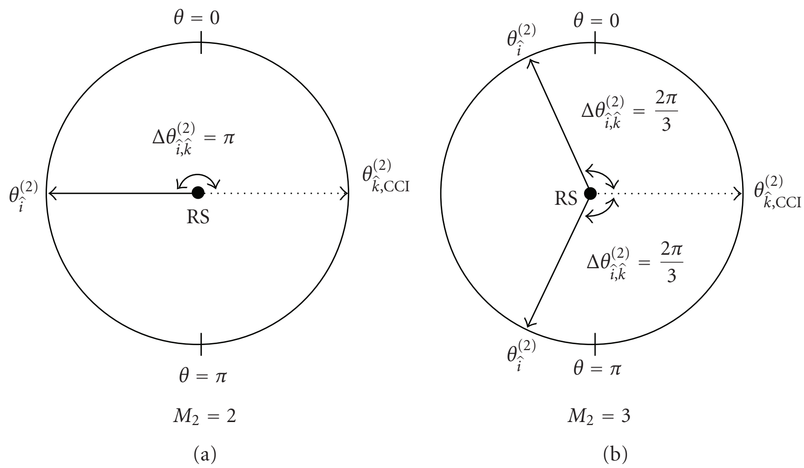


Figure 4: Design concept of the coordinated eigen-beamforming with physical interpretation.

3.1. Eigen-Beamforming (Eig.BF). With the transmit correlation information, the transmitter can determine the eigen-beamforming vector by the principal eigenvector of the transmit correlation matrix (i.e., $\mathbf{w}_k^{(2)} = \mathbf{u}_{k,\max}^{(2)}$), yielding an achievable rate bounded as [28]

$$R_{k,\mathrm{Eig.BF}}^{(2)} \le \log_2\left(1 + \overline{\gamma}_k^{(2)} \lambda_{k,\max}^{(2)}\right), \tag{7}$$

where $\overline{\gamma}_k^{(2)} (= P_{\mathrm{BS}}\Gamma_k^{(2)}/\sigma_k^2)$ denotes the average SNR of user k. However, this scheme may experience the performance degradation in interference-limited environments.

3.2. MMSE Interference Aware-Coordinated Beamforming (MMSE-CBF). The MMSE-CBF designed for a two-cell two-user MIMO cellular system can be applied to a two-user two-hop MIMO relay system where users are equipped with multiple receive antennas [15]. (Unlike our system model, the MMSE-CBF assumes that each user has multiple receive antennas since it jointly optimizes the transmit beamforming and receive combining vector to maximize the SINR [15]. However, the design concept is applicable even when each user has a single receive antenna.) In this case, the SINR of user k can be represented as

$$\begin{aligned}\mathrm{SINR}_k^{(2)} &= \frac{\overline{\gamma}_k^{(2)} \mathbf{f}_k^{(2)*} \mathbf{H}_k^{(2)*} \mathbf{w}_k^{(2)} \mathbf{w}_k^{(2)*} \mathbf{H}_k^{(2)} \mathbf{f}_k^{(2)}}{1 + \overline{\gamma}_{k,\mathrm{CCI}}^{(2)} \mathbf{f}_k^{(2)*} \mathbf{H}_{k,\mathrm{CCI}}^{(2)*} \mathbf{w}_i^{(2)} \mathbf{w}_i^{(2)*} \mathbf{H}_{k,\mathrm{CCI}}^{(2)} \mathbf{f}_k^{(2)}} \\ &= \frac{\overline{\gamma}_k^{(2)} \mathbf{f}_k^{(2)*} \mathbf{H}_k^{(2)*} \mathbf{w}_k^{(2)} \mathbf{w}_k^{(2)*} \mathbf{H}_k^{(2)} \mathbf{f}_k^{(2)}}{\mathbf{f}_k^{(2)*}\left(\mathbf{I}_N + \overline{\gamma}_{k,\mathrm{CCI}}^{(2)} \mathbf{H}_{k,\mathrm{CCI}}^{(2)*} \mathbf{w}_i^{(2)} \mathbf{w}_i^{(2)*} \mathbf{H}_{k,\mathrm{CCI}}^{(2)}\right) \mathbf{f}_k^{(2)}},\end{aligned} \tag{8}$$

where N is the number of receive antennas of each user, $\mathbf{f}_k^{(2)}$ denotes an $(N \times 1)$ receive combining vector of user k, and $\mathbf{H}_k^{(2)}$ and $\mathbf{H}_{k,\mathrm{CCI}}^{(2)}$ denote an $(M_1 \times N)$ channel matrix from the BS to user k and an $(M_2 \times N)$ CCI channel matrix from the relay to user k, respectively. Equation (8) is known as a Rayleigh quotient [40] and is maximized when $\mathbf{f}_k^{(2)}$ (before the normalization) is given by [41]

$$\mathbf{f}_k^{(2)} = \left(\mathbf{I}_N + \overline{\gamma}_{k,\mathrm{CCI}}^{(2)} \mathbf{H}_{k,\mathrm{CCI}}^{(2)*} \mathbf{w}_i^{(2)} \mathbf{w}_i^{(2)*} \mathbf{H}_{k,\mathrm{CCI}}^{(2)}\right)^{-1} \mathbf{H}_k^{(2)*} \mathbf{w}_k^{(2)}, \tag{9}$$

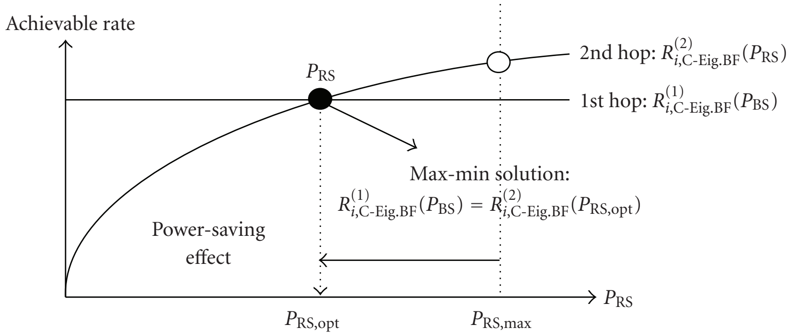


Figure 5: Design concept of the proposed power allocation scheme.

which is the principal singular vector of $\overline{\gamma}_k^{(2)}\mathbf{H}_k^{(2)*}\mathbf{w}_k^{(2)}\mathbf{w}_k^{(2)*} \times\mathbf{H}_k^{(2)}(\mathbf{I}_N + \overline{\gamma}_{k,\text{CCI}}^{(2)}\mathbf{H}_{k,\text{CCI}}^{(2)*}\mathbf{w}_i^{(2)}\mathbf{w}_i^{(2)*}\mathbf{H}_{k,\text{CCI}}^{(2)})^{-1}$. The corresponding SINR and the achievable rate of user k are, respectively, given by

$$\begin{aligned}
&\text{SINR}_k^{(2)} \\
&\quad = \overline{\gamma}_k^{(2)}\mathbf{w}_k^{(2)*}\mathbf{H}_k^{(2)} \\
&\qquad \times \left(\mathbf{I}_N + \overline{\gamma}_{k,\text{CCI}}^{(2)}\mathbf{H}_{k,\text{CCI}}^{(2)*}\mathbf{w}_i^{(2)}\mathbf{w}_i^{(2)*}\mathbf{H}_{k,\text{CCI}}^{(2)}\right)^{-1}\mathbf{H}_k^{(2)*}\mathbf{w}_k^{(2)}, \\
&R_{k,\text{MMSE-CBF}}^{(2)} = \log_2\left(1 + \text{SINR}_k^{(2)}\right).
\end{aligned} \tag{10}$$

Given the receive combing vector $\mathbf{f}_k^{(2)}$, the transmit beamforming vector can be determined by

$$\mathbf{w}_k^{(2)} = \nu_{\max}\left\{\left(\mathbf{H}_{i,\text{CCI}}^{(2)}\mathbf{H}_{i,\text{CCI}}^{(2)*} + \frac{1}{\overline{\gamma}_{i,\text{CCI}}^{(2)}}\mathbf{I}_{M_1}\right)^{-1}\mathbf{H}_k^{(2)}\mathbf{H}_k^{(2)*}\right\}, \tag{11}$$

where $\nu_{\max}\{\mathbf{A}\}$ is the principal singular vector of matrix $\mathbf{A}$ and $\mathbf{H}_{i,\text{CCI}}^{(2)}$ denotes an $(M_1 \times N)$ CCI channel matrix from the BS to user i. However, the channel gain of $\mathbf{H}_{i,\text{CCI}}^{(2)}$ is very small due to large path loss and shadowing effect [2]. The transmit beamforming and receive combining vector for user i can be determined in a similar manner.

4. Proposed Coordinated Transmission

In this section, we design a coordinated transmission strategy for CCI mitigation and power allocation in a correlated two-user two-hop MIMO relay system. To this end, we first investigate the statistical characteristics of the CCI, and then describe the design concept for the CCI mitigation and power allocation. Finally, we derive the performance of the proposed scheme in terms of the achievable sum rate.

4.1. Statistical Characteristics of Cochannel Interference. In a spatially correlated channel, the channel gain is statistically concentrated on a few eigen-dimensions of the transmit correlation matrix [29]. In this case, the eigen-beamforming is known as the optimum beamforming strategy when a single data stream is transmitted to the user [28]. When the eigen-beamforming is applied to the relay (i.e., $\mathbf{w}_i^{(2)} = \mathbf{u}_{i,\max}^{(2)}$), the CCI power from the relay can be represented in terms of the eigenvalue $\lambda_{k,\text{CCI},m}^{(2)}$ and the inner-product between $\mathbf{u}_{i,\max}^{(2)}$ and $\mathbf{u}_{k,\text{CCI},m}^{(2)}$, where $\lambda_{k,\text{CCI},m}^{(2)}$ and $\mathbf{u}_{k,\text{CCI},m}^{(2)}$ denote the mth eigenvalue and eigenvector of $\mathbf{R}_{k,\text{CCI}}^{(2)}$. The following theorem provides the main result of this subsection.

