

Research Article

Space Alignment for Cognitive Transmission in MIMO Uplink Channels

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This paper investigates a new transmission technique for cognitive access in multiple-input multiple-output (MIMO) uplink channels with waterfilling power allocation- (WPA-) based primary transmission. The proposed technique allows a cognitive node to access the common destination simultaneously with the primary node, without affecting the MIMO primary performance. By using an appropriate precoding design, the cognitive node aligns its transmitted signal to the unused primary eigenmodes and ensures an orthogonality between the primary and the cognitive links. In order to overcome the limitation of the conventional WPA at high signal-to-noise ratios, a modified WPA enables the primary node to release some eigenmodes in order to provide a positive cognitive rate for all the cases.

1. Introduction

Cognitive radio (CR) is introduced as an efficient technique in order to use the radio spectrum more efficiently [1]. It is characterized by the capability of the cognitive radios (secondary or unlicensed nodes) to coexist with the spectrum owners (primary or licensed nodes) and share the same frequency band in an opportunistic fashion. The opportunistic access enables the cognitive nodes to use the spectrum when it is idle (spectrum holes) and makes the CR network transparent to the primary network. However, for scenarios with high primary traffic, the number of spectrum holes becomes limited, and thus a positive cognitive data rate cannot be supported.

In order to overcome this limitation of the conventional CR concept, several approaches have been proposed in the literature. In [2], a dirty-paper coding (DPC) approach allows both the primary and the secondary nodes to simultaneously access the channel (interference channel) and protects the primary receiver from interference. However, the DPC design requires a knowledge of the primary signal at the CR as well as a global instantaneous channel knowledge which correspond to a high implementation complexity. On the other hand, in [3] the primary node exchanges its transmission silence with a cooperative (relaying) assistance

from the cognitive node in order to increase both the primary and the cognitive performance. However, this approach requires a cooperation between primary and CR nodes which is not always possible in a “strict” cognitive context where the primary network is not aware of the CR nodes. Another approach incorporates the interference alignment concept [4, 5] with a multiple-input multiple-output (MIMO) waterfilling power allocation (WPA) [6, 7] in order to achieve orthogonality between primary and cognitive networks. This approach uses an appropriate precoding technique in order to align the cognitive transmitted signal to the unused eigenmodes of the primary channel. However, the original work in [6, 7] focuses on an interference channel (2 Tx-2 Rx) and cannot guarantee a cognitive transmission at high signal-to-noise ratios (SNRs), where the number of the unused eigenmodes is limited.

In this paper we extend the technique presented in [6] for a MIMO uplink channel with CR. An appropriate space alignment design enables the CR node to access the common destination simultaneously with the primary node without affecting the primary performance by using the unused primary eigenmodes. We show that due to the uplink topology the cognitive space alignment corresponds to a parallel symmetric Gaussian channel where the number of subchannels is equal to the number of unused primary

eigenmodes. In addition, in order to ensure a positive cognitive data rate for high SNRs, a modified WPA that allows the primary node to control its transmitted power for each subchannel is proposed. We show that an appropriate power threshold can release some primary eigenmodes for cognitive transmission without significantly affecting the primary performance. The average achievable rate for both the primary and the secondary networks is evaluated via theoretical and analytical results. To the best of our knowledge the space alignment design for CR uplink channels as well as the modified WPA scheme are reported in this paper for the first time.

The rest of this paper is organized as follows. In Section 2 we present the system model, and we introduce the main assumptions required for our analysis. In Section 3 we describe the proposed CR space alignment design, and we discuss its achievable rate performance. Numerical results are shown and discussed in Section 4, followed by concluding remarks in Section 5.

Notation. Upper case and lower case bold symbols denote matrices and vectors, respectively. $\text{Trace}(\mathbf{X})$ denotes the trace of a matrix \mathbf{X} , \mathbf{I}_n denotes the identity matrix of order n , $\log(\cdot)$ denotes the logarithm of base 2, $\mathbb{E}[\cdot]$ represents the expectation operator, and the superscript H denotes hermitian transposition operation.

