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Spreading Sequence Design and Theoretical Limits for Quasisynchronous CDMA Systems

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For various quasisynchronous (QS) CDMA systems such as LAS-CDMA system which emerged recently, in order to reduce or eliminate the multiple access interference and multipath interference, it is required to design a set of spreading sequences which are mutually orthogonal within a designed shift zone, called orthogonal zone. For traditional orthogonal sequences, such as Walsh sequences and orthogonal Gold sequences, the orthogonality can only be achieved at the inphase point; in other words, the orthogonality is destroyed whenever there is a relative shift between the sequences, that is, their orthogonal zone is 0. In this paper, new concepts of generalized orthogonality (GO) and generalized quasiorthogonality (GQO) for spreading sequence design in both direct sequence (DS) QS-CDMA systems and time/frequency hopping (TH/FH) QS-CDMA systems are presented. Besides, selected GO/GQO sequence designs and general theoretical periodic and aperiodic limits, together with several applications in QS-CDMA systems, are also reviewed and analyzed.

Keywords and phrases: sequences design, generalized orthogonality, generalized quasiorthogonality, sequence bounds, QS-CDMA.

1. INTRODUCTION

In a typical direct sequence (DS) code division multiple access (CDMA) system, all users use the same bandwidth, but each transmitter is assigned a distinct spreading sequence [1]. The importance of the spreading sequences to spread spectrum CDMA is difficult to overemphasize, for the type of sequences used, its length, and its chip rate set bounds on the capability of the system that can be changed only by changing the spreading sequences [2, 3].

The well-known binary Walsh sequences or variable-length orthogonal sequences have perfect orthogonality at zero time delay, and are ideal for *synchronous CDMA* (S-CDMA) systems, such as the forward link transmission. Orthogonal spreading sequences can be used if all the users of the same channel are synchronized in time to the accuracy of a small fraction of one chip, because the crosscorrelation between different shifts of normal orthogonal sequences is normally not zero. Apart from the synchronization problem, in mobile communication environment, multipath propagation also introduces relatively nonzero time delays that destroy the orthogonality between Walsh or other orthogonal sequences [4, 5, 6, 7, 8].

For *asynchronous CDMA* (A-CDMA) system, no synchronization between transmitted spreading sequences is required, that is, the relative delays between the transmitted spreading sequences are arbitrary [1]. Therefore, in order to eliminate the multiple access interference, it is required to design a set of spreading sequences with impulsive autocorrelation functions (ACFs) and zero crosscorrelation functions (CCFs). Unfortunately, according to Welch bounds [9] and other theoretical limits [3, 10, 11, 12, 13, 14, 15, 16, 17], in theory, it is impossible to construct such an ideal set of sequences. In A-CDMA system, therefore, the spreading sequences are normally designed to have low autocorrelation sidelobes and low crosscorrelations, such as Gold sequences, Kasami sequences, and so forth [2, 3, 18].

To overcome these difficulties, the new concepts, *generalized orthogonality* (GO) and *generalized quasiorthogonality* (GQO) [4], are introduced, which can be employed in *quasisynchronous CDMA* (QS-CDMA) to eliminate the multiple access interference and multipath interference. These ideas, in fact, open a new direction in spreading sequence design. Recently, the investigation of QS-CDMA systems has been very active [19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31], many of the QS-CDMA systems are based on the use of GO/GQO sequences [4, 29, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55]. It should be noted that the GO is also called *zero correlation zone* (ZCZ) [38], *interference free windows* (IFW) or *zero correlation window* (ZCW) [19], *zero correlation duration* (ZCD) [40], or *no hit zone* (NHZ) if applies to frequency/time hopping systems, where the so-called Hamming correlation play a major role on the multiaccess interference [54]; the GQO is also called *low correlation zone* (LCZ) [50]; and the concept is also related to *almost perfect autocorrelation* [32], *pseudoperiodicity* [21], *semiperfect autocorrelation and semiorthogonality* [33] in earlier investigations.

Up to now, a number of GO/GQO sequence sets for QS-CDMA applications have been derived. For single GO sequence design, it is likely that Wolfmann was the first to consider the problem, and he did obtain a list of GO sequences with half sequence length orthogonal zone, that is, the so-called *almost perfect sequences* [32]. Later, more such sequence designs and their applications in channel measurement (estimation) have been considered, such as the work by Popovic [33] and Han, Deng, and so forth [34, 35, 36, 37]. An early work contributed to the set of GO sequences and their applications in QS-CDMA (or AS-CDMA) system was done by Suehiro who proposed a *pseudoperiodicity* concept and gave a construction of pseudoperiodic polyphase sequences [23]. The first systematic investigation on binary GO (or ZCZ) sequence designs was given in [38], where several classes of binary GO sequences with arbitrarily large GO zone are derived based on complementary pairs/sets; independently, Saito, Cha, and Matsufuji et al. also obtained a couple of binary GO sequence sets [29, 40, 41]. In order to provide an alternative CDMA technology, Li proposed a set of large area (LA) ternary sequences and a set of loosely synchronous (LS) ternary sequences having generalized orthogonal zone (or IFW) [42, 43, 44]. Based on LA and LS sequences, a so-called large area synchronous CDMA (LAS-CDMA) system, which was chosen by 3gpp2 as a candidate for next generation mobile communication technology, is proposed [19, 20, 21]. Later, other ternary GO sequence sets were proposed by a number of researchers [45, 46, 47]. Similarly, nonbinary GO sequences can also be derived [4, 48, 49]. In order to provide larger number of sequences, based on the GQO (or LCZ) concept, Tang and Fan constructed several classes of GQO sequences [50, 51]. By extending the GO concept to the two-dimensional case, families of GO arrays, where the one-dimensional GO zone becomes a rectangular GO zone, can also be synthesized [52, 53]. For the application of frequency/time hopping CDMA systems, similar ideas can be employed, forming the GO (or NHZ) hopping sequences [54, 55].

