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Research Article

Analysis of the Tradeoff between Delay and Source Rate in Multiuser Wireless Systems

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This work addresses the limits on the information that can be transmitted over the wireless channel under the conditions stated by the MAC layer: a selected scheduling discipline and an ensured level of QoS. Based on the effective bandwidth theory, the joint influence of the channel fading, the data outsourcing process, and the scheduling discipline in the QoS metrics are studied. We obtain a closed-form expression of the vector of attainable users' rates $R^u_{D^t,\varepsilon}$ for several scheduling algorithms, representing the maximum constant rate that the uth user can transmit under the selected discipline and fulfilling a target Bit Error Rate (BER) and the delay constraint given by the pair (D^t, ε), where D^t is the target delay and ε is the probability of exceeding D^t.

1. Introduction

Providing Quality of Service (QoS) guarantees to different applications is an important issue in the design of next generation of high-speed networks. The QoS metrics of interest are likely to vary from one application to another, but are predicted to include measures such as throughput, Bit Error Rate (BER), and delay. Unlike traditional data communication, where system performance is largely measured in terms of the average overall throughput and loss rate, real-time communications may require QoS metrics expressed in terms of the mean delay or its variance (jitter).

Traditional networking approaches design separately, the physical and the medium access layer (MAC). Instead, in future wireless networks the physical knowledge of the wireless medium is shared with higher layers on a cross-layer basis [1], an increasingly important topic for the evolving wireless build-out.

User multiplexing for QoS guarantees is an active research topic [2] in wireless systems, under different names such as, subcarrier and slot allocation, resource allocation or scheduling. Exploiting both the source diversity and the variations in channel conditions can increase the system throughput. A scheduling scheme ideally should be able not only to handle the uncertainty of the channel but also to exploit it, that is, opportunistically serve users with good channels. Using such an approach leads to a system capacity that increases with the number of users (multiuser diversity) [3].

Many questions regarding the performance of most used opportunistic algorithms are still open. For example, very few works consider the delay or study the treatment given to each user [4, 5]. The main difficulty in obtaining analytical results comes from the fact that the classical queueing theory is no longer suitable. Moreover, the result is linked to the scheduling discipline and the analysis has to be done algorithm by algorithm. To the best of our knowledge, the papers with analytical results found in the literature either use simple channel models [6, 7] or only provide bounds on the QoS metrics [8, 9].

Within this context, we explore the limits on the information that can be transmitted over the wireless channel under the conditions stated by the MAC layer: an ensured level of QoS and a selected scheduling discipline. In particular, both the channel fading and the scheduling algorithm determine the maximum source rate to be transmitted under some statistical QoS guarantees.

In single user systems, ergodic capacity [10] is not a suitable information-theoretic measure for delay sensitive applications over fading channels. On the other hand, delay-limited capacity becomes zero for Rayleigh channels. In this case, its probabilistic version, the Capacity with

Probabilistic Delay Constraint, becomes useful [11]. Then, rather than ensuring a deterministic delay, a probabilistic delay constraint is defined as the pair (D^t, ε), where D^t is the target delay and ε is the probability of exceeding it.

On the other hand, in multiuser communications the capacity of the channel is no longer fully characterized by a single number. Instead, the capacity should be redefined to consider each user's data rate separately. Thus, in a system with U users, the capacity should take the form of a U dimensional vector representing rates allocated to the U users [12]. A capacity region is then defined as the set of all U dimensional rate vectors that are achievable in the channel. Furthermore, when a QoS constraint is imposed (in the form of a probabilistic delay constraint), the data rate attainable by each user will be closely linked to the scheduling strategy.

In this paper, we obtain a closed-form expression of the vector of users' data rates $\mathrm{R}^u_{D^t,\varepsilon}$ for several scheduling algorithms, representing the maximum constant rate that the uth user can transmit under the selected discipline and fulfilling a target BER and the delay constraint given by the pair (D^t, ε). The total system capacity will be the sum of the individual users' rates, where each user can have a different delay constraint and can experience a different channel. The procedure to obtain these rates, based on the effective bandwidth theory [13], is similar to some previous results in a single-user system [11]. Three simple and widely employed disciplines have been analyzed: Round Robin [14], Best Channel [3], and Proportional Fair [15]. For simplicity, the results are obtained for a CBR (Constant Bit Rate) data source but they can be extended to any other traffic model as explained in [16].

The remainder of the paper is organized as follows. Section 2 describes the multiuser system model. Section 3 first details the derivation of the maximum users' rates subject to a delay constraint for an uncorrelated Rayleigh channel (Section 3.1). Later on, the expressions are particularized to Round Robin, Best Channel, and Proportional Fair disciplines in Sections 3.2, 3.3, and 3.4, respectively. The time-correlated channel is examined in Section 4, first of all the achievable rates (Section 4.1) and then the same three particularizations for the three examples of discipline (Sections 4.2, 4.3, and 4.4). The validation of the results by comparison with simulations is presented in Section 5. Finally, concluding remarks are given in Section 6.

2. Multiuser System Model

2.1. Queueing Model. Figure 1 illustrates the system model considered in this paper. The channel is shared among U users, whose incoming traffics are characterized by U source processes, respectively. Each user has its own queue where the data are stored before being transmitted. The server represents the information transmitted to the shared channel, which is decided at each instant by the scheduler. The instantaneous response of the wireless channel is, in general, a time-variant and autocorrelated random process.

Physical time is divided into units, hereinafter referred to as symbol periods, which represent the transmission discrete time unit, n. The channel response of each user is assumed to be constant over the symbol. Moreover, the scheduler allocates the channel to users in a symbol per symbol basis: every new symbol, a user is selected for transmission.

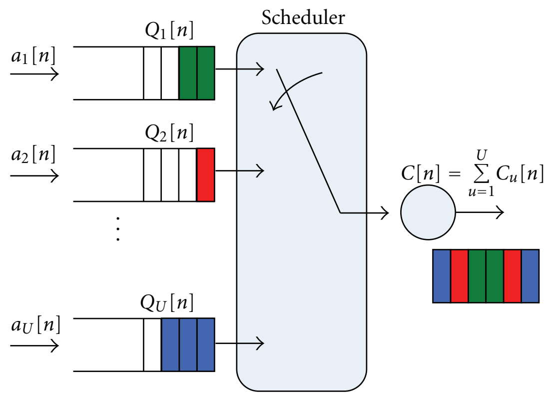

Figure 1: Multiuser system model.

It is assumed that the transmitter employs adaptive techniques, so that the transmission rate is modified dynamically, seeking to adapt to the time-varying conditions of the physical channel.

Each incoming user traffic has an instantaneous rate $a_u[n]$. On his side, the wireless channel transmits at an instantaneous rate $c[n]$, that is, every symbol n, $c[n]$ bits can be transmitted by the channel.

Each user has a potential rate $r_u[n]$, which represents the channel rate that he may use if the channel is assigned to him, and which depends on his channel conditions as explained in Section 2.2. Moreover, the instantaneous channel rate of user uth, $c_u[n]$, is given by

$$c_u[n] = \begin{cases} r_u[n] & \text{if channel is assigned to user } u, \\ 0 & \text{in other case.} \end{cases} \tag{1}$$

Since the channel is shared among U users, $c[n]$ can be expressed as

$$c[n] = \sum_{u=1}^{U} c_u[n]. \tag{2}$$

Notice that in the sum above only one of the terms is nonzero, corresponding to the user allocated to the channel.

The effective bandwidth analysis of the queueing system is done by means of the processes $a_u[n]$ and $c_u[n]$. The processes $a_u[n]$ and $c_u[n]$ are not necessarily white and represent the amount of bits per symbol generated by user u and the amount of bits per symbol of the uth user transmitted by the server, respectively. In addition, the accumulated source rate $A_u[n]$ is the amount of bits generated by user u from 0 to instant $n-1$:

$$A_u[n] = \sum_{m=0}^{n-1} a_u[m]. \tag{3}$$

And similarly the accumulated channel process of uth user is

$$C_u[n] = \sum_{m=0}^{n-1} c_u[m]. \tag{4}$$

The queue size is assumed to be infinite and $Q_u[n]$ denotes the length of the queue at time n. The dynamics of the queueing system seen by user u is characterized by the equation $Q_u[n] = (Q_u[n-1] + a_u[n] - c_u[n])^+$, with $(x)^+ \triangleq \max(0, x)$.

2.2. Channel Model. Every user experiences a flat Rayleigh channel with complex channel response $h_u[n]$. The envelope of the channel response is denoted by $z_u[n] = |h_u[n]|$. Furthermore, users are independent among them, that is, the channel response seen by one user is independent from the rest.

