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Research Article

Iterative Soft Decision Interference Cancellation for DS-CDMA Employing the Distribution of Interference

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A well-known receiver strategy for direct-sequence code-division multiple-access (DS-CDMA) transmission is iterative soft decision interference cancellation. For calculation of soft estimates used for cancellation, the distribution of residual interference is commonly assumed to be Gaussian. In this paper, we analyze matched filter-based iterative soft decision interference cancellation (MF ISDIC) when utilizing an approximation of the actual probability density function (pdf) of residual interference. In addition, a hybrid scheme is proposed, which reduces computational complexity by considering the strongest residual interferers according to their pdf while the Gaussian assumption is applied to the weak residual interferers. It turns out that the bit error ratio decreases already noticeably when only a small number of residual interferers is regarded according to their pdf. For the considered DS-CDMA transmission the bit error ratio decreases by 80% for high signal-to-noise ratios when modeling all residual interferers but the strongest three to be Gaussian distributed.

1. Introduction

The demands on the data rates provided by future mobile communications systems are further increasing especially for the downlink. Higher data rates in the downlink require for example, higher-order modulation schemes and more efficient receiver algorithms to overcome intersymbol interference (ISI) and, additionally for direct-sequence code-division multiple access (DS-CDMA) systems, multiple access interference (MAI).

In this paper we consider the downlink of a DS-CDMA system. Although the optimum solution for the receiver of a user terminal is known [1], its application is prohibitively complex, because the computational effort increases exponentially with the number of users. Therefore, when applying more powerful receiver algorithms than the standard Rake receiver [2], one has to consider suboptimum schemes.

A promising approach was presented in [3], where successive interference cancellation is proposed. This scheme was refined in for example, [4–7] by utilizing soft decisions for cancellation. For calculation of soft decisions, the distribution of residual interference is commonly assumed to be Gaussian. This assumption is accurate according to the central limit theorem [8], when a successive interference cancellation algorithm starts and the number of noteworthy residual interferers is high. But the Gaussian model gets inaccurate as the algorithm converges and the number of relevant residual interferers decreases.

A general discussion of the accuracy of the Gaussian assumption can be found in [9, 10]. In the noniterative approach [11] the distribution of interference is not approximated as Gaussian but modeled via uniform triangular densities which, however, leads to an only minor performance gain. The benefit of employing the actual probability density function (pdf) is for example, recognized in [12, 13] where the distribution of interference is approximated by kernel smoothing, modeling the pdf of interference by a Gaussian mixture density (sum of Gaussian densities) which is then adjusted according to the occurrence of interference. Approximating the pdf of interference with a Gaussian mixture is also proposed in [14], where the approximation is fixed for the entire transmission.

In contrast to these approaches we derive an approximation of the pdf of interference based on probabilities of interfering symbols calculated in the receiver. Our approach uses the matched filter-based iterative soft decision interference cancellation (MF ISDIC) algorithm which was introduced in [6] and extended in [15]. In [6], the algorithm is derived for synchronous DS-CDMA systems with random spreading sequences and binary phase-shift keying (BPSK) symbols considering transmission over an additive white Gaussian noise (AWGN) channel. In [15], MF ISDIC is designed for quadrature amplitude modulation (QAM) transmission over multipath channels. Extensions to the ISDIC algorithm for linear modulation are proposed in [16].

The paper is organized as follows. First, we introduce the system model in Section 2. We review the MF ISDIC receiver for transmission over multipath channels with general square QAM constellations using the Gaussian assumption according to [15] in Section 3.1 and extend it for use of an approximation of the actual pdf of residual interference in Section 3.2. In Section 3.3, a graphical comparison of the pdf calculated via the Gaussian assumption and the approximation of the actual interference distribution is given. A hybrid scheme is derived in Section 3.4. In Section 4, performance of the introduced algorithms is compared by means of simulations.

2. System Model

In the following, all signals and systems are represented by their complex-valued baseband equivalents. A DS-CDMA transmission with general square QAM constellations and Gray mapping is considered. The discrete-time transmit signal $e[l']$ at chip time l' of a base station is expressed as

$$e[l'] = \sum_{k=1}^{K}\sum_{l=1}^{L} s_{l,k}[l' - (l-1)N] \cdot a_k[l], \qquad (1)$$

where K is the number of served users, L is the number of transmitted QAM symbols per user during the considered time interval, and N is the spreading factor which is assumed to be identical for all users. $s_{l,k}[l']$ denotes the spreading sequence of user $k \in \{1,\ldots,K\}$ at symbol time $l \in \{1,\ldots,L\}$ which is nonzero for $0 \le l' \le N-1$. $a_k[l] \in \{x_1 = x_{I,1} + j\,x_{Q,1},\ldots,x_i = x_{I,i} + j\,x_{Q,i},\ldots,x_M = x_{I,M} + j\,x_{Q,M}\}$ is the transmitted QAM coefficient of user k at symbol time l which is taken from a square QAM set of cardinality M. The average power of the transmitted coefficients $\mathcal{E}\{|a_k[l]|^2\}$ ($\mathcal{E}\{\cdot\}$: expectation) is denoted by σ_a^2. The coefficients $x_i = x_{I,i} + j\,x_{Q,i}$ of the QAM set are assumed to be equiprobable and the values $x_{I,i}$ and $x_{Q,i}$ are taken from the set $\{c_1,\ldots,c_j,\ldots c_{M_x}\}$ of cardinality $M_x = \sqrt{M}$. If a user is served with several spreading sequences simultaneously we denote this as multicode transmission. Obviously, this case is also covered by the system model because one user may comprise several virtual users which are served with one spreading sequence each.

The discrete-time received signal is given by

$$r[l'] = \sum_{\kappa=0}^{q_h} h[\kappa]e[l'-\kappa] + n[l'], \qquad (2)$$

where $h[\cdot]$ denotes the causal discrete-time channel impulse response of order q_h including the effects of transmit filtering, channel, and continuous-time receiver input filtering. $n[l']$ is additive complex white Gaussian noise with variance σ_n^2.