Theorem 1. *The average CCI from the relay with the use of eigen-beamforming can be represented as*

$$\overline{\sigma}_{k,CCI}^{(2)} = \overline{\gamma}_{k,CCI}^{(2)}\sum_{m=1}^{M_2}\lambda_{k,CCI,m}^{(2)}\cos^2\Delta_{i,k,m}^{(2)}, \tag{12}$$

where $\Delta_{i,k,m}^{(2)}(= \measuredangle(\mathbf{u}_{i,\max}^{(2)}, \mathbf{u}_{k,CCI,m}^{(2)}))$ denotes the angle difference between $\mathbf{u}_{i,\max}^{(2)}$ and $\mathbf{u}_{k,CCI,m}^{(2)}$.

Proof. When $\mathbf{w}_k^{(2)} = \mathbf{u}_{k,\max}^{(2)}$ and $\mathbf{w}_i^{(2)} = \mathbf{u}_{i,\max}^{(2)}$, the instantaneous SINR of user k can be represented as

$$\text{SINR}_k^{(2)} = \frac{\overline{\gamma}_k^{(2)}\left|\mathbf{h}_k^{(2)*}\mathbf{u}_{k,\max}^{(2)}\right|^2}{1 + \overline{\gamma}_{k,\text{CCI}}^{(2)}\left|\mathbf{h}_{k,\text{CCI}}^{(2)*}\mathbf{u}_{i,\max}^{(2)}\right|^2}. \tag{13}$$

It can easily be shown that the average CCI can be represented as

$$\overline{\sigma}_{k,\text{CCI}}^{(2)} = \overline{\gamma}_{k,\text{CCI}}^{(2)}E\left\{\left|\mathbf{h}_{k,\text{CCI}}^{(2)*}\mathbf{u}_{i,\max}^{(2)}\right|^2\right\}. \tag{14}$$

Since $\mathbf{h}_{k,\text{CCI}}^{(2)} = \mathbf{R}_{k,\text{CCI}}^{(2)/2}\widetilde{\mathbf{h}}_{k,\text{CCI}}^{(2)}$ and $E\{\widetilde{\mathbf{h}}_{k,\text{CCI}}^{(2)*}\mathbf{A}\widetilde{\mathbf{h}}_{k,\text{CCI}}^{(2)}\} = \text{tr}(\mathbf{A})$ [40], (14) can be rewritten as

$$\overline{\sigma}_{k,\text{CCI}}^{(2)} = \overline{\gamma}_{k,\text{CCI}}^{(2)}\,\text{tr}\left(\mathbf{R}_{k,\text{CCI}}^{(2)}\mathbf{u}_{i,\max}^{(2)}\mathbf{u}_{i,\max}^{(2)*}\right). \tag{15}$$

It can be shown from $\mathbf{R}_{k,\text{CCI}}^{(2)} = \mathbf{U}_{k,\text{CCI}}^{(2)}\mathbf{\Lambda}_{k,\text{CCI}}^{(2)}\mathbf{U}_{k,\text{CCI}}^{(2)*}$ and

$\operatorname{tr}(\mathbf{AB}) = \operatorname{tr}(\mathbf{BA})$ [40] that

$$\overline{\sigma}_{k,\mathrm{CCI}}^{(2)} = \overline{\gamma}_{k,\mathrm{CCI}}^{(2)} \operatorname{tr}\left(\mathbf{\Lambda}_{k,\mathrm{CCI}}^{(2)} \mathbf{U}_{k,\mathrm{CCI}}^{(2)*} \mathbf{u}_{i,\max}^{(2)} \mathbf{u}_{i,\max}^{(2)*} \mathbf{U}_{k,\mathrm{CCI}}^{(2)}\right) = \overline{\gamma}_{k,\mathrm{CCI}}^{(2)} \sum_{m=1}^{M_2} \lambda_{k,\mathrm{CCI},m}^{(2)} \left| \mathbf{u}_{i,\max}^{(2)*} \mathbf{u}_{k,\mathrm{CCI},m}^{(2)} \right|^2. \tag{16}$$

Since

$$|\mathbf{u}_{i,\max}^{(2)*} \mathbf{u}_{k,\mathrm{CCI},m}^{(2)}| = \|\mathbf{u}_{i,\max}^{(2)}\| \cdot \|\mathbf{u}_{k,\mathrm{CCI},m}^{(2)}\| \cos \measuredangle(\mathbf{u}_{i,\max}^{(2)}, \mathbf{u}_{k,\mathrm{CCI},m}^{(2)}) \tag{17}$$

and $\|\mathbf{u}_{i,\max}^{(2)}\| = \|\mathbf{u}_{k,\mathrm{CCI},m}^{(2)}\| = 1$, thus, we can get

$$\overline{\sigma}_{k,\mathrm{CCI}}^{(2)} = \overline{\gamma}_{k,\mathrm{CCI}}^{(2)} \sum_{m=1}^{M_2} \lambda_{k,\mathrm{CCI},m}^{(2)} \cos^2 \measuredangle\left(\mathbf{u}_{i,\max}^{(2)}, \mathbf{u}_{k,\mathrm{CCI},m}^{(2)}\right). \tag{18}$$

This completes the proof of the theorem. □

It can be seen that the CCI is associated with the eigenvalue $\lambda_{k,\mathrm{CCI},m}^{(2)}$ and the angle difference $\Delta_{i,k,m}^{(2)}$ between $\mathbf{u}_{i,\max}^{(2)}$ and $\mathbf{u}_{k,\mathrm{CCI},m}^{(2)}$ for $m = 1, 2, \ldots, M_2$. This implies that the CCI can be controlled by adjusting $\lambda_{k,\mathrm{CCI},m}^{(2)}$ and $\Delta_{i,k,m}^{(2)}$ in a statistical manner. In a highly correlated channel, the CCI can be minimized (or maximized) by making $\Delta_{i,k,\max}^{(2)} (= \Delta_{i,k,1}^{(2)}) = \pi/2$ (or $\Delta_{i,k,\max}^{(2)} = 0$) since $\lambda_{k,\mathrm{CCI},m}^{(2)} = 0$ for $m = 2, \ldots, M_2$. However, in a weakly correlated channel, even when $\Delta_{i,k,\max}^{(2)} = \pi/2$, the CCI cannot perfectly be eliminated since $\mathbf{u}_{i,\max}^{(2)}$ and $\mathbf{u}_{k,\mathrm{CCI},m}^{(2)}$ are not orthogonal to each other and $\lambda_{k,\mathrm{CCI},m}^{(2)}$ is not zero for $m = 2, \ldots, M_2$ (i.e., $\Delta_{i,k,m}^{(2)} \neq \pi/2$ and $\lambda_{k,\mathrm{CCI},m}^{(2)} \neq 0$ for $m = 2, \ldots, M_2$).

Corollary 2. *When $M_2 = 2$, the average CCI can be simplified to*

$$\overline{\sigma}_{k,CCI}^{(2)} \Big|_{M_2=2} = \overline{\gamma}_{k,CCI}^{(2)} \left(1 + \alpha_{k,CCI}^{(2)} \cos \Delta\theta_{i,k}^{(2)}\right), \tag{19}$$

where $\overline{\sigma}_{k,CCI}^{(2)}|_{M_2=2}$ denotes the CCI power when $M_2 = 2$ and $\Delta\theta_{i,k}^{(2)} (= |\theta_i^{(2)} - \theta_{k,CCI}^{(2)}|)$ denotes the phase difference between the transmit correlation coefficients of $\mathbf{h}_i^{(2)}$ and $\mathbf{h}_{k,CCI}^{(2)}$.

Proof. Since the eigenvalues and the corresponding eigenvectors of $\mathbf{R}_{k,\mathrm{CCI}}^{(2)}$ for $M_2 = 2$ can be, respectively, represented as [8]

$$\mathbf{\Lambda}_{k,\mathrm{CCI}}^{(2)} = \begin{bmatrix} \lambda_{k,\mathrm{CCI},1}^{(2)} & 0 \\ 0 & \lambda_{k,\mathrm{CCI},2}^{(2)} \end{bmatrix} = \begin{bmatrix} 1 + \alpha_{k,\mathrm{CCI}}^{(2)} & 0 \\ 0 & 1 - \alpha_{k,\mathrm{CCI}}^{(2)} \end{bmatrix},$$

$$\mathbf{U}_{k,\mathrm{CCI}}^{(2)} = \left[\mathbf{u}_{k,\mathrm{CCI},1}^{(2)} \quad \mathbf{u}_{k,\mathrm{CCI},2}^{(2)}\right] = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ e^{-j\theta_{k,\mathrm{CCI}}^{(2)}} & -e^{-j\theta_{k,\mathrm{CCI}}^{(2)}} \end{bmatrix}, \tag{20}$$

(12) can be rewritten as

$$\overline{\sigma}_{k,\mathrm{CCI}}^{(2)} \Big|_{M_2=2} = \overline{\gamma}_{k,\mathrm{CCI}}^{(2)} \left[\left(1 + \alpha_{k,\mathrm{CCI}}^{(2)}\right) \left| \frac{1}{2} + \frac{e^{j|\theta_i^{(2)} - \theta_{k,\mathrm{CCI}}^{(2)}|}}{2} \right|^2 + \left(1 - \alpha_{k,\mathrm{CCI}}^{(2)}\right) \left| \frac{1}{2} - \frac{e^{j|\theta_i^{(2)} - \theta_{k,\mathrm{CCI}}^{(2)}|}}{2} \right|^2 \right]. \tag{21}$$

