Joint Performance Study of Channel Estimation and Multiuser Detection for Uplink Long-Code CDMA Systems

Ping Liu
Department of Electrical Engineering, University of California, Riverside, CA 92521, USA
Email: pliu@ee.ucr.edu

Zhengyuan Xu
Department of Electrical Engineering, University of California, Riverside, CA 92521, USA
Email: dxu@ee.ucr.edu

Received 1 November 2003; Revised 10 March 2004

Although numerous channel estimation and multiuser detection approaches have appeared for long-code uplink CDMA systems, joint performance study of channel estimators and symbol detectors remains largely open. In this paper, we construct three typical symbol-level linear receivers upon existing channel estimation method, known as zero-forcing (ZF), minimum mean-square-error (MMSE), and RAKE receivers, for symbol detection. Since the channel estimation error is rippled to the linear receivers, performance of all receivers is thus jointly analyzed with the channel estimator from a perturbation perspective. Extensive simulation examples involving different communication environments demonstrate high consistency between our analysis and experimental results.

Keywords and phrases: long codes, channel estimation, multiuser detection, perturbation analysis.

1. INTRODUCTION

Direct sequence (DS) code division multiple access (CDMA) technology has become an appealing solution to third-generation wireless systems [1, 2]. It provides a communication-system unique capabilities of simultaneous spectrum sharing, mitigation of jamming, interception, and multipath fading [3], and can easily support transmission of multirate information streams.

Multiuser interference (MUI) is a typical obstacle to be obviated in detection of input signals in a DS/CDMA system, which has attracted substantial efforts in recent years [4]. In DS/CDMA systems, the bandwidth of the input signal is spread by a sequence with a much higher rate in order to effectively suppress the interference. There are two kinds of spreading codes: the periodic spreading sequence (short codes) which repeats from symbol to symbol, and the aperiodic spreading sequence (long codes) which has a much longer period compared with the symbol duration. In recent years, many efforts have been focused on the CDMA system with short codes, resulting in simplified methods, and very importantly, tractable analysis of the system performance. However, long spreading codes feature the new CDMA-based wireless standards [5]. Employment of long codes exhibits certain superiority to short codes in terms of increasing immunity of the system to MUI and channel fading on the average [6], improving the spectrum efficiency through the uniform distribution of the signal bandwidth, and moreover ensuring a secure communication link in a hostile environment, protecting users’ information against intentional interception.

Despite various advantages, adopted long spreading codes inevitably destroy cyclostationarity of CDMA signals, making many of the existing channel estimation and detection approaches for short-code CDMA systems not directly applicable. Therefore, solutions for long-code CDMA systems are still under extensive investigation. It has been witnessed that various solutions for downlink communications have been developed [7, 8, 9, 10]. However, uplink communications incur new problems due to asynchronism and particular code assignment strategies. Given pilot symbols of all users, least squares (LS)-fitting or iterative maximum likelihood (ML) approaches have been reported [6, 11, 12]. Blind methods have also been derived. Correlation matching techniques have been successfully applied for channel estimation [13, 14]. Employing a space-time
2D RAKE receiver structure to maximize the output signal-to-interference-plus-noise ratio (SINR), both channel estimation and minimum mean-square-error (MMSE)/zero-forcing (ZF) receivers are presented in [15, 16]. Built upon the structure of convolutive channels, a Toeplitz displacement channel estimation method has been proposed [17]. A novel parallel factor analysis technique is applied to multiuser detection [18]. Even in the presence of colored Gaussian noise in addition to unknown multipath fading and MUI, turbo multiuser detectors can be developed [19]. Based on LS fitting, a blind decorrelating RAKE receiver is recently proposed [20]. If all users’ spreading codes and propagation delays are known, a subspace technique can be applied to estimate multipath parameters [21]. Performance of long-code CDMA systems and detection methods has also been studied under the assumption of perfect channel estimate. A heuristic approximation of SINR for the decorrelator and simulation results for the MMSE receiver are provided [22]. Spectral efficiency of some linear receivers are explored [23, 24]. Near-far resistance of MMSE detection is investigated [25] and trade-offs of long-versus-short spreading sequences are demonstrated [26, 27]. Performance of ML detectors is studied for multicarrier CDMA systems [28].

In this paper, we study joint performance of the channel estimator and multiuser receivers for long-code uplink CDMA systems. The channel estimator considered in our analysis is based on [21], which extends [10] to uplink communications with slight modification. It is also closely related to [20] where a low-complexity algorithm is further developed. Based on the estimated channel, different linear detection techniques such as ZF, MMSE, and RAKE are applied. Both ZF and MMSE receivers have complexity which is cubic in the processing gain. Due to the time-varying property of a long-code system, designing new low-complexity multiuser detectors remains a challenging issue. Since in practice, the channel estimation is based on processing of finite data samples, it will deviate from its theoretical value, resulting in estimation errors. Perturbation in channel estimate will be further carried on to the receivers, causing receivers’ output SINRs and BERs all perturbed. To quantify the effect of sample size, we apply perturbation theory to obtain the channel mean-square error (MSE), detectors’ perturbed SINR, and finite-data-based bit-error rate (BER). Unlike previous work [22, 23], which studied the effect of randomness of codes on the fluctuation of the SINR of receivers constructed from perfect signature vectors, our analysis considers the effect of imperfect signature waveforms caused by finite data samples on the performance of receivers. Although joint performance has been studied for short-code DS/CDMA systems [29] and long-code downlink CDMA systems [30], long-code uplink communications adopt completely different channel estimation and symbol detection schemes and the results in [29, 30] are not applicable. Therefore, new joint performance analysis for long-code uplink systems is necessary and important. Moreover, high consistency is observed between our experimental results and analytical results.

This paper is organized as follows. A long-code CDMA uplink system model is described in Section 2. Subspace-based channel estimation method is reviewed, together with addition of noise-power estimation method and implementation of three typical linear detectors are presented in Section 3. In Section 4, joint performance of channel estimator and detectors in terms of channel MSE, receivers’ SINR and BER is evaluated. Finally, various simulation examples are provided in Section 5 and conclusions are drawn in Section 6.

2. CDMA UPLINK WITH LONG CODES

Consider an uplink CDMA system [14], where J mobile stations are communicating with a base station. The jth user’s bit \( w_j(n) \) is first spread by aperiodic codes \( c_{j,n}(k) \) \( (k = 0, \ldots, P-1) \), and then transmitted through a multipath channel \( g_j(m) \). All channels are assumed to have maximum order \( q (q \ll P) \). The signal from user \( j \) is assumed to arrive at the base station with delay \( d_j \). Then, after considering rectangle pulse shaping, the received chip-rate signal is a superposition of signals from \( J \) users corrupted by noise [13]

\[
y(k) = \sum_{j=1}^{J} \sum_{m=0}^{q} g_j(m) s_j(k - m - d_j) + v(k),
\]

where

\[
s_j(k) = \sum_{n=-\infty}^{\infty} w_j(n) c_{j,n}(k - nP),
\]

\( v(n) \) is zero-mean AWGN with variance \( \sigma_v^2 = E[|v(n)|^2] \). If we assume that the system is quasi-synchronous, where transmission delays from all users to the base station are within a small fraction of a chip duration, that is, \( 0 \leq d_j \ll P \). As a result, the intersymbol interference could be eliminated if we collect only \( L = P - \mu \) samples in the nth bit interval into a vector \( y(n) = [y(nP + \mu), \ldots, y(nP + P - 1)]^T \) with \( \mu = \max(q + d_j) \). Let \( C_j(n) \) be the truncated version of the following code filtering matrix of dimensions \( (P + q) \) by \( (q + 1) \) for user \( j \):

\[
C_j(n) = \begin{bmatrix}
c_{j,n}(0) & 0 & \cdots & 0 \\
\vdots & \ddots & \vdots & \vdots \\
c_{j,n}(P-1) & \ddots & c_{j,n}(P-1) \\
0 & \cdots & c_{j,n}(P-1)
\end{bmatrix} + 1.
\]

(3)

that is, \( C_j(n) = [C_j(n)]_{\mu+1:P,1:q+1} \). Then according to (1), a simple matrix form follows:

\[
y(n) = \sum_{j=1}^{J} C_j(n) g_j w_j(n) + v(n)
\]

\[
= C(n) g w(n) + v(n) = H(n) w(n) + v(n),
\]

(4)
where $C(n)_{(P-\mu)\times(J(q+1))}$, $\mathcal{g}$, and $H(n)_{(P-\mu)\times J}$ are defined as

$$C(n) = [C_1(n), \ldots, C_J(n)],$$

$$\mathcal{g} = \text{diag} \{ \mathcal{g}_1, \ldots, \mathcal{g}_J \},$$

$$H(n) = [h_1(n), \ldots, h_J(n)] = [C_1(n)\mathcal{g}_1, \ldots, C_J(n)\mathcal{g}_J] = C(n)\mathcal{g},$$

$\mathcal{g}_j$ with $(q+1)$ elements is the channel vector of user $j$, $w(n)_{J\times 1}$ and $v(n)_{(P-\mu)\times 1}$ are given by

$$w(n) = [w_1(n), \ldots, w_J(n)]^T,$$

$$v(n) = [v(nP+\mu), \ldots, v(nP+P-1)]^T.$$ (5)

Throughout this paper, we make the following assumptions.