For the ISDIC algorithm we use a matrix vector notation for simplicity, which is introduced in the following. First, we define a convolution matrix $\mathbf{H}$ of size $(LN + q_h) \times LN$ whose entries in the ith row and jth column are $\mathbf{H}_{(i,j)} = h[i-j]$. The received signal vector $\mathbf{r} = [r[1],\ldots,r[LN+q_h]]^{\mathrm{T}}$ ($(\cdot)^{\mathrm{T}}$: transposition) can be expressed as

$$\mathbf{r} = \mathbf{H}\cdot\mathbf{S}\cdot\mathbf{a} + \mathbf{n} = \mathbf{T}\cdot\mathbf{a} + \mathbf{n}, \qquad (3)$$

where $\mathbf{a} = [a_1[1],\ldots,a_K[1],\ldots,a_k[l],\ldots,a_1[L],\ldots,a_K[L]]^{\mathrm{T}}$ which may also be written as $\mathbf{a} = [a_1,\ldots,a_\nu,\ldots,a_{KL}]^{\mathrm{T}}$ with

$$\nu = k + (l-1)K \qquad (4)$$

and $\mathbf{n} = [n[1],\ldots,n[LN+q_h]]^{\mathrm{T}}$. The matrix $\mathbf{S}$ contains the accordingly stacked spreading sequences and $\mathbf{HS}$ is abbreviated by $\mathbf{T}$.

Furthermore, a truncated version of the model according to (3) is used to derive the sliding window filters of MF ISDIC. For the truncated system model we consider a time interval of the received signal in vector $\mathbf{r}_{\nu_0}$ which exactly matches the influence of a transmitted symbol $a_{k_0}[l_0]$, a_{ν_0}, respectively, according to (4),

$$\mathbf{r}_{\nu_0} = \mathbf{T}_{\nu_0}\cdot\mathbf{a}_{\nu_0} + \mathbf{n}_{\nu_0}, \qquad (5)$$

with $\mathbf{r}_{\nu_0} = [r[(l_0-1)N],\ldots,r[l_0N-1+q_h]]^{\mathrm{T}}$, and $\mathbf{n}_{\nu_0} = [n[(l_0-1)N],\ldots,n[l_0N-1+q_h]]^{\mathrm{T}}$. $\mathbf{T}_{\nu_0}$ and $\mathbf{a}_{\nu_0} = [a_{\alpha_{\nu_0}},\ldots,a_{\nu_0},\ldots,a_{\beta_{\nu_0}}]^{\mathrm{T}}$ denote the according parts of the matrix $\mathbf{T}$ and the vector $\mathbf{a}$, respectively. As γ_{ν_0} we denote the index of the entry a_{ν_0} in the vector $\mathbf{a}_{\nu_0}$. Therefore, the γ_{ν_0}th column of matrix $\mathbf{T}_{\nu_0}$ is equivalent to the effective spreading sequence of the symbol a_{ν_0}. The indices α_{ν_0} and β_{ν_0} lead to the first symbol $a_{\alpha_{\nu_0}}$ and the last symbol $a_{\beta_{\nu_0}}$, respectively, in vector $\mathbf{a}$ which has any influence on the time interval covered by the effective spreading sequence of the symbol a_{ν_0}. Note that this system model also comprises transmission with linear modulation for one user ($K=1$) employing spreading factor $N=1$. Hence, the algorithm analyzed in the following can also be utilized as a receiver for transmission with linear modulation over dispersive channels.

3. Matched Filter-Based Iterative Soft Decision Interference Cancellation (MF ISDIC)

In each iteration of MF ISDIC, soft-decision feedback is performed for cancellation of ISI and MAI in a sequential manner starting from symbol index $\nu = 1$ up to $\nu = KL$.

$\mu \in \{0, \dots, \mu_{\max}\}$ is the index of the current iteration and $\hat{a}^{\mathrm{s}}_{\nu,\mu}$, cf. (4), denotes the soft decision on a_ν calculated in iteration μ.

The soft decisions $\hat{a}^{\mathrm{s}}_{\nu,\mu}$ are initialized according to $\hat{a}^{\mathrm{s}}_{\nu,0} = 0$ for $\nu \in \{1, \dots, KL\}$. For derivation of MF ISDIC, we introduce the vector $\bar{\mathbf{a}}^{\mathrm{s}}_{\nu,\mu} = [\hat{a}^{\mathrm{s}}_{\alpha_\nu,\mu}, \dots, \hat{a}^{\mathrm{s}}_{\nu-1,\mu}, 0, \hat{a}^{\mathrm{s}}_{\nu+1,\mu-1}, \dots, \hat{a}^{\mathrm{s}}_{\beta_\nu,\mu-1}]^{\mathrm{T}}$, which is used for calculation of the soft estimate $\hat{a}^{\mathrm{s}}_{\nu,\mu}$.

In each iteration μ and for each $\nu \in \{1, \dots, KL\}$ the following steps have to be done. In order to obtain an estimate for the desired coefficient a_ν, ISI and MAI caused by other coefficients are removed from the vector $\mathbf{r}_\nu$ in the best possible way using the latest soft estimates in vector $\bar{\mathbf{a}}^{\mathrm{s}}_{\nu,\mu}$

$$\mathbf{r}_{\nu,\mu} = \mathbf{r}_\nu - \mathbf{T}_\nu \cdot \bar{\mathbf{a}}^{\mathrm{s}}_{\nu,\mu} = \mathbf{T}_\nu \cdot \left(\mathbf{a}_\nu - \bar{\mathbf{a}}^{\mathrm{s}}_{\nu,\mu}\right) + \mathbf{n}_\nu. \tag{6}$$

The resulting vector $\mathbf{r}_{\nu,\mu}$ contains significantly less interference compared to $\mathbf{r}_\nu$ when the soft estimates in $\bar{\mathbf{a}}^{\mathrm{s}}_{\nu,\mu}$ get better from iteration to iteration.

Subsequently, the vector $\mathbf{r}_{\nu,\mu}$ is filtered with a vector $\mathbf{w}^{\mathrm{H}}_\nu$ ($(\cdot)^{\mathrm{H}}$: Hermitian transposition) acting as a matched filter adjusted to the effective spreading sequence of symbol a_ν

$$\mathbf{w}^{\mathrm{H}}_\nu = \frac{1}{\rho_{\nu,\nu}} \left(\mathbf{T}_{\nu(:,\gamma_\nu)}\right)^{\mathrm{H}}. \tag{7}$$

