Hindawi Publishing Corporation
EURASIP Journal on Wireless Communications and Networking
Volume 2010, Article ID 595431, 17 pages
doi:10.1155/2010/595431

Research Article

Opportunistic Multicasting Scheduling Using Erasure-Correction Coding over Wireless Channels

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Received 9 August 2010; Accepted 3 November 2010

Academic Editor: Zhiqiang Liu

This paper proposes an opportunistic multicast scheduling scheme using erasure-correction coding to jointly explore the multicast gain and multiuser diversity. For each transmission, the proposed scheme sends only one copy to all users in the multicast group at a transmission rate determined by a SNR threshold. Analytical framework is developed to establish the optimum selection of the SNR threshold and coding rate for given channel conditions to achieve the best throughput in both cases of full channel knowledge and only partial channel knowledge of the average SNR and fading type. Numerical results show that the proposed scheme outperforms both the worst-user and best-user schemes for a wide range of average SNR and multicast group size. Our study indicates that full channel knowledge is only significantly beneficial at small multicast group size. For a large multicast group, partial channel knowledge is sufficient to closely approach the achievable throughput in the case of full channel knowledge while it can significantly reduce the overhead required for channel information feedback. Further extension of the proposed scheme applied to OFDM system to exploit frequency diversity in a frequency-selective fading environment illustrates that a considerable delay reduction can be achieved with negligible degradation in multicast throughput.

1. Introduction

Multicast services over wireless communications have recently become more and more popular. Multicast gain has been explored in a worst-user (WU) approach by transmitting only one copy to all users in the multicast group, for example, [1]. In wireline networks, since user channels are fixed, the multicast throughput increases linearly with the multicast group size, N. However, due to a possible large difference in the instantaneous channel gains of the various links from the base-station (BS) to users in a wireless fading environment, the BS may have to apply the lowest supportable rate (corresponding to the worst BS-user link), which results in very low bandwidth efficiency. Based on this fact, opportunistic scheduling for *unicast* transmission to explore the multiuser diversity by sending one copy to the user with the highest instantaneous channel gain has been extensively researched. Unfortunately, such a best-user (BU) approach does not make use of the multicast gain, which can yield low utility of resources, especially for a large multicast group size. While opportunistic scheduling for unicast transmission has been extensively researched, the idea of exploiting multiuser diversity into multicast has not been studied in an equivalent extent yet. In [2], a threshold-T multicast scheduling scheme at MAC layer is proposed in which in each time-slot the BS sends multicast packet if there are at least T users that have sufficiently good channel to receive the packet. Another opportunistic multicast approach is proposed in [3–6], in which, in each time-slot, one copy is sent to only $T \leq N$ users with the best channel quality of the multicast group. The transmission rate is selected as the supportable rate of the worst user in these T best users. In this way, in each transmission, only T user can reliably receive the packet while the other $(N-T)$ users with insufficient channel gains cannot. To cover the whole multicast group, the use of retransmission has been discussed and proposed in [4–6]; however, those schemes are either inefficient or too complex to implement. In [7–9], proportional fair schemes have been studied aiming to maximize throughput while maintaining the fairness between multicast users and multicast group.

These studies assume perfect knowledge of the channel responses of all users in the multicast group at the BS.

In this paper, we propose an opportunistic multicast scheduling scheme that can jointly explore multicast gain, multiuser diversity, and time/frequency diversity in a wireless fading environment. In the proposed scheme, each packet is sent only once to all users in the multicast group at a transmission rate determined by a selected channel gain threshold and an erasure-correction coding is used to deal with possible erasures when the instantaneous signal-to-noise ratio (SNR) of a BS-user link happens to be inadequate. Reed-Solomon (n, k) erasure-correction code is applied to a block of transmitted packets such that erased packets can be recovered as long as the number of erased packets in a block does not exceed the erasure correction capability, that is, $(n - k)$. As each packet can be transmitted in a time or a frequency slot, erasure-correction coding to a block of transmitted packets effectively explores the time/frequency diversity in a wireless fading environment. The selection of channel gain threshold and erasure correction code parameters are jointly optimized for best multicast throughput. Furthermore, to study the role of channel knowledge, the proposed scheme is considered in two cases: (i) with full channel gain knowledge and (ii) with only partial knowledge of fading type and average SNR. An analytical framework has been developed to evaluate the multicast throughput of the proposed erasure-correction coding opportunistic multicast scheduling (ECOM) scheme as well as the BU and WU approaches. We prove that the effective multicast throughput (i.e., the multicast rate that each user can receive) of WU and BU asymptotically converges to zero as the group size increases while that of our proposed scheme is bounded from zero depending on the SNR. Numerical results illustrate that for small multicast group size, full channel gain knowledge can offer better multicast throughput than partial channel knowledge; however, for large group size, the difference in multicast rates of these two cases is just negligible. Besides, performance evaluation shows that with the ability of combining both gains, the proposed scheme outperforms both BU and WU for a wide range of SNRs.

Furthermore we consider extending ECOM for applications to Orthogonal Frequency Division Multiplexing (OFDM) systems. In particular, we explore frequency diversity in a frequency-selective fading environment by sending coded packets over subcarriers. However, since there is a correlation in channel gains among the subcarriers, deep fade on one subcarrier may result in insufficient instantaneous SNR on neighbouring subcarriers. Hence, we have investigated the effects of correlation in subcarrier channel gains on the achievable multicast throughput of the proposed scheme. Numerical results indicate that by exploring frequency diversity, we can significantly reduce the delay with negligible degradation in multicast throughput.

The rest of this paper is organized as follows. In Section 2 the proposed ECOM schemes are described and the analytical framework on multicast throughput of the proposed schemes and BU and WU is provided. Then performance evaluation and comparisons are discussed to illustrate the trade-off between multicast gain and multiuser diversity and the significance of full and partial channel knowledge. In Section 3, ECOM scheme is extended for applications to OFDM systems. The effects of correlation in subcarrier channel responses in a frequency-selective fading environment on the multicast throughput and the trade-off between throughput and delay are discussed. Finally, Section 4 provides concluding remarks.

2. ECOM Scheme over Block Flat Fading Channels

2.1. System Model. Consider a wireless point-to-multipoint downlink system supporting multicast service for a group of N users. For simplicity, without loss of generality, downlink transmission from the BS to users is assumed to consist of nonoverlapping time-slots; each slot can accommodate one equal-length packet. Let $x(t)$ be the transmitted signal in the time-slot t; let $n_i(t) \sim CN(0, \mathcal{N}_0)$ be additive white Gaussian thermal noise with $\mathcal{N}_0$ noise power. The average SNR, which is denoted by $\overline{\gamma}$, represents the average link quality of the channel assumed to be the same for all BS-user links, (As our main focus in this paper is to study opportunistic multicast schemes for wireless communications in presence of small-scale fading, we consider the homogenous case in which users in a multicasting group have similar average SNR and independent and identically distributed (i.i.d.) small-scale fading. The analytical framework can be extended for the nonhomogeneous case.) The received signal $y_i(t)$ at user i is then given by

$$y_i(t) = h_i(t) * x(t) + n_i(t), \tag{1}$$

where $h_i(t)$ is the instantaneous channel gain in the time-slot t on the link from the BS to user i. $h_i(t)$ represents the instantaneous channel gain on wireless link from the BS to user i with normalized power of $E\{|h_i(t)|^2\} = 1$. Further, fades over BS-user links in each time-slot are assumed to be block frequency-flat fading channels; that is, the channel impulse response can be expressed as $h(t) = h_t\delta(t-\tau)$, where h_t is assumed to be independent and identically distributed (i.i.d.) and quasistatic; that is, any BS-user link fade remain unchanged during a given time-slot and varies independently from one time-slot to another.

In the case of perfect channel knowledge at the transmitter, that is, the BS knows exactly the instantaneous channel gains, $h_i(t)$'s, of all BS-user links, adaptive modulation and coding (AMC) can be applied to achieve the maximum transmission rate, in terms of bandwidth efficiency, b/s/Hz for user i at time-slot t as

$$r_i(t) = \log_2[1 + \overline{\gamma}\rho_i(t)], \qquad \rho_i(t) \triangleq |h_i(t)|^2. \tag{2}$$

Since wireless environment is broadcast in its nature, the BS can transmit each multicast packet to the whole multicast group using only one transmission by sending at the supportable rate of the user with lowest channel response, that is,

$$r_{\mathrm{WU}}(t) = \log_2\left(1 + \overline{\gamma} \min_{i=1,2,\ldots,N}\{\rho_i(t)\}\right). \tag{3}$$

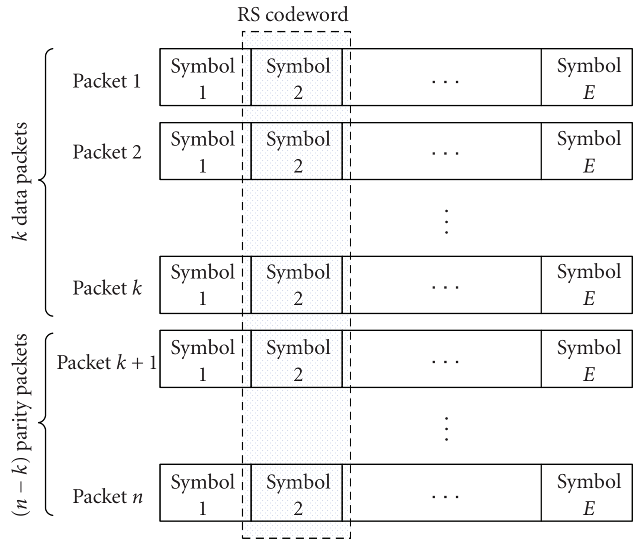


Figure 1: Packet-level coding structure using an RS(n, k) code.

This is known as the worst-user (WU) approach. In a line-of-sight (LOS) environment, the wireless links only suffered from path loss and shadowing, which result in small difference among the channel gains, that is, $\rho_i(t) \approx 1$. In this scenario, it can be seen that by using WU approach, the *full* multicast gain can be achieved. However, when taking into account small-scale multipath fading, instantaneous channel gains of various user links at a given time can be largely different. Hence, $\min_{i=1,2,\dots,N}\{\rho_i(t)\}$ and accordingly, $r_{\mathrm{WU}}(t)$ is likely to be very low when N is large, which may lead to inefficient use of available resource (bandwidth) although multicast gain is exploited (As to be shown in Section 2.3.1, $r_{\mathrm{WU}}(t)$ asymptotically converges to zero as N increases).

In fact, this difference in instantaneous channel responses among the users promotes multiuser diversity that has been explored in unicast services by sending information to the best-user (BU), that is, the user with the best instantaneous channel gain. This opportunistic approach can be also used to support multicast services with the transmission rate of

$$r_{\mathrm{BU}}(t) = \log_2\left(1 + \overline{\gamma}\max_{i=1,2,\dots,N}\{\rho_i(t)\}\right). \tag{4}$$

In this way, the resource utilization can be maximized in each time slot at the cost of sending each packet N times. Since each packet requires at least N transmissions to cover the whole multicast group, the *effective multicast rate* that each user receives can be expressed as

$$r_{\mathrm{BU_{eff}}}(t) = \frac{1}{N}\log_2\left(1 + \overline{\gamma}\max_{i=1,2,\dots,N}\{\rho_i(t)\}\right). \tag{5}$$

As shown in (5), this effective multicast rate of the BU opportunistic approach is likely to be reduced when N increases. (As to be shown in Section 2.3.2, $r_{\mathrm{BU_{eff}}}(t)$ asymptotically converges to zero as N increases.)