(A1) All users’ information sequences are mutually independent and temporally i.i.d. with unit power.

(A2) Each user’s codes and delay are known.

(A3) All users’ long codes are pseudorandom variables.

(A4) The number of active users in the system satisfies $J < (P-q)/(q+1)$. The first assumption is adopted for most systems and convenient for analysis. However, the effect of different interfering power is studied in the simulation in the paper. For the second assumption, it is reasonable to assume that the base station knows all users’ spreading codes. On the other hand, all users’ delays are assumed to be estimated in advance by some delay estimator such as [16]. However, if they are not available, we can over-parameterize the channel vector to absorb the delay ambiguity [20]. Long-code sequence can be categorized as deterministic or stochastic. To facilitate our analysis, the latter is adopted as (A3). (A4) is the condition required for channel estimation in [10, 21] upon which our receivers are built.

3. CHANNEL ESTIMATION AND MULTIUSER DETECTION IN CDMA UPLINK

Before we start joint performance analysis, we first briefly present the channel estimation method, which extends [10] to CDMA uplink with some alterations. The tailored method, instead of using the whole code-decorrelated data vector, utilizes only a portion pertaining to the desired user to estimate channel in a reduced complexity. Whitening is performed before subspace method is applied, and noise power is estimated from the whitened data. Then, three most commonly used symbol-level linear receivers, ZF, MMSE, and RAKE receivers, are constructed in order to detect the desired user’s symbols.

3.1. Subspace-based channel and noise-power estimation

According to (4), the original data vector $y(n)$ contains nonstationary signal contribution and stationary noise contribution. Consequently, the conventional unconditional covariance of $y(n)$, which is estimated from sample average of $y(n)y(n)^H$, contains no noise subspace. The signal subspace in fact spans the whole operational space, as mentioned in [30]. Therefore, subspace approach cannot be applied to the unconditional covariance of $y(n)$.

As in [10], we first decorrelate the data vector $y(n)$ in (4) using the pseudoinverse of code matrix $C(n)$ at each symbol to obtain an approximately stationary sequence of $J(q+1)$ elements as

$$u(n) = C(n)^\dagger y(n) = \mathcal{g}w(n) + C(n)^\dagger v(n).$$ (7)

It is observed that after code decorrelation, $u(n)$ contains a stationary signal process and a nonstationary noise process. The unconditional covariance of the decorrelated sequence becomes

$$\hat{\mathbf{R}} = \mathcal{g}\mathcal{g}^H + \sigma_s^2 \mathbf{A}_j,$$ (8)

where

$$\mathbf{A}_j_{(q+1)\times(q+1)} = E\{C(n)\dagger(C(n)^\dagger)^H\} = E\{[C(n)^H C(n)]^{-1}\}. \quad (9)$$

Noticing that $\mathcal{g}$ contains $J$ users’ channel vector as its diagonal block, we then partition both $\hat{\mathbf{R}}$ and $\mathbf{A}_j$ into $J \times J$ submatrices of dimensions $(q+1) \times (q+1)$. If we denote their $(j,j)$th submatrices as $\hat{\mathbf{R}}_j$ and $\mathbf{A}_j$, respectively, then

$$\hat{\mathbf{R}}_j \triangleq S_j^\dagger \hat{\mathbf{R}} S_j = \mathcal{g}_j\mathcal{g}_j^H + \sigma_s^2 \mathbf{A}_j,$$ \hspace{1em} where $j = 1, \ldots, J, \quad (10)$$

and $\mathbf{A}_j \triangleq S_j^\dagger \mathbf{A} S_j$, where $S_j$ is a selection matrix of dimensions $(q+1) \times (q+1)$ by $J(q+1)$ and defined as

$$S_j = \left[ \mathbf{0}_{(j-1)(q+1)\times(q+1)}; \mathbf{I}_{(q+1)\times(q+1)}; \mathbf{0}_{(J-j)(q+1)\times(q+1)} \right]^T.$$ (11)

The correlation matrix $\hat{\mathbf{R}}_j$ after decorrelation now contains the desired subspace spanned by the $j$th user’s channel vector, but it is corrupted by a colored matrix $\mathbf{A}_j$. Therefore, whitening is performed to yield

$$\hat{\mathbf{R}}_j \triangleq A_j^{-1/2} \hat{\mathbf{R}}_j A_j^{-1/2} = \mathcal{g}_j\mathcal{g}_j^H A_j^{-1/2} + \sigma_s^2 \mathbf{I}.$$ (12)

It is observed that $A_j^{-1/2} \mathcal{g}_j$ constitutes the unique signal subspace of $\hat{\mathbf{R}}_j$, and $\sigma_s^2$ is the least eigenvalue of $\hat{\mathbf{R}}_j$ with multiplicity $q$. For convenience, denote a new column vector of $(q+1)$ elements as $x_j \triangleq A_j^{-1/2} \mathcal{g}_j || A_j^{-1/2} \mathcal{g}_j ||$, the subspace orthogonal to $x_j$, as the noise subspace $U_l^n$, and the eigenvalues of $\hat{\mathbf{R}}_j$ as $\lambda_i$ ($i = 1, \ldots, q+1$) in a descending order. Applying the subspace technique to $\hat{\mathbf{R}}_j$ immediately yields the following channel estimation method for the channel of user $j$ and
the noise power

\[ x_j = \arg \max_{|\beta| = 1} \beta^H R_j \beta, \]

\[ g_j = \frac{A_j^{1/2} x_j}{||A_j^{1/2} x_j||}, \quad \sigma_i^2 = \frac{1}{q} \sum_{i=2}^{q+1} \lambda_i. \tag{13} \]

It can easily be verified from (12) that

\[ \sigma_i^2 = \frac{1}{q} \text{tr} \left\{ \left( U_n^H R U_n \right) \right\}. \tag{14} \]

where “tr” is a trace operator.

It is necessary to mention that the above derivations use (AS3). However, without any assumption on codes, the averaged covariance of the decorrelated sequence \( u(n) \) can still be obtained as (8), if we define \( A_j^{(q+1) \times (q+1)} = \frac{1}{N} \sum_n \left( C(n) H C(n) \right)^{-1} \). Clearly, in this situation, \( A \) can be calculated in advance given all sets of codes, and therefore modeled as a deterministic quantity. As a result, the above mentioned approach is still applicable.

### 3.2. Symbol detection

We now turn to symbol detection based on estimated channel vectors. Without loss of generality, we focus on our desired user, user 1, and typical linear receivers to detect the desired user’s symbols \( w_1(n) \).

#### 3.2.1. ZF receiver

Once all users’ channel vectors are estimated, the signature matrix can be constructed at each time instance \( n \) using all users’ spreading codes as (5). Then, the ZF receiver at time instant \( n \) is defined as a column vector with \((P-q)\) elements

\[ f_{ZF}(n) = H(n) H(n)^H H(n)^{-1} e, \tag{15} \]

where \( e \) is a unitary vector with the first element as 1. Then \( w_1(n) \) is estimated by

\[ \hat{w}_{1,ZF}(n) = f_{ZF}^H(n) y(n). \tag{16} \]

#### 3.2.2. MMSE receiver

The MMSE receiver, which is a column vector with \((P-q)\) elements, can be defined as

\[ f_{MMSE}(n) = R(n)^{-1} C_1(n) g_i, \tag{17} \]

where \( R(n) \) is the conditional correlation matrix of \( y(n) \) at time \( n \), which is conditioned on all users’ codes. Notice that \( R(n) \) cannot be estimated by conventional sample average. However, it can be constructed by

\[ R(n) = H(n) H(n)^H + \sigma_i^2 I, \tag{18} \]

once \( H(n) \) is constructed as (5) from all estimated channel vectors and \( \sigma_i^2 \) is estimated according to (14). Correspondingly, the detected symbol is given by

\[ \hat{w}_{1,MMSE}(n) = \tilde{f}_{MMSE}^H(n) y(n). \tag{19} \]

### 3.2.3. RAKE receiver

The RAKE receiver at time \( n \) is constructed as the following column vector with \((P-q)\) elements [8]:

\[ f_{RAKE}(n) = h_1(n) = C_i(n) g_i, \tag{20} \]

and the desired user’s symbol is estimated as

\[ \hat{w}_{1,RAKE}(n) = f_{RAKE}^H(n) y(n). \tag{21} \]

Linear receivers are coupled with estimated channel vectors. Their performance will be investigated jointly with channel estimators next.

