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Research Article

Bit Error Rate Approximation of MIMO-OFDM Systems with Carrier Frequency Offset and Channel Estimation Errors

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The bit error rate (BER) of multiple-input multiple-output (MIMO) orthogonal frequency-division multiplexing (OFDM) systems with carrier frequency offset and channel estimation errors is analyzed in this paper. Intercarrier interference (ICI) and interantenna interference (IAI) due to the residual frequency offsets are analyzed, and the average signal-to-interference-and-noise ratio (SINR) is derived. The BER of equal gain combining (EGC) and maximal ratio combining (MRC) with MIMO-OFDM is also derived. The simulation results demonstrate the accuracy of the theoretical analysis.

1. Introduction

Spatial multiplexing multiple-input multiple-output (MIMO) technology significantly increases the wireless system capacity [1–4]. These systems are primarily designed for flat-fading MIMO channels. A broader band can be used to support a higher data rate, but a frequency-selective fading MIMO channel is met, and this channel experiences intersymbol interference (ISI). A popular solution is MIMO-orthogonal frequency-division multiplexing (OFDM), which achieves a high data rate at a low cost of equalization and demodulation. However, just as single-input single-output-(SISO-) OFDM systems are highly sensitive to frequency offset, so are MIMO-OFDM systems. Although one can use frequency offset correction algorithms [5–10], residual frequency offsets can still increase the bit error rate (BER).

The BER of SISO-OFDM systems impaired by frequency offset is analyzed in [11], in which the frequency offset is assumed to be perfectly known at the receiver, and, based on the intercarrier interference (ICI) analysis, the BER is evaluated for multipath fading channels. Many frequency offset estimators have been proposed [8, 12–14]. A synchronization algorithm for MIMO-OFDM systems is proposed in [15], which considers an identical timing offset and frequency offset with respect to each transmit-receive antenna pair. In [10], where frequency offsets for different transmit-receive antennas are assumed to be different, the Cramer-Rao lower bound (CRLB) for either the frequency offsets or channel estimation variance errors for MIMO-OFDM is derived. More documents on MIMO-OFDM channel estimation by considering the frequency offset are available at [16, 17].

However, in real systems, neither the frequency offset nor the channel can be perfectly estimated. Therefore, the residual frequency offset and channel estimation errors impact the BER performance. The BER performance of MIMO systems, without considering the effect of both the frequency offset and channel estimation errors, is studied in [18, 19].

This paper provides a generalized BER analysis of MIMO-OFDM, taking into consideration both the frequency offset and channel estimation errors. The analysis exploits the fact that for unbiased estimators, both channel and frequency offset estimation errors are zero-mean random variables (RVs). Note that the exact channel estimation algorithm design is not the focus of this paper, and the main parameter of interest is the channel estimation error. Many channel estimation algorithms developed for either SISO or MIMO-OFDM systems, for example, [20–22], can be used to perform channel estimation. The statistics of these RVs are used to derive the degradation in the receive SINR and the BER. Following [10], the frequency offset of each transmit-receive antenna pair is assumed to be an independent and identically distributed (i.i.d.) RV.

This paper is organized as follows. The MIMO-OFDM system model is described in Section 2, and the SINR degradation due to the frequency offset and channel estimation errors is analyzed in Section 3. The BER, taking into consideration both the frequency offset and channel estimation errors, is derived in Section 4. The numerical results are given in Section 5, and the conclusions are presented in Section 6.

Notation. $(\cdot)^T$ and $(\cdot)^H$ are transpose and complex conjugate transpose. The imaginary unit is $j = \sqrt{-1}$. $\Re\{x\}$ and $\Im\{x\}$ are the real and imaginary parts of x, respectively. $\arg\{x\}$ represents the angle of x, that is, $\arg\{x\} = \arctan(\Im\{x\}/\Re\{x\})$. A circularly symmetric complex Gaussian RV with mean m and variance σ^2 is denoted by $w \sim \mathcal{CN}(m, \sigma^2)$. $\mathbf{I}_N$ is the $N \times N$ identity matrix, and $\mathbf{O}_N$ is the $N \times N$ all-zero matrix. $\mathbf{0}_N$ is the $N \times 1$ all-zero vector. $\mathbf{a}[i]$ is the ith entry of vector $\mathbf{a}$, and $[\mathbf{B}]_{mn}$ is the mnth entry of matrix $\mathbf{B}$. $\mathbb{E}\{x\}$ and $\mathrm{Var}\{x\}$ are the mean and variance of x.

2. MIMO-OFDM Signal Model

Input data bits are mapped to a set of N complex symbols drawn from a typical signal constellation such as phase-shift keying (PSK) or quadrature amplitude modulation (QAM). The inverse discrete fourier transform (IDFT) of these N symbols generates an OFDM symbol. Each OFDM symbol has a useful part of duration T_s seconds and a cyclic prefix of length T_g seconds to mitigate ISI, where T_g is longer than the channel-response length. For a MIMO-OFDM system with N_t transmit antennas and N_r receive antennas, an $N \times 1$ vector $\mathbf{x}_{n_t}$ represents the block of frequency-domain symbols sent by the n_tth transmit antenna, where $n_t \in \{1, 2, \dots, N_t\}$. The time-domain vector for the n_tth transmit antenna is given by $\mathbf{m}_{n_t} = \sqrt{E_s/N_t}\mathbf{F}\mathbf{x}_{n_t}$, where E_s is the total transmit power and $\mathbf{F}$ is the $N \times N$ IDFT matrix with entries $[\mathbf{F}]_{nk} = (1/\sqrt{\mathrm{N}})e^{j2\pi nk/N}$ for $0 \le n, k \le N-1$. Each entry of $\mathbf{x}_{n_t}$ is assumed to be i.i.d. RV with mean zero and unit variance; that is, $\sigma_x^2 = \mathbb{E}\{|x_{n_t}[n]|^2\} = 1$ for $1 \le n_t \le N_t$ and $0 \le n \le N-1$.

The discrete channel response between the n_rth receive antenna and n_tth transmit antenna is $\mathbf{h}_{n_r,n_t} = [h_{n_r,n_t}(0), h_{n_r,n_t}(1), \dots, h_{n_r,n_t}(L_{n_r,n_t}-1), \mathbf{0}^T_{L_{\max}-L_{n_r,n_t}}]^T$, where L_{n_r,n_t} is the maximum delay between the n_tth transmit and the n_rth receive antennas, and $L_{\max} = \max\{L_{n_r,n_t} : 1 \le n_t \le N_t, 1 \le n_r \le N_r\}$. Uncorrelated channel taps are assumed for each antenna pair (n_r, n_t); that is, $\mathbb{E}\{h^*_{n_r,n_t}(m)h_{n_r,n_t}(n)\} = 0$ when $n \ne m$. The corresponding frequency-domain channel response matrix is given by $\mathbf{H}_{n_r,n_t} = \mathrm{diag}\{H^{(0)}_{n_r,n_t}, H^{(1)}_{n_r,n_t}, \dots, H^{(N-1)}_{n_r,n_t}\}$ with $H^{(n)}_{n_r,n_t} = \sum_{d=0}^{L_{n_r,n_t}-1} h_{n_r,n_t}(d)e^{-j2\pi nd/N}$ representing the channel attenuation at the nth subcarrier. In the sequel, the channel power profiles are normalized as $\sum_{d=0}^{L_{n_r,n_t}-1}\mathbb{E}\{|h_{n_r,n_t}(d)|^2\} = 1$ for all (n_r, n_t). The covariance of channel frequency response is given by

$$C_{H^{(n)}_{n_r,n_t} H^{(l)}_{p,q}} = \sum_{d=0}^{L_{\max}-1} \mathbb{E}\left\{h^*_{n_r,n_t}(d)h_{p,q}(d)\right\} e^{-j2\pi d(l-n)/N}, \qquad 0 \le d \le L_{\max}, 0 \le l, n \le N-1. \tag{1}$$

Note that if $n_r \ne p$ and $n_t \ne q$ are satisfied simultaneously, we assume that there is no correlation between h_{n_r,n_t} and $h_{p,q}$. Otherwise the correlation between h_{n_r,n_t} and $h_{p,q}$ is nonzero.

In this paper, ψ_{n_r,n_t} and ε_{n_r,n_t} are used to represent the initial phase and normalized frequency offset (normalized to the OFDM subcarrier spacing) between the oscillators of the n_t-th transmit and the n_rth receive antennas. The frequency offsets ε_{n_r,n_t} for all (n_r, n_t) are modeled as zero-mean i.i.d. RVs. (Multiple rather than one frequency offset are assumed in this paper, with each transmit-antenna pair being impaired by an independent frequency offset. This case happens when the distance between different transmit or receive antenna elements is large enough, and this big distance results in a different angle-of-arrive (AOA) of the signal received by each receive antenna element. In this scenario, once the moving speed of the mobile node is high, the Doppler Shift related to different transmit-receive antenna pair will be different.)

By considering the channel gains and frequency offsets, the received signal vector can be represented as

$$\mathbf{y} = \left[\mathbf{y}_1^T, \mathbf{y}_2^T, \dots, \mathbf{y}_{N_r}^T\right]^T, \tag{2}$$

where $\mathbf{y}_{n_r} = \sqrt{E_s/N_t}\sum_{n_t=1}^{N_t}\mathbf{E}_{n_r,n_t}\mathbf{F}\mathbf{H}_{n_r,n_t}\mathbf{x}_{n_t} + \mathbf{w}_{n_r}$, $\mathbf{E}_{n_r,n_t} = \mathrm{diag}\{e^{j\psi_{n_r,n_t}}, \dots, e^{j(2\pi\varepsilon_{n_r,n_t}(N-1)/N+\psi_{n_r,n_t})}\}$ and $\mathbf{w}_{n_r}$ is a vector of additive white Gaussian noise (AWGN) with $\mathbf{w}_{n_r}[n] \sim \mathcal{CN}(0, \sigma_w^2)$. Note that the channel state information is available at the receiver, but not at the transmitter. Consequently, the transmit power is equally allocated among all the transmit antennas.

3. SINR Analysis in MIMO-OFDM Systems

This paper treats spatial multiplexing MIMO, where independent data streams are mapped to distinct OFDM symbols and are transmitted simultaneously from transmit antennas. The received vector $\mathbf{y}_{n_r}$ at the n_rth receive antenna is thus a superposition of the transmit signals from all the N_t transmit antennas. When demodulating $\mathbf{x}_{n_t}$, the signals from the transmit antennas other than the n_tth transmit antenna constitute interantenna interference (IAI). The structure of MIMO-OFDM systems is illustrated in Figure 1, where Δf represents the subcarrier spacing.

