

## Research Article

# Amplitude PDF Analysis of OFDM Signal Using Probabilistic PAPR Reduction Method

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To reduce the peak-to-average power ratio (PAPR) of an orthogonal frequency division multiplexing (OFDM) modulation scheme, one class of methods is to generate several OFDM symbols (candidates) carrying the same information and to select for transmission the one having the lowest PAPR. We derive a theoretical amplitude probability density function (PDF) of the selected OFDM symbol using order statistics. This amplitude PDF enables one to derive the signal-to-noise-plus-distortion ratio (SNDR) as a function of the number of candidates. Based on the SNDR derivation, theoretical error performance and statistical channel capacity are provided for this class of methods. The results match the simulations and make the system design easier.

## 1. Introduction

Orthogonal frequency division multiplexing (OFDM) is a multicarrier multiplexing technique, where data is transmitted through several parallel frequency subchannels at a lower rate. It has been popularly standardized in many wireless applications such as Digital Video Broadcasting (DVB), Digital Audio Broadcasting (DAB), High Performance Wireless Local Area Network (HIPERLAN), IEEE 802.11 (WiFi), and IEEE 802.16 (WiMAX). It has also been employed for wired applications as in the Asynchronous Digital Subscriber Line (ADSL) and power-line communications.

A significant drawback of the OFDM-based system is its high Peak-to-Average Power Ratio (PAPR) at the transmitter, requiring the use of a highly linear amplifier which leads to low power efficiency [1]. Moreover, when an OFDM signal level works on the nonlinear area of amplifier, the OFDM signals go through nonlinear distortions and degrade the error performance.

The various approaches to alleviate this problem in OFDM-based systems can be classified into five categories: clip effect transformation [2], coding [3], frame superposition using reserved tones [4], expansible constellation point: tone injection [4] and active constellation extension [5], and probabilistic solutions [6–13].

The principle of probabilistic methods is to reduce the probability of high PAPR by generating several OFDM symbols (multiple candidates) carrying the same information and by selecting the one having the lowest PAPR. The probabilistic method can also be classified into two strategies: subblock partitioning strategy and entire block strategy. The subblock partitioning strategy, such as partial transmit sequence (PTS) [6–8], divides frequency domain signals into several subblocks. On the other hand, the entire block strategy, such as selected mapping (SLM) [8–10] and interleaving [11–13], considers the entire block for generating multiple candidates.

In this paper, we consider the entire block strategy of the probabilistic methods to generate multiple candidates. First, the probability density function (PDF) for the multiple candidate system is analyzed. When the candidate having the lowest PAPR is selected, the PDF of the amplitude of a selected OFDM symbol becomes the function of the number of candidates  $n$ . We apply the analyzed PDF (as a function of  $n$ ) to Ochiai's method [13] for obtaining the signal-to-noise-plus-distortion ratio (SNDR) as a function of  $n$ . Then, the SNDR (as a function of  $n$ ) can be used for analytical error performance. Note that in [13], the authors used the Rayleigh PDF (single candidate) for obtaining the error performance of multiple candidate cases. However, we suggest using our

PDF (multiple candidates) to obtain the theoretical error performance and also the statistical channel capacity for the multiple candidate system.

The paper is organized as follows: in Section 2, we describe the multiple candidate OFDM system, and analyze the PDF for the system. In Section 3, we derive the theoretical performance, such as the SNDR (as a function of  $n$ ), and error rate, and also statistical channel capacity. In Section 4, an extension of the results to an oversampled SLM model, implementing the “clipping and filtering” technique [14], is tackled. Finally, we conclude this paper in Section 5.

## 2. Multiple Candidate System

*2.1. Description.* In this section, we describe the multiple candidate solution for reduction of PAPR. Figure 1 describes the multiple candidate system and our PDF notation for several variables.  $n$  candidates (frequency domain signal) are generated by the candidate generator, where this candidate generator represents a class of probabilistic methods such as the SLM method [8–10] or the interleaving method [11–13]. After the  $N$ -point Inverse Discrete Fourier Transform (IDFT), we get the  $n$  OFDM candidates (time domain signal),  $\mathbf{x}_i = \{x_{i,1}, x_{i,2}, \dots, x_{i,N}\}$ ,  $i \in \{1, \dots, n\}$ . When we define  $r_{i,j} \triangleq |x_{i,j}|$ , then

