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Research Article

Joint Sensing Period and Transmission Time Optimization for Energy-Constrained Cognitive Radios

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Under interference constraint and energy consumption constraint, to maximize the channel utilization, an opportunistic spectrum access (OSA) strategy for a slotted secondary user (SU) overlaying an unslotted ON/OFF continuous time Markov chain (CTMC) modeled primary network is proposed. The OSA strategy is investigated via a cross-layer optimization approach, with joint consideration of sensing period (related to PHY layer) and transmission time (related to MAC layer), which will affect both interference and energy consumption. Two access policies are investigated in this paper; that is, SU transmits only in "OFF slots" (i.e., the slots that the sensing results are OFF) and transmits in both "OFF slots" and "ON slots". The allocation of sensing period and transmission time for two access policies is investigated and analyzed by means of geometric methods. The closed form solutions are derived, which show that SU should transmit in "OFF slots" as much as possible, and that the proposed OSA strategy has low computational cost. Numerical results also show that with the proposed policies, SU can efficiently access the channel and meanwhile consume less energy and time to sense.

1. Introduction

With the wide deployment of wireless communication systems, the precious radio spectrum is becoming more and more crowded. On the other hand, the report published by Federal Communication Commission (FCC) shows that the current spectrum management policy has resulted in an underutilized spectrum [1].

Thus, cognitive radio (CR) [2] and opportunistic spectrum access (OSA) [3], as the means to deal with the spectrum underutilization problem, are proposed. The basic idea of OSA is to allow secondary users (SUs) to search for, identify, and exploit instantaneous spectrum opportunities while limiting the interference perceived by primary users (PUs). The interference depends on SU's access policy (namely, when and how to access the spectrum and the corresponding transmission time), power, rate, and so on. And to solve the interference problem, there are many works of the literature using all kinds of methods. In [4–6], the authors propose power control strategies for different system scenarios. In [7–9], the optimal access policies have been studied. And in [10, 11], the authors consider the effect of both power and rate.

On the other hand, in most practical situation, the SU is a battery-powered device. Thus, the energy consumption, as one of the most important problems that, affecting cognitive radio networks, should also be considered. Unfortunately, none of the former works take into account the energy consumption, and to the best of our knowledge, there are only a few work literature focus on this problem. In [12], the authors consider the impact of transmit power consumption, and the power consumption is integrated into the objective function named power efficiency, which is defined as throughput divided by power consumption. But, the work in [12] does not take into consideration the sensing energy consumption. In [13, 14], the authors take into account both sensing and transmission energy consumption, and within the framework of partially observable Markov decision process (POMDP), the optimal MAC policies for energy-constrained OSA have been studied. However, all of these works assume PU is time slotted. Under this assumption, SU is required to have a knowledge of slot time and synchronization of PU and SU is necessary. The synchronization request will cause more overhead and the time offset is fatal for the MAC policy. Therefore, we relax this assumption, which means PU is not time slotted, and part of this work has been presented in a previous paper [15].

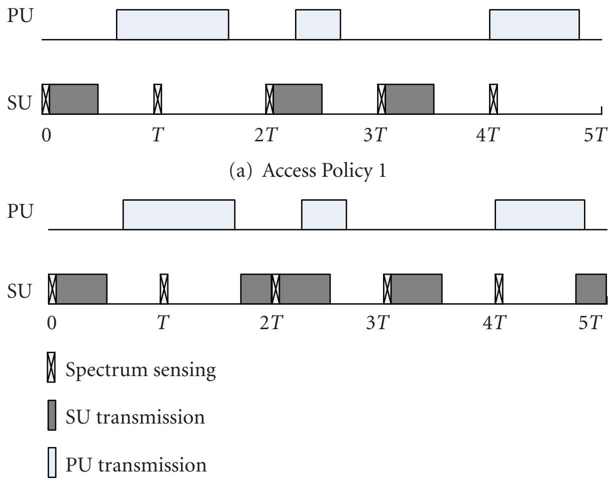
In this paper, to maximize the channel utilization, we address an OSA strategy for a slotted SU overlaying an unslotted primary network under interference constraint (IC) and energy consumption constraint (ECC). We consider the simplest cognitive radio system model which has only one channel available for transmissions by a pair of PU and SU. For the model of channel occupancy by the PU, we assume that it is a not-time-slotted two-state ON/OFF continuous time Markov chain (CTMC), which arise from [7]. While for SU, a time slotted (periodic) communication protocol is used. At the beginning of each slot, the SU senses the channel and then a specified volume of transmission time is allocated depends on the sensing results. Two access policies are investigated in this paper; that is, SU transmits only in "OFF slots" (i.e., the slots that the sensing results are OFF) and transmits in both "OFF slots" and "ON slots".

Since the SU is periodic, thus the period will affect the energy consumption and the identified result of channel state, which will affect the spectrum utilization. For example, smaller period will cause more energy consumption for sensing but better identified results for more transmission, and vice versa. Thus, suitable period should be chosen for better spectrum utilization and energy consumption. Then, we define the ECC as a function of the sensing period, according to the idea of battery life. On the other hand, the IC is modeled by the average temporal overlap between SU and PU.

The allocation of sensing period and transmission time for two access policies are investigated and analyzed by means of geometric methods. The closed-form solutions are derived, which show that SU should transmit in "OFF slots" as much as possible. Numerical results show that with the proposed policies, SU can efficiently access the channel, and meanwhile consumes less energy and time to sense the channel's state than the reference policies.

The rest of the paper is organized as follows. After introducing the system model and problem formulation in Section 2, the mathematical model and its the optimal allocation of sensing period and transmission time for two different access policies are derived in Sections 3 and 4, respectively. In Section 5, two reference access policies are introduced in order to put the performance of our proposed policies in perspective. And then, in Section 6, the simulation results are present and discussed. Finally, conclusions are stated in Section 7.

Throughout this paper, we use the following notation. $\Pr(\cdot)$ denotes the probability. $\mathcal{W}(\cdot)$ denotes the Lambert W function. $(\cdot)^{-}$ denotes that $(\cdot)$ minus an infinitesimal. $\min\{a, b\}$ and $\max\{a, b\}$ denote the minimal and maximal value of a and b, respectively.



(a) Access Policy 1

(b) Access Policy 2

Figure 1: Illustration of the time behavior of PU and SU under two different access policies.

2. System Model and Problem Formulation

In this section, we first present the system model and the time behaviors of PU and SU, and then introduce the interference constraint and energy consumption constraint.

2.1. System Model. We consider the simplest cognitive radio system model which has only one channel available for transmissions by a pair of PU and SU. The time behavior of both PU and SU is shown in Figure 1. We assume that the PU exhibits nontime slotted ON/OFF behavior, while the SU employs a time slotted communication protocol with period $T > 0$ (e.g., Bluetooth). In each slot, the SU senses the channel at the beginning of the slot, and then access this channel according to the sensing results.

Besides, we assume perfect sensing, and the sensing time is short enough to be ignored.

2.2. The Time Behavior of the PU. As mentioned above, the behavior of the PU is not time slotted and switches between ON and OFF states. Furthermore, we model the time behavior of the PU by a two-state ON/OFF CTMC, which arise from [7]. And this modeling approach has been used in related publications [8, 16] and solidified by a measurement-based analysis of WLAN traffic [17]. The CTMC assumption strikes a good tradeoff between model accuracy and the analytical tractability that is needed in the subsequent sections.

Based on stochastic theory [18], the holding times in both ON and OFF state are exponentially distributed with parameters $\mu > 0$ for the ON state and $\lambda > 0$ for the OFF state, and the transition matrix is given by

$$\mathbf{P}(\tau) = \frac{1}{\lambda + \mu} \begin{bmatrix} \mu + \lambda e^{-(\lambda+\mu)\tau} & \lambda - \lambda e^{-(\lambda+\mu)\tau} \\ \mu - \mu e^{-(\lambda+\mu)\tau} & \lambda + \mu e^{-(\lambda+\mu)\tau} \end{bmatrix}. \quad (1)$$

When the sensing result is in OFF state at time t_0, then the probability of PU being ON at time $t_0+\tau$ is given by the upper right entry in the transition matrix, that is, $(1/(\lambda+\mu))\ (\lambda - \lambda e^{-(\lambda+\mu)\tau})$.

We assume the channel state information μ and λ are known to SU.

2.3. SU's Access Policies. The SU's access policy, that is, the allocation of transmission time, will affect the channel utilization and the interference between PU and SU. For example, more transmission time can improve the spectrum usage, meanwhile may cause more interferences. Geirhofer et al. [7] has proved that it is optimal to transmit at the beginning (the end) of the slot if the sensing outcome is OFF (ON). Based on this result, we consider two access policies.

(1) Policy π_1: a ρ_0 fraction of period T transmission time is allocated at the beginning of the slot if and only if the sensing result is OFF, as shown in Figure 1(a).

(2) Policy π_2: if the sensing result is OFF (ON), a ρ_0 (ρ_1) fraction of period T transmission time is allocated at the beginning (the end) of the slot, as shown in Figure 1(b).

The policy π_1 is more intuitive, that is, it allows SU to transmit in "OFF slots", and the policy π_2 allows SU to try to utilize both "OFF slots" and "ON slots". Since policy π_1 can be seen as a special case of policy π_2 as $\rho_1 = 0$, thus, policy π_2 is no worse than policy π_1.

2.4. Interference Constraint. The interference between PU and SU is modeled by the average temporal overlap. Consider the activity of the PU is given by the CTMC $\{X(\xi), \xi \geq 0\}$ with parameters μ and λ. Based on the sensing result and access policy, from [7], we can get that if the sensing result is OFF, the expected time overlap $\phi_0(\rho_0, T)$ is given by

$$\begin{aligned}\phi_0(\rho_0, T) &= \frac{1}{T}\int_0^{\rho_0 T} \Pr(X(\xi) = 1 \mid X(0) = 0)\, d\xi \\ &= \frac{1}{T}\int_0^{\rho_0 T} \frac{1}{\lambda+\mu}\left(\lambda - \lambda e^{-(\lambda+\mu)\tau}\right) d\tau \\ &= \frac{\lambda}{(\lambda+\mu)T}\left(\rho_0 T + \frac{1}{\lambda+\mu}\left(e^{-(\lambda+\mu)\rho_0 T} - 1\right)\right),\end{aligned} \tag{2}$$

and if the sensing result is ON, the expected time overlap $\phi_1(\rho_1, T)$ is given by

$$\begin{aligned}&\phi_1(\rho_1, T) \\ &\quad = \frac{\lambda}{(\lambda+\mu)T}\left(\rho_1 T + \frac{\mu/\lambda}{\lambda+\mu} e^{-(\lambda+\mu)T}\left(e^{(\lambda+\mu)\rho_1 T} - 1\right)\right).\end{aligned} \tag{3}$$

Now, let $C \in [0, 1]$ be the maximum interference level tolerable by PU, namely, the percentage of interference time in the total time is no more than C. Then, the IC under policy π_1 can be written as

$$k \times \phi_0(\rho_0, T) \leq C \tag{4}$$

and the IC under policy π_2 can be written as

$$k \times \phi_0(\rho_0, T) + (1-k) \times \phi_1(\rho_1, T) \leq C, \tag{5}$$

where $k = \mu/(\mu+\lambda)$, which is the probability of the sensing result being OFF.