In order to evaluate the theoretical performance of the GO/GQO sequences, it is important to find the tight theoretical limits that set bounds among the sequence length, sequence set size, quasiorthogonal zone (or orthogonal zone), and the maximum value of correlations within quasiorthogonal zone (or low correlation zone LCZ). First, Tang and Fan established bounds on the periodic and aperiodic correlations of the GO/GQO sequences based on Welch's technique [56, 57], which include Welch bounds as special cases. In 2001, Peng and Fan [3, pages 99–106] obtained new lower bounds on aperiodic correlation of the GO/GQO sequences, which are stronger than the Tang-Fan bounds. Further study shows that even tighter aperiodic bounds for GO/GQO sequences can be derived [58]. Recently, periodic bound named generalized Sarwate bounds, for GO/GQO sequence design was obtained [59]. It has been shown that all the previous periodic and aperiodic sequence bounds, such as Welch bound [9], Sarwate bound [11], Levenshtein bounds [13], and previous GO/GQO bounds [3, 56, 57], are special cases of the new bounds [14, 58, 59]. As for the frequency/time hopping sequences, early in 1974, Lempel and Greenberger established some bounds on the periodic Hamming correlation of FH sequences for single or pair of hopping sequences [15]. Several years later, Seay derived a bound for set of hopping sequences [16]. Recently, several new periodic and aperiodic lower bounds that are more general and tighter than the known Lempel-Greenberger and Seay bounds for hopping sequences have been derived [17]. By using similar technique, the corresponding GQO hopping bounds have also been obtained, which includes the GO hopping bound (NHZ bound) presented in [54] as a special case.

In QS-CDMA systems, also called *approximately synchronous CDMA* (AS-CDMA) systems [21], the correlation functions of the GO spreading sequences employed take zero or very low values for a continuous correlation shift zone (GO zone or GQO zone) around the in-phase shift. The significance of GO sequences to QS-CDMA systems is that, even there are relative delays between the received spreading signals due to the inaccurate access synchronization and the multipath propagation, the orthogonality between the signals is still maintained as long as the relative delay does not exceed certain limit [27]. It has been shown that the GO sequences are indeed more robust in the multipath propagation channels, compared with the normal spreading sequences [4, 19, 21, 24, 27, 28].

There are several promising QS-CDMA technologies employing GO/GQO spreading sequences, which have attracted much attention and research interests in recent years [19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31]. The typical example of QS-CDMA system is the well-known LAS-CDMA system employing LA and LS spreading sequences or *smart code sequences* [19, 21]. Due to its high system capacity and spectral efficiency, it is claimed that LAS-CDMA technology would become a competitive candidate for 4G technologies [19]. Besides, a lot of attention have been paid to quasisynchronous multicarrier CDMA (QS-MC-CDMA) or quasisynchronous multicarrier direct sequence CDMA (QS-MC DS-CDMA), quasisynchronous orthogonal frequency division multiplexing CDMA (QS-OFDM-CDMA) or other derivatives [24, 25, 26, 47]. Since multicarrier CDMA is generally believed to be a promising technology [60, 61, 62] and it appears that the GO/GQO spreading sequences are suitable for time and frequency domain spreading in multicarrier CDMA in order to eliminate or reduce interference, therefore, the author has confidence in quasisynchronous multicarrier CDMA for future mobile communications. Furthermore, other QS-CDMA systems that are different from LAS-CDMA systems and MC-CDMA systems are also in research [22, 24, 27, 28, 29, 30, 31]. Similarly, it is also possible to design quasisynchronous time/frequency hopping (TH/FH) CDMA systems by employing GO spreading hopping sequences, that is, *NHZ hopping sequences*, with potential applications to areas such as ultrawide bandwidth (UWB) TH-CDMA radio systems, multiuser radar and sonar systems

[63]. Besides, GO/GQO sequences can also be used to accurately and efficiently perform channel estimation in single and multiple antenna communication systems [34, 35, 36, 37].

Based on the GO/GQO concepts, it is the aim of this paper to present recent advances in GO/GQO sequence design and the related theoretical limits, as well as several applications in QS-CDMA systems. The rest of the paper is organized as follows. In Section 2, basic concepts, that is, orthogonality, quasiorthogonality, GO, and GQO are given; then Section 3 presents various binary and nonbinary GO/GQO spreading sequences. In Sections 4, 5, and 6, periodic and aperiodic bounds for GO/GQO spreading sequences including GO/GQO hopping sequences are reviewed and analyzed, respectively; in Section 7, several applications of GO/GQO spreading sequences in QS-CDMA systems are discussed; and finally Section 8 concludes the paper with some remarks.