Here, we first assume that $\varepsilon_{n_r,i}$ and $\mathbf{H}_{n_r,i}$ for each $(1 \le i \le N_t, i \ne n_t)$ have been estimated imperfectly; that is, $\hat{\varepsilon}_{n_r,i} = \varepsilon_{n_r,i} + \Delta\varepsilon_{n_r,i}$ and $\hat{\mathbf{H}}_{n_r,i} = \mathbf{H}_{n_r,i} + \Delta\mathbf{H}_{n_r,i}$, where $\Delta\varepsilon_{n_r,i}$ and $\Delta\mathbf{H}_{n_r,i} = \mathrm{diag}\{\Delta H^{(0)}_{n_r,i}, \dots, \Delta H^{(N-1)}_{n_r,i}\}$ are the estimation errors of $\varepsilon_{n_r,i}$ and $\mathbf{H}_{n_r,i}$ ($\Delta H^{(n)}_{n_r,i} = \hat{H}^{(n)}_{n_r,i} - H^{(n)}_{n_r,i}$ represents the estimation error of $H^{(n)}_{n_r,i}$), respectively. We also assume that each $\mathbf{x}_{i \ne n_t}$ is demodulated with a negligible error. After

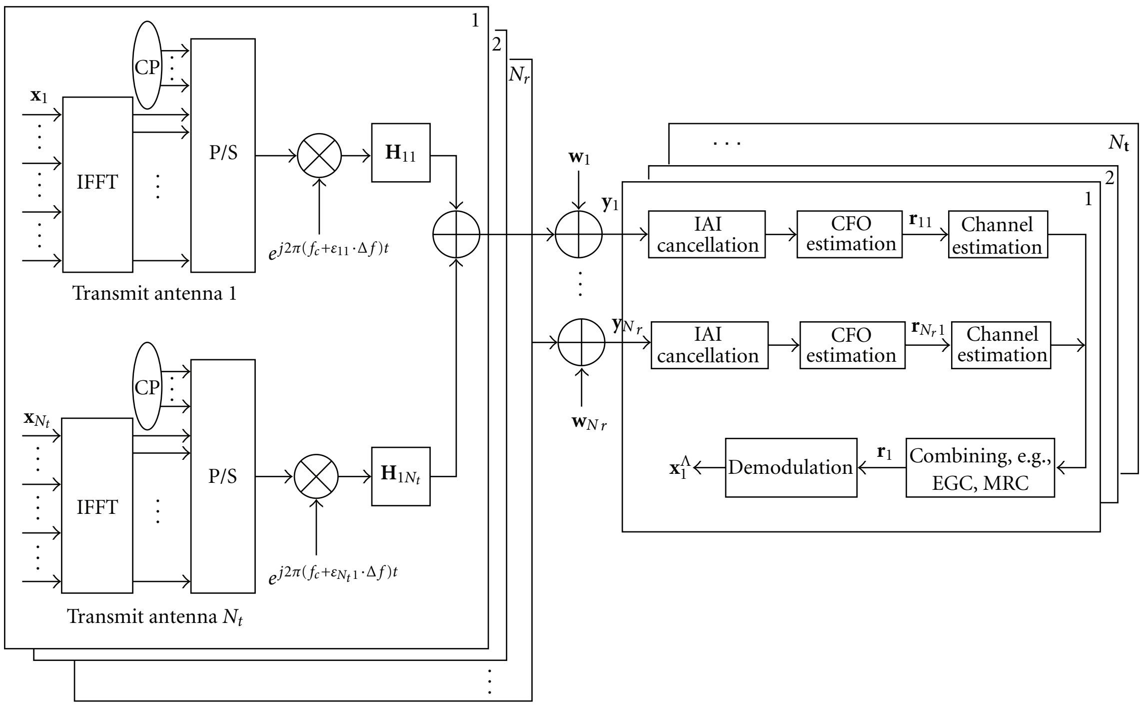

Figure 1: Structure of MIMO-OFDM transceiver.

estimating ε_{n_r,n_t}, that is, $\hat{\varepsilon}_{n_r,n_t} = \varepsilon_{n_r,n_t} + \Delta\varepsilon_{n_r,n_t}$, ε_{n_r,n_t} can be compensated for and $\mathbf{x}_{n_t}$ can be demodulated as

$$\begin{aligned}
\mathbf{r}_{n_r,n_t} &= \mathbf{F}^H\hat{\mathbf{E}}^H_{n_r,n_t}\left(\mathbf{y}_{n_r} - \sqrt{\frac{E_s}{N_t}}\sum_{i=1,i\neq n_t}^{N_t}\hat{\mathbf{E}}_{n_r,i}\mathbf{F}\hat{\mathbf{H}}_{n_r,i}\mathbf{x}_i\right)\\
&= \sqrt{\frac{E_s}{N_t}}\underbrace{\mathbf{F}^H\hat{\mathbf{E}}^H_{n_r,n_t}\mathbf{E}_{n_r,n_t}\mathbf{F}\mathbf{H}_{n_r,n_t}\mathbf{x}_{n_t}}_{\mathbf{s}_{n_r,n_t}}\\
&\quad+\underbrace{\sqrt{\frac{E_s}{N_t}}\sum_{i=1,i\neq n_t}^{N_t}\mathbf{F}^H\hat{\mathbf{E}}^H_{n_r,n_t}\left(\mathbf{E}_{n_r,i}\mathbf{F}\mathbf{H}_{n_r,i}-\hat{\mathbf{E}}_{n_r,i}\mathbf{F}\hat{\mathbf{H}}_{n_r,i}\right)\mathbf{x}_i}_{\mathbf{\Upsilon}_{n_r,n_t}}\\
&\quad+\underbrace{\mathbf{F}^H\hat{\mathbf{E}}^H_{n_r,n_t}\mathbf{w}_{n_r}}_{\tilde{\mathbf{w}}_{n_r,n_t}},
\end{aligned}\tag{3}$$

where $\hat{\mathbf{E}}_{n_r,i}$ is derived from $\mathbf{E}_{n_r,i}$ by replacing $\varepsilon_{n_r,i}$ with $\hat{\varepsilon}_{n_r,i}$ and $\mathbf{\Upsilon}_{n_r,n_t}$ and $\hat{\mathbf{w}}_{n_r,n_t}$ are the residual IAI and AWGN components of $\mathbf{r}_{n_r,n_t}$, respectively (When N_t is large enough and the frequency offset is not too big (e.g., $\ll 1$), from the Central-Limit Theorem (CLT) [23, Page 59], the IAI can be approximated as Gaussian noise.).

3.1. SINR Analysis without Combining at Receive Antennas. The SINR is derived for the n_tth transmit signal at the n_rth receive antenna. The signals transmitted by antennas other than the n_tth antenna are interference, which should be eliminated before demodulating the desired signal of the n_tth transmit antenna. Existing interference cancelation algorithms [24–27] can be applied here.

Let us first define the parameters $m^{(n,l)}_{n_r,n_t} = (\sin[\pi(l-n-\Delta\varepsilon_{n_r,n_t})]/N\sin[\pi(l-n-\Delta\varepsilon_{n_r,n_t})/N])e^{j\pi(N-1)(l-n)/N}$, $m^{(n,l)}_{n_r,i\neq n_t} = (\sin[\pi(l-n+\varepsilon_{n_r,i}-\hat{\varepsilon}_{n_r,n_t})]/N\sin[\pi(l-n+\varepsilon_{n_r,i}-\hat{\varepsilon}_{n_r,n_t})/N])e^{j\pi(N-1)(l-n)/N}$, and $\hat{m}^{(n,l)}_{n_r,i\neq n_t} = (\sin[\pi(l-n+\hat{\varepsilon}_{n_r,i}-\hat{\varepsilon}_{n_r,n_t})]/N\sin[\pi(l-n+\hat{\varepsilon}_{n_r,i}-\hat{\varepsilon}_{n_r,n_t})/N])e^{j\pi(N-1)(l-n)/N}$, $0\le l\le N-1$. Based on (3), the nth subcarrier ($0\le n\le N-1$) of the n_tth transmit antenna can be demodulated as

$$\begin{aligned}
\mathbf{r}_{n_r,n_t}[n] &= \sqrt{\frac{E_s}{N_t}}\mathbf{s}_{n_r,n_t}[n] + \mathbf{\Upsilon}_{n_r,n_t}[n] + \tilde{\mathbf{w}}_{n_r,n_t}[n]\\
&= \sqrt{\frac{E_s}{N_t}}m^{(n,n)}_{n_r,n_t}H^{(n)}_{n_r,n_t}\mathbf{x}_{n_t}[n]\\
&\quad+\underbrace{\sqrt{\frac{E_s}{N_t}}\sum_{l\neq n}m^{(n,l)}_{n_r,n_t}H^{(l)}_{n_r,n_t}\mathbf{x}_{n_t}[l]}_{\eta^{(n)}_{n_r,n_t}=H^{(n)}_{n_r,n_t}\alpha^{(n)}_{n_r,n_t}+\beta^{(n)}_{n_r,n_t}}\\
&\quad+\underbrace{\sqrt{\frac{E_s}{N_t}}\sum_{i=1,i\neq n_t}^{N_t}m^{(n,n)}_{n_r,i}H^{(n)}_{n_r,i}\mathbf{x}_i[n]}_{\lambda^{(n)}_{n_r,n_t}}\\
&\quad-\underbrace{\sqrt{\frac{E_s}{N_t}}\sum_{i=1,i\neq n_t}^{N_t}\hat{m}^{(n,n)}_{n_r,i}\hat{H}^{(n)}_{n_r,i}\mathbf{x}_i[n]}_{\hat{\lambda}^{(n)}_{n_r,n_t}}
\end{aligned}$$

$$+\underbrace{\sqrt{\frac{E_s}{N_t}}\sum_{l\neq n}\sum_{i=1,i\neq n_t}^{N_t} m_{n_r,i}^{(n,l)} H_{n_r,i}^{(l)}\mathbf{x}_i[l]}_{\xi_{n_r,n_t}^{(n)}}$$

$$-\underbrace{\sqrt{\frac{E_s}{N_t}}\sum_{l\neq n}\sum_{i=1,i\neq n_t}^{N_t} \widehat{m}_{n_r,i}^{(n,l)} \widehat{H}_{n_r,i}^{(l)}\mathbf{x}_i[l]}_{\widehat{\xi}_{n_r,n_t}^{(n)}}+\widetilde{\mathbf{w}}_{n_r,n_t}[n]$$

$$=\sqrt{\frac{E_s}{N_t}} m_{n_r,n_t}^{(n,n)} H_{n_r,n_t}^{(n)}\mathbf{x}_{n_t}[n]+H_{n_r,n_t}^{(n)}\alpha_{n_r,n_t}^{(n)}$$

$$+\beta_{n_r,n_t}^{(n)}+\Delta\lambda_{n_r,n_t}^{(n)}+\Delta\xi_{n_r,n_t}^{(n)}+\widetilde{\mathbf{w}}_{n_r,n_t}[n], \quad (4)$$

where $\eta_{n_r,n_t}^{(n)}$ is decomposed as $\eta_{n_r,n_t}^{(n)} = H_{n_r,n_t}^{(n)}\alpha_{n_r,n_t}^{(n)} + \beta_{n_r,n_t}^{(n)}$, which is the ICI contributed by subcarriers other than the nth subcarrier of transmit antenna n_t. (The decomposition of ICI into the format of $H\alpha+\beta$ is referred to [11].) We can easily prove that $\alpha_{n_r,n_t}^{(n)}$ and $\beta_{n_r,n_t}^{(n)}$ are zero-mean RVs subject to the following assumptions.

(1) ε_{n_r,n_t} is an i.i.d. RV with mean zero and variance σ_ϵ^2 for all (n_r, n_t).

(2) $\Delta\varepsilon_{n_r,n_t}$ is an i.i.d. RV with mean zero and variance σ_{res}^2 for each (n_r, n_t).

(3) $H_{n_r,n_t}^{(n)} \sim \mathcal{CN}(0,1)$ for each (n_r, n_t, n).

(4) $\Delta H_{n_r,n_t}^{(n)}$ is an i.i.d. RV with mean zero and variance $\sigma_{\Delta H}^2$ for each (n_r, n_t, n).

(5) ε_{n_r,n_t}, $\Delta\varepsilon_{n_r,n_t}$, $H_{n_r,n_t}^{(n)}$, and $\Delta H_{n_r,n_t}^{(n)}$ are independent of each other for each (n_r, n_t).

