Throughput-Efficient Dynamic Coalition Formation in Distributed Cognitive Radio Networks

<table>
<thead>
<tr>
<th>ArticleInfo</th>
</tr>
</thead>
<tbody>
<tr>
<td>ArticleID   : 1983</td>
</tr>
<tr>
<td>ArticleDOI  : 10.1155/2010/653913</td>
</tr>
<tr>
<td>ArticleCitationID : 653913</td>
</tr>
<tr>
<td>ArticleSequenceNumber : 196</td>
</tr>
<tr>
<td>ArticleCategory : Research Article</td>
</tr>
<tr>
<td>ArticleFirstPage : 1</td>
</tr>
<tr>
<td>ArticleLastPage : 1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>ArticleHistory</th>
</tr>
</thead>
<tbody>
<tr>
<td>RegistrationDate : 2010–2–12</td>
</tr>
<tr>
<td>Received : 2010–2–12</td>
</tr>
<tr>
<td>Revised : 2010–7–25</td>
</tr>
<tr>
<td>Accepted : 2010–10–30</td>
</tr>
<tr>
<td>OnlineDate : 2010–11–3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>ArticleCopyright</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zaheer Khan et al.2010</td>
</tr>
<tr>
<td>This article is published under license to BioMed Central Ltd. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>ArticleGrants</th>
</tr>
</thead>
<tbody>
<tr>
<td>:</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>ArticleContext</th>
</tr>
</thead>
<tbody>
<tr>
<td>136382010201011</td>
</tr>
</tbody>
</table>
Abstract

We formulate the problem of distributed throughput-efficient sensing in cognitive radio (CR) networks as a dynamic coalition formation game based on a Markovian model. The proposed coalition formation enables the CRs to increase their achievable throughput, under the detection probability constraint, while also taking into account the overhead in sensing reports combining. The dynamic model of coalition formation is used to express and model the behavior of the coalition forming CRs over time. In the proposed game, CRs form coalitions either to increase their individual gains (selfish coalition formation) or to maximize the overall gains of the group (altruistic coalition formation). We show that the proposed coalition formation solutions yield significant gains in terms of reduced average false alarm probability and increased average throughput per CR as compared to the non-cooperative solutions. Given a target detection probability for a coalition, we adopt a weighted target detection probability for individual CRs in a coalition. We find that the weighted target detection probability for individual CRs results in increased average throughput per CR as compared to when each CR is assigned the same target detection probability in a coalition.
1. Introduction

Access to radio spectrum has traditionally been regulated by government agencies. This approach to radio spectrum management has been mostly successful in interference avoidance, but at the expense of efficiency in overall spectrum utilization. Cognitive radio networks are envisioned to utilize the radio spectrum more efficiently through opportunistic access to licensed frequency spectrum. A CR utilizes spectrum opportunistically by monitoring the licensed frequency spectrum to reliably detect the licensed (primary) user signals and operating whenever the primary user is absent. The detection of primary user signals is called spectrum sensing. As with any detection problem, the two types of errors associated with spectrum sensing are false alarms and missed detections. The lower the missed detection probability, the better the primary user is protected. However, to increase the achievable throughput of the CRs, the false alarm probability must also be low. Thus, there exists an inherent tradeoff between protection of primary users and achievable throughput for the cognitive radio network [1, 2].

Cooperation in sensing can reduce the false alarm probability of the cooperating CRs [3, 4]. In cooperative spectrum sensing, each CR performs spectrum sensing and sends its sensing report to a data collector known as the fusion center. To reduce signaling cost, the report may be binary (hard decision), consisting of zeros (primary user not present) and ones (primary user present). Hard decisions may be combined at the fusion center using, for example, "OR", "AND", and "MAJORITY" rules [1, 2]. However, in distributed CR networks, individual CRs need to interact with each other without a centralized fusion center.

This paper formulates the problem of distributed throughput-efficient sensing in cognitive radio networks as a dynamic coalition formation game based on a Markovian model. Our model is dynamic in the sense that distributed CRs reach self-organizing stable coalition structures (where no two coalitions have an incentive to merge) through a time-evolving sequence of steps. We model coalition structures as a sequence of random variables describing the state of the CR network, and the transition mechanism between
coalition structures is modeled as a Markov chain [5–7]. The dynamics of the coalition formation enable us to analyze (1) How are the coalitions formed? and (2) How do CRs arrive at equilibrium? We choose a coalition formation framework as it provides tools for the radios to decide which coalitions to form to achieve their goals more efficiently via cooperation [8–10]. In coalition formation, a coalition is a set of distinct, autonomous agents or players which may cooperate in order to increase their individual gains, which we call as selfish cooperation [11, 12]. Or they may cooperate to maximize the overall gains of the group, which we call altruistic cooperation. A key question this paper tries to address is how to coordinate distributed cognitive radios to perform cooperative sensing to minimize their false alarm probabilities and therefore increase their achievable throughput, under a probability of detection constraint. Our model also takes into account the overhead in combining sensing reports.

Two different game theoretic analyses of distributed cooperative spectrum sensing are presented in [13, 14]. In [14], an evolutionary game theoretic framework is used to analyze the interactions among distributed selfish CRs in cooperative sensing. It is assumed in [14] that the selfish CRs overhear the detection results from the other CRs and can free ride by refusing to take part in spectrum sensing. Hence [14] models the spectrum sensing problem as a noncooperative game. Distributed coalition formation for cooperative spectrum sensing CRs is the topic of [13]. Using a merge-and-split based coalition formation game model, the authors in [13] analyze the average missed detection probability per CR. However, unlike [13], we propose a value function that encourages collaborating CRs to minimize their false alarm probabilities for a given target primary user detection probability ($P_d$). This is an important requirement for coexistence with primary users; otherwise, CRs will not be allowed to operate in the primary user band [15]. Moreover, in contrast with [13], the coalition formation model proposed in this paper also takes into account the overhead in combining sensing reports within a coalition. In [5], a dynamic coalition formation game based on a Markovian model is used to analyze the selfish interactions among distributed nodes for spectrum sharing in an interference channel. Unlike [5], we analyze the interactions among CRs for the problem of distributed throughput-efficient sensing in CR networks. Moreover, selfish coalition formation proposed in [5] for distributed networks may lead to a suboptimal equilibrium where nodes, through their interactions, reach an undesirable coalition structure from a network point of view. Hence, we extend the dynamic model of coalition formation proposed in [5] to determine whether and how the coalitional behavior of CRs will change if coalition formation is "not entirely selfish".