$$\begin{aligned} |\mathbf{x}_1| &= \mathbf{r}_1 = \{r_{1,1}, r_{1,2}, \dots, r_{1,N-1}, r_{1,N}\}, \\ |\mathbf{x}_2| &= \mathbf{r}_2 = \{r_{2,1}, r_{2,2}, \dots, r_{2,N-1}, r_{2,N}\}, \\ &\vdots \\ |\mathbf{x}_{i_0}| &= \mathbf{r}_{i_0} = \{r_{i_0,1}, r_{i_0,2}, \dots, r_{i_0,N-1}, r_{i_0,N}\}, \\ &\vdots \\ |\mathbf{x}_n| &= \mathbf{r}_n = \{r_{n,1}, r_{n,2}, \dots, r_{n,N-1}, r_{n,N}\}, \end{aligned} \quad (1)$$

and the peak detector selects the  $i_0$ th candidate, where  $i_0 = \operatorname{argmin}_i (\max_j \{r_{i,j}\})$  for  $i \in \{1, \dots, n\}$  and  $j \in \{1, \dots, N\}$ . Then, the selected ( $i_0$ th) OFDM signal candidate is clipped by a nonlinear amplifier, where we consider the soft clipping model [13] as follows:

$$\tilde{x}_{i_0,j} = g(x_{i_0,j}) \triangleq \begin{cases} x_{i_0,j}, & \text{for } |x_{i_0,j}| \leq \bar{A}, \\ \bar{A} \cdot \frac{x_{i_0,j}}{|x_{i_0,j}|}, & \text{for } |x_{i_0,j}| > \bar{A}, \end{cases} \quad (2)$$

where  $\bar{A}$  is the maximum permissible amplitude for the clipping model.

The clipped  $i_0$ th candidate is transmitted to the receiver with its side information, where the side information contains the information of  $i_0$  and it is used for recovering the original data. The side information protection depends on the various protection strategies, such as no side information method [9, 10] or coded side information method [12].

However, in this paper, for analyzing the pure effect of increasing  $n$  for the multiple candidate system, we assume that the side information is sent without errors.

Throughout this paper, the following are also assumed: according to the central limit theorem, the complex OFDM signal, which consists of a number of independent orthogonal subcarriers, is modeled as a complex Gaussian process with Rayleigh envelope distribution. In addition, since the OFDM modulation is strictly band limited, we consider only in-band distortion.

*2.2. PDF Analysis.* Based on the assumption that the OFDM signal  $x_{i,j}$  for  $i \in \{1, \dots, n\}$  and  $j \in \{1, \dots, N\}$  is complex Gaussian distributed with mean 0 and variance 1, the envelope  $r_{i,j} = |x_{i,j}|$  is Rayleigh distributed with PDF  $f_r$  given by

$$f_r(r) = \begin{cases} 2r \cdot \exp(-r^2), & \text{for } r \geq 0, \\ 0, & \text{for } r < 0. \end{cases} \quad (3)$$

According to the largest order statistics [15], the distribution of the maximum of the amplitude values  $\max_j \{r_{i,j}\} \sim f_{r_{\max}}$  is given by

$$\begin{aligned} f_{r_{\max}}(r) &= N f_r(r) \left( \int_{-\infty}^r f_r(x) dx \right)^{N-1} \\ &= N f_r(r) (1 - \exp(-r^2))^{N-1}. \end{aligned} \quad (4)$$

When we select the candidate having a minimum peak amplitude among  $n$  candidates, according to the smallest order statistics [15], we obtain the PDF of the peak amplitude of the selected candidate  $\min_i [\max_j \{r_{i,j}\}] \sim f_{r_{\max}^*}(r)$ , using  $f_{r_{\max}}(r)$ :

$$\begin{aligned} f_{r_{\max}^*}(r) &= n \cdot f_{r_{\max}}(r) \cdot \left( \int_r^{\infty} f_{r_{\max}}(x) dx \right)^{n-1} \\ &= 2nNr (S(r)^{N-1} - S(r)^N) \cdot (1 - S(r)^N)^{n-1}, \end{aligned} \quad (5)$$

where  $\int_r^{\infty} f_{r_{\max}}(x) dx = 1 - (1 - \exp(-r^2))^N$  and  $S(r) = 1 - \exp(-r^2)$ .