Given these assumptions, let us first define $\Delta\lambda_{n_r,n_t}^{(n)} = \lambda_{n_r,n_t}^{(n)} - \widehat{\lambda}_{n_r,n_t}^{(n)}$ as the interference contributed by the nth subcarrier of the interfering transmit antennas, that is, the co-subcarrier inter-antenna-interference (CSIAI), and define $\Delta\xi_{n_r,n_t}^{(n)} = \xi_{n_r,n_t}^{(n)} - \widehat{\xi}_{n_r,n_t}^{(n)}$ as the ICI contributed by the subcarriers other than the nth subcarrier of the interfering transmit antennas, that is, the intercarrier-interantenna interference (ICIAI). Then we derive $\text{Var}\{\alpha_{n_r,n_t}^{(n)}\}$ and $\text{Var}\{\beta_{n_r,n_t}^{(n)}\}$ as

$$\begin{aligned}\text{Var}\left\{\alpha_{n_r,n_t}^{(n)}\right\} &= \frac{E_s}{N_t}\cdot\mathbb{E}\left\{\left|C_{H_{n_r,n_t}^{(n)}H_{n_r,n_t}^{(n)}}^{-1}\right|^2\sum_{l\neq n}\left|m_{n_r,n_t}^{(n,l)}C_{H_{n_r,n_t}^{(l)}H_{n_r,n_t}^{(n)}}\right|^2\right\}\\ &\cong \frac{E_s}{N_t}\cdot\mathbb{E}\left\{\sum_{l\neq n}\left|\frac{\sin(\pi\Delta\varepsilon_{n_r,n_t})}{N\sin[\pi(l-n)/N]}\right|^2\right.\\ &\quad\left.\cdot\left|\sum_{d=0}^{L_{\max}-1}\mathbb{E}\left\{\left|h_{n_r,n_t}(d)\right|^2\right\}e^{-j2\pi d(l-n)/N}\right|^2\right\}\\ &= \frac{\pi^2\sigma_{\text{res}}^2E_s}{N_t}\sum_{l\neq n}\frac{\left|C_{H_{n_r,n_t}^{(n)}H_{n_r,n_t}^{(l)}}\right|^2}{N^2\sin^2[\pi(l-n)/N]},\end{aligned} \quad (5)$$

$$\begin{aligned}\text{Var}\left\{\beta_{n_r,n_t}^{(n)}\right\} &= \frac{E_s}{N_t}\\ &\quad\cdot\mathbb{E}\left\{\sum_{l\neq n}\left|m_{n_r,n_t}^{(n,l)}\right|^2\right.\\ &\quad\left.\times\left(C_{H_{n_r,n_t}^{(l)}H_{n_r,n_t}^{(l)}} - C_{H_{n_r,n_t}^{(n)}H_{n_r,n_t}^{(n)}}^{-1}\left|C_{H_{n_r,n_t}^{(l)}H_{n_r,n_t}^{(n)}}\right|^2\right)\right\}\\ &\cong \frac{\pi^2\sigma_{\text{res}}^2E_s}{3N_t} - \text{Var}\left\{\alpha_{n_r,n_t}^{(n)}\right\},\end{aligned} \quad (6)$$

where $C_{H_{n_r,n_t}^{(l)}H_{n_r,n_t}^{(n)}}$ is given by (1). The demodulation of $\mathbf{x}_{n_t}[n]$ is degraded by either $\eta_{n_r,n_t}^{(n)}$ or IAI (CSIAI plus ICIAI). In this paper, we assume that the integer part of the frequency offset has been estimated and corrected, and only the fractional part frequency offset is considered. Considering small frequency offsets, the following requirements are assumed to be satisfied:

(1) $|\varepsilon_{n_r,i}| \ll 1$ for all (n_r, i),

(2) $|\varepsilon_{n_r,n_t}| + |\varepsilon_{n_r,i}| < 1$ for all (n_r, n_t, i),

(3) $|\widehat{\varepsilon}_{n_r,n_t}| + |\widehat{\varepsilon}_{n_r,i}| < 1$ for all (n_r, n_t, i).

Condition 1 requires that each frequency offset should be much smaller than 1, and conditions 2 and 3 require that the sum of any two frequency offsets (and the frequency offset estimation results) should not exceed 1. The last two conditions are satisfied only if the estimation error does not exceed 0.5. If all these three conditions are satisfied simultaneously, we can represent $\lambda_{n_r,n_t}^{(n)}$, $\widehat{\lambda}_{n_r,n_t}^{(n)}$, $\xi_{n_r,n_t}^{(n)}$, and $\widehat{\xi}_{n_r,n_t}^{(n)}$ as

$$\begin{aligned}\lambda_{n_r,n_t}^{(n)} &= \sqrt{\frac{E_s}{N_t}}\sum_{i=1,i\neq n_t}^{N_t} m_{n_r,i}^{(n,n)}H_{n_r,i}^{(n)}\mathbf{x}_i[n]\\ &= \sqrt{\frac{E_s}{N_t}}\sum_{i=1,i\neq n_t}^{N_t}\frac{\sin[\pi(\varepsilon_{n_r,i}-\widehat{\varepsilon}_{n_r,n_t})]}{N\sin[\pi(\varepsilon_{n_r,i}-\widehat{\varepsilon}_{n_r,n_t})/N]}H_{n_r,i}^{(n)}\mathbf{x}_i[n],\end{aligned} \quad (7)$$

$$\begin{aligned}\widehat{\lambda}_{n_r,n_t}^{(n)} &= \sqrt{\frac{E_s}{N_t}}\sum_{i=1,i\neq n_t}^{N_t} \widehat{m}_{n_r,i}^{(n,n)}\widehat{H}_{n_r,i}^{(n)}\mathbf{x}_i[n]\\ &= \sqrt{\frac{E_s}{N_t}}\sum_{i=1,i\neq n_t}^{N_t}\frac{\sin[\pi(\widehat{\varepsilon}_{n_r,i}-\widehat{\varepsilon}_{n_r,n_t})]}{N\sin[\pi(\widehat{\varepsilon}_{n_r,i}-\widehat{\varepsilon}_{n_r,n_t})/N]}\widehat{H}_{n_r,i}^{(n)}\mathbf{x}_i[n],\end{aligned} \quad (8)$$

$$\xi^{(n)}_{n_r,n_t} = \sqrt{\frac{E_s}{N_t}}\sum_{l\neq n}\sum_{i=1,i\neq n_t}^{N_t} m^{(n,l)}_{n_r,i} H^{(l)}_{n_r,i}\mathbf{x}_i[l]$$
$$\cong \sqrt{\frac{E_s}{N_t}}\sum_{l\neq n}\sum_{i=1,i\neq n_t}^{N_t} \frac{(-1)^{(l-n)}\sin[\pi(\varepsilon_{n_r,i}-\hat{\varepsilon}_{n_r,n_t})]}{N\sin[\pi(l-n)/N]} \times e^{j\pi(N-1)(l-n)/N} H^{(l)}_{n_r,i}\mathbf{x}_i[l], \quad (9)$$

$$\hat{\xi}^{(n)}_{n_r,n_t} = \sqrt{\frac{E_s}{N_t}}\sum_{l\neq n}\sum_{i=1,i\neq n_t}^{N_t} \hat{m}^{(n,l)}_{n_r,i} \hat{H}^{(l)}_{n_r,i}\mathbf{x}_i[l]$$
$$\cong \sqrt{\frac{E_s}{N_t}}\sum_{l\neq n}\sum_{i=1,i\neq n_t}^{N_t} \frac{(-1)^{(l-n)}\sin[\pi(\hat{\varepsilon}_{n_r,i}-\hat{\varepsilon}_{n_r,n_t})]}{N\sin[\pi(l-n)/N]} \times e^{j\pi(N-1)(l-n)/N} \hat{H}^{(l)}_{n_r,i}\mathbf{x}_i[l]. \quad (10)$$

Therefore, the interference due to the nth subcarrier of transmit antennas (other than the n_tth transmit antenna, i.e., the interfering antennas) is

$$\Delta\lambda^{(n)}_{n_r,n_t} = \lambda^{(n)}_{n_r,n_t} - \hat{\lambda}^{(n)}_{n_r,n_t} = \sqrt{\frac{E_s}{N_t}} \cdot \sum_{i=1,i\neq n_t}^{N_t}\left[\frac{\pi^2(\varepsilon_{n_r,i}-\hat{\varepsilon}_{n_r,n_t}+(\Delta\varepsilon_{n_r,i}/2))H^{(n)}_{n_r,i}\Delta\varepsilon_{n_r,i}}{3} - \left(1-\frac{\pi^2(\hat{\varepsilon}_{n_r,i}-\hat{\varepsilon}_{n_r,n_t})^2}{6}\right)\Delta H^{(n)}_{n_r,i}\right]\mathbf{x}_i[n] + o(\Delta\varepsilon_{n_r,i},\Delta H_{n_r,i}), \quad (11)$$

$$\Delta\xi^{(n)}_{n_r,n_t} = \xi^{(n)}_{n_r,n_t} - \hat{\xi}^{(n)}_{n_r,n_t} = \sqrt{\frac{E_s}{N_t}}\sum_{l\neq n}\sum_{i=1,i\neq n_t}^{N_t}\frac{(-1)^{l-n+1}e^{j\pi(N-1)(l-n)/N}}{N\sin[\pi(l-n)/N]} \cdot\left[\pi\cos\left(\pi\left(\varepsilon_{n_r,i}-\hat{\varepsilon}_{n_r,n_t}+\frac{\Delta\varepsilon_{n_r,i}}{2}\right)\right)H^{(l)}_{n_r,i}\Delta\varepsilon_{n_r,i} + \sin(\pi(\hat{\varepsilon}_{n_r,i}-\hat{\varepsilon}_{n_r,n_t}))\Delta H^{(l)}_{n_r,i}\right]\mathbf{x}_i[l] + o(\Delta\varepsilon_{n_r,i},\Delta H_{n_r,i}) \quad (12)$$

with $o(\Delta\varepsilon_{n_r,i},\Delta H_{n_r,i})$ representing the higher-order item of $\Delta\varepsilon_{n_r,i}$ and $\Delta H_{n_r,i}$. It is easy to show that $\Delta\lambda^{(n)}_{n_r,n_t}$ and $\Delta\xi^{(n)}_{n_r,n_t}$ are zero-mean RVs and that their variances are given by

$$\mathbb{E}\left\{\left|\Delta\lambda^{(n)}_{n_r,n_t}\right|^2\right\} = \frac{E_s}{N_t}\sum_{i=1,i\neq n_t}^{N_t} \times\mathbb{E}\left\{\left[\frac{\pi^2(\varepsilon_{n_r,i}-\hat{\varepsilon}_{n_r,n_t}+(\Delta\varepsilon_{n_r,i}/2))H^{(n)}_{n_r,i}\Delta\varepsilon_{n_r,i}}{3}\right]^2\right\} + \frac{E_s}{N_t}\sum_{i=1,i\neq n_t}^{N_t}\mathbb{E}\left\{\left[\left(1-\frac{\pi^2(\hat{\varepsilon}_{n_r,i}-\hat{\varepsilon}_{n_r,n_t})^2}{6}\right)\Delta H^{(n)}_{n_r,i}\right]^2\right\}$$
$$\cong \frac{(N_t-1)\pi^4E_s}{9N_t}\left(2\sigma^2_\epsilon\sigma^2_{\text{res}}+\sigma^4_{\text{res}}+\frac{\mathbb{E}\left\{\Delta\varepsilon^4_{n_r,i}\right\}}{4}\right)+\frac{(N_t-1)E_s}{N_t} \cdot\sigma^2_{\Delta H}\cdot\left[1+\frac{\pi^4\left(\mathbb{E}\left\{\varepsilon^4_{n_r,i}\right\}+8\sigma^2_\epsilon\sigma^2_{\text{res}}+2\sigma^4_\epsilon+2\sigma^4_{\text{res}}\right)}{18} - \frac{2\pi^2(\sigma^2_\epsilon+\sigma^2_{\text{res}})}{3}\right], \quad (13)$$

$$\mathbb{E}\left\{\left|\Delta\xi^{(n)}_{n_r,n_t}\right|^2\right\} = \frac{E_s}{N_t}\sum_{l\neq n}\sum_{i=1,i\neq n_t}^{N_t}\frac{1}{N^2\sin^2[\pi(l-n)/N]} \cdot\mathbb{E}\left\{\left[\pi\cos\left(\pi\left(\varepsilon_{n_r,i}-\hat{\varepsilon}_{n_r,n_t}+\frac{\Delta\varepsilon_{n_r,i}}{2}\right)\right)H^{(l)}_{n_r,i}\Delta\varepsilon_{n_r,i} + \sin(\pi(\hat{\varepsilon}_{n_r,i}-\hat{\varepsilon}_{n_r,n_t}))\Delta H^{(l)}_{n_r,i}\right]^2\right\}$$
$$\cong\frac{(N_t-1)E_s}{3N_t}\left[\pi^2\sigma^2_{\text{res}}-\pi^4\left(2\sigma^2_\epsilon\sigma^2_{\text{res}}+\sigma^4_{\text{res}}+\frac{\mathbb{E}\left\{\Delta\varepsilon^4_{n_r,i}\right\}}{4}\right)\right] + \frac{2(N_t-1)\pi^2E_s}{3N_t}(\sigma^2_\epsilon+\sigma^2_{\text{res}})\sigma^2_{\Delta H}, \quad (14)$$

respectively. After averaging out frequency offset ε_{n_r,n_t}, frequency offset estimation error $\Delta\varepsilon_{n_r,n_t}$, and channel estimation error $\Delta H^{(n)}_{n_r,n_t}$ for all (n_r,n_t), the average SINR of $r_{n_r,n_t}[n]$

(parameterized by only $H^{(n)}_{n_r,n_t}$) is

$$\overline{\gamma}_{n_r,n_t}\left(n \mid H^{(n)}_{n_r,n_t}\right)$$

$$\triangleq \frac{\mathbb{E}\left\{\left|\sqrt{E_s/N_t}m^{(n,n)}_{n_r,n_t}H^{(n)}_{n_r,n_t}\mathbf{x}_i[n]\right|^2\right\}}{\mathbb{E}\left\{\left|\eta^{(n)}_{n_r,n_t}+\Delta\lambda^{(n)}_{n_r,n_t}+\Delta\xi^{(n)}_{n_r,n_t}+\widetilde{\mathbf{w}}_{n_r,n_t}[n]\right|^2\right\}}$$

$$\cong \frac{E_s/N_t\cdot\sigma_m^2\cdot\left|H^{(n)}_{n_r,n_t}\right|^2}{\left|H^{(n)}_{n_r,n_t}\right|^2\cdot\mathrm{Var}\left\{\alpha^{(n)}_{n_r,n_t}\right\}+\nu},$$

$$\nu = \pi^2\sigma^2_{\text{res}}E_s/3N_t$$

$$-\mathrm{Var}\left\{\alpha^{(n)}_{n_r,n_t}\right\}+\mathbb{E}\left\{\left|\Delta\lambda^{(n)}_{n_r,n_t}\right|^2\right\}$$

$$+\mathbb{E}\left\{\left|\Delta\xi^{(n)}_{n_r,n_t}\right|^2\right\}+\sigma_w^2 \qquad (15)$$

where $\sigma_m^2 = \mathbb{E}\{|m^{(n,n)}_{n_r,n_t}|^2\} \cong 1-\pi^2\sigma^2_{\text{res}}/3+\pi^4\mathbb{E}\{\Delta\varepsilon^4_{n_r,i}\}/36$ and ν, independent of (n_r, n_t, n).