In this paper, using a coalition game-theoretic framework we devise distributed cooperative strategies for CRs that are either selfish or altruistic. We propose a coalition formation model for these CRs to utilize primary user spectrum efficiently, under the constraint of a target probability of detection. We take into account the cost of distributed cooperative sensing in terms of overhead in combining sensing reports within a coalition. We analyze the impact of this cost on the distributed cooperative strategies of CRs. Given a target detection probability for a coalition, we also adopt a weighted target detection probability for individual CRs in a coalition and analyze its impact on the average throughput per CR. We model the stable network structure that emerged as a result of the dynamic coalition formation as the absorbing state of a Markov chain. Using absorbing Markov chain theory, we analyze the mean $\mu$ and the variance $\sigma^2$ of the time for the game to reach the
stable coalition structures. We propose a technique to reduce the time taken by the CRs to form a stable coalition. Using simulations we assess the performance of the proposed altruistic and selfish coalition formation solutions in terms of gains in increased average throughput per CR. We compare these results to the throughput achieved by a noncooperative strategy and by the grand coalition (when all the CRs cooperate). We also determine average maximum coalition sizes for both altruistic and selfish coalition formation solutions. Finally, we evaluate the impact of the distance between the primary user transmitter and the distributed CR network on cooperative strategies of the distributed CRs.

The rest of the paper is organized as follows. Section 2 presents the system setup, while Section 3 introduces the proposed coalition formation game for distributed spectrum sensing. Section 4 introduces the dynamic coalition formation model for the proposed game. Section 5 presents simulation results and an analysis of the proposed dynamic coalition formation model, and Section 6 concludes the paper.

2. System Setup

The system setup used in this paper includes a primary user transmitter and a distributed CR network of $n$ active CRs (transmitter/receiver pairs). The CRs are uniformly and independently distributed in a circle with radius $\bar{R}$ and centered at the coordinates $(0, 0)$. The PU (primary user transmitter) is at coordinates $(0, 0)$ as shown in Figure 1. This corresponds to, for example, using only downlink frequencies for CR access. The primary user and CRs are both assumed to use a time-slotted system with perfect time synchronization, and one transmission by primary user corresponds to one time slot, as in [16, 17]. In this approach, the primary user is either present for the whole time slot, or absent for the whole time slot. The CRs use the beginning of each slot for sensing. We assume that $n$ cognitive radios employ energy detection to make primary user detection observations in the frequency band they are monitoring. The probability of primary user present is denoted by $P_1$. In order to detect the primary user, each CR can either sense the spectrum on its own (noncooperative strategy) or it can perform cooperative sensing by forming coalitions with other CRs (cooperative strategy).

![Figure 1 Topology of a cognitive radio network.](image)
Let us represent the received signal-to-noise ratio (SNR) from the primary user to the \(i\)th CR by

\[ \gamma_i = \frac{P_i}{\sigma^2}, \]  

(1)

where \(\sigma^2\) represents noise power and \(P_i = \kappa P_{PU} / d_i^{\alpha}\) is the signal power received by CR \(i\). \(P_{PU}\) is the primary user's signal power, \(d_i\) is the distance between the primary user and the \(i\)th CR, \(\alpha\) is the path-loss exponent and \(\kappa\) is a scalar. In our system setup we assume a complex-valued PSK primary user signal and circular symmetric complex Gaussian (CSCG) noise. For the CSCG noise case the probability of false alarm of CR \(i\) for a chosen detection threshold \(\lambda\), is given by [4, 18]

\[ P_{fa}(\lambda) = Q((\frac{\lambda}{\sigma^2} - 1)\sqrt{N}). \]  

(2)

where \(Q()\) is the tail probability for the standard normal distribution and \(N\) represents the time-bandwidth
product, given as $\hat{N} = \tau_s \nu$, where $\tau_s$ is the sensing duration and $\nu$ is the measurement bandwidth. For a chosen threshold $\hat{\lambda}$, the probability of detection of CR $i$ is approximated by [4, 18]

$$
\hat{P}_{d,i}(\hat{\lambda}, y_i) = Q\left(\frac{\hat{\lambda}}{2\nu} - \sqrt{y_i - 1} \frac{\hat{N}}{2\nu N_i + 1}\right).
$$

(3)

To protect the primary user against harmful interference from the CRs, the detection probability is fixed at a desired target value, $\hat{P}_{d,\mathcal{C}}$. In practice, $\hat{P}_{d,\mathcal{C}}$ is required to be close to 1 [2]. The probability of false alarm of each CR $i$ for the targeted $\hat{P}_{d,\mathcal{C}}$ can be rewritten as

$$
\hat{P}_{f,i}(\hat{\lambda}, y_i) = Q\left(\sqrt{2y_i - 1} Q^{-1}(\hat{P}_{d,\mathcal{C}}) + \sqrt{N_i y_i}\right).
$$

(4)

It may be seen from (4) that a high detection probability requirement may lead to a high false alarm probability for an individual CR if its $y_i$ is low, thus reducing the achievable throughput of that CR. In this case, the individual CRs may interact to form coalitions to help decrease the false alarm probability. Within a coalition, sensing decisions by individual CRs are transmitted over the narrowband common control channel to a CR selected as a coalition head. Although the optimal decision fusion rule is the Chair-Varshney rule [19], for simplicity of implementation we assume that an OR fusion rule is used by the coalition head to combine the individual CR sensing decisions within a coalition. The OR rule is a simple decision rule explained as follows: if one out of $|\mathcal{S}|$ CRs in a coalition detects the primary user, the final decision for the coalition declares a primary user is present, where $|\mathcal{S}|$ represents the number of CRs in a coalition $\mathcal{S}$. CRs in any coalition $\mathcal{S}$ decide to transmit or not, based on the final combined sensing decision of the coalition head. Therefore, the probabilities of detection and false alarm of a coalition head are also the probabilities of detection and false alarm of each CR $i$ that is the member of $\mathcal{S}$.