We now want to know the PDF of amplitude of the selected candidate  $r_{i_0} \sim f_{r^*}$ . In (5), we have obtained  $f_{r_{\max}^*}(r)$  from  $f_{r_{\max}}(r)$  using the smallest order statistics. Furthermore, since  $\max_j \{r_{i_0,j}\} = \min_i [\max_j \{r_{i,j}\}] \sim f_{r_{\max}^*}(r)$ , we can also express  $f_{r_{\max}^*}(r)$  as a function of  $f_{r^*}(r)$  using the largest order statistics. Then,

$$\begin{aligned} f_{r_{\max}^*}(r) &= N \cdot f_{r^*}(r) \cdot \left[ \int_0^r f_{r^*}(x) dx \right]^{N-1} \\ &= \frac{d[\mathcal{F}_{r^*}(r)^N]}{dr}, \end{aligned} \quad (6)$$

where  $\mathcal{F}_{r^*}(r) = \int_0^r f_{r^*}(x) dx$ . From (6), we can obtain

$$\mathcal{F}_{r^*}(r) = \left[ \int_0^r f_{r_{\max}^*}(x) dx \right]^{1/N}. \quad (7)$$

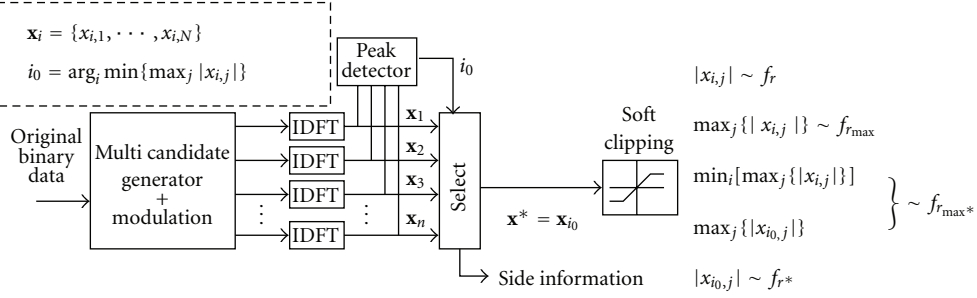


FIGURE 1: Multiple candidate system and its PDF notation for several variables. We define  $r_{i,j} \triangleq |x_{i,j}|$ , and the peak detector selects the  $i_0$ th candidate, where  $i_0 = \operatorname{argmin}_i(\max_j\{r_{i,j}\})$  for  $i \in \{1, \dots, n\}$  and  $j \in \{1, \dots, N\}$ .

So, the PDF of the amplitude of the selected candidate is given by

$$\begin{aligned} f_{r^*}(r) &= \mathcal{F}_{r^*}(r)' \\ &= \frac{1}{N} \left( \int_0^r f_{r_{\max^*}}(x) dx \right)^{1/N-1} \cdot f_{r_{\max^*}}(r) \\ &= n f_r(r) \cdot \left( 1 - [1 - S(r)^N]^n \right)^{1/N-1} \\ &\quad \cdot S(r)^{N-1} \cdot \left( 1 - S(r)^N \right)^{n-1}, \end{aligned} \quad (8)$$

where  $S(r) = 1 - \exp(-r^2)$ .

Figure 2 gives a comparison between the analytical and the simulation PDF in logarithm scale. Notice that the analytical line fits the simulation points.

### 3. Theoretical Performance

**3.1. SNDR<sup>(n)</sup> for Multiple Candidate System.** Now, we apply (8) to obtaining the signal-to-noise-plus-distortion ratio (SNDR) as a function of  $n$ , by using Ochiai's method [13]. The authors in [13] used the Rayleigh PDF,  $f_r$ , to obtain the SNDR of a multiple candidate system. However, as shown in Figure 2, the PDF of amplitude of the selected candidate is not Rayleigh PDF anymore, being the function of  $n$ . Therefore, we use the PDF of (8),  $f_{r^*}$ , to obtain the SNDR of multiple candidate system, and hereafter we will use SNDR<sup>(n)</sup> as a function of  $n$ , instead of SNDR.

