Hindawi Publishing Corporation
EURASIP Journal on Wireless Communications and Networking
Volume 2011, Article ID 653506, 9 pages
doi:10.1155/2011/653506

Research Article

Full-Diversity Space-Time Error Correcting Codes with Low-Complexity Receivers

Mohamad Sayed Hassan and Karine Amis

Signals and Communications Department, TELECOM Bretagne, 29238 BREST, France

Correspondence should be addressed to Mohamad Sayed Hassan, mohamad-sayed@hotmail.com

Received 3 August 2010; Revised 9 November 2010; Accepted 11 January 2011

Academic Editor: Wolfgang H. Gerstacker

We propose an explicit construction of full-diversity space-time block codes, under the constraint of an error correction capability. Furthermore, these codes are constructed in order to be suitable for a serial concatenation with an outer linear forward error correcting (FEC) code. We apply the binary rank criterion, and we use the threaded layering technique and an inner linear FEC code to define a space-time error-correcting code. When serially concatenated with an outer linear FEC code, a product code can be built at the receiver, and adapted iterative receiver structures can be applied. An optimized hybrid structure mixing MMSE turbo equalization and turbo product code decoding is proposed. It yields reduced complexity and enhanced performance compared to previous existing structures.

1. Introduction

Space-time block (STB) code designs have recently attracted considerable attention, since they improve the reliability of communication systems over fading channels. Tarokh et al. [1] developed some criteria for designing STB codes (for the high SNR regime), in order to minimize the pairwise error probability. Among the resulting proposed schemes (based on these criteria), orthogonal space-time block (OSTB) codes, introduced by Alamouti [2] and generalized by Tarokh et al. [3], are attractive due to their low optimal decoding complexity. Their practical use is limited to the Alamouti scheme (two transmit antennas) as their rate decreases rapidly with an increase in the number of transmit antennas, and they cannot achieve the MIMO system capacity. Hassibi and Hochwald proposed the linear dispersion codes (LDCs) [4] that maximize the mutual information between transmitted and received signals in order to achieve the maximum ergodic capacity of the equivalent MIMO system.

Then full rate and full diversity STB codes were designed. The application of the threaded layering principle yielded the threaded algebraic space-time (TAST) codes [5]. Belfiore et al. added a nonvanishing determinant constraint [6–8] to achieve the optimal diversity/multiplexing tradeoff [9] and defined the perfect space-time block codes [10, 11].

However, in any transmission system the forward error correction coding is used in conjunction with interleaving. All the above STB codes deal with the forward error correcting code as an independent entity of the transmitter scheme. A joint design of FEC, modulation, and space-time scheme was considered in [1], in order to construct the space-time trellis codes (STTCs) that provide maximum diversity and maximum coding gain. STTCs exhibit higher coding gains than STB codes, but due to their trellis nature, the optimal decoding has a high computational cost incompatible with a practical implementation.

An interesting method to design full diversity space-time block codes with an error correction capability has been developed in [12–14]. In the current paper, these codes are referred as space-time error correcting codes (STECCs) in order to stress on their ability to correct errors due to the transmission. We name also concatenated STECC the serial concatenation of a STECC and an outer linear FEC code. In [12], a binary rank criterion has been introduced in order to construct full diversity STECCs for binary phase shift keying (BPSK) modulation. A generalization of this criterion to design full diversity STECCs with higher spectral efficiency using quadrature amplitude modulation (QAM), has been considered in [14]. But, since the generalized rank criterion implies that the error correcting code must be defined over a finite field where its dimension depends on the modulation order, the construction of full diversity STECCs for higher-order modulations cannot be realized in practice using this approach. In [13], an unified construction of STECCs achieving the optimal rate-diversity tradeoff [15] from binary error correcting codes for different types and orders of modulation has been presented. Independently, an explicit construction of concatenated STECCs for 2 transmit antennas based on explicit linear combinations of FEC codewords has been presented in [16]. Thanks to the linearity of FEC codes, it has been shown in [17] that STECCs seem more adapted than other STB codes to be concatenated with linear forward error correcting codes.

The contributions of this paper are as follows.

(i) Compared to [16], a theoretical analysis of STECC is given resulting in the design of full diversity STECCs for two transmit antennas.

(ii) It is proved that a product code can be reconstructed from a concatenated STECC. An optimized hybrid receiver associating a turbo equalizer (interference canceller) based on the minimum mean square error (MMSE) criterion and a turbo product code decoder is proposed, yielding reduced complexity and enhanced performance.

In Section 2, we describe the system model and show through theoretical analysis the suboptimality of STECCs proposed in [16] for 2 transmit antennas. In Section 3, we apply the threaded layering principle and the binary rank criterion to design full transmit diversity STECCs. In Section 4, considering STECCs concatenated to an outer FEC code, we first show how a product code can be reconstructed at the receiver. Then we combine the turbo equalization and turbo product code decoding principles to develop an adapted receiver with reduced complexity and enhanced performance. In Section 5, we present simulation results and finally, we give our conclusions in Section 6.

Notations. Column vectors (resp., matrices) are denoted by boldface lower (resp., capital) case letters. Superscripts $(\cdot)^{\mathrm{T}}$ and $(\cdot)^{\mathrm{H}}$ stand for transpose and conjugate transpose, respectively. $\mathbf{I}_n$ represents the $n \times n$ identity matrix. $\mathbb{Z}$, $\mathbb{C}$, and $\mathbb{Z}[i]$ denote, respectively, the ring of rational integers, the field of complex numbers and the ring of complex integers. $\mathbf{0}_{n\times n}$ (resp., $\mathbf{1}_{n\times n}$) denotes the $n \times n$ matrix having all its elements equal to 0 (resp., 1). Subscripts of matrices indicate their dimensions.

2. System Model

We first recall the usual criteria applied to design space-time codes for a MIMO system with n_t transmit antennas and n_r receive antennas considering a transmission over a nonfrequency selective block fading channel. We assume that the channel state information (CSI) is known at the receiver. In the second part, we give a theoretical analysis of the family of rectangular STECCs presented in [16] for $n_t = 2$ transmit antennas to show that they do not achieve full transmit diversity.

The $n_r \times T$ received signal matrix can then be expressed as

$$\mathbf{Y}_{n_r\times T} = \mathbf{H}_{n_r\times n_t}\mathbf{X}_{n_t\times T} + \mathbf{N}_{n_r\times T}, \tag{1}$$

where $\mathbf{X}$ is the $n_t \times T$ transmitted space-time error correcting codeword, $\mathbf{H}$ is the $n_r \times n_t$ channel matrix with independent and identically distributed (i.i.d.) zero-mean complex circular Gaussian entries and $\mathbf{N}$ is the $n_r \times T$ i.i.d. zero-mean complex circular Gaussian noise.