2. System Model

We assume a three-node cognitive topology consisting of one primary node P , one secondary (cognitive) node S , and a common destination R (e.g., base station) as shown in Figure 1 (a similar uplink configuration is assumed in [3]). All the nodes are equipped with $M > 1$ antennas, and both nodes operate in the same frequency band following the rules of the cognitive radio. More specifically, the node P (licensed node) exclusively uses the channel and “enjoys” a point-to-point MIMO channel while the node S (unlicensed node) is looking for transmission opportunities and can access the channel subject to the constraint of not affecting the primary link. The common receiver has been optimized (designed) for the primary network and cannot handle Multiple-access interference (MAI) by using advanced interference cancelation schemes. (The goal of most cognitive radio applications is to allow unlicensed nodes to opportunistically use a licensed band. Thus, in many cases, the signaling format of the cognitive nodes may not be known, and thus interference mitigation will generally not be possible.) All the nodes always have data to transmit, and therefore the primary link never becomes idle which results in no-transmission opportunities for the cognitive node. However, an appropriate linear precoding enables the cognitive node to simultaneously access the channel and ensures a positive cognitive rate without affecting the primary network. The received signal at R is given by

$$\mathbf{y} = \mathbf{H}_{RP}\mathbf{G}_P\mathbf{x}_P + \mathbf{H}_{RS}\mathbf{G}_S\mathbf{x}_S + \mathbf{n}, \quad (1)$$

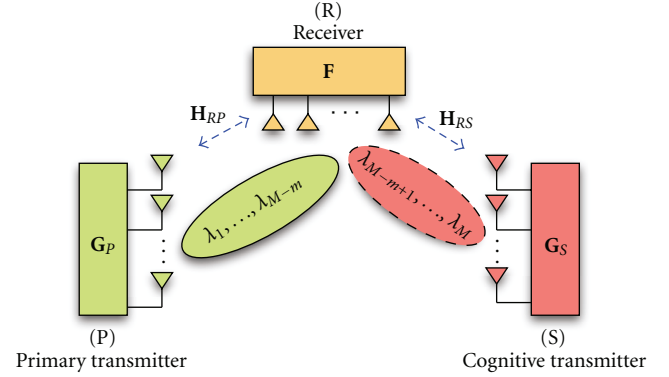


FIGURE 1: Cognitive transmission in an $M \times M$ MIMO channel via SVD and space alignment.

where $\mathbf{x}_i \in \mathbb{C}^{M \times 1}$ denotes the transmit vector for the i th node with $i \in \{P, S\}$, the matrix $\mathbf{G}_i \in \mathbb{C}^{M \times M}$ denotes the linear precoder used at the i th node (unitary matrix with $\text{Trace}[\mathbf{G}_i\mathbf{G}_i^H] = M$), $\mathbf{H}_{Ri} \in \mathbb{C}^{M \times M}$ is the $M \times M$ channel matrix for the link $i \rightarrow R$, and $\mathbf{n} \in \mathbb{C}^{M \times 1}$ indicates an additive white Gaussian noise (AWGN) vector at the receiver with a covariance matrix \mathbf{I}_M . Both nodes are subject to a power constraint $\text{Trace}[\mathbf{P}_i] \leq p_0$, where $\mathbf{P}_i \triangleq \mathbb{E}[\mathbf{x}_i^H\mathbf{x}_i]$ denotes the covariance matrix of the transmit vector \mathbf{x}_i and indicates the power allocation at each node. The entries of the channel matrices \mathbf{H}_{Ri} are independently and identically distributed (i.i.d.) complex Gaussian circularly symmetric random variables with unit variance (without loss of generality); this means that both channel matrices are almost surely of full rank (i.e., $\text{rank}[\mathbf{H}_{Ri}] = M$). In addition, the channel matrices remain constant for the whole transmission and change to an independent realization for the next transmission. At the receiver R , the received signal is linearly processed with the postprocessing matrix $\mathbf{F} \in \mathbb{C}^{M \times M}$ which results in the output signal $\mathbf{r} = \mathbf{F}\mathbf{y}$.