Here, $\mathbf{T}_{\nu(:,\gamma_\nu)}$ is the γ_νth column of the matrix $\mathbf{T}_\nu$ which is equal to the effective spreading sequence of symbol a_ν and $\rho_{\nu,\nu}$ is the energy of the effective spreading sequence of symbol a_ν. The output of the matched filter is, compare with (6) and (7),

$$\begin{aligned}
\hat{a}_{\nu,\mu} &= \mathbf{w}^{\mathrm{H}}_\nu \cdot \mathbf{r}_{\nu,\mu} = \frac{1}{\rho_{\nu,\nu}} \left(\mathbf{T}_{\nu(:,\gamma_\nu)}\right)^{\mathrm{H}} \cdot \mathbf{r}_{\nu,\mu} \\
&= \frac{1}{\rho_{\nu,\nu}} \left(\mathbf{T}_{\nu(:,\gamma_\nu)}\right)^{\mathrm{H}} \cdot \left(\mathbf{T}_\nu \cdot \left(\mathbf{a}_\nu - \bar{\mathbf{a}}^{\mathrm{s}}_{\nu,\mu}\right) + \mathbf{n}_\nu\right) \\
&= a_\nu + \sum_{\xi=\alpha_\nu}^{\nu-1} \frac{\rho_{\xi,\nu}}{\rho_{\nu,\nu}} \left(a_\xi - \hat{a}^{\mathrm{s}}_{\xi,\mu}\right) + \sum_{\xi=\nu+1}^{\beta_\nu} \frac{\rho_{\xi,\nu}}{\rho_{\nu,\nu}} \left(a_\xi - \hat{a}^{\mathrm{s}}_{\xi,\mu-1}\right) \\
&\quad + \frac{\left(\mathbf{T}_{\nu(:,\gamma_\nu)}\right)^{\mathrm{H}}}{\rho_{\nu,\nu}} \cdot \mathbf{n}_\nu
\end{aligned} \tag{8}$$

$$= a_\nu + n_{\nu,\mu}, \tag{9}$$

where we have used

$$\left(\mathbf{T}_{\nu(:,\gamma_\nu)}\right)^{\mathrm{H}} \cdot \mathbf{T}_\nu = \left[\, \rho_{\alpha_\nu,\nu}, \dots, \rho_{\nu,\nu}, \dots, \rho_{\beta_\nu,\nu} \,\right]. \tag{10}$$

$n_{\nu,\mu}$ is an abbreviation for residual ISI and MAI and noise.

After the current iteration has been finished with processing of the last data symbol a_{KL}, a new iteration starts. The algorithm stops if soft decisions remain essentially unchanged from one iteration to the next, that is,

$$\begin{aligned}
&\max_{\nu \in \{1,2,\dots,KL\}} \left| \mathrm{Re}\left\{\hat{a}^{\mathrm{s}}_{\nu,\mu}\right\} - \mathrm{Re}\left\{\hat{a}^{\mathrm{s}}_{\nu,\mu-1}\right\} \right| < \epsilon, \\
&\max_{\nu \in \{1,2,\dots,KL\}} \left| \mathrm{Im}\left\{\hat{a}^{\mathrm{s}}_{\nu,\mu}\right\} - \mathrm{Im}\left\{\hat{a}^{\mathrm{s}}_{\nu,\mu-1}\right\} \right| < \epsilon,
\end{aligned} \tag{11}$$

with a small constant ϵ, or the iteration number exceeds a prescribed limit $\mu_{\max}$.

3.1. MF ISDIC with Gaussian Assumption. For MF ISDIC with Gaussian assumption, we model the residual ISI and MAI and noise $n_{\nu,\mu}$ as a random variable with a complex Gaussian pdf [8] with zero mean and variance $\hat{\sigma}^2_{\nu,\mu}/2$ in real and imaginary part and zero correlation coefficient between them. The power $\hat{\sigma}^2_{\nu,\mu}$ of $n_{\nu,\mu}$ can be calculated via conditioned expectations of the latest matched filter outputs for refinement of estimation as

$$\begin{aligned}
\hat{\sigma}^2_{\nu,\mu} &= \mathcal{E}\left\{ \left|n_{\nu,\mu}\right|^2 \mid \hat{\mathbf{a}}_{\nu,\mu} \right\} \\
&= \sum_{\xi=\alpha_\nu}^{\nu-1} \frac{\left|\rho_{\xi,\nu}\right|^2}{\rho^2_{\nu,\nu}} \\
&\quad \times \left(\mathcal{E}\left\{ |a_\xi|^2 \mid \hat{a}_{\xi,\mu} \right\} - 2 \cdot \mathcal{E}\left\{ a_\xi \mid \hat{a}_{\xi,\mu} \right\} \left(\hat{a}^{\mathrm{s}}_{\xi,\mu}\right)^* + \left|\hat{a}^{\mathrm{s}}_{\xi,\mu}\right|^2 \right) \\
&\quad + \sum_{\xi=\nu+1}^{\beta_\nu} \frac{\left|\rho_{\xi,\nu}\right|^2}{\rho^2_{\nu,\nu}} \left(\mathcal{E}\left\{ |a_\xi|^2 \mid \hat{a}_{\xi,\mu-1} \right\} - 2 \cdot \mathcal{E}\left\{ a_\xi \mid \hat{a}_{\xi,\mu-1} \right\} \right. \\
&\quad \left. \times (\hat{a}^{\mathrm{s}}_{\xi,\mu-1})^* + \left|\hat{a}^{\mathrm{s}}_{\xi,\mu-1}\right|^2 \right) + \frac{\sigma^2_n}{\rho_{\nu,\nu}} \\
&= \sum_{\xi=\alpha_\nu}^{\nu-1} \frac{\left|\rho_{\xi,\nu}\right|^2}{\rho^2_{\nu,\nu}} \left(\mathcal{E}\left\{ |a_\xi|^2 \mid \hat{a}_{\xi,\mu} \right\} - \left|\hat{a}^{\mathrm{s}}_{\xi,\mu}\right|^2 \right) \\
&\quad + \sum_{\xi=\nu+1}^{\beta_\nu} \frac{\left|\rho_{\xi,\nu}\right|^2}{\rho^2_{\nu,\nu}} \left(\mathcal{E}\left\{ |a_\xi|^2 \mid \hat{a}_{\xi,\mu-1} \right\} - \left|\hat{a}^{\mathrm{s}}_{\xi,\mu-1}\right|^2 \right) + \frac{\sigma^2_n}{\rho_{\nu,\nu}},
\end{aligned} \tag{12}$$

where we have used

$$\hat{a}^{\mathrm{s}}_{\nu,\mu} = \mathcal{E}\left\{ a_\nu \mid \hat{a}_{\nu,\mu} \right\}, \tag{13}$$

