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Research Article

Appropriate Algorithms for EstimatingFrequency-Selective Rician Fading MIMO Channels and Channel Rice Factor: Substantial Benefits of Rician Model and Estimator Tradeoffs

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The training-based channel estimation (TBCE) scheme in multiple-input multiple-output (MIMO) frequency-selective Rician fading channels is investigated. We propose the new technique of shifted scaled least squares (SSLS) and the minimum mean square error (MMSE) estimator that are suitable to estimate the above-mentioned channel model. Analytical results show that the proposed estimators achieve much better minimum possible Bayesian Cramér-Rao lower bounds (CRLBs) in the frequency-selective Rician MIMO channels compared with those of Rayleigh one. It is seen that the SSLS channel estimator requires less knowledge about the channel and/or has better performance than the conventional least squares (LS) and MMSE estimators. Simulation results confirm the superiority of the proposed channel estimators. Finally, to estimate the channel Rice factor, an algorithm is proposed, and its efficiency is verified using the result in the SSLS and MMSE channel estimators.

1. Introduction

In wireless communications, multiple-input multiple-output (MIMO) systems provide substantial benefits in both increasing system capacity and improving its immunity to deep fading in the channel [1, 2]. To take advantage of these benefits, special space-time coding techniques are used [3, 4]. In most previous research on the coding approaches for MIMO systems, however, the accurate channel state information (CSI) is required at the receiver and/or transmitter. Moreover, in the coherent receivers [1], channel equalizers [5], and transmit beamformers [6], the perfect knowledge of the channel is usually needed.

In the literature, three classes of methods for channel identification are presented. They include training-based channel estimation (TBCE) [7, 8], blind channel estimation (BCE) [9, 10], and semiblind channel estimation (SBCE) [11, 12]. Due to low complexity and better performance, TBCE is widely used in practice for quasistatic or slow fading channels, for instance, indoor MIMO channels. However, in outdoor MIMO channels where channels are under fast fading, the channel tracking and estimating algorithms as the Wiener least mean squares (W-LMS) [13], the Kalman filter [14], recursive least squares (RLS) [15], generalized RLS (GRLS) [16], and generalized LMS (GLMS) [17] are used.

TBCE schemes can be optimal at high signal-to-noise ratios (SNRs) [18]. Moreover, it is shown in [19] that at high *SNRs*, training-based capacity lower bounds coincide with the actual Shannon capacity of a block fading finite impulse response (FIR) channel. Nevertheless, at low SNRs, training-based schemes are suboptimal [18].

The optimal training signals are usually obtained by minimizing the channel estimation error. For MIMO flat fading channels, the design of optimal training sequences to satisfy the required semiunitary condition in the channel estimator error, given in [7, (9)], is straightforward. For instance, a properly normalized submatrix of the discrete Fourier transform (DFT) matrix has been used in [7] to estimate the Rayleigh flat fading MIMO channel. In this case, a Hadamard matrix can also be applied.

On the other hand, to estimate MIMO frequency-selective or MIMO intersymbol interference (ISI) channels, training sequences are designed considering a few aspects. For MIMO ISI channel estimation, training sequences should have both good autocorrelations and cross correlations. Furthermore, to separate the transmitted data and training symbols, one of the zero-padding- (ZP-) based guard period or cyclic prefix- (CP-) based guard period is inserted. In order to estimate the Rayleigh fading MIMO ISI channels, the delta sequence has been used in [20] as optimal training signal. This sequence satisfies the semiunitary condition in the mean square error (MSE) of channel estimator. However, it may result in high peak to average power ratio (PAPR) that is important in practical communication systems.

The optimal training sequences of [21–25] not only satisfy the semiunitary condition but also introduce good PAPR. In [21], a set of sequences with a zero correlation zone (ZCZ) is employed as optimal training signals. In [26–28], to find these sequence sets, some algorithms are presented. In [22], different phases of a perfect polyphase sequence such as the Frank sequence or Chu sequence are proposed. Furthermore, in [23–25], uncorrelated Golay complementary sets of polyphase sequences have been used. Since both ZCZ and perfect polyphase sequences have periodic correlation properties, the CP-based guard period is employed with them. On the other hand, uncorrelated Golay complementary sets of polyphase sequences have both aperiodic and periodic types that are used with ZP- and CP-based guard periods, respectively.

Since all sequences under their conditions attain the same channel estimation error [25] and also our goal is not comparing them in this paper (this work is done in [24, 25]), we will use ZCZ sequences here.

In [25], the performance of the best linear unbiased estimator (BLUE) and linear minimum mean square error (LMMSE) estimator is studied in the frequency-selective Rayleigh fading MIMO channel. It is observed that the LMMSE estimator has better performance than the BLUE, because it can employ statistical knowledge about the channel. Nevertheless, all estimators of [23–25] are optimal since they achieve the minimum possible classical (or Bayesian) Cramér-Rao Lower Bound (CRLB) in the Rayleigh fading channels.

In most previous research on the MIMO channel estimation, the channel fading is assumed to be Rayleigh. In [29], the SLS and minimum mean square error (MMSE) estimators of [7] have been used to estimate the Rician fading MIMO channel. It is notable that these estimators are appropriate to estimate the Rayleigh fading channels, and hence the results of [29] are controversial. In [30], to estimate the channel matrix in the Rician fading MIMO systems, the MMSE estimator is analyzed. It is proved in [30] analytically that the MSE improves with the spatial correlation at both the transmitter and the receiver side. An interesting result in this paper is that the optimal training sequence length can be considerably smaller than the number of transmitter antennas in systems with strong spatial correlation.

In [31–33], the TBCE scheme is investigated in MIMO systems when the Rayleigh fading model is replaced by the more general Rician model. By the new methods of shifted scaled least squares (SSLS) and LMMSE channel estimators, it is shown that increasing the Rice factor improves the performance of channel estimation. In [31], it is assumed that the Rician fading channel has spatial correlation. It has also been shown that the error of the LMMSE channel estimator decreases when the Rice factor and/or the correlation coefficient increase.

In this paper, we extend the results of [31–33] in flat fading to the frequency-selective fading case. For channel estimator error, the new formulations are obtained so that in the special case where the channel has flat fading, the results reduce to the previous results in [31–33]. The substantial benefits of Rician fading model are investigated in the MIMO channel estimation. It is seen that Rician fading not only can increase the capacity of a MIMO system [2] but it also may be helpful for channel estimation. It is notable that the aforementioned channel model is suitable for suburban areas where a line of sight (LOS) path often exists. This may also be true for microcellular or picocellular systems with cells of less than several hundred meters in radius.

First, the traditional least squares (LS) method is probed. It is notable that for linear channel model with Gaussian noise, the maximum likelihood (ML), LS, and BLUE estimators are identical [34]. Simulation results show that the LS estimator achieves the minimum possible classical CRLB. Clearly, the performance of this estimator is independent of the Rice factor. Then, the SSLS and MMSE channel estimators are proposed. Simulation results show that these estimators attain their minimum possible Bayesian CRLBs. Furthermore, analytical and numerical results show that the performance of these estimators is improved when the Rice factor increases. It is also seen that in the frequency-selective Rician fading MIMO channels, the MMSE estimator outperforms the LS and SSLS estimators. However, it requires that both the power delay profile (PDP) of the channel and the receiver noise power as well as the Rice factor be known a priori. In general, the SSLS technique requires less knowledge about the channel statistics and/or has better performance than the LS and MMSE approaches.

Moreover, to estimate the channel Rice factor, we propose an algorithm which is important in practical usages of the proposed SSLS and MMSE estimators. In single-input single-output (SISO) channels, different methods have been proposed for estimation of the Rice factor. In [35], the ML estimate of the Rice factor is obtained. In [36], a Rice factor estimation algorithm based on the probability distribution function (PDF) of the received signal is proposed. In [37–41], the moment-based methods are used for the Rice factor estimation. Besides, to estimate the Rice factor in low SNR environments, the phase information of received signal has been used in [42]. Moreover, in [43, 44], the Rice factor along with some other parameters is estimated in MIMO systems using weighted LS (WLS) and ML criteria.

In the above-mentioned references, the channel Rice factor is estimated using the received signals. However, in this paper, we suggest an algorithm based on training signal and LS technique. Simulation results corroborate the good performance of this algorithm in channel estimation. In practice, such algorithms are required to identify the type of environment (Rayleigh or Rician) in several applications, for instance, adaptive modulation for MIMO antenna systems.

The next section describes the MIMO channel model underlying our framework and some assumptions on the fading process. The performance of the LS, SSLS, and MMSE estimators in the frequency-selective Rician fading MIMO channel estimation and optimal choice of training sequences are investigated in Sections 3, 4, and 5, respectively. Numerical examples and simulation results are presented in Section 6. Finally, concluding remarks are presented in Section 7.

Notation: $(\cdot)^H$ is reserved for the matrix Hermitian, $(\cdot)^{-1}$ for the matrix inverse, $(\cdot)^T$ for the matrix (vector) transpose, $(\cdot)^*$ for the complex conjugate, $\otimes$ for the Kronecker product, $\mathrm{tr}\{\cdot\}$ for the trace of a matrix, $\mathrm{mean}(\cdot)$ for the mean value of the elements in a matrix, $\mathrm{mode}(\cdot)$ for the mode value of the elements in a vector and $\mathrm{abs}(\cdot)$ for the absolute value of the complex number. $\mathrm{vec}(\cdot)$ stacks all the columns of its matrix argument into one tall column vector. $E\{\cdot\}$ is the mathematical expectation, $\mathbf{I}_m$ denotes the $m \times m$ identity matrix, and $\|\cdot\|_F$ denotes the Frobenius norm.