As for the channel side information (CSI), we assume that both the primary and the secondary networks have a global knowledge of their channels (a similar assumption is considered in [2, 6, 7]). More specifically, we assume that the instantaneous primary channel \mathbf{H}_{RP} is perfectly known at the nodes P, S , and the receiver R while the instantaneous secondary channel \mathbf{H}_{RS} is perfectly known at the cognitive node S . It is worth noting that although this assumption provides *an upper bound* on the achievable rate for both the primary and the cognitive networks, several techniques make this assumption reasonable: (a) in some contexts channel reciprocity can be exploited to acquire CSI at the transmitters (Time Division Duplex (TDD) mode), (b) feedback channels are often available in wireless communications (in several modern wireless standards, e.g., Long Term Evolution (LTE)), and (c) learning mechanisms [8] can be exploited to iteratively track the required CSI. The impact of an imperfect channel knowledge on the achievable rates is beyond the scope of this paper and will be considered for future investigation.

3. A Space Alignment Technique for Cognitive Access

In this section we introduce a space alignment technique that enables the cognitive node to communicate with the common destination R , simultaneously with the primary node, without affecting the MIMO primary network. The performance of the system is determined in terms of the achievable data rate for both the primary and the secondary networks.

3.1. Primary Network. According to the principles of the cognitive radio, the primary network has an exclusive use of the spectrum and ignores the existence of the cognitive network. Given that both the primary node and the common receiver have a knowledge of the instantaneous channel \mathbf{H}_{RP} , the combination of a spatial multiplexing transmission based on the single value decomposition (SVD) of the channel, matrix with a WPA maximizes the primary achievable rate [9, Section 7.1]. The SVD technique allows the primary node to send parallel data streams (without interference between them) along the eigenmodes of the channel and the WPA allocates the power based on the instantaneous strength of the subchannels. More specifically, if $\mathbf{H}_{RP} = \mathbf{U}\mathbf{\Lambda}\mathbf{V}^H$ denotes the SVD of the matrix \mathbf{H}_{RP} , where $\mathbf{V}, \mathbf{U} \in \mathbb{C}^{M \times M}$ are (rotation) unitary matrices and $\mathbf{\Lambda} \in \mathbb{C}^{M \times M}$ is a rectangular matrix whose diagonal elements $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_M$ are the ordered singular values of the matrix \mathbf{H}_{RP} , the SVD design requires

$$\mathbf{G}_P = \mathbf{V}, \quad \mathbf{F} = \mathbf{U}^H \quad (2)$$

and transforms the initial MIMO channel to M scalar parallel channels defined as

$$\begin{aligned} \mathbf{r} &= \mathbf{U}^H \mathbf{y} \\ &= \mathbf{U}^H \left[\left(\mathbf{U}\mathbf{\Lambda}\mathbf{V}^H \right) \mathbf{v}\mathbf{x}_P + \mathbf{H}_{RS}\mathbf{G}_S\mathbf{x}_S + \mathbf{n} \right] \\ &= \mathbf{\Lambda}\mathbf{x}_P + \mathbf{U}^H \mathbf{H}_{RS}\mathbf{G}_S\mathbf{x}_S + \mathbf{n}', \end{aligned} \quad (3)$$

where $\mathbf{n}' \triangleq \mathbf{U}^H \mathbf{n}$ is an AWGN with covariance matrix \mathbf{I}_M . If we ignore the interference-related term (the proposed technique forces this interference to zero), the maximum average achievable primary rate is given as

$$R_P(\delta) = \mathbb{E} \left[\log \det \left(\mathbf{I}_M + \mathbf{P}_P \mathbf{\Lambda} \mathbf{\Lambda}^H \right) \right], \quad (4)$$

where $\mathbb{E}(\cdot)$ denotes the expectation operator and the power allocation covariance matrix \mathbf{P}_P is given by the modified WPA defined as

$$P_P(k, k) = \begin{cases} \left[\mu - \frac{1}{\lambda_k^2} \right] & \text{if } \left[\mu - \frac{1}{\lambda_k^2} \right] > \delta, \\ 0, & \text{elsewhere,} \end{cases} \quad (5)$$

where $k = 1 \dots M$, the parameter μ is chosen to satisfy the total power constraint $\sum_k^M P_P(k, k) = p_0$, $\delta \in [0, p_0)$ is a constant, and the noise power is equal to 1. In contrast

to the conventional WPA technique, which corresponds to $\delta = 0$ and maximizes the achievable rate, the introduced modified waterfilling policy gives the primary node the flexibility to control the power allocated at each eigenmode. More specifically, the system parameter δ depends on the *degradation tolerance* (maximum degradation level without affecting the required system quality of service (QoS)) that characterizes the system and enables the primary node to release some spatial directions without significantly affecting its performance (e.g., if $\mu - 1/\lambda_k^2 \approx 0$, the release of the k th spatial channel does not modify the achievable performance).