For signal demodulation in MIMO-OFDM, signal received in multiple receive antennas can be exploited to improve the receive SINR. In the following, equal gain combining (EGC) and maximal ratio combining (MRC) are considered.

3.2. SINR Analysis with EGC at Receive Antennas. In order to demodulate the signal transmitted by the n_tth transmit antenna, the N_r received signals are cophased and combined to improve the receiving diversity. Therefore, the EGC output is given by

$$\mathbf{r}^{\text{EGC}}_{n_t}[n] = \sum_{n_r=1}^{N_r} e^{-j\theta^{(n)}_{n_r,n_t}}\mathbf{r}_{n_r,n_t}[n]$$

$$= \sum_{n_r=1}^{N_r}\sqrt{\frac{E_s}{N_t}}e^{-j\theta^{(n)}_{n_r,n_t}}m^{(n,n)}_{n_r,n_t}H^{(n)}_{n_r,n_t}\mathbf{x}_{n_t}[n]$$

$$+\sum_{n_r=1}^{N_r}e^{-j\theta^{(n)}_{n_r,n_t}}\left(\eta^{(n)}_{n_r,n_t}+\Delta\lambda^{(n)}_{n_r,n_t}+\Delta\xi^{(n)}_{n_r,n_t}+\widetilde{\mathbf{w}}_{n_r,n_t}[n]\right), \qquad (16)$$

where $\theta^{(n)}_{n_r,n_t} = \arg\{m^{(n,n)}_{n_r,n_t}H^{(n)}_{n_r,n_t}\}$. After averaging out ε_{n_r,n_t}, $\Delta\varepsilon_{n_r,n_t}$, and $\Delta H^{(n)}_{n_r,n_t}$ for each (n_r, n_t), the average SINR of $r^{\text{EGC}}_{n_t}[n]$ is given by

$$\overline{\gamma}^{\text{EGC}}_{n_t}\left(n \mid H^{(n)}_{1,n_t},\ldots,H^{(n)}_{N_r,n_t}\right)$$

$$\triangleq \frac{\mathbb{E}\left\{\left|\sum_{n_r=1}^{N_r}\sqrt{E_s/N_t}e^{-j\theta^{(n)}_{n_r,n_t}}m^{(n,n)}_{n_r,n_t}H^{(n)}_{n_r,n_t}\mathbf{x}_{n_t}[n]\right|^2\right\}}{\mathbb{E}\left\{\left|\sum_{n_r=1}^{N_r}e^{-j\theta^{(n)}_{n_r,n_t}}\left(\eta^{(n)}_{n_r,n_t}+\Delta\lambda^{(n)}_{n_r,n_t}+\Delta\xi^{(n)}_{n_r,n_t}+\widetilde{\mathbf{w}}_{n_r,n_t}[n]\right)\right|^2\right\}}$$

$$\cong \frac{E_s/N_t\cdot\sigma_m^2\cdot\left(\sum_{n_r=1}^{N_r}\left|H^{(n)}_{n_r,n_t}\right|^2+\sum_{n_r\neq l}\left|H^{(n)}_{n_r,n_t}\right|\cdot\left|H^{(n)}_{l,n_t}\right|\right)}{\sum_{n_r=1}^{N_r}\left|H^{(n)}_{n_r,n_t}\right|^2\cdot\mathrm{Var}\left\{\alpha^{(n)}_{n_r,n_t}\right\}+N_r\nu}. \qquad (17)$$

When N_r is large enough, (17) can be further simplified as

$$\overline{\gamma}^{\text{EGC}}_{n_t}\left(n \mid H^{(n)}_{1,n_t},\ldots,H^{(n)}_{N_r,n_t}\right)$$

$$\cong \frac{E_s/N_t\cdot\sigma_m^2\cdot\left(\sum_{n_r=1}^{N_r}\left|H^{(n)}_{n_r,n_t}\right|^2+N_r(N_r-1)\pi/4\right)}{\sum_{n_r=1}^{N_r}\left|H^{(n)}_{n_r,n_t}\right|^2\cdot\mathrm{Var}\left\{\alpha^{(n)}_{n_r,n_t}\right\}+N_r\nu}. \qquad (18)$$

3.3. SINR Analysis with MRC at Receive Antennas. In a MIMO-OFDM system with N_r receive antennas, based on the channel estimation $\hat{H}^{(n)}_{n_r,n_t} = H^{(n)}_{n_r,n_t}+\Delta H^{(n)}_{n_r,n_t}$ for each (n_r, n_t, n), the received signal at all the N_r receive antennas can be combined by using MRC, and therefore the combined output is given by

$$\mathbf{r}^{\text{MRC}}_{n_t}[n]$$

$$= \frac{\sum_{n_r=1}^{N_r}\omega_{n_r,n_t}\mathbf{r}_{n_r,n_t}[n]}{\sum_{n_r=1}^{N_r}\left|\omega_{n_r,n_t}\right|^2}$$

$$= \frac{\sqrt{E_s/N_t}\sum_{n_r=1}^{N_r}\left|H^{(n)}_{n_r,n_t}\right|^2\left|m^{(n,n)}_{n_r,n_t}\right|^2\mathbf{x}_{n_t}[n]}{\sum_{n_r=1}^{N_r}\left|\omega_{n_r,n_t}\right|^2}$$

$$+\frac{\sqrt{E_s/N_t}\sum_{n_r=1}^{N_r}\Delta H^{(n)H}_{n_r,n_t}H^{(n)}_{n_r,n_t}\left|m^{(n,n)}_{n_r,n_t}\right|^2\mathbf{x}_{n_t}[n]}{\sum_{n_r=1}^{N_r}\left|\omega_{n_r,n_t}\right|^2}$$

$$+\frac{\sum_{n_r=1}^{N_r}\omega_{n_r,n_t}\left(\eta^{(n)}_{n_r,n_t}+\Delta\lambda^{(n)}_{n_r,n_t}+\Delta\xi^{(n)}_{n_r,n_t}+\widetilde{\mathbf{w}}_{n_r,n_t}[n]\right)}{\sum_{n_r=1}^{N_r}\left|\omega_{n_r,n_t}\right|^2}, \qquad (19)$$

where $\omega_{n_r,n_t} = (\hat{H}^{(n)}_{n_r,n_t} m^{(n,n)}_{n_r,n_t})^*$. After averaging out ε_{n_r,n_t}, $\Delta\varepsilon_{n_r,n_t}$, and $\Delta H^{(n)}_{n_r,n_t}$ for each (n_r, n_t), the average SINR of $r^M_{n_t}[n]$ is

$$\begin{aligned}
&\overline{\gamma}^{\mathrm{MRC}}_{n_t}\left(n \mid H^{(n)}_{1,n_t},\dots,H^{(n)}_{N_r,n_t}\right)\\
&\triangleq \frac{\mathbb{E}\left\{\left|\sqrt{E_s/N_t}\mathfrak{A}\left|m^{(n,n)}_{n_r,n_t}\right|^2 \mathbf{x}_{n_t}[n]\right|^2\right\}}{\mathbb{E}\left\{\left|\sqrt{E_s/N_t}\sum_{n_r=1}^{N_r}\Delta H^{(n)*}_{n_r,n_t}H^{(n)}_{n_r,n_t}\left|m^{(n,n)}_{n_r,n_t}\right|^2\mathbf{x}_{n_t}[n]\right|^2\right\}+\aleph'}\\
&\cong \frac{E_s/N_t\cdot\sigma_m^2\cdot\mathfrak{A}}{\left(\mathfrak{A}-\sum_{n_r\neq l}\mathfrak{A}\left|H^{(n)}_{l,n_t}\right|^2/\mathfrak{A}\right)\mathrm{Var}\left\{\alpha^{(n)}_{n_r,n_t}\right\}+\nu'+N_r\cdot\nu\cdot\sigma^2_{\Delta H}/\mathfrak{A}},\\
&\mathfrak{A} = \sum_{n_r=1}^{N_r}\left|H^{(n)}_{n_r,n_t}\right|^2
\end{aligned}\tag{20}$$

where we have defined $\nu' = [\nu + (E_s/N_t + \mathrm{Var}\{\alpha^{(n)}_{n_r,n_t}\})\sigma^2_{\Delta H}]$, and the noise part can be represented as $\aleph' = \mathbb{E}\{|\sum_{n_r=1}^{N_r}\omega^*_{n_r,n_t}(\eta^{(n)}_{n_r,n_t}+\Delta\lambda^{(n)}_{n_r,n_t}+\Delta\xi^{(n)}_{n_r,n_t}+\mathbf{w}_{n_r,n_t}[n])|^2\}$. When N_r is large enough, (20) can be further simplified as

$$\begin{aligned}
&\overline{\gamma}^{\mathrm{MRC}}_{n_t}\left(n \mid H^{(n)}_{1,n_t},\dots,H^{(n)}_{N_r,n_t}\right)\\
&\cong \frac{E_s/N_t\cdot\sigma_m^2\cdot\mathfrak{A}}{\left(\mathfrak{A}-\sum_{n_r\neq l}\left|H^{(n)}_{n_r,n_t}\right|^2\left|H^{(n)}_{l,n_t}\right|^2/\mathfrak{A}\right)\mathrm{Var}\left\{\alpha^{(n)}_{n_r,n_t}\right\}+\nu'+N_r\cdot\nu\cdot\sigma^2_{\Delta H}/\mathfrak{A}}\\
&\cong \frac{E_s/N_t\cdot\sigma_m^2\cdot\mathfrak{A}}{(\mathfrak{A}-(N_r-1))\,\mathrm{Var}\left\{\alpha^{(n)}_{n_r,n_t}\right\}+\nu'+\nu\cdot\sigma^2_{\Delta H}}.\\
&\mathfrak{A} = \sum_{n_r=1}^{N_r}\left|H^{(n)}_{n_r,n_t}\right|^2
\end{aligned}\tag{21}$$

4. BER Performance

The BER as a function of SINR in MIMO-OFDM is derived in this section. We consider M-ary square QAM with Gray bit mapping. In the work of Rugini and Banelli [11], the BER of SISO-OFDM with frequency offset is developed. The BER analysis in [11] is now extended to MIMO-OFDM.

As discussed in [11, 28, 29], the BER for the n_tth transmit antenna with the input constellation being M-ary square QAM (Gray bit mapping) can be represented as

$$P_{\mathrm{BER}}(\gamma_{n_t}) = \sum_{i=1}^{\sqrt{M}-1} a_i^M \,\mathrm{erfc}\left(\sqrt{b_i^M\gamma_i}\right),\tag{22}$$

where a_i^M and b_i^M are specified by signal constellation, γ_{n_t} is the average SINR of the n_tth transmit antenna, and $\mathrm{erfc}(x) = (2/\sqrt{\pi})\int_x^\infty e^{-u^2}du$ is the error function (Please refer to [28] for the meaning of a_i^M and b_i^M.).