2. Signal and Channel Models

We assume block transmission over block fading Rician MIMO channel with N_T transmit and N_R receive antennas. The frequency-selective fading subchannels between each pair of Tx-Rx antenna elements are modeled by $L+1$ taps as $\mathbf{h}_{rt} = [h_{r,t}(0)\ h_{r,t}(1)\ \cdots\ h_{r,t}(L)]^T$, for all $r \in [1, N_R]$ and $t \in [1, N_T]$. We suppose identical PDP as $(b_0, b_1, \ldots, b_L)$ for all subchannels. Then, the lth taps of all the subchannels have the same power b_l, that is, $E\{|h_{r,t}(l)|^2\} = b_l$; for all l, t, r. It is also assumed unit power for each sub-channel, that is, $\sum_{l=0}^{L} b_l = 1$.

The discrete-time base-band model of the received training signal at symbol time m can be described by

$$\mathbf{y}(m) = \sum_{l=0}^{L} \mathbf{H}_l \mathbf{x}(m-l) + \mathbf{v}(m), \tag{1}$$

where $\mathbf{y}(i)$ and $\mathbf{x}(i)$ are the $N_R \times 1$ complex vector of received symbols on the N_R-Rx antennas and the $N_T \times 1$ vector of transmitted training symbols on the N_T-Tx antennas at symbol time i, respectively. The $N_R \times 1$ vector $\mathbf{v}(i)$ in (1) is the complex additive Rx noise at symbol time i. The $L+1$ matrices $N_R \times N_T$, $\{\mathbf{H}_l\}_{l=0}^{L}$, constitute the $L+1$ taps of the multipath MIMO channel.

For Rician frequency-selective fading channels, the elements of the matrix $\mathbf{H}_l$, for all $l \in [0, L]$, are defined similar to [45, 46] in the following form:

$$\mathbf{H}_l = \sqrt{b_l \frac{\kappa}{\kappa+1}} \widetilde{\mathbf{M}}_l + \sqrt{\frac{b_l}{\kappa+1}} \widetilde{\mathbf{H}}_l, \tag{2}$$

where κ is the channel Rice factor. The matrices $\widetilde{\mathbf{M}}_l$ and $\widetilde{\mathbf{H}}_l$ describe the LOS and scattered components, respectively. We assume that the elements of $\widetilde{\mathbf{M}}_l$, for all l are complex as $(1+j)/\sqrt{2}$ and the elements of the matrix $\widetilde{\mathbf{H}}_l$, for all l, are independently and identically distributed (i.i.d.) complex Gaussian random variables with the zero mean and the unit variance. The frequency-selective fading MIMO channel can be defined as the $N_R \times N_T(L+1)$ matrix $\mathbf{H} = \{\mathbf{H_0}, \mathbf{H_1}, \ldots, \mathbf{H_L}\}$, where $\mathbf{H_l}$ has the following structure

$$\mathbf{H}_l = \begin{bmatrix} h_{11}(l) & h_{12}(l) & \cdots & h_{1N_T}(l) \\ h_{21}(l) & h_{22}(l) & \cdots & h_{2N_T}(l) \\ \vdots & \vdots & \cdots & \vdots \\ h_{N_R1}(l) & h_{N_R2}(l) & \cdots & h_{N_RN_T}(l) \end{bmatrix}, \quad \forall l \in [0, L]. \tag{3}$$

Moreover, it is assumed that the elements of matrices $\widetilde{\mathbf{H}}_{l_1}$ and $\widetilde{\mathbf{H}}_{l_2}$, for all l_1, l_2 are independent of each other. Hence, the elements of the matrix $\mathbf{H}$ are also independent of each other. Using (2), the mean value and the variance of the elements $h_{r,t}(l)$ of $\mathbf{H}$ can be computed as follows:

$$\begin{aligned} E\{h_{r,t}(l)\} &= \sqrt{b_l \frac{\kappa}{\kappa+1}} \frac{(1+j)}{\sqrt{2}} + \sqrt{\frac{b_l}{\kappa+1}} \times 0 \\ &= \sqrt{b_l \frac{\kappa}{\kappa+1}} \frac{(1+j)}{\sqrt{2}} \\ &= \frac{\mu_l}{\sqrt{2}}(1+j), \end{aligned} \tag{4}$$

$$\begin{aligned} \sigma_l^2 &= E\left\{|h_{r,t}(l)|^2\right\} - |E\{h_{r,t}(l)\}|^2 \\ &= b_l - b_l \frac{\kappa}{\kappa+1} = \frac{b_l}{\kappa+1}, \end{aligned} \tag{5}$$

where $\mu_l = \sqrt{b_l \kappa/(1+\kappa)}$. According to (4) and (5), the channel Rice factor can vary the mean value and the variance of the channel in the defined model.

Suppose that $\mathbf{h} = \mathrm{vec}(\mathbf{H})$. The $N_R N_T(L+1) \times N_R N_T(L+1)$ covariance matix of $\mathbf{h}$ can be obtained as follows:

$$\mathbf{C}_h = \mathbf{R}_h - E\{\mathbf{h}\}E\{\mathbf{h}\}^H = \mathbf{C}_\Sigma \otimes \mathbf{I}_{N_R N_T}, \tag{6}$$

where

$$\begin{aligned} \mathbf{C}_\Sigma &= \begin{bmatrix} \sigma_0^2 & 0 & 0 & \cdots & 0 \\ 0 & \sigma_1^2 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & \sigma_L^2 \end{bmatrix} \\ &= \frac{1}{1+\kappa} \begin{bmatrix} b_\circ & 0 & 0 & \cdots & 0 \\ 0 & b_1 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & b_L \end{bmatrix}. \end{aligned} \tag{7}$$

Note that the latter one is written using (5).

In order to estimate the channel matrix $\mathbf{H}$, the $N_P \geq N_T(L+1) + L$ symbols are transmitted from each Tx antenna. The L first symbols are CP guard period that are used to avoid the interference from symbols before the first training symbols. At the receiver, because of their pollution by data, due to interference, these symbols are discarded. Hence, by collecting the last $N_P - L$ received vectors of (1) into the $N_R \times (N_P - L)$matrix $\mathbf{Y} = [\mathbf{y}(L+1), \mathbf{y}(L+2), \ldots, \mathbf{y}(N_P)]$, the compact matrix form of received training symbols can be represented in a linear model as

$$\mathbf{Y} = \mathbf{H}\mathbf{X} + \mathbf{V}, \tag{8}$$

where $\mathbf{X}$ is the $N_T(L+1) \times (N_P - L)$ training matrix. The matrix $\mathbf{X}$ is constructed by the N_P-vector of transmitted symbols in the form of $\mathbf{x}(i) = [x_1(i), x_2(i), \ldots, x_{N_T}(i)]^T$ as follows:

$$\mathbf{X} = \begin{bmatrix} \mathbf{x}(L+1) & \mathbf{x}(L+2) & \cdots & \mathbf{x}(N_P) \\ \mathbf{x}(L) & \mathbf{x}(L+1) & \cdots & \mathbf{x}(N_P-1) \\ \vdots & \vdots & \vdots & \vdots \\ \mathbf{x}(2) & \mathbf{x}(3) & \cdots & \mathbf{x}(N_P-L+1) \\ \mathbf{x}(1) & \mathbf{x}(2) & \cdots & \mathbf{x}(N_P-L) \end{bmatrix}. \tag{9}$$

Note that $x_t(i)$ is the transmitted symbol by the tth Tx antenna at symbol time i. The matrix $\mathbf{V}$ in (8) is the complex N_R-vector of additive Rx noise. The elements of the noise matrix are i.i.d. complex Gaussian random variables with zero-mean and σ_n^2 variance, and we have

$$\mathbf{R}_V = E\left\{\mathbf{V}^H\mathbf{V}\right\} = \sigma_n^2 N_R \mathbf{I}_{N_P-L}. \tag{10}$$

The elements of $\mathbf{H}$ and noise matrix are independent of each other.

The matrix $\mathbf{H}$ is a complex normally distributed matrix and its $N_R \times N_T(L+1)$ mathematical expectation matrix can be written as $\mathbf{M} = E\{\mathbf{H}\} = \{\mathbf{M}_0, \mathbf{M}_1, \ldots, \mathbf{M}_L\}$, where the elements of the matrix $\mathbf{M}_l$ are

$$m_{r,t}(l) = \frac{\mu_l}{\sqrt{2}}(1+j). \tag{11}$$

Using (5) and (11), it is straightforward to show that the elements of the columns of $\mathbf{H}$ have the following $N_T(L+1) \times N_T(L+1)$ covariance matrix

$$\begin{aligned} \mathbf{C}_H &= \mathbf{R}_H - \mathbf{M}^H\mathbf{M} = E\left\{\mathbf{H}^H\mathbf{H}\right\} - \mathbf{M}^H\mathbf{M} \\ &= N_R(\mathbf{C}_\Sigma \otimes \mathrm{I}_{N_T}). \end{aligned} \tag{12}$$

In a particular case, when the uniform PDP is used, that is, $b_0 = b_1 = \cdots = b_L = 1/(L+1)$, we have

$$\mathbf{C}_H = \frac{N_R}{(1+\kappa)(1+L)}\mathbf{I}_{N_T(L+1)}. \tag{13}$$

When $\kappa = 0$, (12) reduces to the Rayleigh fading channel introduced in [24, 25].