2. ORTHOGONALITY, GENERALIZED ORTHOGONALITY, QUASIORTHOGONALITY, AND GENERALIZED QUASIORTHOGONALITY

Given a sequence set $\{a_n^{(r)}\}$ with family size M, $r = 1, 2, 3, \ldots, M$, $n = 0, 1, 2, 3, \ldots, N-1$, each sequence $a^{(r)}$ is of length N, and each sequence element a_n is a complex number with unity amplitude. Then a sequence set is said to be orthogonal and generalized orthogonal (GO or Z_o-orthogonal) if the set has the following periodic correlation characteristics, respectively, [4],

$$\phi_{r,s}(\tau) = \sum_{n=0}^{N-1} a_n^{(r)} a_{n+\tau}^{*(s)} = \begin{cases} N, & \text{for } \tau = 0,\ r = s, \\ 0, & \text{for } \tau = 0,\ r \neq s, \end{cases} \tag{1}$$

$$\phi_{r,s}(\tau) = \sum_{n=0}^{N-1} a_n^{(r)} a_{n+\tau}^{*(s)} = \begin{cases} N, & \text{for } \tau = 0,\ r = s, \\ 0, & \text{for } \tau = 0,\ r \neq s, \\ 0, & \text{for } 0 < |\tau| \leq Z_o, \end{cases} \tag{2}$$

where the subscript addition $n + \tau$ is performed *modulo* N, a_n^* denotes the complex conjugate of sequence element a_n. The corresponding sequence sets are denoted by $G(N, M)$ and $\mathrm{GO}(N, M, Z_o)$, respectively. Obviously, $\mathrm{GO}(N, M, 0) = G(N, M)$.

For normal orthogonality defined in (1), it is clear that the value $\phi_{r,s}(\tau)$ between rth and sth members of the set is equal to zero only at zero-time delay. The $\phi_{r,s}(\tau)$ at nonzero time delay is normally nonzero, as is the case of Walsh sequences. This will cause problems in sequence acquisition and tracking, and generate large amounts of multipath interference.

For GO defined in (2), the zero zone Z_o represents the degree of the GO. It is clear that the bigger the length Z_o, the better the sequence set, and hence the more general the orthogonality. When $Z_o = 0$, the GO becomes the normal orthogonality, and the GO sequence set becomes the normal orthogonal sequence set. In addition, $\phi_{r,s}(\tau)$ can be of any value when τ is outside the range $(-Z_o, Z_o)$.

In order to obtain larger set of sequences with minimum interference between users, another concept, named quasiorthogonality (QO), is defined by Yang et al. [8]. The major condition for a sequence set, $\{a_n^{(r)}\}$, which should contain Walsh sequences as a subset, to be quasiorthogonal is

$$\phi_{r,s}(\tau) = \sum_{n=0}^{N-1} a_n^{(r)} a_{n+\tau}^{*(s)} \begin{cases} = N, & \text{for } \tau = 0,\ r = s, \\ \leq \varepsilon, & \text{for } \tau = 0,\ r \neq s, \end{cases} \tag{3}$$

where, ε is a very small number compared with N. It is required that the inner product between any two distinct sequences in the QO set, denoted by $\mathrm{QO}(N, M, \varepsilon)$, should be as small as possible.

In practice, it may be difficult to synthesize a set of GO sequences with the desired parameters because of the strict condition of GO. Therefore, based on the QO concept, a more general concept, called GQO, is defined in this paper, that is,

$$\phi_{r,s}(\tau) = \sum_{n=0}^{N-1} a_n^{(r)} a_{n+\tau}^{*(s)} \begin{cases} = N, & \text{for } \tau = 0,\ r = s, \\ \leq \varepsilon, & \text{for } \tau = 0,\ r \neq s, \\ \leq \varepsilon, & \text{for } 0 < |\tau| \leq L_o, \end{cases} \tag{4}$$

where L_o is called the periodic generalized quasiorthogonal zone. It is clear that the GQO set, denoted by $\mathrm{GQO}(N, M, \varepsilon, L_o)$, becomes a QO set when $L_o = 0$, a GO set when $\varepsilon = 0$, and a normal orthogonal set when $L_o = 0$ and $\varepsilon = 0$. Similar to autocorrelation and crosscorrelation functions, it is necessary in some occasions to differentiate the maximum value ε as ϕ_a for all $r = s$, and ϕ_c for all $r \neq s$, $\phi_m = \max\{\phi_a, \phi_c\}$.

As for the aperiodic GQO case, we have the following similar definition,

$$\delta_{r,s}(\tau) = \sum_{n=0}^{N-\tau} a_n^{(r)} a_{n+\tau}^{*(s)} \begin{cases} = N, & \text{for } \tau = 0,\ r = s, \\ \leq \varepsilon, & \text{for } \tau = 0,\ r \neq s, \\ \leq \varepsilon, & \text{for } 0 < \tau \leq L_o, \end{cases} \tag{5}$$

where, for simplicity, only positive time shifts are considered in this paper. The aperiodic GQO becomes aperiodic GO when $\varepsilon = 0$. It is clear that the aperiodic QO and periodic QO are the same, so they are the normal aperiodic orthogonality and periodic orthogonality, as there is no relative shift between the sequences.