Assuming that all decisions are conditionally independent (this means that the sensing measurements performed by CRs are independent, but that for each CR the same hypotheses $\{H_0, H_1\} = PU$ not present and $\hat{H}_1 = PU$ present, apply) then using the OR rule, the detection probability of the coalition $\mathcal{S}$ is given as

$$
\hat{P}_{d,\mathcal{S}} = 1 - \prod_{i=1}^{\mathcal{S}} (1 - \hat{P}_{d,i})
$$

(5)

For a given $\hat{P}_{d,\mathcal{S}}$, the individual CR's target probability of detection in a coalition using OR fusion rule is written as (assuming same target probability of detection for every CR, as in [4, 14])
The probability of false alarm of each CR \(i\) for the \(\hat{P}_{d,i}\) can be written as

\[
\hat{P}_{f,i} = \frac{1}{1 - \left(1 - \hat{P}_{d,i}\right)}.
\]  

(6)

However, it may happen that some of the CRs have better received SNR from the PU than the others. To gain from this SNR diversity, we adopt a weighted target probability of detection for any CR \(i \in S\). The weighted target probability of detection modifies \(\hat{P}_{d,i}\) in (6) and takes a sensing CR's received SNR \(\gamma_i\) into consideration. We propose to use weighted target probability of detection for CR \(i\) given by

\[
\hat{P}_{d,i}^w = \frac{1}{1 - \left(1 - \hat{P}_{d,i}\right)^{\gamma_i}}.
\]

(7)

While we make no claims as to the optimality of the selected weighting method, these probabilities preserve the relationship in (5) and assign a higher expectation on detection accuracy to the CRs that experience higher SNR. We justify the use of \(\hat{P}_{d,i}^w\) as follows. For a given SNR \(\gamma_i\) and \(\gamma\), false alarm probability of CR \(i\) is a function of the target detection probability for that CR (see (7) and Figure 2). It can be seen from Figure 2 that the impact of target detection probability on the false alarm probability of individual CRs varies for the different values of \(\gamma_i\). If a CR has high \(\gamma_i\), for instance \(\gamma_i = 0.05\), then the target detection probability has little impact on its false alarm probability. However, if a CR has low \(\gamma_i\), for instance \(\gamma_i = 0.005\), then the target detection probability has strong impact on its false alarm probability.

![Figure 2 Probability of false alarm for different detection probabilities and for different values of SNR, \(N = 6000\).](image-url)
In this approach, the CR with high $\gamma_i$ is assigned relatively high target detection probability. However, due to high $\gamma_i$, the assignment of high target detection probability has little impact on the false alarm probability of that CR. The CR with low $\gamma_i$ is assigned relatively low target detection probability which results in low false alarm probability for that CR (see Figure 2), as compared to when each CR is assigned the same target detection probability in a coalition.
The probability of false alarm of each CR $i$ for the $P_{d,i}$ is written as

$$P_{f,i}(P_{d,i}, y_i) = Q\left(\sqrt{y_i} + \frac{1}{Q^{-1}(P_{d,i})} + \frac{1}{y_i}\right).$$  \hspace{1cm} (9)

Using the OR rule, the false alarm probability $P_{f,s}$ of the coalition $s$ is given as

$$P_{f,s} = 1 - \prod_{i=1}^{\left|s\right|} (1 - P_{f,i}).$$ \hspace{1cm} (10)

It may be seen from (7) and (9) that, for a given $P_{d,i}$ and $N$, the CRs with low values of $y_i$ have incentives to form coalitions as it helps to decrease $P_{f,i}$ (due to the increase in $Q^{-1}(P_{d,i})$ or $Q^{-1}(P_{d,i})$ term which in turn decreases $P_{f,i}$). However, for a given $P_{d,i}$ the coalitional false alarm probability given by (10) is an increasing function of coalition size $|s|$.

3. Coalition Game-Theoretic Framework for Distributed Cooperative Sensing

Coalition game theory provides useful tools to decide which group of players will cooperate with each other to efficiently achieve their goals [8, 20]. Therefore, to analyze cooperative interactions among CRs performing distributed spectrum sensing, we model the problem as a coalition game.

3.1. Preliminaries

Let $N = \{1, 2, \ldots, n\}$ denote the set of players (CRs) playing the coalition game. A coalition, $S$, is a subset of $N$, $S \subseteq N$. An individual non-cooperating player is called a singleton coalition and the set $N$ is called the grand coalition, where all players cooperate. The utility of a coalition in a coalition game is called the coalition value, and is denoted by $v$. Coalitions are assumed to be non-overlapping, that is, CRs are members of at most one coalition.

The most common form of a coalition game is the characteristic function form. In the characteristic function form (CFF) of coalition games, utilities achieved by the players in a coalition are unaffected by those outside it.
Definition 1.

A nontransferable utility (NTU) game is a coalition game in CFF, in which the value $v(S)$ of a coalition $S$ cannot be arbitrarily divided among the coalition's players. In such games, each player will have its own value within a coalition $S$. The value function $\phi_i(S)$ represents the value of player $i$ that belongs to a coalition $S$.

### 3.2. Throughput of a CR

For a noncooperative CR network with periodic spectrum sensing, each slot consists of sensing duration $T_s$ and data transmission duration $T_f - T_s$. To focus on the coalition formation model for the sensing-throughput tradeoff problem in distributed CR networks; we assume that the entire primary user band is divided into $\kappa$ subbands and, when the primary user is absent each CR operates exclusively in one of the $\kappa$ subbands. (We do not analyze scheduling policies in this work and assume a very simple and predetermined orthogonal subband allocation policy for the CRs when the primary user band is free for access. Our ongoing work, outside the scope of this paper, addresses competition among CRs for subbands that are free.) This assumption is in line with the other models in the literature $[14, 21]$. For the noncooperative case, the average throughput of the CR $i$ is approximated by $[1, 14]$

\[
\mathcal{R}_i = P_{H_0} (1 - \frac{r_i}{T_f} ) \frac{(1 - P_{T_i})}{r_i},
\]

(11)

where $P_{H_0}$ is the probability of primary user absent, $T_f$ represents the total frame length and $r_i$ represents the transmission rate of the CR $i$ to its receiver when the primary user is absent.

In distributed CR networks when CRs decide to perform cooperative sensing by forming coalitions with one another they may incur overhead costs in terms of time delay due to the process of coalition formation and due to combining sensing reports within a formed coalition. Note that the time delay due to the process of coalition formation depends on how frequently the coalition process is initiated which in turn depends on changes in the network configuration (for, e.g., CR mobility, etc.). However, the time delay in data transmissions of a coalition due to the overhead in combining sensing reports is periodic (since this overhead is incurred in every time slot). In this work, we analyze the impact of overhead costs in terms of time spent during combining sensing reports on the throughput of distributed CRs. We ignore the overhead in terms of time delay due to the process of coalition formation. One of the extensions we envision for our current work is the consideration of the effects due to overhead in terms of spending time and resources due to the process of coalition formation and also the effects due to overhead in terms of energy consumed by a coalition head to collect and combine sensing reports.