3. LS Channel Estimator

In this section, $\mathbf{H}$ is assumed to be an unknown but deterministic matrix. The LS channel estimator minimizes $\mathrm{tr}\{(\mathbf{Y} - \mathbf{H}\mathbf{X})^H(\mathbf{Y} - \mathbf{H}\mathbf{X})\}$ and is given by

$$\widehat{\mathbf{H}}_{\mathrm{LS}} = \mathbf{Y}\mathbf{X}^H\left(\mathbf{X}\mathbf{X}^H\right)^{-1}. \tag{14}$$

This estimator utilizes only received and transmitted signals that are given at the receiver. It has no knowledge about channel statistics. The channel estimation error is defined by $E\{\|\mathbf{H} - \widehat{\mathbf{H}}_{\mathrm{LS}}\|_F^2\}$ that results in

$$J_{\mathrm{LS}} = \sigma_n^2 N_R \,\mathrm{tr}\left\{\left(\mathbf{X}\mathbf{X}^H\right)^{-1}\right\}. \tag{15}$$

Let us find $\mathbf{X}$ which minimizes the error of (15) subject to a power constraint on $\mathbf{X}$. This is equivalent to the following optimization problem

$$\min_{\mathbf{X}} \mathrm{tr}\left\{\left(\mathbf{X}\mathbf{X}^H\right)^{-1}\right\} \quad \text{S.T. } \mathrm{tr}\left\{\mathbf{X}\mathbf{X}^H\right\} = P, \tag{16}$$

where P is a given constant value considered as the total power of training matrix $\mathbf{X}$. To solve (16), the Lagrange multiplier method is used. The problem can be written as

$$L\left(\mathbf{X}\mathbf{X}^H, \eta\right) = \mathrm{tr}\left\{\left(\mathbf{X}\mathbf{X}^H\right)^{-1}\right\} + \eta\left[\mathrm{tr}\left\{\mathbf{X}\mathbf{X}^H\right\} - P\right], \tag{17}$$

where η is the Lagrange multiplier. By differentiating this equation with respect to $\mathbf{X}\mathbf{X}^H$ and setting the result equal to zero as well as using the constraint $\mathrm{tr}\{\mathbf{X}\mathbf{X}^H\} = P$, we obtain that the optimal training matrix should satisfy

$$\mathbf{X}\mathbf{X}^H = \frac{P}{N_T(L+1)}\mathbf{I}_{N_T(L+1)}. \tag{18}$$

Substituting the semiunitary condition (18) back into (15), the error under optimal training is

$$(J_{\mathrm{LS}})_{\min} = \frac{\sigma_n^2 (N_T(L+1))^2 N_R}{P}. \tag{19}$$

For flat fading, $L = 0$, (19) is similar to that of [7]. In order to achieve the minimum error of (19), the training sequences should satisfy the semiunitary condition (18). Due to the structure of $\mathbf{X}$ in (9), it means that the optimal training sequence in each Tx antenna has to be orthogonal not only to its shifts within L taps, but also to the training sequences in other antennas and their shifts within L taps. Here, we consider the ZCZ sequences as optimal training signals without loss of generality.

It is supposed that the transmitted power of any Tx antennas at all times is p. Then,

$$P = pN_T(L+1)(N_P - L). \tag{20}$$

Substituting (20) back into (19), the minimum error can be rewritten as

$$(J_{\mathrm{LS}})_{\min} = \frac{\sigma_n^2 N_T N_R (L+1)}{p(N_P - L)} \tag{21}$$

From (21), holding L constant, the minimum error of the LS estimator decreases when N_P increases. On the other hand, holding N_P constant, the minimum error of this estimator increases when L increases.

For optimal training which satisfies (18), the LS channel estimator (14) reduces to

$$\widehat{\mathbf{H}}_{\mathrm{LS}} = \frac{N_T(L+1)}{P}\,\mathbf{Y}\mathbf{X}^H. \tag{22}$$

This estimator obtains the minimum possible classical CRLB (21). However, the error of (21) is independent of the Rice factor. Clearly, the LS estimator cannot exploit any statistical knowledge about the frequency-selective Rayleigh or Rician fading MIMO channels. In the next sections, we derive new results in the frequency-selective Rician channel model by the proposed SSLS and MMSE estimators.

4. Shifted Scaled Least Squares Channel Estimator

The SSLS channel estimator of [33] is an optimally shifted type of the presented scaled LS (SLS) method of [7, 21]. The motivation of using it is the further reduction of the error in the MIMO frequency-selective Rician fading channel estimation. This estimator has been expressed in the following general form

$$\widehat{\mathbf{H}}_{\mathrm{SSLS}} = \gamma\widehat{\mathbf{H}}_{\mathrm{LS}} + \mathbf{B}, \tag{23}$$

where γ and $\mathbf{B}$ are the scaling factor and the shifting matrix, respectively. They are obtained so that the total mean square error (TMSE), $E\{\|\mathbf{H} - \widehat{\mathbf{H}}_{\mathrm{SSLS}}\|_F^2\}$, is minimized. The results are [33]

$$\begin{aligned} \widehat{\mathbf{H}}_{\mathrm{SSLS}} &= \gamma\widehat{\mathbf{H}}_{\mathrm{LS}} + (1-\gamma)\mathbf{M}, \\ \gamma &= \frac{\mathrm{tr}\{\mathbf{C}_H\}}{J_{\mathrm{LS}} + \mathrm{tr}\{\mathbf{C}_H\}}. \end{aligned} \tag{24}$$

Note that in the special case, $\kappa = 0$, the Rayleigh fading model, this estimator is identical to the SLS estimator of [7, 21]. Here, J_{LS} is given by (15). The minimum TMSE with respect to γ and $\mathbf{B}$ can be given by

$$\min_{\gamma,\mathbf{B}} J_{\mathrm{SSLS}} = \frac{J_{\mathrm{LS}}\,\mathrm{tr}\{\mathbf{C}_H\}}{J_{\mathrm{LS}} + \mathrm{tr}\{\mathbf{C}_H\}}. \tag{25}$$

The minimum TMSE obtained from (25) is lower than the presented J_{SLS} in [21], because always $\mathrm{tr}\{\mathbf{C}_H\} \le \mathrm{tr}\{\mathbf{R}_H\}$. Therefore, it is derived from [21] and (25) that

$$J_{\mathrm{SSLS}} < J_{\mathrm{SLS}} < J_{\mathrm{LS}}, \quad \kappa > 0. \tag{26}$$

It means that the SSLS estimator has the lowest error among the LS, SLS, and SSLS estimators. In order to choose the optimal training sequences, let us to find $\mathbf{X}$ which minimizes J_{SSLS} subject to a transmitted power constraint. Clearly, such an optimization problem and (16) are equivalent. Since $\mathrm{tr}\{\mathbf{C}_H\} > 0$, from (25) it is obvious that J_{SSLS} is a monotonically increasing function of J_{LS}. Note that $\mathrm{tr}\{\mathbf{C}H\}$ is not a function of $\mathbf{X}$ and so J_{LS} is the only term in (25) which depends on $\mathbf{X}$. Therefore, the optimal choice of training matrix for the SSLS channel estimator is the same as for the LS approach. Using (12), (21), and (25), we obtain that the minimum possible Bayesian CRLB (Since all of the estimators utilized in this paper attain the minimum possible CRLB, we use CRLB and TMSE interchangeably.) under the optimal training is given by

$$(J_{\mathrm{SSLS}})_{\min} = \frac{\sigma_n^2 N_R N_T (L+1)}{\sigma_n^2 (L+1)(1+\kappa) + p(N_P - L)}. \tag{27}$$

From (27), it is seen that increasing the Rice factor leads to decreasing TMSE in the introduced SSLS estimator. In other words, the SSLS channel estimator achieves lower minimum possible CRLB compared with the traditional LS estimator. The SSLS channel estimator under the optimal training can be rewritten in the following form using (20)–(24)

$$\begin{aligned} \widehat{\mathbf{H}}_{\mathrm{SSLS}} = {} & \frac{\mathrm{tr}\{\mathbf{C}_H\}}{\sigma_n^2 N_R N_T (L+1) + p(N_P - L)\,\mathrm{tr}\{\mathbf{C}_H\}} \mathbf{Y}\mathbf{X}^H \\ & + \frac{\sigma_n^2 N_R N_T (L+1)}{\sigma_n^2 N_R N_T (L+1) + p(N_P - L)\,\mathrm{tr}\{\mathbf{C}_H\}} \mathbf{M}. \end{aligned} \tag{28}$$

This estimator offers a more significant improvement than the LS and SLS methods. However, from (28), it requires that $\mathrm{tr}\{\mathbf{C}_H\}$ and $\mathbf{M}$ or equivalently the Rice factor as well as σ_n^2 be known a priori. The required knowledge of the channel statistics can be estimated by some methods. For instance, the problem of estimating the MIMO channel covariance, based on limited amounts of training sequences, is treated in [47]. Moreover, in [48], the channel autocorrelation matrix estimation is performed by an instantaneous autocorrelation estimator that only one channel estimate (obtained by a very low complexity channel estimator) has been used as input.