From the previous discussion, it can be seen that if we try to take advantage of multicast gain by using WU approach, the BS needs to send multicast packets only once but the consequence is that the transmission rate must be chosen as the lowest rate of all the users. On the other hand, if we try to make use of multiuser diversity by using BU approach, the BS can maximize its transmission rate at each time slot; however, each packet needs to be sent many times.

2.2. Proposed ECOM Schemes. Taking into account both multiuser diversity and multicast gain, the proposed ECOM schemes try to maximize the achievable multicast throughput. ECOM schemes make use of an erasure-correcting code, for example, Reed-Solomon (RS) code, to encode the transmitted packets as shown in Figure 1. (A similar packet-level coding structure used for a different purpose has been proposed for DVB-S2, e.g., see [10].)

Each *information* packet is partitioned into E symbols; each symbol has q bits. Organizing the k *information* equal-length packets (to be sent) in a rowwise manner, they are encoded in a columnwise manner by using a Reed-Solomon code RS(n, k) defined over the Galois field GF(2^q), as follows. Each RS codeword contains k information q-bit symbols and $(n - k)$ *parity* q-bit symbols. The k information symbols of the RS codeword e, $e = 1, 2, \dots, E$, are the eth symbols of the k *information* packets and are used to generate the $(n - k)$ *parity* symbols of the RS codeword e. Each of these $(n - k)$ parity symbols forms the eth symbol of one of $(n - k)$ parity packets. In other words, for k *information* packets, the proposed ECOM scheme sends n packets, in which $(n - k)$ additional packets contain parity symbols as overhead.

The transmission rate (in b/s/Hz) to send n packets is selected as

$$r_{\mathrm{ECOM}} = \log_2(1 + \overline{\gamma}\rho_*), \tag{6}$$

where ρ_* is the predetermined channel gain threshold. Taking into account the overhead of the *parity* packets, the *effective* transmission rate in the proposed ECOM scheme is $(k/n)r_{\mathrm{ECOM}}$. The choice of ρ_* for certain criterion will be discussed later.

It can be seen that, in the time-slot t, users with $\rho_i(t) \geq \rho_*$ can correctly receive the packet. For other users with $\rho_i(t) < \rho_*$, the packet is likely in error due to insufficient instantaneous SNR. In this case, the erroneous packets can be assumed to be erased and this event can be denoted at the receiver. It is well known that an RS(n, k) code can correct up to $(n - k)$ erased symbols, for example, [11]. Therefore, in the proposed ECOM scheme, user i can correctly decode all k packets when the number of events that $\rho_i(t) < \rho_*$ is not exceeding $(n - k)$-within the n time-slots. It can be seen that the proposed ECOM schemes explore multicasting gain by sending only one copy to all N users while making use of both multiuser diversity (by selecting ρ_*) and time diversity (with erasure-correcting codes). Although RS code is used as an illustrative example in this paper, other erasure-correcting codes can be applied in the proposed ECOM schemes.

Regarding the choice of ρ_*, interesting questions are raised: whether possessing exact channel gain knowledge of all users can help to increase multicast throughput? And if it can, in which case channel gain knowledge is most pronounced and in which case the gain provided by this side information is negligible. Motivated by these questions, the selection of ρ_* is considered for two following scenarios.

2.2.1. ECOM with Full Channel Knowledge (ECOMF). Inspired by WU and BU as extreme cases of multicast gain and multiuser diversity and threshold-T scheme, if the base-station transmitter has full knowledge of the instantaneous channel gains, $\rho_i(t)$'s, of all users in every timeslot, the BS can sort users in the descending order of their instantaneous channel gains, that is, $\rho_1 > \rho_2 > \cdots > \rho_{N'} > \cdots > \rho_N$, and selects a subgroup of N' users ($N' \leq N$) that have the highest channel gains and ρ_* as $\rho_* = \rho_{N'}$.

Interestingly, WU and BU can be considered as two specific cases of ECOMF; that is, WU is ECOMF with $N' = N$ (all users), $k = n$ (no coding), while BU is ECOMF with $N' = 1$ (best user), $k = 1$ (repetition code).

The choice of the subgroup size N' and code rate k/n is crucial in optimizing the required transmission rate and will be discussed in Section 2.3.3 (1).

2.2.2. ECOM with Partial Channel Knowledge (ECOMP). As the full knowledge of the instantaneous channel gains, $\rho_i(t)$, of all users at any time-slot t comes at the costs of required fast and accurate channel measurements and signalling between the BS and users, it is interesting to consider the case without perfect channel information at transmitter. In particular, we investigate an approach called ECOMP to select $\rho_* = \rho_{\text{th}}$ that maximizes the average multicast rate based on the partial knowledge of the channel stochastic properties of the BS-user links, for example, the fading *type* and average SNR $\overline{\gamma}$. The throughput analysis of ECOMP is to be discussed in Section 2.3.3 (2).

2.3. Throughput Analysis. For the considered quasistatic i.i.d. fading environment, the channel gain $\rho_i(t)$ can be represented by a random variable ρ with the probability density function (pdf) $f_\rho(\rho)$ and the instantaneous SNR is denoted by the random variable $\gamma \triangleq \overline{\gamma}\rho$.

2.3.1. Throughput of Worst-User (WU) Scheme. In the WU scheme, only one copy is sent to all N users using the transmission rate corresponding to the channel gain of the worst user. The cumulative distribution (cdf) of the channel gain of the worst user is given by

$$F_{\rho_{\text{WU}}}(\rho) = 1 - \left(1 - F_\rho(\rho)\right)^N, \tag{7}$$

where $F_\rho(\rho)$ is the cdf of ρ.

As only one copy is sent to all N users, effectively, the average achievable *multicast* rate of the WU scheme is N times the average transmission rate, that is,

$$\overline{r}_{\text{WU}} = N \int_0^\infty \log_2(1 + \overline{\gamma}\rho) f_{\rho_{\text{WU}}}(\rho)\, d\rho, \tag{8}$$

where the pdf $f_{\rho_{\text{WU}}}(\rho) = N(1 - F_\rho(\rho))^{N-1} f_\rho(\rho)$.

According to (8), the effective average throughput of WU for each user is given by

$$\overline{r}_{\text{WU}_{\text{eff}}} = \frac{\overline{r}_{\text{WU}}}{N} = \int_0^\infty \log_2(1 + \overline{\gamma}\rho) f_{\rho_{\text{WU}}}(\rho)\, d\rho. \tag{9}$$

For Rayleigh fading channel, $f_{\rho_{\text{WU}}}(\rho) = Ne^{-N\rho}$ and, therefore, according to Jensen's inequality

$$\begin{aligned} \overline{r}_{\text{WU}_{\text{eff}}} &= E_{\rho_{\text{WU}}}\left[\log_2(1 + \overline{\gamma}\rho)\right] \\ &\leq \log_2\left(1 + \overline{\gamma} E_{\rho_{\text{WU}}}[\rho]\right) = \log_2\left(1 + \frac{\overline{\gamma}}{N}\right) \leq \frac{\overline{\gamma}}{N}. \end{aligned} \tag{10}$$

Since $\overline{\gamma}/N \underset{N \to \infty}{\longrightarrow} 0$, the effective throughput of WU approaches zero as the multicast group size N grows large; therefore, for large multicast group, exploiting only multicast gain is not an efficient way to do multicast.

2.3.2. Throughput of Best-User (BU) Scheme. In the BU scheme, each packet is sent N times at the rate of the user with best channel condition. Under the assumption of a quasistatic i.i.d. fading environment, the cdf of the instantaneous SNR of the best user is given by

$$F_{\rho_{\text{BU}}}(\rho) = \prod_{i=1}^{N} F_{\rho_i}(\rho) = \left(F_\rho(\rho)\right)^N. \tag{11}$$

The expected transmission rate for the best user in any given time-slot is given by

$$E_{\rho_{\text{BU}}}[r_{\text{BU}}] = \int_0^\infty \log_2(1 + \overline{\gamma}\rho) f_{\rho_{\text{BU}}}(\rho)\, d\rho, \tag{12}$$

where the pdf $f_{\rho_{\text{BU}}}(\rho) = N(F_\rho(\rho))^{N-1} f_\rho(\rho)$.

As one copy is sent to each user, effectively, the average achievable *multicast* rate of BU scheme over n time-slots can be expressed as

$$\begin{aligned} \overline{r}_{\text{BU}} &= \frac{N}{n} \sum_{x=1}^{n} \binom{n}{x} E_{\rho_{\text{BU}}}[r_{\text{BU}}] x p^x (1-p)^{n-x} \\ &= \frac{N}{n} E_{\rho_{\text{BU}}}[r_{\text{BU}}] np. \end{aligned} \tag{13}$$

With $p = 1/N$ being the probability that a given user can receive the packet, (23) becomes

$$\overline{r}_{\text{BU}} = N E_{\rho_{\text{BU}}}[r_{\text{BU}}] \frac{1}{N} = E_{\rho_{\text{BU}}}[r_{\text{BU}}]. \tag{14}$$

According to (24), the effective average throughput of BU for each user is given by

$$\overline{r}_{\text{BU}_{\text{eff}}} = \frac{1}{N} E_{\rho_{\text{BU}}}[r_{\text{BU}}]. \tag{15}$$

It is noted that since $p = 1/N$, the probability that a given user can receive the packet after N consecutive transmissions is not 1. Hence, further implementation is needed for BU to achieve (15). One of such implementations is illustrated in [6] with a separated queue for each user.

For Rayleigh fading channel,

$$f_{\rho_{\text{BU}}}(\rho) = N(1 - e^{-\rho})^{N-1} e^{-\rho}, \tag{16}$$

and therefore, according to Jensen's inequality,

$$\begin{aligned}\overline{r}_{\mathrm{BU_{eff}}} &= \frac{1}{N}E_{\rho_{\mathrm{BU}}}\left[\log_2(1+\overline{\gamma}\rho)\right]\\ &\le \frac{1}{N}\log_2\left(1+\overline{\gamma}E_{\rho_{\mathrm{BU}}}[\rho]\right)\\ &= \frac{1}{N}\log_2\left(1+\overline{\gamma}\sum_{i=1}^{N}\frac{1}{i}\right)\\ &\le \frac{1}{N}\log_2\left(1+\overline{\gamma}+\overline{\gamma}\frac{N-1}{2}\right).\end{aligned} \tag{17}$$

Using L'Hospital rule for (17) at the limit $N \to \infty$, we have

$$\begin{aligned}&\lim_{N\to\infty}\frac{1}{N}\log_2\left(1+\overline{\gamma}+\overline{\gamma}\frac{N-1}{2}\right)\\ &\quad= \lim_{N\to\infty}\frac{\overline{\gamma}/2}{1+\overline{\gamma}+\overline{\gamma}(N-1)/2} = 0.\end{aligned} \tag{18}$$

Equations (17)-(18) prove that the effective throughput of BU approaches zero as the multicast group size N grows large; therefore, for large multicast group, exploiting only multiuser diversity is also not an efficient way for multicasting.