Let us define $\gamma_u[n]$ as the instantaneous Signal-to-Noise Ratio of user u at the receiver. With Additive White Gaussian Noise (AWGN), $\gamma_u[n]$ is exponentially distributed:

$$f(\gamma_u) = \frac{1}{\overline{\gamma}_u} e^{-\gamma_u/\overline{\gamma}_u}, \tag{5}$$

where $\overline{\gamma}_u$ is the average Signal-to-Noise Ratio of user u.

Moreover, $\gamma_u[n]$ is proportional to the square of $z_u[n]$:

$$\gamma_u[n] = z_u[n]^2 \frac{E_s}{N_0}, \tag{6}$$

where E_s is the average energy per symbol and N_0 is the noise power spectral density.

We consider constant transmitted power and a continuous rate policy. Then, the potential channel rate of user u, $r_u[n]$, is a function of $\gamma_u[n]$:

$$r_u[n] = \log_2(1 + \beta_u \gamma_u[n]), \tag{7}$$

where β_u, under adaptive modulation, is a constant related to the target BER. Its value for uncoded QAM is [17] $\beta_u \approx (1.6/ - \log(5\text{BER}^t))$, where BER^t is the target BER. Further, the value $\beta_u = 1$ represents the upper bound corresponding to the evaluation of the AWGN channel capacity (in Shannon's sense).

The variability of the channel over time is usually reflected through its autocorrelation function (ACF). This second-order statistic generally depends on the propagation geometry, the velocity of the mobile, and the antenna characteristics [18]. The autocorrelation function of the envelope of the channel response is denoted by $\mathcal{R}_z(m)$. In Section 3, the channel response of each user is assumed to be uncorrelated, that is, $\mathcal{R}_z(m)$ is zero except for $m = 0$. Later, in Section 4, the time-correlation is considered. In particular, a very simple model of correlation is employed: the ACF is assumed to decay exponentially with a parameter ρ, $0 < \rho < 1$:

$$\mathcal{R}_z(m) = \rho^m. \tag{8}$$

The use of the exponential model simplifies and speeds up the simulations and numerical evaluations along this paper without altering the conclusions. Nevertheless, any other correlation function can be used (i.e., the classical Jakes' model [18]).

2.3. Effective Bandwidth Analysis. The asymptotic log-moment generating function of $Q_u[n]$ is defined as [13]

$$\Lambda_u(v) = \lim_{n\to\infty} \frac{1}{n} \log \mathrm{E}\left[e^{vQ_u[n]}\right]. \tag{9}$$

Since $a_u[n]$ and $c_u[n]$ are independent of each other, $\Lambda_u(v)$ may be decomposed into two terms, $\Lambda_u(v) = \Lambda_{A_u}(v) + \Lambda_{C_u}(-v)$, where $\Lambda_{A_u}(v)$ and $\Lambda_{C_u}(v)$ are the log-moment generating functions of the accumulated source process and the accumulated channel process of user u, respectively.

If the source and channel processes are stationary and the steady state queue length exists, then the workload process $Q_u[n]$ satisfies a Large Deviation Principle and the following steady state solution for the queue length exceeding B_u is satisfied [13]:

$$\Pr\{Q_u(\infty) > B_u\} \asymp e^{-\theta B_u}, \quad B_u \longrightarrow \infty, \tag{10}$$

where $f(x) \asymp g(x)$ means that $\lim_{x\to\infty} f(x)/g(x) = 1$ and θ, known as the QoS exponent, is the solution to $\Lambda_{A_u}(v) + \Lambda_{C_u}(-v)|_{v=\theta} = 0$

Defining the effective bandwidth function (EBF) $\alpha_u(v) = \Lambda_u(v)/v$, the equation to obtain θ can be expressed as

$$\alpha_u(v) = \alpha_{A_u}(v) - \alpha_{C_u}(-v)\,|_{v=\theta} = 0, \tag{11}$$

where $\alpha_{A_u}(v)$ and $\alpha_{C_u}(v)$ are the effective bandwidth functions of the source process and the channel process for the uth user, respectively.

Let η_u be the probability that the queue of user u is not empty, $\eta_u = \Pr\{Q_u[n] > 0\}$. By the inclusion of this term in the analysis, the following less conservative approximation for the tail probability of the queue is satisfied:

$$\Pr\{Q_u(\infty) > B_u\} \approx \eta_u \cdot e^{-\theta B_u}. \tag{12}$$

The delay of the bits leaving the queue of user u at symbol n is denoted as $D_u[n]$. For simplicity, assume that the source traffic from the uth user arrives to the buffer at a constant rate:

$$a_u[n] = \lambda_u. \tag{13}$$

It leads to a constant EBF for the following source process [19]:

$$\alpha_{A_u}(v) = \lambda_u. \tag{14}$$

The procedure to generalize the results to other traffic sources can be found in [16].

As in the queue length process, the steady state solution for the delay process exists. In addition, for constant sources the delay at the queue of user u can be calculated:

$$D_u[n] = \frac{Q_u[n]}{\lambda_u[n]}. \tag{15}$$

Thus, the probability of exceeding D^t, denoted throughout this paper as target delay, can be written as follows [20]:

$$\varepsilon = \Pr\{D_u(\infty) > D^t\} \approx \eta_u \cdot e^{-\theta \cdot \lambda_u D^t}, \tag{16}$$

where the probability of exceeding the target delay D^t is denoted by ε.

3. Uncorrelated Channel

We start the analysis of the multiuser system presented above with the case of users experiencing an uncorrelated Rayleigh channel (block fading model). Part of these results can be found in [21].

3.1. Achievable Users' Rates with a Delay Constraint. Starting from (16), the set of individual users' rates that accomplish a delay constraint can be calculated. Each user has his own delay constraint. The effective bandwidth of the channel is needed, $\alpha_{C_u}(v)$.

With no time-correlation among samples, the accumulated transmission rate for the uth user, $C_u[n]$, is simply the addition of n uncorrelated and identically distributed random variables. As $n \to \infty$, the Central Limit Theorem can be applied and $C_u[n]$ can be considered a Gaussian random variable with average $n \cdot m_u$ and variance $n \cdot \sigma_u^2$, where m_u and σ_u^2 are the mean and the variance of $c_u[n]$, the instantaneous channel rate for the uth user. Then, the effective bandwidth function for the resulting Gaussian distribution of $C_u[n]$ is computed as [19]

$$\alpha_{C_u}(v) = \lim_{n\to\infty} \frac{1}{n \cdot u} \log \mathrm{E}\left[e^{vC_u[n]}\right] = m_u + \frac{u}{2}\sigma_u^2. \tag{17}$$

In a high load scenario, the probability that the buffer is not empty approaches one, that is, $\eta_u \to 1$. Under this assumption, the delay constraint is worked out from (16):

$$-\frac{\log(\varepsilon)}{D^t} = \theta \cdot \lambda_u \cdot \tag{18}$$

Assume that the uncorrelated channel has parameters m_u and σ_u^2, then the QoS exponent is obtained by solving (11):

$$\begin{aligned} \lambda_u - \alpha_{C_u}(-\theta) = 0 \Longrightarrow \theta(\lambda_u) &\triangleq \theta(m_u, \sigma_u^2, \lambda_u) \\ &= \frac{2(m_u - \lambda_u)}{\sigma_u^2}. \end{aligned} \tag{19}$$

With (19) substituted into (18), the value of λ_u is worked out and it is the achievable user rate that we were seeking. It represents the maximum source rate that may be supported for user u with a probability ε of exceeding a delay bound D^t. We denote it by $\mathrm{R}^u_{D^t,\varepsilon}$:

$$\mathrm{R}^u_{D^t,\varepsilon} = \frac{m_u}{2} + \frac{1}{2}\sqrt{m_u^2 - 2\sigma_u^2 \frac{(-\log \varepsilon)}{D^t}}. \tag{20}$$

The delay constraint formed by the pair (D^t, ε) can be different for each user. Nevertheless, we maintain the notation above for the sake of simplicity. Equation (20) shows explicitly the tradeoff between user source rate ($\mathrm{R}^u_{D^t,\varepsilon}$) and delay requirements (D^t, ε). It can be observed that for high D^t values or $\varepsilon \to 1$, the QoS requirement relaxes and $\mathrm{R}^u_{D^t,\varepsilon}$ approaches m_u. On the other hand, as the target delay D^t or ε become lower, the user has to transmit at a lower rate in order to guarantee its own delay constraint. Moreover, the influence of the scheduling algorithm and the channel conditions (SNR, target BER) are captured in the mean and the variance m_u and σ_u^2. Thus, the evaluation of the user rates comes down to obtaining the mean and the variance of the channel process seen by user u:

$$\begin{aligned} m_u &= E[c_u[n]], \\ \sigma_u^2 &= E[c_u^2[n]] - m_u^2. \end{aligned} \tag{21}$$

These statistics depend on the distribution of $c_u[n]$ which in turn depends on the scheduling algorithm. Three scheduling disciplines will be detailed in next sections.