Note that in MIMO-OFDM systems, the SINR at each subcarrier is an RV parameterized by the frequency offset and channel attenuation. In order to derive the average SINR of MIMO-OFDM systems, (22) should be averaged over the distribution of γ_i as

$$\begin{aligned}
\overline{P}_{\mathrm{BER}}(\gamma_{n_t}) &= \sum_{i=1}^{\sqrt{M}-1} a_i^M \int_{\gamma_{n_t}} \mathrm{erfc}\left(\sqrt{b_i^M\gamma_{n_t}}\right) f(\gamma_{n_t})\,d\gamma_{n_t}\\
&= \sum_{i=1}^{\sqrt{M}-1} a_i^M \int_{\mathbf{H}_{n_t}}\int_{\mathcal{E}_{n_t}}\int_{\mathbf{v}_{n_t}}\int_{\mathbf{\Phi}_{n_t}}\\
&\quad \mathrm{erfc}\left(\sqrt{b_i^M\gamma_{n_t}}\right)\cdot f(\mathbf{H}_{n_t}) f(\mathcal{E}_{n_t}) f(\mathbf{v}_{n_t})\\
&\quad \times f(\mathbf{\Phi}_{n_t})\,d\mathbf{H}_{n_t}\,d\mathcal{E}_{n_t}\,d\mathbf{v}_{n_t}\,d\mathbf{\Phi}_{n_t},
\end{aligned}\tag{23}$$

where $\mathbf{H}_{n_t} = [\mathbf{H}_{1,n_t},\dots,\mathbf{H}_{N_r,n_t}]$, $\mathcal{E}_{n_t} = [\varepsilon_{1,n_t},\dots,\varepsilon_{N_r,n_t}]^T$, $\mathbf{v}_{n_t} = [\Delta\varepsilon_{1,n_t},\dots,\Delta\varepsilon_{N_r,n_t}]^T$, and $\mathbf{\Phi}_{n_t} = [\Delta\mathbf{H}_{1,n_t},\dots,\Delta\mathbf{H}_{N_r,n_t}]$. Since obtaining a close-form solution of (23) appears impossible, an infinite-series approximation of $\overline{P}_{\mathrm{BER}}$ is developed. In [11], the average is expressed as an infinite series of generalized hypergeometric functions.

From [30, page 939], $\mathrm{erfc}(x)$ can be represented as an infinite series:

$$\mathrm{erfc}(x) = \frac{2}{\sqrt{\pi}}\sum_{m=1}^{\infty}(-1)^{(m+1)}\frac{x^{(2m-1)}}{(2m-1)(m-1)!}.\tag{24}$$

Therefore, (23) can be rewritten as

$$\begin{aligned}
\overline{P}_{\mathrm{BER}}(\gamma_{n_t}) &= \frac{2}{\sqrt{\pi}}\sum_{i=1}^{\sqrt{M}-1} a_i^M \sum_{m=1}^{\infty}\frac{(-1)^{(m+1)}\left(b_i^M\right)^{(m-1/2)}}{(2m-1)(m-1)!}\cdot D_{n_t;m},\\
D_{n_t;m} &= \int_{\mathbf{H}_{n_t}}\int_{\mathcal{E}_{n_t}}\int_{\mathbf{v}_{n_t}}\int_{\mathbf{\Phi}_{n_t}}(\gamma_{n_t})^{(m-1/2)} f(\mathbf{H}_{n_t})\\
&\quad\times f(\mathcal{E}_{n_t}) f(\mathbf{v}_{n_t}) f(\mathbf{\Phi}_{n_t})\,d\mathbf{H}_i\,d\mathcal{E}_{n_t}\,d\mathbf{v}_{n_t}\,d\mathbf{\Phi}_{n_t}
\end{aligned}\tag{25}$$

where $D_{n_t;m}$ depends on the type of combining. Note that γ_{n_t} has been derived in Section 3 and that for the nth subcarrier $(0 \le n \le N-1)$, ε_{n_r,n_t}, $\Delta\varepsilon_{n_r,n_t}$ and $\Delta H^{(n)}_{n_r,n_t}$ for each (n_r, n_t) have been averaged out. Therefore, γ_{n_t} in (25) can be replaced by $\overline{\gamma}_{n_t}(n)$; that is, the average BER can be expected over subcarrier n $(0 \le n \le N-1)$, and finally $\overline{P}_{\mathrm{BER}}$ can be simplified as

$$\begin{aligned}
&\overline{P}_{\mathrm{BER}}\left(\overline{\gamma}_{n_t}(n)\right)\\
&\quad= \frac{2}{\sqrt{\pi}}\sum_{i=1}^{\sqrt{M}-1} a_i^M \sum_{m=1}^{\infty}\frac{(-1)^{(m+1)}\left(b_i^M\right)^{(m-(1/2))}}{(2m-1)(m-1)!}\cdot D_{n_t;m},
\end{aligned}\tag{26}$$

where $D_{n_t;m}$ is based on $\overline{\gamma}_{n_t}(n)$ instead of γ_{n_t}. We first define $\varpi = E_s/N_t\cdot\sigma_m^2$ and $\mu = \mathrm{Var}\{\alpha^{(n)}_{n_r,n_t}\}$, which will be used in the following subsections. We next give a recursive definition for

$D_{n_t;m}$ for the following reception methods: (1) demodulation without combining, (2) EGC, and (3) MRC.

Note that the SINR for each combining scenario (i.e., without combining, EGC, or MRC) is a function of the second-order statistics of the channel and frequency offset estimation errors (although the interference also comprises the fourth-order statistics of the frequency offset estimation errors, they are negligible as compared to the second-order statistics for small estimation errors). Any probability distribution with zero mean and the same variance will result in the same SINR. Therefore, the exact distributions need not be specified. However, when the BER is derived by using an infinite-series approximation, the actual distribution of the frequency offset estimation errors is required. In [31], it is shown that both the uniform distribution and Gaussian distribution are amenable to infinite-series solutions with closed-form formulas for the coefficients. In the following sections, the frequency offset estimation errors are assumed to be i.i.d. Gaussian RVs with mean zero and variance σ_ϵ^2 [10].

4.1. BER without Receiving Combining. The BER measured at the n_rth receive antenna for the n_tth transmit antenna can be approximated by (25) with $D_{n_t;m}^{n_r}$ instead of $D_{n_t;m}$ being used here; that is,

$$\overline{P}_{\mathrm{BER}}^{n_r}\left(\overline{\gamma}_{n_r,n_t}\left(n \mid H_{n_r,n_t}^{(n)}\right)\right) = \frac{2}{\sqrt{\pi}}\sum_{i=1}^{\sqrt{M}-1} a_i^M \sum_{m=1}^{\infty}\frac{(-1)^{(m+1)}\left(b_i^M\right)^{(m-1/2)}}{(2m-1)(m-1)!}\cdot D_{n_t;m}^{n_r}. \tag{27}$$

When $m > 2$, we have $D_{n_t;m}^{n_r} = \varpi[(2m-3)\mu+\nu]/\mu^2(m-3/2)\cdot D_{n_t;m-1}^{n_r} - \varpi^2/\mu^2\cdot D_{i;m-2}^{n_r}$, as derived in Appendix A. The initial condition is given by

$$D_{n_t;1}^{n_r} = \int_0^\infty \frac{\varpi^{1/2}h^{1/2}}{(\mu h+\nu)^{1/2}}e^{-h}\mathrm{d}h. \tag{28}$$

4.2. BER with EGC. For a MIMO-OFDM system with EGC reception, the average BER can be approximated by (25) with $D_{n_t;m}^{\mathrm{EGC}}$ instead of $D_{n_t;m}$ being used here; that is,

$$\overline{P}_{\mathrm{BER}}^{\mathrm{EGC}}\left(\gamma_{n_t}^{\mathrm{EGC}}\left(n \mid H_{1,n_t}^{(n)},\dots,H_{N_r,n_t}^{(n)}\right)\right) = \frac{2}{\sqrt{\pi}}\sum_{i=1}^{\sqrt{M}-1} a_i^M \sum_{m=1}^{\infty}\frac{(-1)^{(m+1)}\left(b_i^M\right)^{(m-1/2)}}{(2m-1)(m-1)!}\cdot D_{n_t;m}^{\mathrm{EGC}}. \tag{29}$$

Defining $\nu^E = N_r\nu$, $\sigma_{\mathrm{EGC}}^2 = (N_r!)^2/8[(N_r-(1/2))\cdots 1/2]^2$, $\widetilde{\nu}^E = \nu^E - \mu N_r(N_r-1)\pi/4$, and $\widetilde{\mu} = 2\sigma_{\mathrm{EGC}}^2\cdot\mu$, when $m > 2$, we have

$$D_{n_t;m}^{\mathrm{EGC}} = \frac{2\sigma_{\mathrm{EGC}}^2\varpi\left[(2m+N_r-4)\widetilde{\mu}(N_r-1)!+\widetilde{\nu}^E\right]}{\widetilde{\mu}^2(m-3/2)(N_r-1)!}\cdot D_{n_t;m-1}^{\mathrm{EGC}} - \frac{\left(2\sigma_{\mathrm{EGC}}^2\varpi\right)^2(m+N_r-5/2)}{\widetilde{\mu}^2(m-3/2)}\cdot D_{n_t;m-2}^{\mathrm{EGC}} \tag{30}$$

Table 1: Parameters for BER simulation in MIMO-OFDM systems.

Subcarrier modulation	QPSK; 16QAM
DFT length	128
σ_{res}^2	10^{-3};10^{-4}
$\sigma_{\Delta H}^2$	10^{-4}
MIMO parameters	($N_t = 1, 2$; $N_r = 1, 2, 4$)
Receiving combining	Without combining; EGC; MRC

as derived in Appendix B. The initial condition is given by

$$D_{n_t;1}^{\mathrm{EGC}} = \frac{\left(2\sigma_{\mathrm{EGC}}^2\varpi\right)^{1/2}}{(N_r-1)!}\int_0^\infty \frac{h^{(N_r-1/2)}}{\left(\widetilde{\mu}h+\widetilde{\nu}^E\right)^{1/2}}e^{-h}\mathrm{d}h. \tag{31}$$

4.3. BER with MRC. For a MIMO-OFDM system with channel knowledge at the receiver, the receiving diversity can be optimized by using MRC, and the average BER can be approximated by (25) with $D_{n_t;m}^{\mathrm{MRC}}$ instead of $D_{n_t;m}$ being used here; that is,

$$\overline{P}_{\mathrm{BER}}^{\mathrm{MRC}}\left(\gamma_{n_t}^{\mathrm{MRC}}\left(n \mid H_{1,n_t}^{(n)},\dots,H_{N_r,n_t}^{(n)}\right)\right) = \frac{2}{\sqrt{\pi}}\sum_{i=1}^{\sqrt{M}-1} a_i^M \sum_{m=1}^{\infty}\frac{(-1)^{(m+1)}\left(b_i^M\right)^{(m-1/2)}}{(2m-1)(m-1)!}\cdot D_{n_t;m}^{\mathrm{MRC}}. \tag{32}$$

By defining $\nu^M = \nu' + \nu\cdot\sigma_{\Delta H}^2$, $D_{n_t;m}^{\mathrm{MRC}}$ with $m > 2$ is given by

$$D_{n_t;m}^{\mathrm{MRC}} = \frac{\varpi\left[(2m+N_r-4)\mu(N_r-1)!+\widetilde{\nu}^M\right]}{\mu^2(m-3/2)(N_r-1)!}\cdot D_{n_t;m-1}^{\mathrm{MRC}} - \frac{\varpi^2(m+N_r-5/2)e^{-(N_r-1)}}{\mu^2(m-3/2)}\cdot D_{n_t;m-2}^{\mathrm{MRC}}, \tag{33}$$

as derived in Appendix C. The initial condition is given by

$$D_{n_t;1}^{\mathrm{MRC}} = \frac{e^{-(N_r-1)}\varpi^{1/2}}{(N_r-1)!}\int_0^\infty \frac{h^{(N_r-1/2)}}{\left(\mu h+\widetilde{\nu}^M\right)^{1/2}}e^{-h}\mathrm{d}h. \tag{34}$$

4.4. Complexity of the Infinite-Series Representation of BER. Infinite-series BER expression (27), (29), or (32) must be truncated in practice. The truncation error is negligible if the number of terms is large enough: Reference [31] shows that when the number of terms is as large as 50, the finite-order approximation is good. In this case, a total of $151\sqrt{M}$ multiplication and $101\sqrt{M}$ summation operations are needed to calculate the BER for each combining scheme.

5. Numerical Results

Quasistatic MIMO wireless channels are assumed; that is, the channel impulse response is fixed over one OFDM symbol period but changes across the symbols. The simulation parameters are defined in Table 1.