2.3.3. Throughput of Proposed ECOM Schemes. In the ECOM schemes, a user can correctly decode its information if it can receive k or more nonerased packets within n transmitted packets. Under the assumption of a quasistatic i.i.d. fading environment, the probability p that channel gain of a certain user is greater than channel gain threshold ρ_* is the same for all users i in all time-slots, and the probability that each user can receive at least k nonerased packets can be expressed as

$$\Pr\{x \ge k\} = \sum_{x=k}^{n}\binom{n}{x}p^x(1-p)^{n-x}. \tag{19}$$

(1) Throughput of ECOMF. As previously discussed, the ECOMF selects a subgroup of N' users ($N' \le N$) that have the highest channel gains and ρ_* as $\rho_{N'}$ the lowest instantaneous channel gain of the N'th user. Under the assumption of a quasistatic i.i.d. fading environment, according to order statistics, the cdf of is given by

$$F_{\rho_{N'}}(\rho) = \sum_{i=N-N'+1}^{N}\binom{N}{i}F_\rho(\rho)^i\left(1-F_\rho(\rho)\right)^{N-i}, \tag{20}$$

and the corresponding pdf is

$$\begin{aligned}f_{\rho_{N'}}(\rho) &= \frac{N}{(N'-1)!(N-N')!}\,F_\rho(\rho)^{N-N'}\\ &\quad\times\left(1-F_\rho(\rho)\right)^{N'-1}f_\rho(\rho).\end{aligned} \tag{21}$$

It is obvious that, in a given time-slot, the channel gain of a certain user is greater than channel gain threshold $\rho_{N'}$ if this user belongs to the selected subgroup of N' users. Since the user channel gains distributions are i.i.d., the probability that a user is in this selected subgroup is N'/N. In other words, the probability p_{ECOMF} that the channel gain of a certain user exceeds the threshold $\rho_{N'}$ is

$$p_{\mathrm{ECOMF}} = \frac{N'}{N}. \tag{22}$$

As a result, the average achievable *multicast* rate of the ECOMF scheme with RS(n,k) is given by

$$\overline{r}_{\mathrm{ECOMF}} = \frac{k}{n}N\cdot E_{\rho_{N'}}\left[\log_2(1+\overline{\gamma}\rho)\right]\cdot\Pr\{x\ge k\}, \tag{23}$$

where $E_{\rho_{N'}}[\log_2(1+\overline{\gamma}\rho)] = \int_0^\infty \log_2(1+\overline{\gamma}\rho)f_{\rho_{N'}}(\rho)d\rho$, and $\Pr\{x\ge k\} = \sum_{x=k}^{n}\binom{n}{x}p_{\mathrm{ECOMF}}^x(1-p_{\mathrm{ECOMF}})^{n-x}$.

When $N' = N$, $k = n$, (23) becomes

$$\begin{aligned}\overline{r}_{\mathrm{ECOMF}} &= \frac{n}{n}N\left(\int_0^\infty \log_2(1+\overline{\gamma}\rho)f_{\rho_N}(\rho)d\rho\right)\binom{n}{n}\left(\frac{N}{N}\right)^n\\ &= N\int_0^\infty \log_2(1+\overline{\gamma}\rho)N\left(1-F_\rho(\rho)\right)^{N-1}f_\rho(\rho)d\rho,\end{aligned} \tag{24}$$

and ECOMF becomes WU.

When $N' = 1$ and $k = 1$, (23) becomes

$$\begin{aligned}\overline{r}_{\mathrm{ECOMF}} &= \frac{1}{n}N\left(\int_0^\infty \log_2(1+\overline{\gamma}\rho)f_{\rho_1}(\rho)d\rho\right)\\ &\quad\times\sum_{i=1}^{n}\binom{n}{i}\left(\frac{1}{N}\right)^i\left(1-\frac{1}{N}\right)^{n-i}\\ &= \frac{1}{n}N\left(\int_0^\infty \log_2(1+\overline{\gamma}\rho)N\left(F_\rho(\rho)\right)^{N-1}d\rho\right)\\ &\quad\times\left(1-\left(1-\frac{1}{N}\right)^n\right)\\ &\underset{N\to\infty}{\longrightarrow} \frac{1}{n}N\left(\int_0^\infty \log_2(1+\overline{\gamma}\rho)N\left(F_\rho(\rho)\right)^{N-1}d\rho\right)\\ &\quad\times\left(1-\left(1-\frac{n}{N}\right)\right)\\ &= \int_0^\infty \log_2(1+\overline{\gamma}\rho)N\left(F_\rho(\rho)\right)^{N-1}d\rho.\end{aligned} \tag{25}$$

As shown in (25), for a very large number of users, ECOMF with $N' = 1$ approaches BU.

For a given channel fading $f_\rho(\rho)$, the average achievable multicast rate of ECOMF, $\overline{r}_{\mathrm{ECOMF}}$, can be optimized by selecting N' and k/n.

(2) Throughput of ECOMP. In ECOMP, for a selected channel gain threshold ρ_{th}, the probability p_{ECOMP} that the channel gain of a certain user exceeds the threshold ρ_{th} is

$$\begin{aligned}p_{\mathrm{ECOMP}} &= \Pr\{\rho > \rho_{\mathrm{th}}\}\\ &= \int_{\rho_{\mathrm{th}}}^\infty f_\rho(\rho)d\rho = 1 - F_\rho(\rho_{\mathrm{th}}).\end{aligned} \tag{26}$$

For example, $p_{\text{ECOMP}} = e^{-\rho_{\text{th}}}$ for a Rayleigh fading channel. In average, there are only $N\Pr\{x \geq k\}$ users that can successfully receive the multicast packets at an effective transmission rate of $(k/n)r_{\text{ECOM}}$. Therefore, effectively, the average achievable *multicast* rate of the ECOM scheme with RS(n,k) code is given by

$$\begin{aligned}\overline{r}_{\text{ECOMP}} &= \frac{k}{n} N r_{\text{ECOMP}} \Pr\{x \geq k\} \\ &= \frac{k}{n} N \log_2(1+\overline{\gamma}\rho_{\text{th}}) \sum_{x=k}^{n} \binom{n}{x} p_{\text{ECOMP}}^{x} (1-p_{\text{ECOMP}})^{n-x}.\end{aligned} \tag{27}$$

For a given channel fading $f_\rho(\rho)$, ρ_{th} and k/n can be selected to maximize the above average achievable *multicast* rate of the ECOMP scheme.

From (27), it is straightforward to see that $\overline{r}_{\text{ECOMP}}/N$ does not depend on the multicast group size N; that is, at a given SNR, $\overline{\gamma}$, there always exist k and ρ_{th} so that $\overline{r}_{\text{ECOMP}}$ is bounded from zero regardless of N, and $\overline{r}_{\text{ECOMP}}$ is reduced as $\overline{\gamma}$ reduces.

(3) Comparison between ECOMF and ECOMP. In this part, an analytical derivation is given to compare the average achievable multicast rates of ECOMF and ECOMP.

Using the Jensen inequality, $E_{\rho_{N'}}[\log_2(1+\overline{\gamma}\rho)]$ in (23) can be approximated as

$$E_{\rho_{N'}}\left[\log_2(1+\overline{\gamma}\rho)\right] \approx \log_2\left(1+\overline{\gamma}E_{\rho_{N'}}[\rho]\right). \tag{28}$$

For a Rayleigh fading channel, we have

$$\begin{aligned}E_{\rho_{N'}}[\rho] &= \int_0^\infty \Pr(\rho_{N'} > \rho)\,d\rho \\ &= \int_0^\infty \sum_{i=0}^{N-N'} \binom{N}{i} (1-e^{-\rho})^i e^{-\rho(N-i)}\,d\rho \\ &= \sum_{i=0}^{N-N'} \binom{N}{i} X(N,i),\end{aligned} \tag{29}$$

where

$$\begin{aligned}X(N,i) &\triangleq \int_0^\infty (1-e^{-\rho})^i e^{-\rho(N-i)}\,d\rho \\ &= \sum_{j=0}^{i} (-1)^{i-j} \binom{i}{j} \int_0^\infty e^{-\rho(N-j)}\,d\rho \\ &= \sum_{j=0}^{i} (-1)^{i-j} \binom{i}{j} \left[\frac{-1}{N-j} e^{-\rho(N-j)}\right]_{\rho=0}^{\rho\to\infty} \\ &= -\sum_{j=0}^{i} \frac{(-1)^{i-j}\binom{i}{j}}{N-j}.\end{aligned} \tag{30}$$

It follows that

$$\begin{aligned}X(N,i+1) &= -\sum_{j=0}^{i+1} \frac{(-1)^{1+i-j}\binom{i+1}{j}}{N-j} \\ &= -\frac{(-1)^{1+i}\binom{i+1}{0}}{N} - \sum_{j=1}^{i} \frac{(-1)^{1+i-j}\binom{i+1}{j}}{N-j} \\ &\quad - \frac{\binom{i+1}{i+1}}{N-i-1}.\end{aligned} \tag{31}$$

Using the relation $\binom{i+1}{j} = \binom{i}{j} + \binom{i}{j-1}$, we can write

$$\begin{aligned}X(N,i+1) &= -\frac{(-1)^{1+i}\binom{i+1}{0}}{N} - \sum_{j=1}^{i} \frac{(-1)^{1+i-j}\binom{i}{j}}{N-j} \\ &\quad - \sum_{j=1}^{i} \frac{(-1)^{1+i-j}\binom{i}{j-1}}{N-j} - \frac{\binom{i+1}{i+1}}{N-i-1} \\ &= \sum_{j=0}^{i} \frac{(-1)^{i-j}\binom{i}{j}}{N-j} - \sum_{j=0}^{i} \frac{(-1)^{i-j}\binom{i}{j}}{N-1-j} \\ &= -X(N,i) + X(N-1,i),\end{aligned} \tag{32}$$

with $X(N,0) = 1/N$. Using the above recursive relation, we obtain

$$\begin{aligned}X(N,1) &= X(N-1,0) - X(N,0) \\ &= \frac{1}{N-1} - \frac{1}{N} = \frac{1}{\binom{N}{1}(N-1)}, \\ X(N,2) &= X(N-1,1) - X(N,1) \\ &= \frac{1}{\binom{N-1}{1}(N-2)} - \frac{1}{\binom{N}{1}(N-1)} \\ &= \frac{1}{\binom{N}{2}(N-2)}.\end{aligned} \tag{33}$$

For $i = 3,\ldots,N$ it can be verified that $1/\binom{N-1}{i-1}(N-i) - 1/\binom{N}{i-1}(N-i+1) = 1/\binom{N}{i}(N-i)$, and hence $X(N,i) = X(N-1,i-1) - X(N,i-1) = 1/(\binom{N}{i}(N-i))$.

Hence, $E_{\rho_{N'}}[\rho]$ then becomes

$$\begin{aligned}E_{\rho_{N'}}[\rho] &= \sum_{i=0}^{N-N'} \binom{N}{i} X(N,i) = \sum_{i=0}^{N-N'} \frac{1}{N-i} \\ &= \sum_{i=N'}^{N} \frac{1}{i} > \int_{N'}^{N} \frac{1}{x}\,dx = \ln\left(\frac{N}{N'}\right) \triangleq \rho'.\end{aligned} \tag{34}$$

From (23) and (34), the lower bound of ECOMF multicast rate can be expressed as

$$\overline{r}_{\text{ECOMF}} > \frac{k}{n} N \log_2(1+\overline{\gamma}\rho') \sum_{x=k}^{n} \binom{n}{x} \left(e^{-\rho'}\right)^x \left(1-e^{-\rho'}\right)^{n-x}. \tag{35}$$

It is interesting to see that the right-hand side of inequality (35) is equivalent to the multicast rate of ECOMP as in (27) with $\rho' \equiv \rho_{\text{th}}$. In other words, the multicast rate of ECOMF is lower-bounded by that of ECOMP and therefore is also bounded away from zero. The relationship $e^{-\rho'} = N'/N$ further shows that when the multicast group size N is sufficient large, ECOMP can converge to ECOMF by setting $\rho_{\text{th}} = \ln(N/N')$.

2.4. Illustrative Results. As a figure of merit to evaluate and compare the performance of different schemes, we define the *effective multicast throughput* in units of b/s/Hz/user as the ratio of the average achievable *multicast* rate (as shown in (8), (14), (23) and (27)) to the multicast group population, N. Our numerical results are based on (8), (14), (23), and (27) and are confirmed by simulation at a very good agreement with difference of less than 1%.