When the CRs decide to form a coalition then the entire coalition cannot transmit data until sensing reports
are collected and the final combined sensing decision is transmitted to all the coalition members (see Figure 3). One simple method of sensing reports collection by the coalition head can be stated as follows. The coalition head grants a contention free channel to individual cognitive radios by polling them (using their identity numbers) for transmitting their local decisions. The coalition head may employ a round-robin scheduler [22, 23], and on being polled, a CR transmits its local decision to the coalition head. In this sensing reports collection method there is cost in terms of time delay in data transmissions of a coalition due to the overhead in combining sensing reports. This cost generally increases with the number of coalition forming CRs as more decisions need to be reported to the coalition head. The average throughput of the CR \( i \) considering the cost in terms of overhead in sensing reports combining within a coalition is approximated as

\[
\hat{\gamma}_i = p_{H_0}(1 - \frac{\tau_c}{\tau_f}) - \frac{\tau_c}{\tau_f} (1 - p_{f,s} \gamma_0) \gamma_i,
\]

where \( \tau_c \) is the time spent on reporting a sensing decision to the coalition head.

For a target detection probability, \( p_{H_1} \), the CRs may form coalitions to reduce their false alarm probability and therefore increase their average throughput given by (12). The coalition forming CRs may also increase their average throughput by reducing their sensing time \( \tau_s \) via joint coalitional sensing. However, we note that for the distributed spectrum sensing problem when the coalition forming CRs are allowed to vary sensing time \( \tau_s \), it generates significant uncertainties in the coalition values. For instance, if two or more CRs reduce their sensing time \( \tau_s \) (or in other words increase their data transmission time) via forming the coalition \( s \) then it may happen that the CRs in \( s \) start transmitting data while some CRs outside \( s \) may still be sensing. This may lead to a change in the coalition value (in terms of false alarm probabilities) of the CRs outside the coalition \( s \). To avoid this uncertainty in the coalition value due to CR transmissions, we fix the sensing duration \( \tau_s \) for each individual CR.
3.3. Coalition Value with Nontransferable Utility

For a target detection probability, \( \hat{p}_{t_0} \), and the fixed sensing duration \( t_s \), the coalition value \( \nu(S) \) must characterize the incentives to form coalitions in terms of the decreased false alarm probability \( \hat{p}_{f,S} \) of the coalition. Moreover, the coalition value must also take into account the cost in terms of delay in data transmissions of a coalition due to the overhead in combining sensing reports. A suitable coalition value that satisfies the above requirements is given by

\[
\nu(S) = (\nu_1(S), \nu_2(S), \ldots, \nu_{|S|}(S)) = (s_1, s_2, \ldots, s_{|S|}),
\]

where \( \nu_i(S) \) denotes the average throughput for CR \( i \) given by (12), for \( i = 1, 2, \ldots, |S| \). In the proposed coalition formation game, each CR has its own value within a coalition and its nontransferable due to the following reasons.

Indivisible False Alarm Probability

The probability of false alarm \( \hat{p}_{f,i} \) of each CR \( i \), where \( i \in S \), is also given by the probability of false alarm of the coalition \( S \), that is, \( \hat{p}_{f,S} \) (as explained in Section 2) and it cannot be divided among the radios.

Indivisible Cost

Each CR incurs the same cost in terms of overhead in combining sensing reports, that is, \( (\tau_c / T_c)(|S| - 1) \), and this cost cannot be divided among the CRs.

Indivisible Bandwidth

Finally, each CR operates exclusively in one of the \( K \) subbands. Therefore, they cannot arbitrarily divide the spectrum among themselves.

Therefore, each CR will have its own value within the coalition \( S \) and hence, the proposed game is an NTU game.

When two or more CRs form a coalition \( S \), then any CR within \( S \) is selected as a coalition head to combine the individual CR sensing decisions within the coalition. The decisions to form coalitions by the CRs are based on consensus, that is, a coalition is formed only if it is acceptable to everyone involved. We also assume that CRs are myopic, that is, CRs care only about their current payoffs.
3.4. Selfish Coalition Formation

**Definition 2.**

Internal stability means that no CR has an incentive to leave its coalition to become a singleton (individual noncooperative CR), that is, \( \varphi_i(S_1) \geq \varphi_i(\{i\}) \), for all \( i \in S_1 \).

We assume that CRs are individually rational, that is, CRs seek to maximize their payoffs, conditional on feasibility. Therefore, for selfish coalition formation, the merge of two coalitions only occurs when all CRs in the new coalition \( S \) are at least as well off through the merge as they were before it. Mathematically speaking, coalitions \( S_1 \) and \( S_2 \) will merge to form \( S \) only: if for all \( i, j \) where \( i \in S_1 \) and \( j \in S_2 \), \( (\varphi_i(S) - \varphi_i(S_1)) \geq 0 \) and also \( (\varphi_j(S) - \varphi_j(S_2)) \geq 0 \). Due to this coalition formation condition, whenever CRs agree to form \( S \) the new coalition is **internally stable**, that is, no CR has an incentive to become a singleton (an individual noncooperative CR).

3.5. Altruistic Coalition Formation

The model of selfish coalition formation discussed above is in line with much of the coalition formation literature, which assumes that users form coalitions to maximize their individual payoffs. However, it is equally interesting to investigate the question of whether and how the achievable throughput of CRs will change if CRs are assumed to be "not entirely selfish". Intuitively, we want to model the case when two or more CRs propose to form a new coalition \( S \) by taking into account one another's welfare. Before studying altruistic coalition formation, we first define the concept of altruistic contribution of a coalition as follows [11].

**Definition 3.**

Let \( S_1 \) and \( S_2 \) be two disjoint coalitions. The altruistic contribution

where \( S = S_1 \cup S_2 \), is \( \bar{\varphi}_{S_1}(S) = \sum_{i \in S_2} (\varphi_i(S) - \varphi_i(S_1)) \), the altruistic contribution

is \( \bar{\varphi}_{S_2}(S) = \sum_{j \in S_1} (\varphi_j(S) - \varphi_j(S_2)) \), and the sum of altruistic contributions

is \( \bar{\varphi}(S) = \bar{\varphi}_{S_1}(S) + \bar{\varphi}_{S_2}(S) \).

In simple words, the altruistic contribution of coalition \( S_1 \) to \( S \) represents the change in the value of the CRs in \( S_2 \), when the CRs in \( S_1 \) are added to the coalition \( S \). It can be easily seen that if any proposed coalition \( S \) has \( \bar{\varphi}(S) > 0 \), then the merger of \( S_1 \) and \( S_2 \) to form \( S \) would do more good than harm to the overall value of the coalition \( S \). Formally, we assume that...
to maximize the achievable throughput of the proposed coalition $S$, an altruistic coalition decides to form the coalition $S$ whenever

$$\mathcal{S}(S) > 0.$$  

4. A Markovian Model of Coalition Formation Game

To model all $n$ CR coalitions, we define coalition structures as follows.

Definition 4.

A coalition structure is a partition of $\mathbb{N}$ into exhaustive and disjoint subsets, where each subset is a coalition. The set of all possible coalition structures is denoted as $\mathcal{C}$.