Finally, the total system capacity $\mathrm{C}_{\mathbf{D}^t,\ \boldsymbol{\epsilon}}$ is obtained as the sum of the individual user rates, each of them with its own delay constraint:

$$\mathrm{C}_{\mathbf{D}^t,\ \boldsymbol{\epsilon}} = \sum_{u=1}^{U} \mathrm{R}^u_{D^t,\varepsilon}, \tag{22}$$

with $\mathbf{D}^t$ and $\boldsymbol{\epsilon}$ being the vectors with the target delays and probabilities of violation of each user, respectively.

3.2. Round Robin. First of all, the mean and the variance to compute $\mathrm{R}^u_{D^t,\varepsilon}$ under a Round Robin strategy are calculated.

Round Robin (RR) [14] is a fixed cyclic algorithm without priorities, which dispenses the channel equally among the different flows independently of their priorities or radio channel conditions. Transmission at symbol n is assigned to the following user in a cyclic order and therefore,

$$c_u[n] = \begin{cases} \log_2(1 + \beta_u \gamma_u[n]) & \text{if } \bmod (n, U) = u, \\ 0 \text{ in other case.} \end{cases} \tag{23}$$

It is known that this strategy does not work well over varying channels and a low efficiency in terms of system capacity and QoS differentiation is expected.

The mean and the variance of $c_u[n]$ are required. With only one user ($U = 1$) and continuous rate policy, the mean $c_u[n]$ matches up with the ergodic capacity of the channel. We denote it m_1 for convenience

$$m_1 = E\left[\log_2(1 + \beta\gamma)\right] = \log_2(e) \exp\left(\frac{1}{\beta\overline{\gamma}}\right) E_1\left(\frac{1}{\beta\overline{\gamma}}\right), \tag{24}$$

where $E_1(x)$ is the exponential integral and $\overline{\gamma}$ is the average Signal-to-Noise Ratio of the single user.

Likewise, the expression of the variance σ_1^2 with only one user is [11]

$$\begin{aligned}\sigma_1^2 &= \mathrm{E}\left[\left(\log_2(1+\beta\gamma)\right)^2\right] - m_1^2 \\ &= \left(\log_2(e)\right)^2 e^{1/(\beta\bar{\gamma})} \\ &\quad\times\left[\frac{\pi^2}{6} + g^2 + 2g\log\left(\frac{1}{\beta\bar{\gamma}}\right) + \log^2\left(\frac{1}{\beta\bar{\gamma}}\right)\right. \\ &\quad\quad - 2\left(\frac{1}{\beta\bar{\gamma}}\right){}_3\mathbf{F}_3\left([1,1,1],[2,2,2],-\frac{1}{\beta\bar{\gamma}}\right) \\ &\quad\quad \left. - e^{1/(\beta\bar{\gamma})}\mathrm{E}_1^2\left(\frac{1}{\beta\bar{\gamma}}\right)\right],\end{aligned} \tag{25}$$

where g is the Euler constant and ${}_p\mathrm{F}_q(\mathbf{n},\mathbf{d},z)$ is the hypergeometric function.

When U users share the channel under an RR discipline, we only need to take into account that the channel is equally divided among users. Thus, the expressions of m_u and σ_u^2 are written directly as a function of m_1 and σ_1^2, by just replacing with the average Signal-to-Noise Ratio of each user and dividing by the number of users:

$$\begin{aligned} m_u &= E\left[\log_2(1+\beta_u\gamma_u)\right] = \frac{m_1}{U}, \\ \sigma_u^2 &= \mathrm{E}\left[\left(\log_2(1+\beta_u\gamma_u)\right)^2\right] - m_u^2 = \frac{\sigma_1^2}{U^2}. \end{aligned} \tag{26}$$

The expressions above make it possible to evaluate the individual users' rates in (20) under a Round Robin discipline.

An example is shown in Figure 2. Three users have been considered, with average SNR 5, 7 and 12 dB, respectively. The individual rates $\mathrm{R}^u_{D^t,\varepsilon}$ are plot as a function of the target delay D^t. The other parameter, the violation probability ε, has been set to 0.1 for all users. The parameter β_u is set to 1(Hereinafter β_u is set to 1 in all the numerical evaluations.). The mean m_u of each user is represented with solid line, whereas the dashed line is $\mathrm{R}^u_{D^t,\varepsilon}$. Both m_u and $\mathrm{R}^u_{D^t,\varepsilon}$ are plotted for each user (users marked with triangles, squares and circles). Moreover, the system capacity normalized with the number of users, which corresponds to the "average" rate, is represented with no marks and thicker line.

First of all, let us observe the common behaviour of $\mathrm{R}^u_{D^t,\varepsilon}$ for all the user. As presumed, the curve increases with D^t (the more relaxed the QoS requirements, the higher the maximum attainable rate). Obviously, $\mathrm{R}^u_{D^t,\varepsilon}$ is always below m_u. Moreover, values of the user rate equal to zero must be interpreted as QoS requirements that cannot be fulfilled with that channel conditions, number of users and discipline.

Observing the differences among users, those with better channel conditions obtain higher rates and can demand stringent QoS conditions, as it was expected. Thus, the best user in this example could fix a delay constraint with a target delay of 3 symbols in contrast to the 5 symbols of the worst user. On the other hand, if the same target delay is fixed for the three users the rate to be employed increases for *better* users (users with better channel conditions). For example, for 10 symbols user 1 can transmit 0.5 bits/symbol, user 2, 0.7 bits/symbol, and user 3, 1.1 bits/symbol.

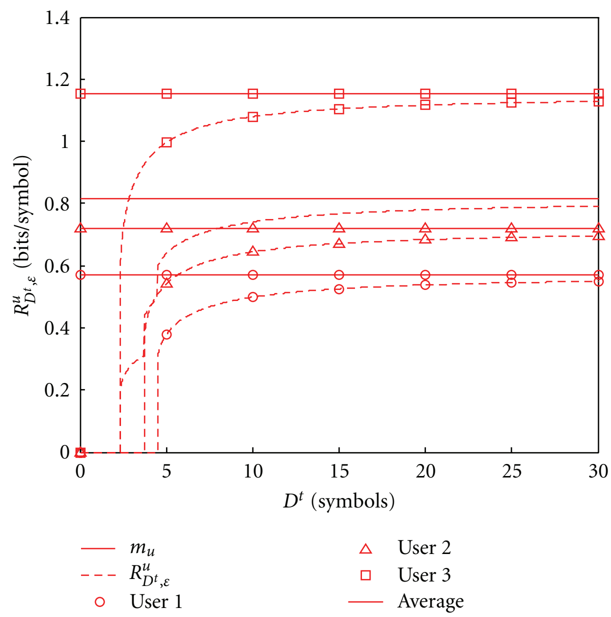

Figure 2: Achievable users' rates with Round Robin scheduling in an uncorrelated channel. Three users with $\bar{\gamma}_u = 5, 7, 12$ dB, respectively. $\varepsilon = 0.1$. $\beta_u = 1$.

3.3. Best Channel. Best Channel (BC) [3] strategy is adaptive to the channel state, giving priority to those users with higher potential transmission rate. The channel is assigned to the user that may transmit with the highest number of bits per symbol

$$c_u[n] = \begin{cases} \log_2(1+\beta\gamma_u[n]) & \text{if } \gamma_u[n] > \gamma_k[n] \quad \forall k \neq u, \\ 0 & \text{in other case.} \end{cases} \tag{27}$$

This algorithm maximizes the total system efficiency. However, under this strategy, good average SNR users get more average throughput than low SNR users.

Let us define $\gamma_{\max}$

$$\gamma_{\max} = \max_u\{\gamma_u\}. \tag{28}$$

The CDF of $\gamma_{\max}$ can be written as

$$F_{\gamma_{\max}}(\gamma) = \Pr(\gamma_{\max} < \gamma) = \Pr(\gamma_1 < \gamma, \gamma_2 < \gamma, \dots, \gamma_U < \gamma). \tag{29}$$

Since the users are i.i.d., it comes down to

$$F_{\gamma_{\max}}(\gamma) = \prod_{u=1}^{U}\left(1 - \exp\left(-\frac{\gamma}{\bar{\gamma}_u}\right)\right). \tag{30}$$

Consider the following effective SNR for the uth user [22]:

$$\gamma_u^* = \begin{cases} \gamma_u, & \gamma_u > \gamma_{-u}, \\ 0, & \gamma_u < \gamma_{-u}, \end{cases} \tag{31}$$

where $\gamma_{-u} = \max_{k \neq u}\{\gamma_k\}$ is the maximum of the average SNR of all the users except u.