Using (12) and (21), the scaling factor in (24) can be rewritten as

$$\gamma = \frac{pN_T/\sigma_n^2}{(1+\kappa) + pN_T/\sigma_n^2}. \tag{29}$$

The SNR is defined as $\mathrm{SNR} = pN_T/\sigma_n^2$. Then, we have

$$\gamma = \frac{\mathrm{SNR}}{(1+\kappa) + \mathrm{SNR}}. \tag{30}$$

From (30), it is seen that increasing SNR leads to increasing γ which is restricted by 1. Then, the SSLS estimator in (24) reduces to the LS estimator when $\mathrm{SNR} \to \infty$. Moreover, decreasing the Rice factor to zero (which implies that $\mu_l = 0$ and hence $\mathbf{M} = 0$) leads to increasing γ which is restricted by $\mathrm{SNR}/(\mathrm{SNR} + 1)$. Hence, the SSLS estimator in (24) reduces to the SLS estimator of [21] when $\kappa = 0$. On the other hand, at $\mathrm{SNR} = 0$ or for $\kappa \to \infty$ (which implies that $\gamma = 0$), the SSLS estimator in (24) reduces to $\widehat{\mathbf{H}}_{\mathrm{SSLS}} = \mathbf{M} = E\{\mathbf{H}\}$.

Generally speaking, the scaling factor in (24) is between 0 and 1. When the channel fading is weak ($\kappa \to \infty$ or AWGN) or the transmitted power is small, that is, $\mathrm{tr}\{\mathbf{C}_H\} \ll J_{\mathrm{LS}}$, the scaling factor $\gamma \to 0$. Also, when the channel fading is strong ($\kappa \to 0$ or Rayleigh) or the transmitted power is large, that is, $\mathrm{tr}\{\mathbf{C}_H\} \gg J_{\mathrm{LS}}$, the scaling factor $\gamma \to 1$. Finally, in the Rician fading channel ($0 < \kappa < \infty$), we have $0 < \gamma < 1$.

5. MMSE Channel Estimator

For the linear model described in Section 2, the MMSE, LMMSE, and maximum a posteriori (MAP) estimators are identical [34]. Hence, we obtain a general form of the linear estimator, appropriate for Rician fading channels, that minimizes the estimation error of channel matrix $\mathbf{H}$. It can be expressed in the following form

$$\begin{aligned}\widehat{\mathbf{H}}_{\text{MMSE}} &= E\{\mathbf{H}\} + (\mathbf{Y} - E\{\mathbf{Y}\})\mathbf{A}_\circ \\ &= \mathbf{M} + (\mathbf{Y} - \mathbf{M}\mathbf{X})\mathbf{A}_\circ,\end{aligned} \tag{31}$$

where $\mathbf{A}_\circ$ has to be obtained so that the following TMSE is minimized

$$J_{\text{MMSE}} = E\left\{\left\|\mathbf{H} - \widehat{\mathbf{H}}_{\text{MMSE}}\right\|_F^2\right\}. \tag{32}$$

The optimal $\mathbf{A}_\circ$ can be found from $\partial J_{\text{MMSE}}/\partial \mathbf{A}_\circ = 0$ and it is given by

$$\mathbf{A}_\circ = \left(\mathbf{X}^H\mathbf{C}_H\mathbf{X} + \sigma_n^2 N_R \mathbf{I}_{N_T-L}\right)^{-1}\mathbf{X}^H\mathbf{C}_H. \tag{33}$$

Proof. See the appendix. □

Substituting $\mathbf{A}_\circ$ back into (31), the linear MMSE estimator of $\mathbf{H}$ can be rewritten as

$$\begin{aligned}\widehat{\mathbf{H}}_{\text{MMSE}} &= \mathbf{M} + (\mathbf{Y} - \mathbf{M}\mathbf{X}) \\ &\quad \cdot \left(\mathbf{X}^H\mathbf{C}_H\mathbf{X} + \sigma_n^2 N_R \mathbf{I}_{N_T-L}\right)^{-1}\mathbf{X}^H\mathbf{C}_H.\end{aligned} \tag{34}$$

It is notable that in the frequency-selective Rayleigh fading MIMO channel, $\mathbf{M} = 0, \mathbf{C}_H = \mathbf{R}_H$. The performance of MMSE channel estimator is measured by the error matrix $\boldsymbol{\varepsilon} = \mathbf{H} - \widehat{\mathbf{H}}_{\text{MMSE}}$, whose pdf is Gaussian with zero mean and

$$\mathbf{C}_\varepsilon = \mathbf{R}_\varepsilon = E\left\{\boldsymbol{\varepsilon}^H\boldsymbol{\varepsilon}\right\} = \left(\mathbf{C}_H^{-1} + \frac{1}{\sigma_n^2 N_R}\mathbf{X}\mathbf{X}^H\right)^{-1}. \tag{35}$$

The MMSE estimation error can also be computed as

$$J_{\text{MMSE}} = E\left\{\left\|\mathbf{H} - \widehat{\mathbf{H}}_{\text{MMSE}}\right\|_F^2\right\} = E\left\{\operatorname{tr}\left(\boldsymbol{\varepsilon}^H\boldsymbol{\varepsilon}\right)\right\} \tag{36}$$

$$= \operatorname{tr}\{\mathbf{C}_\varepsilon\} = \operatorname{tr}\left\{\left(\mathbf{C}_H^{-1} + \frac{1}{\sigma_n^2 N_R}\mathbf{X}\mathbf{X}^H\right)^{-1}\right\}. \tag{37}$$

Let us find $\mathbf{X}$ which minimizes the channel estimation error subject to a transmitted power constraint. This is equivalent to the following optimization problem

$$\begin{aligned}&\min_{\mathbf{X}} \quad \operatorname{tr}\left\{\left(\mathbf{C}_H^{-1} + \frac{1}{\sigma_n^2 N_R}\mathbf{X}\mathbf{X}^H\right)^{-1}\right\} \\ &\text{S.T.} \quad \operatorname{tr}\left\{\mathbf{X}\mathbf{X}^H\right\} = P.\end{aligned} \tag{38}$$

By using ZCZ training sequences that satisfy(18), $\mathbf{C}_H^{-1} + (1/\sigma_n^2 N_R)\mathbf{X}\mathbf{X}^H$ will be a diagonal matrix. Note that $\mathbf{C}_H$ in (12) is a diagonal matrix. Therefore, according to the lemma 1 in [7] (see also the proposition 2 in [24]) and by using (12) and (20), we obtain that the TMSE (37) will be minimized as

$$(J_{\text{MMSE}})_{\min} = \sigma_n^2 N_R N_T \sum_{l=0}^{L} \frac{b_l}{p(N_P - L)b_l + \sigma_n^2(\kappa+1)}. \tag{39}$$

When $\kappa = 0$, (39) is analogous to the acquired result in [24, 25] for LMMSE estimator. For $\kappa > 0$, the minimum CRLB (39) is lower than the minimum CRLB of this channel estimator. Equation (39) will be equal to (27) when the channel has uniform PDP. In this case, using (13), (18), and (20) the MMSE channel estimator (34) reduces to

$$\widehat{\mathbf{H}}_{\text{MMSE}} = \beta\mathbf{M} + \alpha\mathbf{Y}\mathbf{X}^H, \tag{40}$$

where

$$\begin{aligned}\alpha &= \frac{1}{p(N_P - L) + \sigma_n^2(1+L)(1+\kappa)}, \\ \beta &= \frac{\sigma_n^2(L+1)(\kappa+1)}{p(N_P - L) + \sigma_n^2(1+L)(1+\kappa)}.\end{aligned} \tag{41}$$

Then, the SSLS and MMSE channel estimators are identical within the uniform PDP.

6. Simulation Results

In this section, the performance of the LS, SLS, SSLS, and MMSE channel estimators is numerically examined in the frequency-selective Rayleigh and Rician fading channels. It is assumed that each sub-channel has the exponential PDP as

$$b_l = \frac{(1 - e^{-1})e^{-l}}{1 - e^{-L-1}}; \quad l = 0, 1, \ldots, L. \tag{42}$$

As a performance measure, we consider the channel TMSE, normalized by the average channel energy as

$$\text{NTMSE} = \frac{E\left\{\left\|\mathbf{H} - \widehat{\mathbf{H}}\right\|_F^2\right\}}{E\left\{\|\mathbf{H}\|_F^2\right\}}. \tag{43}$$

Here, we denote a ZCZ set with length $N = N_P - L$, size N_T, and ZCZ length $Z = L$ by ZCZ-(N, N_T, Z). In the following subsections, we present several numerical examples to illustrate both the superiority and reasonability of the proposed SSLS and MMSE channel estimators in the frequency-selective Rician fading models.

6.1. The Shorter Training Length to Estimate the Rician Fading Model. Figure 1 shows the normalized TMSE $J_{\text{LS}}/N_R N_T$ of the LS channel estimator versus SNR in the Rayleigh ($\kappa = 0$) and Rician ($\kappa = 1, 10$) fading channels. As it is expected, the performance of the LS estimator is independent of the fading model. In order to improve the performance of this estimator, the training length may be increased. It is notable that the bandwidth is wasted when the training length is increased.