### 4. Performance Analysis

In this section, we will apply perturbation theory to analyze the channel estimation MSE for each user. Performance of three linear receivers for the desired user is then evaluated in terms of SINR and BER. Before we proceed, we clarify some notations used in the following analysis. First, the perturbation is defined as the difference between the estimated quantity based on finite data samples and its asymptotic value based on infinite large data samples. For our subspace-based channel estimation, its perturbation is given by the difference between the estimated channel based on \( N \) data samples and the true channel vector, after noticing that the true channel vector can be obtained when infinite data samples are applied. For receivers, since they are constructed based on estimated channels, their perturbations thus depend on channel perturbations, which will be derived later. Moreover, as we can see later, all perturbations originate from the perturbation of data-covariance matrix, which is estimated from finite data samples. Since the inputs and noise are all random, therefore, the perturbation of data covariance, and thus all other perturbations, can be modeled as random process. In the sequel, the perturbation is denoted by preceding the corresponding quantity by \( \delta \), and the perturbed quantity with \( \tilde{.} \).

For example, \( \delta g_j = \tilde{g}_j - g_j, \delta R_j = \tilde{R}_j - R_j \). Assume the number of data samples \( N \) is sufficiently large such that perturbation technique is applicable.

#### 4.1. Channel mean-square error

When estimated from finite data samples, data-covariance matrix gets perturbed. Its perturbation will be carried over to channel estimate. In the sequel, we will study the statistical performance of the \( j \)th user’s channel estimator under a large sample size assumption by applying perturbation techniques. It is observed from (13) that the \( j \)th user’s channel estimate depends on \( R_j \) and therefore \( \tilde{R} \). When \( \tilde{R} \) is estimated from \( N \) decorrelated data samples as

\[ \tilde{R} = \frac{1}{N} \sum_{n=1}^{N} u(n) u(n)^H, \tag{22} \]

\( R_j \) is perturbed as the following according to (10), (11), and (12):

\[ \tilde{R}_j = A_j^{-1/2} S_j^T \tilde{R} S_j A_j^{-1/2}. \tag{23} \]
Using (22) and (23), and replacing \(u(n)\) with (7), we can further express \(R_j\) as an estimate from \(N\) decorrelated data samples as

\[
\hat{R}_j = \frac{1}{N} \sum_{i=1}^{N} [x_j(n) + v_j(n)] \overline{[x_j(n) + v_j(n)]^H},
\]

(24)

where

\[
x_j(n) = A_j^{-1/2} g_j w_j(n),
\]

\[
v_j(n) = A_j^{-1/2} S_j^j C n^j v(n).
\]

(25)

Noticing that \(v_j(n)\) is a Gaussian vector whose covariance \(E[A_j^{-1/2} S_j^j C n^j v(n)^H (C n^j)^H S_j^j A_j^{-1/2}]\) can be approximated by \(\sigma^2 I\), we can view \(R_j\) to be estimated from \(N\) samples of \(x_j(n)\) corrupted by AWGN \(v_j(n)\), and perturbation analysis can be readily conducted to \(R_j\). The Gaussian property on the noise \(v_j(n)\) is necessary for deriving some statistics later. According to [31], the first-order perturbation of the signal subspace \(\chi_j\) is given by

\[
\delta \chi_j = (1/\gamma_j) U_j^i (U_j^i)^H \delta R_j \chi_j,
\]

(26)

where \(\gamma_j = g_j^H A_j^{-1} g_j\). According to (13), \(\delta \chi_j\) will cause \(g_j\) perturbed as \(\hat{g}_j = (\chi_j^H A_j \chi_j)^{-1/2} A_j^H \chi_j\). Substituting \(\delta \chi_j\) into \(\hat{g}_j\), expanding the power term using Taylor series, and keeping only the first-order terms, we obtain the \(j\)th user’s channel perturbation as

\[
\delta g_j \approx (\chi_j^H A_j \chi_j)^{-1/2} A_j^H \delta \chi_j
\]

(27)

\[= \frac{1}{2} (\chi_j^H A_j \chi_j)^{-3/2} (\delta \chi_j^H A_j \chi_j + \chi_j^H A_j \delta \chi_j) A_j^H \chi_j.
\]

Since in-space error is much smaller than the orthogonal space error by [31], and noticing \(g_j = (\chi_j^H A_j \chi_j)^{-1/2} A_j^H \chi_j\), we further simply (27) to the following by keeping only the orthogonal space error:

\[
\delta g_j \approx \Pi_{\kappa}^j (\chi_j^H A_j \chi_j)^{-1/2} A_j^H \delta \chi_j,
\]

(28)

where \(\Pi_{\kappa}^j = (1 - g_j^H g_j) / (g_j^H g_j)\). On the other hand, \((g_j^H A_j^{-1} g_j)(\chi_j^H A_j \chi_j) = 1\) by (13), which implies \(\chi_j^H A_j \chi_j = 1/\gamma_j^2\). Using these results and replacing \(\delta \chi_j\) with (26), (28) becomes

\[
\delta g_j \approx \frac{1}{\gamma_j} \Pi_{\kappa}^j A_j^{1/2} U_j^i (U_j^i)^H \delta R_j \chi_j.
\]

(29)

Then the covariance of channel estimation error is evaluated according to

\[
E[\delta g_j^H \delta g_j^H]
\]

\[
\approx \frac{1}{\gamma_j} \Pi_{\kappa}^j A_j^{1/2} U_j^i (U_j^i)^H E[\delta R_j \chi_j^H \delta R_j] U_j^i (U_j^i)^H A_j^{1/2} \Pi_{\kappa}^j.
\]

(30)

Clearly, the covariance depends on the term \(E[\delta R_j \chi_j^H \delta R_j]\). Hence, it suffices to determine a \((q + 1)\) by \((q + 1)\) general-form matrix that is also required later:

\[
T_j(D) = E[\delta R_j D \delta R_j^H],
\]

(31)

where \(D\) can be replaced by corresponding deterministic matrices of dimensions \((q + 1)\) by \((q + 1)\), respectively, later. Noticing (12) and (24), and using the results in [32], we have that if all quantities are real, and the noise is Gaussian, then

\[
T_j(D) = \frac{1}{N} \chi_j^i [I \odot (\chi_j^H D \chi_j)] \chi_j^H + \frac{1}{N} \text{tr}(R_j D R_j),
\]

(32)

where \(\odot\) represents elementwise multiplication, and \(\kappa_{\text{sw}}\) is the fourth-order cumulant of \(w_j(n)\). For a complex system,

\[
T_j(D) = \frac{1}{N} \chi_j^i [I \odot (\chi_j^H D \chi_j)] \chi_j^H + \frac{1}{N} \text{tr}(R_j D R_j) + \frac{1}{N} \text{tr}(R_j D^T R_j),
\]

(33)

Therefore, for a given data model, statistical properties of the inputs and additive noise, \(T_j(D)\) can always be evaluated. Applying (32) or (33) to (30), setting \(D = \chi_j^H H\), and noticing \(\chi_j^H U_n^i = 0\), \(\text{tr}(R_j D) = \chi_j^H R_j \chi_j = y_j + \sigma_n^2\), one can verify that in both cases (30) reduces to

\[
E[\delta g_j^H \delta g_j^H] \approx \frac{\sigma_n^2 (y_j + \sigma_n^2)}{N \gamma_j} \Pi_{\kappa}^j A_j^{1/2} U_j^i (U_j^i)^H A_j^{1/2} \Pi_{\kappa}^j.
\]

(34)

It is further simplified if noise is small

\[
E[\delta g_j^H \delta g_j^H] \approx \frac{\sigma_n^2}{N} \Pi_{\kappa}^j A_j^{1/2} U_j^i (U_j^i)^H A_j^{1/2} \Pi_{\kappa}^j.
\]

(35)

The channel MSE is then given by the trace of (35). It can be seen that the \(j\)th user’s channel MSE is proportional to noise power and inversely proportional to data length \(N\). It also depends on its own channel condition and all users’ long codes.