2.1. Space-Time Code Design Criteria. Let us assume a coherent scenario and an optimal maximum likelihood STB code detection. The pairwise error probability (PEP) is defined as the probability of estimating a codeword $\hat{\mathbf{X}} \neq \mathbf{X}$ at the receiver while $\mathbf{X}$ has been sent. To minimize the pairwise error probability, the space-time code must fulfill the following constraints [18].

(i) *The Rank Criterion.* Maximize the minimum rank r of the matrix $\mathbf{A}(\mathbf{X},\hat{\mathbf{X}}) = (\mathbf{X}-\hat{\mathbf{X}})(\mathbf{X}-\hat{\mathbf{X}})^{\mathrm{H}}$.

(ii) *The Determinant Criterion.* Maximize the minimum product of the nonzero eigenvalues, $(\prod_{j=1}^{r}\lambda_j)$, of the matrix $\mathbf{A}(\mathbf{X},\hat{\mathbf{X}})$. This criterion maximizes the coding gain.

The maximum diversity advantage in this context is $n_t \times n_r$. Space-time codes that achieve such a diversity are called full diversity space-time codes [1, 18].

2.2. Definition of a STECC. A STECC is a space-time code defined as an $n_t \times T$ symbol matrix based on the modulation of m_b basic binary matrices $\mathbf{C}^b_{n_t\times T}$, which can be grouped to form a binary matrix $\mathbf{C}_{n_t\times T}$ where each entry is a binary m_b-tuple so that m_b is the modulation efficiency. Matrix $\mathbf{C}^b_{n_t\times T}$ is a spatial rearrangement of a FEC codeword of $\mathcal{C}_I$. $\mathcal{C}_I$ is called the inner code. Its length is equal to $n_t \cdot T$.

2.3. $2 \times T$ STECCs: Theoretical Analysis. We consider, in this subsection, the family of rectangular STECCs proposed in [16] and we verify that they do not fulfill the preceding criteria, and thus do not achieve full diversity.

The $2 \times T$ STECC in [16] is defined from an one-half coding rate systematic block code of length $2T$. Its codewords are given by

$$c_k = m_k, \quad c_{k+T} = \bigoplus_{j=1,\, j\neq k}^{T} m_j \quad 1 \le k \le T, \tag{2}$$

where $\{m_j\}$, $1 \le j \le T$, represent the information bits and $\oplus$ stands for the mod-2 addition. Let $\mathbf{C}_{n_t\times T}$ $(n_t = 2)$ be the binary matrix associated to a space-time error correcting codeword. It is defined as [16]

$$\mathbf{C}_{n_t\times T} = \begin{bmatrix} \mathbf{c}_1 = \mathbf{m}_1 & \mathbf{c}_2 = \mathbf{m}_2 & \cdots & \mathbf{c}_T = \mathbf{m}_T \\ \bigoplus_{j=2}^{T} \mathbf{c}_j & \bigoplus_{j=1, j\neq 2}^{T} \mathbf{c}_j & \cdots & \bigoplus_{j=1}^{T-1} \mathbf{c}_j \end{bmatrix}, \quad (3)$$

where $\mathbf{c}_j$, $1 \leq j \leq T$, represents an information binary m_b-tuple. Assuming a M-ary quadrature amplitude modulation (M-QAM) where $M = 2^{m_b}$, the STECC codeword which corresponds to $\mathbf{C}_{n_t\times T}$ is defined by

$$\mathbf{X} = \begin{bmatrix} x_1 & x_2 & \cdots & x_T \\ x_{2\oplus 3\cdots\oplus T} & x_{1\oplus 3\cdots\oplus T} & \cdots & x_{1\oplus 2\cdots\oplus T-1} \end{bmatrix}, \quad (4)$$

where x_j, $1 \leq j \leq T$, represents the M-QAM symbol associated to the jth binary m_b-tuple denoted by $\mathbf{c}_j$ and $x_{1\oplus 2\cdots\oplus k-1\oplus k+1\oplus\cdots T}$ represents the M-QAM symbol associated to the binary m_b-tuple denoted by $\bigoplus_{j=1, j\neq k}^{T}\mathbf{c}_j$. To prove that the rank criterion is not realized, we verify that it is possible to find two STECC codewords such that their difference matrix has a rank of 1 inferior to the maximum possible rank $r = n_t = 2$.

Let $\mathbf{c}_1 = \mathbf{c}_2 = \cdots = \mathbf{c}_T = \mathbf{1}_{1\times m_b}$, thus

$$\begin{aligned} &\text{if } T \text{ is odd} \bigoplus_{j=1, j\neq k}^{T} \mathbf{c}_j = \mathbf{0}_{1\times m_b}, \quad 1 \leq k \leq T, \\ &\text{if } T \text{ is even} \bigoplus_{j=1, j\neq k}^{T} \mathbf{c}_j = \mathbf{1}_{1\times m_b}, \quad 1 \leq k \leq T. \end{aligned} \quad (5)$$

Let $\hat{\mathbf{c}}_1 = \hat{\mathbf{c}}_2 = \cdots = \hat{\mathbf{c}}_T = \mathbf{0}_{1\times m_b}$ thus $\bigoplus_{j=1, j\neq k}^{T}\hat{\mathbf{c}}_j = \mathbf{0}_{1\times m_b}$. Let us denote by $\mathbf{X}$ and $\hat{\mathbf{X}}$ the STECC codewords associated to $\mathbf{C}$ and $\hat{\mathbf{C}}$, respectively. If T is odd, then the difference matrix $\mathbf{B} = \mathbf{X} - \hat{\mathbf{X}}$ has all entries of the second row equal to 0. In the other case (T is even), all the columns of $\mathbf{B}$ are equal. Therefore, for all values of T, the difference matrix has a rank equal to $1 < n_t$ and as a consequence this family of STECCs does not achieve a full transmit diversity.

Furthermore, if we consider the minimum Hamming distance ($d_{\min}^{(T)}$) of the basic binary code defined in (2), one can see that $d_{\min}^{(3)} = 3$ and for $T \geq 4$, $d_{\min}^{(T)} = 4$. For example, the Hamming weight of this basic binary codeword matrix $\mathbf{C}_{n_t\times T}^{b} = \left[\begin{smallmatrix} 1 & 1 & 0 & \cdots & 0 \\ 1 & 1 & 0 & \cdots & 0 \end{smallmatrix}\right]$ for all $T \geq 4$ is equal to 4. As a consequence, inner FEC codes defined for $T \geq 5$ will not enable to increase the maximum time diversity achieved by the STECC.