The SINR degradation due to the residual frequency offsets is shown in Figure 2 for $\sigma_{\Delta H}^2 = 0.01$ and SNR = 10 dB. The SINR degradation increases with σ_{res}^2. Because of IAI due to the multiple transmit antennas, the SINR performance of

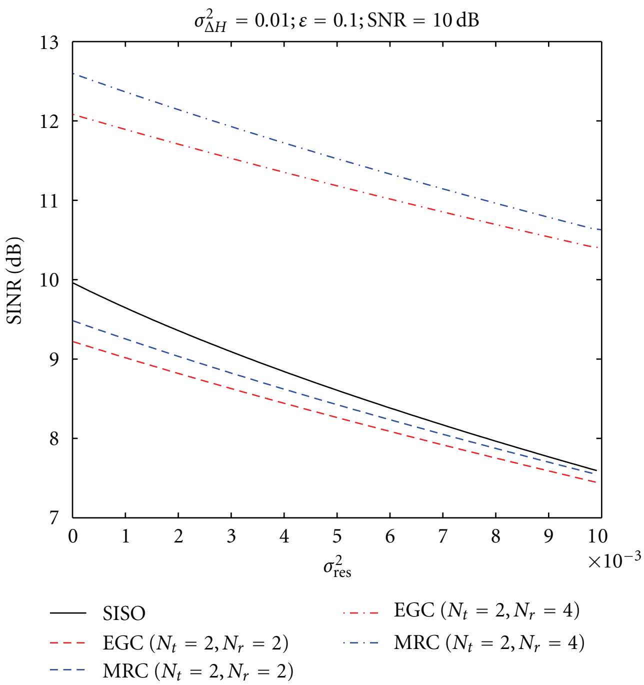

Figure 2: SINR reduction by frequency offset in MIMO-OFDM systems.

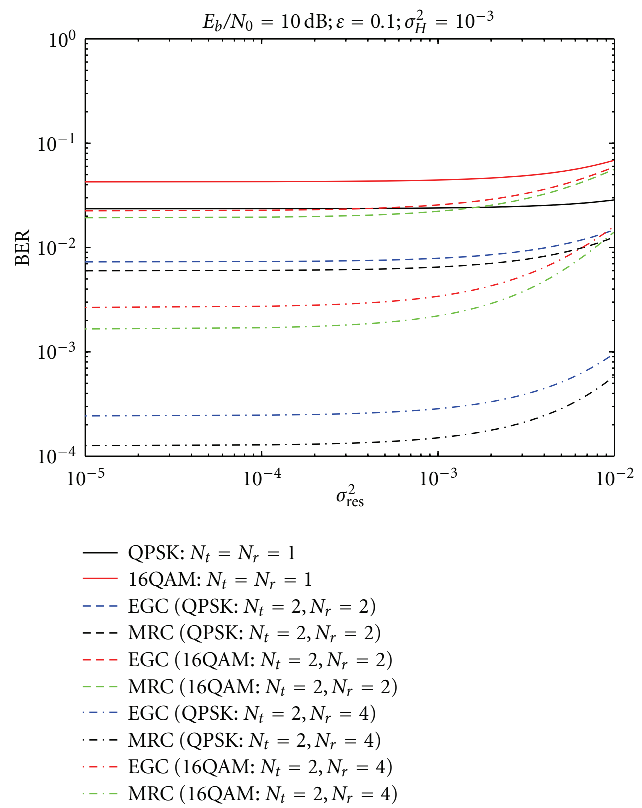

Figure 3: BER degradation due to the residual frequency offset in MIMO-OFDM systems.

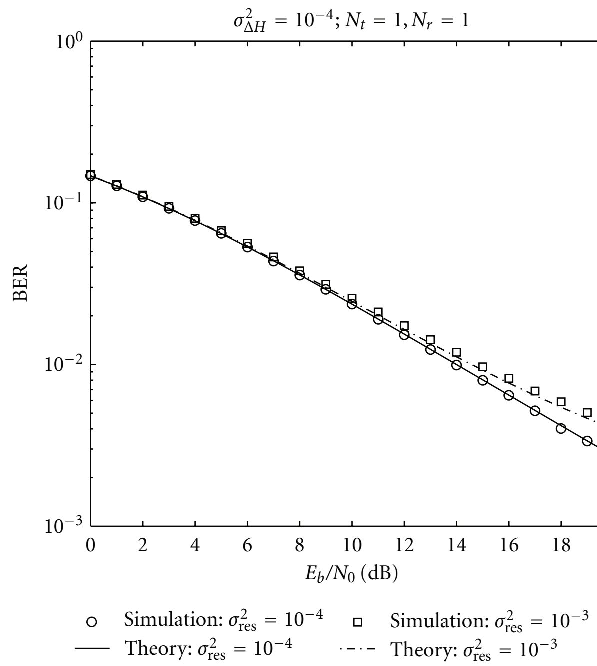

Figure 4: BER with QPSK when ($N_t = 1, N_r = 1$).

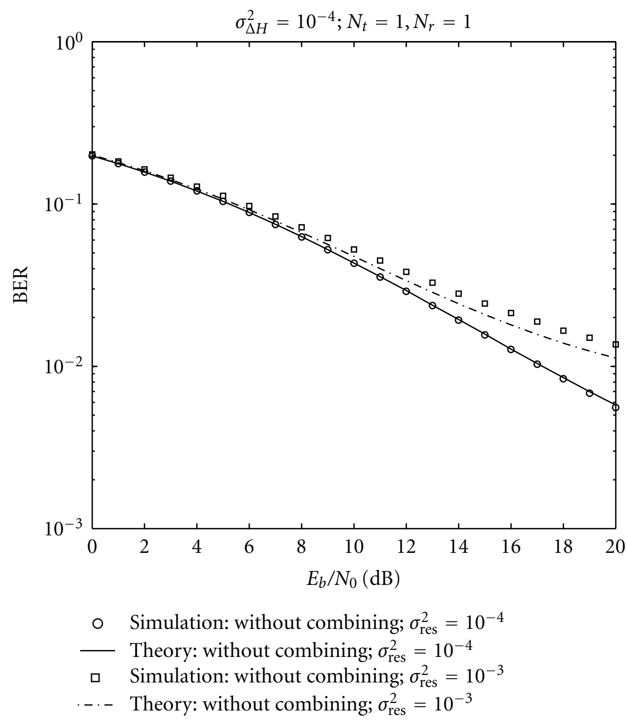

Figure 5: BER with 16QAM when ($N_t = 1, N_r = 1$).

MIMO-OFDM with ($N_t = 2, N_r = 2$) is worse than that of SISO-OFDM, even though EGC or MRC is applied to exploit the receiving diversity. IAI in MIMO-OFDM can be suppressed by increasing the number of receive antennas. In this simulation, when $N_r = 4$, the average SINR with

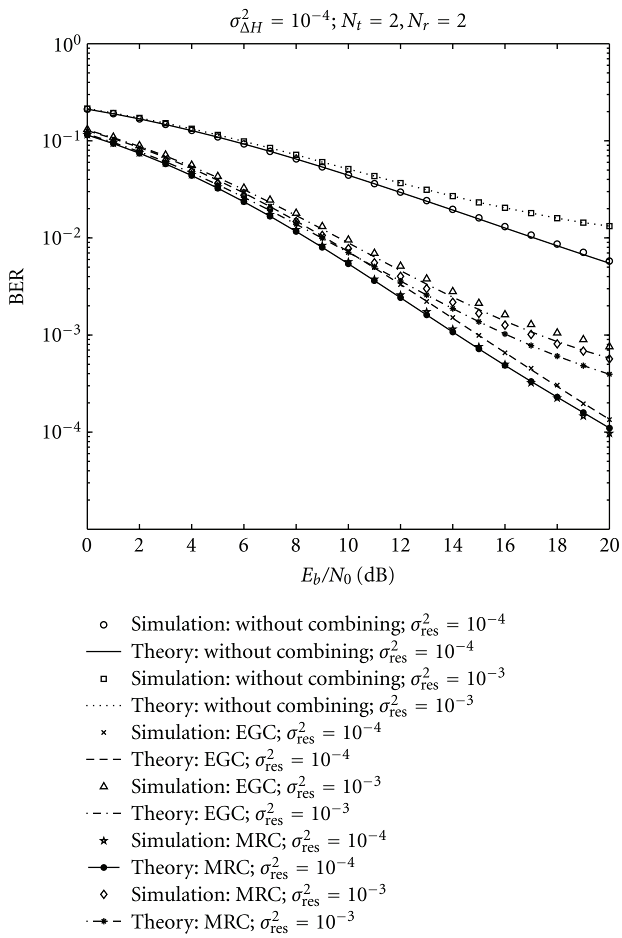

Figure 6: BER with QPSK when ($N_t = 2, N_r = 2$).

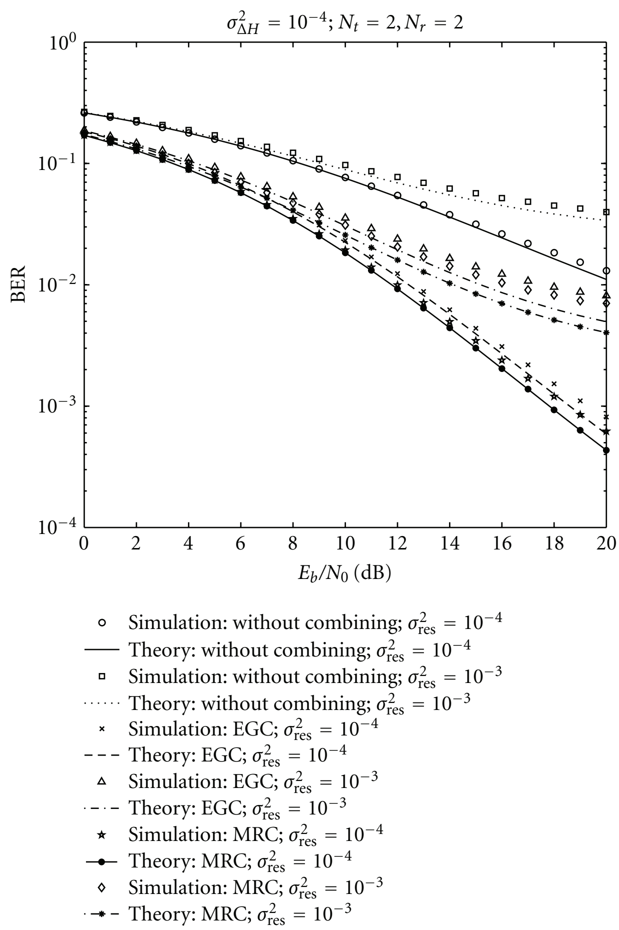

Figure 7: BER with 16QAM when ($N_t = 2, N_r = 2$).

either EGC or MRC will be higher than that of SISO-OFDM system. For each MIMO scenario, MRC outperforms EGC.

The BER degradation due to the residual frequency offsets is shown in Figure 3 for $\sigma^2_{\Delta H} = 10^{-3}$ and $E_b/N_0 = 10$ dB (E_b/N_0 is the bit energy per noise per Hz). The BER for 4-phase PSK (QPSK) or 16QAM subcarrier modulation is considered. Just as with the case of SINR, the BER degrades with large σ^2_{res}. For example, when ($N_t = 2, N_r = 2$) and $\sigma^2_{\text{res}} = 10^{-5}$ for QPSK (16QAM), a BER of 7×10^{-3} (2.5×10^{-2}) or 6×10^{-3} (2×10^{-2}) is achieved with EGC or MRC at the receiver, respectively. When σ^2_{res} is increased to 10^{-2}, a BER of 2×10^{-2} (6×10^{-2}) or 1×10^{-2} (5.5×10^{-2}) can be achieved with EGC or MRC, respectively.

Figures 4 to 9 compare BERs of QPSK and 16QAM with different combining methods. Figures 4 and 5 consider SISO-OFDM. The BER is degraded due to the frequency offset and channel estimation errors. For a fixed channel estimation variance error $\sigma^2_{\Delta H}$, a larger variance of frequency offset estimation error, that is, σ^2_{res}, implies a higher BER. For example, if $\sigma^2_{\Delta H} = 10^{-4}$, $E_b/N_0 = 20$ dB and $\sigma^2_{\text{res}} = 10^{-4}$, the BER with QPSK (16QAM) is about 1.8×10^{-3} (5.5×10^{-3}); when σ^2_{res} increases to 10^{-3}, the BER with QPSK (16QAM) increases to 4.3×10^{-3} (1.5×10^{-2}).