The pdf of the effective SNR γ_u^* can be expressed as follows [22]:

$$f_u^*(\gamma_u^*) = \text{Prob}\{\gamma_u < \gamma_{-u}\}\delta(\gamma_u^*) + f_u(\gamma_u^*)F_{-u}(\gamma_u^*), \tag{32}$$

where $\delta(x)$ is the Dirac delta function, $f_u(x)$ is the exponential pdf in (5), and $F_{-u}(x)$ is the CDF of γ_{-u}:

$$\begin{aligned} F_{-u}(x) &= \prod_{k \neq u}^{U} \left[1 - \exp\left(-\frac{x}{\overline{\gamma}_k}\right)\right] \\ &= \sum_{\mathbf{i} \in \mathfrak{U}} (-1)^{\mathbf{i} \cdot \mathbf{1}} (1 - i_u) \exp(-x\mathbf{b} \cdot \mathbf{i}) \end{aligned} \tag{33}$$

with $\mathbf{b} = [(1/\overline{\gamma}_1)(1/\overline{\gamma}_2) \cdots (1/\overline{\gamma}_U)]$, $\mathbf{1}$ denotes the all-ones U-dimensional vector, $\mathfrak{U}$ is the set of all U-dimensional vectors with entries taking values 0 of 1, that is, $\mathfrak{U}$ contains the 2^U binary words of length U, and i_u is the uth component of $\mathbf{i}$.

The mean m_u to be computed for BC is

$$m_u = E[c(\gamma_u)] = \int_0^\infty c(\gamma_u^*) f_u^*(\gamma_u^*) d\gamma_u^*. \tag{34}$$

From (32) it can be observed that the first addend will be zero in the required expectation and only the second term needs to be integrated:

$$f_u(\gamma_u^*)F_{-u}(\gamma_u^*) = -\sum_{\mathbf{i} \in \mathfrak{U}} (-1)^{\mathbf{i} \cdot \mathbf{1}} \frac{i_u}{\overline{\gamma}_u} \exp(-x\mathbf{b} \cdot \mathbf{i}). \tag{35}$$

Substituting into the mean, it yields

$$m_u = -\int_0^\infty \log_2(1 + \beta_u \gamma_u^*) \sum_{\mathbf{i} \in \mathfrak{U}} (-1)^{\mathbf{i} \cdot \mathbf{1}} \frac{i_u}{\overline{\gamma}_u} \exp(-\gamma_u^* \mathbf{b} \cdot \mathbf{i}) d\gamma_u^*. \tag{36}$$

This integral is analogous to the single user case by simply defining $1/\overline{\gamma} = \mathbf{b} \cdot \mathbf{i}$. The result is then

$$m_u = -\sum_{\mathbf{i} \in \mathfrak{U}} (-1)^{\mathbf{i} \cdot \mathbf{1}} \frac{i_u}{\overline{\gamma}_u \mathbf{b} \cdot \mathbf{i}} \log_2(e) \exp\left(\frac{\mathbf{b} \cdot \mathbf{i}}{\beta_u}\right) E_1\left(\frac{\mathbf{b} \cdot \mathbf{i}}{\beta_u}\right). \tag{37}$$

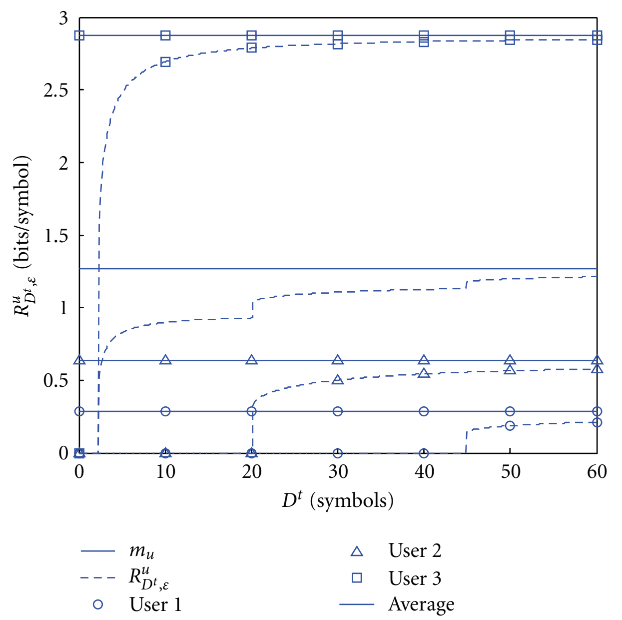

FIGURE 3: Achievable users' rates with Best Channel scheduling in an uncorrelated channel. Three users with $\overline{\gamma}_u = 5, 7, 12$ dB, respectively. $\varepsilon = 0.1$. $\beta_u = 1$.

Likewise, the calculation of the variance is similar to the single user case, obtaining.

$$\begin{aligned} \sigma_u^2 &= E\left[\log_2^2(1 + \beta_u \gamma)\right] - m_u^2 \\ &= -\sum_{\mathbf{i} \in S} (-1)^{\mathbf{i} \cdot \mathbf{1}} \frac{i_s}{\overline{\gamma}_u \mathbf{b} \cdot \mathbf{i}} \left(\log_2(e)\right)^2 e^{(1/\beta_u)\mathbf{b} \cdot \mathbf{i}} \\ &\quad \times \left[\frac{\pi^2}{6} + g^2 + 2g \ln\left(\frac{1}{\beta_u}\mathbf{b} \cdot i\right) + \ln^2\left(\frac{1}{\beta_u}\mathbf{b} \cdot \mathbf{i}\right) \right. \\ &\quad \left. -2\left(\frac{1}{\beta_u}\mathbf{b} \cdot \mathbf{i}\right) {}_3\mathbf{F}_3\left([1, 1, 1], [2, 2, 2], -\frac{1}{\beta_u}\mathbf{b} \cdot \mathbf{i}\right)\right]. \end{aligned} \tag{38}$$

The same evaluation example presented for RR is shown in Figure 3, now for BC allocation.

The differences among users are much more noticeable than for RR. Thus, the best user is better off with the change to BC allocation at the expenses of users with lower average SNR. Notice that not only the differences in the mean m_u are remarkable (the asymptotic behaviour when relaxing the QoS constraint) but also the minimum target delays of each user move away. For example, the worst user cannot demand a target delay below 45 symbols for these channel conditions and scheduling, in contrast to the 2 symbols of the best user. As expected, the average rate is higher than for RR, since this algorithm maximizes the total system efficiency.

It is wellknown that by exploiting the multiuser diversity one can achieve higher system capacity as the number of users increases. This multiuser diversity gain is illustrated in Figure 4. Lognormal shadowing is considered, so that the average SNR of users follows a lognormal distribution, with

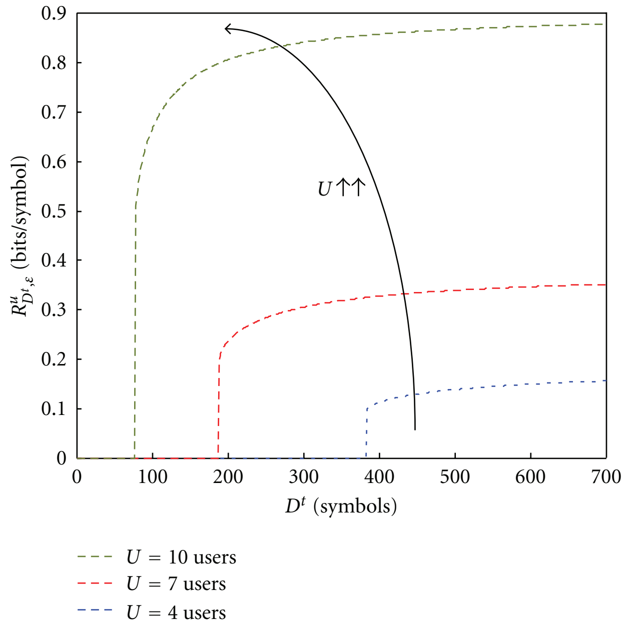

Figure 4: Multiuser diversity for BC and uncorrelated channel: maximum achievable rate of the median user. Average SNR following lognormal shadowing with mean 10 dB and standard deviation 4 dB. $\varepsilon = 0.1$. $\beta_u = 1$.

average 10 dB and standard deviation 4 dB. The violation probability is 0.1 for all users. The maximum achievable rate of the median user is plot, for 4, 7, and 10 users. It can be observed that as the number of users increases, the maximum achievable rate of the median user increases, due to multiuser diversity.