As for TH/FH sequence design, five parameters are normally involved, the size q of the time/frequency slot set F, the sequence length N, the family size M, the maximum Hamming autocorrelation sidelobe H_a, and the maximum Hamming crosscorrelation H_c, where $H_m = \max\{H_a, H_c\}$. Given a hopping sequence set with family size M and sequence length N, that is, $\{a_n^{(r)}\}$, $r = 1, 2, \ldots, M$, $n = 0, 1, 2, \ldots, L-1$, where the sequence elements are over a given alphabet F with size q. Then the periodic Hamming autocorrelation function $(r = s)$ and crosscorrelation function $(r \neq s)$ can be defined as follows:

$$H_{rs}(\tau) = \sum_{n=0}^{N-1} h\Big[a_n^{(r)}, a_{n+\tau}^{(s)}\Big], \quad 0 \leq \tau < N, \tag{6}$$

where the subscript addition is also performed modulo N and the Hamming product $h[x, y]$ is defined as

$$h[x, y] = \begin{cases} 0, & x \neq y, \\ 1, & x = y, \end{cases} \tag{7}$$

and the corresponding GQO (or low hit zone, LHZ) for hopping sequences can be defined similarly as

$$H_{rs}(\tau) = \sum_{n=0}^{N-1} h\left[a_n^{(r)}, a_{n+\tau}^{(r)}\right] \begin{cases} = N, & \text{for } \tau = 0,\ r = s, \\ \leq \varepsilon, & \text{for } \tau = 0,\ r \neq s, \\ \leq \varepsilon, & \text{for } 0 < |\tau| \leq L_o, \end{cases} \tag{8}$$

where the GQO hopping sequence set, denoted by $\text{GQO}(N, M, q, \varepsilon, L_o)$, becomes a GO hopping set, or NHZ set when $\varepsilon = 0$, and a normal orthogonal hopping set when $L_o = 0$ and $\varepsilon = 0$. Similarly, one can also define aperiodic Hamming correlation functions and aperiodic GQO.

In the following sections, the GQO and GO sequence designs and the related periodic and aperiodic bounds will be discussed in details.

3. SPREADING SEQUENCES WITH GO/GQO CHARACTERISTICS

In this section, a number of orthogonal sequences, GO sequences, and GQO sequences are briefly described. Due to the limited space, only basic ideas and selected constructions are given without proofs.

Walsh sequences

The well-known binary orthogonal sequences, that is, Walsh-Hadamard sequences, can be generated from the rows of special square matrices, called Hadamard matrices. These matrices contain one row of all zeros, and the remaining rows each have equal numbers of ones and zeros. The Walsh sequences of length $N = 2^n$ can also be generated recursively.

Variable-length orthogonal sequences

The variable-length orthogonal binary sequences, also called orthogonal variable spreading factor (OVSF) sequences, can be generated recursively by a layered tree diagram [5]. An interesting property of the OVSF sequences is that not only the sequences in the same layer are orthogonal, but also any two sequences of different layers are orthogonal except for the case that one of the two sequences is a mother sequence of the other. In applying these sequences, the number of available sequences is not fixed, but depends on the rate and spreading factor of each physical channel, therefore supporting multirate transmission.

Quadriphase and polyphase orthogonal sequences

Based on a set of quadriphase sequences, a general construction for the orthogonal sets is recently developed [6]. It is shown that a subset of the quadriphase sequences can be transformed into an orthogonal set simply by extending each sequence by the same arbitrary element. The same construction can also be extended to polyphase orthogonal sequences over the integer ring $\mathbb{Z}_{p^k}$ for any prime p and integer k.

It should be noted that for any prime p and even number n, Matsufuji and Suehiro also gave a construction which can generate orthogonal polyphase sequences of length p^n, including binary and quadriphase orthogonal sequences [7].

Generalized orthogonal binary sequences

Given a sequence matrix $F^{(n)}$ with M_n rows, each row consists of M_n sequences, each of length N_n, one can derive a matrix $F^{(n+1)}$ with $2M_n$ rows, each row consists of $2M_n$ sequences, each of length $2N_n$, that is,

$$F^{(n+1)} = \begin{bmatrix} F^{(n)}F^{(n)} & (-F^{(n)})F^{(n)} \\ (-F^{(n)})F^{(n)} & F^{(n)}F^{(n)} \end{bmatrix}, \tag{9}$$

where $-F^{(n)}$ denote the matrix whose ijth entry is the ijth negation of $F^{(n)}$, $F^{(n)}F^{(n)}$ denotes the matrix whose ijth entry is the concatenation of the ijth entry $F^{(n)}$ and the ijth entry of $F^{(n)}$.