which minimizes the mean-squared error $\mathcal{E}\{|a_\nu - \hat{a}^{\mathrm{s}}_{\nu,\mu}|^2\}$, cf. [8, 17]. $(\cdot)^*$ denotes complex conjugate of a complex number. The vector $\hat{\mathbf{a}}_{\nu,\mu}$ is defined as $\hat{\mathbf{a}}_{\nu,\mu} = [\hat{a}_{\alpha_\nu,\mu}, \dots, \hat{a}_{\nu-1,\mu}, 0, \hat{a}_{\nu+1,\mu-1}, \dots, \hat{a}_{\beta_\nu,\mu-1}]^{\mathrm{T}}$. The initialization of $\hat{a}_{\nu,\mu}$ is done according to $\hat{a}_{\nu,0} = 0$ for all ν. With the Gaussian assumption we can evaluate the expectation values in (12). $\mathcal{E}\{|a_\nu|^2 | \hat{a}_{\nu,\mu}\} = \mathcal{E}\{(\mathrm{Re}\{a_\nu\})^2 | \hat{a}_{\nu,\mu}\} + \mathcal{E}\{(\mathrm{Im}\{a_\nu\})^2 | \hat{a}_{\nu,\mu}\}$ is valid, where $\mathrm{Re}\{\cdot\}$ and $\mathrm{Im}\{\cdot\}$ denote the real part and the imaginary part of a complex number, respectively. The first term can be calculated to

$$\begin{aligned}
\mathcal{E}\left\{ (\mathrm{Re}\{a_\nu\})^2 \mid \hat{a}_{\nu,\mu} \right\} &= \mathcal{E}\left\{ (\mathrm{Re}\{a_\nu\})^2 \mid \mathrm{Re}\left\{\hat{a}_{\nu,\mu}\right\} \right\} \\
&= \frac{\sum_{j=1}^{M_x} c_j^2 \cdot \mathrm{e}^{\left(-(\mathrm{Re}\{\hat{a}_{\nu,\mu}\} - c_j)^2 / \hat{\sigma}^2_{\nu,\mu}\right)}}{\sum_{j=1}^{M_x} \mathrm{e}^{\left(-(\mathrm{Re}\{\hat{a}_{\nu,\mu}\} - c_j)^2 / \hat{\sigma}^2_{\nu,\mu}\right)}}.
\end{aligned} \tag{14}$$

Similarly we get

$$\mathcal{E}\left\{(\mathrm{Im}\{a_\nu\})^2 \mid \widehat{a}_{\nu,\mu}\right\} = \mathcal{E}\left\{(\mathrm{Im}\{a_\nu\})^2 \mid \mathrm{Im}\left\{\widehat{a}_{\nu,\mu}\right\}\right\} = \frac{\sum_{j=1}^{M_x} c_j^2 \cdot \mathrm{e}^{(-(\mathrm{Im}\{\widehat{a}_{\nu,\mu}\}-c_j)^2/\widehat{\sigma}_{\nu,\mu}^2)}}{\sum_{j=1}^{M_x} \mathrm{e}^{(-(\mathrm{Im}\{\widehat{a}_{\nu,\mu}\}-c_j)^2/\widehat{\sigma}_{\nu,\mu}^2)}}. \tag{15}$$

Finally, using the mentioned assumptions, the soft estimate $\widehat{a}_{\nu,\mu}^{\mathrm{s}}$ of (13) can be derived as

$$\widehat{a}_{\nu,\mu}^{\mathrm{s}} = \mathcal{E}\left\{a_\nu \mid \widehat{a}_{\nu,\mu}\right\} = \frac{\sum_{i=1}^{M} (x_{I,i} + j\, x_{Q,i})\mathrm{e}^{(-[(\mathrm{Re}\{\widehat{a}_{\nu,\mu}\}-x_{I,i})^2+(\mathrm{Im}\{\widehat{a}_{\nu,\mu}\}-x_{Q,i})^2]/\widehat{\sigma}_{\nu,\mu}^2)}}{\sum_{i=1}^{M} \mathrm{e}^{(-[(\mathrm{Re}\{\widehat{a}_{\nu,\mu}\}-x_{I,i})^2+(\mathrm{Im}\{\widehat{a}_{\nu,\mu}\}-x_{Q,i})^2]/\widehat{\sigma}_{\nu,\mu}^2)}}. \tag{16}$$

The calculation of soft estimates for general complex-valued symbol alphabets according to (16) is also given in [7]. When assuming symbols of a general phase-shift keying (PSK) or cross QAM constellation for transmission instead of a square QAM constellation, only (14)-(15) have to be modified.

For 4QAM transmission with $a_\nu \in \{\pm 1 \pm j\}$ the expectation values in (12) simplify to $\mathcal{E}\{|a_\nu|^2 \mid \widehat{a}_{\nu,\mu}\} = 2$ and (16) reads

$$\widehat{a}_{\nu,\mu}^{\mathrm{s}} = \tanh\left(\frac{2}{\widehat{\sigma}_{\nu,\mu}^2}\mathrm{Re}\left\{\widehat{a}_{\nu,\mu}\right\}\right) + j\tanh\left(\frac{2}{\widehat{\sigma}_{\nu,\mu}^2}\mathrm{Im}\left\{\widehat{a}_{\nu,\mu}\right\}\right), \tag{17}$$

which was also utilized in [18].

For the Gaussian assumption, the pdf $f_{n_{\nu,\mu}}(n)$ of residual ISI and MAI and noise is

$$f_{n_{\nu,\mu}}(n) = \frac{1}{\pi\widehat{\sigma}_{\nu,\mu}^2}\mathrm{e}^{-|n|^2/\widehat{\sigma}_{\nu,\mu}^2}, \tag{18}$$

with $\widehat{\sigma}_{\nu,\mu}^2$ according to (12).

3.2. MF ISDIC Employing an Approximation of the Actual Interference Distribution. Assuming the residual interference to be Gaussian distributed is well justified according to the central limit theorem [8], when a successive interference cancellation algorithm starts and the number of relevant residual interferers is high. However, the assumption gets inaccurate as the algorithm converges in course of the iterations and the number of relevant residual interferers decreases. In this subsection, we modify MF ISDIC for employment of an approximation of the actual pdf of residual interference which has to be derived first.

Hence, the pdf $f_{n_{\nu,\mu}}(n)$ of residual ISI and MAI and noise $n_{\nu,\mu}$ in (9) which was assumed to be a complex Gaussian pdf with zero mean and power $\widehat{\sigma}_{\nu,\mu}^2$ in Section 3.1 is now calculated approximately, resulting in a more accurate expression. For this, we first derive the pdf of a term $\rho_{\xi,\nu}/\rho_{\nu,\nu}(a_\xi - \widehat{a}_{\xi,\mu}^{\mathrm{s}})$ in the first sum in (8), keeping in mind that the pdf $f_{n_{\nu,\mu}}(n)$ is the convolution of several such pdfs and a normal pdf. We assume that $\widehat{a}_{\xi,\mu}$ and $f_{n_{\xi,\mu}}(n)$ are already available and calculate the conditional probability of all symbols x_i given $\widehat{a}_{\xi,\mu}$ using Bayes' theorem [8]