2.5. Energy Consumption Constraint. In most practical situations, the SU is a battery-powered device. Thus, the energy consumption, as one of the most important problem that affecting cognitive radio networks, should be considered. We define the ECC according to the idea of battery life, which means in per unit time, the sum of sensing and transmission energy consumption is less than or equal to some threshold, namely,

$$\frac{1}{T}(Q + p_t \beta T) \leq P, \tag{6}$$

where Q is the sensing energy consumption and p_t is the transmit power of SU, which is assumed to be constant in the following discussion. P is the maximum energy consumption per unit time tolerable by SU, and β is the channel utilization ratio. For policy π_1 and π_2, β is given by

$$\beta = k\rho_0, \tag{7}$$

$$\beta = k\rho_0 + (1-k)\rho_1, \tag{8}$$

respectively.

3. Optimal Allocation under Policy π_1

In the previous section, the system model and two concerned constraints (IC and ECC) have been established. In this section, we study the tradeoff of sensing period and transmission time under policy π_1, which leads to a convex problem. Since convex techniques could not get closed-form solutions, thus, through investigating the properties of IC and ECC, we transform this convex problem to a more intuitive geometrical problem, which leads to a closed-form solution. And we also given some intuitive explanations.

3.1. Problem Formulation. For policy π_1, under interference constraint (4) and energy consumption constraint (6), we try to find the optimal sensing period T and transmission time $\rho_0 T$ to maximize the channel utilization (7).

Mathematically, this leads to the problem **P1**

$$\begin{aligned}\max_{\rho_0, T} \quad & \beta = k \times \rho_0 \\ \text{s.t.} \quad & k \times \phi_0(\rho_0, T) \leq C, \\ & \frac{Q}{T} + k \times \frac{p_t \times \rho_0 T}{T} \leq P, \\ & T > 0, \\ & 0 \leq \rho_0 \leq 1.\end{aligned} \tag{9}$$

This problem leads to a convex optimization problem, thus convex optimization techniques can be used. However,

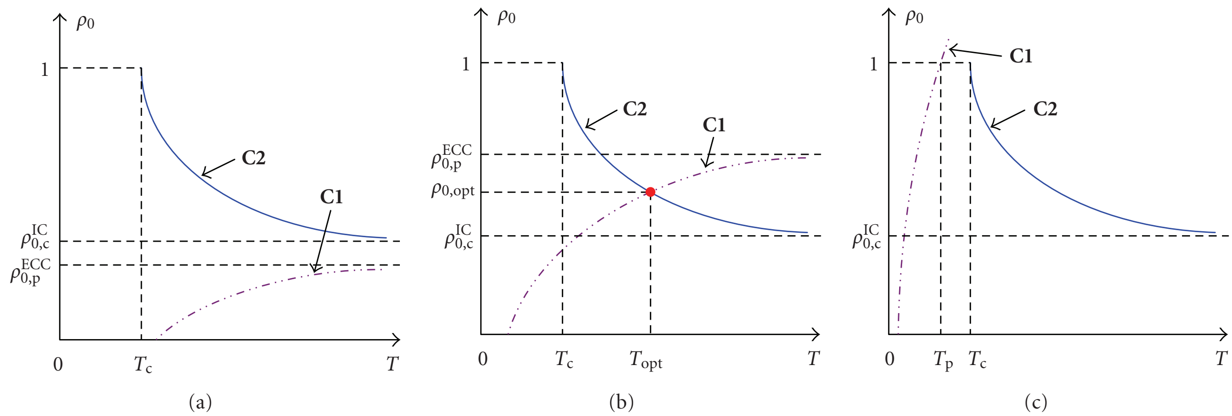


FIGURE 2: Illustration of the relationship of IC and ECC under access policy π_1. **C1** and **C2** are the curves when ECC and IC hold with equality, respectively.

these techniques could not get closed-form solutions. We study the properties of this model and transform it into a geometrical problem, which is more intuitive. And based on this geometrical model, we get its closed-form solutions.

Since k is constant for a given channel, maximizing the goal $k \times \rho_0$ equals to maximizing ρ_0. Thus, our goal equals to finding the maximal ρ_0.

3.2. Properties of IC and ECC. First, we recall the following Lemma from [7].

Lemma 1. *For any given $T > 0$, the expected time overlap $\phi_0(\rho_0, T)$ is strictly increasing in ρ_0.*

This lemma is easy to understand, because more transmission time causes more interference. Based on Lemma 1, merely under the IC, for any given sensing period T_1, one has that

(1) the maximal $\rho_0^{IC}(T_1)$ is obtained when the IC (4) holds with equality, that is, $k \times \phi_0(\rho_0, T_1) = C$,

(2) and when $\rho_0 \leq \rho_0^{IC}(T_1)$, the IC is always satisfied.

Similarly, we have the following lemma.

Lemma 2. *For any given $T > 0$, the energy consumption $(1/T)(Q + kp_t\rho_0 T)$ is strictly increasing in ρ_0.*

Since it is obvious, one omits the proof of this Lemma. Based on Lemma 2, one can obtain the same results that merely under the ECC, for any given sensing period T_1,

(1) the maximal $\rho_0^{ECC}(T_1)$ is obtained when the ECC (6) holds with equality, that is, $(1/T_1)(Q + kp_t\rho_0 T_1) = P$,

(2) and when $\rho_0 \leq \rho_0^{ECC}(T_1)$, the ECC is always satisfied.

Based on these lemmas, merely under IC or ECC, the maximal ρ_0 depends on T, which means given a sensing period T, we can get a maximal ρ_0, and for another T, we have another one. Thus, we have a question directly: Which T makes ρ_0 achieve the global optimum?

To answer this question, we first introduce the following two lemmas.

Lemma 3. *ρ_0^{IC}, the solution of $k \times \phi_0(\rho_0, T) = C$, is strictly decreasing in T and the infimum of ρ_0^{IC} is $\rho_{0,c}^{IC} = C/k(1-k)$.*

Proof. See Appendix A. □

Lemma 4. *ρ_0^{ECC}, the solution of $(1/T)(Q + kp_t\rho_0 T) = P$, is strictly increasing in T and the supremum of ρ_0^{ECC} is $\rho_{0,p}^{ECC} = P/kp_t$.*

Since it is obvious, we omit the proof of this lemma.

3.3. Optimal Allocation and Solution Structure. Based on Lemma 3, when $\rho_{0,c}^{IC} = C/k(1-k) \geq 1$ (i.e., $C \geq k(1-k)$), for all $\rho_0 \in [0, 1]$, the IC (4) is always satisfied, which means that when the sensing result is OFF, for any $T > 0$, the maximum interference level C is large enough for SU to transmit in the whole period T. Thus, in this situation, the problem **P1** only consists of ECC, and is simplified to

$$\begin{aligned}
\max_{\rho_0, T} \quad & \beta = k \times \rho_0 \\
\text{s.t.} \quad & \frac{Q}{T} + k \times \frac{p_t \times \rho_0 T}{T} \leq P, \\
& T > 0, \\
& 0 \leq \rho_0 \leq 1.
\end{aligned} \tag{10}$$

Thus, based on Lemmas 2 and 4, the optimal solutions of **P1** depend on the relationship of $\rho_{0,p}^{ECC}$ and 1.

Then, we focus on the situation of $C < k(1-k)$. Based on Lemma 3, we can obtain that maximizing ρ_0 is equal to minimizing T, while based on Lemma 4, maximizing ρ_0 is equal to maximizing T. Thus, as shown in Figure 2, under the IC and the ECC, the optimal ρ_0 can be obtained by the relationship of curves **C1**:

$$\frac{1}{T}(Q + kp_t\rho_0 T) = P \tag{11}$$

and **C2**:

$$k \times \phi_0(\rho_0, T) = C. \tag{12}$$

Therefore, considering the scope of C and P, we have obtained the optimal solution.

(1) When $C \geq k(1-k)$, C is large enough for SU to transmit in the whole "OFF slots". (since k is the probability of the sensing result is OFF, and $1-k$ is the probability of PU being ON, thus $k(1-k)$ can be interpreted as the average time overlap when SU transmits in the whole "OFF slots".) Thus, as we discussed, the optimal solutions of **P1** depend on the relationship of $\rho_{0,\mathrm{p}}^{\mathrm{ECC}}$ and 1.

   (a) When $\rho_{0,\mathrm{p}}^{\mathrm{ECC}} \leq 1$, each ρ_0, which satisfies the ECC, is less than 1. Similar to the situation of Figure 2(b) without curve **C2**, the optimal $\rho_{0,\mathrm{opt}}$ is obtained when $T = +\infty$. In practice, when $P \geq 100Q/T$, ρ_0 is close to $\rho_{0,\mathrm{opt}}$, so we can choose $T_{\mathrm{opt}} \approx 100Q/P$.

   (b) When $\rho_{0,\mathrm{p}}^{\mathrm{ECC}} > 1$, similar to the situation of Figure 2(c) without curve **C2**, we have that $\rho_{0,\mathrm{opt}} = 1$ and $T_{\mathrm{opt}} \geq T_{\mathrm{p}}$.

(2) When $C < k(1-k)$, the optimal solution is restrict by both IC and ECC.

   (a) When P is small, as shown in Figure 2(a), the ECC plays a major role. Thus, the optimal ρ_0 is obtain when $T = +\infty$.

   (b) As P increases, both IC and ECC take effect, as shown in Figure 2(b). Thus, the optimal ρ_0 is the intersection of **C1** and **C2**.

   (c) When the ECC is loose enough, the IC will play a major role, as shown in Figure 2(c). Thus, $\rho_{0,\mathrm{opt}} = 1$ and $T_{\mathrm{opt}} \in [T_{\mathrm{p}}, T_{\mathrm{c}}]$.

Expressed in mathematical language, the optimal solution can be obtained in the following theorem.