Our construction of generalized orthogonal binary sequences is based on a starter $F^{(0)}$ consisting of a pair of complementary sequence mates [2] defined below [38],

$$F^{(0)} = \begin{bmatrix} F_{11}^{(0)} & F_{12}^{(0)} \\ F_{21}^{(0)} & F_{22}^{(0)} \end{bmatrix} = \begin{bmatrix} -X_m & Y_m \\ -\overleftarrow{Y_m} & -\overleftarrow{X_m} \end{bmatrix}_{2\times 2^{m+1}}, \tag{10}$$

where $\overleftarrow{Y_m}$ denotes the reverse of sequence Y_m and $-Y_m$ is the binary complement of Y_m. The two sequences X_m and Y_m, each of length $N_0 = N'_m$, are defined recursively by

$$\begin{aligned} [X_0, Y_0] &= [1, 1], \\ [X_m, Y_m] &= [X_{m-1}Y_{m-1}, (-X_{m-1})Y_{m-1}], \end{aligned} \tag{11}$$

where the length of X_0 and Y_0 is $N'_0 = 2^0 = 1$, and the length of X_m and Y_m, is $N'_m = 2^m$.

If $m = 2$, $n = 1$, then we can generate the following $F^{(1)}(N, M, Z_o)$, that is, $\text{GO}(N, M, Z_o) = \text{GO}(32, 4, 4)$,

$$\begin{aligned} a_n^{(1)} &= \{+ + - + + + - + - - - + - - - + - - + - + \\ &\quad + - + + + + - - - - +\}, \\ a_n^{(2)} &= \{- - + - + + - + + + + - - - - + + + - + + \\ &\quad + - + - - - + - - - +\}, \\ a_n^{(3)} &= \{+ - - - + - - - - + - - - + - - - + + + + \\ &\quad - - - + - + + - + - -\}, \\ a_n^{(4)} &= \{- + + + + - - - + - + + - + - - + - - - + \\ &\quad - - - - + - - - + - -\}, \\ \phi_{r,r} &= \{\text{xxxxxxxxxxxx}0000 \quad 32 \quad 0000\text{xxxxxxxxxxxx}\}, \\ \phi_{r,s} &= \{\text{xxxxxxxxxxxx}0000 \quad 0 \quad 0000 \ \ \text{xxxxxxxxxxxx}\}. \end{aligned} \tag{12}$$

Table 1: Primary LA code sequences (N, M, N_0).

N	M	N_0	Basic Intervals
18	4	3	3, 4, 6, 5
156	8	16	16, 17, 18, 20, 19, 22, 23, 21
731	16	38	52, 53, 54, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50
760	16	40	45, 46, 47, 48, 49, 50, 51, 41, 40, 42, 43, 44, 52, 53, 54, 55
792	16	42	47, 48, 49, 50, 51, 52, 53, 43, 42, 44, 45, 46, 54, 55, 56, 57
826	16	44	49, 50, 51, 52, 53, 54, 55, 45, 44, 46, 47, 48, 56, 57, 58, 61
856	16	46	51, 52, 53, 54, 55, 56, 57, 47, 46, 48, 49, 50, 58, 59, 60, 61
2473	32	32	44, 45, 46, 47, 48, 32, 33, 34, 37, 35, 38, 36, 41, 40, 42, 52, 49, 56, 51, 97, 43, 55, 63, 126, 75, 142, 176, 58, 79, 66, 122, 565
2562	32	34	47, 48, 49, 50, 51, 52, 53, 54, 55, 34, 35, 37, 36, 38, 40, 41, 44, 42, 80, 59, 45, 65, 61, 57, 39, 173, 70, 58, 91, 264, 60, 634
...	...	...	...

From a generalized orthogonal sequence set

$$F^{(n+1)}(N, M, Z_o) = \text{GO}\left(2^{2n+m+1}, 2^{n+1}, 2^{n+m-1}\right), \quad (13)$$

we can construct a shorter generalized orthogonal sequence set $F^{(n-t+1)}(N, M, Z_o) = \text{GO}(2^{2n+m-t+1}, 2^{n+1}, 2^{n+m-t-1})$ with the same number of sequences by truncation technique, that is, by simply halving each sequence t times in set $F^{(n+1)}$, where $t < n$ for $n > 0$, or $t < m$ for $n = 0$. When $N = M$, we have $Z_o = 0$, thus $F^{(n+1)}(N, M, Z_o) = \text{GO}(N, N, 0)$, which is a Walsh sequence set. For any GO binary and GO polyphase sequences, it can be shown later that $Z_o \le N/M - 1$.

Further study shows that the above construction can be extended to a larger class of generalized orthogonal binary sequences, by using a set of complementary binary mates, instead of a pair of complementary binary mates, as a starter [2]. Other binary GO sequences can be obtained from Gold sequences, Hadamard matrices, and so forth [29, 40].

The generalized orthogonal sequences can also be extended to higher-dimensional generalized orthogonal arrays [52, 53].

Generalized orthogonal quadriphase sequences

In order to synthesize generalized orthogonal quadriphase sequences, the same methods, as shown in the construction of generalized orthogonal binary sequences, can be employed. Unlike the binary complementary pairs, the quadriphase complementary pairs exist for many more sequence lengths. For lengths up to 100, only the quadriphase complementary pairs of lengths 7, 9, 11, 15, 17 do not exist.