In the following, we focus only on the cases $T = 3$ and $T = 4$ in order to illustrate the construction of full diversity STECCs.

3. Full-Diversity STECCs

To determine the maximum rate that can be realized when a full spatial diversity is achieved, we need to define the optimal rate-diversity tradeoff [13] which can be characterized by the following equation:

$$R_{\text{symbol}} = K[n_t - d + 1], \quad (6)$$

where R_{symbol} is the modulation symbol rate per channel use (p.c.u.), the quantity d is termed the transmit diversity gain, and K is the extension degree of the transmission symbol set drawn from the modulation symbol set. In our construction, a transmission symbol is drawn from a constellation set, thus $K = 1$. Moreover, a full transmit diversity is quantified by $d = n_t$, therefore the optimal rate-diversity tradeoff implies a maximum rate of one modulation symbol p.c.u., that is, a full diversity STECC has a maximum rate equals to $1/n_t$. As a consequence, we use for the construction $1/n_t$-rate inner codes.

As the maximum rate of the inner code is determined, it is important to note that in the following for $T = 3$ and $T = 4$, the selected inner codes have the maximum of the minimum Hamming distance for a combination of n_t transmit antennas and T time periods.

3.1. Full-Diversity STECC Designs. In order to realize the full spatial diversity, we recall that the binary rank criterion is a sufficient but not necessary condition to guarantee a full spatial diversity [12]. We apply the unified construction proposed in [13] to ensure a full spatial diversity for high-order modulations.

In [12], the authors prove that if every nonzero codeword of a linear binary code matrix has a maximum rank over the binary field $\mathbb{F} = \{0, 1\}$, then for a binary phase shift keying (BPSK) transmission, the STECC achieves full spatial diversity, that is, $d = n_t$. Moreover, it is demonstrated in [13] that if a STECC achieves full spatial diversity for BPSK transmission then using the Lu and Kumar construction (unified construction), we can obtain a full diversity STECC for high-order modulations based on full rank linear binary code matrices.

It yields a sufficient but not necessary condition on the linear binary inner code matrix to guarantee full diversity STECCs.

3.2. Threaded Layering Approach. To maximize the STECC diversity, we consider the threaded layering approach [5]. We assume rectangular STECCs of size $n_t \times T$. To design full diversity STB code, the threaded layering approach consists in splitting information symbols into disjoint threads. The threads must be active over the T transmission intervals. For each layer (thread) and at each transmission interval, symbols of this layer are transmitted. Threads use equally often the transmit antennas. To ensure a maximum diversity, each thread must achieve a maximum diversity when symbols corresponding to the other threads are put to zero and threads must be transparent to each other. This can be realized by affecting weighting numbers to each thread such that resulting threads span disjoint algebraic subspaces. These numbers are "Diophantine number".

In the case of STECCs, linear combinations are applied on binary elements which greatly relaxes the constraints to achieve full diversity STECCs. The number of threads is taken equal to the number of transmit antennas. The threaded layering set $\mathbb{L} = \{\ell_1, \ell_2\}$ is defined, for $1 \leq j \leq n_t = 2$, by

$$\ell_j = \left\{\left(\lfloor t + j - 1\rfloor_{n_t} + 1, t\right) : 0 \leq t < T\right\}, \tag{7}$$

where $\lfloor \cdot \rfloor_{n_t}$ denotes the mod-n_t operation. Table 1 shows the threaded layering for 2×3 and 2×4 structures using 2 threads. We associate the information symbols (resp., redundancy symbols) to the first layer (resp., second layer). Let us denote by $\mathbf{x} = [x_1, \ldots, x_T]^{\mathrm{T}}$ the information symbol vector and by $\mathbf{y} = [y_1, \ldots, y_T]^{\mathrm{T}} = \mathbf{M}[x_{2\oplus\cdots\oplus T}, \ldots, x_{1\oplus\cdots\oplus T-1}]^{\mathrm{T}}$ the vector associated to the redundancy symbols. $\mathbf{M}_{T\times T}$ is an integer permutation matrix such that the STECC, built up from $\mathbf{x}$, $\mathbf{y}$, and a diophantine number, achieves a full spatial diversity ($\mathbf{M}$ must also keep properties of STECCs defined in (6), i.e., each entry of a space-time codeword matrix is composed of one modulation symbol).

3.2.1. 2×3 *Full-Diversity STECC.* In this case, it was verified in [19] that a full diversity STECC can be defined by

$$\mathbf{X} = \begin{bmatrix} x_1 & \phi y_2 & x_2 \\ \phi y_1 & x_2 & \phi y_3 \end{bmatrix}, \tag{8}$$

where $(y_1, y_2, y_3) \in \{(x_{1\oplus3}, x_{1\oplus2}, x_{2\oplus3}), (x_{2\oplus3}, x_{1\oplus2}, x_{1\oplus3})\}$, whatever $\phi^2 \notin \mathbb{Z}[i]$ and $|\phi| = 1$ (to ensure an energy efficiency). As the determinant of the difference codeword matrix depends on the value of ϕ, thus to maximize the coding gain we must carefully select the diophantine number. In [19], it was proved that $\phi = 1$ is the optimal value for 4-QAM and 16-QAM. In this paper, by applying the binary rank criterion and the unified construction of [13] we verify that a full spatial diversity can be achieved without the necessity of a diophantine number ($\phi = 1$). For $(y_1, y_2, y_3) = (x_{1\oplus3}, x_{1\oplus2}, x_{2\oplus3})$, (the same result can be obtained for the other possible 3-tuple) the associated linear binary code matrix is defined by

$$\mathbf{C}_{2\times3} = \begin{bmatrix} \mathbf{c}_1 & \mathbf{c}_1 \oplus \mathbf{c}_2 & \mathbf{c}_3 \\ \mathbf{c}_1 \oplus \mathbf{c}_3 & \mathbf{c}_2 & \mathbf{c}_2 \oplus \mathbf{c}_3 \end{bmatrix}. \tag{9}$$

It clearly appears that the first and the second row of $\mathbf{C}_{2\times3}$ are linearly independent over the binary field $\mathbb{F}$. Thus for a BPSK transmission, the STECC constructed from $\mathbf{C}_{2\times3}$ achieves a full spatial diversity. Therefore, using the unified construction presented in [13], the STECC built up from this linear binary code matrix ensures a full spatial diversity for any order QAM modulation. We also note that this construction can be extended to phase shift keying (PSK) and pulse amplitude (PAM) modulations.