### 4.2. SINRs of different receivers

SINR is an important performance indicator for receivers. The average SINR can be defined as

\[
\text{SINR} = \frac{E[f(n)^H R_i(n) f(n)]}{E[f(n)^H R_{\text{int}}(n) f(n)]},
\]

(36)

where \(f(n)\) is any symbol-level receiver,

\[
R_i(n) = h_i(n) h_i(n)^H,
\]

\[
R_{\text{int}}(n) = R(n) - R_i(n).
\]

(37)
Although fluctuation of SINR can be analyzed under perfect conditions, perturbation in channel estimation induced by finite data samples inevitably causes the receiver to be perturbed as \( \hat{f}(n) = f(n) + \delta f(n) \), where the first-order perturbation \( \delta f(n) \) is assumed to have zero mean. This is a reasonable assumption. According to [32], it can be assumed that \( E[\delta R_j] = 0 \) and \( E[\delta \sigma^2] = 0 \), when they are estimated from finite data. Therefore one can verify that \( E[\delta g_j] = 0 \), and then \( E[\delta H] = 0 \). Consequently, from later analysis, we have \( E[\delta f(n)] = 0 \). Based on it, the perturbed SINR has the following form:

\[
\tilde{\text{SINR}} = \frac{E[f(n)H\mathcal{R}_1(n)f(n)] + E[\delta f(n)H\mathcal{R}_1(n)\delta f(n)]}{E[f(n)H\mathcal{R}_m(n)f(n)] + E[\delta f(n)H\mathcal{R}_m(n)\delta f(n)]}
\]

(38)

It depends on both unperturbed terms (signal power, interference-plus-noise power) and corresponding perturbed terms. They follow particular forms of \( \Phi(X) = E[f(n)H\mathcal{Xf}(n)] \) for unperturbed terms and of \( \Psi(X) = E[\delta f(n)H\mathcal{X\delta f}(n)] \) for perturbed terms, where \( \mathcal{X} \) can be replaced by \( \mathcal{R}_1(n) \) or \( \mathcal{R}_m(n) \). Since different receivers take different forms, these quantities need to be evaluated for each receiver respectively. For shorter notations, all receiver subscripts are dropped later.

### 4.2.1. SINR of the ZF receiver

First, replacing the ZF receiver with (15), it can be shown that

\[
\Phi(\mathcal{R}_1(n)) = 1
\]

and \( \Phi(\mathcal{R}_m(n)) = \sigma_r^2 \| f(n) \|^2 = \sigma_r^2 e^H E[|H(n)H(n)|^{-1}] e \). It is much involved to further simplify \( \Phi(\mathcal{R}_m(n)) \). Therefore, time average over codes is performed for approximation:

\[
\Phi(\mathcal{R}_m(n)) \approx \sigma_r^2 e^H \frac{1}{N} \sum_{n=1}^{N} \| H(n)H(n) \|^{-1} e.
\]

(40)

To evaluate the perturbation term, that is, \( \Psi(X) \), we first obtain \( \delta f(n) \) according to (15). Since perturbation in the signature matrix \( H(n) \) is given by

\[
\delta H(n) = \left[ C_1(n)\delta g_1, \ldots, C_J(n)\delta g_J \right],
\]

(41)

according to (5), noticing \( \hat{H}(n) = H(n) + \delta H(n) \), expanding \( |H(n)H(n)|^{-1} \) using Taylor series, and keeping only the first-order terms, we obtain

\[
\delta f(n) = \Gamma(n)\delta H(n)(H(n)H(n))^{-1} e
\]

\[
- (H(n)H(n))^{-1} H(n)H(n)(H(n)^{-1})^H e,
\]

where

\[
\Gamma(n) = I - H(n)H(n)^{-1},
\]

\[
H(n)^{-1} = (H(n)H(n))^{-1} H(n)^H.
\]

(43)

Then we obtain

\[
\Psi(X) \approx E \left[ e^H [H(n)H(n)]^{-1} \delta H(n)H(n) \Gamma(n) \times X \Gamma(n) [H(n)^{-1}]^H e \right]
\]

\[
- E \left[ e^H [H(n)H(n)]^{-1} \delta H(n)H(n) \times X \Gamma(n) [H(n)^{-1}]^H e \right]
\]

\[
- E \left[ e^H \delta H(n) [H(n)^{-1}]^H H(n) \times \delta H(n)H(n) \times X \Gamma(n) [H(n)^{-1}]^H e \right]
\]

\[
+ E \left[ e^H \delta H(n) [H(n)^{-1}]^H H(n) \times [H(n)^{-1}]^H e \right].
\]

(44)

\( \delta g_j \) could be regarded as independent of \( C_j(n) \) for \( j = 1, \ldots, J \), since the former depends on \( A_j \) which can be viewed as independent of \( C_j(n) \) when the number of samples used to estimate \( A_j \) is sufficiently large. As a result, the expectation in (44) will be performed in two steps: first with respect to \( \delta g_j \) while regarding each \( C_j(n) \) as a constant, and then with respect to \( C_j(n) \) for the result obtained in the first step. Similarly, ensemble average can be replaced by sample average for large \( N \), resulting in the approximation of (44) by

\[
\Psi(X) \approx \frac{1}{N} \sum_{n=1}^{N} \left[ e^H [H(n)^H H(n)]^{-1} E \left[ \delta H(n)H(n) \Gamma(n) X \Gamma(n) \delta H(n) \right] \times [H(n)^H H(n)]^{-1} e - e^H [H(n)^H H(n)]^{-1} e \right.
\]

\[
- E \left[ \delta H(n) [H(n)^{-1}]^H H(n) \times \delta H(n)H(n) \times X \Gamma(n) [H(n)^{-1}]^H e \right]
\]

\[
- e^H \delta H(n) [H(n)^{-1}]^H H(n) \times \delta H(n)H(n) \times X \Gamma(n) [H(n)^{-1}]^H e
\]

\[
+ E \left[ \delta H(n) [H(n)^{-1}]^H H(n) \times [H(n)^{-1}]^H e \right].
\]

(45)

where each unperturbed quantity inside the expectation (underlined) terms is regarded as deterministic. Noticing that \( \Gamma(n) \) is orthogonal to \( \mathcal{R}_1(n) \) and \( H(n)^{-1} \mathcal{R}_1(n) [H(n)^{-1}]^H = \sigma^2 e^H \), the perturbation of the desired power is then simplified as

\[
\Psi(\mathcal{R}_1(n)) = e^H \delta H(n) [H(n)^{-1}]^H e\left. E \left[ \delta H(n)H(n) [H(n)^{-1}]^H e \right] \right| \left( H(n)^{-1} \right)^H e.
\]

(46)

Computation of the underlined term is given by Appendix A. If we apply results there, (46) becomes

\[
\Psi(\mathcal{R}_1(n))
\]

\[
\approx \frac{1}{N} \sum_{n=1}^{N} e^H H(n)C_1(n) E \left[ \delta g_1, \delta g_2 \right] C_1(n)^H [H(n)^{-1}]^H e.
\]

(47)

Similarly, if we decompose \( \mathcal{R}_m(n) = H(n)(1 - e^H [H(n)]^H \sigma_r^2 I + [H(n)H(n)]^{-1}) \) and apply the orthogonality between \( \Gamma(n) \) and \( H(n)H(n)^{-1} \), the second and third terms in (45) for computing the perturbation of noise power disappear. Moreover, it
can be shown that \( \Gamma(n) R_{\text{int}}(n) \Gamma(n) = \sigma_c^2 \Gamma(n) \), and
\[
H(n)^\dagger R_{\text{int}}(n) (H(n)^\dagger)^H = I - \mathbf{e} \mathbf{e}^H + \sigma_c^2 (H(n)^\dagger H(n))^{-1}.
\]
Applying the above results and using Appendix A, we obtain
\[
\Psi(R_{\text{int}}(n)) \approx \frac{1}{N} \sum_{n=1}^{N} \left\{ \sigma_c^2 \mathbf{e}^H [H(n)^\dagger H(n)]^{-1} E[\delta H(n)^\dagger \Gamma(n) \delta H(n)] \right. \\
\times [H(n)^\dagger H(n)]^{-1} \mathbf{e} + \sigma_c^2 \mathbf{e}^H H(n)^\dagger \\
\left. \times E[\delta H(n) [H(n)^\dagger H(n)]^{-1} \delta H(n)^H] (H(n)^\dagger)^H \mathbf{e} + \mathbf{e}^H H(n)^\dagger \sum_{j=2}^{r} C_j(n) E[\delta g_j \delta g_j^H] \right\} C_j(n)^H (H(n)^\dagger)^H \mathbf{e}.
\] (49)