LA and LS sequences used by LAS-CDMA systems

LA sequences are derived from the so-called primary code, whose construction is similar (but not equivalent) to the method used for optical sequences with small sidelobes of aperiodic correlation functions, but with a GO zone Z_o [42, 43, 44]. A partial list of primary LA code sequences, each of length N, having M intervals (pulses) with the minimum interval length being N_0, is given in Table 1.

Here, given parameters M and N_0, a theoretical proposition is how to generate a primary code sequence with the minimum length. In general, the shorter the length N for the fixed number of intervals, M, and the minimum interval length N_0, the better the LA code constructed. For this theoretical aspect, related bounds have been derived and, based on an efficient algorithm, more efficient primary codes have been obtained, which will be reported later on.

In definition, LA code is a class of ternary GO sequences $\text{GO}(N, M, Z_o)$, $Z_o = N_0$, which is constructed from a given primary code (N, M, N_0). The generation of LA code can be done in two steps, firstly, choose an orthogonal sequence set of length M, and secondly, insert zero strings between the elements (pulses) of the orthogonal sequences with different intervals (length) according to the primary code listed in Table 1.

The resultant LA sequences have the following characteristics, (1) all but one length of intervals between nonzero elements are even; (2) each length of interval between nonzero elements can only appear once; (3) no length or length summation of intervals between nonzero elements can be a summation of others; (4) the periodic/aperiodic autocorrelation sidelobes and crosscorrelations take only three possible values, +1, 0, and −1; (5) there is an orthogonal zone of length Z_o around the in-phase position.

It is clear that LA code sequences have large intervals (zero gaps) between two adjacent pulses, where the minimal interval is equal to N_0. For instance, choosing $(N, M, N_0) = (18, 4, 3)$ and an orthogonal Walsh set of order 4, one can obtain the following four LA sequences $\text{GO}(18, 4, 3)$:

$$\begin{aligned}
a^{(1)} &= \{1\ 0\ 0\ 1\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\},\\
a^{(2)} &= \{1\ 0\ 0\ -1\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ -1\ 0\ 0\ 0\ 0\},\\
a^{(3)} &= \{1\ 0\ 0\ 1\ 0\ 0\ 0\ -1\ 0\ 0\ 0\ 0\ 0\ -1\ 0\ 0\ 0\ 0\},\\
a^{(4)} &= \{1\ 0\ 0\ -1\ 0\ 0\ 0\ -1\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\},
\end{aligned} \quad (14)$$

where each LA sequence has 4 intervals (pulses) and length 18, the minimum interval length is equal to 3, and its duty ratio is equal to 4/18.

$$
\begin{bmatrix} C_1 & S_1 \\ C_2 & S_2 \end{bmatrix} \longrightarrow
\begin{cases}
\begin{bmatrix} C_1C_2 & S_1S_2 \\ C_1-C_2 & S_1-S_2 \end{bmatrix} \longrightarrow
\begin{cases}
\begin{bmatrix} C_1C_2C_1-C_2 & S_1S_2S_1-S_2 \\ C_1C_2-C_1C_2 & S_1S_2-S_1S_2 \end{bmatrix}, \\
\begin{bmatrix} C_1-C_2C_1C_2 & S_1-S_2S_1S_2 \\ C_1-C_2-C_1-C_2 & S_1-S_2-S_1-S_2 \end{bmatrix},
\end{cases} \\
\begin{bmatrix} C_2C_1 & S_2S_1 \\ C_2-C_1 & S_2-S_1 \end{bmatrix} \longrightarrow
\begin{cases}
\begin{bmatrix} C_2C_1C_2-C_1 & S_2S_1S_2-S_1 \\ C_2C_1-C_2C_1 & S_2S_1-S_2S_1 \end{bmatrix}, \\
\begin{bmatrix} C_2-C_1C_2C_1 & S_2-S_1S_2S_1 \\ C_2-C_1-C_2-C_1 & S_2-S_1-S_2-S_1 \end{bmatrix},
\end{cases}
\end{cases}
$$

Figure 1: Construction of $C_i^{(k)}$ and $S_i^{(k)}$ subsequences.

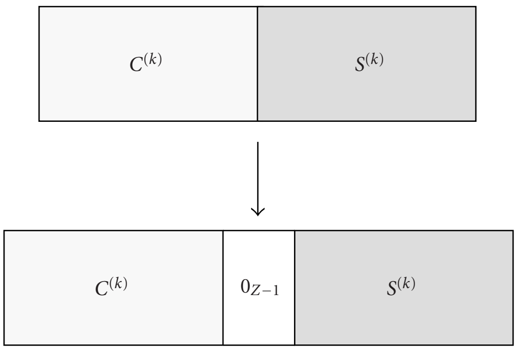

Figure 2: Zero insertion to form an LS sequence.

Due to the large number of zeros existed, or the low duty ratio M/N, in LAS-CDMA, LA code has to be combined with LS code sequences in a way to provide excellent antiinterference behavior.