3.4. Proportional Fair. Proportional Fair (PF) [15] is a compromise-based scheduling algorithm. It is intended to improve Best Channel by maintaining a balance between two competing interests: maximizing the total wireless network throughput while allowing a minimum level of service to all users. Fair sharing will lower the total throughput over the maximum possible, but it will provide more acceptable levels to users with poorer SNR. Instead of using the instantaneous potential transmission rate of BC, PF uses as metrics the ratio $\gamma_u[n]/\overline{\gamma}_u$:

$$c_u[n] = \begin{cases} \log_2\left(1+\beta\gamma_u[n]\right) & \text{if } \gamma_u[n]/\overline{\gamma}_u > \gamma_k[n]/\overline{\gamma}_k \quad \forall k \neq u, \\ 0, & \text{in other case.} \end{cases} \tag{39}$$

Therefore, we just need to do a change of variable in the previous results for BC. Let us define $\Gamma_{\max}$:

$$\Gamma_{\max} = \max_u \left\{ \frac{\gamma_u}{\overline{\gamma}_u} \right\}. \tag{40}$$

Now the effective SNR for the uth user is

$$\Gamma_u^* = \begin{cases} \gamma_u, & \dfrac{\gamma_u}{\overline{\gamma}_u} > \dfrac{\gamma_{-u}}{\overline{\gamma}_{-u}}, \\ 0, & \dfrac{\gamma_u}{\overline{\gamma}_u} < \dfrac{\gamma_{-u}}{\overline{\gamma}_{-u}}. \end{cases} \tag{41}$$

And the second term of the pdf of Γ_u^* is expressed:

$$f_u(x)F_{-u}\left(x\overline{\gamma}_u\right) = -\overline{\gamma}_u \cdot \sum_{\mathbf{i}\in\mathfrak{U}} (-1)^{\mathbf{i}\cdot\mathbf{1}} \frac{i_u}{\overline{\gamma}_u} \exp\left(-x\overline{\gamma}_u \mathbf{1}\right) \tag{42}$$

with $\mathbf{1}$ the all-ones U-dimensional vector.

The result for the mean m_u is analogous to that of BC:

$$m_u = -\sum_{\mathbf{i}\in\mathfrak{U}} (-1)^{\mathbf{i}\cdot\mathbf{1}} \frac{i_u}{\mathbf{1}} \log_2(e) \exp\left(\mathbf{1}\frac{\overline{\gamma}_u}{\beta}\right) E_1\left(\mathbf{1}\frac{\overline{\gamma}_u}{\beta}\right). \tag{43}$$

Likewise, the calculation of the variance yields

$$\begin{aligned} \sigma_u^2 &= E\left[\log_2^2\left(1+\beta_u\gamma\right)\right] - m_u^2 \\ &= -\sum_{\mathbf{i}\in S} (-1)^{\mathbf{i}\cdot\mathbf{1}} \frac{i_s}{\mathbf{q}\cdot\mathbf{i}} \left(\log_2(e)\right)^2 e^{(1/\beta_u)\mathbf{q}\cdot\mathbf{i}} \\ &\quad \times \left[\frac{\pi^2}{6} + g^2 + 2g\ln\left(\frac{1}{\beta_u}\mathbf{q}\cdot\mathbf{i}\right) + \ln^2\left(\frac{1}{\beta_u}\mathbf{q}\cdot\mathbf{i}\right)\right. \\ &\quad \left. -2\left(\frac{1}{\beta_u}\mathbf{q}\cdot\mathbf{i}\right) F\left([1,1,1],[2,2,2],-\frac{1}{\beta_u}\mathbf{q}\cdot\mathbf{i}\right)\right]. \end{aligned} \tag{44}$$

In Figure 5, the maximum achievable users' rates are evaluated under the same conditions as it was done for RR and BC. The three users have average SNR 5, 7, and 12 dB and the violation probability is set to 0.1.

It can be observed that the differences among users reduce if we compare with the BC strategy. That is exactly the goal of this discipline: to maintain a balance between the total throughput and the level of service of all users. Obviously, the average rate reduces to increase the fairness. The achieved fairness is specially noticeable in the behaviour of the target delay, which is 10 symbols for the three users.

4. Correlated Channel

In this section, a time-correlated channel is considered, meaning that the channel response of each user follows the exponential ACF described in (8).

4.1. Achievable Users' Rates with a Delay Constraint. To face the new problem, we split the accumulated transmission rate for the uth user, $C_u[n]$, into b blocks of length k symbols:

$$C_u[n] = \sum_{i=0}^{b-1} C_{u_i}[k] = \sum_{i=0}^{b-1}\sum_{m=0}^{k-1} c_u[k\cdot i + m]. \tag{45}$$

The channel correlation among the elements in the block is considered but, with the proper selection of the block's

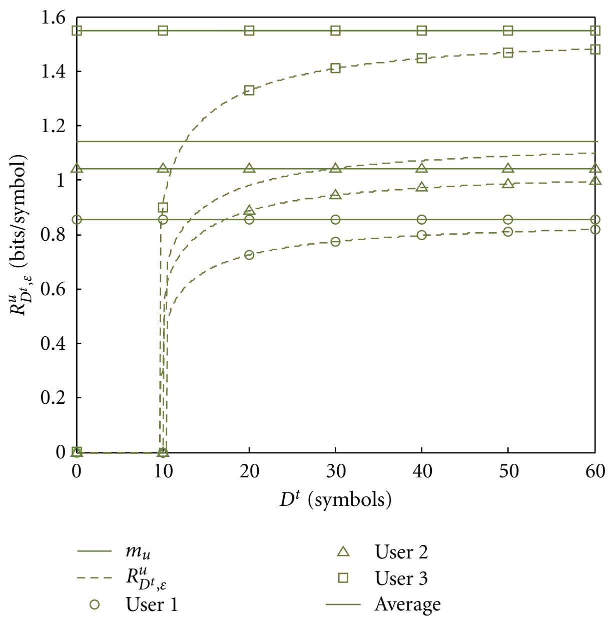

Figure 5: Achievable users' rates with Proportional Fair scheduling in an uncorrelated channel. Three users with $\overline{\gamma}_u = 5, 7, 12$ dB, respectively. $\varepsilon = 0.1$. $\beta_u = 1$.

length, k large enough, independence among blocks may be assumed. The choice of k will be closely related to the correlation of the channel. If the channel is strongly correlated, longer blocks have to be defined in order to assume independent blocks. Whatever the value of k is, there is a residual value of correlation between the last elements of one block and the first elements of next one. Nevertheless, this *border* correlation is negligible when the value of k is large enough. Notice that a decreasing autocorrelation function is required, as it is the case in fading channels.

Under these conditions (sufficiently long k and n), $C_u[n]$ is the sum of a sufficiently large number of independent random variables, and the Central Limit Theorem can be applied. Thus, $C_u[n]$ approximates a Gaussian random variable with average $b \cdot m_{k_u}$ and variance $b \cdot \sigma^2_{k_u}$, where m_{k_u} and $\sigma^2_{k_u}$ are the mean and the variance of a block of size k of the uth user.

On the other hand, the Lilliefors test [23] is used in statistics to test whether an observed sample distribution is consistent with normality. In the numerical results and simulations conducted throughout this paper, the validity of the Gaussian approximation for $C_u[n]$ has been validated by testing for normality with the Lilliefors test for the selected values of k and n.

The effective bandwidth function of the Gaussian distribution of $C_u[n]$ yields

$$\alpha_{C_u}(v) = \lim_{n\to\infty} \frac{1}{n \cdot v} \log \mathrm{E}\left[e^{vC_u[n]}\right] = \frac{m_{k_u}}{k} + \frac{v}{2}\frac{\sigma^2_{k_u}}{k}. \tag{46}$$

And the achievable rate of uth user is

$$R^u_{D^t,\varepsilon} = \frac{m_{k_u}}{2k} + \frac{1}{2}\sqrt{\frac{m^2_{k_u}}{k^2} - 2\frac{\sigma^2_{k_u}}{k}\frac{(-\log\varepsilon)}{D^t}}. \tag{47}$$

Let us examine first the single user system, since this result will be needed later. With only one user ($U = 1$) and continuous rate policy, the mean and variance of the blocks, denoted as m_{k_1} and $\sigma^2_{k_1}$, are calculated as follows.