Theorem 5. *While $C \geq k(1-k)$, the optimal solution of* **P1** *is*

$$\rho_{0,\mathrm{opt}} = \begin{cases} \left(\dfrac{P}{kp_t}\right)^{-}, & 0 < P \leq kp_t \\ 1, & P > kp_t, \end{cases}$$
$$T_{\mathrm{opt}} \begin{cases} = +\infty, & 0 < P \leq kp_t \\ \in \left[T_{\mathrm{p}}, +\infty\right), & P > kp_t. \end{cases} \tag{13}$$

While $C < k(1-k)$, the optimal solution of **P1** *is*

$$\rho_{0,\mathrm{opt}} = \begin{cases} \left(\dfrac{P}{kp_t}\right)^{-}, & 0 < P \leq \dfrac{Cp_t}{1-k} \\ \dfrac{PT_{\mathrm{opt}} - Q}{kp_t T_{\mathrm{opt}}}, & \dfrac{Cp_t}{1-k} < P \leq \dfrac{Q}{T_c} + kp_t \\ 1, & P > \dfrac{Q}{T_c} + kp_t, \end{cases}$$
$$T_{\mathrm{opt}} \begin{cases} = +\infty, & 0 < P \leq \dfrac{Cp_t}{1-k} \\ = -\dfrac{1}{c}\mathcal{W}\left(-\dfrac{c}{a}e^{(ad-bc)/a}\right) - \dfrac{b}{a}, & \dfrac{Cp_t}{1-k} < P \leq \dfrac{Q}{T_c} + kp_t \\ \in \left[T_{\mathrm{p}}, T_{\mathrm{c}}\right], & P > \dfrac{Q}{T_c} + kp_t, \end{cases} \tag{14}$$

where $T_c = (1/(\lambda+\mu))(\mathcal{W}((1/m)e^{1/m}) - 1/m)$, $m = C/k(1-k) - 1$, $T_{\mathrm{p}} = Q/(P - kp_t)$ (while $\rho_{0,\mathrm{p}}^{\mathrm{ECC}} = P/kp_t > 1$) and

$$\begin{aligned} a &= \frac{\lambda+\mu}{k}\left(\frac{C}{1-k} - \frac{P}{p_t}\right), \\ b &= 1 + \frac{(\lambda+\mu)Q}{kp_t}, \\ c &= -\frac{(\lambda+\mu)P}{kp_t}, \\ d &= \frac{(\lambda+\mu)Q}{kp_t}. \end{aligned} \tag{15}$$

Proof. See Appendix B. □

4. Optimal Allocation under Policy π_2

The previous section derived the optimal allocation under policy π_1. The policy only allows SU to access "OFF slots", which may lead to lower spectrum utilization.

According to Theorem 5, under policy π_1, when the thresholds P and C are large enough, the maximal $\rho_{0,\max} = 1$, and the maximal channel utilization ratio $\beta_{\max} = k$. Therefore, when the ECC and IC get looser, the channel utilization ratio β will not increase any more. In an extreme instance, when $C = 1$ and $P = +\infty$, which means there are no interference constraint and energy consumption constraint, the optimal access policy of the SU is transmitting all the time regardless of the sensing result, and then the channel utilization ratio $\beta = 1$. Thus, the access policy π_1 is not suitable any more.

The main reason is SU do not access the "ON slots". Therefore, in this section, we will study the policy π_2, which allows SU access of both "ON slots" and "OFF slots". However, this problem is a nonconvex one. Similar to what we have done in the previous section, we investigate the properties of IC and ECC, and then transform this nonconvex problem to a geometrical problem, and finally obtain the closed-form solution.

4.1. Problem Formulation. Similar to the previous section, we can get the mathematical model **P2** of policy π_2, namely,

$$\begin{aligned} \max_{\rho_0,\rho_1,T} \quad & \beta = k\times\rho_0 + (1-k)\times\rho_1 \\ \text{s.t.} \quad & k\times\phi_0(\rho_0,T) + (1-k)\times\phi_1(\rho_1,T) \le C, \\ & \frac{Q}{T} + p_t\beta \le P, \\ & T>0, \\ & 0\le\rho_0, \quad \rho_1\le 1. \end{aligned} \tag{16}$$

It can be proved that problem **P2** is not a convex optimization problem, due to the fact that IC (5) of problem **P2** is not convex in T. Thus, for this nonconvex problem, we study its properties and transform it into a geometrical problem, and obtain the closed-form solution as we have done in the former section.

4.2. Properties of IC and ECC. First, we give some notation; $\Delta\rho_0$ and $\Delta\rho_1$ are the increments of ρ_0 and ρ_1, respectively. And we define that

$$\begin{aligned} \Delta\phi_0 &= \phi_0(\rho_0+\Delta\rho_0,T) - \phi_0(\rho_0,T), \\ \Delta\phi_1 &= \phi_1(\rho_1+\Delta\rho_1,T) - \phi_1(\rho_1,T). \end{aligned} \tag{17}$$

We recall the following lemma from [7].

Lemma 6. *For any given $T>0$, the expected time overlap $\phi_1(\rho_1,T)$ is strictly increasing in ρ_1.*

Based on Lemmas 1 and 6, increasing ρ_0 and ρ_1 will both increase the interference. On the other hand, increasing ρ_0 and ρ_1 can also raise the channel utilization ratio. However, from the following Lemma, one can know that the effect of increasing interference and channel utilization via increasing ρ_0 or ρ_1 is different.

Lemma 7. *For any given T, to generate the same interference, that is, $k\Delta\phi_0 = (1-k)\Delta\phi_1$, increasing ρ_0 can always make more channel utilization increment than increasing ρ_1, that is, $k\Delta\rho_0 > (1-k)\Delta\rho_1$. And when $\rho_0\neq 0$, $\rho_1\neq 1$ and $T\to+\infty$, the effect of increasing ρ_0 or ρ_1 are almost the same.*

Proof. See Appendix C. □

Based on Lemma 7, we know that increasing ρ_0 is always better than increasing ρ_1. In other words, SU should transmit in the "OFF slot", and if the transmission time could not increase, that is, $\rho_0=1$, then SU transmit in the "ON slot". Thus, we can obtain this following lemma directly.

Lemma 8. *ρ_1 is greater than 0 if and only if $\rho_0=1$.*

Based on Lemma 8, we can infer what follows.

(i) When $\rho_0<1$, $\rho_1=0$, and the optimal solution of **P2** can be obtained by **P1**.

(ii) When $\rho_0=1$, $\rho_1>0$, **P2** change into the following optimization problem **P3**, namely,

$$\begin{aligned} \max_{\rho_1,T} \quad & \beta = k + (1-k)\times\rho_1 \\ \text{s.t.} \quad & k\times\phi_0(1,T) + (1-k)\times\phi_1(\rho_1,T) \le C, \\ & \frac{Q}{T} + p_t\beta \le P, \\ & T>0, \\ & 0\le\rho_0\le 1. \end{aligned} \tag{18}$$

Furthermore, based on Lemma 8, if the IC or ECC is loose enough such that $\rho_0=1$, ρ_1 can be obtained by

$$k\times\phi_0(1,T) + (1-k)\times\phi_1(\rho_1,T) \le C. \tag{19}$$

Since $1-k=\lambda/(\lambda+\mu)$ is the probability of PU being ON, thus, if $C\ge 1-k$, for all $\rho_0,\rho_1\in[0,1]$, the IC (5) is always satisfied. Then, we focus on the situation of $C<1-k$. Based on Lemma 6, the maximal ρ_1 is obtained when (19) holds with equality. And similar to Lemma 3, we can reach the conclusion.

Lemma 9. *When $C<1-k$, ρ_1^{IC}, the solution of $k\times\phi_0(1,T) + (1-k)\times\phi_1(\rho_1,T) = C$, is strictly decreasing in T. The infimum of ρ_1^{IC} is $\rho_{1,c}^{\mathrm{IC}} = \lim_{T\to+\infty}\rho_1 = (C-k(1-k))/(1-k)^2$ and $\lim_{T\to 0}\rho_1^{\mathrm{IC}} = C/(1-k)$.*

Proof. See Appendix D. □

4.3. Optimal Allocation and Solution Structure. Based on Lemmas 3, 9, and 8, we can infer that under IC, the maximal channel utilization ratio β decreases as T increases. On the other hand, under ECC (6), the maximal β increases as T increases. Thus, similar to the form section, we can obtain the optimal solution of **P2** through the relationship of IC (5) and ECC (6), as shown in Figure 3.

(1) When $C\ge 1-k$ (Figure 3(a)), the IC is always satisfied for any ρ_0 and ρ_1. From the ECC (6), we can obtain that $\beta\le P/p_t - Q/p_tT < P/p_t$, therefore the optimal β of **P2** depends on the relationship between P/p_t and 1.

  (a) When $P/p_t\le 1$, the optimal β is obtained when $T=+\infty$. And furthermore, based on Lemma 8, if $P/p_t\le k$, the optimal $\rho_0<1$ and $\rho_1=0$; otherwise, $\rho_0=1$ and $\rho_1>0$.

  (b) When $P/p_t>1$, we have that $\beta=1$ and $T_{\mathrm{opt}}\ge T_{\mathrm{p}}$.

(2) When $k(1-k) \le C < 1-k$ (Figure 3(b)), C is large enough for SU transmits in all "OFF slots", that is, $\rho_0 \equiv 1$.

(a) When $P/p_t \le C/(1-k)$, the optimal β is obtain when $T = +\infty$.

(b) When $P/p_t > C/(1-k)$, the optimal β is the intersection of β_{IC} and β_{ECC}.

(3) When $C < k(1-k)$ (Figure 3(c)), the optimal ρ_0 can be obtained according to Theorem 5 (Figure 2(c)), and the optimal $\rho_1 > 0$ if and only if $T < T_c$. Similarly, we have that

(a) When $P/p_t \le C/(1-k)$, the optimal β is obtain when $T = +\infty$.

(b) When $P/p_t > C/(1-k)$, the optimal β is the intersection of β_{IC} and β_{ECC}.

Mathematically, this leads to the following theorem.