In this section, we introduce a dynamic model of coalition formation for distributed cooperative spectrum sensing in CR networks where multiple distributed CRs coexist and opportunistically access the spectrum. In the proposed dynamic coalition formation game the CRs follow a time evolving sequence of steps to reach self-organizing stable spectrum sensing coalition structures. Coalition structures in the dynamic coalition formation game are modeled as a sequence of random variables describing the state of the system, with the mechanism of transitions between coalition structures represented by a Markov chain. To incorporate slow changes in the network configuration (for, e.g., due to CR mobility), the initial round of coalition formation game restarts some time $\tau$ after reaching equilibrium. This time $\tau$ may be assigned according to variations in the network configuration. We assume that during one round of coalition formation the received primary user's SNR, that is, $\gamma_i$, and the transmission rate $r_i$ of each CR $i$ does not change. At the very beginning of each round of the coalition formation game, the distributed CR network is composed of all singleton coalitions, that is, noncooperative CRs.

4.1. A Coalition Formation Game Model

The coalition formation game involves five steps. The five steps of the proposed model are summarized as follows:

**Node Discovery**

Discover the CRs within the network.

**Initialization**
Each individual CR computes its received SNR $\gamma_i$ from the PU.

Coalition Formation Proposal

(a) At each time slot, each coalition, with probability $\mathcal{P}$, proposes a new coalition structure. In this process, one of the CRs in the coalition acts on its behalf. (In the case of singleton coalitions, each singleton coalition, i.e., each CR, individually proposes a new coalition structure with some probability $\mathcal{P}$; when two or more CRs form a coalition $\mathcal{S}$, then any CR within $\mathcal{S}$ is selected as a coalition head to propose a new coalition structure with some probability $\mathcal{P}$ on behalf of that coalition.)

(b) The evolution from one coalition structure to the next can only occur through the merging of two existing coalitions. For instance, any coalition head of any existing coalition $\mathcal{S}_1$ may propose to merge with another coalition $\mathcal{S}_2$, forming $\mathcal{S}_1 \cup \mathcal{S}_2 = \mathcal{S}$.

Coalition Formation Decision

(a) When the CRs are assumed to be selfish then they form a coalition if for all $i, j$, where $i \in \mathcal{S}_1$ and $j \in \mathcal{S}_2$, $(\varphi_i(\mathcal{S}_1) - \varphi_j(\mathcal{S}_1)) \geq 0$ and also $(\varphi_j(\mathcal{S}_2) - \varphi_j(\mathcal{S}_1)) \geq 0$. (b) When the CRs are assumed to be altruistic then they form a coalition if $\delta(\mathcal{S}) > 0$. (see Sections 3.4 and 3.5 for the details).

Steps (3) and (4) of the coalition formation are repeated until all the coalitions have made their coalition formation decisions, resulting in a final stable coalition structure $\mathcal{C}_F$.

Coalitional Spectrum Sensing

Each CR within a coalition computes its local sensing decision and transmits it to the coalition head over the common control channel. The coalition head combines the local sensing decisions (including its own sensing decision) using an OR decision fusion rule.

The proposed distributed coalition formation can be performed by coalition formation message exchanges between CRs over a common control channel. In practice, if all CRs use the same control channel to report sensing decisions to their respective coalition heads then the reporting time may increase. To decrease the reporting time coalitions may use different control channels. However, since the number of available channels in general is limited and the number of coalitions may be large, control channels must be spatially reused. For simplicity, we have assumed different reporting channels for each coalition.

4.2. Dynamics of Coalition Formation: Game Model

The dynamic coalition process can be modeled using a Markov chain, with each state representing a
different coalition structure. A finite set \( C = \{ C_1, C_2, \ldots, C_l \} \) of all possible coalition structures for \( n \) CRs forms the state space of the coalition formation game. Our work in [5] provides further detail on this modeling, including examples of the Markov chain structure for \( n = 3 \) and \( n = 4 \) transmitter/receiver pairs.

### 4.2.1. Selfish Coalition Formation

Any coalition may propose a coalition structure change to another coalition in the current state \( C_k \), and if all the CRs in the proposed coalition are at least as well off as before the merge, the game moves to \( C_l \). When each of \( m \) prevailing coalitions with some probability \( p \) proposes a new coalition structure then the transition probabilities for the \( n \) CR selfish coalition game with coalition structures as state space \( C \) are given as

\[
P_{C_k C_l} = \frac{2p(1-p)}{C_k - 1} \left( \sum_{C_j \in \Phi} \left[ (v_j(S) - v_j(S_i))\geq 0, (v_j(S) - v_j(S_2))\geq 0 \right] \right)
\]

\[\forall j, \text{ where } i \in S_1, j \in S_2, \]

\[
P_{C_k C_k} = 1 - \sum_{C_j \in \Phi} P_{C_k C_j}
\]

where \( \mathbb{1}[x] \) is an indicator function equal to 1 if condition \( x \) is satisfied, and zero otherwise, \( v(S_1) \) and \( v(S_2) \) are the values of the two coalitions participating in the coalition formation to form the proposed coalition \( S \), \( v(S) \) is the value of the proposed coalition \( S \), and \( \Phi \) represents the set of all new possible coalition structure states to which coalitions can transit from \( C_k \). The set \( \Phi \) is given as \( \Phi = \{ \{ S_1 \cup S_2, S_3, \ldots, S_l | C_k \}, \{ S_1, S_2 \cup S_3, \ldots, S_l | C_k \}, \ldots, \{ S_1, S_2, \ldots, S_l | C_k \} \} \). Given \( |C_k| > 1 \) coalitions in the present state \( C_k \), it is possible to transition from \( C_k \) to one of the \( |\Phi| \) possible states, where \( |\Phi| \) can be calculated as

\[
|\Phi| = \left( \frac{|C_k|}{2} \right)
\]

For instance, if the number of coalitions \( |C_1| = 3 \), then it is possible to move from the state \( C_1 \) to one of \( |\Phi| = 3 \) different states (besides itself), provided that the coalition formation condition is satisfied. As explained previously, two coalitions can merge only if all CRs within the proposed coalition \( S \) are at least as well off as before the merge. The indicator function \( \mathbb{1}[((v_j(S) - v_j(S_1))\geq 0, (v_j(S) - v_j(S_2))\geq 0)] \) in (15) represents the possible agreement or disagreement among the CRs participating in the coalition game to form the proposed coalition \( S \). As the transition probability at any present state \( C_k \) does not depend upon the prior states of the coalition structures, the Markov property holds.
4.2.2. Altruistic Coalition Formation

For the altruistic-cooperation case, the indicator function (coalition formation decision) in (15) is changed from \(1[\mu(S) - \nu_j(S) + \nu_j(S_j) - \nu_j(S_j)] \geq 0\) to \(1[\mu(S) > 0]\). It simply means that each CR now decides to merge if the merger of two coalitions would do more good than harm to the overall value of the merged coalition \(S\).