For that, the PAPR threshold for clipping  $\bar{\lambda}$  is defined as  $\bar{\lambda} \triangleq (\bar{A}^{(n)})^2 / P_{\text{in}}^{(n)}$ , where the input power  $P_{\text{in}}^{(n)} = \int_0^\infty r^2 \cdot f_{r^*}(r) dr$  and  $\bar{A}^{(n)}$  is the maximum permissible amplitude for the multiple candidate system.

Then, based on  $f_{r^*}$  in (8), the total output power for the multiple candidate solution after clipping is obtained as

$$P_{\text{out}}^{(n)} = \int_0^{\bar{A}^{(n)}} r^2 f_{r^*}(r) dr + \int_{\bar{A}^{(n)}}^\infty (\bar{A}^{(n)})^2 f_{r^*}(r) dr, \quad (9)$$

and the signal distortion rate,  $\alpha^{(n)}$ , is given by

$$\alpha^{(n)} = \frac{\left( \int_0^{\bar{A}^{(n)}} r^2 f_{r^*}(r) dr + \int_{\bar{A}^{(n)}}^\infty \bar{A}^{(n)} r f_{r^*}(r) dr \right)}{P_{\text{in}}^{(n)}}. \quad (10)$$

Then,  $K_\gamma^{(n)}$ , total attenuation factor, is the following:

$$K_\gamma^{(n)} = \frac{\mathcal{G}^{(n)}}{P_{\text{out}}^{(n)}} = \frac{(\alpha^{(n)})^2 P_{\text{in}}^{(n)}}{P_{\text{out}}^{(n)}}. \quad (11)$$

Finally, SNDR<sup>(n)</sup> for the multiple candidate technique is given by

$$\text{SNDR}^{(n)} = \frac{K_\gamma^{(n)} E_s / N_0}{(1 - K_\gamma^{(n)}) E_s / N_0 + 1}. \quad (12)$$

**3.2. Error Rate.** Since we assume that the side information is transmitted without errors, the BER of QPSK-modulated signal over the AWGN channel is given by  $P_B = Q(\sqrt{\text{SNDR}^{(n)}})$ . Furthermore, QPSK symbol error rates (SER) are as follows:  $P_S = 1 - (1 - P_B)^2$ .

For the frequency-nonselctive slowly (constant attenuation during one OFDM symbol) Rayleigh-fading channel [16], the BER is given by

$$P_B = \int_0^\infty Q\left(\sqrt{\frac{\kappa^2 K_\gamma^{(n)} E_s / N_0}{\kappa^2 (1 - K_\gamma^{(n)}) E_s / N_0 + 1}}\right) f_r(\kappa) d\kappa, \quad (13)$$

where  $\kappa$  is the channel attenuation which is Rayleigh distributed with  $E[\kappa^2] = 1$ .

Figure 3 shows the error performance comparison over AWGN channel and frequency-nonselctive slowly fading channel, where the analytical approach and the simulation results are compared. For the simulations, 1024-point FFT pairs are considered and the signals are modulated by QPSK. At the transmitter, the OFDM signals are clipped at  $\bar{\lambda} = 0$  dB. In the figure, we can see that the simulated SER is well matched on the analytical line, and an error floor appears at large SNR because of the clipping noise. In addition, we can see better error performance, when  $n$  increases.

Since our theoretical analysis matches well the simulations, we can estimate the analytical frame error floor as a function of the PAPR threshold  $\bar{\lambda}$  (see Figure 4). We can see that the error floor level can decrease, by increasing  $n$  and/or  $\bar{\lambda}$ . Our analytical approach makes it possible to foresee the expected level of the error rate without a time-consuming simulation.

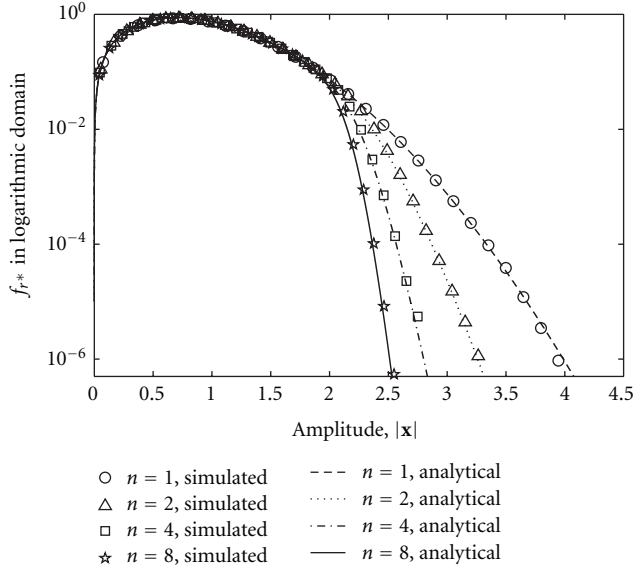


FIGURE 2: Analytical and simulated logarithmic PDF for  $f_{r^*}(x)$  corresponding to  $N = 128$ .