Theorem 10. *While $C \ge 1-k$, the optimal solution of* **P2** *is*

$$\rho_{0,\mathrm{opt}} = \begin{cases} \min\left\{\left(\dfrac{P}{kp_t}\right)^{-}, 1\right\}, & 0 < P \le p_t, \\ 1, & P > p_t, \end{cases}$$

$$\rho_{1,\mathrm{opt}} = \begin{cases} \max\left\{\left(\dfrac{P-kp_t}{(1-k)p_t}\right)^{-}, 0\right\}, & 0 < P \le p_t, \\ 1, & P > p_t, \end{cases} \tag{20}$$

$$T_{\mathrm{opt}} \begin{cases} = +\infty, & 0 < P \le p_t, \\ \in \left[T_{\mathrm{p}}, +\infty\right), & P > p_t. \end{cases}$$

While $k(1-k) \le C < 1-k$, the optimal solution of **P2** *is*

$$\rho_{0,\mathrm{opt}} = \begin{cases} \min\left\{\left(\dfrac{P}{kp_t}\right)^{-}, 1\right\}, & 0 < P \le \dfrac{Cp_t}{1-k}, \\ 1, & P > \dfrac{Cp_t}{1-k}, \end{cases}$$

$$\rho_{1,\mathrm{opt}} = \begin{cases} \max\left\{\left(\dfrac{P-kp_t}{(1-k)p_t}\right)^{-}, 0\right\}, & 0 < P \le \dfrac{Cp_t}{1-k}, \\ \dfrac{(P-kp_t)T_{\mathrm{opt}} - Q}{(1-k)p_t T_{\mathrm{opt}}}, & P > \dfrac{Cp_t}{1-k}, \end{cases}$$

$$T_{\mathrm{opt}} = \begin{cases} +\infty, & 0 < P \le \dfrac{Cp_t}{1-k}, \\ -\dfrac{1}{g}\mathcal{W}\left(-\dfrac{g}{a}e^{(ah-bg)/a}\right) - \dfrac{b}{a}, & P > \dfrac{Cp_t}{1-k},\ P \ne p_t, \\ \dfrac{e^h - b}{a}, & P = p_t. \end{cases} \tag{21}$$

While $C < k(1-k)$, the optimal solution of **P2** *is*

$$\rho_{0,\mathrm{opt}} = \begin{cases} \left(\dfrac{P}{kp_t}\right)^{-}, & 0 < P \le \dfrac{Cp_t}{1-k}, \\ \dfrac{PT_{\mathrm{opt}} - Q}{kp_t T_{\mathrm{opt}}}, & \dfrac{Cp_t}{1-k} < P \le \dfrac{Q}{T_c} + kp_t, \\ 1, & P > \dfrac{Q}{T_c} + kp_t, \end{cases}$$

$$\rho_{1,\mathrm{opt}} = \begin{cases} 0, & 0 < P \le \dfrac{Cp_t}{1-k}, \\ 0, & \dfrac{Cp_t}{1-k} < P \le \dfrac{Q}{T_c} + kp_t, \\ \dfrac{(P-kp_t)T_{\mathrm{opt}} - Q}{(1-k)p_t T_{\mathrm{opt}}}, & P > \dfrac{Q}{T_c} + kp_t, \end{cases}$$

$$T_{\mathrm{opt}} = \begin{cases} +\infty, & 0 < P \le \dfrac{Cp_t}{1-k}, \\ -\dfrac{1}{c}\mathcal{W}\left(-\dfrac{c}{a}e^{(ad-bc)/a}\right) - \dfrac{b}{a}, & \dfrac{Cp_t}{1-k} < P \le \dfrac{Q}{T_c} + kp_t, \\ -\dfrac{1}{g}\mathcal{W}\left(-\dfrac{g}{a}e^{(ah-bg)/a}\right) - \dfrac{b}{a}, & P > \dfrac{Q}{T_c} + kp_t,\ P \ne p_t, \\ \dfrac{e^h - b}{a}, & P = p_t, \end{cases} \tag{22}$$

where $T_c = (1/(\lambda+\mu))(\mathcal{W}((1/m)e^{1/m}) - 1/m)$, $m = C/k(1-k) - 1$, $T_{\mathrm{p}} = Q/(P - kp_t)$ (while $\rho_{0,\mathrm{p}}^{\mathrm{ECC}} = P/kp_t > 1$) and

$$\begin{aligned} a &= \frac{\lambda+\mu}{k}\left(\frac{C}{1-k} - \frac{P}{p_t}\right), \\ b &= 1 + \frac{(\lambda+\mu)Q}{kp_t}, \\ c &= -\frac{(\lambda+\mu)P}{kp_t}, \\ d &= \frac{(\lambda+\mu)Q}{kp_t}, \\ g &= \frac{(\lambda+\mu)(P-p_t)}{(1-k)p_t}, \\ h &= -\frac{(\lambda+\mu)Q}{(1-k)p_t}. \end{aligned} \tag{23}$$

Proof. See Appendix E. □

5. Reference Policies

In this section, in order to put the performance of our proposed policies in perspective, we will introduce two suboptimal reference access policies.

5.1. Probabilistic Access Policy π_{PA}. Here, we consider a probabilistic access policy π_{PA}, with which SU will either choose to access each slot entirely with certain probability or maintain silence. Specifically, in each slot, if the sensing result is OFF, then SU will access this slot with the probability p_0; and if the sensing result is ON, then SU will access this slot with the probability p_1. It is apparent that greedy access policy (namely, in each slot, SU will access the whole slot as long as the sensing result is OFF, or stay silence if the sensing result is ON) is a special case of the probabilistic access policy when $p_0 = 1$ and $p_1 = 0$.

Thus, we have the mathematical model of policy π_{PA}

$$\begin{aligned} \max_{p_0, p_1, T} \quad & \beta = k \times p_0 + (1-k) \times p_1 \\ \text{s.t.} \quad & k p_0 \phi_0(1,T) + (1-k) p_1 \phi_1(1,T) \leq C, \\ & \frac{Q}{T} + p_t \beta \leq P, \\ & T > 0, \\ & 0 \leq p_0, \quad p_1 \leq 1. \end{aligned} \tag{24}$$

Since $\phi_0(1,T) < \phi_1(1,T)$, thus similar to Lemmas 7 and 8, we can obtain the following properties.

(1) For any given T, to generate the same interference, increasing p_0 can always make more channel utilization increment than increasing p_1.

(2) p_1 is greater than 0 if and only if $p_0 = 1$.

Therefore, similar to Theorem 10, we can obtain the optimal p_0 by the relationship of curves $p_0 = C/k\phi_0(1,T)$ and $p_0 = (1/kp_t)(P - Q/T)$, namely,

$$p_0 = \max_{T>0} \left\{ \min \left\{ \frac{C}{k\phi_0(1,T)}, \frac{PT - Q}{kp_t T} \right\} \right\}. \tag{25}$$

Thus, according to (25), we can obtain the maximal p_0^* and the corresponding T^* by linear search algorithm.

If $0 < p_0^* < 1$, then the optimal solution is $p_{0,\text{opt}} = p_0^*$, $p_{1,\text{opt}} = 0$ and $T_{\text{opt}} = T^*$. And if $p_0^* \geq 1$, the optimal solution $p_{0,\text{opt}} = 1$, and $p_{1,\text{opt}}$, T_{opt} can also be obtained by linear search algorithm, according to the following equation:

$$p_1 = \max_{T>0} \left\{ \min \left\{ \frac{C - k\phi_0(1,T)}{(1-k)\phi_1(1,T)}, \frac{PT - kp_t T - Q}{(1-k)p_t T} \right\} \right\}. \tag{26}$$

5.2. No Sensing Access Policy π_{NSA}. We consider an access policy, with which SU does not carry out the sensing event before its transmission. Specifically, in each slot, SU will transmit a ρ fraction of period T without sensing the channel. Thus, we have the mathematical model of policy π_{NSA}

$$\begin{aligned} \max_{\rho, T} \quad & \beta = \rho \\ \text{s.t.} \quad & (1-k)\rho \leq C, \\ & p_t \rho \leq P, \\ & T > 0, \\ & 0 \leq \rho \leq 1. \end{aligned} \tag{27}$$

Thus, the maximal channel utilization is

$$\beta = \rho_{\text{opt}} = \min \left\{ \min \left\{ \frac{C}{1-k}, \frac{P}{p_t} \right\}, 1 \right\} \tag{28}$$

and $T_{\text{opt}} > 0$.

6. Numerical Results

In this section, we will present three numerical simulations: one on the performance of policy π_1, the second on the performance of policy π_2, and the third on the comparison of policy π_2 and reference policies.

6.1. Optimal Allocation under Policy π_1. In this subsection, the simulation results for access policy π_1 are presented. We will illustrate the optimal allocations for different channel states, namely, holding times, and different IC threshold C. Furthermore, We assume that $Q = 0.1$, $p_t = 3$ and P varies from 1 to 1.5, which guarantee $P \in ((Cp_t/(1-k)), Q/T_c + kp_t]$ for any chosen C, λ, and μ in the following simulation.

Figures 4 and 5 show the optimal solutions for different holding times λ and μ, while $C = 10\%$. For any given λ and μ, as P increases; that is, the ECC gets looser, the optimal solution $\rho_{0,\text{opt}}$ increases, and T_{opt} decreases, which means that the channel utilization increases with increasing P. For any given P, as the holding times λ^{-1} and μ^{-1} increase, $\rho_{0,\text{opt}}$ and T_{opt} increase. Thus, the channel utilization increases with increasing holding times.

Figures 6 and 7 show the optimal solutions for different C, while $\lambda = \mu = 0.1$. For any given C, as P increases, the optimal solution $\rho_{0,\text{opt}}$ increases and T_{opt} decreases. Thus, the channel utilization increases as P increases. For any given P, as C increases; that is, the IC gets looser, $\rho_{0,\text{opt}}$ and T_{opt} increase. Thus, the channel utilization increases as C increases.

Furthermore, comparing Figures 4 and 6, it is observed that $\rho_{0,\text{opt}}$ increases near linearly with increase of P. Thus, due to $\rho_{0,\text{opt}} = (1/kp_t)(P - Q/T_{\text{opt}})$, we can obtain that $1/T_{\text{opt}}$ is a near linear equation of P.

6.2. Optimal Allocation under Policy π_2. In this subsection, the simulation results for access policy π_2 are presented. We will illustrate the optimal allocations for different channel states, namely, k and holding times, and different IC threshold C. We also assume that $Q = 0.1$, $p_t = 3$.