Using simulation results in Section 5.3 we will compare the performance of the coalition formation based on altruistic decision with the selfish decision in terms of average throughput per CR, for different network sizes.

Using standard theory of absorbing Markov chains one can calculate how long will it take for the process to reach stable coalition structures, that is, one can calculate the mean time \(\mu\) and its variance \(\sigma^2\) for the dynamic coalition game starting from the initial state of all singleton coalitions to reach stable coalition structures (where no two coalitions have an incentive to merge) [5].

In Section 5.3, we analyze the mean \(\mu\) and the variance \(\sigma^2\) of the time for the proposed coalition formation to reach the stable coalitions.

5. Dynamic Game Analysis and Simulation Results

To perform the coalition formation, CRs need to exchange their received primary user's signal-to-noise ratio (SNR). The amount of information exchange necessary to reach stable coalition structures can be measured by the total number of coalition formation proposals sent by the \(n\) CRs during the coalition formation process. For instance, if \(N\) number of coalition formation proposals are sent during the entire coalition formation process and each proposal requires the exchange of \(D\) messages for coalition heads to take a coalition formation decision then to reach equilibrium \(N \times D\) messages need to be exchanged among the CRs. When we are not taking into account the loss of coalition formation proposals due to collisions among distributed uncoordinated CRs then in the worst case, that is, where almost all proposals are rejected, the number of proposals is only \(\left(\begin{array}{c} n+1 \\ 2 \end{array}\right) + \sum_{i=1}^{n-2} i = \left(\begin{array}{c} n+1 \\ 2 \end{array}\right) + \frac{n^2 - 2n + 1}{2} = n^2 - 2n + 1\). Thus for the worst case, coalition formation proposals can be said to be order of \(O(n^2)\). In the best case, that is, where all proposals are accepted and the coalition formation leads to the formation of the grand coalition, the number of coalition formation proposals necessary is only \(n\). Thus for the best case, coalition formation proposals can be said to be order of \(O(n)\). In practical scenarios the complexity is between these two extremes.

This reduction in the information exchange is due to the reason that when coalitions are formed then instead of all the CRs only the coalition heads exchange proposals on behalf of their respective coalition members.
5.1. Stable Coalition Structures

As CRs form self-organizing spectrum sensing coalition structures, we analyze under what conditions the coalition formation process will reach a stable coalition structure (where no two coalitions have an incentive to merge anymore).

Selfish Coalition Formation

For the proposed selfish coalition formation, a coalition structure state $C^*$ is an equilibrium state if it satisfies the following condition

$$\forall S_j, S_k, k \neq j \in C^*, \quad \text{for some } i \in S_j, (\varphi_i(S_j \cup S_k) - \varphi_i(S_j)) < 0,$$

or for some $i \in S_j, (\varphi_i(S_k \cup S_j) - \varphi_i(S_k)) < 0$.

The above stated condition ensures that no two coalitions in the prevailing coalition structure $C^*$ have an incentive to merge anymore.

The following simple fact proves that the selfish coalition formation process converges to an equilibrium state:
In the proposed selfish coalition formation, if a certain coalition structure is not an equilibrium state, there must exist at least two coalitions that can decide to merge to improve their value functions. As long as such two coalitions exist, the coalition structure changes to another coalition structure, until an equilibrium state is reached.

Altruistic Coalition Formation

For the proposed altruistic coalition formation, a coalition structure $C^*$ is an equilibrium state if it satisfies the following condition

$$\forall S_j, S_k, k \neq j \in C^*, \quad \text{if } \mathcal{A}(S) \leq 0.$$

Following the same reasoning as we did for the selfish coalition formation process, it is easy to see that the altruistic coalition formation process converges to an equilibrium.

5.2. Coalition Sizes
The study of coalition formation in wireless networks has previously focused on cohesive games [24], that is, games where the value of the grand coalition formed by the set of all users $N$ is at least as large as the sum of the values of any partition of $N$. The authors in [24], also assume that there is no cost to the coalition formation process. In such coalition games, coalition structure generation is trivial because the wireless nodes always benefit by forming the grand coalition.

However, many coalition game models of wireless node cooperation are not cohesive (see, e.g., [25]). In such network scenarios, the network welfare maximizing coalition structure varies.

For the proposed selfish coalition formation where CRs seek to maximize their payoffs, conditional on feasibility, it is interesting to note that the higher the $y_i$ (the signal power received by CR $i$), the lower the $\phi_{i,n}$ given by (7). CRs with high values of $y_i$ may have either less or no incentive to cooperate with CRs with low values of $y_i$ (or with high $\phi_{i,n}$). Hence, cooperation among all CRs is desirable only when all CRs experience similar $y_i$. Therefore, in the proposed coalition formation, the grand coalition of all the CRs may not always form.

For the proposed altruistic coalition formation where CRs seek to merge if the merger of two coalitions would do more good than harm to the overall value of the merged coalition, it is interesting to note that the CRs with high $y_i$ values may form coalitions with the CRs having low $y_i$ values as long as the overall value of the newly formed coalition $S$ is increased.

Using simulation results in Section 5.3 we will present average maximum number of CRs per formed coalition for both selfish and altruistic coalition formation.

5.3. Simulation Results

Using simulation our aim is to compare the performance (for, e.g., in terms of average throughput per CR) of the coalition structure that emerges as the outcome of the proposed coalition formation solutions to a noncooperative solution and to the grand coalition.

For simulation illustrations, the following distributed CR network is set up: $n$ CRs are uniformly and independently distributed in a circle with radius $R = 1000$ m and centered at the coordinates $(0, 0)$. The PU transmitter is at coordinates $(0, 0)$ as shown in Figure 1. The sensing time $T_s = 1$ ms, the time-bandwidth product is set as $N = 5000$, and the frame duration is set to be $T_f = 100$ ms. We set the path loss exponent $a = 3$. The PU power $P_i$, scalar $\kappa$ and noise power $\phi_i^2$ are set at a value such that $y_i$ (PU's SNR at CR $i$) at the coordinates $(\theta_i, 0) = (20000, 0)$ is $-15$ dB. The probability of primary user present is assumed to be $p = 0.2$. To keep our simulation analysis simple, we assume that all the CRs have the same transmission rate, that is, $r_i = r = \log(1 + \text{SNR}_i^2) = 3.45$ bits/sec/Hz in (12), where SNR is signal-to-noise ratio from a CR to its receiver. Simulations were performed by "dropping" the CRs randomly around the coordinates $(\theta_i, 0)$. For the
target detection probability $\hat{p}_d = 0.9$ and $\hat{p}_d = 0.95$ in Figures 4(a) and 4(b), we show the average (averaged over the simulation runs) throughput per CR for different network sizes, when all CRs sense independently (noncooperative strategy), and when CRs can form coalitions (selfish and altruistic). It can be seen from Figures 4(a) and 4(b) that the proposed coalition formation (both selfish and altruistic) yields an improvement in the average throughput as compared to the noncooperative solution. However, the selfish coalition formation solution leads to a loss in average throughput as compared to the altruistic coalition formation solution. In Figure 5, we compare the performance of altruistic and selfish coalition formation solutions with the noncooperative solution in terms of average false alarm probability per CR for different network sizes. It can be seen that the altruistic coalition formation solution significantly reduces the average false alarm per CR, as compared to both selfish coalition formation and noncooperative solutions. It can also be seen from Figures 4 and 5 that the weighted target detection probability for individual CRs in a coalition results in better average throughput and reduced average false alarm per CR as compared to when each CR is assigned the same target detection probability.