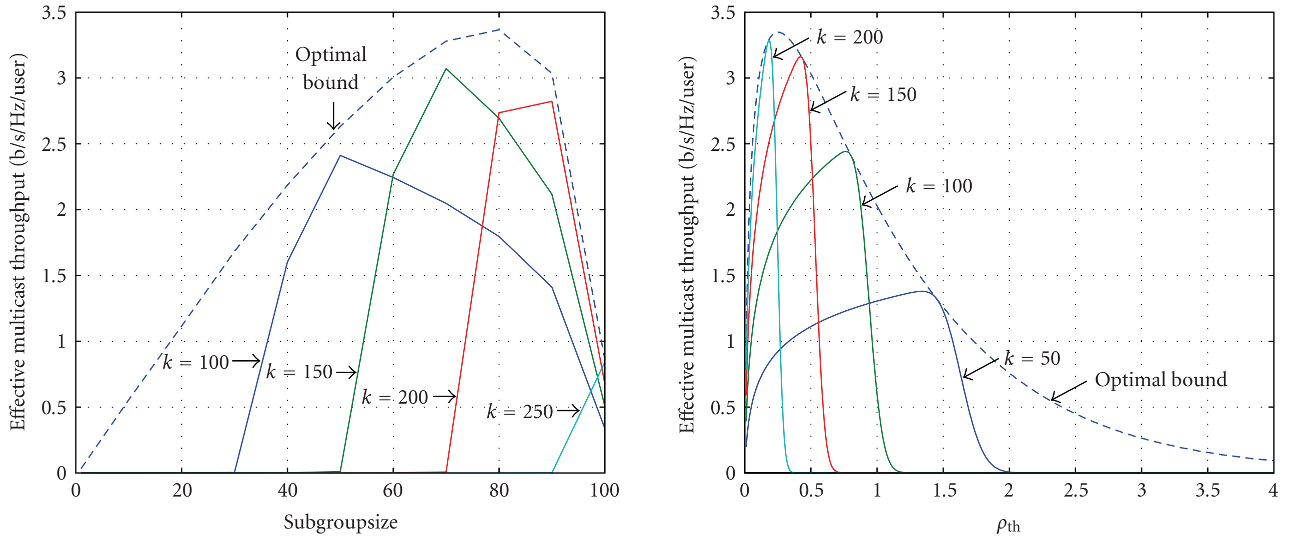
2.4.1. Effect of ρ_* on Throughput. We first analyze the effect of the selected ρ_* on the effective throughput of ECOM schemes over different Rayleigh fading conditions. As an illustrative example, a multicast group size of $N = 100$ and RS(255, k) is considered.

Consider a Rayleigh fading environment with average SNR of 20 dB; the effect of subgroup size and cut-off threshold selection is depicted in Figure 2. It is shown that for a given value of k, there is an optimum value of ρ_* that maximizes the effective multicast rate. From the derived relationship $\rho_* = \rho_{\text{th}} = \ln(N/N')$, increasing the subgroup size N' in ECOMF is equivalent to decreasing the cut-off threshold ρ_{th} in ECOMP. It is observed that the optimum value of ρ_* decreases as k increases. This indicates that multicast gain is preferred over multiuser diversity as more users can receive the packet. It is noted that the normalized throughput of both ECOMF and ECOMP drops sharply after this optimal point when ρ_{th} or ρ_* increases (or, equivalently, N' decreases). In this case, a lower k with its corresponding ρ_* is a better choice since it provides better erasure correction capability at the expense of more coding overhead. The optimal bound (dashed line) presents the maximum achievable multicast throughput over all possible values of k for ECOMF and ECOMP. The results in Figure 2 show that the optimum throughput increases with ρ_* until reaching its peak and decreases afterwards, which implies that if we try to increase a short-term rate in each timeslot, the payoff will be the long-term average throughput as the erasure correction capability has to be high to compensate for packet loss, which makes multicast transmission inefficient after its optimal point. It is shown in Figure 2 that over Rayleigh fading channel at an average SNR of 20 dB, the optimal k for best multicast throughput is 190 for ECOMF and 184 for ECOMP with an appropriate optimum threshold value at $N' \approx 80$ for ECOMF and $\rho_{\text{th}} = 0.25$ for ECOMP. The optimal values of ρ_{th} and k for ECOMP for each channel condition can be found through optimization method as illustrated in the appendix.

We are now extending our observation of the *optimal* throughput versus ρ_* for different SNRs as shown in Figure 3. It is observed that the peak throughput decreases with SNR as expected. As the average SNR decreases, the optimum channel gain threshold ρ_* increases which illustrates that erasures occur more often at lower average SNR and k has to be reduced to increase the erasure-correction capability of RS(255, k) at the expense of lower coding rate (and hence lower achievable throughput). The results also show that as the average SNR increases, the proposed ECOM schemes select a lower transmission rate, as shown in (6), implying that the multicast gain becomes more dominant at higher SNR as more users can receive multicast packet in each timeslot.

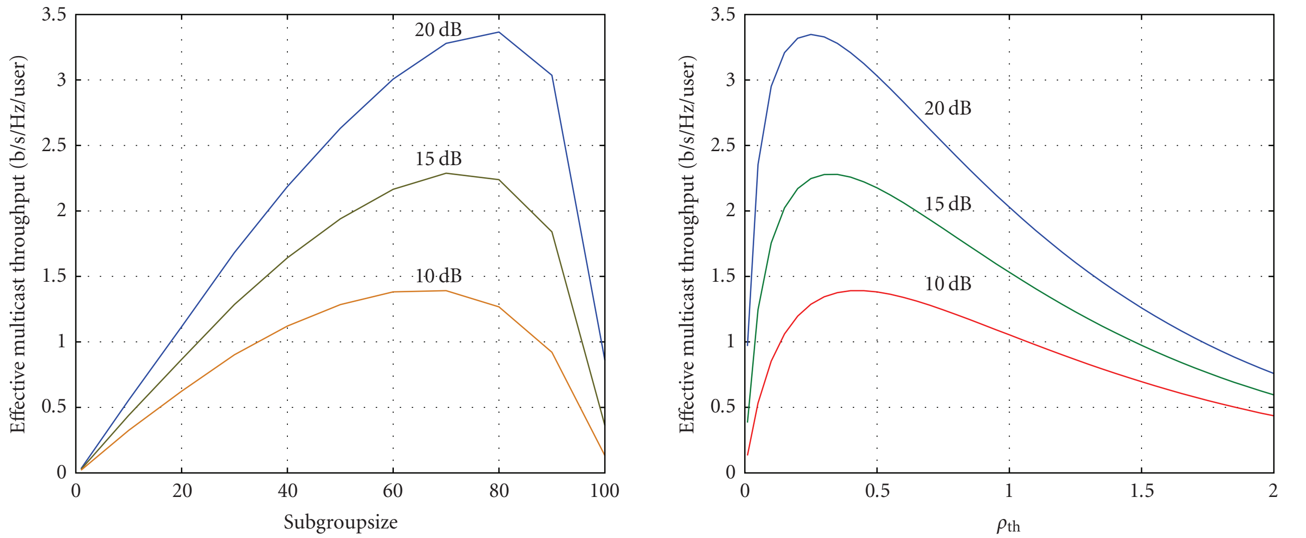
The above results and discussions confirm that the proposed ECOM schemes can flexibly combine the multicast gain with the multiuser diversity and time diversity via the use of erasure correction coding to achieve optimum achievable throughput in various fading conditions.

2.4.2. Effect of Group Size on Multicast Throughput. The effect of multicast group size on multicast throughput on Rayleigh fading channel at 20 dB is shown in Figure 4(a) for WU, BU, and ECOM schemes. As defined at the beginning of Section 2.4, the effective multicast throughput in terms of b/s/Hz/user represents the effective rate *each* user of the multicast group can expect. When the number of users increases, the achievable multicast rate of the WU and BU schemes is quickly reduced to zero, as indicated by (10) and (18), while the proposed ECOM schemes achieve a high multicast rate with the effective multicast throughput of ECOMP unchanged with the multicast group size, shown by (27). This can be explained by the fact that, in the proposed ECOMP scheme, the probability of successful decoding/reception of the multicast copy does not depend on the multicast group size and is the same for every user in the group in an i.i.d. fading environment while the decision for transmission in WU, BU, and ECOMF cases requires the consideration of the whole multicast group for determining transmission rate at each timeslot. Further performance comparisons of the four schemes at low SNR of 0 dB are shown in Figure 4(b). The results confirm the previous observations that WU and accordingly, multicast gain are more favorable at high SNR (Figure 4(a)) while BU or multiuser diversity is superior at low SNR (Figure 4(b)). Furthermore, at any SNR, the performance of both BU and WU quickly decreases as the multicast group size increases, which indicates that for a large group size, neither multicast gain nor multiuser diversity alone can fully exploit multicast capacity and a hybrid treatment is more suitable. Figures 4(a) and 4(b) also indicate that, for a very small multicast group size, ECOMF has the same performance as WU, as shown in (24), which yields the best throughput at high SNR. At very low SNR of 0 dB and for a very small multicast group size (Figure 4(b)), ECOMF has slightly lower performance than BU due to erasure code overhead (i.e., coding rate is less than 1). However, for the case of BU, a complicated queuing system (e.g., in [6]) is needed to guarantee loss-free transmission to achieve (15). The results in Figures 4(a) and 4(b) also confirm that the performance of ECOMF is lower-bounded by that of ECOMP as discussed in Section 2.3.3(3), and as the multicast group size increases, the performance



(a) ECOMF (b) ECOMP

Figure 2: Effective multicast throughput versus ρ_* for ECOM schemes using RS(256, k) in a Rayleigh fading channel with an average SNR of 20 dB.



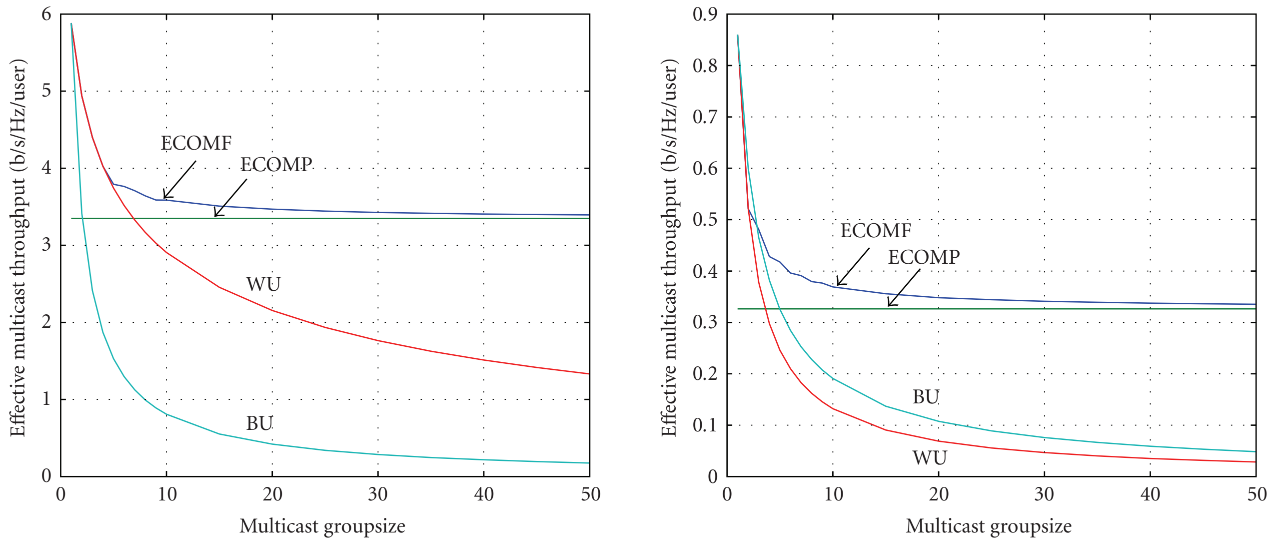
(a) ECOMF (b) ECOMP

Figure 3: Optimal effective multicast throughputs of ECOMF and ECOMP in a Rayleigh fading channel for different average SNRs.

difference between ECOMF and ECOMP is greatly reduced. In other words, the *full* channel knowledge is beneficial to enhance the throughput of ECOMF, but for a large multicast group size, such performance advantage of ECOMF over ECOMP diminishes, and ECOMP with only required *partial* channel knowledge can be a better choice for simplicity.