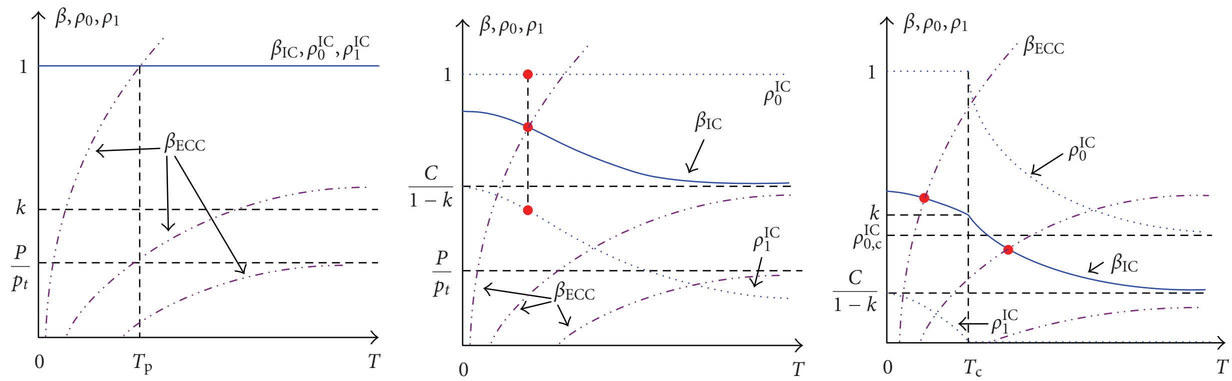


FIGURE 3: Illustration of the relationship of IC and ECC under access policy π_2.β_{ECC} is the maximal β under ECC. β_{IC} is the maximal β under IC and $\rho_0^{\mathrm{IC}}, \rho_1^{\mathrm{IC}}$ are its corresponding transmission allocation.

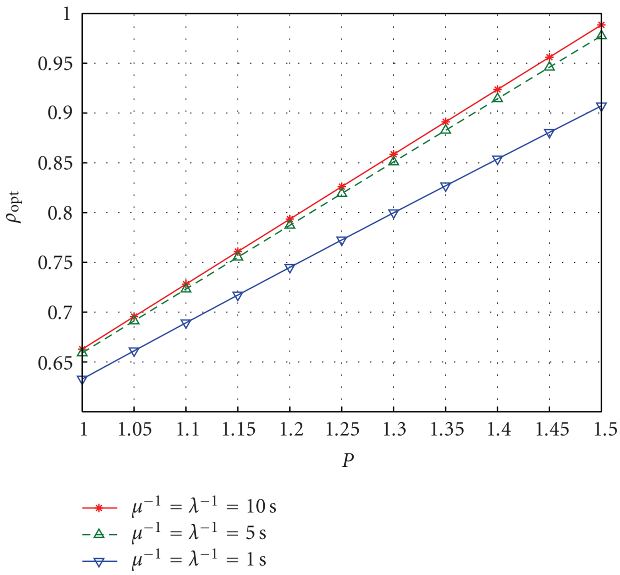


FIGURE 4: The optimal $\rho_{0,\mathrm{opt}}$ versus P for different holding times.

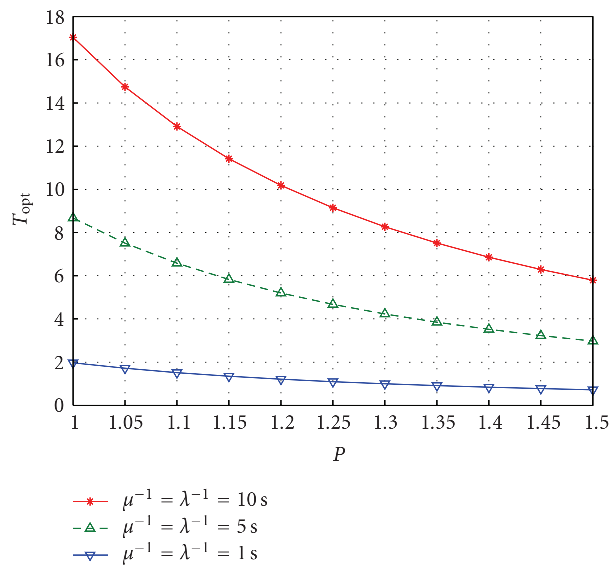


FIGURE 5: The optimal T_{opt} versus P for different holding times.

Figures 8 and 9 show the optimal solutions for different k. We assume the holding times $\mu^{-1} = 2, 2.5, 3$, and $\lambda^{-1} = 3, 2.5, 2$, which make $k = 0.4, 0.5$ and 0.6, respectively. Besides, we assume $C = 10\%$, corresponding to the situation of Figure 3(c). For any given k, the optimal spectrum utilization β increases as P increases and tends to $C+k$. From Figure 3(c), we can know that when P is small, the problem **P2** can be regard as being only restricted by ECC, which make the maximal $\beta = P/p_t$. As P increases, both IC and ECC take effect. Then, the optimal β, the intersection of β_{IC} and β_{ECC}, increases as P increases. Based on Lemma 9 and Figure 3(c), we can obtain that $\lim_{P\to\infty}\beta = k \times 1 + (1-k) \times C/(1-k) = C + k$. Furthermore, Figure 8 shows that $\rho_1 > 0$ if and only if $\rho_0 = 1$, and β increases as k increases. Figure 9 shows that sensing period T decreases as P increases, due to the fact that the intersection of β_{IC} and β_{ECC} moves left as P increases. And we can also observe that T decreases as k decreases.

Figures 10 and 11 show the optimal solutions for different holding times μ^{-1} and λ^{-1}. We assume $C = 10\%$ and $\mu = \lambda = 0.2, 1, 5$, which make k the same. Figure 10 shows that β increases as P increases. This is because based on Theorem 10, when $C < k(1-k)$, both ρ_0 and ρ_1 increase as P increases. Figure 10 also shows that β increases as holding time increases. Since Figure 11 shows that sensing period T increases as holding time increases, and Theorem 10 shows that when $C < k(1-k)$, both ρ_0 and ρ_1 increases as T increases, thus, β increases as holding time increases.

Figures 12 and 13 show the optimal solutions for different C. In Figure 12, we assume $C = 10\%, 40\%, 60\%$, corresponding to the different situations as shown in Figure 3. From Figure 12, we can know that β increases as P increases and increases as C increases, which means when IC or ECC get looser, the spectrum utilization increases. From Figure 13, we can observe that the sensing period T increases as C increases.

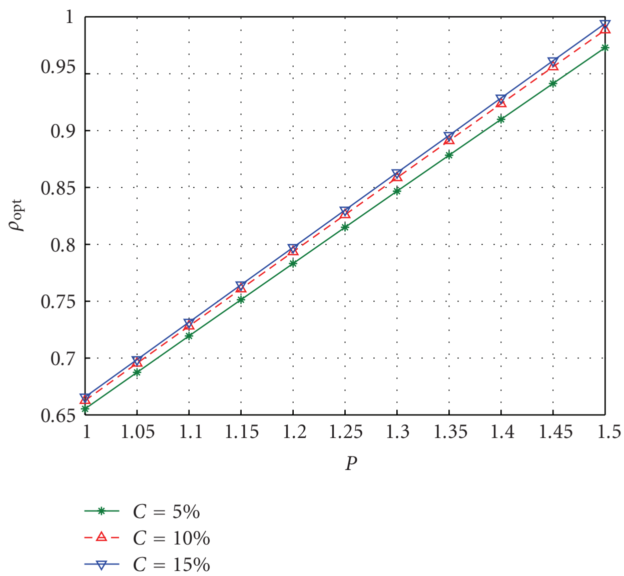


FIGURE 6: The optimal $\rho_{0,\text{opt}}$ versus P for different C.

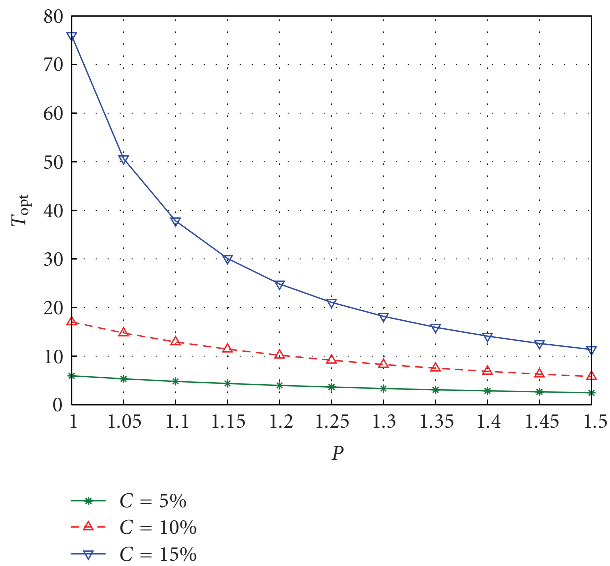


FIGURE 7: The optimal T_{opt} versus P for different C.

Figures 14 and 15 show the optimal solutions for different Q, that is, $Q = 0.1, 1, 3$. We assume that $C = 10\%$ and $\mu = \lambda = 0.2$. Figure 14 shows that the spectrum utilization β increases as Q decreases, but the limit value is the same, which has nothing to do with Q. Figure 15 shows that the sensing period T increases as Q increases.

6.3. Comparison of Access Policy π_2 and Reference Policies. Since policy π_1 is a special case of policy π_2, thus, in this subsection, we will compare the performance of policy π_2 with policy π_{PA} and π_{NSA} for different holding times, namely, $\mu = 1, 10, 30$ and $\lambda = 1, 10, 30$. We also assume that $Q = 0.1$, $p_t = 3$ and $C = 10\%$.

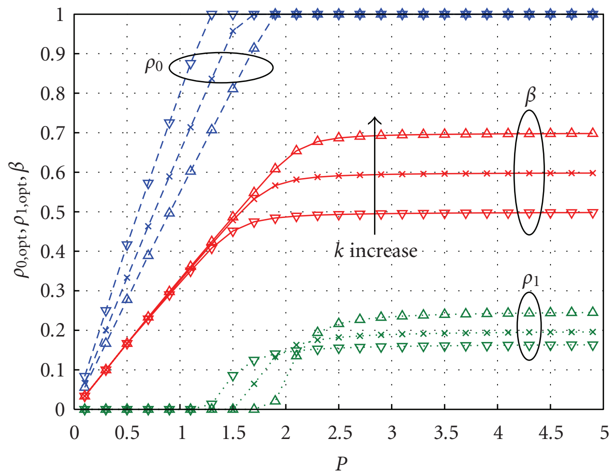


FIGURE 8: The optimal ρ_0, ρ_1, β under policy π_2 for different k.