Since $e^{ja} = \cos a + j \sin a$ for a real-valued a, thus, we can get

$$\overline{\sigma}_{k,\mathrm{CCI}}^{(2)} \Big|_{M_2=2} = \overline{\gamma}_{k,\mathrm{CCI}}^{(2)} \left(1 + \alpha_{k,\mathrm{CCI}}^{(2)} \cos \left|\theta_i^{(2)} - \theta_{k,\mathrm{CCI}}^{(2)}\right|\right). \tag{22}$$

This completes the proof of the corollary. □

It can be seen from Corollary 2 that the CCI depends on the correlation amplitude $\alpha_{k,\mathrm{CCI}}^{(2)}$ and the phase difference $\Delta\theta_{i,k}^{(2)}$ between $\rho_i^{(2)}$ and $\rho_{k,\mathrm{CCI}}^{(2)}$. In a highly correlated channel (i.e., $\alpha_{k,\mathrm{CCI}}^{(2)} = 1$), the CCI can be minimized (or maximized) when $\Delta\theta_{i,k}^{(2)} = \pi$ (or $\Delta\theta_{i,k}^{(2)} = 0$). This implies that the principal eigenvector $\mathbf{u}_{i,\max}^{(2)}$ and $\mathbf{u}_{k,\mathrm{CCI},\max}^{(2)}$ are orthogonal (or parallel) to each other when $\Delta\theta_{i,k}^{(2)} = \pi$ (or $\Delta\theta_{i,k}^{(2)} = 0$) [33].

Corollary 3. *When $M_2 = 3$, the average CCI can be represented as*

$$\overline{\sigma}_{k,CCI}^{(2)} \Big|_{M_2=3} = \overline{\gamma}_{k,CCI}^{(2)} \sum_{m=1}^{3} \lambda_{k,CCI,m}^{(2)} \times \left| 1 + A_{i,\max,2}^{(2)} A_{k,m,2}^{(2)} e^{j\Delta\theta_{i,k}^{(2)}} + A_{i,\max,3}^{(2)} A_{k,m,3}^{(2)} e^{j2\Delta\theta_{i,k}^{(2)}} \right|^2, \tag{23}$$

where

$$A_{i,\max,2}^{(2)} = \frac{\alpha_i^{(2)2} - \left(1 - \lambda_{i,\max}^{(2)}\right)}{\lambda_{i,\max}^{(2)} \alpha_i^{(2)}},$$
$$A_{i,\max,3}^{(2)} = \frac{\left(1 - \lambda_{i,\max}^{(2)}\right)^2 - \alpha_i^{(2)2}}{\lambda_{i,\max}^{(2)} \alpha_i^{(2)2}},$$
$$A_{k,m,2}^{(2)} = \frac{\alpha_{k,CCI}^{(2)2} - \left(1 - \lambda_{k,CCI,m}^{(2)}\right)}{\lambda_{k,CCI,m}^{(2)} \alpha_{k,CCI}^{(2)}},$$
$$A_{k,m,3}^{(2)} = \frac{\left(1 - \lambda_{k,CCI,m}^{(2)}\right)^2 - \alpha_{k,CCI}^{(2)2}}{\lambda_{k,CCI,m}^{(2)} \alpha_{k,CCI}^{(2)2}}. \tag{24}$$

Proof. The eigenvalues and the corresponding eigenvectors of $\mathbf{R}_{k,\mathrm{CCI}}^{(2)}$ for $M_2 = 3$ can be, respectively, represented as (refer to Appendix A)

$$\boldsymbol{\Lambda}_{k,\mathrm{CCI}}^{(2)} = \begin{bmatrix} 1+\dfrac{\alpha_{k,\mathrm{CCI}}^{(2)2}+\sqrt{\alpha_{k,\mathrm{CCI}}^{(2)4}+8\alpha_{k,\mathrm{CCI}}^{(2)2}}}{2} & 0 & 0 \\ 0 & 1-\alpha_{k,\mathrm{CCI}}^{(2)2} & 0 \\ 0 & 0 & 1+\dfrac{\alpha_{k,\mathrm{CCI}}^{(2)2}-\sqrt{\alpha_{k,\mathrm{CCI}}^{(2)4}+8\alpha_{k,\mathrm{CCI}}^{(2)2}}}{2} \end{bmatrix},$$
$$\mathbf{U}_{k,\mathrm{CCI}}^{(2)} = \begin{bmatrix} 1 & 1 & 1 \\ A_{k,1,2}^{(2)}e^{-j\theta_{k,\mathrm{CCI}}^{(2)}} & A_{k,2,2}^{(2)}e^{-j\theta_{k,\mathrm{CCI}}^{(2)}} & A_{k,3,2}^{(2)}e^{-j\theta_{k,\mathrm{CCI}}^{(2)}} \\ A_{k,1,3}^{(2)}e^{-j2\theta_{k,\mathrm{CCI}}^{(2)}} & A_{k,2,3}^{(2)}e^{-j2\theta_{k,\mathrm{CCI}}^{(2)}} & A_{k,3,3}^{(2)}e^{-j2\theta_{k,\mathrm{CCI}}^{(2)}} \end{bmatrix}. \quad (25)$$

It can be shown from (25) that (12) can be represented as

$$\begin{aligned} &\overline{\sigma}_{k,\mathrm{CCI}}^{(2)}\Big|_{M_2=3} \\ &= \overline{\gamma}_{k,\mathrm{CCI}}^{(2)} \sum_{m=1}^{3} \lambda_{k,\mathrm{CCI},m}^{(2)} \\ &\quad \times \left| \begin{bmatrix} 1 & A_{i,\max,2}^{(2)}e^{j\theta_i^{(2)}} & A_{i,\max,3}^{(2)}e^{j2\theta_i^{(2)}} \end{bmatrix} \begin{bmatrix} 1 \\ A_{k,m,2}^{(2)}e^{-j\theta_{k,\mathrm{CCI}}^{(2)}} \\ A_{k,m,3}^{(2)}e^{-j2\theta_{k,\mathrm{CCI}}^{(2)}} \end{bmatrix} \right|^2 \\ &= \gamma_{k,\mathrm{CCI}}^{(2)} \sum_{m=1}^{3} \lambda_{k,\mathrm{CCI},m}^{(2)} \\ &\quad \times \left| 1 + A_{i,\max,2}^{(2)}A_{k,m,2}^{(2)}e^{j|\theta_i^{(2)}-\theta_{k,\mathrm{CCI}}^{(2)}|} + A_{i,\max,3}^{(2)}A_{k,m,3}^{(2)}e^{j2|\theta_i^{(2)}-\theta_{k,\mathrm{CCI}}^{(2)}|} \right|^2. \end{aligned} \quad (26)$$

This completes the proof of the corollary. □

Like Corollary 2, when $M_2 = 3$, the CCI depends on $\alpha_{k,\mathrm{CCI}}^{(2)}$ and $\Delta\theta_{i,k}^{(2)}$ between $\rho_i^{(2)}$ and $\rho_{k,\mathrm{CCI}}^{(2)}$. However, the phase difference minimizing the CCI depends on the number of antennas. Unlike $M_2 = 2$, the CCI can be minimized when $\Delta\theta_{i,k}^{(2)} = 2\pi/3$ for $M_2 = 3$. This implies that the principal eigenvector $\mathbf{u}_{i,\max}^{(2)}$ and $\mathbf{u}_{k,\mathrm{CCI},\max}^{(2)}$ are orthogonal to each other when $\Delta\theta_{i,k}^{(2)} = 2\pi/3$ (refer to Appendix B for the proof). Figure 2 depicts the average CCI power according to $\Delta\theta_{i,k}^{(2)}$ when $\overline{\gamma}_{k,\mathrm{CCI}}^{(2)} = 10$ dB, $\alpha_{k,\mathrm{CCI}}^{(2)} = 1.0, 0.8, 0.6$, and $M_2 = 2, 3$. It can be seen that the CCI is minimized at $\Delta\theta_{i,k}^{(2)} = \pi$ (or $\Delta\theta_{i,k}^{(2)} = 2\pi/3$) when $M_2 = 2$ (or $M_2 = 3$) as $\alpha_{k,\mathrm{CCI}}^{(2)} \to 1$.