The relationship $e^{-\rho_{\text{th}}} = N'/N$ derived in the previous Section is confirmed in Figure 5. As shown in Figure 5 at an average SNR of 0 dB, the ratio N'/N converges to $e^{-\rho_{\text{th}}}$ from above as N increases or, correspondingly, the ECOMF threshold ρ_* approaches the ECOMF threshold ρ_{th} with $\rho_* < \rho_{\text{th}}$. This implies that, in average, ECOMF always obtains higher transmission rate than ECOMP (as the benefit of *full* channel knowledge).

2.4.3. Performance Comparison and Trade-off between Multicast Gain and Multiuser Diversity. Figure 6 compares the effective multicast throughput of the WU, BU, ECOMF, and ECOMP in a Rayleigh fading environment for a wide SNR range from −20 dB to 40 dB with a multicast group size of $N = 100$ users. It is observed that the BU has higher throughput than the WU in the low SNR region, but as the average SNR increases above the crossover point of 5 dB, the BU scheme has inferior performance with an almost



(a) with average SNR of 20 dB

(b) with average SNR of 0 dB

Figure 4: Effective multicast throughputs of different schemes versus number of users (over Rayleigh fading channels).

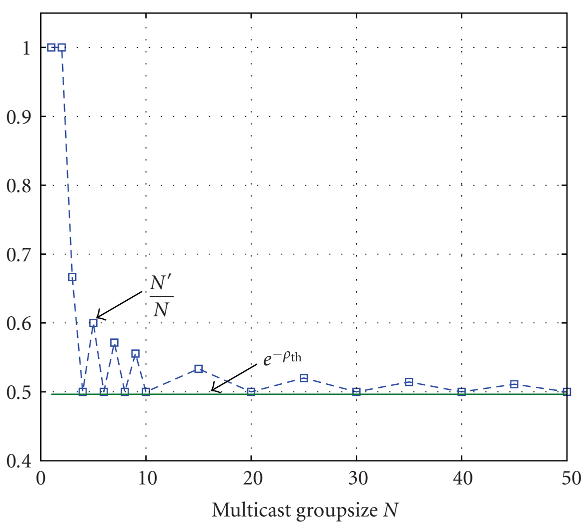


Figure 5: Convergence of N'/N to $e^{-\rho_{\text{th}}}$ (Rayleigh fading channel with average SNR of 0 dB).

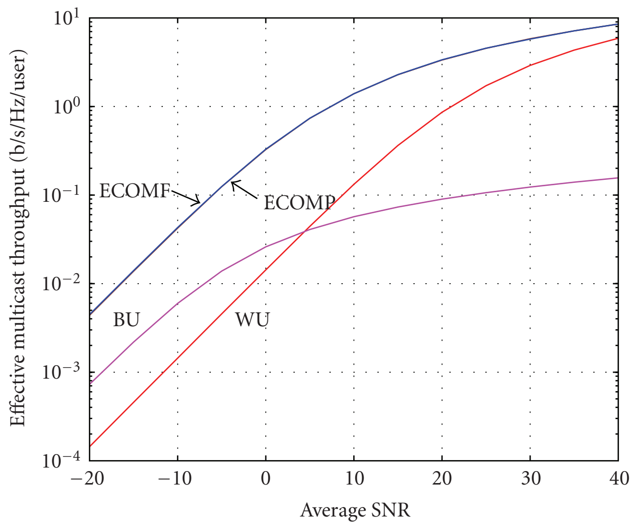


Figure 6: Throughputs given by different schemes versus average SNRs (Rayleigh fading channel, 100 users).

saturating throughput. The results indicate that when the average SNR is sufficiently high, the various BS-user links are sufficiently good, and, as a consequence, it is more likely that all N users in the multicast group are able to successfully receive the transmitted packets. Hence, it is better to explore multicast gain (i.e., transmission only one copy for all N users) to achieve higher normalized throughput in the case of high SNR. However, at a low *average* SNR (e.g., below 5 dB in Figure 6), the *instantaneous* SNRs in various BS-user links are likely more different; that is, some users may be in deep fades while the others have adequate SNRs. This suggests a more pronounced role of multiuser diversity, and hence the BU scheme outperforms the WU scheme as confirmed in Figure 6. It is interesting to note that, by optimizing the subgroup size N' or the threshold value, ρ_{th}, and code rate according to the *average* SNR, as well as fading type (e.g., Rayleigh) of the channel, the proposed ECOM schemes can jointly adjust the use of multicast gain and the multiuser diversity (and time diversity) to obtain a much larger achievable throughput over a wide SNR range, for example, 18 times better than that of the BU and WU schemes at an average SNR of 5 dB. At a very high average SNR, the performance of the WU scheme asymptotically approaches that of the proposed ECOM schemes. This implies that at high average SNR, the proposed ECOM schemes will select a very high coding rate (i.e., k approaches n, or without

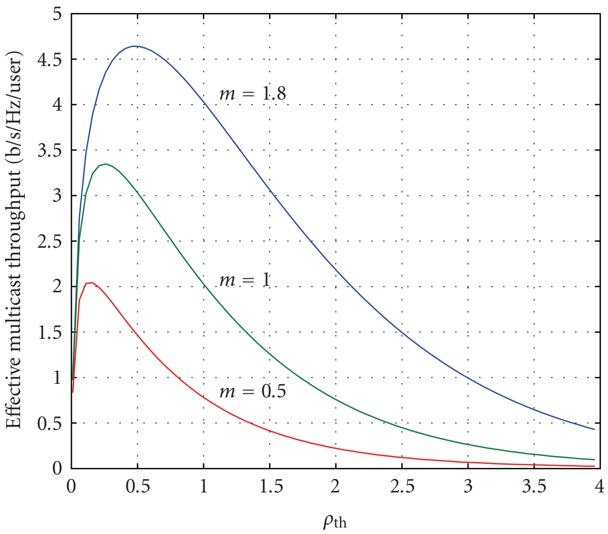


Figure 7: Optimal normalized throughput versus ρ_{th} for ECOMP scheme in different Nakagami-m channels with average SNR of 20 dB.

coding) and essentially explore only the multicast gain. Figure 6 also confirms that for large multicast group size the gain provided by ECOMF is just marginally larger than that provided by ECOMP, and hence, it is sufficiently efficient to have only the *partial* knowledge of the channel distribution which varies much more slowly than the channel itself and is much easier to estimate than the instantaneous channel. Without the required knowledge of the instantaneous user channel responses $h_i(t)$'s, the proposed ECOMP scheme can significantly reduce the system complexity and resources for channel estimation and feedback signaling. Furthermore, it can cope with fast time-varying fading channels, especially in mobile wireless communications systems.

2.4.4. Effect of Different Nakagami-m Fading Environments on ECOM. Consider a quasistatic i.i.d. Nakagami-m fading environment with pdf

$$f_\rho(\rho) = (m)^m \frac{\rho^{m-1}}{\Gamma(m)} \exp(-m\rho) \quad \text{with } E\{\rho\} = 1, \tag{36}$$

and cdf

$$F_\rho(\rho) = \frac{\nu(m, m\rho)}{\Gamma(m)}, \tag{37}$$

where $\Gamma(m)$ is the Gamma function, $\Gamma(m) = \int_0^\infty t^{m-1}e^{-t}dt$, and $\nu(m, m\rho)$ is the lower incomplete Gamma function, $v(m, x) = \int_0^x t^{m-1}e^{-t}dt$.

In this part, the effect of different Nakagami-m fading environments on ECOM is investigated. Since both ECOMP and ECOMF have the same characteristics as shown in the last parts, for simplicity, only the results of ECOMP are illustrated.

In Figure 7, performance comparison of ECOMP on different fading type conditions is investigated. Consider Nakagami-m channels at the same average SNR of 20 dB for different values of m: $m = 1$ for a Rayleigh channel, $m = 1.8$ for a milder situation, equivalent to a Ricean channel, and $m = 0.5$ for a considerably severe fading channel. The results in Figure 7 illustrate that as the fading becomes less severe (i.e., with larger value of m), the optimum achievable throughput is increased as we can expect. Correspondingly, the optimum value of threshold ρ_{th} is increased in a milder fading environment. This can be explained as follows. When m increases, the peak of the Nakagami-m probability density function occurs at a higher value and its variance decreases; in other words, more users have good channels and therefore are less likely to receive erased packets. Hence the proposed ECOMP scheme can select a higher transmission rate, $\overline{r}_{\text{ECOMP}}$, and a higher code rate k/n as shown in (27) for multicast transmission.

3. ECOM Scheme over Frequency-Selective Multipath Fading Wireless Channels

In this section, we consider to extend the application of the proposed ECOM scheme to broadband OFDM systems in a frequency-selective fading environment by exploiting frequency diversity. OFDM divides the entire transmission bandwidth into many subchannels, each with sufficiently narrow bandwidth such that the corresponding subchannel response can be regarded as being frequency-flat. To apply ECOM scheme to OFDM systems, one approach could be transmitting coded packets on *one* selected subcarrier in many independent time-slots as in Section 2. Since packets are coded in blocks and sent in time, each user needs to receive the entire block before decoding, and hence, this introduces a delay. One modification to reduce this delay is to send many coded packets simultaneously on many subcarriers, that is, exploring frequency diversity.

Nevertheless, in multicarrier OFDM systems, fading in neighbor subcarriers can be correlated so that deep fade in one subcarrier can also result in deep fade in other nearby subcarriers. As a consequence, for large fading correlation, the multicast rate can be greatly reduced as packets sent on these subcarriers are likely to be erased. At this point, two questions arise: how this correlation factor affects the multicast throughput and how we can take advantage of frequency diversity with as little as possible degradation in the achievable multicast rate. These two questions will be addressed in this section. As discussed in Section 2, for large group size, the throughput performance of ECOMP is as good as ECOMF. Furthermore, ECOMP requires only the knowledge of average SNR and fading type of BS-user links. Therefore, when applied to OFDM the ECOMP scheme is more suitable than ECOMF as it can significantly reduce feedback signaling. Hence, in this section we focus only on ECOMP scheme. First, using only frequency diversity to study the effect of correlation on multicast rate, all the coded packets are sent in only one time slot, each on one subcarrier. ECOMP is then expanded to make use of both time diversity and frequency diversity to investigate the trade-off between throughput and delay.

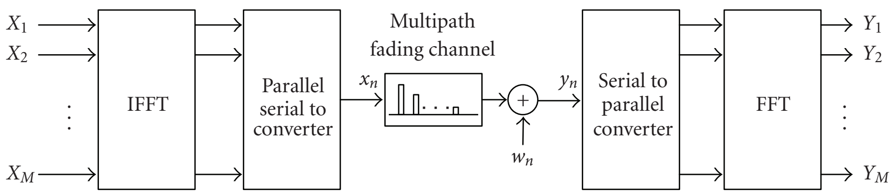


Figure 8: OFDM system model.