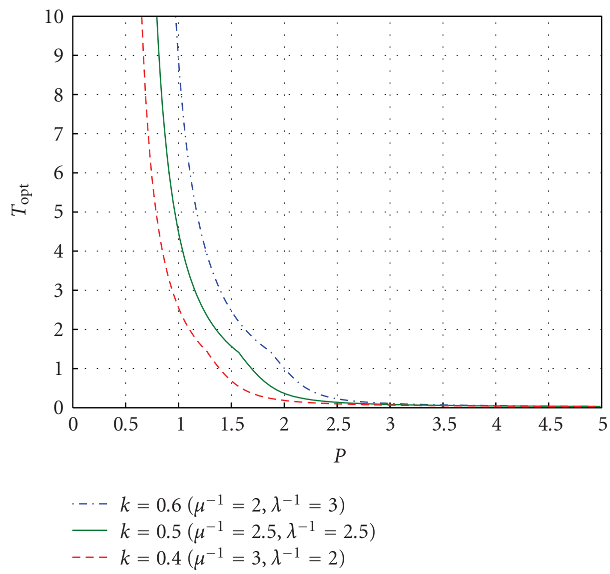


FIGURE 9: The optimal T_{opt} under policy π_2 for different k.

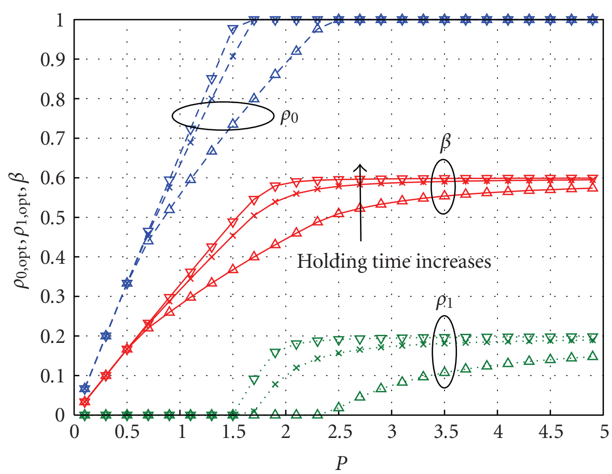


FIGURE 10: The optimal ρ_0, ρ_1, β under policy π_2 for different holding times.

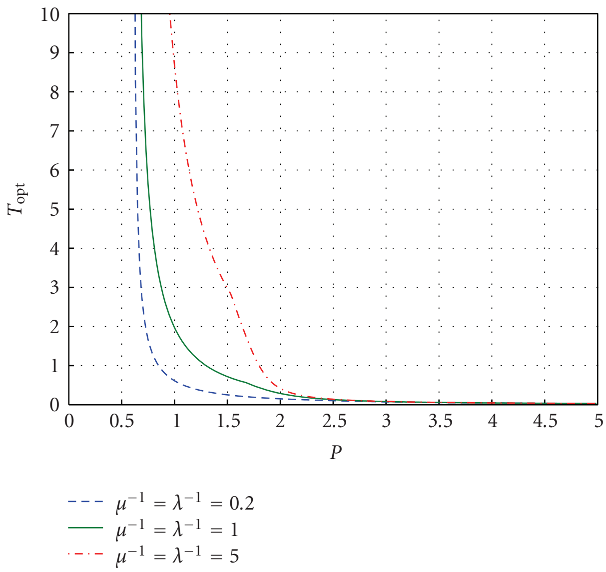


FIGURE 11: The optimal T_{opt} under policy π_2 for different holding times.

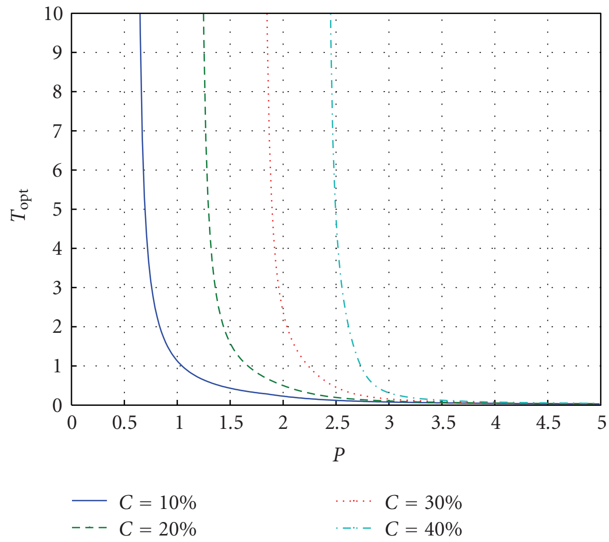


FIGURE 13: The optimal T_{opt} under policy π_2 for different C.

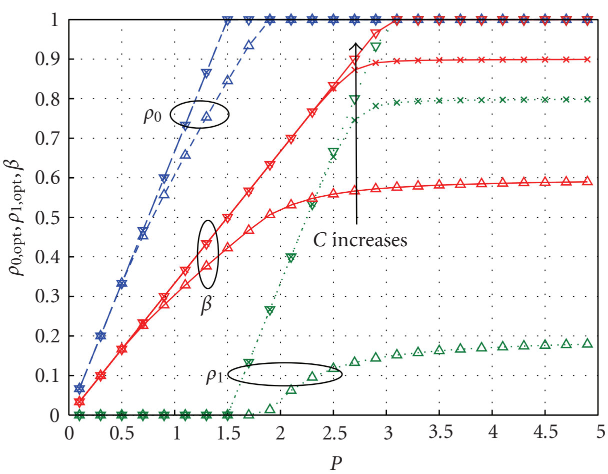


FIGURE 12: The optimal ρ_0, ρ_1, β under policy π_2 for different C.

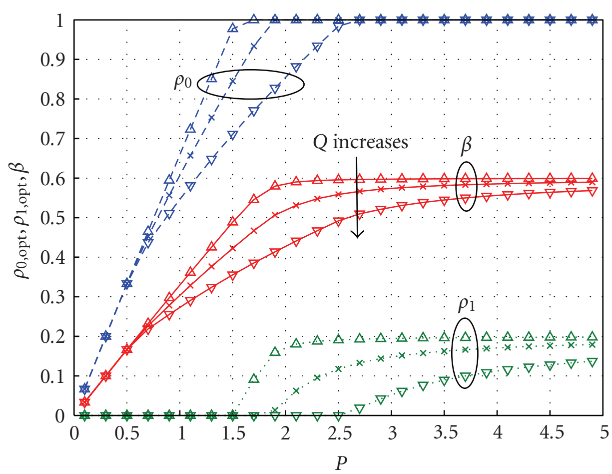


FIGURE 14: The optimal ρ_0, ρ_1, β under policy π_2 for different Q.

Figure 16 illustrates the maximal channel utilization under three different access policies. From this figure, we can obtain that when P is small, the channel utilization under three access policies are the same. This is because the ECC plays a major role and the ECC models for these three access policies is the same. Furthermore, we can obtain that when P is small; that is, the ECC is tight, SU need not sense the channel, since the performance of policy π_{NSA} is the same as policy π_2. When $P > Cp_t/(1-k)$, both IC and ECC will take effect. At that moment, the channel utilization under policy π_2 is much larger than π_{NSA}, especially when the channel's state switches slowly (namely, μ and λ are small), and is always larger than π_{PA}, especially when the channel's state switches fast.

Figure 17 illustrates the optimal sensing period under policies π_2 and π_{PA}. From this figure, we can obtain that the optimal sensing period under policy π_2 is always larger than the one under policy π_{PA}, which means SU can consume less energy and time to sense the channel while adopting our proposed policy π_2. Furthermore, comparing Figure 16 with Figure 17, we can find that when the holding time is large (i.e., μ and λ is small), although the channel utilization under policy π_2 is slightly larger than policy π_{PA}, the sensing period of policy π_2 is much larger than policy π_{PA}.

Therefore, we can obtain that both PA and NSA policies are sub-optimal, and with our proposed policy π_2, SU can efficiently access the channel, and meanwhile consumes less energy and time to sense the channel's state.

7. Conclusion

In this paper, we propose an OSA strategy for a slotted SU overlaying an unslotted ON/OFF CTMC modeled primary network under IC and ECC, where IC is modeled by the average temporal overlap and the ECC is defined by the

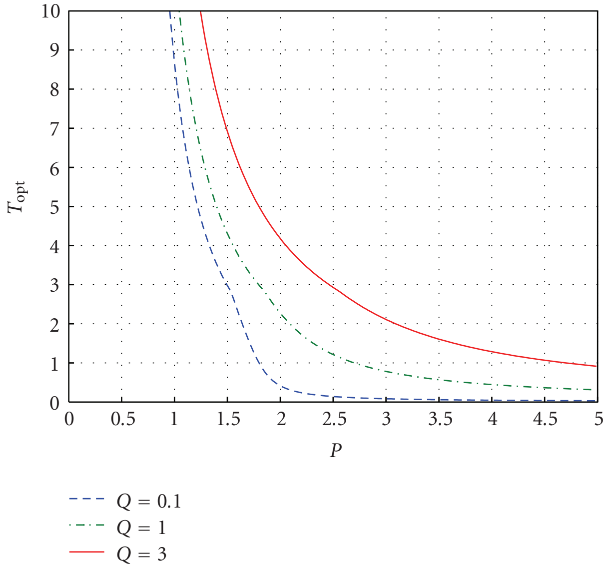


Figure 15: The optimal T_{opt} under policy π_2 for different Q.

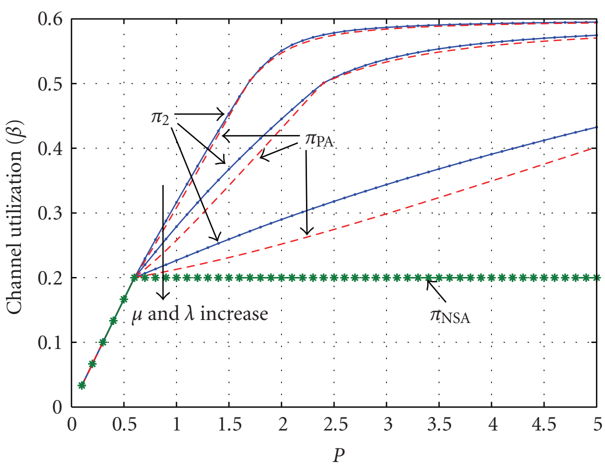


Figure 16: The channel utilization β under access policy π_2, π_{PA} and π_{NSA}.

idea of battery life. Based on the sensing results, two access policies are investigated in this paper; that is, SU transmits only in "OFF slots" (i.e., the slots that the sensing results are OFF) and transmits in both "OFF slots" and "ON slots".

The optimal allocation of sensing period and transmission time for two access policies are formulated and the closed-form solutions are derived, which show that SU should transmit in "OFF slots" as much as possible. Numerical results also show that with the proposed policies, SU can efficiently access the channel, and meanwhile consumes less energy and time to sense the channel's state than the reference policies, namely, probabilistic access policy and no sensing access policy.