4.2. Design Concept of the Proposed Coordinated Eigen-Beamforming. From Theorem 1, we can deduce the design concept of the interference mitigation to minimize the CCI in a statistical manner when the BS's users and relay's users coexist. The main challenge is to determine how to simultaneously schedule the BS's user and the relay's user without major CCI effect. Based on Theorem 1, the reasonable solution is to select a pair of users whose principal eigenvectors $\mathbf{u}_{i,\max}^{(2)}$ and $\mathbf{u}_{k,\mathrm{CCI},\max}^{(2)}$ are orthogonal to each other, that is,

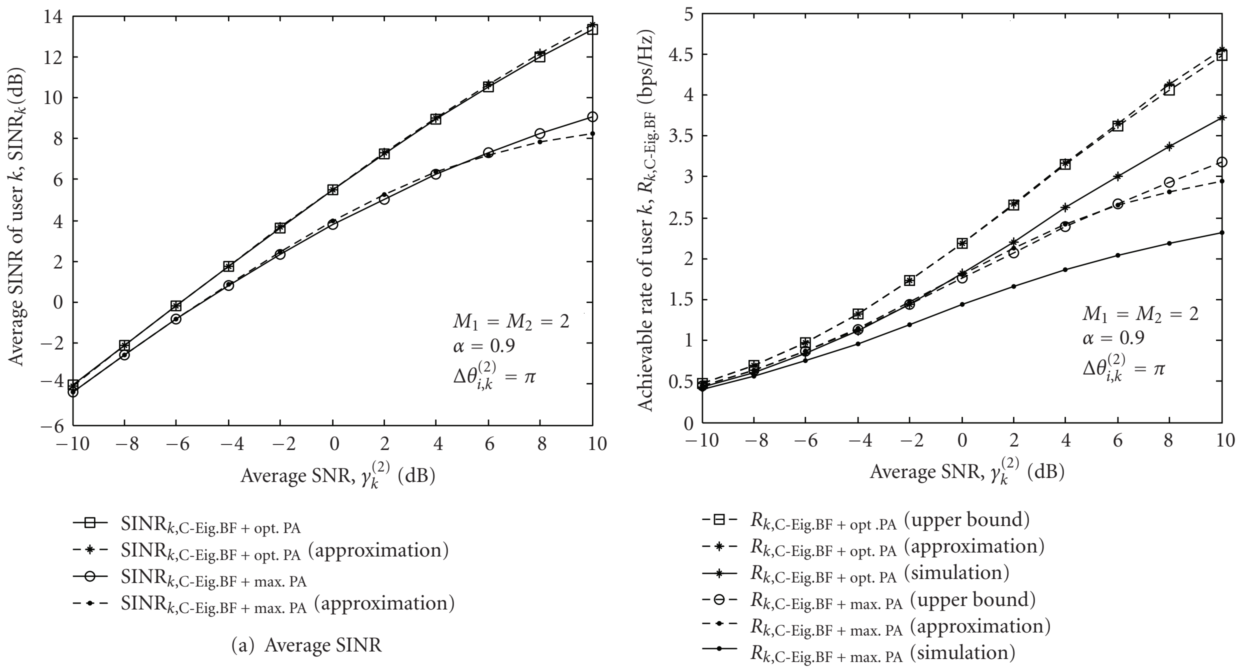
$$\Delta_{\hat{i},\hat{k},\max}^{(2)} = \measuredangle\left(\mathbf{u}_{\hat{i},\max}^{(2)}, \mathbf{u}_{\hat{k},\mathrm{CCI},\max}^{(2)}\right) = \frac{\pi}{2}, \quad (27)$$

where $\hat{i}$ and $\hat{k}$ denote the indices of selected users. We refer this criterion to the coordinated eigen-beamforming. Figure 3 illustrates the design concept of the coordinated eigen-beamforming with geometrical interpretation. It can be shown that the principal eigenvector $\mathbf{u}_{\hat{i},\max}^{(2)}$ is orthogonal to $\mathbf{u}_{\hat{k},\mathrm{CCI},\max}^{(2)}$ regardless of M_2. However, the CCI power has a different behavior according to M_2. When $M_2 = 2$, a pair of users satisfying (27) can uniquely be determined since $\mathbf{u}_{\hat{k},\mathrm{CCI},2}^{(2)}$ is only orthogonal to the principal eigenvector $\mathbf{u}_{\hat{k},\mathrm{CCI},\max}^{(2)}$. This implies that the direction of $\mathbf{u}_{\hat{i},\max}^{(2)}$ should be equal to that of $\mathbf{u}_{\hat{k},\mathrm{CCI},2}^{(2)}$ (i.e., $\mathbf{u}_{\hat{i},\max}^{(2)} || \mathbf{u}_{\hat{k},\mathrm{CCI},2}^{(2)}$), where $||$ denotes a parallel relationship of two complex vectors. It can be inferred that the CCI remains as much as $\lambda_{\hat{k},\mathrm{CCI},2}^{(2)}$ when $M_2 = 2$. On the other hand, when $M_2 = 3$, there may exist many pairs of users since the null-space of $\mathbf{u}_{\hat{k},\mathrm{CCI},\max}^{(2)}$ is two-dimensional. This implies that arbitrary vectors on the null-space are always orthogonal to $\mathbf{u}_{\hat{k},\mathrm{CCI},\max}^{(2)}$. In this case, it is desirable for the relay to select a user with the principal eigenvector minimally inducing the CCI power. This is because $\mathbf{u}_{\hat{i},\max}^{(2)}$ and $\mathbf{u}_{\hat{k},\mathrm{CCI},m}^{(2)}$ are not orthogonal to each other for $m = 2, 3$, and the CCI power remains as $\sum_{m=2}^{3} \lambda_{\hat{k},\mathrm{CCI},m}^{(2)} \cos^2 \Delta_{\hat{i},\hat{k},m}^{(2)}$, which varies according to the user $\hat{i}$ selected by the relay.

The proposed coordinated eigen-beamforming can be fully characterized by the phase difference between $\rho_{\hat{i}}^{(2)}$ and $\rho_{\hat{k},\mathrm{CCI}}^{(2)}$. From Corollaries 2 and 3, it can be inferred that the condition minimizing the CCI for M_2 antennas can be determined as

$$\Delta\theta_{\hat{i},\hat{k}}^{(2)} = \frac{2\pi}{M_2}. \quad (28)$$

(Although we do not consider the case for $M_2 \geq 4$ due to intricate manipulation for the calculation of eigenvalues and



(b) Achievable rate

FIGURE 6: Performance of user k with the proposed scheme according to $\overline{\gamma}_k^{(2)}$.

eigenvectors of $\mathbf{R}_{k,\text{CCI}}^{(2)}$, (28) can straightforwardly be verified by the manner described in Appendices A and B.)

Figure 4 illustrates physical meaning of the coordinated eigen-beamforming. It can be seen that the CCI is minimized when the phases of $\rho_{\hat{i}}^{(2)}$ and $\rho_{\hat{k},\text{CCI}}^{(2)}$ are scattered as much as possible.

4.3. Performance Analysis

Theorem 4. *The average SINR of user $\hat{k}$ with the use of the proposed coordinated eigen-beamforming can be approximated as*

$$\overline{SINR}_{\hat{k},approx}^{(2)} = \frac{\overline{\gamma}_{\hat{k}}^{(2)}\lambda_{\hat{k},\max}^{(2)}}{1+\overline{\sigma}_{\hat{k},CCI}^{(2)}} + \frac{\overline{\gamma}_{\hat{k}}^{(2)}\lambda_{\hat{k},\max}^{(2)}\overline{\sigma}_{\hat{k},CCI}^{(2)2}}{\left(1+\overline{\sigma}_{\hat{k},CCI}^{(2)}\right)^3}, \qquad (29)$$

where $\overline{SINR}_{\hat{k},approx}^{(2)}$ is the approximated average SINR of user $\hat{k}$.

Proof. It can be shown from (13) that

$$\overline{\text{SINR}}_{\hat{k}}^{(2)} = E\left\{\frac{\overline{\gamma}_{\hat{k}}^{(2)}\left|\mathbf{h}_{\hat{k}}^{(2)*}\mathbf{u}_{\hat{k},\max}^{(2)}\right|^2}{1+\overline{\gamma}_{\hat{k},\text{CCI}}^{(2)}\left|\mathbf{h}_{\hat{k},\text{CCI}}^{(2)*}\mathbf{u}_{\hat{i},\max}^{(2)}\right|^2}\right\}. \qquad (30)$$

Letting $x = \overline{\gamma}_{\hat{k}}^{(2)}|\mathbf{h}_{\hat{k}}^{(2)*}\mathbf{u}_{\hat{k},\max}^{(2)}|^2$ and $y = 1 + \overline{\gamma}_{\hat{k},\text{CCI}}^{(2)}|\mathbf{h}_{\hat{k},\text{CCI}}^{(2)*}\mathbf{u}_{\hat{i},\max}^{(2)}|^2$, it can be shown from multivariate Taylor series expansion [42] that (30) can be approximated as

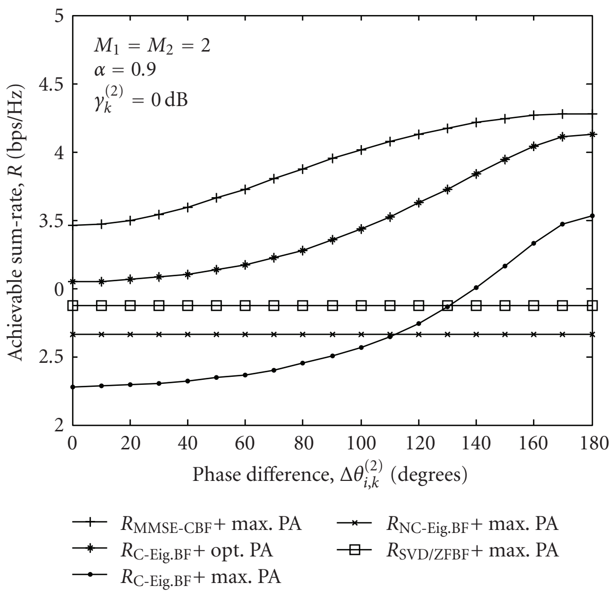
$$\begin{aligned}\overline{\text{SINR}}_{\hat{k}}^{(2)} &= E\left\{\frac{x}{y}\right\} \\ &\approx \frac{E\{x\}}{E\{y\}} - \frac{\text{cov}[x,y]}{E\{y\}^2} + \frac{E\{x\}}{E\{y\}^3}\text{var}[y] \triangleq \overline{\text{SINR}}_{\hat{k},\text{approx}}^{(2)},\end{aligned} \qquad (31)$$

where var$[y]$ denotes the variance of y and cov$[x,y]$ denotes the covariance of x and y. Since x and y are independent random variables (i.e., cov$[x,y] = 0$), (31) can further be simplified to