Figure 4 Average throughput per CR for different network sizes and for different scenarios.
(a) Average throughput (bits/sec/Hz) per CR, $\hat{p}_d = 0.9$, $\beta = 2000$ m and $\gamma_C$ in (12) is set to 0.001ms (the time spent for reporting a sensing decision to the coalition head). Given a target detection probability for a coalition, "same $\hat{p}_d$," means that each CR is required to satisfy the same target probability of detection (see (7)), whereas "weighted $\hat{p}_d$," means that each CR is required to satisfy an SNR-dependent target detection probability (see (9)). (b) Average throughput per CR, $\hat{p}_d = 0.95$, $\beta = 2000$ m and $\gamma_C$ in (12) is set to 0.001 ms.
Figure 5  Average false alarm per CR for different network sizes, \( \hat{\gamma}_c \) in (12) is set to 0.001 ms and \( \beta = 2000 \) m.
When the overhead cost, that is, the cost in terms of collecting and combining sensing reports at the coalition head is not taken into account then the altruistic coalition formation solution yields the same results as if all the CRs perform cooperative sensing, that is, form the grand coalition, see Figure 6. This is because, for no overhead cost, with increasing the number of cooperating CRs, a target detection probability may be achieved by having low detection probability at the individual CRs. The low detection probability at the individual CRs is translated to a low false alarm probability and therefore increase in throughput. However, in practice the reporting of the local CR sensing decisions to the report combining entity (coalition head, fusion center, etc.) incurs overhead in the sense that the entire reporting group cannot transmit until all
the sensing reports are collected and combined by that entity. In the literature (also in IEEE 802.22), polling of CRs by the report combining entity is suggested for the collection of sensing reports [3, 23, 26]. This method has communication overhead that increases linearly with the number of cooperating CRs. When the overhead cost of collecting and combining sensing reports is taken into account (e.g., $T_C$ in (12) is set to 1 ms) then it can be seen in Figure 6 that the performance of the grand coalition degrades significantly as the number of CR increases, as compared to the altruistic coalition formation solution. In Figure 7(a), we compare the performance of altruistic and selfish coalition formation solutions in terms of average throughput per CR for a network size of 20 CRs. In this figure, we show that for small $T_C$, the altruistic coalition formation (same $P_{d,v}$) solution yields significant average throughput gains when compared with the selfish coalition formation solution (same $P_{d,v}$). However, when $T_C$ is large then the selfish coalition formation solution (same $P_{d,v}$) either outperforms or at least matches altruistic solution (same $P_{d,v}$) with respect to average throughput per CR. This is because the average coalition size for the altruistic coalition formation solution (same $P_{d,v}$) is large as compared to the selfish coalition formation solution (same $P_{d,v}$), see Figure 7(b), and for large values of $T_C$, the sensing reporting overhead becomes significant. Due to this reason, the large coalitions can be more costly as compared to the small ones.

**Figure 6** Comparison of the proposed altruistic coalition formation solution with the grand coalition for different scenarios.
Figure 7  Average throughput per CR for different network sizes and for different scenarios.
(a) Average maximum coalition size for selfish and altruistic coalition formation solutions, when the values of $r_c$ (the time spent for reporting a sensing decision to the coalition head) are varied between 0.001 ms to 0.017 s, $\tilde{r}_d = 0.99$ and $\beta = 2000$ m. (b) Average throughput per CR for selfish and altruistic coalition formation solutions, when the values of $r_c$ (the time spent for reporting a sensing decision to the coalition head) are
varied between 0.001 ms to 0.017s, $\hat{d} = 0.99$ and $\beta = 2000$ m.

To illustrate the above situation, we can construct an example, for $n = 3$ CRs, where the altruistic coalition formation solution (same $P_{d,i}$) outperforms selfish solution (same $P_{d,i}$) when $\gamma_c$ is small. However, for the same scenario, when $\gamma_c$ is set to be large then selfish coalition solution (same $P_{d,i}$) may outperform altruistic solution (same $P_{d,i}$).

Example 1.

Let $y_1 = 0.0262$, $y_2 = 0.0148$ and $y_3 = 0.0233$ be the received primary user’s SNRs at $n = 3$ CRs. In this three-CR game, when $\gamma_c = 0.001$ ms and assuming $r_1 = r_2 = r_3 = 2.4594$ bits/sec/Hz then calculating coalition values using (13) and rules explained in Sections 3 and 4 we obtain $v(\{1\}) = 0.9960$, $v(\{2\}) = 0.3076$, $v(\{3\}) = 0.7718$, $v(\{1, 2\}) = (0.9198, 0.9198)$, $v(\{1, 3\}) = (1.4433, 1.4433)$, $v(\{2, 3\}) = (0.8272, 0.8272)$, $v(\{1, 2, 3\}) = (1.3036, 1.3036, 1.3036)$. It may be seen that the altruistic
coalition solution (same $\hat{P}_{C_0}$) for this scenario results in the formation of grand coalition $\{1, 2, 3\}$ of all CRs. However, the selfish coalition solution (same $\hat{P}_{d_{0}}$) results in the formation of either $\{1, 3\}, \{2\}$ or $\{2, 3\}, \{1\}$. Since, $v(\{1, 2, 3\})$ is greater than $v(\{1, 3\}) + v(\{2\})$ and $v(\{2, 3\}) + v(\{1\})$, therefore, the altruistic solution (same $\hat{P}_{d_{0}}$) outperforms the selfish solution (same $\hat{P}_{d_{0}}$) for this scenario. However, when $\tau_c = 15$ ms then we obtain $v(\{1\}) = 0.9968$, $v(\{2\}) = 0.3076$, $v(\{3\}) = 0.7718$, $v(\{1, 2\}) = (0.7805, 0.7805)$, $v(\{1, 3\}) = (1.2247, 1.2247)$, $v(\{2, 3\}) = (0.7019, 0.7019)$, and $v(\{1, 2, 3\}) = (0.9086, 0.9086, 0.9086)$. It may be seen that the selfish coalition solution (same $\hat{P}_{d_{0}}$) for this scenario results in the formation of $\{1, 3\}, \{2\}$. However, the altruistic coalition solution (same $\hat{P}_{d_{0}}$) results in the formation of either $\{1, 2, 3\}$ or $\{1, 3\}, \{2\}$. Since, $v(\{1, 3\}) + v(\{2\})$ is greater than $v(\{1, 2, 3\})$, therefore, the selfish solution (same $\hat{P}_{d_{0}}$) either outperforms or at least matches altruistic solution (same $\hat{P}_{d_{0}}$) for this scenario.