In terms of further work, we will intend to extend our work to more general case of multiple PUs and SUs. We will also consider the effect of imperfect sensing.

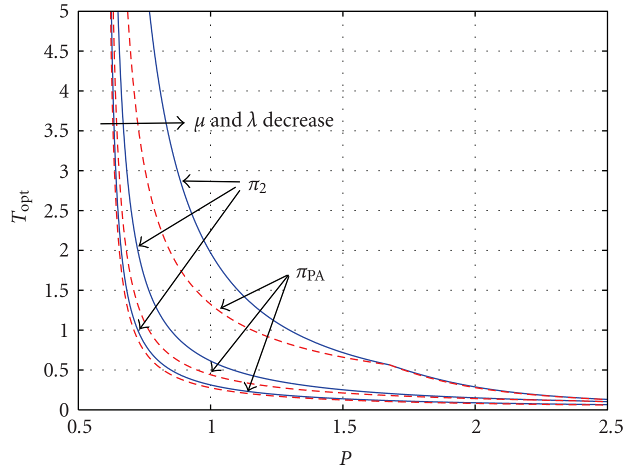


Figure 17: The optimal T_{opt} under access policy π_2 and π_{PA}.

Appendices

A. Proof of Lemma 3

Here, instead of ρ_0^{IC}, we use the notation ρ_0 for convenience.

Let $h(\rho_0, T) = k\phi_0(\rho_0, T) - C$. Because

$$h(\rho_0 + d\rho_0, T + dT) = h(\rho_0, T) + \frac{\partial h(\rho_0, T)}{\partial \rho_0} d\rho_0 + \frac{\partial h(\rho_0, T)}{\partial T} dT \tag{A.1}$$

and $h(\rho_0, T) \equiv 0$, therefore we have

$$\frac{\partial h(\rho_0, T)}{\partial \rho_0} d\rho_0 + \frac{\partial h(\rho_0, T)}{\partial T} dT = 0. \tag{A.2}$$

Substituting (2) into $h(\rho_0, T)$ gives

$$h(\rho_0, T) = \frac{\lambda\mu}{(\lambda+\mu)^2}\left(\rho_0 + \frac{e^{-(\lambda+\mu)\rho_0 T} - 1}{(\lambda+\mu)T}\right) - C. \tag{A.3}$$

Then, we have

$$\frac{\partial h(\rho_0, T)}{\partial \rho_0} = \frac{\lambda\mu}{(\lambda+\mu)^2}\left(1 - e^{-(\lambda+\mu)\rho_0 T}\right), \tag{A.4}$$

$$\begin{aligned}\frac{\partial h(\rho_0, T)}{\partial T} &= \frac{\lambda\mu}{(\lambda+\mu)^3} \times \frac{-(\lambda+\mu)\rho_0 T e^{-(\lambda+\mu)\rho_0 T} - \left(e^{-(\lambda+\mu)\rho_0 T} - 1\right)}{T^2} \\ &= \frac{\lambda\mu}{(\lambda+\mu)^3} \frac{1 - (1 + (\lambda+\mu)\rho_0 T) e^{-(\lambda+\mu)\rho_0 T}}{T^2} \\ &= \frac{\lambda\mu}{(\lambda+\mu)^3} \frac{e^{(\lambda+\mu)\rho_0 T} - (1 + (\lambda+\mu)\rho_0 T)}{T^2 \times e^{(\lambda+\mu)\rho_0 T}}.\end{aligned} \tag{A.5}$$

Because for any $x > 0$, $e^x > 1 + x$ and $e^{-x} \in (0,1)$, thus, for any $\rho_0 \geq 0$ and $T > 0$, we can obtain $\partial h(\rho_0, T)/\partial T > 0$ and $\partial h(\rho_0, T)/\partial \rho_0 > 0$, therefore

$$\frac{d\rho_0}{dT} = -\frac{(\partial h(\rho_0, T)/\partial T)}{(\partial h(\rho_0, T)/\partial \rho_0)} < 0. \tag{A.6}$$

Thus, ρ_0 is strictly decreasing in T.

Because $\lim_{T \to +\infty} h(\rho_0, T) = k(1-k)\rho_0 - C = 0$, thus, the infimum of ρ_0 is $\rho_{0,c}^{\mathrm{IC}} = C/k(1-k)$.

B. Proof of Theorem 5

First, we will discuss the situation of $C < k(1-k)$.

As shown in Figure 2, the optimal solution of **P1** depends on the relationship between curves **C1** and **C2**. Because the curve **C1** is a branch of hyperbola ($\rho_0 = P/kp_t - (Q/kp_t) \times (1/T), T > 0$), thus, as P increases, the relationship between curves **C1** and **C2** will change from not intersecting to intersecting. We will discuss this issue in different situations.

(A-1) When $\rho_{0,p}^{\mathrm{ECC}} \leq \rho_{0,c}^{\mathrm{IC}}$, that is, $0 < P \leq Cp_t/(1-k)$, based on Lemmas 3 and 4, the curves **C1** and **C2** have no intersection, as illustrated in Figure 2(a). Then, when the ECC (6) satisfies, the IC (4) will always satisfy. Thus, the optimal solution of **P1** is $\rho_{0,\mathrm{opt}} = (\rho_{0,p}^{\mathrm{ECC}})^- = (P/kp_t)^-$ and $T_{\mathrm{opt}} = +\infty$.

(A-2) As P increases, the curves **C1** and **C2** will have one intersection (T^*, ρ_0^*) in the first quadrant, as illustrated in Figure 2(b). If $\rho_0^* \leq 1$, based on Lemmas 3 and 4, the intersection (T^*, ρ_0^*) will be the optimal solution $(T_{\mathrm{opt}}, \rho_{0,\mathrm{opt}})$.

Let $(T, \rho_0) = (T_c, 1)$ be the point on the curve **C2** when $\rho_0 = 1$. Substituting $(T_c, 1)$ into (12), we have

$$k(1-k)\left(1 + \frac{1}{(\lambda+\mu)T_c}\left(e^{-(\lambda+\mu)T_c} - 1\right)\right) = C. \tag{B.1}$$

Let $m = C/k(1-k) - 1 \neq 0$ and $x = (\lambda+\mu)T_c$, we have

$$\begin{aligned} mx &= e^{-x} - 1, \\ mxe^x &= 1 - e^x, \\ \left(x + \frac{1}{m}\right)e^x &= \frac{1}{m}, \\ \left(x + \frac{1}{m}\right)e^{x+1/m} &= \frac{1}{m}e^{1/m}. \end{aligned} \tag{B.2}$$

Thus,

$$T_c = \frac{x}{\lambda+\mu} = \frac{1}{\lambda+\mu}\left(\mathcal{W}\left(\frac{1}{m}e^{1/m}\right) - \frac{1}{m}\right). \tag{B.3}$$

Substituting T_c into (11), we have $\rho_{0,\,\mathrm{C1}} = (1/kp_t)(P - Q/T_c)$. Thus, $\rho_0^* \leq 1$ is equal to $\rho_{0,\,\mathrm{C1}} \leq 1$, that is, $P \leq Q/T_c + kp_t$.

Thus, when $Cp_t/(1-k) < P \leq Q/T_c + kp_t$, the curves **C1** and **C2** have one intersection, which is the optimal solution $(T_{\mathrm{opt}}, \rho_{0,\mathrm{opt}})$. By solving (11), we have

$$\rho_{0,\mathrm{opt}} = \frac{1}{kp_t}\left(P - \frac{Q}{T_{\mathrm{opt}}}\right). \tag{B.4}$$

Substituting $\rho_{0,\mathrm{opt}}$ into (12) leads to

$$aT + b = e^{cT+d}, \tag{B.5}$$

where a, b, c, d are given in (15). And when $Cp_t/(1-k) < P \leq Q/T_c + kp_t$, we can prove $a \neq 0$ and $c \neq 0$. Thus, using the similar approach to finding T_c, we can obtain that

$$T_{\mathrm{opt}} = -\frac{1}{c}\mathcal{W}\left(-\frac{c}{a}e^{(ad-bc)/a}\right) - \frac{b}{a}. \tag{B.6}$$

(A-3) As P increases, ρ_0^* will be greater than 1. Furthermore, let $(T, \rho_0) = (T_p, 1)$ be the point on the curve **C1** when $\rho_0 = 1$. By solving (11), we have $T_p = Q/(P - kp_t)$. Thus, when $P > (Q/T_c) + kp_t$, the optimal solution is $T_{\mathrm{opt}} \in [T_p, T_c]$ and $\rho_{0,\mathrm{opt}} = 1$, as illustrated in Figure 2(c).

Second, when $C \geq k(1-k)$, the optimal solution depends on the relationship of $\rho_{0,p}^{\mathrm{ECC}}$ and 1.

When $\rho_{0,p}^{\mathrm{ECC}} \leq 1$, that is, $0 < P \leq kp_t$, which is similar to the situation of (A-1), the optimal solution of **P1** is $\rho_{0,\mathrm{opt}} = (\rho_{0,p}^{\mathrm{ECC}})^- = (P/kp_t)^-$ and $T_{\mathrm{opt}} = +\infty$.

When $\rho_{0,p}^{\mathrm{ECC}} > 1$, that is, $P > kp_t$, which is similar to the situation of (A-3), the optimal solution of **P1** is $T_{\mathrm{opt}} \in [T_p, +\infty)$ and $\rho_{0,\mathrm{opt}} = 1$.

C. Proof of Lemma 7

To prove this lemma equals to prove

$$\frac{\partial \phi_0(\rho_0, T)}{\partial \rho_0} < \frac{\partial \phi_1(\rho_1, T)}{\partial \rho_1}. \tag{C.1}$$

Substituting (2) and (3) into $\partial \phi_0(\rho_0, T)/\partial \rho_0$ and $\partial \phi_1(\rho_1, T)/\partial \rho_1$, we can obtain that

$$\begin{aligned} \frac{\partial \phi_0(\rho_0, T)}{\partial \rho_0} &= (1-k)\left(1 - e^{-(\mu+\lambda)\rho_0 T}\right), \\ \frac{\partial \phi_1(\rho_1, T)}{\partial \rho_1} &= 1 - k + ke^{-(\mu+\lambda)(1-\rho_1)T}. \end{aligned} \tag{C.2}$$

Thus,

$$\frac{\partial \phi_0(\rho_0, T)/\partial \rho_0}{\partial \phi_1(\rho_1, T)/\partial \rho_1} = \frac{(1-k)\left(1 - e^{-(\mu+\lambda)\rho_0 T}\right)}{1 - k + ke^{-(\mu+\lambda)(1-\rho_1)T}} < \frac{1-k}{1-k} = 1. \tag{C.3}$$

And when $\rho_0 \neq 0$ and $\rho_1 \neq 1$,

$$\begin{aligned} \lim_{T \to +\infty} \frac{\partial \phi_0(\rho_0, T)/\partial \rho_0}{\partial \phi_1(\rho_1, T)/\partial \rho_1} &= \lim_{T \to +\infty} \frac{(1-k)\left(1 - e^{-(\mu+\lambda)\rho_0 T}\right)}{1 - k + ke^{-(\mu+\lambda)(1-\rho_1)T}} \\ &= \frac{1-k}{1-k} = 1. \end{aligned} \tag{C.4}$$

Therefore, when $\rho_0 \neq 0$, $\rho_1 \neq 1$ and $T \to +\infty$, the effect of increasing ρ_0 or ρ_1 are almost the same.