3.2.2. 2×4 *Full-Diversity STECC.* The inner code corresponds to the extended Hamming code. Due to the fact that the all-one vector is a codeword, the binary rank criterion is false whatever the spatial arrangement into a 2×4 binary matrix. We thus apply the threaded layering approach.

Table 1: The threaded layering in coherent scenario using 2 layers. The numbers refer to thread indexes. The vertical and horizontal axes correspond to the spatial and temporal dimensions, respectively.

(a)

1	2	1
2	1	2

(b)

1	2	1	2
2	1	2	1

The binary 2×4 matrix is

$$\mathbf{C}_{2\times4} = \begin{bmatrix} \mathbf{c}_1 & \bigoplus_{j=2}^{4} \mathbf{c}_j & \mathbf{c}_3 & \bigoplus_{j=1, j\neq2}^{4} \mathbf{c}_j \\ \bigoplus_{j=1, j\neq3}^{4} \mathbf{c}_j & \mathbf{c}_2 & \bigoplus_{j=1}^{3} \mathbf{c}_j & \mathbf{c}_4 \end{bmatrix}, \tag{10}$$

and the associated space-time error correcting codeword is

$$\mathbf{X}_{2\times4} = \begin{bmatrix} x_1 & \phi x_{2\oplus3\oplus4} & x_3 & \phi x_{1\oplus3\oplus4} \\ \phi x_{1\oplus2\oplus4} & x_2 & \phi x_{1\oplus2\oplus3} & x_4 \end{bmatrix}. \tag{11}$$

To ensure full diversity diophantine number ϕ is chosen such that $\phi^2 \notin \mathbb{Z}[i]$ and $|\phi| = 1$. In that case, the maximization of the coding gain yields an optimum value for ϕ depending of the modulation order. However, in practice the STECC will be serially concatenated to an outer FEC and it was shown in [20] that the asymptotic global coding gain is independent of the choice of the parameter ϕ provided that $\phi^2 \notin \mathbb{Z}[i]$ and $|\phi| = 1$. In [21], we proposed a way to construct full diversity $2 \times T$ STECCs defined from an inner half-rate invertible linear binary codes.

4. Receiver Structures

This section aims at designing reduced complexity and efficient receivers for concatenated STECCs. A maximum likelihood receiver for the concatenated STECC has a prohibitive complexity, and cannot be implemented in practice. We thus consider lower complexity receivers consisting of the cascade of elementary devices and based on the exchange of reliability information from a device to its neighbour in an iterative manner.

For the sake of generality, we consider the unified construction of Lu and Kumar [13] which encompasses, from a decoding point of view, the $2 \times T$ STECCs presented in the previous section (as the diophantine number can be viewed as a rotation of the second thread versus the first one, it does not appear at the decoding stage). We first show how a product code can be built from the concatenated STECC. Hybrid receivers combining both MMSE turbo equalization (successive interference cancellation) and turbo product code decoding will be designed.

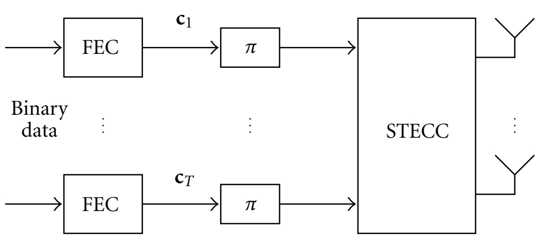

Figure 1: Concatenated STECC transmitter scheme.

4.1. Reconstruction of a Product Code from a Concatenated STECC. As mentioned previously to achieve the optimal rate-diversity tradeoff (6) thus the maximum rate of inner codes are necessarily equal to $1/n_t$ in order to ensure a full diversity STECC. We consider systematic linear binary inner codes. Thus, to construct a full diversity STECC we need T binary m_b-tuples $\mathbf{c}_j$, $j = 1, \ldots, T$ that correspond to the information sequences and $n_t(T-1)$ binary m_b-tuples that correspond to the redundancy sequences. Thus for $r = T+1, \ldots, n_t \cdot T$ the redundancy sequences can be written as

$$\mathbf{c}_r = (p_{1,r}\mathbf{c}_1) \oplus (p_{2,r}\mathbf{c}_2) \oplus \cdots \oplus (p_{T,r}\mathbf{c}_T), \qquad (12)$$

where $p_{j,r} \in \mathbb{F}$, and $\oplus$ stands for mod-2 addition. It can be space-time formatted to provide space-time codes achieving full spatial diversity using the threaded layering approach presented in the previous section or the unified construction [13].

Let $\mathcal{C}_O(N, L)$ be a linear FEC code (outer code), where N denotes the code length and L its dimension. For the concatenated STECC, $\{\mathbf{c}_j\}$, $j = 1, \ldots, T$ represent T information codewords and for $j = T+1, \ldots, n_t T$, $\mathbf{c}_j$, is a linear combination of codewords $\{\mathbf{c}_j\}$ defined by (12). After applying a sophisticated space-time formatter, we obtain $P = N/m_b$ space-time error correcting codewords $\mathbf{X}_{n_t \times T}(k)$, $1 \leq k \leq P$ to be transmitted. The transmitter scheme is represented in Figure 1. An interleaver is used to protect the information against burst errors, and to benefit from the time diversity.

A product code is defined as a serial concatenation of two FEC codes denoted $\mathcal{C}_r(N_r, K_r)$ and $\mathcal{C}_c(N_c, K_c)$. It consists in placing information bits in a $K_c \times K_r$ array. $\mathcal{C}_r$ is applied to encode rows of the array and the columns of the resulting $K_c \times N_r$ array are encoded using $\mathcal{C}_c$. The $N_c \times N_r$ obtained array is a codeword of the product code $\mathcal{C}_r \times \mathcal{C}_c$. Its minimum Hamming distance is the product of minimum Hamming distances of elementary constituent codes. In the case of concatenated STECCs, the $n_t T$ FEC codewords $\{\mathbf{c}_j\}$, $1 \leq j \leq n_t T$, form a product code represented as

$$\mathbf{T}_{n_t T \times N} = \begin{bmatrix} \mathbf{c}_1 \\ \vdots \\ \mathbf{c}_T \\ \mathbf{c}_{T+1} = (p_{1,T+1}\mathbf{c}_1) \oplus \cdots \oplus (p_{T,T+1}\mathbf{c}_T) \\ \vdots \\ \mathbf{c}_{n_t T} = (p_{1,n_t T}\mathbf{c}_1) \oplus \cdots \oplus (p_{T,n_t T}\mathbf{c}_T) \end{bmatrix}, \qquad (13)$$

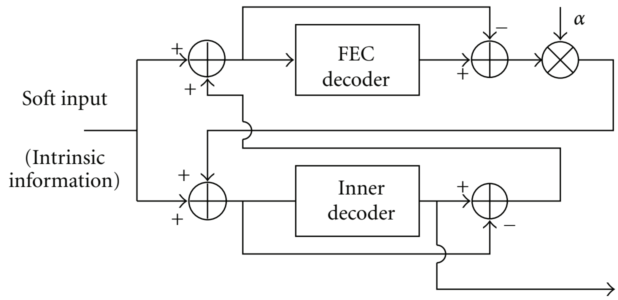

Figure 2: SISO turbo product code decoding structure.

where the inner code (resp., outer code) is applied along columns (resp., rows). We note that the linear outer FEC has no constraints on its rate neither on its length. Moreover, the information rate of the inner code can be increased by reducing the spatial diversity of the STECC according to the optimal rate-diversity tradeoff [13].