**3.3. Channel Capacity.** We consider the channel capacity of selected and clipped OFDM symbols for a multiple candidate system. For this, we take into consideration the M-ary Input AWGN channel models [17]. Suppose that the receiver knows the exact information about which candidate has been transmitted. Then, the channel capacity of transmitted symbols is  $C_{M\text{-ary}} = h(\mathbf{y}_{i_0}) - h(\mathbf{y}_{i_0} | \mathbf{x}_{i_0})$ , where  $\mathbf{y}_{i_0}$  is the received symbol, and from which we may write

$$C_{M\text{-ary}} = - \iint_{-\infty}^{+\infty} p(I, Q) \log_2 p(I, Q) dI dQ - \log_2(2\pi e \sigma^2), \quad (14)$$

where  $p(I, Q)$  is the two-dimensional PDF of received symbol with the Gaussian noise variance  $\sigma^2 = 0.5/\text{SNDR}^{(n)}$  in each dimension.

Figure 5 illustrates the channel capacity for 16-QAM case (up) and 64-QAM case (down) over M-ary Input AWGN channel. The figure implies that, due to the clipped symbol, it is impossible to achieve error-free performance. However, as the number of candidate increases, we can obtain theoretical capacity gains as long as  $\text{SNDR}^{(n)}$  increases. In particular, the channel symbols of M-QAM, where  $M \geq 16$ , are so sensitive to the clipping noise that the multiple candidate system can attain additional channel capacity gains effectively. When  $\text{SNR} = 45$  dB, the measured capacity gain is 0.1284 bits/channel symbol with 16 candidates (64-QAM symbols clipped at  $\bar{\lambda} = 2$  dB).

## 4. Application: Oversampling and Filtering

We present an extension of the multiple candidate system: combination with an oversampling and filtering technique [14]. For the single-candidate system, an OFDM symbol

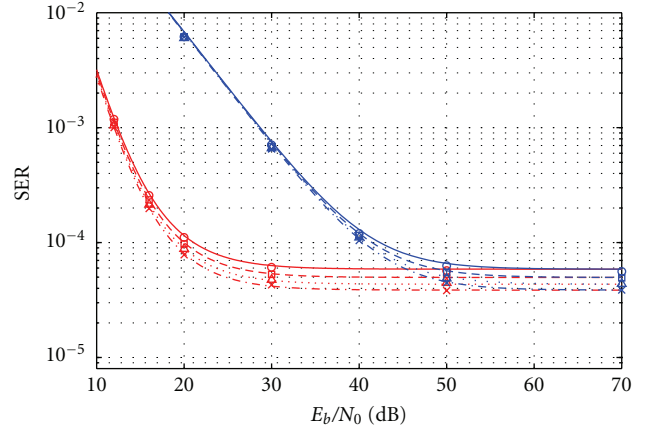


FIGURE 3: Symbol Error Rate (SER) comparison between the analytical approach (line) and the simulation results (marker) over AWGN channel (red) and frequency-nonsselective slowly fading channel (blue), where the data is QPSK modulated, and 1024-point FFT pairs are considered. The OFDM symbols are clipped at  $\bar{\lambda} = 0$  dB. In the figure, the lines represent our analytical approach, such as solid line ( $n = 1$ ), dash line ( $n = 2$ ), dot line ( $n = 4$ ), and dash-dot line ( $n = 8$ ). The markers represent the simulation results, such as ○ ( $n = 1$ ), □ ( $n = 2$ ), △ ( $n = 4$ ), × ( $n = 8$ ).