In the case of the mean, it is straightforward that

$$m_{k_1} = k \cdot m_1 \tag{48}$$

For the variance, it can be written in terms of the autocovariance $\mathcal{K}_c(m)$ [24]:

$$\begin{aligned}\sigma^2_{k_1} &= \sum_{q=0}^{k-1}\sum_{r=0}^{k-1}\mathcal{K}_c(r-q),\\ \mathcal{K}_c(m) &= E[c[n]c[n+m]] - m_c^2.\end{aligned} \tag{49}$$

The bivariate probability density function for Rayleigh distributed variables is needed. It can be expressed as follows in terms of the instantaneous SNR [25]:

$$\begin{aligned}f_\gamma(\gamma_n, \gamma_{n+m}) = {}& \frac{1}{(1-\mathcal{R}_z^2(m))\overline{\gamma}^2}\exp\left(-\frac{(\gamma_n+\gamma_{n+m})}{(1-\mathcal{R}_z^2(m))\overline{\gamma}}\right)\\ &\cdot I_0\left(\frac{2\mathcal{R}_z(m)\sqrt{\gamma_n\gamma_{n+m}}}{(1-\mathcal{R}_z^2(m))\overline{\gamma}}\right),\end{aligned} \tag{50}$$

where $I_0(u)$ is the modified Bessel function of the first kind and $\mathcal{R}_z(m)$ is the value of the ACF of the envelope $z[n]$ for a time lag m. The expectation to be evaluated is

$$\begin{aligned}&\mathrm{E}[c[n]c[n+m]]\\ &\quad= \mathrm{E}[c(\gamma_n)c(\gamma_{n+m})]\\ &\quad= \int_{\gamma_n=0}^{\infty}\int_{\gamma_{n+m}=0}^{\infty} c(\gamma_n)c(\gamma_{n+m})f_\gamma(\gamma_n,\gamma_{n+m})\,d\gamma_n d\gamma_{n+m}.\end{aligned} \tag{51}$$

After some manipulations the autocovariance yields

$$\mathcal{K}_c(m) = \frac{b}{\overline{\gamma}}\sum_{p=0}^{\infty}\left(I_p(\beta, \mathcal{R}_z(m), b)\right)^2 - m_c^2, \tag{52}$$

where $b = 1/((1-\mathcal{R}_z^2(m))\cdot\overline{\gamma})$ and the integral $I_p(\beta, \mathcal{R}_z(m), b)$ has the following form:

$$\begin{aligned}&I_p(\beta, \mathcal{R}_z(m), b)\\ &\quad= \frac{\mathcal{R}_z^p(m)}{p\cdot\log(2)}\\ &\qquad\cdot\left(-\frac{p}{b}(-\psi(1+p)+\log(b)) + {}_2\mathbf{F}_2([1,1],[2,1-p],b)\right)\end{aligned} \tag{53}$$

with $\psi(x) = (d(\log(\Gamma(x))))/dx$ the digamma function.

4.2. Round Robin. When U users share the channel under an RR discipline, the channel is equally divided among users. Like in the uncorrelated channel, the expressions of m_{k_u} and

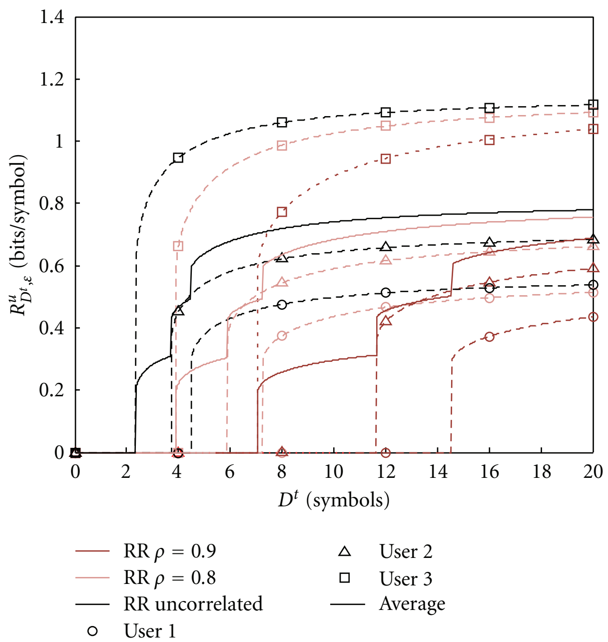

Figure 6: Achievable users' rates with Round Robin scheduling in a time-correlated channel with exponential ACF of parameter ρ. Three users with $\overline{\gamma}_u = 5, 7, 12$ dB, respectively. $\varepsilon = 0.1$. $\beta_u = 1$.

$\sigma^2_{k_u}$ are written directly as a function of m_{k_1} and $\sigma^2_{k_1}$, by just replacing $\overline{\gamma}$ with $\overline{\gamma}_u$, β with β_u and dividing by the number of users:

$$m_{k_u} = \frac{m_{k_1}}{U}, \qquad \sigma_{k_u} = \frac{1}{U}\sigma_{k_1}\left(\overline{\gamma}_u, \beta_u\right). \tag{54}$$

The evaluation of RR in a correlated channel is presented in Figure 6. There are three users with average SNR 5, 7, and 12 dB, the parameter ε is set to 0.1, and β_u is 1 for the three users. The correlation follows an exponential decay and two values of ρ are evaluated, 0.8 and 0.9. The previous result of the uncorrelated channel is also shown with black line. The qualitative behaviour is the same as in the uncorrelated channel. The influence of the parameter ρ is remarkable: as expected, the correlation is harmful to the delay performance, so that when the correlation increases (ρ increases), the achievable users' rates decrease. Moreover, it is observed that the differences among users increase with the time-correlation.

4.3. Best Channel. Similarly as done in RR, the calculation of the maximum attainable rates in the case of Best Channel strategy leads to the computation of the variance of the blocks (the evaluation of the mean of the blocks is straightforward), which comes down to the computation of the expectation $E[c_u[n]c_u[n+m]]$.

The joint pdf and CDF of two correlated Rayleigh variates are needed. We have the expressions in terms of the envelope of the channel response z_n. For normalized power, the pfd is given by [25] (page 142, equation 6.2.):

$$f_{\mathbf{z}}(z_n, z_{n+m}) = \frac{4 z_n z_{n+m}}{(1-\mathcal{R}_z(m))}\exp\left(-\frac{z_n^2 + z_{n+m}^2}{(1-\mathcal{R}_z(m))}\right) \cdot I_0\left(\frac{2\sqrt{\mathcal{R}_z(m)}\,z_n z_{n+m}}{(1-\mathcal{R}_z(m))}\right), \tag{55}$$

where $I_0(u)$ is the modified Bessel function of 0th order.

And the CDF of the envelope of the channel is [25] (page 143, (6.5))

$$\begin{aligned} &F_{\mathbf{z}}(z_n, z_m) \\ &= 1 - \exp(-z_n^2)\,Q_1\left(\sqrt{\frac{2}{(1-\mathcal{R}_z(m))}}z_m, \sqrt{\frac{2\mathcal{R}_z(m)}{(1-\mathcal{R}_z(m))}}z_n\right) \\ &\quad - \exp(-z_m^2)\left[1 - Q_1\left(\sqrt{\frac{2\mathcal{R}_z(m)}{(1-\mathcal{R}_z(m))}}z_m, \sqrt{\frac{2}{(1-\mathcal{R}_z(m))}}z_n\right)\right], \end{aligned} \tag{56}$$

with $Q_1(a,b)$ being the Marcum Q function.

Let the effective envelope of the uth user be

$$z^*_{n_u} = \begin{cases} z_u[n], & z_u[n]\cdot\overline{\gamma}_u > z_{-u}[n]\cdot\overline{\gamma}_{-u}, \\ 0, & z_u[n]\cdot\overline{\gamma}_u > z_{-u}[n]\cdot\overline{\gamma}_{-u}, \end{cases} \tag{57}$$

where $z_{-u}[n] = \max_{k\neq u}\{z_u[n]\}$.

This random variable, equivalent to the effective SNR defined in the uncorrelated channel, indicates the fact that the user only gets the channel if his instantaneous SNR is the highest among all the users. In contrast to the uncorrelated channel, we include the time through the subindex n since it is needed to calculate the expectation evaluated in two different symbols.