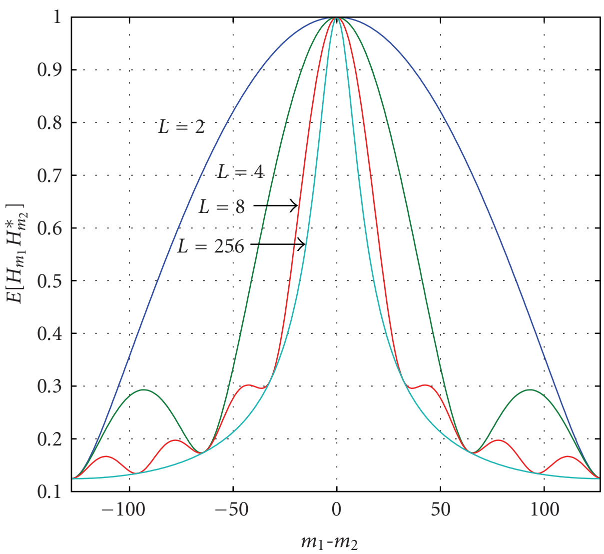


Figure 9: Correlation between OFDM subcarriers.

3.1. OFDM System Model and Correlation Factor among the OFDM Subcarriers. Consider a wireless downlink OFDM system with M subcarriers over a bandwidth $B = M\Delta f$,where Δf is the subcarrier bandwidth, as shown in Figure 8. Cyclic prefix is not shown in the diagram for simplicity, as it does not affect the following study. In a given time slot t, $\{X_m\}_{m=1,\ldots,M}$ is a block of complex signals in frequency domain where X_m is the transmitted signal on frequency m. X_m is assumed to have zero mean and power $E[X^2] = P$. These transmitted signals are transformed into samples $x_n(t)$ in time domain through the discrete inverse Fourier transform with sampling time $\Delta t = 1/(M\Delta f)$. These time samples are serially transmitted through a multipath fading channel and are contaminated by additive white Gaussian noise $w_m(t) \sim CN(0, \mathcal{N}_0)$ with $\mathcal{N}_0$ noise power. The received signal y_n can be expressed as

$$y_n(t) = h(t) * x_n(t) + w_n(t). \quad (38)$$

Let L be the number of resolvable paths in the multipath fading channel, with h_l being the tap gain of the lth resolvable path. Similar to Section 2, h_l's are assumed to remain unchanged in a given time-slot and vary independently from one time-slot to another. The impulse response of the multipath channel in time domain can be written as

$$h(t) = \sum_{l=0}^{L-1} h_l \delta(t - l\Delta t). \quad (39)$$

In our study, h_l's are assumed to follow power delay profile in COST 259 [12], that is, $h_l \sim CN(0, \sigma_l^2)$ with $\sigma_l^2 = e^{-l/4}/\sum_{l=0}^{L-1} e^{-l/4}$.

From (38), the received signal Y_m in frequency domain (after FFT block in Figure 8) can be given by

$$Y_m = H_m X_m + W_m, \quad (40)$$

where W_m is the fast Fourier transform (FFT) of w_m and H_m denotes the channel response in frequency domain, which is the FFT of (39), that is,

$$H_m = \sum_{l=0}^{L-1} h_l e^{-j2\pi(l\Delta t)(m\Delta f)} = \sum_{l=0}^{L-1} h_l e^{-j2\pi lm/M}. \quad (41)$$

H_m indicates the fade level of the signal on subcarrier m and hence determines the channel quality at that frequency. Since $h_l \sim CN(0, \sigma_l^2)$ with $\sigma_l^2 = e^{-l/4}/\sum_{l=0}^{L-1} e^{-l/4}$, $H_m \sim CN(0, 1)$. Since h_l's remain unchanged in a given time-slot and vary independently from one time-slot to another, H_m's also remain unchanged in a given time-slot and vary independently from one time-slot to another.

From (41), the correlation in channel responses of subcarriers m_1 and m_2 can be calculated as follows:

$$E\left[H_{m_1} H_{m_2}^*\right] = E\left[\sum_{l=0}^{L-1}\sum_{l'=0}^{L-1} h_l e^{-j2\pi l m_1/M} h_{l'}^* e^{j2\pi l' m_2/M}\right], \quad (42)$$

and since $E[h_l h_{l'}^*] = 0$,

$$\begin{aligned} E\left[H_{m_1} H_{m_2}^*\right] &= E\left[\sum_{l=0}^{L-1} |h_1|^2 e^{-j2\pi l(m_1-m_2)/M}\right] \\ &= E\left[\sum_{l=0}^{L-1} \alpha_l^2 e^{-j2\pi l(m_1-m_2)/M}\right]. \end{aligned} \quad (43)$$

As shown in (43), this correlation factor depends on the distance between two subcarriers and the number of resolvable paths, L. Figure 9 further illustrates this correlation factor for an OFDM system of 256 subcarriers over a multipath Rayleigh fading channel with 2, 4, 8, and 256 tap gains. It can be seen in Figure 9 that this correlation is large when the frequency separation between m_1 and m_2 is small; hence if one subcarrier is in deep fade, the other is also likely to be in deep fade which may result in packet loss on both subcarriers if we use these two for transmission. On the other hand, when this correlation factor is small, deep fade on one subcarrier is less likely to affect the other subcarrier. Moreover, it is observed from Figure 9 that, for a given level of correlation, when the number of resolvable paths increases, the frequency separation decreases. For example, when the number of resolvable paths increases from 2 to 4, the minimum frequency separation to achieve a correlation of 0.3 decreases from 105 to 52 subcarriers, which is approximately two times. The same observation applies when L increases from 4 to 8. However, when L is larger than 8, this observation is no longer valid as shown in Figure 9. Hence, multipath fading channel introduces frequency diversity that can be used, especially for L from 2 to 8.

3.2. ECOMP for OFDM. Using the system model as described in Section 2.1 to support multicast scenario for N users, we first derive the relationship between average SNR and the instantaneous SNR on each subcarrier and then describe the operation of ECOMP in the case of OFDM.

Applying (41), the instantaneous channel gain for user i on subcarrier m is given by

$$H_{i,m} = \sum_{l=0}^{L-1} h_{i,l} e^{-2\pi ml/M}. \tag{44}$$

Let $X_m(t)$ be the transmitted signal on subcarrier m at timeslot t with normalized power of 1, the average SNR on subcarrier m of user i at time-slot t can be expressed as

$$\begin{aligned}\overline{\gamma}_{i,m}(t) &= \frac{E\left[\left|X_m(t)H_{i,m}(t)\right|^2\right]}{N_0}\\ &= \frac{E\left[|X_m(t)|^2\right]E\left[\left|H_{i,m}(t)\right|^2\right]}{N_0} = \frac{P}{N_0} = \overline{\gamma},\end{aligned} \tag{45}$$

and the instantaneous SNR on subcarrier m of user i at timeslot t is given by

$$\overline{\gamma}_{i,m}(t) = \left|H_{i,m}(t)\right|^2 \frac{E\left[|X_m(t)|^2\right]}{N_0} = \rho_{i,m}(t)\overline{\gamma}, \tag{46}$$

where $\rho_{i,m}(t) \triangleq |H_{i,k}(t)|^2$.

Following the discussions in the previous section, similar to (4) and (5), the effective multicast rates for WU and BU in OFDM system can be given by

$$\begin{aligned} r_{\mathrm{WU}_m}(t) &= \log_2\left(1 + \overline{\gamma} \min_{i=1,2,\ldots,N}\{\rho_{i,m}(t)\}\right),\\ r_{\mathrm{BU}_m}(t) &= \frac{1}{N}\log_2\left(1 + \overline{\gamma} \max_{i=1,2,\ldots,N}\{\rho_{i,m}(t)\}\right).\end{aligned} \tag{47}$$

3.2.1. ECOMP for OFDM Using Only Frequency Diversity. Expanding ECOMP to OFDM system, the transmission rate to send multicast packet on each subcarrier can be selected based on the channel gain threshold ρ_{th} as follows:

$$r_{\mathrm{ECOM}_m} = \log_2(1 + \overline{\gamma}\rho_{\text{th}}). \tag{48}$$

To recover the erased packets for users with inadequate instantaneous SNR, ECOMP makes use of RS-code with the same encoding scheme as described in Section 2. For simplicity, without loss of generality, we assume that the number of RS coded packets is equal to the number of subcarriers, that is, $n = M$, and hence, the whole RS-coded packet block is sent in one time-slot with each coded packet transmitted on one subcarrier as illustrated in Figure 10 for time-slot t.

The optimization problem for multicast rate when applying ECOMP to OFDM then becomes

$$\arg\max_{\rho_{\text{th}},k} \frac{k}{n}\log_2(1 + \overline{\gamma}\rho_{\text{th}})\Pr\{x \geq k\}, \tag{49}$$

where $\Pr\{x \geq k\}$ is the probability that a given user can receive at least k nonerased packets on all the subcarriers. At the first look, this probability is similar to that in the previous section; however, it is noted that, in the scenario of OFDM, channel gains in subcarriers are correlated and a simple expression using binomial distribution as in the previous section is not applicable. For this, we use simulation to study the throughput performance of ECOMP for OFDM. In particular, we investigate the effects of the number of resolvable paths on the effective multicast throughput and compare the performance of the proposed scheme with that of BU and WU. In our simulations, multicasting is done on a multicast group of $N = 100$ users. For simplicity, an OFDM system with $M = 256$ subcarriers and RS$(256, k)$ is considered.

(1) Effects of the Number of Resolvable Paths on Multicast Throughput. In Figure 11, the effect of different fading conditions on the achievable normalized throughput over multipath fading channels is examined at the same average SNR of 20 dB with the number of resolvable paths $L = 2$, 5, 8, and 11. The results in Figure 11(a) show that the effective multicast throughput depends on the number of resolvable paths. When L increases, the correlation among the subcarriers reduces, that is, less chance that packets are erased at the same time, and hence the achievable multicast rate increases as L increases. This is not the case for unicast where the ergodic capacity is independent of the number of resolvable paths as shown in [13–15]. However, as shown in Figure 11(b), this gain comes with a cost: at the same code rate, as the number of resolvable paths increases, the throughput curve becomes more sensitive to the channel gain threshold ρ_{th}.

(2) Performance Comparison. Figure 12 compares the effective multicast rates of the WU, BU, and ECOMP schemes in a 5-tap multipath fading environment for a wide range of SNR.