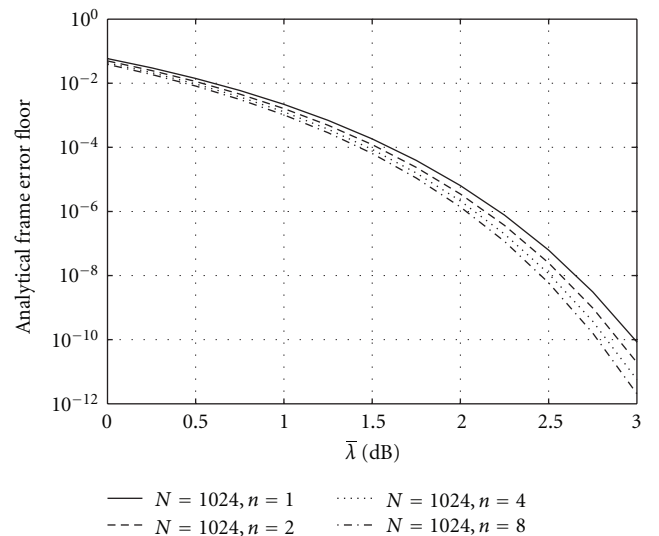


FIGURE 4: Analytical Frame Error Rate at the error floor level for the clipping threshold  $\bar{\lambda}$ . In the case,  $N = 1024$  and the QPSK modulation is considered.

with a large  $N$  is usually assumed to have a Gaussian PDF in the real and imaginary parts. However, for the multiple candidate system, this Gaussian assumption no longer holds. In this section, we show mathematical non-Gaussian PDF for the multiple candidate system.

**4.1. Presentation of Extended Model.** The multiple candidate system in the presence of the soft limiter can be extended to the oversampling and filtering technique [14]. In this case,  $n$  frequency domain OFDM symbols  $\mathbf{X}_i = \{X_{i,1}, \dots, X_{i,N}\}$

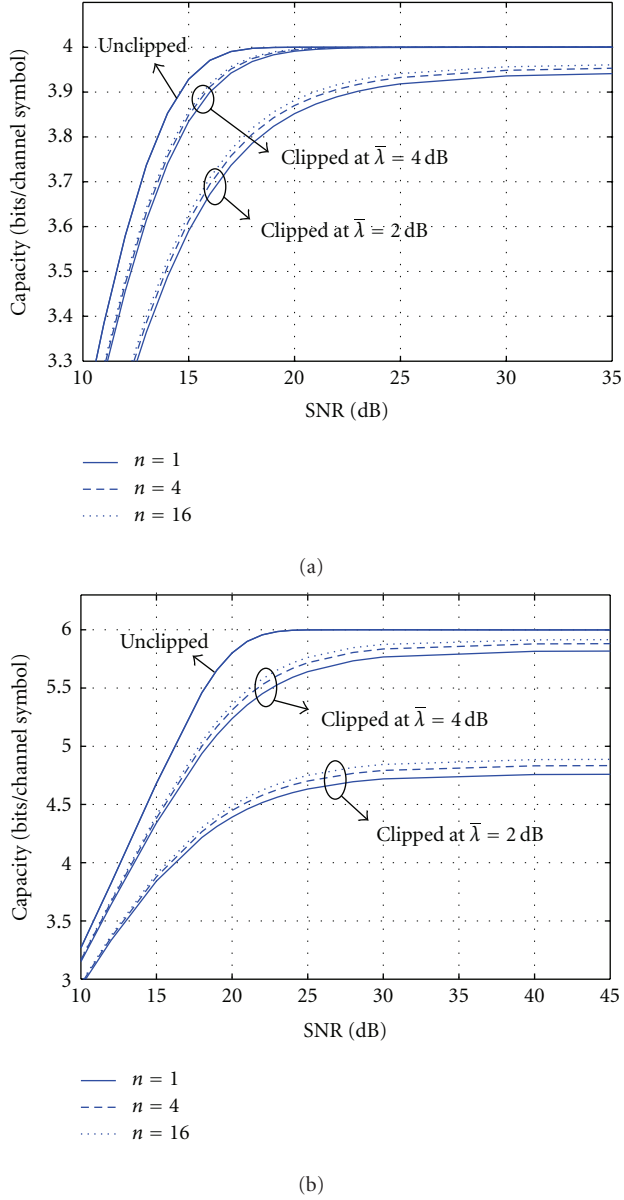


FIGURE 5: Channel capacity for 16-QAM case (a) and 64-QAM case (b), where  $N = 1024$  and M-ary Input AWGN channel (MI-AWGN) is considered.