Figures 2 and 3 show the normalized TMSE of SSLS and MMSE channel estimators, respectively, versus SNR in the Rayleigh ($\kappa = 0$) and Rician ($\kappa = 1, 10$) fading channels. It is observed that for the given length of training sequences, the performance of SSLS and MMSE estimators in the Rician fading channel is significantly better than the Rayleigh one.

In the Rayleigh fading model, increasing the training length improves the normalized TMSE of the estimators. However, in the Rician fading channels, the performance of both SSLS and MMSE estimators with a shorter training length is better than the Rayleigh fading model with a longer training length particularly at low SNRs and high Rice factors. Then, the training length can be reduced in the presence of the Rician channel model. At higher SNRs, the normalized TMSEs of each estimator with various Rice factors are nearly identical. In practice, for the given values of TMSE, SNR, and κ, the optimum training length can be calculated from (27), (39), or these figures.

The sequences under test in Figures 1 through 3 are ZCZ-$(4, 2, 1)$ and ZCZ-$(8, 2, 1)$ sets [26]. It is notable that these results are obtained based on both the channel model and the channel Rice factor which are defined in Section 2.

6.2. Comparing the LS-Based and MMSE Channel Estimators. All estimators are optimal because they achieve their minimum possible CRLB. However, the performance of the estimators is different. This subsection compares the computational complexity and performance of the LS, SLS, SSLS, and MMSE estimators. As illustrated in Table 1 and Figures 4 and 5 due to lower number of multiplications and additions, the LS-based (LS, SLS, and SSLS) estimators have lower computational complexity than MMSE estimator. Moreover, LS-based algorithms do not include the matrix inverse operation. However, the LMMSE channel estimator of [25, 29] cannot fundamentally benefit from the Rice factor of the Rician fading channels. The general form of this estimator has a complexity near to it, while it can fully exploit a priori knowledge of the $\mathbf{C}_H$ and $\mathbf{M}$.

In Figures 6 and 7, the performances of LS-based and MMSE estimators are compared in the cases of $L = 4$ and $L = 8$, respectively. The ZCZ-$(16, 2, 4)$ and ZCZ-$(64, 4, 8)$ sets are used in these figures, respectively. We obtained the ZCZ-$(64, 4, 8)$ set using the algorithm of [28] and the $(P, V, M) = (16, 4, 2)$ code of [26]. Table 2 shows the generated ZCZ-$(64, 4, 8)$ set. As depicted, the MMSE channel estimator has the best performance among all the methods tested. However, it requires that the channel PDP and σ_n^2 as well as κ be known a priori. For the large values of L, the MMSE channel estimator outperforms the SSLS channel estimator. However, for the small values of L, the performances of both estimators are similar. Practically, even small values of L lead to enough accuracy for the channel order approximation if there is a good synchronization. Hence, the SSLS channel estimator that requires less knowledge about the channel statistics and has lower complexity than the MMSE estimator can be used. Furthermore, the normalized TMSEs of the SSLS and MMSE estimators coincide at low SNRs when the Rice factor increases. It is noteworthy that the performances of the two above-mentioned estimators are always identical in uniform PDP.

6.3. The Rician Fading Model with a Higher Number of Antennas. In Figures 8 and 9, the effect of both the channel fading type and the number of Tx-Rx antennas is considered in a joint state. The two sets of ZCZ-$(64, 2, 8)$, that is, $\mathbf{x}_1$ and $\mathbf{x}_2$ of Table 2, and ZCZ-$(64, 4, 8)$ are employed in 2×2 and 4×4 MIMO systems, respectively, the former system has the Rayleigh fading and the latter one has the Rician model. At low SNRs, it is seen that the performance of the SSLS and MMSE estimators in the Rician fading model with a higher number of antennas is still better than the Rayleigh fading model with lower number of antennas especially at high Rice factors. At higher SNRs, the performances of the above mentioned estimators in both models are analogous. It is noteworthy that the capacity of MIMO system increases almost linearly with the number of antennas. It should also be noted that Rician fading can improve capacity, particularly when the value of κ is known at the transmitter [2].

6.4. Increasing Rice Factor. Figure 10 indicates the channel estimation normalized TMSE of the LS, SSLS, and MMSE estimators versus κ for SNR $= 10$ dB. From this figure, it is observed that increasing the Rice factor leads to decreasing the normalized TMSE of the SSLS and MMSE channel estimators. At high Rice factors, the performances of the proposed estimators are analogous particularly at low SNRs and for the small values of L (see also Figures. 6 and 7). It is noteworthy that the TMSE of LS and SLS estimators is independent of κ. The channel will be no fading or AWGN when $\kappa \to \infty$.

6.5. Substantial Benefits of the Rician Fading MIMO Channels. In Tables 3 and 4, substantial benefits of the frequency-selective Rician fading MIMO channels are shown using the SSLS and MMSE estimators. According to these tables, a lower SNR or shorter training length can be used to estimate the channel in the presence of the Rician model. In practice, the Rice factor can be measured at the receiver and fed back to the transmitter to adjust the SNR or training length. Hence, resources can be saved in the interested channel model. As illustrated in these tables, a higher number of antennas may be used in the mentioned channel without increasing TMSE. This means that the capacity of MIMO systems is increased.

It is generally true that the less the channel estimation error, the better the bit error rate (BER) performance for a fixed data detection scheme. The proposed methods can also guarantee the best BER performance for a given detection method.

6.6. A New Algorithm to Estimate the Rice Factor. The difference of the proposed estimators with the other estimators such as SLS of [7, 21] or LMMSE of [25] is that the performance of our proposed estimators can be improved because of exploiting the Rice factor, while the other methods cannot use this factor. In order to perform the proposed SSLS and MMSE channel estimators in the Rician fading MIMO channels, it is required that the channel Rice factor be known at the receiver. In this subsection, we propose an algorithm to estimate κ. This algorithm has the following steps.

Table 1: Computational complexity of the LS-based and MMSE channel estimators ($N_P = N_T(L+1)+L$).

Channel estimation algorithm	Number of real multiplications	Number of real additions	Matrix inverse operation
LS, SLS	$2N_RN_T^2(L+1)^2$	$2N_RN_T^2(L+1)^2 - 2N_RN_T(L+1)$	No
SSLS	$2N_RN_T^2(L+1)^2$	$2N_RN_T^2(L+1)^2$	No
MMSE ($\kappa = 0$)	$3N_T^3(L+1)^3 + 2N_RN_T^2(L+1)^2$	$3N_T^3(L+1)^3 - 2N_T^2(L+1)^2 + 2N_RN_T^2(L+1)^2 - 2N_RN_T(L+1)$	Yes
General MMSE	$3N_T^3(L+1)^3 + 4N_RN_T^2(L+1)^2$	$3N_T^3(L+1)^3 - 2N_T^2(L+1)^2 + 4N_RN_T^2(L+1)^2 - 2N_RN_T(L+1)$	Yes

Table 2: ZCZ-(64, 4, 8) set.

$\mathbf{x}_1 = [1\ -1\ -1\ -1\ -1\ -1\ 1\ -1\ 1\ -1\ -1\ -1\ -1\ -1\ 1\ -1\ -1\ 1\ 1\ 1\ -1\ -1\ 1\ -1\ -1\ 1\ 1\ 1\ -1\ -1\ 1\ -1\ 1\ -1\ -1\ -1\ -1\ -1\ 1\ -1\ -1$ $1\ 1\ 1\ 1\ 1\ -1\ 1\ -1\ 1\ 1\ 1\ -1\ -1\ 1\ -1\ 1\ -1\ -1\ -1\ 1\ 1\ -1\ 1]$
$\mathbf{x}_2 = [1\ -1\ 1\ 1\ -1\ -1\ -1\ 1\ 1\ -1\ 1\ 1\ -1\ -1\ -1\ 1\ -1\ 1\ -1\ -1\ -1\ -1\ -1\ 1\ -1\ 1\ -1\ -1\ -1\ -1\ -1\ 1\ 1\ -1\ 1\ 1\ -1\ -1\ -1\ 1\ -1\ 1$ $-1\ -1\ 1\ 1\ 1\ -1\ -1\ 1\ -1\ -1\ -1\ -1\ -1\ 1\ 1\ -1\ 1\ 1\ 1\ 1\ 1\ -1]$
$\mathbf{x}_3 = [1\ -1\ -1\ -1\ -1\ -1\ 1\ -1\ -1\ 1\ 1\ 1\ 1\ 1\ -1\ 1\ -1\ 1\ 1\ 1\ -1\ -1\ 1\ -1\ 1\ -1\ -1\ -1\ 1\ 1\ -1\ 1\ 1\ -1\ -1\ -1\ -1\ -1\ 1\ -1$ $1\ -1\ -1\ -1\ -1\ -1\ 1\ -1\ -1\ 1\ 1\ 1\ -1\ -1\ 1\ -1\ -1\ 1\ 1\ 1\ -1\ -1\ 1\ -1]$
$\mathbf{x}_4 = [1\ -1\ 1\ 1\ -1\ -1\ -1\ 1\ -1\ 1\ -1\ -1\ 1\ 1\ 1\ -1\ -1\ 1\ -1\ -1\ -1\ -1\ -1\ 1\ 1\ -1\ 1\ 1\ 1\ 1\ 1\ -1\ 1\ -1\ 1\ 1\ -1\ -1\ -1\ 1\ 1\ -1\ 1$ $1\ -1\ -1\ -1\ 1\ -1\ 1\ -1\ -1\ -1\ -1\ -1\ 1\ -1\ 1\ -1\ -1\ -1\ -1\ -1\ 1]$