Since the underlined terms computed by Appendix A are at the order of \( O(\sigma_c^2/N) \), the first and second terms in (49) are at the order of \( O(\sigma_c^2/N) \) and can be omitted in the presence of very small noise power. Then (49) reduces to
\[
\Psi(R_{\text{int}}(n)) \approx \frac{1}{N} \sum_{n=1}^{N} \mathbf{e}^H H(n)^\dagger \left[ \sum_{j=2}^{r} C_j(n) E[\delta g_j \delta g_j^H] \right] C_j(n)^H (H(n)^\dagger)^H \mathbf{e}.
\] (50)

To summarize, the perturbed SINR can be predicted based on (38), where unperturbed terms are computed by (39) and (40), and perturbed terms are computed by (47) and (50). According to (35), (47) and (50), both \( \Psi(R_{\text{int}}(n)) \) and \( \Psi(R_{\text{int}}(n)) \) are approximated at the order of \( O(\sigma_c^2/N) \), which is inversely proportional to \( N \) and proportional to \( \sigma_c^2 \). Moreover, it is observed that \( \Psi(R_{\text{int}}(n)) \) is caused by the desired user’s channel perturbation, while \( \Psi(R_{\text{int}}(n)) \) is caused by all interfering users’ channel perturbations.

### 4.2.2. SINR of the MMSE receiver
Substituting (17) for the receiver, the unperturbed term can be shown to be
\[
\Phi(X) = g_1^H \left[ \frac{1}{N} \sum_{n=1}^{N} C_1(n)^H R(n)^{-1} X R(n)^{-1} C_1(n) \right] g_1.
\] (51)
To evaluate the perturbed term, perturbation of the receiver is necessary. Obtaining \( \delta R(n) \) from (18) and then using (17), one can verify the perturbation of the MMSE receiver by
\[
\delta f(n) = R(n)^{-1} \delta \mathbf{e}.
\] (52)
Correspondingly, \( \Psi(X) \) can be approximated by two-step expectations in the same way as for the ZF receiver as
\[
\Psi(X) = \frac{1}{N} \sum_{n=1}^{N} \left\{ \mathbf{e}^H E[\delta H(n)^H R(n)^{-1} X R(n)^{-1} \delta H(n)] \mathbf{e} + f(n)^H H(n)^\dagger H(n) \right. \\
+ f(n)^H E[\delta H(n)^H R(n)^{-1} X R(n)^{-1} \delta H(n)] H(n)^H f(n) \\
+ f(n)^H E[\delta H(n)^H R(n)^{-1} X R(n)^{-1} H(n)^H H(n)^H f(n)] \\
+ f(n)^H R(n)^{-1} X R(n)^{-1} f(n) E[\delta g_1^2 \delta g_1^2] \\
+ f(n)^H E[\delta H(n)^H R(n)^{-1} X R(n)^{-1} \delta H(n)] H(n)^H f(n) + \alpha_1^2 \\
+ \frac{f(n)^H R(n)^{-1} X R(n)^{-1} E[\delta H(n)^H H(n)^H f(n)] + \alpha_2^2}{2} \right. \\
- \frac{f(n)^H E[\delta H(n) \delta g_1^2] H(n)^H f(n) - \alpha_2^2}{2} \left. \right. \\
- \frac{f(n)^H E[\delta H(n)^H R(n)^{-1} X R(n)^{-1} \delta H(n)] H(n)^H f(n) - \alpha_2^2}{2} \right. \\
- \frac{f(n)^H E[\delta H(n)^H \delta g_1^2] R(n)^{-1} X R(n)^{-1} f(n) - \alpha_2^2}{2} \left. \right. \\
- \frac{E[\delta H(n)^H \delta g_1^2] \mathbf{e} - \alpha_2^2}{2} \right\},
\] (53)
where for shorter expression \( a_1 \) up to \( a_q \) have been defined. All underlined terms follow general forms derived in Appendix A except \( E\{\delta \sigma^2 \} \) and \( E\{\delta \sigma^4 \} \) which are derived in Appendix B. Since each expectation term is shown to be at the order of \( O(\sigma^2/N) \), the final perturbations for both the desired signal and interference plus noise are thus all at the order of \( O(\sigma^2/N) \). To summarize, the perturbed SINR for the MMSE receiver can be predicted based on (38), where unperturbed terms are computed by (51), and perturbed terms are computed by (53), with \( X \) replaced by \( R_1(n) \) or \( R_{\text{int}}(n) \) respectively.

### 4.2.3. SINR of the RAKE receiver

Closed-form SINR of the RAKE receiver will be derived here due to the simplicity of the receiver. We first rewrite the signature matrix as the following:

\[
H(n) = [C_1(n)g_1, \ldots, C_j(n)g_j]
\]

(54)

where

\[
c_{j,n} = [c_{j,n}(0), \ldots, c_{j,n}(P-1)]^T,
\]

(55)

\[
G_j = [G_j]_{[\nu-1-d_j, \nu-d_j, \nu-P+1]} = \delta_j G_j,
\]

\[
G_j \text{ with dimensions of } (P+q) \times P \text{ is defined as}
\]

\[
G_j(n) = \begin{bmatrix}
g_j(0) & 0 \\
\vdots & \ddots & 0 \\
g_j(q+1) & \vdots & \ddots & 0 \\
& & & & &
\end{bmatrix},
\]

(56)

and \( \delta_j = [0_{(P-\nu) \times (P-d_j)}, I_{(P-\nu) \times (P-\nu)}, 0_{(P-\nu) \times (P-d_j)}] \). Then all subsequent analysis will be based on (54). First, the unperturbed desired power is derived after replacing the RAKE receiver with (20) and \( \Phi_1(n) \) with \( G_i c_{i,n} \), and applying trace, vec, and kronecker product operations [33]. These steps are summarized by

\[
\Phi(R_1(n)) = E\{ c_{i,n}^H G_i^H c_{i,n} c_{i,n}^H G_i^H c_{i,n} \} 
= E\{ \text{tr} [G_i^H c_{i,n} c_{i,n}^H G_i^H c_{i,n} c_{i,n}^H] \} 
= \text{vec}^H (G_i^H G_i) \text{vec}(E \{c_{i,n} c_{i,n}^H G_i^H c_{i,n}^H G_i^H c_{i,n} \}) 
= \text{vec}^H (G_i^H G_i) E\{ (c_{i,n}^H \otimes c_{i,n}) (c_{i,n}^H \otimes c_{i,n}^H) \} \text{vec}(G_i^H G_i).
\]

(57)

The underlined term is given by [34] for real codes as

\[
E\{ (c_{i,n}^H \otimes c_{i,n}) (c_{i,n}^H \otimes c_{i,n}^H) \}
= \kappa_{4c} X_1 + \sigma_4^2 X_2 + \sigma_4^2 \text{vec}(I) \text{vec}(I)^H + \sigma_4^4 I,
\]

(58)

and for complex codes as

\[
E\{ (c_{i,n}^H \otimes c_{i,n}) (c_{i,n}^H \otimes c_{i,n}^H) \}
\]

\[
= \kappa_{4c} X_1 + \sigma_4^2 X_2 + \sigma_4^2 \text{vec}(I) \text{vec}(I)^H + \sigma_4^4 I,
\]

(59)

where \( \kappa_{4c} \) is the fourth-order cumulant of the spreading codes, \( \sigma_4^2 = E\{c_j(n)^2\} \),

\[
X_1 = \text{diag} \{a_1 a_1^T, \ldots, a_p a_p^T\},
\]

(60)

and \( X_2 \) is partitioned into \( P \times P \) subblocks with the \( (i,j) \)-th subblock \( a_i a_i^T \).