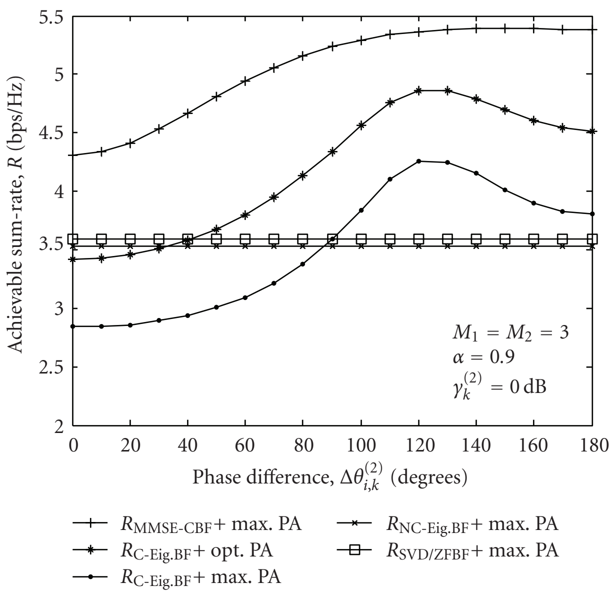
$$\overline{\text{SINR}}_{\hat{k},\text{approx}}^{(2)} = \frac{E\{x\}}{E\{y\}} + \frac{E\{x\}}{E\{y\}^3}\text{var}[y]. \qquad (32)$$

It can be shown from $E\{x\} = \overline{\gamma}_{\hat{k}}^{(2)}\lambda_{\hat{k},\max}^{(2)}$ and $E\{y\} = 1+\overline{\sigma}_{\hat{k},\text{CCI}}^{(2)}$ that

$$\begin{aligned}\overline{\text{SINR}}_{\hat{k},\text{approx}}^{(2)} &= \frac{\overline{\gamma}_{\hat{k}}^{(2)}\lambda_{\hat{k},\max}^{(2)}}{1+\overline{\sigma}_{\hat{k},\text{CCI}}^{(2)}} + \frac{\overline{\gamma}_{\hat{k}}^{(2)}\lambda_{\hat{k},\max}^{(2)}}{\left(1+\overline{\sigma}_{\hat{k},\text{CCI}}^{(2)}\right)^3} \\ &\quad\times E\left\{\overline{\gamma}_{\hat{k},\text{CCI}}^{(2)2}\left|\mathbf{h}_{\hat{k},\text{CCI}}^{(2)*}\mathbf{u}_{\hat{i},\max}^{(2)}\right|^4 - \overline{\sigma}_{\hat{k},\text{CCI}}^{(2)2}\right\}.\end{aligned} \qquad (33)$$



(a) $M_2 = 2$



(b) $M_2 = 3$

Figure 7: Performance comparison according to $\Delta\theta_{i,k}^{(2)}$.

It can be shown after some mathematical manipulation that [41]

$$\overline{\text{SINR}}_{\hat{k},\text{approx}}^{(2)} = \frac{\overline{\gamma}_{\hat{k}}^{(2)}\lambda_{\hat{k},\max}^{(2)}}{1+\overline{\sigma}_{\hat{k},\text{CCI}}^{(2)}} + \frac{\overline{\gamma}_{\hat{k}}^{(2)}\lambda_{\hat{k},\max}^{(2)}\overline{\sigma}_{\hat{k},\text{CCI}}^{(2)2}}{\left(1+\overline{\sigma}_{\hat{k},\text{CCI}}^{(2)}\right)^{3}}. \tag{34}$$

This completes the proof of the theorem. □

It can be seen from (12) and (29) that $\overline{\text{SINR}}_{\hat{k},\text{approx}}^{(2)}$ depends on the eigenvalues $\lambda_{\hat{k},\max}^{(2)}$ and $\lambda_{\hat{k},\text{CCI},m}^{(2)}$, and the angle difference $\Delta_{\hat{i},\hat{k},m}^{(2)}$. Although $\overline{\text{SINR}}_{\hat{k},\text{approx}}^{(2)}$ depends on M_2 as seen in Corollaries 2 and 3, it is maximized by selecting a pair of users whose angle difference $\Delta_{\hat{i},\hat{k},\max}^{(2)}$ is $\pi/2$ in a highly correlated channel.

Theorem 5. *The proposed coordinated eigen-beamforming can provide an achievable sum rate approximately represented as*

$$\begin{aligned}\widehat{R}_{C\text{-}Eig.BF} &\approx \log_2\left[1 + \frac{\overline{\gamma}_{\hat{k}}^{(2)}\lambda_{\hat{k},\max}^{(2)}}{1+\overline{\sigma}_{\hat{k},CCI}^{(2)}} + \frac{\overline{\gamma}_{\hat{k}}^{(2)}\lambda_{\hat{k},\max}^{(2)}\overline{\sigma}_{\hat{k},CCI}^{(2)2}}{\left(1+\overline{\sigma}_{\hat{k},CCI}^{(2)}\right)^{3}}\right] \\ &\quad + \frac{1}{2}\min\left[\log_2\left(1+\overline{\gamma}_1^{(1)}M_2\lambda_{1,\max}^{(1)}\right), \log_2\left(1+\overline{\gamma}_{\hat{i}}^{(2)}\lambda_{\hat{i},\max}^{(2)}\right)\right],\end{aligned} \tag{35}$$

where $\widehat{R}_{C\text{-}Eig.BF}$ *denotes an approximated upper bound of the achievable sum rate.*

Proof. Using the Jensen's inequality [39], the achievable sum rate is bounded as

$$\begin{aligned} R_{\text{C-Eig.BF}} &= R_{\hat{k},\text{C-Eig.BF}} + R_{\hat{i},\text{C-Eig.BF}} \\ &\le \log_2\left(1 + E\left\{\frac{\overline{\gamma}_{\hat{k}}^{(2)}\left|\mathbf{h}_{\hat{k}}^{(2)*}\mathbf{u}_{\hat{k},\max}^{(2)}\right|^2}{1+\overline{\gamma}_{\hat{k},\text{CCI}}^{(2)}\left|\mathbf{h}_{\hat{k},\text{CCI}}^{(2)*}\mathbf{u}_{\hat{i},\max}^{(2)}\right|^2}\right\}\right) \\ &\quad + \frac{1}{2}\min\left[\log_2\left(1+E\left\{\overline{\gamma}_1^{(1)}\left|\mathbf{f}_1^{(1)*}\mathbf{H}_1^{(1)*}\mathbf{u}_{1,\max}^{(1)}\right|^2\right\}\right),\right. \\ &\qquad \left.\log_2\left(1+E\left\{\overline{\gamma}_{\hat{i}}^{(2)}\left|\mathbf{h}_{\hat{i}}^{(2)*}\mathbf{u}_{\hat{i},\max}^{(2)}\right|^2\right\}\right)\right] \\ &\triangleq \widehat{R}_{\text{C-Eig.BF}}, \end{aligned} \tag{36}$$

where $R_{\hat{k},\text{C-Eig.BF}}$ and $R_{\hat{i},\text{C-Eig.BF}}$ denote the achievable rate of user $\hat{k}$ and $\hat{i}$, respectively, $\widehat{R}_{\text{C-Eig.BF}}$ denotes the upper bound of the achievable sum rate, and $\mathbf{f}_1^{(1)}$ denotes an $(M_2 \times 1)$ combining vector of the relay. From (29), the upper bound of user $\hat{k}$ can be approximated as

$$\widehat{R}_{\hat{k},\text{C-Eig.BF}} \approx \log_2\left[1 + \frac{\overline{\gamma}_{\hat{k}}^{(2)}\lambda_{\hat{k},\max}^{(2)}}{1+\overline{\sigma}_{\hat{k},\text{CCI}}^{(2)}} + \frac{\overline{\gamma}_{\hat{k}}^{(2)}\lambda_{\hat{k},\max}^{(2)}\overline{\sigma}_{\hat{k},\text{CCI}}^{(2)2}}{\left(1+\overline{\sigma}_{\hat{k},\text{CCI}}^{(2)}\right)^{3}}\right]. \tag{37}$$

Assuming that maximum ratio combining (MRC) is used at the relay [15], the achievable rate of user $\hat{i}$ is bounded as

$$\begin{aligned} R_{\hat{i},\text{C-Eig.BF}} &\le \frac{1}{2}\min\left\{\begin{matrix}\log_2\left[1+\overline{\gamma}_1^{(1)}\operatorname{tr}\left(\mathbf{G}_1^{(1)}\right)\operatorname{tr}\left(\mathbf{R}_1^{(1)}\mathbf{u}_{1,\max}^{(1)}\mathbf{u}_{1,\max}^{(1)*}\right)\right], \\ \log_2\left[1+\overline{\gamma}_{\hat{i}}^{(2)}\operatorname{tr}\left(\mathbf{R}_{\hat{i}}^{(2)}\mathbf{u}_{\hat{i},\max}^{(2)}\mathbf{u}_{\hat{i},\max}^{(2)*}\right)\right]\end{matrix}\right\}. \end{aligned} \tag{38}$$

Since $\mathrm{tr}(\mathbf{G}_1^{(1)}) = \sum_{m=1}^{M_2} \lambda_{1,m}^{(1)} = M_2$ and $\mathrm{tr}(\mathbf{R}_1^{(1)}\mathbf{u}_{1,\max}^{(1)}\mathbf{u}_{1,\max}^{(1)*}) = \lambda_{1,\max}^{(1)}$, (38) can be represented as

$$R_{\hat{i},\text{C-Eig.BF}} \leq \frac{1}{2}\min\left[\log_2\left(1+\overline{\gamma}_1^{(1)}M_2\lambda_{1,\max}^{(1)}\right), \log_2\left(1+\overline{\gamma}_{\hat{i}}^{(2)}\lambda_{\hat{i},\max}^{(2)}\right)\right]. \tag{39}$$

Thus, it can be shown from (37) and (39) that

$$\hat{R}_{\text{C-Eig.BF}} \approx \log_2\left[1+\frac{\overline{\gamma}_{\hat{k}}^{(2)}\lambda_{\hat{k},\max}^{(2)}}{1+\overline{\sigma}_{\hat{k},\text{CCI}}^{(2)}}+\frac{\overline{\gamma}_{\hat{k}}^{(2)}\lambda_{\hat{k},\max}^{(2)}\overline{\sigma}_{\hat{k},\text{CCI}}^{(2)2}}{\left(1+\overline{\sigma}_{\hat{k},\text{CCI}}^{(2)}\right)^3}\right] + \frac{1}{2}\min\left[\log_2\left(1+\overline{\gamma}_1^{(1)}M_2\lambda_{1,\max}^{(1)}\right), \log_2\left(1+\overline{\gamma}_{\hat{i}}^{(2)}\lambda_{\hat{i},\max}^{(2)}\right)\right]. \tag{40}$$

This completes the proof of the theorem. □

It can be seen that the proposed coordinated eigen-beamforming maximizes the achievable sum rate when $\Delta_{\hat{i},\hat{k},\max}^{(2)} = \pi/2$ (i.e., yielding zero interference and large beamforming gain) in a highly correlated channel.