are zero-padded, and  $L$ -times oversampled IDFT processes are performed, generating  $n$  candidates  $\mathbf{x}_i = \{x_{i,1}, \dots, x_{i,LN}\}$  in the time domain, where the  $L$ -times oversampled IDFT operation is denoted as

$$\mathbf{x}_i = \text{IDFT} \left( L, \left\{ \mathbf{X}_i, \underbrace{0, \dots, 0}_{(L-1)N} \right\} \right). \quad (15)$$

Then, the candidate  $\mathbf{x}_{i_0} = \{x_{i_0,1}, \dots, x_{i_0,LN}\}$  with the minimum PAPR is selected, and clipped by the soft limiter, where  $|x_{i_0,k}| \sim f_r^*$ ,  $k \in \{1, \dots, LN\}$ , as in (8). The clipped signal  $\tilde{\mathbf{x}}_{i_0} = \{\tilde{x}_{i_0,1}, \dots, \tilde{x}_{i_0,LN}\}$  goes through a band pass filter (BPS) which removes out-of-band frequency components,

yielding a filtered signal  $\hat{\mathbf{x}}_{i_0} = \{\hat{x}_{i_0,1}, \dots, \hat{x}_{i_0,N}\}$  which will be converted into an analog signal  $\hat{x}_{i_0}(t)$ .

Let  $\text{SNDR}_k^{(n)}$  be the SNDR of the  $k$ th subcarrier for  $n$  candidate system, then its inverse can be expressed as [14]

$$\frac{1}{\text{SNDR}_k^{(n)}} = \frac{1}{\text{SDR}_k^{(n)}} + \frac{1}{\text{SNR}} \left( 1 + \frac{1}{N} \sum_{k=0}^{N-1} \frac{1}{\text{SDR}_k^{(n)}} \right), \quad (16)$$

where SNR denotes the signal-to-noise ratio for the channel, and  $\text{SDR}_k^{(n)}$  denotes the signal-to-distortion ratio of the  $k$ th subcarrier for  $n$  candidate system.

In (16),  $\text{SDR}_k^{(n)}$  can be expressed as [14]

$$\text{SDR}_k^{(n)} = \frac{K_y^{(n)}}{\left( \sqrt{N}/\sqrt{L} \right) \text{DFT} \left( L, \left\{ R_{\tilde{x}_{i_0}}[m]/P_{\text{out}}^{(n)} \right\}_k - K_y^{(n)} \right)}, \quad (17)$$

where  $P_{\text{out}}^{(n)}$  is given in (9) and  $R_{\tilde{x}_{i_0}}[m]$  is the autocorrelation function of the clipped signal.

Let  $x_{i_0,k} \triangleq a_1 + jb_1$  and  $x_{i_0,k+m} \triangleq a_2 + jb_2$ , then the clipped signals are given by  $\tilde{x}_{i_0,k} \triangleq g(a_1 + jb_1)$  and  $\tilde{x}_{i_0,k+m} \triangleq g(a_2 + jb_2)$ , and the autocorrelation function  $R_{\tilde{x}_{i_0}}[m]$  is given by

$$\begin{aligned} R_{\tilde{x}_{i_0}}[m] &= \Re \left\{ E \left[ \tilde{x}_{i_0,k}^* \cdot \tilde{x}_{i_0,k+m} \right] \right\} \\ &= E \left[ g^*(a_1 + jb_1) g(a_2 + jb_2) \right] \\ &= \iiint \int_{D(a_1, b_1, a_2, b_2)} g^*(a_1 + jb_1) g(a_2 + jb_2) \\ &\quad \cdot f(a_1, a_2, b_1, b_2) da_1 db_1 da_2 db_2, \end{aligned} \quad (18)$$

where  $E[\cdot]$  denotes the expectation operation.

**4.2. Inaccuracy of Gaussian Assumption.** For the single candidate case, since  $\{a_1, a_2, b_1, b_2\}$  are assumed to be Gaussian distributed,  $f(a_1, a_2, b_1, b_2)$  is expressed as a joint Gaussian PDF [14, 18]. However, for the multiple candidate case ( $n > 1$ ), since the amplitude of the selected candidate is not Rayleigh distributed, such as (8), this Gaussian assumption no longer holds. In the rest of this paper, we consider the PDF of  $\{a_1, b_1, a_2, b_2\} \sim f_a$  for the multiple candidate case.