Similarly, the unperturbed interference-plus-noise power can be computed as the following after noticing that \( c_{i,n} \) is independent of \( c_{j,n} \), and \( E\{c_{j,n} c_{j,n}^H\} = \sigma^2 I \) for \( j = 1, \ldots, J \): \n
\[
\Phi(R_{\text{int}}(n)) = \sum_{j=2}^{J} \sum_{j=2}^{J} E\{ c_{i,n}^H G_i c_{j,n} c_{j,n}^H G_j^H c_{i,n} \}
+ E\{ \sigma_4^H \text{vec}(I) \text{vec}(I)^H \}
+ \sigma_4^2 \text{tr} \{G_i^H G_i\}.
\]

(61)

We now proceed to evaluate the perturbed term \( \Psi(X) \) by first obtaining the perturbation of the RAKE receiver. According to (20) and (54), we have \( \delta f(n) = \delta G_1 c_{i,n} \). Noticing (56), \( \delta G_1 \) is calculated as

\[
\delta G_1 = [B_0 \delta g_1, \ldots, B_{P-1} \delta g_1],
\]

(62)

where

\[
B_j \triangleq \delta_j \Omega^j B,
B \triangleq \begin{bmatrix} I_{M(q+1)} \end{bmatrix},
\]

(63)

\( \Omega \) is a shifting matrix with all 1’s in the first subdiagonal, and \( \Omega^j \) is defined as an identity matrix for convenience. Based on the above results, following the same steps for deriving (57), and noticing that \( \delta G_1 \) is independent of codes, then the perturbed signal power is first obtained as

\[
\Psi(R_1(n)) = E\{ c_{i,n}^H \delta G_i c_{i,n} c_{i,n}^H G_i^H c_{i,n} \}
= \text{tr} \left\{ E\{ (c_{i,n}^H \otimes c_{i,n}) (c_{i,n}^H \otimes c_{i,n}^H) \} \text{vec}(G_i^H G_i) \right\},
\]

(64)

where the first underlined term is given by (58) or (59) as explained before, and the second underlined term is given in Appendix C. Finally, the perturbed interference-plus-noise
power is computed by

\[ \Psi (R_{\text{int}}(n)) = \sum_{j=2}^{J} E\{c_{i,n}^H \delta G_j^H G_j c_{j,n} c_{j,n}^H G_j^H \delta G_j c_{i,n} \} + E[\sigma_i^2 c_{i,n}^H \delta G_j^H \delta G_j c_{i,n}] \]

\[ = \sigma_i^2 \sum_{j=2}^{J} \text{tr} \left[ G_j^H \mathbb{E}[\delta G_j \delta G_j^H] \right] + \sigma_i^2 \sigma_j^2 \text{tr} \left[ \mathbb{E}[\delta G_j \delta G_j^H] \right] \tag{65} \]

with \( E[\delta G_j \delta G_j^H] \) given in Appendix C.

To summarize, the perturbed SINR for the RAKE receiver can be obtained analytically based on (38), where the unperturbed signal power of the RAKE receiver, given by (57), depends on the desired user’s channel conditions as well as the second- and fourth-order statistics of its own code sequence, while the unperturbed interference plus noise power given by (61) depends on the second-order moment of the codes, and cross channel conditions between each interfering user and the desired user. Finally, perturbed signal power and interference are given by (64) and (65), respectively. From them and Appendix C, we can conclude that both the desired user’s power and interference-plus-noise power are perturbed at the order of \( O(\sigma_i^2/N) \).

4.3. BER performance of different receivers

For each receiver, once its output SINR is evaluated, BER can be obtained by assuming that the interference is Gaussian distributed. This may not necessarily be correct, but this approximation has been shown to be relatively good [8, 35], especially when the number of interfering users is large. The BER for BPSK information symbol is

\[ \text{BER} = Q\left( \sqrt{\text{SINR}} \right), \tag{66} \]

where \( Q(x) = (1/\sqrt{2\pi}) \int_x^{\infty} e^{-t^2/2} dt \).

5. SIMULATION EXAMPLES

In this section, we verify our performance analysis by simulation examples. The average channel MSE, and the average SINR and BER of each receiver over 100 independent realizations are used as performance indicators. All analytical results are obtained based on the results in Section 4. As theoretical limits, SINRs and BERs of all ideal linear receivers are also presented. Those ideal receivers are constructed from all users’ perfect channel vectors as well as noise power. In the simulation setup, we consider an uplink CDMA system, where each user transmit BPSK signals through a respective multipath channel, whose parameters are randomly generated with equal power for each path according to Gaussian distribution. All users are assumed to have multipath delay spread as \( q = 2 \). All users are assumed to have the same transmission power, if not stated otherwise. The spreading factor is set to be 32 for all simulations. To validate our analysis, we study joint performance of the channel estimator and detectors under the effect of different system parameters, such as different data length \( N \), various input signal-to-noise ratio SNR, varied number of users \( J \), and signal to each interfering user’s power ratio (SIR).

Effect of \( N, J = 8, \text{SNR} = 20 \text{dB}, \text{SIR} = 0 \text{dB} \)

MSEs of channel estimation over different \( N \) are plotted in Figure 1a. Experimental and analytical curves are highly consistent for all examined \( N \), and as expected, the MSE level decreases when \( N \) becomes large. SINRs of all receivers are illuminated in Figure 1b, where experimental, analytical, and theoretical SINRs start to overlap from \( N = 50 \), indicating that the proposed receivers require very small data size to achieve their theoretical limit at 20 dB SNR environment. On the other hand, the ZF and MMSE receivers show much better performance than the RAKE receiver.

Effect of \( \text{SNR}, J = 8, N = 500, \text{SIR} = 0 \text{dB} \)

We consider various SNR from 0 dB to 12.5 dB at a step of 2.5 dB. Figure 2a illustrates the channel estimation MSE, which monotonically decreases as SNR increases. It is also
seen that the experimental MSE converges to its analytical value at large SNR. Slight differences are caused by the first-order approximation error. Figures 2b and 2c plot SINRs and BERs of each receiver. The MMSE receiver shows slightly better performance than the ZF receiver. Both of them are significantly superior to the RAKE receiver. The convergence between the experimental and analytical values can also be observed for each receiver, indicating that our analytical SINR and BER can serve as good performance predictors. Moreover, the experimental values are found to be very close to their theoretical ones, showing that each proposed receiver constructed from the proposed channel estimator and estimated noise behaves as well as its ideal counterpart.

Effect of $J$, $N = 500$, $SNR = 10$ dB, $SIR = 0$ dB

The number of equally powered users in the system varies from 2 to 8. Figures 3a to 3c illustrate the MSE, SINR, and BER performance of the proposed methods, respectively. It can be observed that the MSE slightly degrades when $J$ increases. Although an explicit relationship between the MSE and $J$ is not obtained in (35), $J$’s effect can still be found to be caused by the matrix $A_{j}^{1/2}$. Due to the same reason, the SINRs and BERs of the ZF and MMSE receivers also slightly degrade for large $J$. The RAKE receiver’s performance degrades drastically, which can easily be explained from our analysis that the interference-plus-noise power in (61) significantly increases with $J$. Again, for either MSE, SINR, or BER, experimental values are highly consistent with their corresponding analytical ones for large $J$ where Gaussian assumption for interference signals is well suitable. Each receiver’s experimental SINRs and BERs are overlapped with their theoretical values.

Near-far effect, $N = 500$, $SNR = 10$ dB and $J = 8$

The power ratio of each interfering user over the desired user ranges from 0 dB to 10 dB at a step of 2 dB. Analytical and experimental MSEs plotted in Figure 4a are found to be a constant, indicating that channel estimation MSE is not affected by interference power. This conclusion complies with (35), where $A_{j}$, as the only term containing interfering users’ effect, is independent of interfering user’s power. The near-far resistance of the channel estimator can be accredited to the code correlation operation which significantly removes MUI in the partial data covariance matrix. Similar conclusion can be drawn to SINRs and BERs of both ZF and MMSE receivers since they explicitly remove MUI. As expected, RAKE receiver’s performance degrades significantly when interference power increases.

6. CONCLUSION

We derived a closed form covariance for channel estimation error when channel is estimated using code decorrelation and subspace techniques as [10, 20, 21] based on finite data samples in long-code CDMA uplink. The performance of three symbol-level linear receivers constructed from the estimated channel, known as ZF, MMSE, and RAKE receivers, is also studied. Simulation examples involving different communication environments are provided and demonstrate high consistency between our analysis and experimental results.
Figure 3: Effect of number of users. (a) MSE. (b) Output SINR. (c) BER. The legend of (b) and (c) is the same as Figure 1b.