$$\begin{aligned}\Pr\left(a_\xi = x_i \mid \widehat{a}_{\xi,\mu}\right) &= \frac{f_{n_{\xi,\mu}}\left(\widehat{a}_{\xi,\mu} - x_i\right)\Pr(a_\xi = x_i)}{\sum_{i'=1}^{M} f_{n_{\xi,\mu}}\left(\widehat{a}_{\xi,\mu} - x_{i'}\right)\Pr(a_\xi = x_{i'})} \\ &= \frac{(1/M) f_{n_{\xi,\mu}}\left(\widehat{a}_{\xi,\mu} - x_i\right)}{(1/M)\sum_{i'=1}^{M} f_{n_{\xi,\mu}}\left(\widehat{a}_{\xi,\mu} - x_{i'}\right)} \\ &= \frac{f_{n_{\xi,\mu}}\left(\widehat{a}_{\xi,\mu} - x_i\right)}{\sum_{i'=1}^{M} f_{n_{\xi,\mu}}\left(\widehat{a}_{\xi,\mu} - x_{i'}\right)}.\end{aligned} \tag{19}$$

Here, $\Pr(a_\xi = x_i)$ denotes the probability that a_ξ equals x_i.

For derivation of the pdf $f_{n_{\nu,\mu}}(n)$ we have to define the Dirac pulse in the complex plane for a complex variable z in dependence of the Dirac pulse $\delta(\cdot)$ for real numbers

$$\delta_{\mathrm{c}}(z) := \delta(\mathrm{Re}\{z\}) \cdot \delta(\mathrm{Im}\{z\}). \tag{20}$$

Because the transmitted symbols are equally probable, any transmitted symbol a_ξ has the pdf

$$f_{a_\xi}(x) = \sum_{i=1}^{M}\frac{1}{M}\delta_{\mathrm{c}}(x - x_i). \tag{21}$$

The conditional pdf $f_{a_\xi|\widehat{a}_{\xi,\mu}}(x)$ can be expressed as

$$f_{a_\xi|\widehat{a}_{\xi,\mu}}(x) = \sum_{i=1}^{M}\Pr\left(a_\xi = x_i| \widehat{a}_{\xi,\mu}\right)\delta_{\mathrm{c}}(x - x_i), \tag{22}$$

where (19) can be used for $\Pr(a_\xi = x_i \mid \widehat{a}_{\xi,\mu})$. From (22), the conditional expectation value for the symbol a_ξ is obtained

$$\widehat{a}_{\xi,\mu}^{\mathrm{s}} = \mathcal{E}\left\{a_\xi \mid \widehat{a}_{\xi,\mu}\right\} = \sum_{i=1}^{M} x_i \cdot \Pr\left(a_\xi = x_i \mid \widehat{a}_{\xi,\mu}\right). \tag{23}$$

The subtraction of the corresponding expectation value in (8) leads to a modified symbol

$$a'_\xi = a_\xi - \widehat{a}_{\xi,\mu}^{\mathrm{s}}, \tag{24}$$

whose pdf is given by

$$\begin{aligned}f_{a'_\xi|\widehat{a}_{\xi,\mu}}(x) &= f_{a_\xi-\widehat{a}_{\xi,\mu}^{\mathrm{s}}|\widehat{a}_{\xi,\mu}}(x) \\ &= f_{a_\xi|\widehat{a}_{\xi,\mu}}\left(x + \widehat{a}_{\xi,\mu}^{\mathrm{s}}\right) \\ &= \sum_{i=1}^{M}\Pr\left(a_\xi = x_i \mid \widehat{a}_{\xi,\mu}\right)\delta_{\mathrm{c}}\left(x - x_i + \widehat{a}_{\xi,\mu}^{\mathrm{s}}\right).\end{aligned} \tag{25}$$

Obviously, the pdf $f_{a'_\xi|\widehat{a}_{\xi,\mu}}(x)$ has zero mean due to the subtraction of the expectation value in (24). Weighting a'_ξ

with a factor $\rho'_{\xi,\nu} := \rho_{\xi,\nu}/\rho_{\nu,\nu}$ according to (8) yields $a''_{\xi} = \rho'_{\xi,\nu} a'_{\xi}$. The pdf of a''_{ξ} is, cf. [8],

$$f_{a''_{\xi}|\hat{a}_{\xi,\mu}}(x) = \frac{1}{\left|\rho'_{\xi,\nu}\right|^2} f_{a'_{\xi}|\hat{a}_{\xi,\mu}}\left(\frac{x}{\rho'_{\xi,\nu}}\right)$$
$$= \frac{1}{\left|\rho'_{\xi,\nu}\right|^2} \sum_{i=1}^{M} \Pr\left(a_{\xi} = x_i \mid \hat{a}_{\xi,\mu}\right) \delta_{\mathrm{c}}\left(\frac{x}{\rho'_{\xi,\nu}} - x_i + \hat{a}^{\mathrm{s}}_{\xi,\mu}\right) \tag{26}$$

$$= \sum_{i=1}^{M} \Pr\left(a_{\xi} = x_i \mid \hat{a}_{\xi,\mu}\right) \delta_{\mathrm{c}}\left(x + \left(\hat{a}^{\mathrm{s}}_{\xi,\mu} - x_i\right)\rho'_{\xi,\nu}\right). \tag{27}$$

Note that the squared value $1/|\rho'_{\xi,\nu}|^2$ in (26) is founded by the fact that the considered random variables are complex-valued, cf. for example, (20). Therefore, the pdf $f_{n_{\nu,\mu}}(n)$ of residual ISI and MAI and noise $n_{\nu,\mu}$ in (9) is the convolution (symbol "$\star$") of a complex Gaussian pdf and pdfs according to (27) (the convolution of two pdfs of complex-valued random variables, $f_{n_1}(n)$ and $f_{n_2}(n)$ with $n = n_{\mathrm{R}} + j\, n_{\mathrm{I}}$, is defined as $f_{n_1}(n) \star f_{n_2}(n) = \iint_{-\infty}^{+\infty} f_{n_1}((n_{\mathrm{R}} - x) + j(n_{\mathrm{I}} - y)) f_{n_2}(x + jy) \mathrm{d}x\, \mathrm{d}y$)

$$f_{n_{\nu,\mu}}(n) = \frac{1}{\pi \sigma_n'^2} \mathrm{e}^{-|n|^2/\sigma_n'^2} \star f_{a''_{\alpha_\nu}|\hat{a}_{\alpha_\nu,\mu}}(n) \star \cdots \star f_{a''_{\nu-1}|\hat{a}_{\nu-1,\mu}}(n)$$
$$\star f_{a''_{\nu+1}|\hat{a}_{\nu+1,\mu-1}}(n) \star \cdots \star f_{a''_{\beta_\nu}|\hat{a}_{\beta_\nu,\mu-1}}(n) \tag{28}$$