IAI appears with multiple transmit antennas, and the BER will degrade as IAI increases. Note that since IAI cannot be totally eliminated in the presence of the frequency offset and channel estimation errors, a BER floor occurs at the high SNR. IAI can be reduced considerably by exploiting the receiving diversity by using either EGC or MRC, as shown in Figures 6, 7, 8, and 9. Without receiver combining, the BER is much worse than that in SISO-OFDM, simply because of the SINR degradation due to IAI. For example, when $N_t = N_r = 2$ and $\sigma^2_{\Delta H} = 10^{-4}$, the BER with QPSK is about 5.5×10^{-3} when $\sigma^2_{\text{res}} = 10^{-4}$, which is three times of that of SISO-OFDM (which is about 1.8×10^{-3}), as shown in Figure 6. For a given number of receive antennas, MRC can achieve a lower BER than that achieved with EGC, but the receiver requires accurate channel estimation. For example, in Figure 7, when $\sigma^2_{\Delta H} = 10^{-4}$ with $N_t = N_r = 2$ and 16QAM, the performance improvement of EGC (MRC) over that without combining is about 5.5 dB (6 dB), and that performance improvement increases to 7.5 dB (8.5 dB) if σ^2_{res} is increased to 10^{-3}. By increasing the number of receive antennas to 4, this performance improvement is about 8.2 dB (9 dB) for EGC (MRC), with $\sigma^2_{\Delta H} = 10^{-4}$, or 11 dB (13.9 dB) for EGC (MRC), with $\sigma^2_{\Delta H} = 10^{-3}$, as shown in Figure 9.

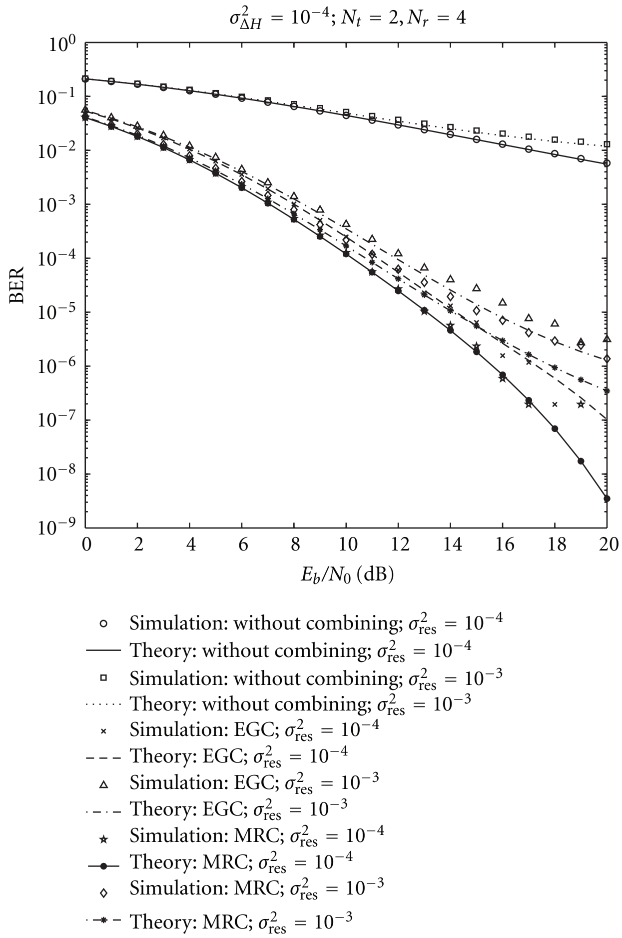

Figure 8: BER with QPSK when ($N_t = 2, N_r = 4$).

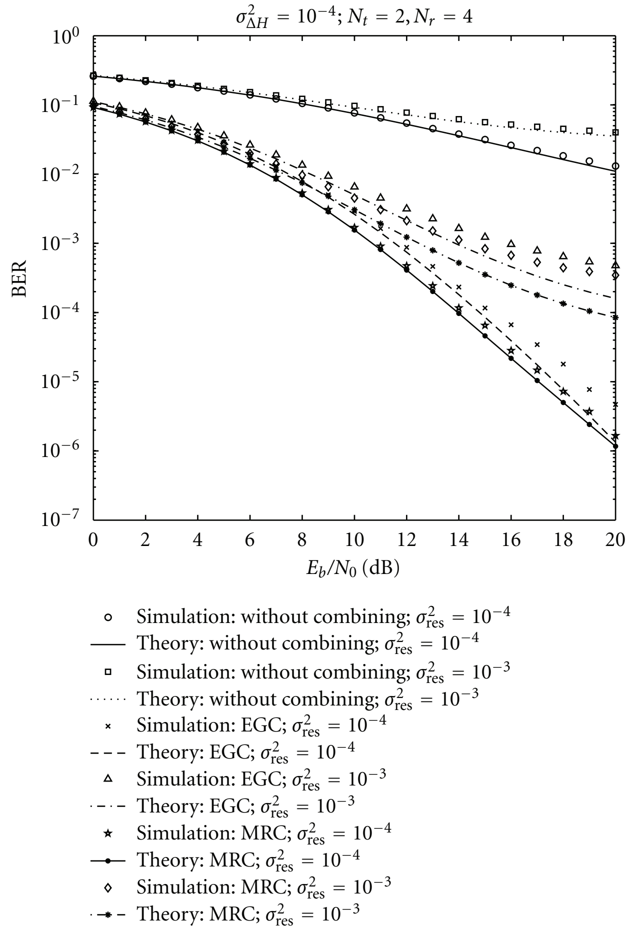

Figure 9: BER with 16QAM when ($N_t = 2, N_r = 4$).

Our theoretical BER approximations are accurate at low SNR with/without diversity combining. However, the simulation and theory results diverge as the SNR increases, especially when σ^2_{res} is large. For example, in Figure 9, with 16QAM, when ($N_t = 2$, $N_r = 4$) and $\sigma^2_{\text{res}} = 10^{-3}$, about 1 dB difference exists between the simulation and the theoretical result for either EGC or MRC at high SNR. This discrepancy is due to several reasons. As the SNR increases, the system becomes interference limited. When N, N_t, and N_r are not large enough, the interferences may not be well approximated as Gaussian RVs with zero mean. In addition, with either EGC or MRC reception, the phase rotation or channel attenuation of the receive substreams should be estimated, and their estimation accuracy will also affect the combined SINR. The instant large phase or channel estimation error also contributes a deviation to the BER when using EGC or MRC.

6. Conclusions

The BER of MIMO-OFDM due to the frequency offset and channel estimation errors has been analyzed. The BER expressions for no combining, EGC, and MRC were derived. These expressions are in infinite-series form and can be truncated in practice. The simulation results show that the truncation error is negligible if the number of terms is large than 50.

Appendices

A. BER without Combining

Without loss of generality, the signal transmitted by the n_tth transmit antenna is assumed in this subsection to be demodulated at the n_rth receive antenna. For each (n_r, n_t, n), $H = |H^{(n)}_{n_r,n_t}|$ has a probability density function (PDF) $f(H) = 2H \cdot e^{-H^2}$. When the number of receive antennas m is larger than 2, $D^{n_r}_{n_t;m}$ can be represented as

$$\begin{aligned}
D^{n_r}_{n_t;m} &= \int_0^\infty \left(\sqrt{\overline{\gamma}_{n_r,n_t}\left(n \mid H^{(n)}_{n_r,n_t}\right)}\right)^{2m-1} f(H)\mathrm{d}H \\
&= \int_0^\infty \frac{\varpi^{(m-1/2)} H^{(2m-1)}}{(\mu H^2 + \nu)^{(m-1/2)}} e^{-H^2} \mathrm{d}H^2 \\
&= \frac{\varpi(m-1/2)}{\mu(m-3/2)} \cdot D^{n_r}_{n_t;m-1} \\
&\quad - \frac{\varpi^{(m-1/2)}}{\mu(m-3/2)} \int_0^\infty \frac{h^{(m-1/2)} e^{-h}}{(\mu h + \nu)^{(m-3/2)}} \mathrm{d}h,
\end{aligned} \tag{A.1}$$

where ν is defined in (15), $h = H^2$, $\varpi = E_s/N_t \cdot \sigma_m^2$, and $\mu = \mathrm{Var}\{\alpha_{n_r,n_t}^{(n)}\}$. Equation (A.1) can be further derived as

$$\begin{aligned} D_{n_t;m}^{n_r} &= \frac{\varpi(m-(1/2))}{\mu(m-(3/2))} \cdot D_{n_t;m-1}^{n_r} \\ &\quad + \frac{\varpi^{(m-1/2)}\nu}{\mu^2(m-3/2)} \int_0^\infty \frac{h^{(m-3/2)}e^{-h}}{(\mu h+\nu)^{(m-3/2)}} \mathrm{d}h \\ &\quad - \frac{\varpi^{(m-1/2)}}{\mu^2(m-3/2)} \underbrace{\int_0^\infty \frac{h^{(m-3/2)}e^{-h}}{(\mu h+\nu)^{(m-5/2)}} \mathrm{d}h}_{Z_{n_t}^{n_r}} \\ &= \frac{\varpi(m-1/2)}{\mu(m-3/2)} \cdot D_{n_t;m-1}^{n_r} + \frac{\varpi\nu}{\mu^2(m-3/2)} \\ &\quad \cdot D_{n_t;m-1}^{n_r} - \frac{\varpi^{(m-1/2)}}{\mu^2(m-3/2)} \cdot Z_{n_t}^{n_r}. \end{aligned} \tag{A.2}$$

From the last step of (A.1), $D_{n_t;m-1}^{n_r}$ can be represented as a function of $D_{n_t;m-2}^{n_r}$ and $Z_{n_t}^{n_r}$:

$$D_{n_t;m-1}^{n_r} = \frac{\varpi(m-3/2)}{\mu(m-5/2)} \cdot D_{n_t;m-2}^{n_r} - \frac{\varpi^{(m-3/2)}}{\mu(m-5/2)} \cdot Z_{n_t}^{n_r}. \tag{A.3}$$

By resolving (A.3), $Z_{n_t}^{n_r}$ can be represented as

$$Z_{n_t}^{n_r} = \frac{\varpi(m-3/2) \cdot D_{n_t;m-2}^{n_r} - \mu(m-5/2) \cdot D_{n_t;m-1}^{n_r}}{\varpi^{(m-3/2)}}. \tag{A.4}$$

By replacing $Z_{n_t}^{n_r}$ in (A.2) with (A.4), $D_{n_t;m}^{n_r}$ can be finally simplified as

$$\begin{aligned} D_{n_t;m}^{n_r} &= \frac{\varpi(m-1/2)}{\mu(m-3/2)} \cdot D_{n_t;m-1}^{n_r} + \frac{\varpi\nu}{\mu^2(m-3/2)} \\ &\quad \cdot D_{n_t;m-1}^{n_r} - \frac{\varpi^{(m-1/2)}}{\mu^2(m-3/2)} \cdot Z \\ &= \frac{\varpi[(2m-3)\mu+\nu]}{\mu^2(m-3/2)} \cdot D_{n_t;m-1}^{n_r} - \frac{\varpi^2}{\mu^2} \cdot D_{n_t;m-2}^{n_r}. \end{aligned} \tag{A.5}$$