Parameter δ . From a CR point of view, the maximization of the parameter δ in respect to the primary performance degradation tolerance, gives more opportunities for a secondary transmission (as the parameter δ is increased, the primary eigenmodes are released with a higher probability). Therefore, if $0 < \mathcal{T} \leq 1$ denotes the primary performance degradation tolerance (its value depends on the QoS of the application), the optimal δ is expressed by the following optimization problem:

$$\delta^* = \max_{\delta \in [0, p_0)} \delta, \quad (6)$$

$$\text{subject to } R_P(\delta) \geq \mathcal{T} R_P(0), \quad (7)$$

where $R_P(0)$ denotes the maximum achievable primary rate corresponding to $\delta = 0$. Due to the iterative nature of the WPA [9], the above optimization problem can be solved by using a simple iterative algorithm that continuously increases δ (with a constant step) until the constraint in (7) is satisfied. Although a further theoretical analysis of δ is beyond the scope of this paper, an interesting remark holds for the high SNR regime. More specifically, given that the WPA converges to a uniform PA scheme at high SNRs, a parameter δ with $p_0/m \leq \delta < p_0/(m-1)$ (where $m = 2, 3, \dots, M$) ensures the release of $M - m + 1$ primary eigenmodes for all the cases. The parameter δ is introduced as a critical system parameter, and its importance is evaluated via simulation results in the next section.

3.2. Primary Network. The proposed technique allows the cognitive node to access the channel simultaneously with the primary node by using the unused primary spatial directions. An appropriate space alignment concentrates the cognitive transmission to the unused primary spatial eigenmodes (with $P_P(k, k) = 0$) and enables a positive cognitive rate without affecting the primary performance. The proposed space alignment requirement is defined as

$$\mathbf{U}^H \mathbf{H}_{RS} \mathbf{G}_S = \bar{\mathbf{P}}_P, \quad (8)$$

where the matrix $\bar{\mathbf{P}}_P$ models the unused primary subspace (eigenmodes) and is an $M \times M$ diagonal matrix with entries

$$\bar{P}_P(k, k) = \begin{cases} 1 & \text{if } P_P(k, k) = 0, \\ 0, & \text{elsewhere.} \end{cases} \quad (9)$$

Based on (8) and (9), the secondary precoding matrix that aligns the cognitive signal to the available primary subspace is equal to

$$\mathbf{G}_S = (\mathbf{H}_{RS})^{-1} \mathbf{U} \bar{\mathbf{P}}_P. \quad (10)$$

(The cognitive node perfectly senses the available primary eigenmodes, or a simple primary signaling feeds the matrix $\bar{\mathbf{P}}_P$ to the cognitive node.) The average rate achieved by the secondary node becomes equal to

$$\begin{aligned} R_S &= \max_{\mathbf{P}_S} \mathbb{E} \left[\log \det \left(\mathbf{I}_M + \mathbf{H}_{RS} \mathbf{G}_S \mathbf{P}_S \mathbf{G}_S^H \mathbf{H}_{RS}^H \right) \right] \\ &\quad \text{(the noise variance is equal to 1)} \\ &= \max_{\mathbf{P}_S} \mathbb{E} \left[\log \det \left(\mathbf{I}_M + \bar{\mathbf{P}}_P \mathbf{P}_S \right) \right] \\ &= \max_{\mathbf{P}_S} \mathbb{E} \left[\sum_{k=1}^M \log \left(1 + \bar{P}_P(k, k) P_S(k, k) \right) \right] \\ &= \mathbb{E} \left[m \log \left(1 + \frac{p_0}{m} \right) \right] \quad \left(\text{with } \mathbf{P}_S = \frac{p_0}{m} \bar{\mathbf{P}}_P \right) \\ &\leq \mathbb{E}[m] \log \left(1 + \frac{p_0}{\mathbb{E}[m]} \right) \quad \left(\text{Jensen's inequality} \right), \end{aligned} \quad (11)$$