When full diversity algebraic STB codes (like the perfect STB codes, LDCs,...) are serially concatenated with linear FEC codes, the maximum order of diversity that can be achieved is equal to $n_t d_r^{\mathrm{H}} n_r$ where d_r^{H} is the minimum Hamming distance of the outer code. For concatenated full diversity STECCs, the maximum diversity order is equal to $n_t d_r^{\mathrm{H}} d_c^{\mathrm{H}} n_r$, where d_c^{H} is the minimum Hamming distance of the inner code. Thus, concatenated STECCs are better adapted to a serial concatenation with linear outer forward error correcting codes.

4.2. Turbo Product Code Decoding for Concatenated STECC. The existence of a product code for concatenated STECCs enables a better information exchange between the inner code decoder, the outer code decoder and the STECC detector through the application of turbo product code decoding principles [22].

As it is summarized in Figure 2, the turbo product code decoding is based on the reliability information exchange between two elementary soft input soft output (SISO) decoders: the inner one and the maximum *a posteriori* (MAP) FEC decoder. Each elementary decoder benefits from two inputs: the soft values (*intrinsic* information) delivered by a detector, and the previous elementary decoder *extrinsic* information (*a priori* information). So as to ensure the convergence, weighting coefficients α ($\alpha < 1$) are applied to the *a priori* information generated by the FEC decoder.

Assuming a nonfrequency selective block fading channel and perfect synchronization, at each receive antenna, the observation is the superposition of attenuated symbols simultaneously transmitted from transmit antennas and a circular complex Additive White Gaussian Noise. To detect the symbols transmitted from a given antenna, one has to take into account the interference of symbols corresponding to the other antennas. Maximum likelihood detection is possible but its complexity may be high and depends on the modulation order. Linear equalization such as successive interference cancellation enables to reduce the computation cost [23–25]. It consists in providing the equalizer with the estimation of the interfering symbols and cancelling it

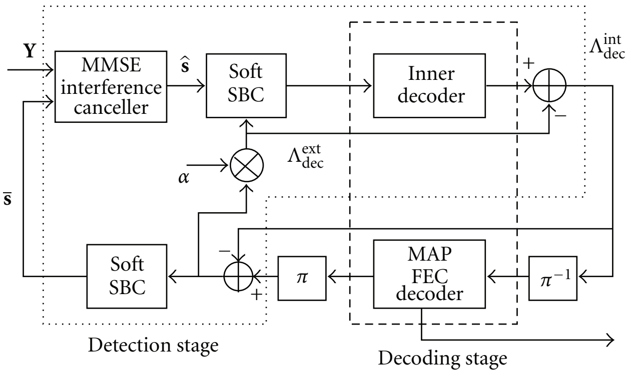

FIGURE 3: Proposed turbo receiver scheme for concatenated STECCs.

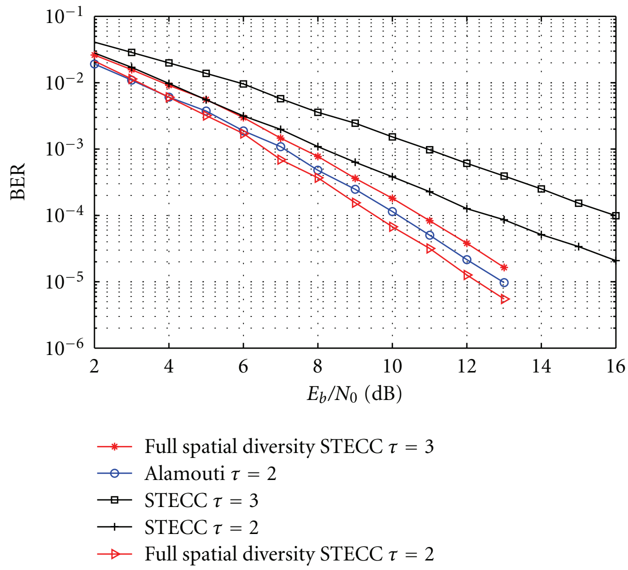

FIGURE 4: Performance of the full diversity STECC compared to suboptimal-diversity STECC [16] and Alamouti OSTBC [2].

in the observation. The equalizer consists of a feedforward filter and a feedback filter that can be optimized so as to minimize the mean square error at its output (in such case it is referred as MMSE interference canceller). In [17], we proposed for the concatenated STECC an iterative receiver structure composed by a MAP symbol detector over one symbol duration, which takes *a priori* soft information from the SISO turbo product code decoder. This structure exploits the channel diversity. But unfortunately, the complexity of MAP detector remains high especially for high-order modulations. To reduce the complexity of the detector, we propose an hybrid structure mixing the MMSE interference canceller and the turbo product code decoder. The resulting receiver scheme is given in Figure 3. The MMSE interference canceller does not take into account the STECC existence. It considers each symbol duration independently of the others and assumes that each received sample only depends on symbols simultaneously transmitted. It thus works as if symbols were transmitted using a spatial multiplexing scheme. The time correlation between transmitted symbols is taken into account in the product code decoder.

Complexity Analysis. In the MMSE turbo equalizer presented in [25], the MMSE interference canceller is the same as the one used in this paper. It is followed by an inner code decoder to take into account the STECC structure, an outer code decoder and another inner code decoder before going back to the equalizer. The structure proposed in this paper, based on the turbo product code decoding algorithm, has a lower complexity since one iteration consists of only one inner code decoding instead of two. Nevertheless, it outperforms the receiver given in [25] thanks to the identification of a turbo product code and to an optimization of the *extrinsic* information exchange (see Figure 3).