Figure 4: Near-far effect. (a) MSE. (b) Output SINR. (c) BER. The legend of (b) and (c) is the same as Figure 1b.
APPENDICES

A. DERIVATION OF EXPECTATION QUANTITIES FOR THE ZF RECEIVER

A.1. Derivation of $E[\delta H(n)H^T Z_0 \delta H(n)]$

In this case, $Z$ is a deterministic matrix of dimensions $(P - \mu)$ by $(P - \mu)$. By (41), the $(i, j)$th element of $E[\delta H(n)^H Z_0 \delta H(n)]$ is given by $E[\delta g_i^H C_j(n)^H Z_0 C_i(n)\delta g_j]$. The diagonal term (i.e., $i = j$) can readily be obtained as follows after replacing $E[\delta g_i^H \delta g_j^H]$ by (35):

$$E[\delta g_i^H C_i(n)^H Z_0 C_i(n)\delta g_i] = \text{tr} \{ C_i(n)^H Z_0 C_i(n) E[\delta g_i^H \delta g_i^H] \}$$

$$\approx \frac{\sigma_v^2}{N} \text{tr} \{ C_i(n)^H Z_0 C_i(n) \Pi_{g_i}^H A_{g_i}^{1/2} U_n^H A_{g_i}^{1/2} \Pi_{g_i} \}$$

(A.1)

which is clearly at the order of $O(\sigma_v^2/N)$. On the other hand, off-diagonal terms $E[\delta g_i^H C_j(n)^H Z_0 C_i(n)\delta g_j] = \text{tr}(C_j(n)^H Z_0 C_i(n) E[\delta g_i^H \delta g_j^H])$ for $i \neq j$ depend on $E[\delta g_i^H \delta g_j^H]$ and will be shown next to be at the order of $O(\sigma_v^2/N)$ and thus are negligible.

Replacing $\delta g_i$ by (29), the covariance between different channel perturbations is computed as

$$E[\delta g_i^H \delta g_j^H] = \frac{1}{N} \sum_{i,j} \Pi_{g_i}^H A_{g_i}^{1/2} U_n^H E[\delta R_i X_i^H \delta R_i] U_n^H A_{g_i}^{1/2} \Pi_{g_j}$$

(A.2)

which depends on $E[\delta R_i X_i^H \delta R_i]$. Following similar steps in [32], one can verify that

$$E[\delta R_i D \delta R_i] = \frac{1}{N} E[v_i(n) v_i(n)^T D v_i(n)^T] - \sigma_v^2 D$$

(A.3)

where $v_i(n) = A_{i}^{1/2} S_i^H C(n)^T \nu(n)$ with $\nu(n)$ independent of $C(n)$, applying the results in [34] for $E[v_i(n) v_i(n)^T D v_i(n)^T]$, it is found that $E[v_i(n) v_i(n)^T D v_i(n)^T]$ can be approximated by $O(\sigma_v^2)$, causing $E[\delta R_i D \delta R_i]$ in (A.3) at the order of $O(\sigma_v^2/N)$. As a result, $E[\delta g_i^H \delta g_j^H]$ in (A.2) can be approximated at most by the order of $O(\sigma_v^2/N)$, making all off-diagonal terms negligible.

To summarize, $E[\delta H(n) Z_0 \delta H(n)]$ can be approximated by a diagonal matrix with its $(i, i)$th element given by (A.1), which is at the order of $O(\sigma_v^2/N)$.

A.2. Derivation of $E[\delta H(n) Z_0 \delta H(n)^T]$  

In this case, $Z$ is a deterministic matrix of dimensions $J$ by $J$. According to (41), we immediately have

$$E[\delta H(n) Z_0 \delta H(n)^T] = \sum_{i,j=1}^{J} z_{ij} C_i(n) E[\delta g_i^H \delta g_j^H] C_j(n)^T$$

$$+ \sum_{i,j=1, i \neq j}^{J} z_{ij} C_i(n) E[\delta g_i^H \delta g_j^H] C_j(n)^T,$$

(A.4)

where $z_{ij}$ denotes the $(i, j)$th entry of $Z$. According to our previous analysis, the first term on the right-hand side of (A.4) involves the covariance of the perturbation between the same channel, which is at the order of $O(\sigma_v^2/N)$, while the second term involves cross covariance between two different channel perturbations, which is at the order of $O(\sigma_v^2/N)$, therefore, (A.4) can be simplified to (A.5), after applying (35), as follows:

$$E[\delta H(n) Z_0 \delta H(n)^T] = \frac{\sigma_v^2}{N} \sum_{i=1}^{J} z_{ii} C_i(n) \Pi_{g_i}^H A_{g_i}^{1/2} U_n^H A_{g_i}^{1/2} \Pi_{g_i} C_i(n)^T.$$  

(A.5)

A.3. Derivation of $E[\delta H(n) Z_0 \delta H(n)^T]$ and $E[\delta H(n)^H Z_0^H \delta H(n)^T]$

Rewriting (41) as $\delta H = C(n) \text{diag}[\delta g_1, \ldots, \delta g_J]$, then

$$E[\delta H(n) Z_0 \delta H(n)^T] = \frac{\sigma_v^2}{N} \sum_{i=1}^{J} z_{ii} C_i(n) \Pi_{g_i}^H A_{g_i}^{1/2} U_n^H A_{g_i}^{1/2} \Pi_{g_i} C_i(n)^T.$$  

(A.6)

In this case, the matrix $Z$ should have dimensions of $J \times (P - \mu)$, and $Z C(n)$ has dimensions of $J \times J(q + 1)$. Partitioning each row of $Z C(n)$ into $J$ subvectors of equal length of $q + 1$ elements, and denoting the $j$th subvector at the $i$th row as $z_{ij}^T$, we have

$$E[\delta H(n) Z_0 \delta H(n)^T] = C(n) E[\text{diag}[\delta g_1, \ldots, \delta g_J] Z C(n)^T \text{diag}[\delta g_1, \ldots, \delta g_J]^T].$$

(A.7)

For a complex system, (A.7) clearly becomes a zero matrix. For a real system, as before, if we ignore the cross covariance between different channel perturbations, then (A.7) reduces to the following after using (35):

$$E[\delta H(n) Z_0 \delta H(n)^T] = \frac{\sigma_v^2}{N} \sum_{i=1}^{J} z_{ii} C_i(n) \Pi_{g_i}^H A_{g_i}^{1/2} U_n^H A_{g_i}^{1/2} \Pi_{g_i} z_{ii}^T.$$  

(A.8)

Once $E[\delta H(n) Z_0 \delta H(n)^T]$ is computed, $E[\delta H(n)^H Z_0^H \delta H(n)^T]$ can be readily obtained as the Hermitian of $E[\delta H(n) Z_0 \delta H(n)^T]$. It can also be observed that both $E[\delta H(n) Z_0 \delta H(n)]$ and its Hermitian are at the order of $O(\sigma_v^2/N)$ for a real system and $\mathbf{0}$ for a complex system.
B. DERIVATION OF EXPECTATION QUANTITIES FOR THE MMSE RECEIVER

B.1. Derivation of $E\{\delta \sigma_i^2 \delta \sigma_i^2\}$

The perturbation of the $q$ least eigenvalues of $R_1$ is given by $\delta \sigma_i^2 = (1/q) \text{tr}((U_n^i)^H R_1 U_n^i)$ according to (14). Using vec and tr operations, it is straightforward to show

$$E\{\delta \sigma_i^2 \delta \sigma_i^2\} = \frac{1}{q^2} E\{ \text{tr} \{(U_n^i)^H \delta R_1 U_n^i\} \text{tr} \{(U_n^i)^H \delta R_1 U_n^i\} \}$$

$$= \frac{1}{q^2} \text{vec}^H \left(U_n^i(U_n^i)^H\right) E\{ \text{vec}(\delta R_1) \text{vec}^H(\delta R_1) \}$$

$$\times \text{vec} \left(U_n^i(U_n^i)^H\right)$$

(B.1)

which depends on $E\{\text{vec}(\delta R_1) \text{vec}^H(\delta R_1)\}$. If we denote the $i$th column of $\delta R_1$ as $\delta r_i$, then we have $\delta r_i = \delta R_i \theta_i$, where $\theta_i$ is a unit vector with its $i$th element as 1. Correspondingly, the $(i, j)$th submatrix of $E\{\text{vec}(\delta R_1) \text{vec}^H(\delta R_1)\}$, which has size $(q+1) \times (q+1)$, becomes $E\{\delta R_i^T \theta_j^H \delta R_1\} = T_i(\theta_i^H \theta_j^H)$ according to our definition (31). Applying those results, and noticing that

$$\text{vec} \left(U_n^i(U_n^i)^H\right) = \left[\theta_i^T U_n^i(U_n^i)^H, \ldots, \theta_q^T U_n^i(U_n^i)^H\right]^H$$

(B.2)

(B.1) reduces to

$$E\{\delta \sigma_i^2 \delta \sigma_i^2\} = \frac{1}{q^2} \sum_{j=1}^{q+1} \theta_i^T U_n^i(U_n^i)^H T_1(\theta_i^H \theta_j^H) U_n^i(U_n^i)^H \theta_j.$$  

(B.3)

Substituting $T_1(\theta_i^H \theta_j^H)$ by (32) or (33), and noticing that $(U_n^i)^H \chi_1 = 0$, and $(U_n^i)^H R_1 = \sigma_i^2 (U_n^i)^H$, we have that for a real system,

$$E\{\delta \sigma_i^2 \delta \sigma_i^2\} = \frac{\sigma_i^2}{Nq^2} \left[ \sum_{j=1}^{q+1} \left(\theta_j^T R_1 \theta_i\right) \theta_i^T U_n^i(U_n^i)^H \theta_j \right]$$

$$+ \sigma_i^2 \sum_{i,j=1}^{q+1} \left(\theta_i^T U_n^i(U_n^i)^H \theta_j\right)^2 ,$$

and for a complex system

$$E\{\delta \sigma_i^2 \delta \sigma_i^2\} = \frac{\sigma_i^2}{Nq^2} \left[ \sum_{j=1}^{q+1} \left(\theta_j^T R_1 \theta_i\right) \theta_i^T U_n^i(U_n^i)^H \theta_j \right].$$

(B.4)

(B.5)

Clearly, $E\{\delta \sigma_i^2 \delta \sigma_i^2\}$ is at the order of $O(\sigma_i^2/Nq^2)$ for either a real or complex system.