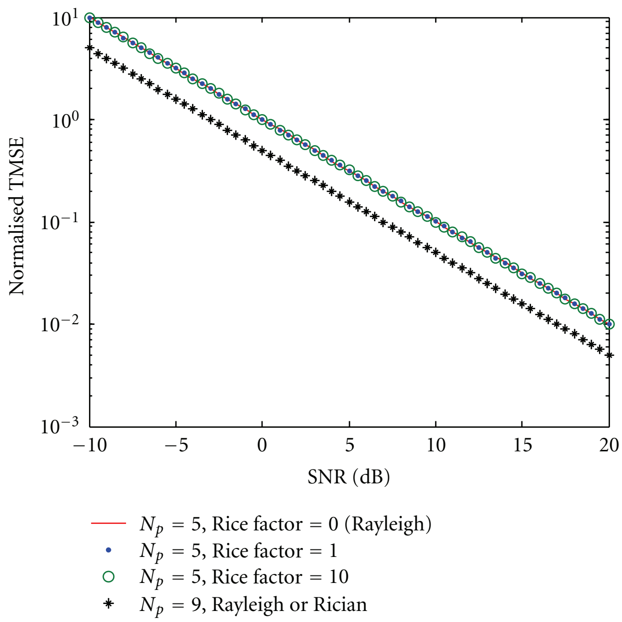

Figure 1: Normalized TMSE of the LS estimator in the Rayleigh and Rician fading channels ($N_T = N_R = 2,\ L = 1,\ N_P = 5, 9$).

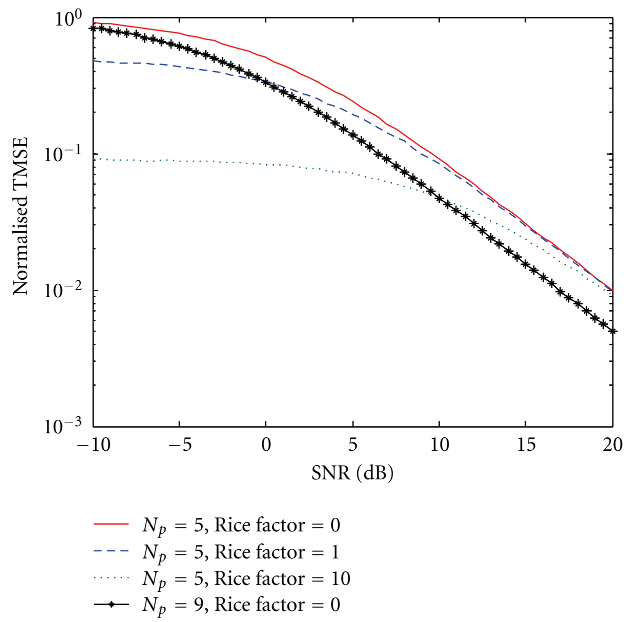

Figure 2: Normalized TMSE of the SSLS estimator in the Rayleigh and Rician fading channels ($N_T = N_R = 2,\ L = 1,\ N_P = 5, 9$).

Table 3: Substantial benefits of Rician fading MIMO channel by using SSLS estimator ($L = 8$).

N_T	N_P	SNR (dB)	κ	Normalized TMSE
2	72	5	0	8.17×10^{-2}
2	72	−4.56	10	8.17×10^{-2}
2	40	5	10	6.02×10^{-2}
4	72	5	10	6.02×10^{-2}

Table 4: Substantial benefits of Rician fading MIMO channel by using MMSE estimator ($L = 8$).

N_T	N_P	SNR (dB)	κ	Normalized TMSE
2	72	5	0	4.60×10^{-2}
2	72	−0.58	10	4.60×10^{-2}
2	40	5	10	3.46×10^{-2}
4	72	5	10	3.46×10^{-2}

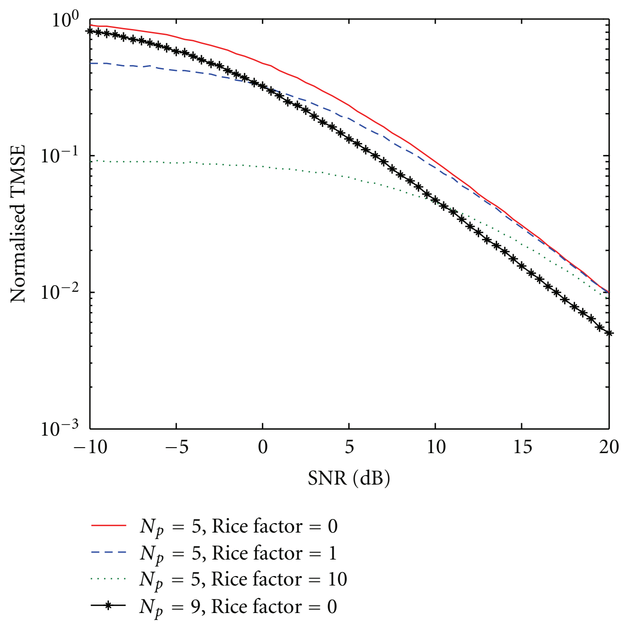

FIGURE 3: Normalized TMSE of the MMSE estimator in the Rayleigh and Rician fading channels ($N_T = N_R = 2$, $L = 1$, $N_P = 5, 9$).

Step 1. Calculate the mathematical expectation matrix of the channel by using the LS estimates of $\mathbf{H}$ during the observed $\mathbf{N}$ previous blocks as follows:

$$\widehat{\mathbf{M}}_n = \frac{1}{n}\sum_{i=1}^{n}\widehat{\mathbf{H}}_{\mathrm{LS}_{(i)}} \quad (n = 1, 2, \ldots, N). \tag{44}$$

Step 2. Partition $\widehat{\mathbf{M}}_n$ to $\widehat{\mathbf{M}}_n = [\widehat{\mathbf{M}}_{n0} \quad \widehat{\mathbf{M}}_{n1} \quad \cdots \quad \widehat{\mathbf{M}}_{nL}]$, where $\widehat{\mathbf{M}}_{nl} = E\{\mathbf{H}_l\}$.

Step 3. Estimate the $\boldsymbol{\mu}$ parameter (based on (11)) for all paths of the multipath channel as

$$\hat{\boldsymbol{\mu}}_{\mathbf{nl}} = \mathbf{abs}\left[\mathbf{mean}\left(\widehat{\mathbf{M}}_{\mathbf{nl}}\right)\right], \quad \forall \mathbf{l} \in [\mathbf{0}, \mathbf{1}, \ldots, \mathbf{L}], \quad \mathbf{n} \in [\mathbf{1}, \mathbf{2}, \ldots, \mathbf{N}]. \tag{45}$$

Step 4. Calculate the Rice factor for all paths of the multipath channel as

$$\hat{\boldsymbol{\kappa}}_{\mathbf{nl}} = \hat{\boldsymbol{\mu}}_{\mathbf{nl}}^{2} / \left(\mathbf{b_l} - \hat{\boldsymbol{\mu}}_{\mathbf{nl}}^{2}\right), \quad \forall \mathbf{l} \in [\mathbf{0}, \mathbf{1}, \ldots, \mathbf{L}], \quad \mathbf{n} \in [\mathbf{1}, \mathbf{2}, \ldots, \mathbf{N}]. \tag{46}$$

Step 5. Calculate the channel Rice factor by calculating the mean value of the several paths' Rice factors in the following form:

$$\hat{\kappa}_n = \frac{1}{L}\sum_{l=0}^{L}\hat{\kappa}_{nl}, \quad \forall n \in [1, 2, \ldots, N]. \tag{47}$$

Step 6. Estimate the final Rice factor by calculating the mode value of the several estimated Rice factors during the observed N consecutive blocks as

$$\hat{\kappa} = \mathrm{mode}(\mathbf{K}), \quad \mathbf{K} = [\hat{\kappa}_1, \hat{\kappa}_2, \ldots, \hat{\kappa}_N]. \tag{48}$$

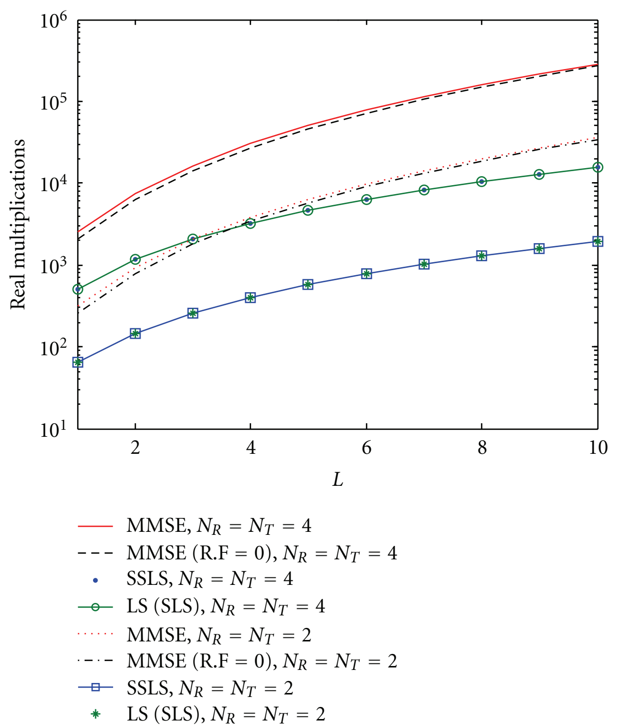

FIGURE 4: Computational complexity of the LS-based and MMSE channel estimators (Real multiplications for $N_T = N_R = 2$ and $N_T = N_R = 4$).