$$= \frac{1}{\pi \sigma_n^2/\rho_{\nu,\nu}} \sum_{i_{\alpha_\nu}=1}^{M} \cdots \sum_{i_{\nu-1}=1}^{M} \sum_{i_{\nu+1}=1}^{M} \cdots$$
$$\sum_{i_{\beta_\nu}=1}^{M} \Pr\left(a_{\alpha_\nu} = x_{i_{\alpha_\nu}} \mid \hat{a}_{\alpha_\nu,\mu}\right) \cdots$$
$$\cdot \Pr\left(a_{\nu-1} = x_{i_{\nu-1}} \mid \hat{a}_{\nu-1,\mu}\right)$$
$$\cdot \Pr\left(a_{\nu+1} = x_{i_{\nu+1}} \mid \hat{a}_{\nu+1,\mu-1}\right) \cdots$$
$$\cdot \Pr\left(a_{\beta_\nu} = x_{i_{\beta_\nu}} \mid \hat{a}_{\beta_\nu,\mu-1}\right) \cdot \mathrm{e}^{(-1/(\sigma_n^2/\rho_{\nu,\nu}))\eta}, \tag{29}$$

where η denotes $(|n + (\hat{a}^{\mathrm{s}}_{\alpha_\nu,\mu} - x_{i_{\alpha_\nu}})\rho'_{\alpha_\nu,\nu} + \cdots + (\hat{a}^{\mathrm{s}}_{\nu-1,\mu} - x_{i_{\nu-1}})\rho'_{\nu-1,\nu} + (\hat{a}^{\mathrm{s}}_{\nu+1,\mu-1} - x_{i_{\nu+1}})\rho'_{\nu+1,\nu} + \cdots + (\hat{a}^{\mathrm{s}}_{\beta_\nu,\mu-1} - x_{i_{\beta_\nu}})\rho'_{\beta_\nu,\nu}|^2)$. Here we have used, that the power of the noise is $\sigma_n'^2 = \sigma_n^2/\rho_{\nu,\nu}$ as can be seen from the last term in (12) and assumed independence of random variables corresponding to the pdfs to keep mathematical tractability. The latter assumption is an approximation which is fulfilled perfectly only for the a priori pdfs of different symbols but not for their a posteriori pdfs conditioned on the computed soft symbols. With help of the pdf $f_{n_{\nu,\mu}}(n)$ a new soft estimate $\hat{a}^{\mathrm{s}}_{\nu,\mu}$ according to (23) can be calculated utilizing (19).

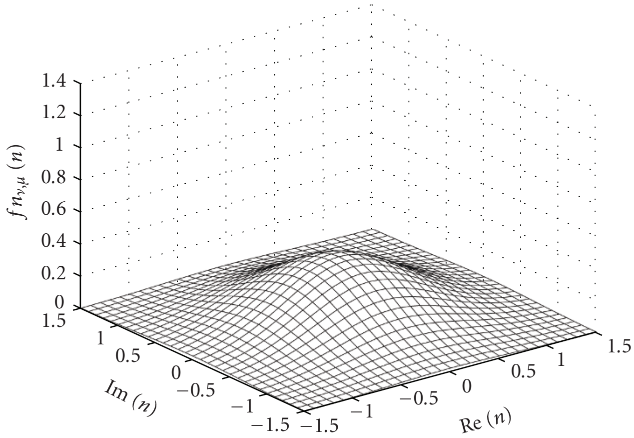

Figure 1: Pdf of ISI, MAI, and noise with Gaussian assumption for $10\log_{10}(E_b/\mathcal{N}_0) = 25$ dB.

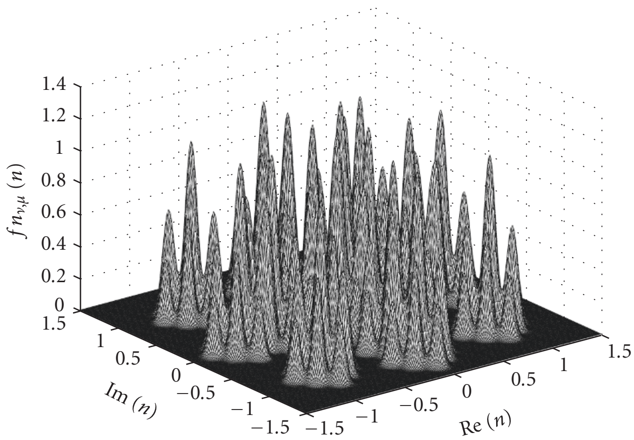

Figure 2: Approximation of actual pdf of ISI, MAI, and noise for $10\log_{10}(E_b/\mathcal{N}_0) = 25$ dB.

3.3. Comparison of the Gaussian Model with the Approximation of the Actual Interference Distribution. To visualize the difference between the pdf with Gaussian assumption, cf. (18), and the approximation of the actual distribution of interference according to (29), some graphs are given in the following. We assume that no prior knowledge is available, that is, MF ISDIC starts with all prior estimates being zero, $\hat{a}_{\nu,\mu} = 0$, for all ν, for all μ and $\hat{a}^{\mathrm{s}}_{\nu,\mu} = 0$, for all ν, for all μ. The crosscorrelation values according to (10) are selected to $(\mathbf{T}_{\nu(:,\gamma_\nu)})^{\mathrm{H}} \cdot \mathbf{T}_\nu = [-0.0563 - 0.0734j, -0.0843 - 0.4293j, 1, -0.0843 + 0.4293j, -0.0563 + 0.0734j]$ and 4QAM ($a_\nu \in \{\pm 1 \pm j\}$) is used.

Figure 1 gives the pdf of interference and noise with Gaussian assumption, cf. (18), and in Figure 2 the approximation of the actual pdf of interference and noise, cf. (29), is shown. For both figures, the assumed signal-to-noise ratio (SNR) is $10\log_{10}(E_b/\mathcal{N}_0) = 25$ dB. Here, E_b is the average received energy per bit and $\mathcal{N}_0$ stands for the single-sided power spectral density of the Gaussian channel noise. Obviously, the Gaussian assumption is not justified for the considered SNR although both pdfs have the same variance.