where m is the number of the available primary eigenmodes (e.g., $m \triangleq \sum_{k=1}^M \bar{P}_P(k, k)$) and the upper bound in (12) yields by Jensen's inequality for the concave function $f(x) = x \log(1 + C/x)$, where C is a constant. The above expressions demonstrate that the cognitive transmission is transformed to m symmetric parallel Gaussian channels (without a channel fading degradation), and thus a PA scheme, which symmetrically allocates the available power p_0 among the m subchannels, maximizes the instantaneous achievable rate. In addition, the average number of the unused primary eigenmodes can be written as

$$\mathbb{E}[m] = \sum_{i=1}^{M-1} \mathbb{P}[m = i] i, \quad (13)$$

$$\mathbb{P}[m = M - i] = \mathbb{P} \left[\frac{1}{\lambda_i} \leq \frac{1}{i} \left(p_0 + \sum_{l=1}^i \lambda_l^{-1} - \delta \right) \leq \frac{1}{\lambda_{i+1}} \right], \quad (14)$$

where the above expression is based on the ordered singular values of the matrix \mathbf{H}_{RP} as well as the WPA applied on the primary network. The probability in (14) can be evaluated numerically, and its general closed form is beyond the scope of this paper. (For small values of M a closed form expression is possible; for example, for $M = 2$, $\delta = 0$ the joint probability density function of the ordered eigenvalues is $f(\lambda_1, \lambda_2) = (\lambda_1 - \lambda_2)^2 \exp(-\lambda_1 - \lambda_2)$ [10] which results in $\mathbb{P}[m = 1] = \mathbb{P}[\lambda_2 < \lambda_1 / (\lambda_1 p_0 + 1)] = \int_0^{\lambda_1 / (\lambda_1 p_0 + 1)} \int_0^\infty f(\lambda_1, \lambda_2) d\lambda_1 d\lambda_2$.) It is worth noting that, due to the channel inversion that is involved in the cognitive precoder matrix \mathbf{G}_S in (10), the achievable cognitive rate is independent of the channel \mathbf{H}_{RS} and the related fading/shadowing effects.

4. Numerical Results

Computer simulations were carried out in order to evaluate the performance of the proposed scheme. The simulation environment follows the system model of Section 2 and the adopted performance metric is the average data rate expressed in bits per channel use (BPCU).

In Figure 2 we plot the achievable data rate for both the primary (4) and the secondary (11) network versus the SNR ($\triangleq p_0$) for different numbers of antennas. (The SNR is similar for both the primary and the secondary networks due to the normalized assumed system model and the precoding process at the cognitive node (the primary channel matrix is a unitary matrix and the cognitive link is independent of the channel fading degradation.) A conventional WPA ($\delta = 0$) is assumed for the primary link while the primary performance with a uniform power allocation ($P_P(k, k) = p_0/M$) is used as a reference curve. The first important observation is that the proposed scheme enables a positive cognitive data rate without any modification at the primary network. More specifically, the cognitive data rate achieves its maximum value at the intermediate SNRs ($p_0 \approx 10$ dB) where the number of the unused eigenmodes as well as their SNR strength provides the best combination. On the other hand, at low SNRs, although the number of the unused eigenmodes is the maximum one (the primary transmitted power is concentrated to the eigenmode with the maximum singular value), the channel quality results in a poor cognitive performance. For high SNRs, the conventional WPA spreads the transmitted power symmetrically along the M eigenmodes of the channel, and therefore the cognitive data rate converges to zero, as no eigenmode is available to convey the cognitive data [9, Section 8.2.2]. In order to validate this remark we can see that the primary performance matches the one achieved by a uniform PA at high SNRs. Furthermore, it can be seen that as the number of antennas increases, the performance is improved for both the primary and the secondary networks. An increase in the number of antennas increases the number of eigenmodes (M) which further increases the number of unused eigenmodes.