Consider the following vector of decision:

$$\left(z^*_{n_u}, z^*_{m_u}\right) = \begin{cases} (0,0), & z_u[n]\overline{\gamma}_u < z_{-u}[n]\overline{\gamma}_{-u} \\ & \quad\text{and } z_u[m]\overline{\gamma}_u < z_{-u}[m]\overline{\gamma}_{-u}, \\ (z_u[n],0), & z_u[n]\overline{\gamma}_u > z_{-u}[n]\overline{\gamma}_{-u} \\ & \quad\text{and } z_u[m]\overline{\gamma}_u < z_{-u}[m]\overline{\gamma}_{-u}, \\ (0,z_u[m]), & z_u[n]\overline{\gamma}_u < z_{-u}[n]\overline{\gamma}_{-u} \\ & \quad\text{and } z_u[m]\overline{\gamma}_u > z_{-u}[m]\overline{\gamma}_{-u}, \\ (z_u[n],z_u[m]), & z_u[n]\overline{\gamma}_u > z_{-u}[n]\overline{\gamma}_{-u} \\ & \quad\text{and } z_u[m]\overline{\gamma}_u > z_{-u}[m]\overline{\gamma}_{-u}. \end{cases} \tag{58}$$

Notice that to calculate the expectation $E[c_u[n]c_u[n+m]]$, only the last case is needed, as the other three options will result in zero in the evaluation of $E[c_u[n]c_u[n+m]]$.

Therefore, only the case in which the channel is assigned to user u in both symbols n and m is required. The joint pdf is

$$f_u\left(z_{n_u}^*, z_{m_u}^*\right) F_{-u}\left(z_{n_u}^* \cdot \overline{\gamma}_u, z_{m_u}^* \cdot \overline{\gamma}_u\right), \quad (59)$$

where $f_u(z_{n_u}^*, z_{m_u}^*)$ is the pdf in (55) and the CDF:

$$F_{-u}\left(z_{n_u}^*, z_{m_u}^*\right) = \prod_{k \neq u}^{U} F_k\left(z_{n_u}^*, z_{m_u}^*\right), \quad (60)$$

with $F_k(z_{n_u}^*, z_{m_u}^*)$ the CDF in (56).

Gathering together the previous expressions, the expectation to be calculated is

$$\begin{aligned}
\mathrm{E}[c_u[n]c_u[m]] &= \mathrm{E}\left[c\left(z_{n_u}^*\right)c\left(z_{m_u}^*\right)\right] \\
&= \int_{z_{n_u}^*=0}^{\infty}\int_{z_{m_u}^*=0}^{\infty} c\left(z_{n_u}^*\right)c\left(z_{m_u}^*\right) f_u\left(z_{n_u}^*, z_{m_u}^*\right) \\
&\quad \cdot F_{-u}\left(z_{n_u}^* \cdot \overline{\gamma}_u, z_{m_u}^* \cdot \overline{\gamma}_u\right) dz_{n_u}^* dz_{m_u}^* \\
&= \left\{x = z_{n_u}^*; y = z_{m_u}^*; p = m - n\right\} \\
&= \int_{x=0}^{\infty}\int_{y=0}^{\infty} \log_2(1+\beta_u x^2)\log_2(1+\beta_u y^2) \\
&\quad \cdot \frac{4xy}{(1-\mathcal{R}_z(p))} \cdot \exp\left(-\frac{(x^2+y^2)}{(1-\mathcal{R}_z(p))}\right) \\
&\quad \cdot I_0\left(\frac{2\sqrt{\mathcal{R}_z(p)}xy}{(1-\mathcal{R}_z(p))}\right) \\
&\quad \cdot \Bigg\{1 - \exp\left(-\overline{\gamma}_u^2 x^2\right) Q_1\left(\sqrt{\frac{2}{(1-\mathcal{R}_z(p))}}\overline{\gamma}_u y,\right. \\
&\qquad \left.\sqrt{\frac{2\mathcal{R}_z(p)}{(1-\mathcal{R}_z(p))}}\overline{\gamma}_u x\right) - \exp\left(-\overline{\gamma}_u^2 y^2\right) \\
&\quad \cdot \Bigg[1 - Q_1\left(\sqrt{\frac{2\mathcal{R}_z(p)}{(1-\mathcal{R}_z(p))}}\overline{\gamma}_u y,\right. \\
&\qquad \left.\sqrt{\frac{2}{(1-\mathcal{R}_z(p))}}\overline{\gamma}_u x\right)\Bigg]\Bigg\}^{U-1} dx dy.
\end{aligned} \quad (61)$$

The evaluation of the users' rates is presented in Figure 7 for the same conditions as in RR. In this case, it is not straightforward to evaluate the variance in (61). Thus, it has been obtained by simulation methods. A long trace of the instantaneous transmission rate process is generated and the sample variance is got from it. The qualitative behaviour already observed in the uncorrelated channel is highlighted here: the differences among users increase significantly with the time correlation of the channel.

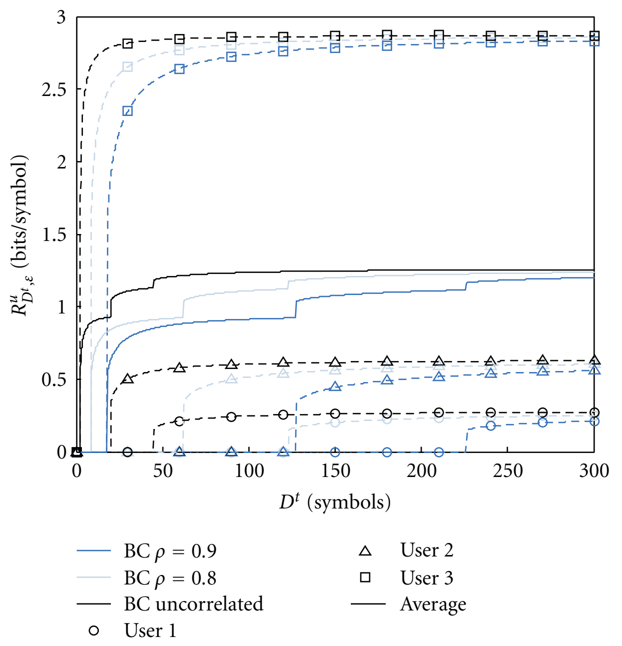

Figure 7: Achievable users' rates with Best Channel scheduling in a time-correlated channel with exponential ACF of parameter ρ. Three users with $\overline{\gamma}_u = 5, 7, 12$ dB, respectively. $\varepsilon = 0.1$. $\beta_u = 1$.

4.4. Proportional Fair. The calculation of the variance in the PF discipline is very similar to the BC algorithm. The effective envelope of the uth user yields

$$z_{n_u}^* = \begin{cases} z_u[n], & z_u[n] > z_{-u}[n], \\ 0, & z_u[n] < z_{-u}[n]. \end{cases} \quad (62)$$

Now the vector of decision simplifies

$$\left(z_{n_u}^*, z_{m_u}^*\right) = \begin{cases} (0,0), & z_u[n] < z_{-u}[n] \\ & \quad \text{and } z_u[m] < z_{-u}[m], \\ (z_u[n], 0), & z_u[n] > z_{-u}[n] \\ & \quad \text{and } z_u[m] < z_{-u}[m], \\ (0, z_u[m]), & z_u[n] < z_{-u}[n] \\ & \quad \text{and } z_u[m] > z_{-u}[m], \\ (z_u[n], z_u[m]), & z_u[n] > z_{-u}[n] \\ & \quad \text{and } z_u[m] > z_{-u}[m]. \end{cases} \quad (63)$$

Like in the BC discipline, only the joint pdf of the last case is needed, as the other three options will result in zero in the expression of $E[c_u[n]c_u[n+m]]$. This joint pdf is

$$f_u\left(z_{n_u}^*, z_{m_u}^*\right) F_{-u}\left(z_{n_u}^*, z_{m_u}^*\right), \quad (64)$$

where $f_u(z^*_{n_u}, z^*_{m_u})$ is the pdf in (55) and the CDF $F_{-u}(z^*_{n_u}, z^*_{m_u})$ is

$$F_{-u}\left(z^*_{n_u}, z^*_{m_u}\right) = \prod_{k \neq u}^{U} F_k\left(z^*_{n_u}, z^*_{m_u}\right), \tag{65}$$

where $F_k(z^*_{n_u}, z^*_{m_u})$ is the CDF in (56).