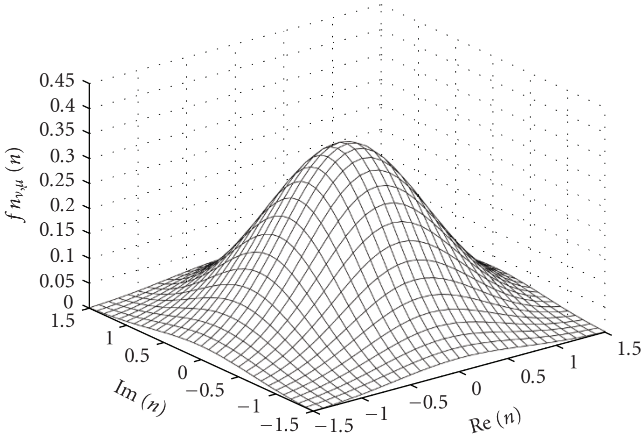

Figure 3: Pdf of ISI, MAI, and noise with Gaussian assumption for $10\log_{10}(E_b/\mathcal{N}_0) = 10$ dB.

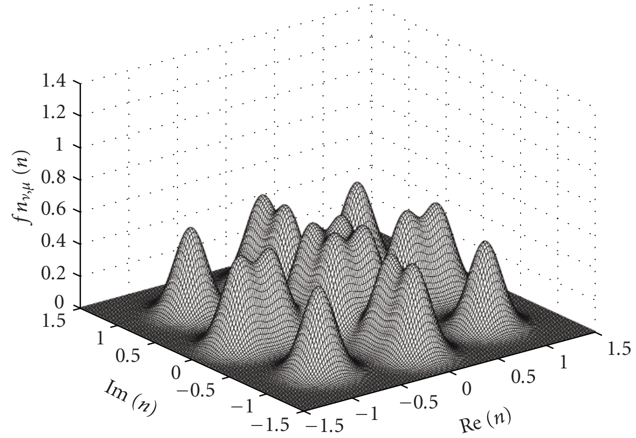

Figure 5: Hybrid pdf approximation of ISI, MAI, and noise for $10\log_{10}(E_b/\mathcal{N}_0) = 25$ dB with $t' = 2$.

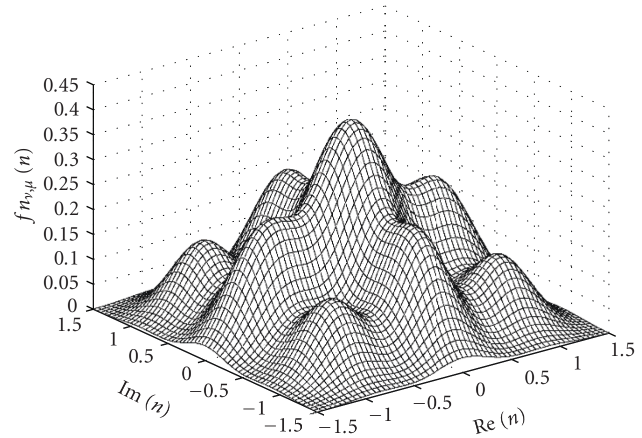

Figure 4: Approximation of actual pdf of ISI, MAI, and noise for $10\log_{10}(E_b/\mathcal{N}_0) = 10$ dB.

However, the approximation of the actual pdf gets closer to a Gaussian pdf for lower SNR as can be seen from Figures 3 and 4 for $10\log_{10}(E_b/\mathcal{N}_0) = 10$ dB. The Gaussian assumption is also justified for a higher number of interfering symbols because in this case the number of convolved pdfs according to (28) increases. However, in MF ISDIC the number of residually interfering symbols is supposed to be low with increasing number of iterations μ.

3.4. Hybrid MF ISDIC. Calculation of $f_{n_{\nu,\mu}}(n)$ according to (29) is very complex. To reduce complexity we propose a hybrid scheme, where the approximation of the pdf is simplified by considering only the strongest t' residual interferers according to their pdf and applying the Gaussian assumption to the weak residual interferers. Obviously, for the hybrid scheme, the number of terms in (29) decreases and σ_n^2 additionally includes the power of the weak residual interferers. The strongest interferer is defined as the one that has the highest value $|\rho'_{\xi,\nu}|^2$.

First, the power $\sigma^2_{\xi,\mu,\nu}$ of each pdf according to (27) in (28) has to be calculated for each $\xi \in \{\alpha_\nu, \dots, \nu-1, \nu+1, \dots, \beta_\nu\}$

$$\sigma^2_{\xi,\mu,\nu} = \sum_{i=1}^{M} \left| \left(\hat{a}^{s}_{\xi,\mu} - x_i\right)\rho'_{\xi,\nu} \right|^2 \cdot \Pr\left(a_\xi = x_i \mid \hat{a}_{\xi,\mu}\right). \qquad (30)$$

It is sufficient to approximate the pdfs $f_{a''_\xi|\hat{a}_{\xi,\mu}}(x)$ which have the weakest power with Gaussian pdfs. As all of these pdfs have zero mean, the Gaussian approximation can be performed in (28) by substituting the corresponding pdf $f_{a''_\xi|\hat{a}_{\xi,\mu}}(n)$ by a Dirac pulse $\delta_c(n)$ and incrementing σ'^2_n by $\sigma^2_{\xi,\mu,\nu}$. The number of pdfs $f_{a''_\xi|\hat{a}_{\xi,\mu}}(x)$ which are taken into account according to their pdf is denoted with t'.

The resulting pdf $f_{n_{\nu,\mu}}(n)$ for hybrid MF ISDIC consists of a sum of Gaussian densities and therefore has the same form like the pdfs in [12–14]. But in contrast to these approaches the resulting pdf of interference is derived based on probabilities of interfering symbols calculated in the receiver. Thus, a better approximation is expected.

Figure 5 shows the hybrid pdf approximation of the pdf in Figure 2. Here, the strongest $t' = 2$ interfering symbols have been taken into account. It gets apparent that the complex pdf of Figure 2 is approximated quite well by that of Figure 5. The hybrid pdf approximation of the pdf in Figure 4 is shown in Figure 6. Again, the strongest $t' = 2$ interfering symbols have been taken into account. In this case the complex pdf of Figure 4 is approximated almost perfectly by that of Figure 6. In both cases the scenario of Section 3.3 has been considered.