B.2. Derivation of $E\{\delta H(n) \delta \sigma_i^2\}$

Replacing $\delta \sigma_i^2$ with $(1/q) \text{tr}((U_n^i)^H \delta R_1 U_n^i)$, and expressing $E\{\delta H(n) \delta \sigma_i^2\}$ in columns, we obtain

$$E\{\delta H(n) \delta \sigma_i^2\} = \frac{1}{q} \left[ E\{ C_i(n) \delta g_i \text{tr} \{(U_n^i)^H \delta R_1 U_n^i\}\}, \ldots, \right.$$

$$E\{ C_q(n) \delta g_i \text{tr} \{(U_n^i)^H \delta R_1 U_n^i\}\} \right].$$

(B.6)

It is observed that except the first column, all other ones involve cross-correlation between $\delta g_i$ and $\delta R_1$. That cross-correlation depends on the cross-correlation between $\delta R_1$ and $\delta R_j$ for $j \neq 1$, and has been shown to be at the order of $O(\sigma_i^2/N)$ by our previous analysis. Then we only focus on the first column next, and approximate all the other columns as 0. Replacing $\delta g_i$ by (29) and applying trace property, the first column of (B.6) becomes

$$E\{\delta h_1(n) \delta \sigma_i^2\} = \frac{1}{q \gamma_1} C_1(n) \Pi_b^A \left[ U_n^i(U_n^i)^H \right] \chi_1$$

(B.7)

which depends on $E\{\delta R_1 \text{vec}^H(\delta R_1) \text{vec}(U_n^i(U_n^i)^H)\}$. Noticing $\delta R_1 = [\delta r_1, \ldots, \delta r_{q+1}]$, we have vec($\delta R_1$) = $[\delta r_1^H, \ldots, \delta r_{q+1}^H]^H$. Applying (B.2), it can be shown that the following holds:

$$E\{\delta R_1 \text{vec}^H(\delta R_1) \text{vec}(U_n^i(U_n^i)^H)\}$$

$$= \sum_{j=1}^{q+1} T_1(\theta_i^H \theta_j^H) U_n^i(U_n^i)^H \theta_j,$$

(B.8)

Replacing (B.8) back into (B.7), substituting $T_1(\theta_i^H \theta_j^H)$ by (32) or (33), and noticing that $(U_n^i)^H \chi_1 = 0$ and $(U_n^i)^H R_1 = \sigma_i^2 (U_n^i)^H$, (B.7) reduces to the following after neglecting the higher order terms of $O(\sigma_i^2/N)$:

$$E\{\delta h_1(n) \delta \sigma_i^2\}$$

$$\approx \frac{\sigma_i^2}{Nq \gamma_1} C_1(n) \Pi_b^A \left[ \sum_{j=1}^{q+1} \left(\theta_j^H R_1 \theta_i\right) U_n^i(U_n^i)^H \theta_j, \ldots, \right.$$

$$\sum_{j=1}^{q+1} \left(\theta_j^H R_1 \theta_{q+1}\right) U_n^i(U_n^i)^H \theta_j \right] \chi_1$$

(B.9)

which is clearly at the order of $O(\sigma_i^2/Nq)$.

To summarize, $E\{\delta H(n) \delta \sigma_i^2\}$ can be approximated by a matrix with its first column given by (B.9) and all the other columns by zeros.
C. DERIVATION OF EXPECTATION QUANTITIES FOR THE RAKE RECEIVER

C.1. Derivation of $E\{\text{vec}(G_{H}^H\delta G_{1})\text{vec}^H(G_{H}^H\delta G_{1})\}$

Rewriting $\text{vec}(G_{H}^H\delta G_{1})$ as $\text{vec}(G_{H}^H\delta G_{1}I)$ and using vec operations, then the following holds:

$$E\{\text{vec}(G_{H}^H\delta G_{1})\text{vec}^H(G_{H}^H\delta G_{1})\} = (I \otimes G_{H}^H)E\{\text{vec}(\delta G_{1})\text{vec}(\delta G_{1})^H\}(I \otimes G_{1}). \tag{C.1}$$

Directly replacing $\delta G_{1}$ by (62), applying the result in (35), (C.1) is further computed as

$$E\{\text{vec}(G_{H}^H\delta G_{1})\text{vec}^H(G_{H}^H\delta G_{1})\} = (I \otimes G_{H}^H)\mathcal{B}E\{\delta g_{1}\text{vec}(\delta g_{1})^H\}H(I \otimes G_{1}),$$

where $\mathcal{B}$ is defined as

$$\mathcal{B} = [B_{0}^T, \ldots, B_{p-1}^T]^T. \tag{C.3}$$

C.2. Derivation of $E\{G_{1}\text{vec}^H G_{1}^H\}$

Noticing (62), applying the result in (34), and ignoring $O(\sigma_{g_{1}}^H/N)$ terms, we have

$$E\{G_{1}\text{vec}^H G_{1}^H\} = \sum_{i=0}^{P-1} B_{i}E\{\delta g_{1}\text{vec}(\delta g_{1})^H\}B_{i}^H,$$

$$= \frac{\sigma_{g_{1}}^2}{N} \sum_{i=0}^{P-1} B_{i}I_{\vec{g}_{1}} A_{i}^{1/2}U_{n_{i}}^H(U_{n_{i}}^H A_{i}^{1/2}I_{\vec{g}_{1}})^H B_{i}^H. \tag{C.4}$$

ACKNOWLEDGMENTS

The authors would like to thank Professor L. Tong of Cornell University for his valuable suggestions. This work was supported by the US National Science Foundation under Grant CCR 0207931.

REFERENCES


Ping Liu received the B.S. and M.S. degrees in electronic engineering from Sichuan University, Chendu, China, in 1990 and 1993, respectively, the M.Eng. degree in electrical and electronic engineering from Nanyang Technological University, Singapore, in 1999, and the Ph.D. degree in electrical engineering from University of California, Riverside, in 2004. From 1993 to 1997, she worked as a senior software engineer with International Software Development Company, Shenzhen, China. From April 1999 to August 1999, she worked as a research engineer with Kent Ridge Digital Labs, Singapore. Her research interests are in the area of blind channel estimation and equalization, wireless multiuser communications, and digital signal processing.

Zhengyuan Xu received both the B.S. and M.S. degrees in electronic engineering from Tsinghua University, Beijing, China, in 1989 and 1991, respectively, and the Ph.D. degree in electrical engineering from Stevens Institute of Technology, Hoboken, NJ, USA, in 1999. From 1991 to 1996, he worked as an engineer and Department Manager at the Tsinghua Unisplendour Group Corp. of Tsinghua University. Since 1999, he has been with the Department of Electrical Engineering, University of California, Riverside, as an Assistant Professor. His current research interests include detection and estimation theory, spread spectrum and ultra-wideband wireless technology, multiuser communications, and ad hoc and wireless sensor networking. Dr. Xu received the Outstanding Student Award and the Motorola Scholarship from Tsinghua University, and the Peskin Award from Stevens Institute of Technology. He also received the Academic Senate Research Award and the Regents’ Faculty Award from University of California, Riverside. He has served as a session chair and technical program committee member for international conferences. He is an IEEE Senior Member, a Member of the IEEE Signal Processing Society’s Technical Committee on Signal Processing for Communications, an Associate Editor for the IEEE Transactions on Vehicular Technology and the IEEE Communications Letters.