Finally, the expression of the expectation is

$$\begin{aligned}
&\mathrm{E}[c_u[n]c_u[m]] \\
&= \mathrm{E}\left[c\left(z^*_{n_u}\right)c\left(z^*_{m_u}\right)\right] \\
&= \int_{x=0}^{\infty}\int_{y=0}^{\infty} \log_2(1+\beta x^2)\log_2(1+\beta y^2)\frac{4xy}{(1-\mathcal{R}_z(p))} \\
&\quad\cdot \exp\left(-\frac{(x^2+y^2)}{(1-\mathcal{R}_z(p))}\right)\cdot I_0\left(\frac{2\sqrt{\mathcal{R}_z(p)}xy}{(1-\mathcal{R}_z(p))}\right) \\
&\quad\cdot\Bigg\{1-\exp(-x^2)Q_1\left(\sqrt{\frac{2}{(1-\mathcal{R}_z(p))}}y, \sqrt{\frac{2\mathcal{R}_z(p)}{(1-\mathcal{R}_z(p))}}x\right) \\
&\qquad - \exp(-y^2)\Bigg[1-Q_1\Bigg(\sqrt{\frac{2\mathcal{R}_z(p)}{(1-\mathcal{R}_z(p))}}y, \\
&\qquad\qquad \sqrt{\frac{2}{(1-\mathcal{R}_z(p))}}x\Bigg)\Bigg]\Bigg\}^{U-1} dxdy.
\end{aligned} \tag{66}$$

Figure 8 shows the evaluation of the users' rates for a PF discipline and with the same parameters defined before. In the uncorrelated channel, the algorithm was able to equal the users in terms of minimum target delay. When the time-correlation of the channel comes on, the algorithm cannot maintain the fairness anymore and differences among users can be observed. Like in the other two algorithms, the minimum target delay that each user can demand is related to the quality of his channel. In spite of not maintaining the fairness among users anymore, it is still the fairest of them all.

5. Simulation Comparison

The analytical results presented in Sections 3 and 4 are validated by comparison with simulations. In particular, the queueing system in Figure 1 is simulated. Each user sends bits to its buffer of queue length $Q_u[n]$ in the nth symbol. The selected user depends on the scheduling algorithm: Round Robin, Best Channel, or Proportional Fair. D^t, ε, and β_u are fixed (the same for all the users, for simplicity) and the users' rates are evaluated with (20). Then, the the arrival process of each user generates source data at the (constant) rate $\mathrm{R}^u_{D^t,\varepsilon}$. The simulation is run and the tail probability of exceeding the target delay is measured based on the measurements of the delay suffered by bits leaving the queue. Notice that the expected value of this tail probability, $\Pr\{D(\infty) > D^t\}$, is ε.

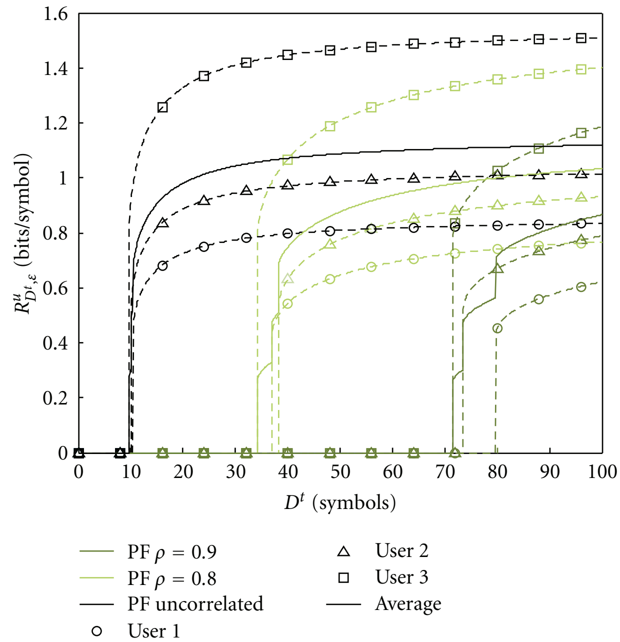

Figure 8: Achievable users' rates with Proportional Fair scheduling in a time-correlated channel with exponential ACF of parameter ρ. Three users with $\overline{\gamma}_u = 5, 7, 12$ dB, respectively. $\varepsilon = 0.1$. $\beta_u = 1$.

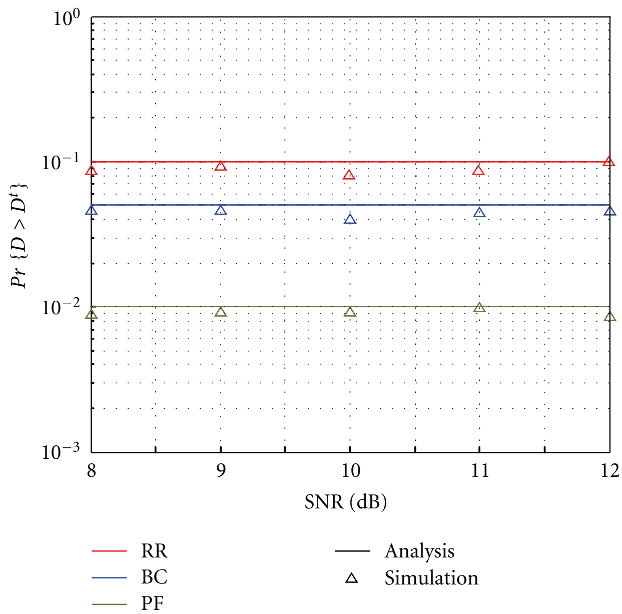

Figure 9: Simulation comparison for uncorrelated channel. 5 users with $\overline{\gamma}_u = 8, 9, 10, 11, 12$ dB, respectively. $D^t = 60$ symbols. $\varepsilon = 0.10$ (RR), 0.05 (BC) and 0.01 (PF).

5.1. Uncorrelated Channel. The simulation of the uncorrelated channel is shown in Figure 9. 5 users with average SNR's 8, 9, 10, 11, and 12 dB are simulated. $\beta_u = 1$. A target delay of 60 symbols is set. The probability of exceeding the target delay, ε, has been set to 0.10 (RR), 0.05 (BC), and 0.01 (PF).

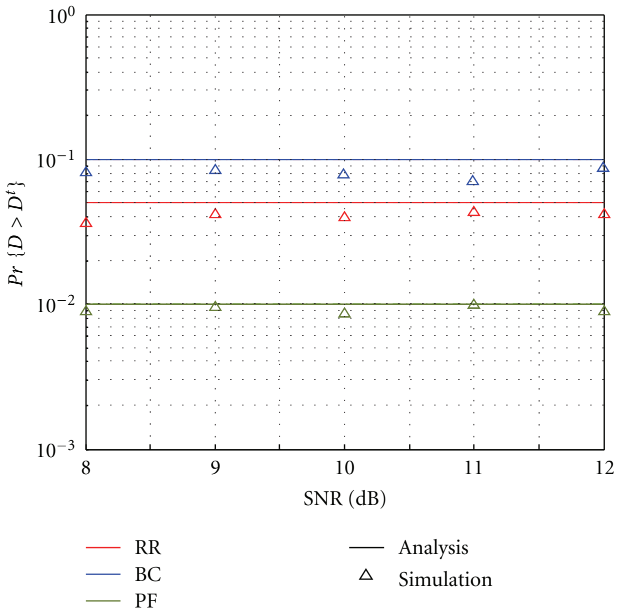

Figure 10: Simulation comparison for a time-correlated channel with exponential ACF of parameter $\rho = 0.8$. 5 users with $\overline{\gamma} = 8, 9, 10, 11, 12$ dB, respectively. $D^t = 250$ symbols. $\varepsilon = 0.10$ (RR), 0.05 (BC), and 0.01 (PF).

$\Pr\{D(\infty) > D^t\}$ is measured and compared to the analytical ε.

The results show that the QoS requirements are accurately reached with the result for $\mathrm{R}^u_{D^t,\varepsilon}$ in (20). The users' rates were obtained under the assumption that $\eta_u = 1$, a high load scenario that constitutes an upper bound. It has been checked in the simulations that the measured probability of a nonempty queue η_u is very close to one in our simulations.

5.2. Correlated Channel. Finally, a time-correlated channel has been simulated under the same conditions of the uncorrelated channel. The only difference is that the target delay is now 250 symbols and the correlation parameter ρ is 0.8. Figure 10 plots the results.

The conclusions from the uncorrelated channel apply also here: the simulation results are satisfactory, approaching accurately the analytical results.

6. Conclusions

In this paper, we have obtained analytical expressions of the achievable users' rates in a wireless system under the conditions stated by the MAC layer: a selected scheduling discipline and a QoS constraint given in terms of a delay constraint and a BER. The delay constraint consists of a target delay D^t and the probability of exceeding it, ε. Three simple and widely employed disciplines have been analyzed: Round Robin, Best Channel and Proportional Fair. The method to calculate these rates is based on the effective bandwidth theory. The analysis is done first for an uncorrelated channel and later for a time-correlated channel. The evaluation of the individual rates and the total capacity confirms the expected qualitative behaviour of the three algorithms. It is also observed that the correlation is harmful for the delay, as expected, and the maximum achievable rates decrease as the correlation increases. Moreover, the differences among users become more noticeable for more correlated channels. Finally, simulations of the algorithms were conducted to validate our outcomes.

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