To judge the accuracy of the hybrid scheme in comparison to the scheme with the Gaussian assumption, an analysis according to the Kullback Leibler distance (KLD) [19] is conducted in the following. The KLD is a measure for the similarity of two pdfs, where a value of KLD = 0 corresponds to a perfect match. First, $10\log_{10}(E_b/\mathcal{N}_0) = 25$ dB is considered. The KLD of the approximation of the actual pdf in Figure 2 and the corresponding pdf with Gaussian assumption in Figure 1 is 1.2784. In contrast, the KLD of the approximation of the actual pdf in Figure 2 and the

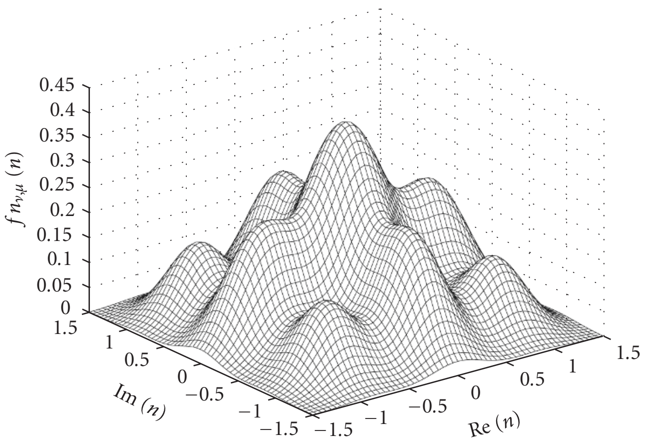

Figure 6: Hybrid pdf approximation of ISI, MAI, and noise for $10\log_{10}(E_b/\mathcal{N}_0) = 10$ dB with $t' = 2$.

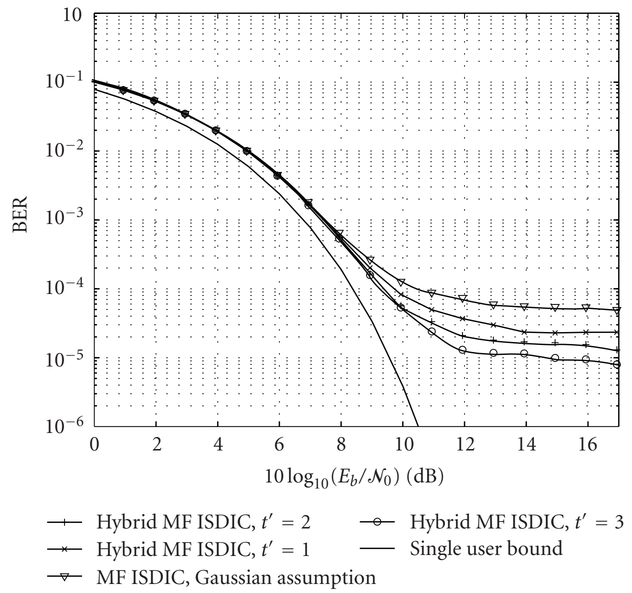

Figure 7: BER versus $E_b/\mathcal{N}_0$ for MF ISDIC employing the Gaussian assumption and hybrid MF ISDIC. $\mu_{\max} = 40$, $\epsilon = 0.05$, 4QAM with $\sigma_a^2 = 2$, uncoded DS-CDMA downlink transmission with $K = 10$ spreading sequences having spreading factor $N = 16$ over a Rayleigh multipath channel, ideal power control.

corresponding pdf with hybrid approximation in Figure 5 is 0.4290. Obviously, the approximation of the actual pdf is matched more accurately by the hybrid approach than by the Gaussian assumption. For $10\log_{10}(E_b/\mathcal{N}_0) = 10$ dB, the following observations can be made. The KLD of the approximation of the actual pdf in Figure 4 and the corresponding pdf with Gaussian assumption in Figure 3 is 0.1557. In contrast, the KLD of the approximation of the actual pdf in Figure 4 and the corresponding pdf with hybrid approximation in Figure 6 is $2.6575 \cdot 10^{-4}$. Here, the approximation of the actual pdf is matched almost perfectly by the hybrid scheme—much better than with the Gaussian assumption.

4. Numerical Results and Discussion

MF ISDIC employing the common Gaussian assumption and hybrid MF ISDIC as introduced in Sections 3.1 and 3.4, respectively, are compared in the following by means of simulations. The results are shown in Figure 7. We consider a Rayleigh multipath channel consisting of ten chip-spaced paths with decreasing average powers according to $\mathcal{E}\{|h[0]|^2\} = 0.4850$, $\mathcal{E}\{|h[1]|^2\} = 0.2813$, $\mathcal{E}\{|h[2]|^2\} = 0.1095$, $\mathcal{E}\{|h[3]|^2\} = 0.0555$, $\mathcal{E}\{|h[4]|^2\} = 0.0260$, $\mathcal{E}\{|h[5]|^2\} = 0.0225$, $\mathcal{E}\{|h[6]|^2\} = 0.0140$, $\mathcal{E}\{|h[7]|^2\} = 0.0013$, $\mathcal{E}\{|h[8]|^2\} = 0.0035$, and $\mathcal{E}\{|h[9]|^2\} = 0.0014$.

This power delay profile is an approximation of the power delay profile of the vehicular A test channel [20] for chip-spaced paths. An ideal power control algorithm is assumed resulting in a normalization of the sum of all instantaneous tap powers to one. Uncoded multicode transmission is applied using $K = 10$ spreading sequences with spreading factor $N = 16$, representing the whole load of the base station. The channel is constant during the transmission of one block, that is, a block fading channel model is used. For simulations, L has been chosen to 48, the number of iterations maximally tolerated to $\mu_{\max} = 40$, and $\epsilon = 0.05$ ($\sigma_a^2 = 2$ for 4QAM). Note that this scenario is related to a high speed downlink packet access (HSDPA) transmission with UMTS.

In Figure 7 simulation results for MF ISDIC employing the common Gaussian assumption and hybrid MF ISDIC are shown for 4QAM transmission. For hybrid MF ISDIC, all but the $t' \in \{1, 2, 3\}$ strongest residual interferers are modeled to be Gaussian distributed. Obviously, the bit error ratio (BER) can be lowered by 80 for high $E_b/\mathcal{N}_0$ by applying the proposed hybrid MF ISDIC, assuming that all residual interferers but the strongest three are Gaussian distributed.

5. Concluding Remarks

In this paper, a receiver algorithm performing matched filter-based iterative soft decision interference cancellation (ISDIC) which was proposed recently has been analyzed for a DS-CDMA downlink transmission over multipath channels when employing general square QAM constellations. The commonly used Gaussian assumption for the pdf of residual interference has been replaced by a better approximation of the exact pdf. Additionally, a hybrid scheme has been proposed, which provides less computational complexity by considering only the strongest residual interferers according to their pdf while the Gaussian assumption is applied to the weak residual interferers. The algorithms have been compared by means of simulations for an UMTS scenario, and it has been shown, that the bit error ratio decreases already noticeably when only a small number of residual interferers is processed according to their pdf. In fact, for the considered DS-CDMA transmission we have been able to lower the bit error ratio by 80% for high signal-to-noise ratios when modeling all residual interferers but the strongest three to be Gaussian distributed.

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