Interestingly, LS sequences can also be constructed from Golay complementary pairs [42, 43, 44]. Given a Golay pair $(C_1\ S_1)$, each sequence is of length L_o, one can find another Golay pair $(C_2\ S_2)$, so that two pairs are mates [2]. An LS sequence set of length $N' = 2^k L_o$ has 2^k sequences, each consists of two subsequences, $C^{(k)}$ and $S^{(k)}$, which can be generated recursively by a starter $(C_1\ S_1) = (++-+, +---)$ and $(C_2\ S_2) = (+++-, +-++)$, $L_o = 4$, $k = 1$, $N' = 8$, as shown in Figure 1. At level k in Figure 1, the arrows split each Golay pair $(C^{(k)}\ S^{(k)})$ into two Golay pairs (mates) $(C^{(k+1)}\ S^{(k+1)})$, $(C'^{(k+1)}\ S'^{(k+1)})$ for the next level $k + 1$.

In fact, the actual LS sequence LS_i, $0 \le i < 2^k$, is defined as the concatenation of $C^{(k)}$ and $S^{(k)}$ subsequences with $Z - 1$ zeros inserted between them, as shown in Figure 2. The reason for the zero insertion is to avoid overlapping between the subsequences so as to form the desired aperiodic orthogonal zone.

Therefore, an LS code set $\text{GO}(N, M, Z_o)$ is a class of aperiodic ternary GO sequences of length $N = 2^k L_o + Z - 1$, family size $M = 2^k$ and aperiodic orthogonal zone $Z_o = \min(\lfloor N/M \rfloor, Z)$, where $\lfloor \cdot \rfloor$ denotes the integer part of a real number, each sequence has $2^n L_o$ nonzeros, and $Z - 1$ zeros. When $L_o = 4$, $k = 5$, $Z = Z_o = 4$ (i.e., 3 zeros should be inserted), $N = 128 + 3$, $M = 32$, which is the recommended LS sequence set for LAS-CDMA system.

The fact that there are only 32 LS sequences of length $128 + 3$ and $Z_o = 4$ (or $Z_o = 7$ if double-sided orthogonal zone is defined) is known as a bottleneck for LAS-CDMA technology. Unfortunately, from the theoretical bounds to be discussed later, one can hardly obtain more LS sequences while maintaining the orthogonal zone, since the current LS family is already nearly optimal. In order to provide larger system capacity and higher adjacent cell/sector interference reduction for LAS-CDMA, one solution is to try to construct several LS code sets, each with the same GO property but having minimum crosscorrelation between any two sequences from different LS code sets. Fortunately, it has been shown theoretically that one can construct a number of such LS code sets, each set having 32 LS codes of length 128 and $Z_o = 4$, and the crosscorrelation function between any two generalized LS codes from different set is zero within the orthogonal zone except for a small in-phase crosscorrelation value in some cases, as will be reported later on. In addition, the connection between the LS codes, Hadamard matrices, bent function, and the Kerdock codes is also established.

Other generalized orthogonal nonbinary sequences

Based on the GO concept, it is also possible to generate other generalized orthogonal nonbinary sequences, such as the GO polyphase or GO multilevel sequences [41, 48, 49].

Generalized quasiorthogonal sequences

In addition to the GO sequences, several classes of GQO sequences have also been constructed.

In [50], a new class of GQO sequences over GF(p), based on GMW sequences, is constructed. This GQO sequence set is a set with length $N = p^n - 1$, $n = p^m - 1$, small nonzero value $\varepsilon = -1$, and GQO zone $L_o = (p^n - 1)/(p^m - 1)$. As for GQO set size M, it has been shown that, for two special cases, we have $M = p^m - p^{m-f}$ and $M = (p^m - p^{m/2})/2$, f is an intermediate parameter as explained in [50]. For $p = 2$, as a special case, a class of binary GQO sequence set GQO$(2^n - 1, (2^m - 2^{m/2})/2, -1, (2^n - 1)/(2^m - 1))$ can be obtained.

Recently, other interesting GQO sequence sets have been obtained based on interleaving, multiplication, and other techniques [51].

It is believed that there are still lots of work which can be done in various GQO sequence constructions and the related theory.

GO hopping or NHZ hopping sequences

There are many ways to construct GQO hopping sequences for applications in quasisynchronous TH/FH CDMA systems. One way is by mapping a set of known binary GO sequences with elements in the field GF(2) to the sequence set with elements in the extension field GF(p^m) = GF(2^{2n+1}) [55]. Another construction is based on the known conventional FH sequences and many-to-one mapping [54]. An NHZ sequence set GO(12, 5, 3) is given below,

$$\begin{aligned} a^{(1)} &= \{1\ 6\ 11\ 2\ 7\ 12\ 4\ 9\ 14\ 3\ 8\ 13\}, \\ a^{(2)} &= \{2\ 7\ 12\ 3\ 8\ 13\ 0\ 5\ 10\ 4\ 9\ 14\}, \\ a^{(3)} &= \{3\ 8\ 13\ 4\ 9\ 14\ 1\ 6\ 11\ 0\ 5\ 10\}, \\ a^{(4)} &= \{4\ 9\ 14\ 0\ 5\ 10\ 2\ 7\ 12\ 1\ 6\ 11\}, \\ a^{(5)} &= \{0\ 5\ 10\ 1\ 6\ 11\ 3\ 8\ 13\ 2\ 7\ 12\}. \end{aligned} \quad (15)$$

Besides, one can also construct GO and GQO hopping sequences by using directly matrix permutation and other algorithms.