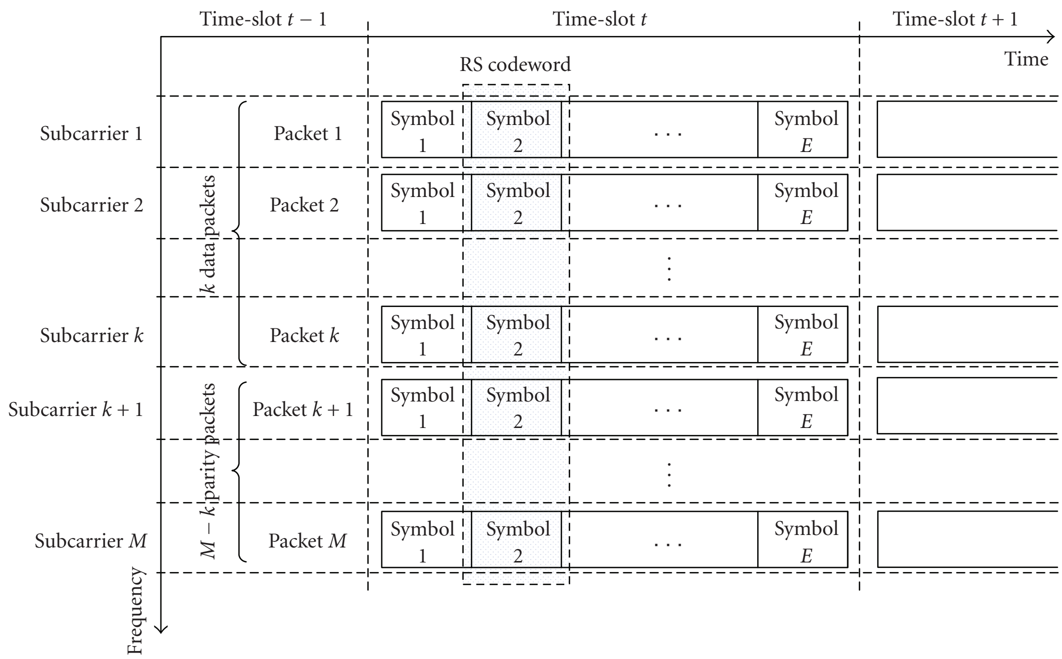
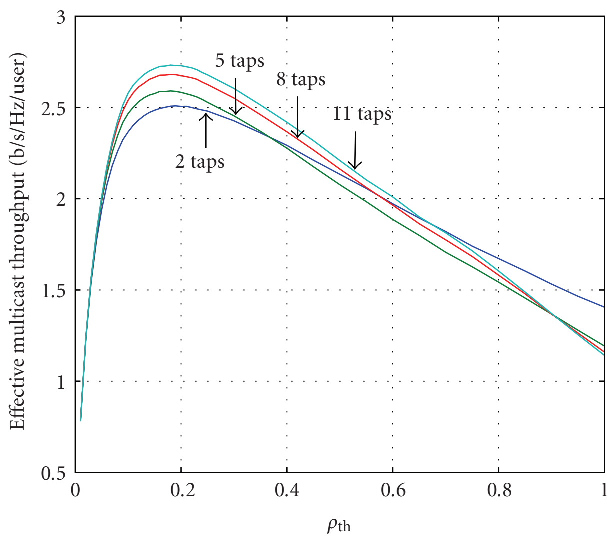
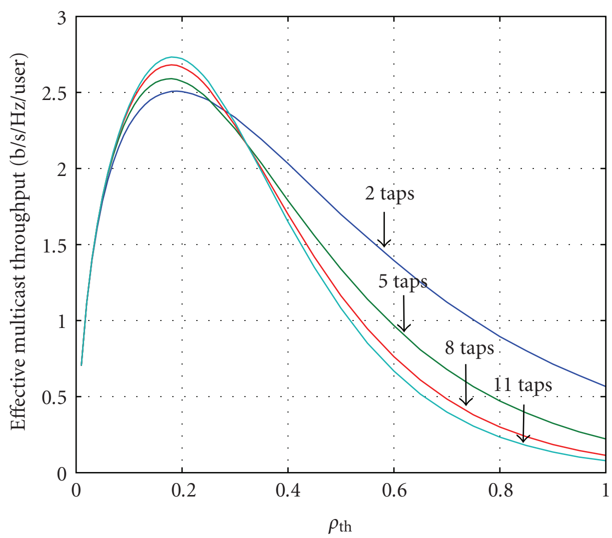


Figure 10: Transmission of RS-coded packets of ECOMP over OFDM subcarriers.



(a) Optimal effective multicast throughput



(b) Effective multicast throughput with RS(256,180)

Figure 11: Effective multicast throughput versus ρ_{th} for ECOMP scheme in different numbers of resolvable paths with average SNR of 20 dB.

Similar to the performance comparison in Section 2, BU is better than WU in the low SNR region while WU outperforms BU after the crossover point of around 5 dB. Combining both multicast gain and multiuser diversity, the throughput performance of ECOMP is superior to both BU and WU in the considered SNR range and asymptotically converges to WU at high SNR. However, it is noted that the improvement in throughput of ECOMP on OFDM system is smaller than that in the case of single carrier. For instance, at 5 dB, as compared to BU and WU, ECOMP offers an improvement in effective multicast rate of 10 times in the case of multicarrier (Figure 12) and 18 times in the case of single carrier (Figure 6). This can be explained by the fact that, due to the high correlation in frequency responses, channel gains of adjacent subcarriers are similar, and consequently, changes in the SNR threshold ρ_{th} to adjust multicast gain and multiuser diversity give less effect on the multicast throughput than in the case of independent time slots. In other words, correlation in channel responses will decrease the benefits of combining multicast gain and multiuser diversity.

3.2.2. ECOMP for OFDM Using Both Time and Frequency Diversity. When applying ECOMP to OFDM by sending all coded packets on all subcarriers, it can be seen that we gain n times reduction in the delay as each RS block can be sent in only one time-slot. However, as the channel gains of OFDM subcarriers are correlated, if one subcarrier of a given user is in deep fade (i.e., $\rho_{i,m}(t)$ is very low), it is likely that the subcarriers close to it are also in deep fade and the packets that are sent on these subcarriers will likely be erased. To compensate for the erased packets ECOMP has to select a lower transmission rate and lower RS code rate k to gain more erasure correction capability and hence this reduces the multicast rate. To enhance this throughput performance, it is necessary to keep the correlation among the subcarriers in use as low as possible by increasing their frequency separation. As shown in Figure 9, this required frequency separation depends on the number of resolvable paths. In other words, by transmitting coded packets on subcarriers far from each other we can achieve lower correlation for higher multicast throughput. However, fewer packets can be transmitted on one time-slot and as a consequence more time-slots are needed for transmitting each RS block, introducing more delay.

Based on the above discussions, we consider the extended erasure-correction coding-based opportunistic multicast (EECOM) scheme, in which the BS encodes k data packets using RS erasure code to form a block of n-coded packets in the same way as in Section 2, but, instead of sending all n-coded packets on all M subcarriers in one timeslot (as in Figure 11), these n-coded packets are sent over S timeslots on only a subset of C OFDM subcarriers equally spaced with a frequency separation of S. For the same assumption of $n = M$, $CS = M$. Figure 13 illustrates the transmission mechanism of EECOMP for $S = 2$.

The optimization problem for EECOM can be modified from (49) as in the following:

$$\arg\max_{\rho_{\text{th}},k} \frac{k}{n}\log_2(1+\overline{\gamma}\rho_{\text{th}})\Pr\{x \geq k\}_S, \tag{50}$$

where $\Pr\{x \geq k\}_S$ is the probability that a given user can receive at least k nonerased packets over S time-slots. For the same reason as in the last part, the probability $\Pr\{x \geq k\}_S$ does not have a closed-form mathematical expression and the throughput of EECOM is investigated by simulations. Similar to Section 3.2, our simulation results are based on a group of 100 users in an OFDM system with $M = 256$ subcarriers.

Figure 14 depicts the performance of EECOMP for different numbers of timeslots. In this graph, the horizontal axis represents the number of time-slot S in $\log_2$ scale. It is observed that as the number of time-slot S increases, the multicast throughput monotonically increases, which illustrates the trade-off between throughput and delay. When $S = 1$, EECOMP exploits only frequency diversity by sending all coded packets of one RS block in one time-slot on all subcarriers. The effective multicast throughput for EECOMP is the same as in Figure 11(a). When $S = 256$, EECOMP exploits only time diversity by sending only one coded packet in each time-slot on one subcarrier and the multicast rate in this case is the same as in Figure 2(b), that is, around 3.34 b/s/Hz which is about 25% higher than that in the case of $S = 1$. Moreover, when S is large enough, the effective throughput for EECOMP is approximately equal to that of $S = 256$. For example, when S is larger than 32 for 8-tap channels, 64 for 4-tap channels, or 128 for 2-tap channels, the achievable multicast rates for EECOMP are roughly the same as the case of $S = 256$, which correspond to a correlation factor of 0.3 or less (by calculating the frequency separations in these cases and referring to Figure 9 for the correlation values). This indicates that there is a significant delay reduction with virtually no penalty in multicast rate at these points and S is reduced when the number of resolvable paths increases; for example, S is reduced by 2, 4, 8 times when $L = 2, 4, 8$, respectively. However, when L is very large, as shown in Figure 9, the correlation between two subcarriers of a given frequency separation is not much changed with L. Hence, for larger L, the delay cannot be reduced further.

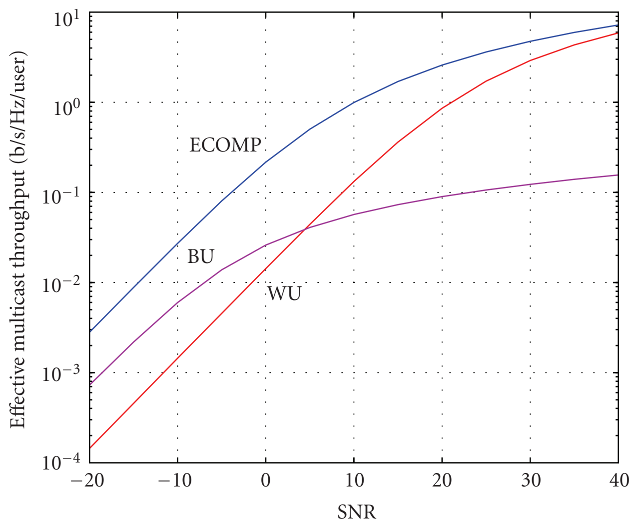


Figure 12: Throughputs of different schemes versus average SNRs (5 taps, 100 users).

In addition, it is observed that for points in Figure 14 with the same multicast rate, they yield the same correlation level shown in Figure 9. For instance, at $S = 32$ for $L = 4$ and $S = 64$ for $L = 2$, the multicast rate is about 3.3 b/s/Hz/user (Figure 14). At these points, the subcarrier separations are 32 and 64 subcarriers when $L = 4$ and $L = 2$, respectively, for the same correlation factor of about 0.7 (Figure 9). The same observation applies for other points with approximately the same effective multicast throughput in Figure 14.

4. Conclusion

We have proposed and studied an erasure-correction coding-based opportunistic multicast scheduling scheme aiming at exploiting multicast gain, multiuser diversity, and time/frequency diversity to enhance the throughput performance over wireless fading channels. In the proposed

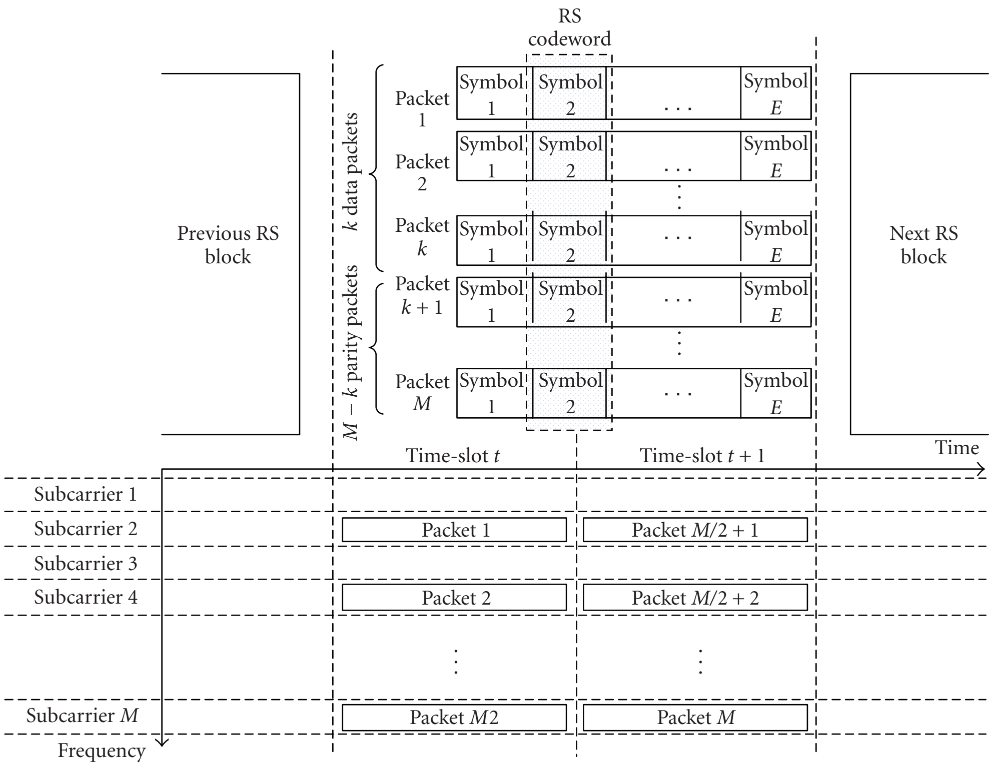


FIGURE 13: Transmission of RS-coded packets for EECOMP.