At the detection stage, the MMSE interference canceller is a linear filter, thus its complexity is determined by the size of matrices to be inverted. Moreover, the proposed receiver structure deals with concatenated STECCs as a spatial multiplexing scheme concatenated with an outer product code. Therefore, the MMSE interference canceller must invert a square matrix of dimension $n_r \times n_r$ at each iteration. An $n_t \times T$ STECC detection thus requires T inversions of matrices of size $n_r \times n_r$, that can be carried out in parallel. Considering a similar transmitter and receiver scheme involving an usual $n_t \times T$ full rate algebraic STB code (like perfect STB code, LDC, TAST code), we need one inversion of a matrix of size $n_rT \times n_rT$. Indeed, at the receiver, an algebraic STB code is equivalent to a spatial multiplexing scheme with n_rT receive antennas and $n_t \times T$ transmit antennas. Concatenated STECCs thus enable lower complexity receivers compared to linear dispersion codes concatenated to the same outer code.

5. Simulation Results

In this section, different transmission schemes are compared via simulations. Furthermore, we evaluate the performance of the iterative receiver structure based on the turbo product code decoding algorithm with a MMSE interference canceller versus the first MMSE turbo equalizer [25].

For our simulations, we consider $n_t = 2$ transmit antennas using a QAM Gray-mapped constellation over a Rayleigh nonfrequency selective block fading channel, constant over τ symbol durations, and $n_r = 2$ receive antennas. The channel is also assumed to be perfectly estimated at the receiver. For the outer FEC code, we use an half-rate convolutional code which is decoded using the SISO BCJR algorithm [26]. The inner code is maximum likelihood decoded according to the rules given in [25]. Plotted curves correspond to the convergence state of the iterative process.

5.1. Performance of the 2×3 *Full-Diversity STECC.* Figure 4 compares the performance of the 2×3 full diversity STECC with those of the 2×3 STECC [16] and the Alamouti scheme [2], without concatenation with an outer code.

In that case, a maximum likelihood (ML) detection is considered at the receiver and as space-time error correcting

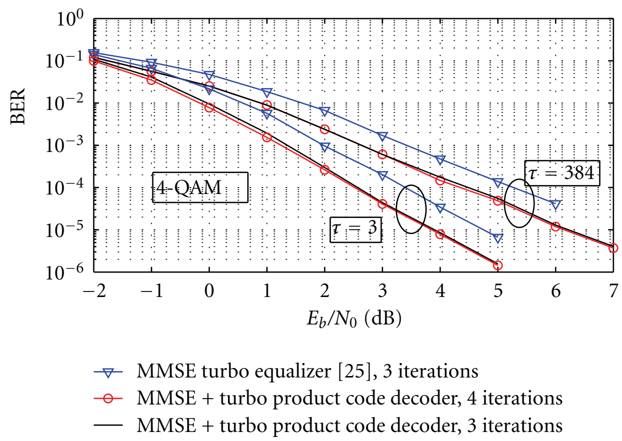

Figure 5: Performance comparison between the MMSE turbo receiver of [25] and the proposed iterative receiver (see Figure 3). Concatenated 2×3 full diversity STECC. 1530 informations bits. $n_r = 2$. $CC(7,5)_{\text{oct}}$.

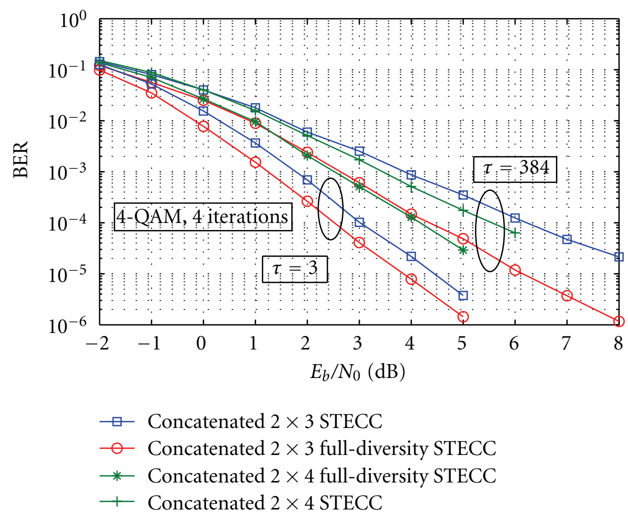

Figure 6: Performance comparison of the concatenated full diversity STECC and the concatenated STECC. 1530 information bits. $n_r = 2$. $CC(7,5)_{\text{oct}}$.

codewords (resp., Alamouti space-time codewords) are independent from each others, the BER performance remains the same for every τ multiple of $T = 3$ (resp., 2). One can see that the BER slope of the 2×3 full diversity STECC for $\tau = 2$ and $\tau = 3$ is equal to $n_t \times n_r = 4$, which confirms the full spatial diversity of this structure. On the other hand, as it was expected by the theoretical analysis in the Section 2.3, the BER slope of the 2×3 STECC is equal to $n_r = 2$ which means that this transmission scheme cannot fully exploit the transmit diversity. For $\tau = 3$ a gain of 5.3 dB is achieved at a BER of 10^{-4} with the full spatial diversity STECC. Moreover, the full diversity STECC performs 0.5 dB worse compared to the Alamouti scheme which is satisfactory as it is obtained without taking into account the outer FEC code. When $\tau = 2$, the full diversity STECC benefits from the additional time diversity of the channel. For a BER $= 10^{-5}$, the gain over the Alamouti scheme is equal to 0.5 dB.

5.2. Optimization of the Concatenated STECC Iterative Receivers. In Figure 5, we compare the performance of the MMSE interference canceller with a SISO turbo product code decoder versus that of the MMSE turbo equalizer [25] for the concatenated 2×3 full diversity STECC. Simulations are carried out for $\tau = 3$ (roughly quasifast fading channel) and $\tau = 384$ (slow fading channel), respectively, in order to observe the influence of the time diversity on the performance of the receiver structures. In addition to the computation cost reduction per iteration (one inner decoding instead of two per iteration), the hybrid scheme enables a SNR gain for a same BER value. Another advantage of product code identification is the stopping criterion [22] that can be applied to stop the iterative process as soon as a product codeword is detected, yielding a power consumption saving.

For a BER $= 10^{-4}$ and for $\tau = 3$ (resp., $\tau = 384$) a gain roughly equal to 0.9 dB (resp., 0.8 dB) is offered by the proposed iterative receiver with respect to the existing one. This hybrid receiver scheme is thus selected.

5.3. Performance of Concatenated Full-Diversity STECCs. We compare for $T = 3$ and $T = 4$ the performance of the concatenated $2 \times T$ full diversity STECC with the concatenated $2 \times T$ STECC [16] in order to further highlight the advantage of optimizing STECCs from a space-time point of view.