Figure 3 demonstrates the impact of the PA parameter δ on the performance of the proposed scheme. The simulation setup is similar to the one used in Figure 2 with $\delta = p_0/M$, which is motivated by the behavior of the conventional WPA at high SNRs. More specifically, given that the conventional WPA becomes equivalent to a uniform PA for the high SNR regime, a $\delta = p_0/M$ ensures that at least one primary eigenmode will always be available in order to convey cognitive data. As can be seen in Figure 3, the cognitive data rate does not converge to zero for high SNRs and continues to increase as the SNR increases. An important observation is that for intermediate SNRs the cognitive data rate significantly increases without significantly affecting the primary data rate (i.e., for $M = 6$ and $p_0 = 15$ dB, the cognitive rate increased from 3 BPCU to 7 BPCU while the primary performance remains almost equal to 24.5 BPCU). On the other hand, for high SNRs, the release of some eigenmodes (due to the constraint $\delta = p_0/M$) yields a performance degradation for the primary link. This performance degradation should

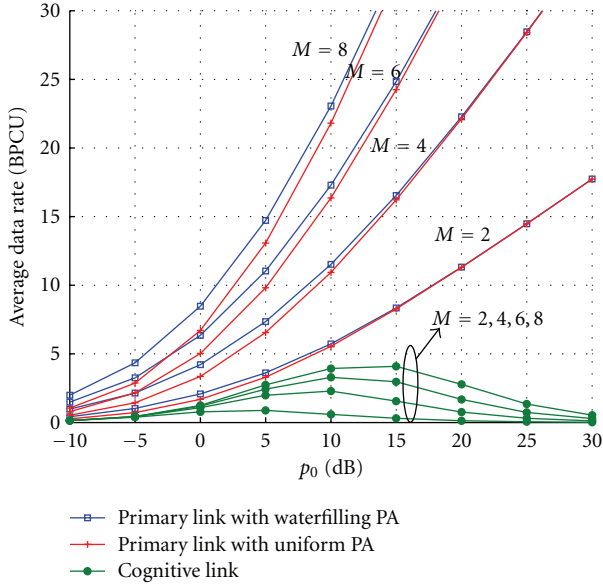


FIGURE 2: Average data rate for both the primary and the secondary networks versus SNR for different numbers of antennas; $M = 2, 4, 6, 8$ and, $\delta = 0$ (conventional WPA).

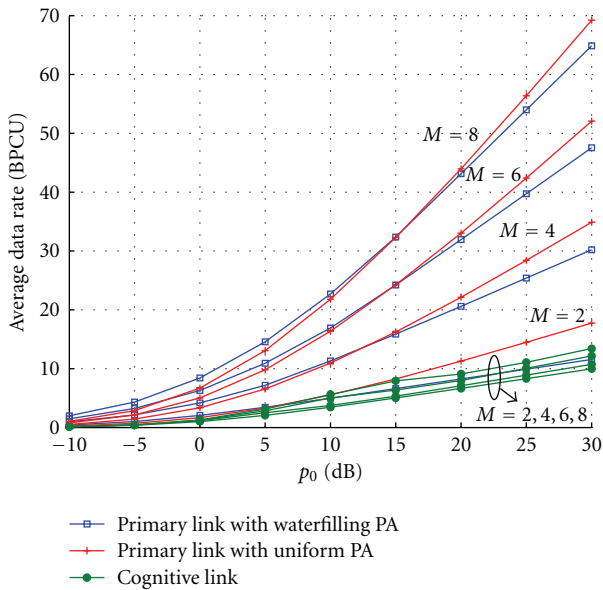


FIGURE 3: Average data rate for both the primary and the secondary networks versus SNR for different numbers of antennas; $M = 2, 4, 6, 8$, and $\delta = p_0/M$.

satisfy the tolerance of the system and is the cost for the cognitive transmission. However, it can be seen that as the number of antennas increases, the number of the released eigenmodes becomes negligible in comparison to M , and thus the performance degradation decreases.