4.4. Allocation of Transmit Power. Although the CCI can effectively be controlled by adjusting the angle difference between the principal eigenvectors of two users, it cannot be minimized in an instantaneous sense. This issue can be alleviated by allocating the relay transmit power as low as possible since the CCI power is proportional to the relay transmit power. However, the transmit power needs to be allocated to maximize the minimum rate of two hops. It may be desirable to allocate the transmit power considering the CCI mitigation in a joint manner. The main goal is to allocate the transmit power to reduce the CCI while maximizing the achievable rate of the relay link.

Suppose that $P_{\text{BS}} \leq P_{\text{BS,max}}$ and $P_{\text{RS}} \leq P_{\text{RS,max}}$ since the BS and relay are not geographically colocated [35], where $P_{\text{BS,max}}$ and $P_{\text{RS,max}}$ denote the maximum power of the BS and the relay, respectively, and that the transmit power of the BS is given by P_{BS}. Then, it is desirable to determine the minimum transmit power of the relay to achieve the rate of the first hop. Figure 5 illustrates the concept of the proposed power allocation. When $P_{\text{RS}} = P_{\text{RS,max}}$, the achievable rate of the relay link is determined as $R_{i,\text{C-Eig.BF}}^{(1)}(P_{\text{BS}})$ since $R_{i,\text{C-Eig.BF}}^{(1)}(P_{\text{BS}}) < R_{i,\text{C-Eig.BF}}^{(2)}(P_{\text{RS,max}})$. Thus, the transmit power of the relay can be determined by the crossing point between the achievable rate of the first and the second hop.

Theorem 6. *The transmit power of the relay can be determined with the consideration of CCI mitigation as*

$$\kappa_{opt} \triangleq \frac{P_{RS,opt}}{P_{BS}} = \frac{\Gamma_1^{(1)}\sigma_{\hat{i}}^2}{\Gamma_{\hat{i}}^{(2)}\sigma_1^2}\frac{M_2\lambda_{1,\max}^{(1)}}{\lambda_{\hat{i},\max}^{(2)}}, \tag{41}$$

where κ_{opt} $(0 \leq \kappa_{opt} \leq P_{RS,\max}/P_{BS})$ is the transmit power ratio between the BS and the relay.

Proof. By means of max-min optimization [35], the achievable rate of the relay link can be maximized when

$$\overline{\gamma}_1^{(1)}M_2\lambda_{1,\max}^{(1)} = \overline{\gamma}_{\hat{i}}^{(2)}\lambda_{\hat{i},\max}^{(2)}. \tag{42}$$

Since $\overline{\gamma}_1^{(1)} = P_{\text{BS}}\Gamma_1^{(1)}/\sigma_1^2$ and $\overline{\gamma}_{\hat{i}}^{(2)} = P_{\text{RS}}\Gamma_{\hat{i}}^{(2)}/\sigma_{\hat{i}}^2$, (42) can be rewritten as

$$\frac{P_{\text{BS}}\Gamma_1^{(1)}}{\sigma_1^2}M_2\lambda_{1,\max}^{(1)} = \frac{P_{\text{RS}}\Gamma_{\hat{i}}^{(2)}}{\sigma_{\hat{i}}^2}\lambda_{\hat{i},\max}^{(2)}. \tag{43}$$

After simple manipulation, it can be seen that

$$\kappa_{\text{opt}} \triangleq \frac{P_{\text{RS,opt}}}{P_{\text{BS}}} = \frac{\Gamma_1^{(1)}\sigma_{\hat{i}}^2}{\Gamma_{\hat{i}}^{(2)}\sigma_1^2}\frac{M_2\lambda_{1,\max}^{(1)}}{\lambda_{\hat{i},\max}^{(2)}}. \tag{44}$$

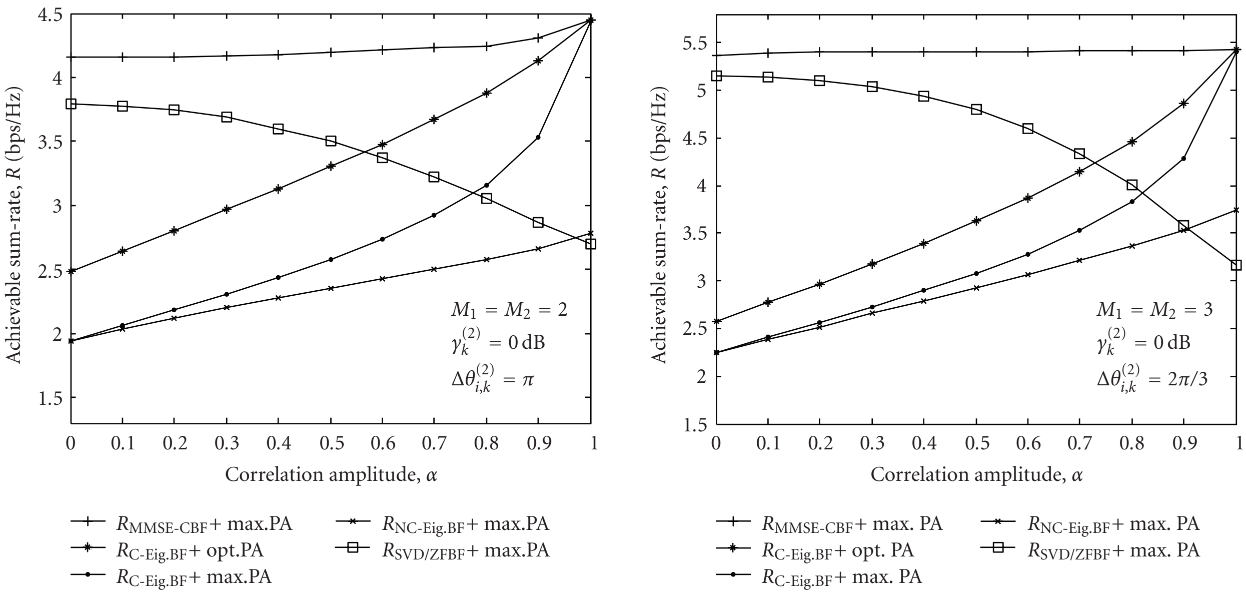
This completes the proof of the theorem. □

It can be seen that the optimum power allocation is associated with the path loss ratio $\Gamma_1^{(1)}/\Gamma_i^{(2)}$ and the principal eigenvalue ratio $\lambda_{1,\max}^{(1)}/\lambda_{\hat{i},\max}^{(2)}$ between the two hops. In fact, κ_{opt} is inversely proportional to the achievable rate of each hop. For example, as $\alpha_{\hat{i}}^{(2)}$ increases, $R_{\hat{i},\text{C-Eig.BF}}^{(2)}$ increases due to large beamforming gain. In this case, it is desirable to decrease κ_{opt} to balance the rate between two-hops, or vice versa.

4.5. Scheduling Complexity. We define the complexity measurement as the number of the required user pairs and compare the scheduling complexity for two user-scheduling schemes; the proposed and the instantaneous CSI-based user-scheduling schemes. For ease of description, we assume that T_{ST} and T_{LT} denote the feedback period of short-term and long-term CSI, respectively, where T_{LT} is a multiple of T_{ST}. We also assume that the BS and the relay have an equal number of users (i.e., $K/2$). To provide fair scheduling opportunities to all K users during T_{LT}, the proposed user-scheduling scheme needs to consider $(K/2)^2$ cases at the first scheduling instant and $(K/2-1)^2$ cases at the second scheduling instant. Thus, it needs to consider S_{LT} scheduling cases during T_{LT}, given by