In Figure 6, we compare the performance of the concatenated full diversity STECC versus the concatenated STECC for $\tau = 3$ (the time duration to transmit one STECC codeword) and $\tau = 384$ using 4-QAM Gray-mapped constellation. The higher the coherence time ($\tau = 384$), the more significant the spatial diversity effect on performance. For a BER $= 10^{-5}$ and for $\tau = 3$ the proposed transmission scheme outperforms by 0.6 dB the one presented in [16]. The gain increases with τ. Indeed for $\tau = 384$ and $T = 3$ (resp., $T = 4$), the gain becomes equal to 2.6 dB for a BER of 10^{-5} (resp., 1.7 dB for a BER $= 10^{-4}$). Note that the computation cost at the receiver side is the same for both schemes.

Figure 7 shows the effect of the error correction capability of the outer code on the performance measured in terms of BER. We consider a slow fading channel ($\tau = 1020$). At a BER $= 10^{-4}$ and using the 1/2-rate convolutional code $CC(7,5)_{\text{oct}}$ as an outer code, a gain of 0.9 dB is offered by exploiting the spatial diversity. At the same BER and taking into account the 1/2-rate convolutional code $CC(133,171)_{\text{oct}}$ as an outer code, the gain achieved by the concatenated full diversity STECC with respect to the suboptimal-diversity concatenated STECC is reduced to 0.7 dB. When the error correction capability of the FEC outer code increases, the time diversity becomes more influent than spatial diversity on performance. Concatenated STECCs exploit efficiently the time diversity. Furthermore, considering the scheme with the outer code $CC(7,5)_{\text{oct}}$, the gain observed in Figure 7 for $\tau = 1020$ is less than the gain observed in Figure 6 for $\tau = 384$. The information length is indeed higher in the scheme with $\tau = 1020$.

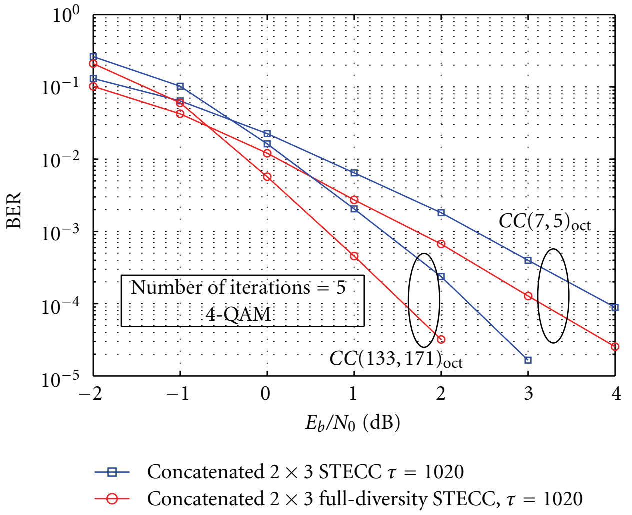

FIGURE 7: Performance comparison of the concatenated full diversity STECC and the concatenated STECC. 3×4074 information bits. $n_r = 2$.

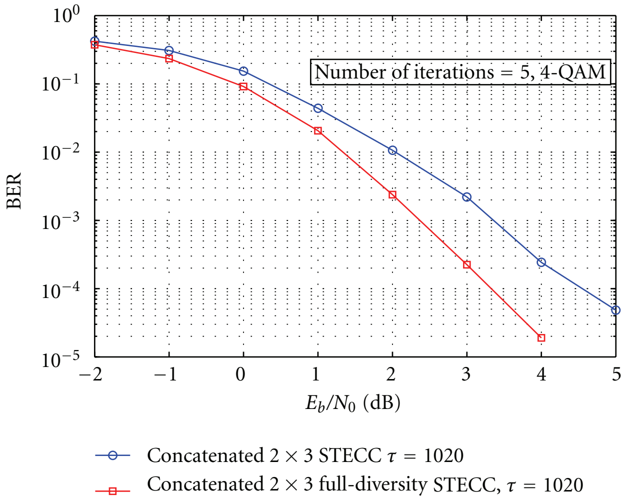

FIGURE 8: Performance comparison of the concatenated full diversity STECC and the concatenated suboptimal-diversity STECC. 3×4074 information bits. $n_r = 2$. Outer code: the 2/3-rate punctured $CC(133, 171)_{oct}$.

5.4. Puncturing. A drawback of concatenated STECCs is their low coding rate. However, puncturing can be used to increase the system throughput. Unfortunately, puncturing the inner code induces a degradation of the transmit diversity. In order to guarantee a full spatial diversity, we apply puncturing on the outer code. We consider 2/3-rate and 7/8-rate punctured convolutional codes generated from the half-rate parent convolutional code $CC(133, 171)_{oct}$.

In Figure 8, the outer code is punctured so as to get a coding rate of 2/3. For a BER of 10^{-4} and $\tau = 1020$, the gain of the full diversity scheme compared to the suboptimal one is equal to 1.2 dB. In Figure 9, the outer code coding rate is 7/8. For a BER of 10^{-3}, the full diversity scheme outperforms the suboptimal one by 4 dB.

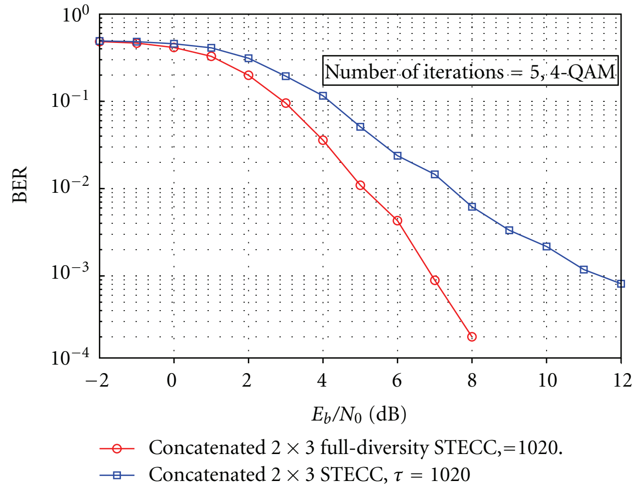

FIGURE 9: Performance comparison of the concatenated full diversity STECC and the concatenated suboptimal-diversity STECC. 3×4074 information bits. $n_r = 2$. Outer code: the 7/8-rate punctured $CC(133, 171)_{oct}$.

The outer code puncturing yields a decrease of its error-correction capability and thus a lower exploitation of the available time diversity. Maximization of the space-diversity is all the more essential.

6. Conclusion

In this paper, we have presented the construction of space-time codes with error correction capability and we have optimized the receiver in case of serial concatenation with an outer FEC code.