B. BER of EGC

Without loss of generality, consider the demodulation of the signal transmitted by the n_tth transmit antenna. Define

$$\begin{aligned} \nu^E &= \sum_{n_r=1}^{N_r} \left(\frac{\pi^2\sigma_{\mathrm{res}}^2 E_s}{3N_t} - \mathrm{Var}\left\{\alpha_{n_r,n_t}^{(n)}\right\} + \mathbb{E}\left\{\left|\Delta\lambda_{n_r,n_t}^{(n)}\right|^2\right\} \right. \\ &\quad \left. + \mathbb{E}\left\{\left|\Delta\xi_{n_r,n_t}^{(n)}\right|^2\right\} + \sigma_w^2 \right) = N_r\nu \end{aligned} \tag{B.1}$$

and $H_{\mathrm{EGC}} = \sum_{n_r=1}^{N_r} |H_{n_r,n_t}^{(n)}|$. As in Appendix A, when $m > 2$, $D_{n_t;m}^{\mathrm{EGC}}$ can be represented as

$$\begin{aligned} D_{n_t;m}^{\mathrm{EGC}} &= \int_0^\infty \left(\sqrt{\overline{\gamma}_{n_t}^E \left(n \mid H_{1,n_t}^{(n)}, \dots, H_{N_r,n_t}^{(n)}\right)} \right)^{2m-1} \\ &\quad \times f(H_{\mathrm{EGC}})\mathrm{d}H_{\mathrm{EGC}} \\ &= \int_0^\infty \frac{\varpi^{(m-1/2)} H_{\mathrm{EGC}}^{(2m-1)}}{\left(\mu(H_{\mathrm{EGC}}^2 - N_r(N_r-1)\pi/4) + \nu^E\right)^{(m-1/2)}} \\ &\quad \cdot \frac{H_{\mathrm{EGC}}^{(2N_r-2)}}{2^{N_r}\sigma_{\mathrm{EGC}}^{2N_r}(N_r-1)!} \cdot e^{-H_{\mathrm{EGC}}^2/2\sigma_{\mathrm{EGC}}^2} \mathrm{d}H_{\mathrm{EGC}}^2 \\ &= \frac{2\sigma_{\mathrm{EGC}}^2\varpi(m+N_r-3/2)}{\widetilde{\mu}(m-3/2)} \cdot D_{n_t;m-1}^{\mathrm{EGC}} \\ &\quad - \frac{(2\sigma_{\mathrm{EGC}}^2\varpi)^{(m-1/2)}}{\widetilde{\mu}(m-3/2)(N_r-1)!} \int_0^\infty \frac{h^{(m+N_r-3/2)}e^{-h}}{(\widetilde{\mu}h+\widetilde{\nu}^E)^{(m-3/2)}} \mathrm{d}h, \end{aligned} \tag{B.2}$$

where $\widetilde{\nu}^E = \nu^E - \mu N_r(N_r-1)\pi/4$, $h = H_{\mathrm{EGC}}^2/2\sigma_{\mathrm{EGC}}^2$, $\sigma_{\mathrm{EGC}}^2 = (N_r!)^2/8[(N_r-1/2)\cdots 1/2]^2$, and $\widetilde{\mu} = 2\sigma_{\mathrm{EGC}}^2 \cdot \mu$. Equation (B.2) can be further simplified as

$$\begin{aligned} D_{n_t;m}^{\mathrm{EGC}} &= \frac{2\sigma_{\mathrm{EGC}}^2\varpi(m+N_r-3/2)}{\widetilde{\mu}(m-3/2)} \cdot D_{n_t;m-1}^{\mathrm{EGC}} \\ &\quad - \frac{(2\sigma_{\mathrm{EGC}}^2\varpi)^{(m-1/2)}\widetilde{\nu}^E}{\widetilde{\mu}^2(m-3/2)(N_r-1)!} \int_0^\infty \frac{h^{(m+N_r-5/2)}e^{-h}}{(\widetilde{\mu}h+\widetilde{\nu}^E)^{(m-3/2)}} \mathrm{d}h \\ &\quad - \frac{\varpi^{(m-1/2)}}{\mu^2(m-3/2)(N_r-1)!} \underbrace{\int_0^\infty \frac{h^{(m+N_r-5/2)}e^{-h}}{(\mu h+\nu^E)^{(m-5/2)}} \mathrm{d}h}_{Z_i^{\mathrm{EGC}}} \\ &= \frac{2\sigma_{\mathrm{EGC}}^2\varpi(m+N_r-3/2)}{\widetilde{\mu}(m-3/2)} \cdot D_{n_t;m-1}^{\mathrm{EGC}} \\ &\quad + \frac{2\sigma_{\mathrm{EGC}}^2\varpi\widetilde{\nu}^E}{\widetilde{\mu}^2(m-3/2)(N_r-1)!} \cdot D_{n_t;m-1}^{\mathrm{EGC}} \\ &\quad - \frac{(2\sigma_{\mathrm{EGC}}^2\varpi)^{(m-1/2)}}{\widetilde{\mu}^2(m-3/2)(N_r-1)!} \cdot Z_{n_t}^{\mathrm{EGC}}. \end{aligned} \tag{B.3}$$

From the last step of (B.2), $D_{n_t;m-1}^{\mathrm{EGC}}$ can be represented as a function of $D_{n_t;m-2}^{\mathrm{EGC}}$ and $Z_{n_t}^{\mathrm{EGC}}$:

$$\begin{aligned} D_{n_t;m-1}^{\mathrm{EGC}} &= \frac{2\sigma_{\mathrm{EGC}}^2\varpi(m+N_r-5/2)}{\widetilde{\mu}(m-5/2)} \cdot D_{n_t;m-2}^{\mathrm{EGC}} \\ &\quad - \frac{(2\sigma_{\mathrm{EGC}}^2\varpi)^{(m-3/2)}}{\widetilde{\mu}(m-5/2)(N_r-1)!} \cdot Z_{n_t}^{\mathrm{EGC}}. \end{aligned} \tag{B.4}$$

By resolving (B.4), $Z_{n_t}^{\mathrm{EGC}}$ can be represented as

$$\begin{aligned} Z_{n_t}^{\mathrm{EGC}} &= \frac{2\sigma_{\mathrm{EGC}}^2\varpi(m+N_r-5/2)(N_r-1)! \cdot D_{n_t;m-2}^{\mathrm{EGC}}}{(2\sigma_{\mathrm{EGC}}^2\varpi)^{(m-3/2)}} \\ &\quad - \frac{\widetilde{\mu}(m-5/2)(N_r-1)! \cdot D_{n_t;m-1}^{\mathrm{EGC}}}{(2\sigma_{\mathrm{EGC}}^2\varpi)^{(m-3/2)}} \end{aligned} \tag{B.5}$$

By replacing $Z_{n_t}^{\mathrm{EGC}}$ in (B.3) with (B.5), $D_{n_t;m}^{\mathrm{EGC}}$ can be finally simplified as

$$D_{n_t;m}^{\mathrm{EGC}} = \frac{2\sigma_{\mathrm{EGC}}^2\varpi\left[(2m+N_r-4)\tilde{\mu}(N_r-1)!+\tilde{\gamma}^E\right]}{\tilde{\mu}^2(m-3/2)(N_r-1)!}\cdot D_{n_t;m-1}^{\mathrm{EGC}} - \frac{\left(2\sigma_{\mathrm{EGC}}^2\varpi\right)^2(m+N_r-5/2)}{\tilde{\mu}^2(m-3/2)}\cdot D_{n_t;m-2}^{\mathrm{EGC}}. \tag{B.6}$$

C. BER of MRC

Without loss of generality, consider the demodulation of the signal transmitted by the n_tth transmit antenna. Define $H_{\mathrm{MRC}} = \sqrt{\sum_{n_r=1}^{N_r}|H_{n_r,n_t}^{(n)}|^2}$. When $m > 2$, $D_{n_t;m}^{\mathrm{MRC}}$ can be represented as

$$\begin{aligned} D_{n_t;m}^{\mathrm{MRC}} &= \int_0^\infty \left(\sqrt{\overline{\gamma}_i^M\left(n|H_{1,i}^{(n)},\ldots,H_{N_r,i}^{(n)}\right)}\right)^{2m-1} f(H_{\mathrm{MRC}})\mathrm{d}H_{\mathrm{MRC}} \\ &= 2^{N_r}\int_0^\infty \frac{\varpi^{(m-1/2)}H_{\mathrm{MRC}}^{(2m-1)}}{\left(\mu[H_{\mathrm{MRC}}^2-(N_r-1)]+\nu^M\right)^{(m-1/2)}} \cdot \frac{H_{\mathrm{MRC}}^{(2N_r-2)}}{(N_r-1)!}\cdot e^{-H_{\mathrm{MRC}}^2}\mathrm{d}H_{\mathrm{MRC}}^2 \\ &= \frac{\varpi(m+N_r-3/2)e^{-(N_r-1)}}{\mu(m-3/2)}\cdot D_{n_t;m-1}^{\mathrm{MRC}} - \frac{2^{N_r}e^{-(N_r-1)}\varpi^{(m-1/2)}}{\mu(m-3/2)(N_r-1)!}\int_0^\infty \frac{h^{(m+N_r-3/2)}e^{-h}}{\left(\mu h+\tilde{\nu}^M\right)^{(m-3/2)}}\mathrm{d}h, \end{aligned} \tag{C.1}$$

where $h = H_{\mathrm{MRC}}^2$ and $\tilde{\nu}^M = \nu^M - \mu(N_r-1)$. (C.1) can be further simplified as

$$\begin{aligned} D_{n_t;m}^{\mathrm{MRC}} &= \frac{\varpi(m+N_r-3/2)e^{-(N_r-1)}}{\mu(m-3/2)}\cdot D_{n_t;m-1}^{\mathrm{MRC}} - \frac{2^{N_r}e^{-(N_r-1)}\varpi^{(m-1/2)}\tilde{\nu}^M}{\mu^2(m-3/2)(N_r-1)!}\int_0^\infty \frac{h^{(m+N_r-5/2)}e^{-h}}{\left(\mu h+\tilde{\nu}^M\right)^{(m-3/2)}}\mathrm{d}h \\ &\quad - \frac{2^{N_r}e^{-(N_r-1)}\varpi^{(m-1/2)}}{\mu^2(m-3/2)(N_r-1)!}\underbrace{\int_0^\infty \frac{h^{(m+N_r-5/2)}e^{-h}}{\left(\mu h+\tilde{\nu}^M\right)^{(m-5/2)}}\mathrm{d}h}_{Z_{n_t}^{\mathrm{MRC}}} \\ &= \frac{\varpi(m+N_r-3/2)e^{-(N_r-1)}}{\mu(m-3/2)}\cdot D_{n_t;m-1}^{\mathrm{MRC}} + \frac{\varpi\tilde{\nu}^M e^{-(N_r-1)}}{\mu^2(m-3/2)(N_r-1)!}\cdot D_{n_t;m-1}^{\mathrm{MRC}} - \frac{2^{N_r}e^{-(N_r-1)}\varpi^{(m-1/2)}}{\mu^2(m-3/2)(N_r-1)!}\cdot Z_{n_t}^{\mathrm{MRC}}. \end{aligned} \tag{C.2}$$

From the last step of (C.1), $D_{n_t;m-1}^{\mathrm{MRC}}$ can be represented as a function of $D_{n_t;m-2}^{\mathrm{MRC}}$ and $Z_{n_t}^{\mathrm{MRC}}$:

$$D_{n_t;m-1}^{\mathrm{MRC}} = \frac{\varpi(m+N_r-5/2)e^{-(N_r-1)}}{\mu(m-5/2)}\cdot D_{n_t;m-2}^{\mathrm{MRC}} - \frac{2^{N_r}e^{-(N_r-1)}\varpi^{(m-3/2)}}{\mu(m-5/2)(N_r-1)!}\cdot Z_{n_t}^{\mathrm{MRC}}. \tag{C.3}$$

By resolving (C.3), $Z_{n_t}^{\mathrm{MRC}}$ can be represented as

$$Z_{n_t}^{\mathrm{MRC}} = \frac{\varpi(m+N_r-5/2)(N_r-1)!\cdot D_{n_t;m-2}^{\mathrm{MRC}}}{2^{N_r}\varpi^{(m-3/2)}} - \frac{\mu(m-5/2)(N_r-1)!e^{(N_r-1)}\cdot D_{n_t;m-1}^{\mathrm{MRC}}}{2^{N_r}\varpi^{(m-3/2)}} \tag{C.4}$$

By replacing $Z_{n_t}^{\mathrm{MRC}}$ in (C.2) with (C.4), $D_{n_t;m}^{\mathrm{MRC}}$ can be finally simplified as

$$D_{n_t;m}^{\mathrm{MRC}} = \frac{\varpi\left[(2m+N_r-4)\mu(N_r-1)!+\tilde{\nu}^M\right]}{\mu^2(m-3/2)(N_r-1)!}\cdot D_{n_t;m-1}^{\mathrm{MRC}} - \frac{\varpi^2(m+N_r-5/2)e^{-(N_r-1)}}{\mu^2(m-3/2)}\cdot D_{n_t;m-2}^{\mathrm{MRC}}. \tag{C.5}$$

Acknowledgments

This paper has been presented in part at the IEEE Globecom 2007 [32]. Although the conference paper was a brief version of this journal paper and they have the same results and conclusion, this journal paper provides a more detailed proof to each result appeared in the IEEE ICC 2007 paper.

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