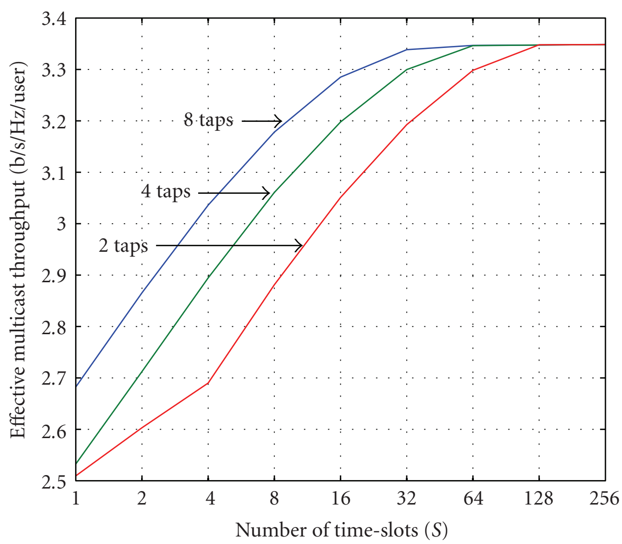


FIGURE 14: Performance of EECOMP for different numbers of timeslots in multipath fading channel with average SNR of 20 dB.

scheme, the BS sends each packet only once at a transmission rate determined by a channel gain threshold and using erasure correction capability of $\mathrm{RS}(n,k)$ to recover erased packets due to insufficient instantaneous SNR on BS-user links. RS coding scheme is applied to a block of packets and coded packets are sent in time or frequency slots to effectively explore time/frequency diversity. The channel gain threshold and erasure code rate are jointly optimized for best multicast throughput.

On frequency-flat fading channels, the selection of channel gain threshold is considered in two cases of full channel knowledge and partial knowledge of average SNR and fading type of wireless channel. An analytical framework has developed to analyze the effective multicast throughput of BU, WU, and of the proposed scheme. In this framework, we prove that while the effective multicast rates of both BU and WU asymptotically converge to zero as the multicast group size increases, this effective multicast rate of the proposed scheme is bounded from zero and depends on the average SNR. We further prove that, for the proposed ECOM scheme, the benefit of full channel knowledge is only pronounced at small multicast group sizes. As the group size increases, partial knowledge of channel response is sufficient in providing approximately the same throughput performance but significantly reducing resources (bandwidth, power) for feedback signalling.

In addition, numerical results illustrate that multiuser diversity is most pronounced at low SNR region since the difference in supportable rates of various users is large while multicast gain is superior at high SNR region where the difference in channel gain is compressed by the log-function that results in small difference in supportable rates among the users. The throughput comparison illustrates that with the ability of combining multicast gain and multiuser diversity, the proposed scheme outperforms both BU and WU for a wide range of SNR.

Furthermore, in this paper, we have extended ECOM for applications to OFDM system aiming at exploiting both time and frequency diversity in a frequency-selective fading environment. The effects of frequency correlation on multicast rate are investigated and our study shows that by exploiting both time and frequency diversity, we can significantly reduce transmission delay with negligible degradation in multicast throughput.

Appendix

Rate Optimization for ECOMP

Using Normal approximation to $\sum_{j=k}^{n}\binom{n}{k}p^j(1-p)^{n-j}$ in (27), the effective multicast rate of ECOMP can be rewritten as

$$\overline{r}_{\mathrm{ECOMP}} = \frac{k}{n}N\log_2(1+\overline{\gamma}\rho_{\mathrm{th}})\left(1-\frac{1}{2}\left(1+\operatorname{erf}\left(\frac{k-np}{\sqrt{2npq}}\right)\right)\right). \tag{A.1}$$

The optimization problem for ECOMP can be expressed as follows:

$$\begin{aligned}&\min_{k,\rho_{\mathrm{th}}}\{-\overline{r}_{\mathrm{ECOMP}}\}\\ &= \min_{k,\rho_{\mathrm{th}}}\left\{-\frac{k}{n}N\log_2(1+\overline{\gamma}\rho_{\mathrm{th}})\left(1-\frac{1}{2}\left(1+\operatorname{erf}\left(\frac{k-np}{\sqrt{2npq}}\right)\right)\right)\right\}.\end{aligned} \tag{A.2}$$

Subject to

$$\begin{aligned}&\rho_{\mathrm{th}} > 0,\\ &k > 0,\\ &k < n+1.\end{aligned} \tag{A.3}$$

The Larangian function can be defined as

$$L(\rho,k) = -\overline{r}_{\mathrm{ECOMP}} - \lambda_1\rho - \lambda_2 k - \lambda_3(n-k+1). \tag{A.4}$$

Since all the constrains (51–53) are inactive constrains, according to the Karush-Kuhn-Tucker conditions [16], $\lambda_1 = \lambda_2 = \lambda_3 = 0$; therefore, the derivative of the Larangian is given by

$$\begin{aligned}\nabla_{\rho_{\mathrm{th}}}L(\rho_{\mathrm{th}},k) &= -\nabla_{\rho_{\mathrm{th}}}\overline{r}_{\mathrm{ECOMP}},\\ \nabla_k L(\rho_{\mathrm{th}},k) &= -\nabla_k\overline{r}_{\mathrm{ECOMP}}.\end{aligned} \tag{A.5}$$

Considering Rayleigh fading channel, $p = e^{-\rho_{\mathrm{th}}}$, we then solve $\nabla L(\rho_{\mathrm{th}},k) = 0$ for the peak rate

$$\begin{aligned}&\nabla_k L(\rho_{\mathrm{th}},k)\\ &= -\frac{1}{2}\frac{\ln(1+\overline{\gamma}\rho_{\mathrm{th}})}{n\ln(2)}\\ &\quad\times\left[1-\operatorname{erf}\left(\frac{\sqrt{2}}{2}\frac{(k-ne^{-\rho_{\mathrm{th}}})}{(ne^{-\rho_{\mathrm{th}}}(1-e^{-\rho_{\mathrm{th}}}))^{1/2}}\right)\right]\\ &\quad+\frac{\sqrt{2}}{2}\frac{k\ln(1+\overline{\gamma}\rho_{\mathrm{th}})}{n\ln(2)\sqrt{\pi}(ne^{-\rho_{\mathrm{th}}}(1-e^{-\rho_{\mathrm{th}}}))^{1/2}}\\ &\quad\times\exp\left(-\frac{1}{2}\frac{(k-ne^{-\rho_{\mathrm{th}}})^2}{(ne^{-\rho_{\mathrm{th}}}(1-e^{-\rho_{\mathrm{th}}}))}\right) = 0,\\ &\nabla_{\rho_{\mathrm{th}}}L(\rho_{\mathrm{th}},k)\\ &= -\frac{1}{2}\frac{k\overline{\gamma}}{n\ln(2)(1+\overline{\gamma}\rho_{\mathrm{th}})}\\ &\quad\times\left[1-\operatorname{erf}\left(\frac{\sqrt{2}}{2}\frac{(k-ne^{-\rho_{\mathrm{th}}})}{(ne^{-\rho_{\mathrm{th}}}(1-e^{-\rho_{\mathrm{th}}}))^{1/2}}\right)\right]\\ &\quad+\frac{\sqrt{2}}{2}\frac{k\ln(1+\overline{\gamma}\rho_{\mathrm{th}})}{n\ln(2)\sqrt{\pi}(ne^{-\rho_{\mathrm{th}}}(1-e^{-\rho_{\mathrm{th}}}))^{1/2}}\\ &\quad\times\exp\left(-\frac{1}{2}\frac{(k-ne^{-\rho_{\mathrm{th}}})^2}{(ne^{-\rho_{\mathrm{th}}}(1-e^{-\rho_{\mathrm{th}}}))}\right)\\ &\quad\times\left[ne^{-\rho_{\mathrm{th}}}-\frac{1}{2}\right.\\ &\qquad\left.\times\frac{(k-ne^{-\rho_{\mathrm{th}}})(-ne^{-\rho_{\mathrm{th}}}(1-ne^{-\rho_{\mathrm{th}}})+ne^{-2\rho_{\mathrm{th}}})}{(ne^{-\rho_{\mathrm{th}}}(1-e^{-\rho_{\mathrm{th}}}))}\right] = 0.\end{aligned} \tag{A.6}$$

Solveing $\nabla_k L(\rho_{\mathrm{th}},k) = 0$ gives us

$$\begin{aligned}&1-\operatorname{erf}\left(\frac{\sqrt{2}}{2}\frac{(k-ne^{-\rho_{\mathrm{th}}})}{(ne^{-\rho_{\mathrm{th}}}(1-e^{-\rho_{\mathrm{th}}}))^{1/2}}\right)\\ &\quad= \frac{k\sqrt{2}}{\sqrt{\pi}(ne^{-\rho_{\mathrm{th}}}(1-e^{-\rho_{\mathrm{th}}}))^{1/2}}\\ &\qquad\times\exp\left(-\frac{1}{2}\frac{(k-ne^{-\rho_{\mathrm{th}}})^2}{(ne^{-\rho_{\mathrm{th}}}(1-e^{-\rho_{\mathrm{th}}}))}\right).\end{aligned} \tag{A.7}$$

Pluging this relationship into $\nabla_{\rho_{\mathrm{th}}}L(\rho_{\mathrm{th}},k) = 0$ gives us

$$k = \frac{(1+\overline{\gamma}\rho_{\mathrm{th}})\ln(1+\overline{\gamma}\rho_{\mathrm{th}})ne^{-\rho_{\mathrm{th}}}}{2\overline{\gamma}(1-e^{-\rho_{\mathrm{th}}})+(1+\overline{\gamma}\rho_{\mathrm{th}})\ln(1+\overline{\gamma}\rho_{\mathrm{th}})(-1+2e^{-\rho_{\mathrm{th}}})}. \tag{A.8}$$

The above equation gives the relationship between k and ρ_{th} at the peak rate of $\overline{r}_{\mathrm{ECOMP}}$. Plugging (A.8) back to (A.6), subject to (A.3), the optimal pairs of k and ρ_{th} can be found numerically; since in (27) the code rate is integer number, the nearest integer of k is the result code rate. Another way of finding this optimal pair of k and ρ_{th} is using the relationship in (A.8), doing the search on ρ_{th} to find the peak multicast rate and using the constraints on (A.3) to limit the search.

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