The application of both binary rank criterion, associated to the Lu and Kumar unified construction, and threaded layering technique yields a systematic procedure for developing full diversity STECCs. When concatenated with an outer FEC code, an equivalent product code can be constructed at the receiver side enabling an hybrid iterative receiver combining both MMSE turbo equalization and turbo product code decoding. Compared to equivalent linear dispersion codes, STECCs are better adapted for a serial concatenation with an outer FEC code and the optimized receiver exhibits a lower computation cost, associated to a better convergence state and a simple stopping criterion yielding power consumption saving.

Acknowledgments

The authors would like to thank Orange Labs for supporting their research. Part of the content of this manuscript has been published in IEEE ISWCS 2009 and in IEEE PIMRC 2009.

References

[1] V. Tarokh, N. Seshadri, and A. R. Calderbank, "Space-time codes for high data rate wireless communication: performance

criterion and code construction," *IEEE Transactions on Information Theory*, vol. 44, no. 2, pp. 744–765, 1998.

[2] S. M. Alamouti, "A simple transmit diversity technique for wireless communications," *IEEE Journal on Selected Areas in Communications*, vol. 16, no. 8, pp. 1451–1458, 1998.

[3] V. Tarokh, H. Jafarkhani, and A. R. Calderbank, "Space-time block codes from orthogonal designs," *IEEE Transactions on Information Theory*, vol. 45, no. 5, pp. 1456–1467, 1999.

[4] B. Hassibi and B. M. Hochwald, "High-rate codes that are linear in space and time," *IEEE Transactions on Information Theory*, vol. 48, no. 7, pp. 1804–1824, 2002.

[5] H. El Gamal and M. O. Damen, "Universal space-time coding," *IEEE Transactions on Information Theory*, vol. 49, no. 5, pp. 1097–1119, 2003.

[6] F. Oggier, J. C. Belfiore, and E. Viterbo, "Cyclic division algebras: a tool for space-time coding," *Foundations and Trends in Communications and Information Theory*, vol. 4, no. 1, pp. 1–95, 2007.

[7] H. Liao and X. G. Xia, "Some designs of full rate space-time codes with nonvanishing determinant," *IEEE Transactions on Information Theory*, vol. 53, no. 8, pp. 2898–2908, 2007.

[8] G. Wang and X. G. Xia, "On optimal multilayer cyclotomic space-time code designs," *IEEE Transactions on Information Theory*, vol. 51, no. 3, pp. 1102–1135, 2005.

[9] L. Zheng and D. N. C. Tse, "Diversity and multiplexing: a fundamental tradeoff in multiple-antenna channels," *IEEE Transactions on Information Theory*, vol. 49, no. 5, pp. 1073–1096, 2003.

[10] J. C. Belfiore, G. Rekaya, and E. Viterbo, "The Golden code: a 2×2 full-rate space-time code with nonvanishing determinants," *IEEE Transactions on Information Theory*, vol. 51, no. 4, pp. 1432–1436, 2005.

[11] F. Oggier, G. Rekaya, J. C. Belfiore, and E. Viterbo, "Perfect space-time block codes," *IEEE Transactions on Information Theory*, vol. 52, no. 9, pp. 3885–3902, 2006.

[12] A. R. Hammons and H. E. Gamal, "On the theory of space-time codes for PSK modulation," *IEEE Transactions on Information Theory*, vol. 46, no. 2, pp. 524–542, 2000.

[13] H. F. Lu and P. V. Kumar, "A unified construction of space-time codes with optimal rate-diversity tradeoff," *IEEE Transactions on Information Theory*, vol. 51, no. 5, pp. 1709–1730, 2005.

[14] Y. Liu, M. P. Fitz, and O. Y. Takeshita, "A rank criterion for QAM space-time codes," *IEEE Transactions on Information Theory*, vol. 48, no. 12, pp. 3062–3079, 2002.

[15] H. F. Lu and P. V. Kumar, "Rate-diversity tradeoff of space-time codes with fixed alphabet and optimal constructions for PSK modulation," *IEEE Transactions on Information Theory*, vol. 49, no. 10, pp. 2747–2751, 2003.

[16] M. Lalam, K. Amis, and D. Leroux, "Space-time error correcting codes," *IEEE Transactions on Wireless Communications*, vol. 7, no. 5, Article ID 4524302, pp. 1472–1476, 2008.

[17] M. S. Hassan and K. Amis, "Turbo product code decoding for concatenated space-time error correcting codes," in *Proceedings of the 20th IEEE Personal, Indoor and Mobile Radio Communications Symposium (PIMRC '09)*, Tokyo, Japan, September 2009.

[18] B. Vucetic and J. Yuan, *Space-Time Coding*, Willey, New York, NY, USA, 2003.

[19] M. S. Hassan and K. Amis, "Multilayer space-time error correcting codes," in *Proceedings of the 6th International Symposium on Wireless Communication Systems (ISWCS '09)*, pp. 318–322, Sienna, Italy, September 2009.

[20] M. S. Hassan and K. Amis, "Constellation labeling optimization for iteratively decoded concatenated multilayer space-time error correcting codes," in *Proceedings of the 5th International Conference on Wireless and Mobile Communications (ICWMC '09)*, pp. 196–199, Cannes, France, August 2009.

[21] M. S. Hassan and K. Amis, "Construction of full diversity space-time codes based on half-rate invertible linear binary codes," in *Proceedings of the 6th International Symposium on Turbo Codes and Iterative Information Processing (ISTC '10)*, pp. 117–121, Brest, France, September 2010.

[22] R. M. Pyndiah, "Near-optimum decoding of product codes: block turbo codes," *IEEE Transactions on Communications*, vol. 46, no. 8, pp. 1003–1010, 1998.

[23] M. Tüchler, R. Koetter, and A. C. Singer, "Turbo equalization: principles and new results," *IEEE Transactions on Communications*, vol. 50, no. 5, pp. 754–767, 2002.

[24] C. Laot, R. Le Bidan, and D. Leroux, "Low-complexity MMSE turbo equalization: a possible solution for EDGE," *IEEE Transactions on Wireless Communications*, vol. 4, no. 3, pp. 965–974, 2005.

[25] M. Lalam, K. Amis, and D. Leroux, "Iterative decoding of space-time error correcting codes," in *Proceedings of the 18th Annual IEEE International Symposium on Personal, Indoor and Mobile Radio Communications (PIMRC '07)*, September 2007.

[26] L. R. Bahl, J. Cocke, F. Jelinek, and J. Raviv, "Optimal decoding of linear codes for minimizing symbol error rate," *IEEE Transactions on Information Theory*, vol. 20, no. 2, pp. 284–287, 1974.