Without loss of generality, we consider  $a = a_1$  and  $b = b_1$ , where  $a$  and  $b$  are assumed to be independent and identically distributed. Then, the amplitude is defined as

$$h \triangleq \sqrt{a^2 + b^2} \sim f_h(r) = f_r^*(r), \quad (19)$$

where  $f_r^*(r)$  is given by (8).

Defining a power variable  $y \triangleq h^2 \geq 0$ , its characteristic function [15] is given by

$$\phi_y(\omega) = \int_0^\infty \exp(j\omega r^2) f_h(r) dr, \quad (20)$$

and let  $y_1 \triangleq a^2$  and  $y_2 \triangleq b^2$ , such that  $y = y_1 + y_2$ . Since  $y_1$  and  $y_2$  are independent and have an identical PDF  $f_{y_1} = f_{y_2}$ , we get

$$\begin{aligned}\varphi_y(\omega) &= \varphi_{y_1}(\omega) \cdot \varphi_{y_2}(\omega) = [\varphi_{y_1}(\omega)]^2 \\ \therefore \varphi_{y_1}(\omega) &= [\varphi_y(\omega)]^{1/2}.\end{aligned}\quad (21)$$

Then,  $f_{y_1}(r) = f_{y_2}(r)$  is given by

$$\begin{aligned}f_{y_1}(r) &= f_{y_2}(r) \\ &= \frac{1}{2\pi} \int_{-\infty}^{+\infty} [\varphi_y(\omega)]^{1/2} \exp(-j\omega r) d\omega.\end{aligned}\quad (22)$$

Now, we consider the case  $a \geq 0$  with the PDF of  $f_a(r)$ . Notice that a negative value of  $a$  will have its symmetrical PDF. The characteristic function of  $y_1 = a^2$  is given by

$$\begin{aligned}\varphi_{y_1}(\omega) &= \int_0^{\infty} \exp(j\omega r^2) f_a(r) dr \\ &= \int_0^{\infty} \exp(j\omega y_1) \cdot \underbrace{\frac{f_a(r)}{2r}}_{=f_{y_1}(y_1)=f_{y_1}(r^2)} \cdot dy_1.\end{aligned}\quad (23)$$

Finally, by using (22) and (23), we get

$$\begin{aligned}f_a(r) &= 2r f_{y_1}(r^2) \\ &= \frac{r}{\pi} \int_{-\infty}^{+\infty} [\varphi_y(\omega)]^{1/2} \exp(-j\omega r^2) d\omega,\end{aligned}\quad (24)$$

which denotes the mathematical non-Gaussian PDF of  $a = \mathfrak{R}\{x_{i_0,k}\}$  or  $b = \mathfrak{I}\{x_{i_0,k}\}$ .

## 5. Conclusion

We study the probability density function (PDF) analysis and the signal-to-noise-plus-distortion ratio (SNDR) of a multiple candidate system for reducing the PAPR in OFDM modulation system. Since the selected OFDM symbol (candidate) has an amplitude PDF which is function of the number of candidates  $n$ , the derived SNDR<sup>(n)</sup> is also the function of  $n$ , and it can be used for estimation of theoretical error performance and statistical channel capacity.

In this paper, the side information is assumed not to be erroneous for analyzing the pure effect of multiple candidates. The analytical estimation matches well the simulation results, and with this study, we conclude that the more the candidates, not only the better PAPR reduction performance, but also the better error performance and the more gain of channel capacity, under the assumption of side information transmission without error, and at the expense of computational complexity for  $n$  IFFT circuits.

Furthermore, the amplitude PDF analysis enables one to apply to a probabilistic PAPR reduction system jointly with "oversampling and filtering" technique. In this application, since the selected candidate is not complex Gaussian distributed, more investigation for SNDR is required.

Our analytical approach to obtaining the SNDR<sup>(n)</sup> implies that the estimation of error rate is achievable without time-consuming simulation, making system level design easier. Note that the error floor level is usually decreased by implementing channel coding techniques. In our future work, we will take channel coding into account for error performance analysis.

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