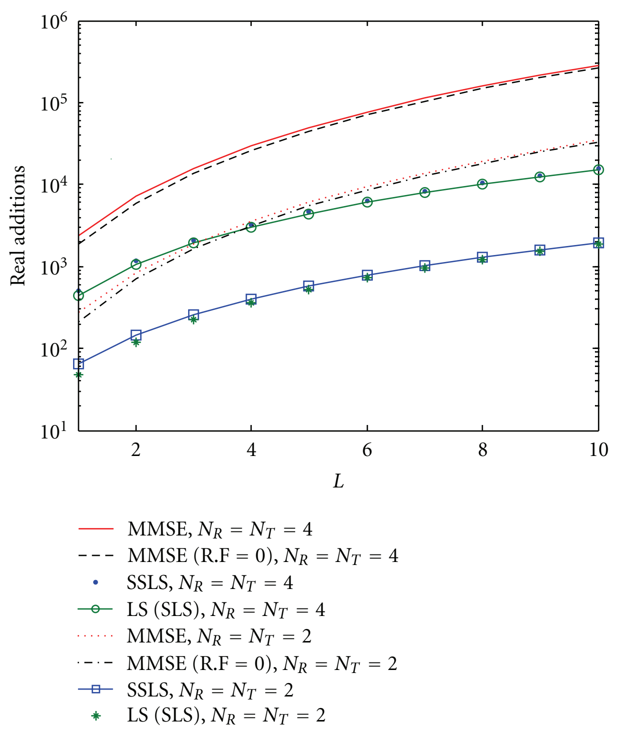

FIGURE 5: Computational complexity of the LS-based and MMSE channel estimators (Real additions for $N_T = N_R = 2$ and $N_T = N_R = 4$).

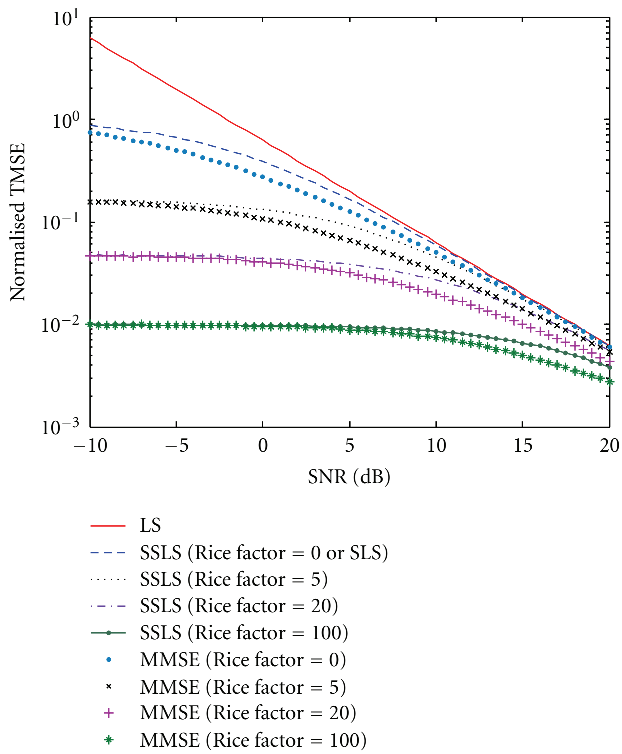

Figure 6: Normalized TMSEs of LS-based and MMSE estimators for various Rice factors in the case of $L = 4$, $N_T = N_R = 2$, $N_P = 20$.

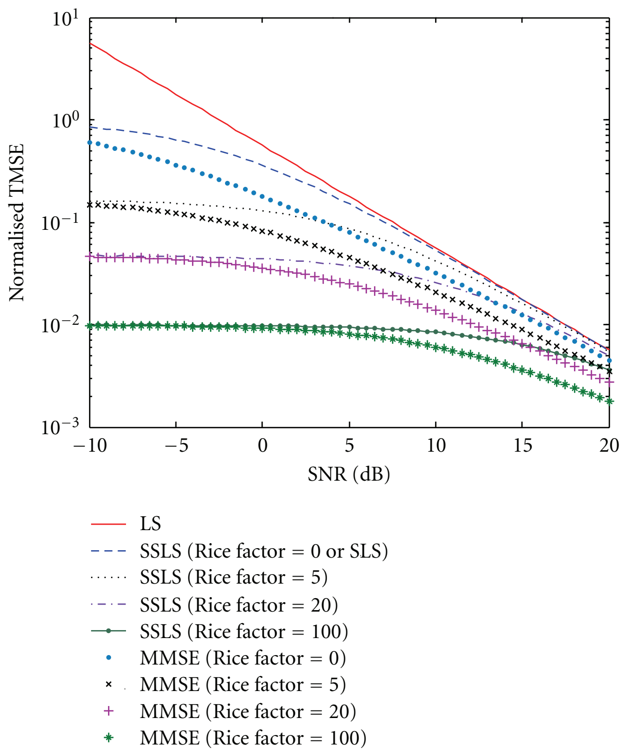

Figure 7: Normalized TMSEs of LS-based and MMSE estimators for various Rice factors in the case of $L = 8$, $N_T = N_R = 4$, $N_P = 72$.

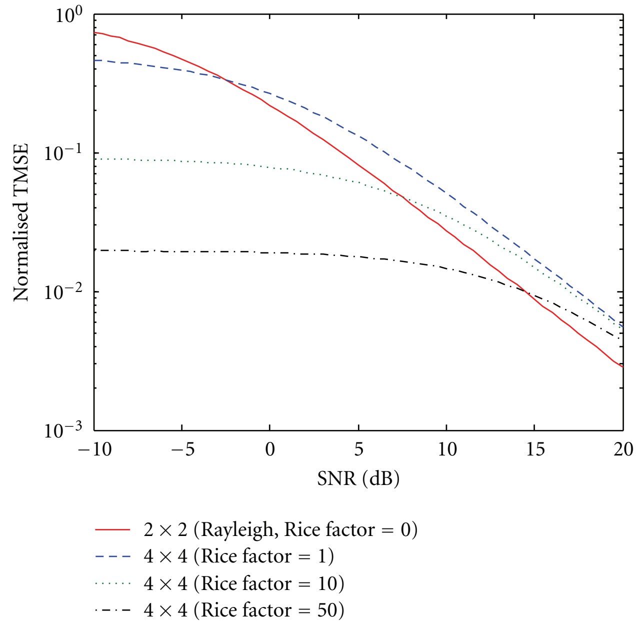

Figure 8: Normalized TMSEs of the SSLS estimator versus SNR in Rayleigh and Rician fading MIMO systems with $L = 8$, $N_P = 72$.

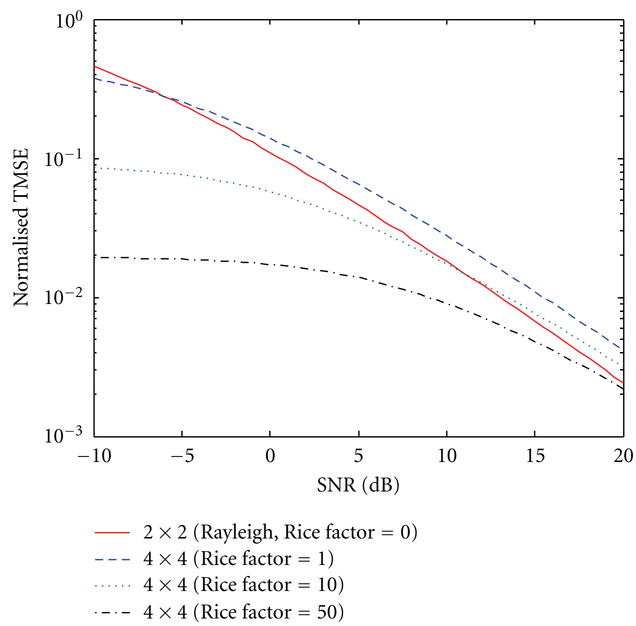

Figure 9: Normalized TMSEs of the MMSE estimator versus SNR in Rayleigh and Rician fading MIMO systems with $L = 8$, $N_P = 72$.