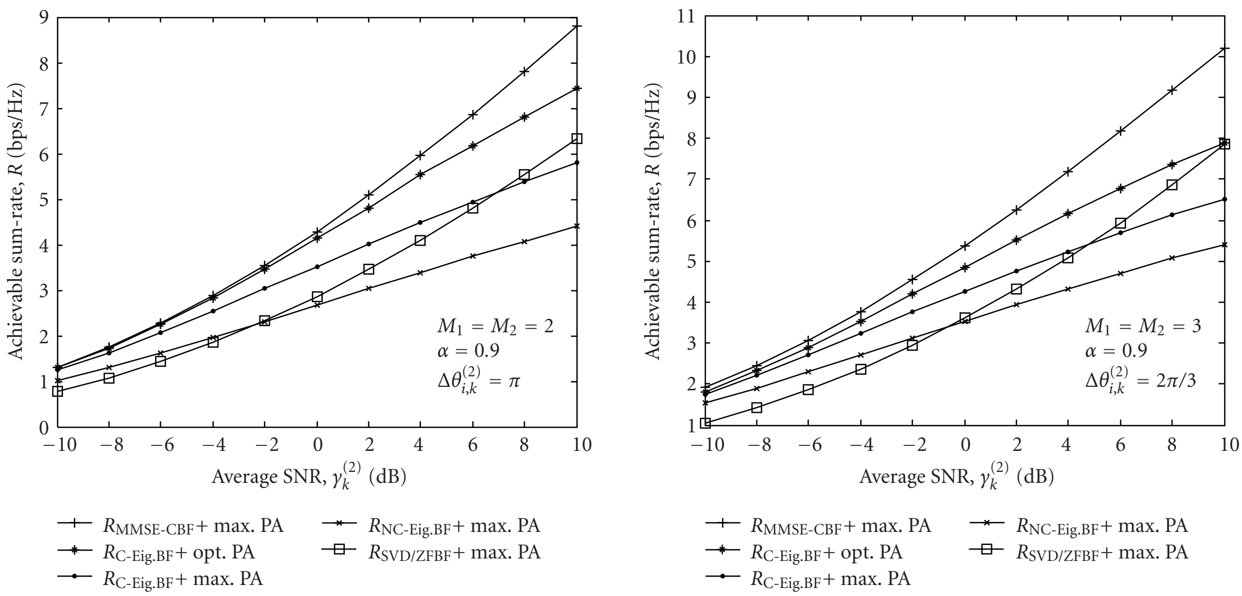
$$S_{\text{LT}} = \sum_{l=1}^{T_{\text{LT}}/T_{\text{Frame}}D_{\text{LT}}}\left[\frac{K}{2}-(l-1)\right]^2, \tag{45}$$

where T_{Frame} denotes the time duration of a single frame and D_{LT} denotes the portion allocated to a pair of users during T_{LT}, that is, $D_{\text{LT}} = 2T_{\text{LT}}/KT_{\text{Frame}}$. On the other hand, the instantaneous CSI-based user-scheduling scheme needs to consider $(K/2)^2$ cases to maximize the sum rate per $T_{\text{ST}}(= T_{\text{Frame}})$. This is because it requires K signals for



(a) $M_2 = 2$ (b) $M_2 = 3$

Figure 8: Performance comparison according to α.



(a) $M_2 = 2$ (b) $M_2 = 3$

Figure 9: Performance comparison according to $\overline{\gamma}_k^{(2)}$.

the feedback per each frame [43, 44]. It can be shown that the instantaneous CSI-based user-scheduling scheme needs to consider S_{ST} scheduling cases during T_{LT}, given by

$$S_{\text{ST}} = \frac{T_{\text{LT}}}{T_{\text{ST}}}\left(\frac{K}{2}\right)^2. \qquad (46)$$

For example, when $T_{\text{LT}} = 300$ ms, $T_{\text{ST}} = T_{\text{Frame}} = 5$ ms, and $K = 10$, the proposed scheme needs to consider $S_{\text{LT}} = \sum_{l=1}^{5}[5-(l-1)]^2 = 55$ scheduling cases. Here, $D_{\text{LT}} = 12$. On the other hand, the instantaneous CSI-based scheme needs to consider $S_{\text{ST}} = 60 \cdot 5^2 = 1500$ scheduling cases. Table 1 depicts the scheduling complexity comparison according to K when $T_{\text{LT}} = 300$ ms and $T_{\text{ST}} = T_{\text{Frame}} = 5$ ms. It can be seen that the proposed scheme requires lower scheduling complexity and smaller feedback signal overhead than the instantaneous CSI-based scheme as K increases.

Table 1: Scheduling complexity according to K when $T_{\mathrm{LT}} = 300$ ms and $T_{\mathrm{ST}} = T_{\mathrm{Frame}} = 5$ ms.

Number of users (K)	2	4	6	8	10	12
Instantaneous-CSI based scheduling	60	240	540	960	1500	2160
Proposed scheduling	1	5	14	30	55	91

5. Performance Evaluation

The analytic results and performance of the proposed coordinated eigen-beamforming with the optimum power allocation ($R_{\text{C-Eig.BF+Opt.PA}}$) and maximum power allocation ($R_{\text{C-Eig.BF+Max.PA}}$) are verified by computer simulation. For comparison, we consider three MIMO relay transmission schemes; the MMSE-CBF with maximum power allocation ($R_{\text{MMSE-CBF+Max.PA}}$) [15], the noncoordinated eigen-beamforming with maximum power allocation ($R_{\text{NC-Eig.BF+Max.PA}}$) [28], and the singular value decomposition and ZF beamforming (SVD/ZFBF) with maximum power allocation ($R_{\text{SVD/ZFBF+Max.PA}}$) [11, 20]. (User k can select the relay as its serving node through a cell selection algorithm [45]. Then, our system can be converted into a concatenated MIMO system comprising single-relay MIMO channel for the first hop and MIMO broadcast channels (MIMO-BC) for the second hop. In this case, the MIMO-SVD and linear precoding such as ZF beamforming can be employed to achieve the multiplexing gain, respectively [11, 20, 37].) We assume that each link has the same correlation amplitude (i.e., $\alpha_1^{(1)} = \alpha_i^{(2)} = \alpha_k^{(2)} = \alpha_{k,\text{CCI}}^{(2)} = \alpha$), and that the relay is placed at 0.7 km from the BS.

Figures 6(a) and 6(b), respectively, depict the average SINR and the achievable rate of user k with the use of the proposed scheme according to $\overline{\gamma}_k^{(2)}$ when $M_1 = M_2 = 2$, $\alpha = 0.9$, and $\Delta\theta_{i,k}^{(2)} = \pi$. It can be seen from Figure 6(a) that the approximated average SINR in (29) agrees well with the real average SINR in (30), implying that the upper bound of the achievable rate can be precisely approximated to (37). Although the analytic and simulation results have somewhat discrepancy in terms of the achievable rate due to the Jensen's loss (in a Rayleigh fading channel, the Jensen's loss is 0.83 bps/Hz at high SNR region [39]) (e.g., this gap is about 0.79 bps/Hz at $\overline{\gamma}_k^{(2)} = 10$ dB as seen in Figure 6(b)), the analytic results still keep the behavior of actual performance.

Figures 7(a) and 7(b), respectively, depict the achievable sum rate of the proposed scheme according to $\Delta\theta_{i,k}^{(2)}$ for $M_1 = M_2 = 2$ and 3 when $\alpha = 0.9$ and $\overline{\gamma}_k^{(2)} = 0$ dB. It can be seen that $R_{\text{C-Eig.BF+Opt.PA}}$ approaches to $R_{\text{MMSE-CBF+Max.PA}}$ when $\Delta\theta_{i,k}^{(2)} = \pi$ for $M_2 = 2$ (or $\Delta\theta_{i,k}^{(2)} = 2\pi/3$ for $M_2 = 3$). This is mainly because the proposed scheme eliminates the most of CCI by making the angle difference between $\mathbf{u}_{i,\max}^{(2)}$ and $\mathbf{u}_{k,\text{CCI},\max}^{(2)}$ orthogonal. It can also be seen that $R_{\text{C-Eig.BF+Opt.PA}}$ is larger than $R_{\text{C-Eig.BF+Max.PA}}$ regardless of $\Delta\theta_{i,k}^{(2)}$, which means that the CCI can also be reduced by minimizing the transmit power of the relay. The sum rate gap between $R_{\text{C-Eig.BF+Max.PA}}$ and $R_{\text{NC-Eig.BF+Max.PA}}$ has a different behavior according to M_2. When $M_2 = 2$, the sum rate gap increases as $\Delta\theta_{i,k}^{(2)}$ increases. On the other hand, when $M_2 = 3$, it somewhat decreases for $2\pi/3 < \Delta\theta_{i,k}^{(2)} \le \pi$ as $\Delta\theta_{i,k}^{(2)}$ increases. This is because the condition minimizing the CCI depends on M_2 as well as $\Delta\theta_{i,k}^{(2)}$. Although the SVD/ZFBF-based relay scheme provides the multiplexing gain without the CCI effect [11], its sum rate is somewhat limited due to the rank deficiency of the first hop in a spatially correlated channel [37, 46].

Figure 8 depicts the achievable sum rate according to α when $M_1 = M_2 = 2, 3$, $\Delta\theta_{i,k}^{(2)} = 2\pi/M_2$ and $\overline{\gamma}_k^{(2)} = 0$ dB. It can be seen that as α increases, the proposed scheme provides the sum rate comparable to that of the MMSE-CBF. This is mainly because the most of CCI is concentrated on the direction of the principal eigenvector of $\mathbf{R}_{k,\text{CCI}}^{(2)}$ in a highly correlated channel. In this case, the most of CCI can be eliminated by selecting users whose angle difference $\Delta_{i,k,\max}^{(2)}$ close to $\pi/2$. It can also be seen that $R_{\text{C-Eig.BF+Opt.PA}}$ is larger than $R_{\text{C-Eig.BF+Max.PA}}$ regardless of α, and that $R_{\text{C-Eig.BF+Max.PA}}$ is nearly equal to $R_{\text{NC-Eig.BF+Max.PA}}$ when α is small. This implies that the proposed power-allocation rule yields less interference than the maximum one. The SVD/ZFBF-based relay scheme outperforms the proposed scheme by having more degrees of freedom in a weakly correlated channel. This is mainly because the bottleneck effect due to the rank deficiency is eased as α decreases.