It can also be seen from Figure 7(a) that for very large values of $\tau_c$, the altruistic solution (same $\hat{P}_{C_0}$) matches the selfish solution (same $\hat{P}_{d_{0}}$) in terms of average throughput per CR. This is because for very large values of $\tau_c$ there are either very little or no gains in terms of average throughput per CR for coalition formation, and network structure is mostly composed of individual noncooperative CRs (see Figure 7(b)).

In Figure 8, we vary the parameter $\beta$ (distance between the primary user base station and the center of the distributed CR network, see Figure 1), and show average maximum coalition sizes for the selfish and altruistic coalition formation solutions. It can be seen from Figure 8 that for large $\beta$, the network for such scenarios is composed of coalitions of large sizes. In this figure, we also show that for the given simulation scenario the altruistic coalition formation solution results in the formation of the grand coalition of all CRs whenever $\beta \geq 2000$ m. As the value of parameter $\beta$ is decreased, the network for such scenarios is composed of independent disjoint coalitions of smaller sizes. It can also be seen from Figure 8 that as the value of the parameter $\beta$ is reduced to 500 m then the coalition formation solution results in a network structure mostly composed of individual noncooperative CRs, that is, all singleton coalitions.

**Figure 8** Average maximum coalition size for selfish and altruistic coalition formations, when parameter $\beta$ (distance between the primary base station and the center of the CR network, see Figure 1) is varied between 0 m to 4500 m, $\tau_c = 0.001$ msec and $n = 30$ CRs.
The two figures in Figures 9(a) and 9(b) illustrate the mean and variance of the time for the altruistic coalition formation to converge to an absorbing state with different probabilities \( p \) of coalition formation proposal. In Figures 9(a) and 9(b), simulation results are generated by dropping \( n = 6 \) CRs in a circle with radius \( R = 1000 \) m and centered at the coordinates (2000,0) (other simulation parameters, for instance sensing time, slot duration, and so forth, are set to be the same as explained in the beginning of Section 5.3). The stable coalition structure that emerges as the outcome of the coalition formation may be different for the different drops of CRs in the distributed CR network. Therefore, we evaluate the mean and variance of the time to converge to the stable coalition structure (absorbing state) for any single random drop. For the target detection probability \( \lambda_d = 0.95 \) and \( \gamma = 0.001 \) ms, when CRs propose altruistic coalitions with some
probability $p$ then the process converges to the grand coalition (absorbing state for a single random drop for the given simulation scenario). The mean and variance of the time to converge to the grand coalition are evaluated for different $p$ as follows: We repeat the coalition formation for the same drop of CRs with different probability $p$ of coalition formation proposal. Figure 9(a) shows that for small $p$, $\mu$ is high because the mean time between coalition formation messages is too long. If $p$ is too high then the mean time between coalition formation messages is shorter but the number of coalition message collisions is higher, resulting in a longer time between coalition structure changes. This suggests that depending on the number of radios in the game there is an optimum value for $p$. We illustrate this optimum value for $\mathcal{N} = 6$ CRs in Figure 9(a).

**Figure 9** Mean time and its variance for the altruistic coalition formation of $\mathcal{N} = 6$ CRs to form the grand coalition (absorbing state for a single random drop for the given simulation scenario) from the transient state of all singleton coalitions for different probabilities $p$ of coalition formation proposal. The dwell time in each state is set to 0.001 ms.

6. Conclusions

We apply a coalitional game-theoretic framework to the study of stable network partitions for the problem of distributed throughput-efficient sensing in cognitive radio (CR) networks. We devise distributed cooperative strategies for cognitive radios that are either selfish or altruistic. We propose a coalition
formation model for these CRs to utilize primary user spectrum efficiently, under the constraint of probability of detection. The proposed coalition formation model also takes into account the overhead in sensing reports combining. We demonstrate that when each CR is assigned the same target detection probability in a coalition then for small to moderate values of sensing reporting time, the altruistic solution yields significant gains in terms of average throughput per CR as compared to the selfish and noncooperative solutions. However, for large values of sensing reporting time, the selfish solution either outperforms or at least matches altruistic solution in terms of average throughput per CR. We then adopt a weighted target detection probability for individual CRs in a coalition. We find that the weighted target detection probability for individual CRs results in higher average throughput as compared to when each CR is assigned the same individual target detection probability in a coalition. We also find that when CRs are assigned weighted target detection probabilities in a coalition then the altruistic coalition formation solution significantly increases the average throughput per CR, as compared to noncooperative solution, selfish coalition formation, and the grand coalition of all CRs. Our work employs an absorbing Markov chain model to model the equilibrium state (where no two coalitions have an incentive to merge) of the proposed coalition formation. Using a Markovian model of the coalition game, we analyze the dynamics of the coalition formation game and the stability of different network partitions in a distributed cognitive radio network. We also analyze the mean \( \mu \) and variance \( \sigma^2 \) of the time for the game to reach the stable coalition structures. Finally, we also show the impact of the distance between the primary user transmitter and the distributed CR network on cooperative strategies of the distributed CRs.

**Funding**

This work was funded by the Finnish Funding Agency for Technology and Innovation (TEKES) and by Academy of Finland.

**Acknowledgment**

The material in this paper was presented in part at the 5th International Conference on Cognitive Radio Oriented Wireless Networks and Communications (CrownCom), Cannes, France, June, 2010.
References


5. Khan Z, Glisic S, DaSilva L, Lehtomäki J: Modeling the dynamics of coalition formation games for cooperative spectrum sharing in an interference channel. to appear in *IEEE Transactions on Computational Intelligence and AI in Games*


15. Cordeiro S, Sofer E, Chouinard G: **Functional requirements for the 802.22 WRAN standard r47.** January 2006


This PDF file was created after publication.