In simulation processes, it is seen that for some restricted values of N, the estimated Rice factors in Step 5 deviate from the actual values of the Rice factor randomly (not shown). This event especially occurs at low SNRs and high values of κ. Step 6 is used to remove this deficiency. In this step, we use MATLAB FUNCTION (HIST and MAX) to calculate the mode value of the elements in vector $\mathbf{K}$. Hence, the accurate Rice factor can be obtained. It is assumed that the channel

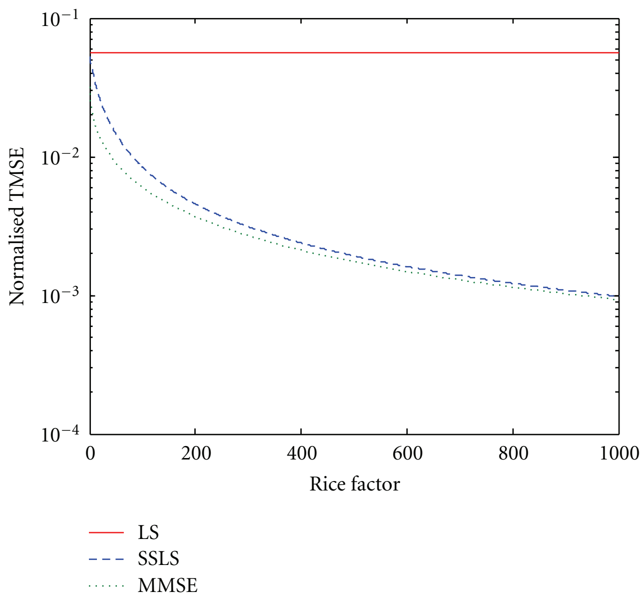

Figure 10: Normalized TMSE of the LS, SSLS, and MMSE estimators versus Rice factor for SNR = 10 dB, $N_T = N_R = 4$, $L = 8$, $N_P = 72$.

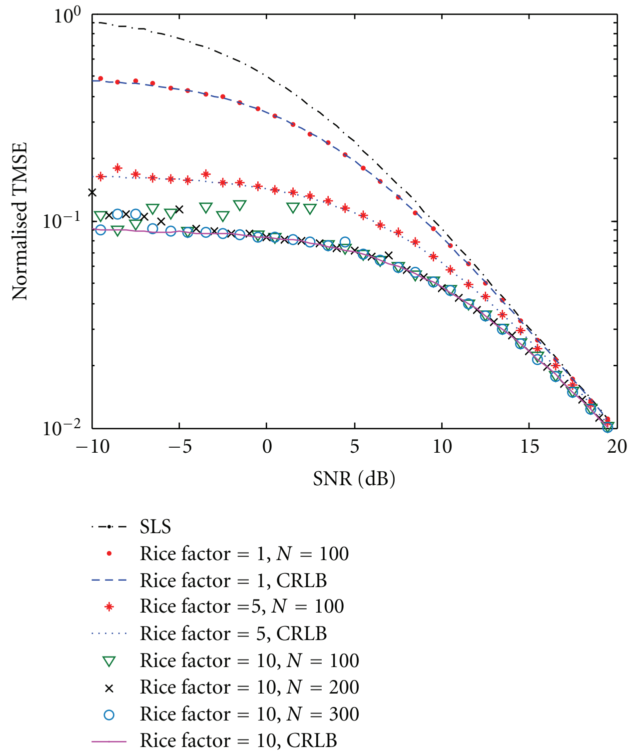

Figure 11: Normalized TMSE of the SSLS estimator by using the Rice factor estimation algorithm ($N_T = N_R = 2$, $L = 1$, $N_P = 5$).

Rice factor is stable during the received N consecutive blocks. It should be noted that the channel Rice factor estimator can be updated using a sliding window comprising N blocks, which would be useful in real-time estimation of κ.

For example, the performance of the SSLS and MMSE channel estimators using the aforementioned algorithm is probed in Figures 11 and 12. First, the channel Rice factor is estimated using the proposed algorithm. Then, the result is applied to the channel estimator. In order to compare the results with other works, normalized TMSE of the SLS estimator of [21] and the MMSE estimator in the case of $\kappa = 0$ is plotted in Figures 11 and 12, respectively. Also, the CRLB of the channel estimators is displayed as a reference. As depicted, the normalized TMSE of the channel estimators using the proposed algorithm is very close to the CRLB, especially for low values of κ. However, for high values of κ, the results diverge from CRLB, particularly at low SNRs. Nevertheless, it is observed that increasing the number of received blocks, N, leads to a better result for normalized TMSE of the channel estimators.

7. Conclusion

In this paper, the performance of training-based channel estimators in the frequency-selective Rician fading MIMO channels is investigated. The conventional LS technique and proposed SSLS and MMSE approaches have been probed. The MMSE channel estimator has better performance among the tested estimators, but it requires more knowledge about the channel. For channels with uniform PDP or a lower number of taps, the SSLS estimator is acceptable. However, for nonuniform PDP with a higher number of taps, the MMSE channel estimator is required to attain a lower TMSE.

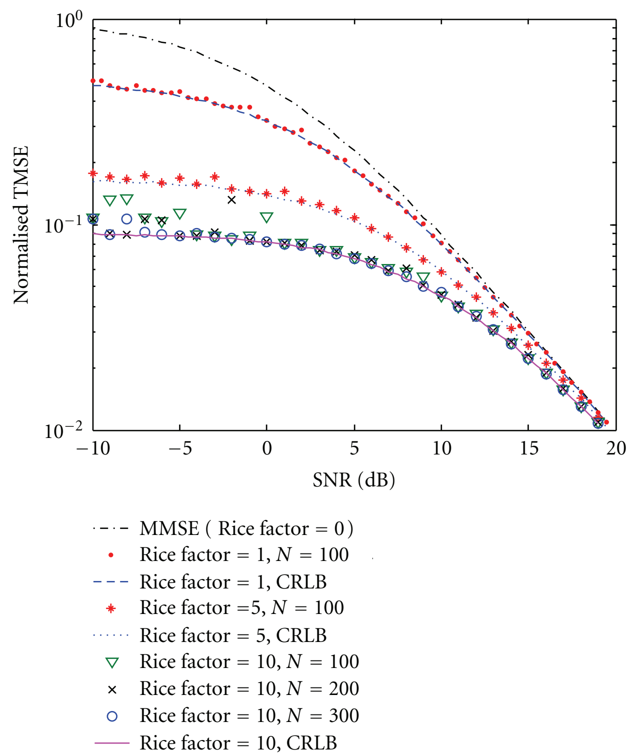

Figure 12: Normalized TMSE of the MMSE estimator by using the Rice factor estimation algorithm ($N_T = N_R = 2$, $L = 1$, $N_P = 5$).

In general, the SSLS technique provides a good tradeoff between the TMSE performance and the required knowledge about the channel. Moreover, the computational complexity of this estimator is lower than that of MMSE and near to that of LS estimator. Finally, we proposed an algorithm to estimate the channel Rice factor. Numerical results validate the good performance of this algorithm in Rician fading MIMO channel estimation.

The estimators suggested in this paper can be practically used in the design of MIMO systems. For instance, in order to obtain a given value of TMSE in the Rician channel model, either the required SNR may be decreased or the training length can be reduced. Then, resources will be saved. Besides, for the given values of the SNR, training length, and TMSE in the aforementioned channel model, the number of antennas can be increased. It is worthwhile to note that the excess of antenna numbers in MIMO systems leads to a higher capacity. It is also remarkable that the Rician fading is known as a more appropriate model for wireless environments with a dominant direct LOS path. This type of the fading model, especially in the microcellular mobile systems and LOS mode of WiMAX, is more suitable than the Rayleigh one.

Appendix

Proof of (33)

Using (31), the TMSE (32) yields

$$\begin{aligned} J_{\mathrm{MMSE}} &= E\{\|\mathbf{H}-\mathbf{M}-(\mathbf{Y}-\mathbf{MX})\mathbf{A}_\circ\|_F^2\} \\ &= E\Big\{\mathrm{tr}\Big\{[\mathbf{H}-\mathbf{M}-(\mathbf{Y}-\mathbf{MX})\mathbf{A}_\circ]^H \\ &\qquad \cdot [\mathbf{H}-\mathbf{M}-(\mathbf{Y}-\mathbf{MX})\mathbf{A}_\circ]\Big\}\Big\}. \end{aligned} \tag{A.1}$$

With some calculations, the TMSE (A.1) is given by

$$\begin{aligned} J_{\mathrm{MMSE}} &= \mathrm{tr}\Big\{\Big(\mathbf{I}_{N_T-L}-\mathbf{A}_\circ{}^H\mathbf{X}^H\Big)\mathbf{C}_H\cdot(\mathbf{I}_{N_T-L}-\mathbf{X}\mathbf{A}_\circ)\Big\} \\ &\quad + \sigma_n^2 N_R\,\mathrm{tr}\Big\{\mathbf{A}_\circ{}^H\mathbf{A}_\circ\Big\}. \end{aligned} \tag{A.2}$$

The optimal $\mathbf{A}_\circ$ can be found from

$$\frac{\partial J_{\mathrm{MMSE}}}{\partial \mathbf{A}_\circ} = -\mathbf{X}^T\mathbf{C}_H + \mathbf{X}^T\mathbf{C}_H\mathbf{X}^*\mathbf{A}_\circ{}^* + \sigma_n^2 N_R\mathbf{A}_\circ{}^* = 0. \tag{A.3}$$

Finally, we have

$$\mathbf{A}_\circ = \Big(\mathbf{X}^H\mathbf{C}_H\mathbf{X}+\sigma_n^2 N_R\mathbf{I}_{N_T-L}\Big)^{-1}\mathbf{X}^H\mathbf{C}_H. \tag{A.4}$$

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