Figure 9 depicts the achievable sum rate of the proposed scheme according to $\overline{\gamma}_k^{(2)}$ when $M_1 = M_2 = 2, 3$, $\alpha = 0.9$, and $\Delta\theta_{i,k}^{(2)} = 2\pi/M_2$. It can be seen that $R_{\text{MMSE-CBF+Max.PA}}$ is always larger than $R_{\text{C-Eig.BF+Opt.PA}}$ regardless of $\overline{\gamma}_k^{(2)}$ since it can maximize the SINR by optimally handling the tradeoff between the CCI cancellation and the noise suppression [19]. Nevertheless, it can be seen that the sum rate gap between $R_{\text{MMSE-CBF+Max.PA}}$ and $R_{\text{C-Eig.BF+Opt.PA}}$ is very marginal in low SNR region, (e.g., 0.08 bps/Hz gap (or 0.31 bps/Hz gap) for $M_2 = 2$ (or $M_2 = 3$)). This implies that the proposed scheme is quite effective near the cell boundary or coverage hole.

6. Conclusions

In this paper, we have considered the use of coordinated transmission for the interference mitigation and power allocation in a correlated two-user two-hop MIMO relay system. We have analytically investigated the statistical characteristics of CCI and the condition minimizing the CCI. Then, we have considered coordinated transmission with the use of eigen-beamforming with power allocation. We have shown that the proposed scheme can maximize the achievable sum rate when the principal eigenvectors of the transmit correlation matrix of the intended and the CCI channel are orthogonal to each other. The numerical results show that the proposed scheme provides the maximum sum rate similar to that of the MMSE-CBF, while reducing the complexity and feedback signaling overhead. To extend this work, including multiple antennas at each user is a topic for future work.

Appendices

A. Derivation of Eigenvalues and Eigenvectors of R When $M_2 = 3$

By the definition in [40], an eigenvector $\mathbf{u}$ of a transmit correlation matrix $\mathbf{R}$ satisfies the equation

$$(\mathbf{R} - \lambda \mathbf{I}_3)\mathbf{u} = \mathbf{0}, \tag{A.1}$$

where user and time slot index are dropped for simple notation. Since a necessary and sufficient condition for (A.1) is

$$\det(\mathbf{R} - \lambda \mathbf{I}_3) = 0, \tag{A.2}$$

it can be explicitly expressed as

$$\det\begin{bmatrix} 1-\lambda & \rho & \rho^2 \\ \rho^* & 1-\lambda & \rho \\ \rho^{2*} & \rho^* & 1-\lambda \end{bmatrix} = 0. \tag{A.3}$$

Letting $A = 1 - \lambda$ and $s = \rho\rho^* = \alpha^2$, and after some mathematical development, (A.3) can be rewritten as

$$A^3 - (s^2 + 2s)A + 2s^2 = 0. \tag{A.4}$$

Since (A.4) has three solutions $A = (-s \pm \sqrt{s^2 + 8s})/2$ and s, the eigenvalues of $\mathbf{R}$ can be obtained as

$$\lambda_1 = 1 + \frac{\alpha^2 + \sqrt{\alpha^4 + 8\alpha^2}}{2}, \qquad \lambda_2 = 1 - \alpha^2, \qquad \lambda_3 = 1 + \frac{\alpha^2 - \sqrt{\alpha^4 + 8\alpha^2}}{2}, \tag{A.5}$$

where $\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq 0$. To get the eigenvector corresponding to the mth eigenvalue of $\mathbf{R}$, let us plug λ_m back into (A.2) and solve $\mathbf{u}_m = \begin{bmatrix} u_{m,1} & u_{m,2} & u_{m,3} \end{bmatrix}^T$. It can be shown after some manipulation that the mth eigenvector $\mathbf{u}_m$ can be determined as

$$\mathbf{u}_m = \begin{bmatrix} 1 & \dfrac{\alpha^2 - (1-\lambda_m)}{\lambda_m \alpha} e^{-j\theta} & \dfrac{(1-\lambda_m)^2 - \alpha^2}{\lambda_m \alpha^2} e^{-j2\theta} \end{bmatrix}^T. \tag{A.6}$$

This completes the proof of (25).

B. Orthogonal Property of Two Principal Eigenvectors When $M_2 = 3$

Without the loss of generality, the time slot index is dropped for simple notation. It can be shown from (A.6) that the principal eigenvector of user i can be represented as

$$\mathbf{u}_{i,\max} = \begin{bmatrix} 1 & \dfrac{\alpha_i^2 - (1-\lambda_{i,\max})}{\lambda_{i,\max} \alpha_i} e^{-j\theta_i} & \dfrac{(1-\lambda_{i,\max})^2 - \alpha_i^2}{\lambda_{i,\max} \alpha_i^2} e^{-j2\theta_i} \end{bmatrix}^T. \tag{B.1}$$

From [40], we have

$$\begin{aligned} &\cos\measuredangle(\mathbf{u}_{i,\max}, \mathbf{u}_{k,\mathrm{CCI},\max}) \\ &= \frac{\left|\mathbf{u}_{i,\max}^* \mathbf{u}_{k,\mathrm{CCI},\max}\right|}{\|\mathbf{u}_{i,\max}\| \cdot \|\mathbf{u}_{k,\mathrm{CCI},\max}\|} \\ &= \frac{\left| 1 + \dfrac{\alpha_i^2 - (1-\lambda_{i,\max})}{\lambda_{i,\max}\alpha_i} \dfrac{\alpha_{k,\mathrm{CCI}}^2 - (1-\lambda_{k,\mathrm{CCI},\max})}{\lambda_{k,\mathrm{CCI},\max}\alpha_{k,\mathrm{CCI}}} e^{j|\theta_i - \theta_{k,\mathrm{CCI}}|} + \dfrac{(1-\lambda_{i,\max})^2 - \alpha_i^2}{\lambda_{i,\max}\alpha_i^2} \dfrac{(1-\lambda_{k,\mathrm{CCI},\max})^2 - \alpha_{k,\mathrm{CCI}}^2}{\lambda_{k,\mathrm{CCI},\max}\alpha_{k,\mathrm{CCI}}^2} e^{j2|\theta_i - \theta_{k,\mathrm{CCI}}|} \right|}{\|\mathbf{u}_{i,\max}\| \cdot \|\mathbf{u}_{k,\mathrm{CCI},\max}\|}. \end{aligned} \tag{B.2}$$

By letting

$$\begin{aligned} A &= \frac{\alpha_i^2 - (1-\lambda_{i,\max})}{\lambda_{i,\max}\alpha_i} \frac{\alpha_{k,\mathrm{CCI}}^2 - (1-\lambda_{k,\mathrm{CCI},\max})}{\lambda_{k,\mathrm{CCI},\max}\alpha_{k,\mathrm{CCI}}}, \\ B &= \frac{(1-\lambda_{i,\max})^2 - \alpha_i^2}{\lambda_{i,\max}\alpha_i^2} \frac{(1-\lambda_{k,\mathrm{CCI},\max})^2 - \alpha_{k,\mathrm{CCI}}^2}{\lambda_{k,\mathrm{CCI},\max}\alpha_{k,\mathrm{CCI}}^2}, \end{aligned} \tag{B.3}$$

(B.2) can be rewritten as

$$\begin{aligned} &\cos\measuredangle(\mathbf{u}_{i,\max}, \mathbf{u}_{k,\mathrm{CCI},\max}) \\ &= \frac{\left| 1 + Ae^{j|\theta_i - \theta_{k,\mathrm{CCI}}|} + Be^{j2|\theta_i - \theta_{k,\mathrm{CCI}}|} \right|}{\|\mathbf{u}_{i,\max}\| \cdot \|\mathbf{u}_{k,\mathrm{CCI},\max}\|} \\ &= \frac{\sqrt{1 + A^2 + B^2 + 2A(1+B)\cos|\mathcal{C}| + 2B\cos 2|\mathcal{C}|}}{\|\mathbf{u}_{i,\max}\| \cdot \|\mathbf{u}_{k,\mathrm{CCI},\max}\|}, \end{aligned} \tag{B.4}$$

where $\mathcal{C}$ denotes $\theta_i - \theta_{k,\mathrm{CCI}}$, which yields

$$\begin{aligned} &\measuredangle(\mathbf{u}_{i,\max}, \mathbf{u}_{k,\mathrm{CCI},\max}) \\ &= \cos^{-1}\left(\frac{\sqrt{1 + A^2 + B^2 + 2A(1+B)\cos|\mathcal{C}| + 2B\cos 2|\mathcal{C}|}}{\|\mathbf{u}_{i,\max}\| \cdot \|\mathbf{u}_{k,\mathrm{CCI},\max}\|} \right). \end{aligned} \tag{B.5}$$

Since $A, B \rightarrow 1$ as $\alpha_i, \alpha_{k,\mathrm{CCI}} \rightarrow 1$, it can be shown that

$$\begin{aligned} &\measuredangle(\mathbf{u}_{i,\max}, \mathbf{u}_{k,\mathrm{CCI},\max}) \\ &= \cos^{-1}\left(\frac{\sqrt{3 + 4\cos|\theta_i - \theta_{k,\mathrm{CCI}}| + 2\cos 2|\theta_i - \theta_{k,\mathrm{CCI}}|}}{\|\mathbf{u}_{i,\max}\| \cdot \|\mathbf{u}_{k,\mathrm{CCI},\max}\|} \right). \end{aligned} \tag{B.6}$$

When $|\theta_i - \theta_{k,\mathrm{CCI}}| = 2\pi/3$, we have $\measuredangle(\mathbf{u}_{i,\max}, \mathbf{u}_{k,\mathrm{CCI},\max}) = \pi/2$.

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