D. Proof of Lemma 9

Here, instead of ρ_1^{IC}, we use the notation ρ_1 for convenience.

Substituting (2) and (3) into

$$k \times \phi_0(1, T) + (1-k) \times \phi_1(\rho_1, T) = C, \tag{D.1}$$

and through simplifying, we have

$$-(1-k)(1-\rho_1)x + ke^{-(1-\rho_1)x} = \left(\frac{C}{1-k} - 1\right)x + k, \tag{D.2}$$

where $x = (\mu+\lambda)T$.

Let $y = -(1-\rho_1)x$, $A = 1-k$ and $B = C/(1-k)-1)x+k$, then we have

$$\begin{aligned} ke^y &= B - Ax, \\ \frac{k}{A}e^y &= \frac{B}{A} - y, \\ \frac{k}{A}e^{B/A} &= \left(\frac{B}{A} - y\right)e^{B/A-y}. \end{aligned} \tag{D.3}$$

Thus,

$$y = \frac{B}{A} - \mathcal{W}\left(\frac{k}{A}e^{B/A}\right). \tag{D.4}$$

Thus,

$$\begin{aligned} \rho_1 &= 1 + \frac{y}{x} \\ &= 1 + \frac{1}{x}\left(\frac{B}{A} - \mathcal{W}\left(\frac{k}{A}e^{B/A}\right)\right) \\ &= 1 + \frac{1}{x}\left(\frac{k}{1-k} - \mathcal{W}\left(\frac{k}{1-k}e^{B/(1-k)}\right)\right) + \frac{C-(1-k)}{(1-k)^2}. \end{aligned} \tag{D.5}$$

Since $C < 1-k$, when T increases, x increases and B decreases and $B < k$.

Let $z = (k/(1-k))e^{B/(1-k)}$. Due to $z > 0$, thus, $\mathcal{W}(z)$ decreases as B decreases and $\mathcal{W}(z) < W((k/(1-k))e^{k/(1-k)}) = k/(1-k)$. Thus, as x increases, $k/(1-k) - \mathcal{W}(z)$ increases and $k/(1-k) - \mathcal{W}(z) > 0$. However, from the following equation we can obtain that as T increases, $k/(1-k) - \mathcal{W}(z)$ increases slowly than x

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{k}{1-k} - \mathcal{W}(z)\right) &= -\frac{\mathrm{d}}{\mathrm{d}z}(\mathcal{W}(z))\frac{\mathrm{d}z}{\mathrm{d}x} \\ &= -\frac{\mathcal{W}(z)}{z(1+\mathcal{W}(z))} \times z \times \frac{C-(1-k)}{(1-k)^2} \\ &= \frac{\mathcal{W}(z)}{(1+\mathcal{W}(z))} \times \frac{(1-k)-C}{(1-k)^2} \\ &< \frac{1}{(1-k)} < 1. \end{aligned} \tag{D.6}$$

Therefore, when $C < 1-k$, ρ_1, the solution of $k \times \phi_0(1, T) + (1-k) \times \phi_1(\rho_1, T) = C$, is strictly decreasing in T.

Taking the limit of (D.1), we have

$$\begin{aligned} \lim_{T \to +\infty} (k \times \phi_0(1, T) + (1-k) \times \phi_1(\rho_1, T)) &= C, \\ k(1-k) + (1-k)^2 \times \rho_1 &= C. \end{aligned} \tag{D.7}$$

Thus,

$$\lim_{T \to +\infty} \rho_1 = \frac{C - k(1-k)}{(1-k)^2}. \tag{D.8}$$

Similarly, we have that

$$\begin{aligned} \lim_{T \to 0} (k \times \phi_0(1, T) + (1-k) \times \phi_1(\rho_1, T)) &= C, \\ k(1-k)\left(1 - \frac{\lambda+\mu}{\lambda+\mu}\right) + (1-k)^2 \times \left(\rho_1 + \frac{\mu}{\lambda}\rho_1\right) &= C, \\ (1-k)\rho_1 &= C. \end{aligned} \tag{D.9}$$

Thus,

$$\lim_{T \to 0} \rho_1 = \frac{C}{1-k}. \tag{D.10}$$

E. Proof of Theorem 10

Since when $C \geq 1-k$, the IC is looser enough for SU to transmit all the time regardless of the sensing results, namely, for all $\rho_0, \rho_1 \in [0, 1]$, the IC (5) is always satisfied. Thus, the problem **P2** is only constrained by ECC. From (6), we can obtain that

$$\beta \leq \frac{P}{p_t} - \frac{Q}{p_t T} < \frac{P}{p_t}. \tag{E.1}$$

Thus, considering the scope of β, we can easily get the optimal solution of **P2**, as shown in Figure 3(a).

(1) When $0 < P/p_t \leq 1$, that is, $0 < P \leq p_t$, based on Lemma 8, when $\beta \leq k$, $\rho_{0,\mathrm{opt}} \leq 1$ and $\rho_{0,\mathrm{opt}} = 0$, and on the other hand, when $k < \beta < 1$, $\rho_{0,\mathrm{opt}} = 1$ and $\rho_{0,\mathrm{opt}} > 0$. Thus,

$$\begin{aligned} \rho_{0,\mathrm{opt}} &= \min\left\{\left(\frac{P}{kp_t}\right)^{-}, 1\right\}, \\ \rho_{1,\mathrm{opt}} &= \max\left\{\left(\frac{P - kp_t}{(1-k)p_t}\right)^{-}, 0\right\}, \\ T_{\mathrm{opt}} &= +\infty. \end{aligned} \tag{E.2}$$

(2) When $P/p_t > 1$, that is, $P > p_t$,

$$\rho_{0,\text{opt}} = 1,$$
$$\rho_{1,\text{opt}} = 1, \tag{E.3}$$
$$T_{\text{opt}} \in \left[\frac{Q}{P - p_t}, +\infty\right).$$

When $k(1-k) \le C < 1-k$, as shown in Figure 3(b), the IC is looser enough for SU to transmit all the time when the sensing result is OFF.

Based on Lemmas 3, 9, and 8, we can infer that under IC, the maximal channel utilization ratio β_{IC} decreases as T increases, and its infimum is

$$\lim_{T\to+\infty}\beta_{\text{IC}} = k \times 1 + (1-k) \times \frac{C - k(1-k)}{(1-k)^2} = \frac{C}{1-k}. \tag{E.4}$$

Furthermore, based on Theorem 5, we can obtain the following.

(1) When $0 < P/p_t \le k$, that is, $0 < P \le kp_t$, $\rho_{0,\text{opt}} < 1$,

$$\rho_{0,\text{opt}} = \left(\frac{P}{kp_t}\right)^{-},$$
$$\rho_{1,\text{opt}} = 0, \tag{E.5}$$
$$T_{\text{opt}} = +\infty.$$

(2) When $k < P/p_t \le C/(1-k)$, that is, $kp_t < P \le Cp_t/(1-k)$,

$$\rho_{0,\text{opt}} = 1,$$
$$\rho_{1,\text{opt}} = \frac{(P/p_t)^{-} - k}{1-k} = \left(\frac{P - kp_t}{(1-k)p_t}\right)^{-}, \tag{E.6}$$
$$T_{\text{opt}} = +\infty.$$

(3) When $P/p_t > C/(1-k)$, that is, $P > Cp_t/(1-k)$, $\rho_{0,\text{opt}} = 1$ and the optimal $T_{\text{opt}}, \rho_{1,\text{opt}}$ can be obtained by solving **P3**. In other words, T_{opt} and $\rho_{1,\text{opt}}$ satisfy

$$k\phi_0\left(1, T_{\text{opt}}\right) + (1-k)\phi_1\left(\rho_{1,\text{opt}}, T_{\text{opt}}\right) = C,$$
$$\frac{Q}{T_{\text{opt}}} + p_t\left(k + (1-k)\rho_{1,\text{opt}}\right) = P. \tag{E.7}$$

Thus, $\rho_{1,\text{opt}} = (P - kp_t)T_{\text{opt}} - Q/(1-k)p_tT_{\text{opt}}$, and T_{opt} satisfies

$$aT + b = e^{gT+h}, \tag{E.8}$$

where a, b, g, h are given in (23). Due to $g \ne 0$, thus when $g \ne 0$, that is, $P \ne p_t$, using the similar method to solve (B.5), we can directly get that

$$T_{\text{opt}} = -\frac{1}{g}\mathcal{W}\left(-\frac{g}{a}e^{(ah-bg)/a}\right) - \frac{b}{a}. \tag{E.9}$$

When $P = p_t$, the optimal $T_{\text{opt}} = (e^h - b)/a$.

When $C < k(1-k)$, based on Theorem 5, we know that $\rho_{0,\text{opt}} = 1$ and $\rho_{0,\text{opt}} > 0$ if and only if $P > Q/T_c + kp_t$, as shown in Figure 3(c). Otherwise, $\rho_{0,\text{opt}}$ and T_{opt} are given by Theorem 5 and $\rho_{0,\text{opt}} = 0$.

Therefore, when $P > Q/T_c + kp_t$, T_{opt} and $\rho_{1,\text{opt}}$ can be obtained by solving **P3**. Thus, similar to the situation of $k(1-k) \le C < 1-k$ and $P/p_t > C/(1-k)$, we have that $\rho_{1,\text{opt}} = ((P - kp_t)T_{\text{opt}} - Q)/(1-k)p_tT_{\text{opt}}$ and

$$T_{\text{opt}} = \begin{cases} \dfrac{e^h - b}{a}, & P = p_t \\ -\dfrac{1}{g}\mathcal{W}\left(-\dfrac{g}{a}e^{(ah-bg)/a}\right) - \dfrac{b}{a}, & P \ne p_t, \end{cases} \tag{E.10}$$

where a, b, g, h are given in (23).

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