Finally, Figures 4 and 5 plot the average data rate for both the primary and the secondary nodes ((4) and (11)) versus the power allocation level δ for $p_0 = 10$ dB and $p_0 = 30$ dB, respectively, with $M = 4$ antennas. As it can be

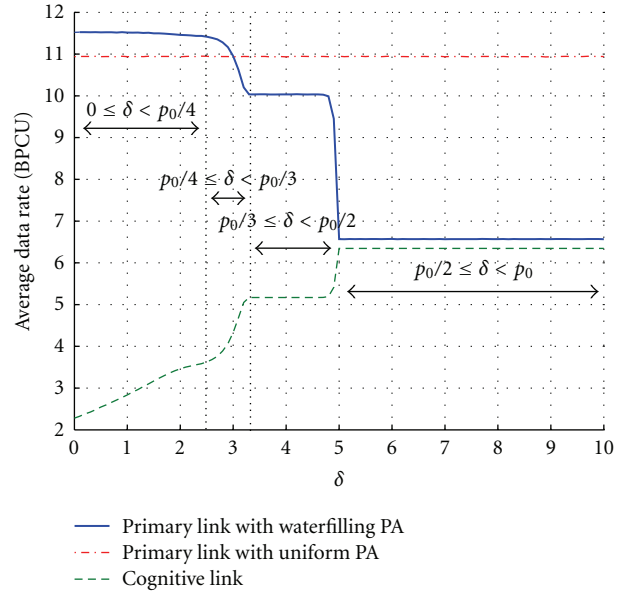


FIGURE 4: Average data rate for both the primary and the secondary networks versus the power allocation level δ (linear scale); $M = 4$, and $p_0 = 10$ dB.

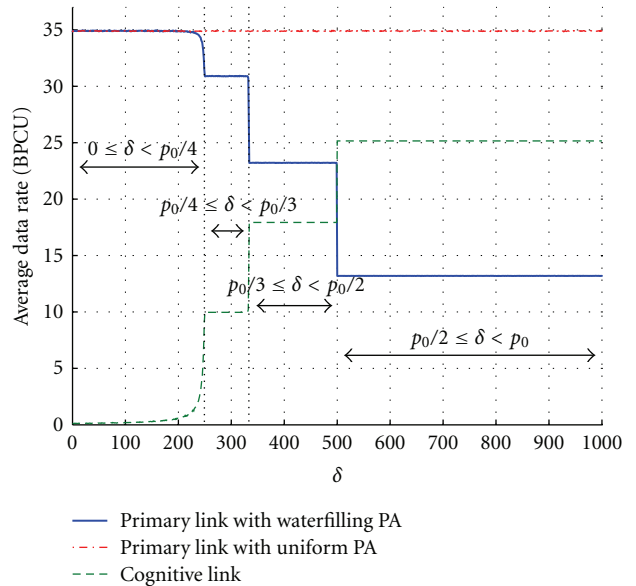


FIGURE 5: Average data rate for both the primary and the secondary networks versus the power allocation level δ (linear scale); $M = 4$, and $p_0 = 30$ dB.

seen, the parameter δ significantly affects the performance for both nodes and introduces a trade-off between them. More specifically, as the parameter δ is increased, the primary eigenmodes are released with a higher probability which results in a degradation of the primary performance while it increases the CR performance. An interesting observation is that the average performance for both nodes is divided in some rate areas where the achievable data rate is almost constant. For high SNRs (Figure 5) this observation is in

line with the discussion in Section 3.1, and therefore a δ that takes values in the interval $[(p_0/m)(p_0/(m-1))]$ (with $m = 2, 3, 4$) ensures that $5 - m$ spatial directions are used for CR transmission while the remaining $m - 1$ spatial directions are used for primary transmission. On the other hand, at low SNRs and for high values of m (e.g., $m = 4$) the parameter δ does not guarantee a continuous release of the corresponding primary eigenmodes as the WPA levels $P_p(k, k)$ strongly depend on the related single values of the primary channel (δ is of the same order with the inverse of the single values). However, as δ increases the WPA levels converge to a uniform PA (δ becomes significantly larger than the inverse of the single values), and therefore the system performance follows our remarks for the high SNR regime.

5. Conclusion

A new cognitive transmission technique for MIMO uplink channels has been proposed. The new technique incorporates space alignment with WPA and results in an orthogonality between primary and cognitive transmissions. We have shown that a conventional WPA provides an efficient cognitive performance for intermediate SNRs but results in a zero cognitive data rate for high SNRs. A modified WPA that allows the primary node to release some eigenmodes without affecting its required QoS and ensures a positive cognitive data rate for all the cases has been also investigated.

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