4. THEORETICAL PERIODIC LIMITS FOR GO/GQO SEQUENCES

Because the traditional bounds, such as Welch bounds [9], Sidelnikov bounds [10], Sarwate bounds [11], Massey bounds [12], Levenshtein bounds [13], and so forth, cannot directly predict the existence of the GO and GQO sequences, it is important to derive the theoretical bounds for GO and GQO sequences, which are not previously known because of the new concept.

This section discusses mainly the periodic bounds for the new sequence design, such as Tang-Fan bounds [56] and Peng-Fan bounds [14, 59], and points out the generality of the new bounds which include the previous periodic bounds for normal sequence design as special cases.

For binary sequences, we have derived a new periodic bound for GQO sequences [59],

$$\frac{1}{M}\left(1 - \sum_{s=0}^{L_o} w_s^2\right)\phi_a^2 + \left(1 - \frac{1}{M}\right)\phi_c^2 \geq N - \frac{N^2}{M}\sum_{s=0}^{L_o} w_s^2, \quad (16)$$

where $w = (w_0, w_1, \ldots, w_{L_o})$, and

$$w_i \geq 0,\ i = 0, 1, \ldots, L_o, \quad \sum_{i=0}^{L_o} w_i = 1. \quad (17)$$

In particular, let $\phi_m = \max\{\phi_a, \phi_c\}$; choose w_s such that

$$\sum_{s=0}^{L_o} w_s^2 = \frac{1}{L_o + 1}, \quad (18)$$

then for binary sequences, we have

$$\phi_m^2 \geq \frac{ML_o + M - N}{ML_o + M - 1}N, \quad (19)$$

which was derived by Tang and Fan and is suitable for any sequences with equal energy [56].

In addition, for binary sequences, we have

$$\frac{1}{M}\left(1 - \frac{1}{L_o + 1}\right)\phi_{\mathrm{a}}^2 + \left(1 - \frac{1}{M}\right)\phi_{\mathrm{c}}^2 \geq N - \frac{N^2}{M(L_o + 1)}. \quad (20)$$

In particular, let $L_o = N - 1$, we have

$$\frac{N - 1}{(M - 1)N^2}\phi_{\mathrm{a}}^2 + \frac{1}{N}\phi_{\mathrm{c}}^2 \geq 1, \quad (21)$$

which was derived by Sarwate, that is, Sarwate bound [11]. Further, let $\phi_m = \max\{\phi_a, \phi_c\}$ then (21) becomes

$$\phi_m^2 \geq \frac{(M - 1)N^2}{MN - 1}, \quad (22)$$

which is the famous Welch bound [9]. It is worth notice that from Welch bound equation (22), ϕ_m can be zero if and only if $M = 1$ *and* $N \neq 1$; for binary case, there is only one sequence of length 4 satisfying $\phi_m = 0$, that is, $\{a_n\} = (1110)$. However, from Tang-Fan GQO-bound equation (19), ϕ_m may take the zero value for all $M(L_o + 1) \leq N$. By replacing the GQO zone L_o with GO zone Z_o, we have the following periodic bound for GO sequences,

$$Z_o \leq \frac{N}{M} - 1. \quad (23)$$

In addition, if the length N is a multiple of 4, in most cases, there exist binary sequences with $Z_o = N/2 - 1$ [4], which is not covered by Welch bound.

5. THEORETICAL APERIODIC LIMITS FOR GO/GQO SEQUENCES

In this section, in addition to reviewing the existing results, our focus is on the new aperiodic correlation bounds which are much tighter than other known bounds. It is noted that all the new bounds, named generalized Sarwate bounds, presented here are in a form which is quite similar to that of Sarwate bounds, but contain different coefficients.

Peng-Fan bound (2002) [14]:

$$\begin{aligned}
&3L\delta_a^2 + 3(L+1)(M-1)\delta_c^2 \\
&\quad \geq 3MN - 3N^2 + 3MNL \\
&\qquad - 2ML - ML^2, \quad 0 \leq L \leq L_o, \\
&2(4^L - 1)\delta_a^2 + 3(M-1)4^L\delta_c^2 \\
&\quad \geq (3MN - N^2 - 4M)4^L + 6(L-2)M2^L \\
&\qquad + 6ML + 16M - 2N^2, \quad 0 \leq L \leq L_o.
\end{aligned} \tag{24}$$

Peng-Fan bound (2001) [3, pages 99–106]:

$$\begin{aligned}
\delta_m^2 &\geq \frac{3MN - 3N^2 + 3MNL - 2ML - ML^2}{3(ML + M - 1)}, \\
&\qquad 0 \leq L \leq L_o \\
\delta_m^2 &\geq \frac{\sqrt{3M} - 2}{\sqrt{3M}}N, \qquad L_o > \sqrt{3/M}N - 1.
\end{aligned} \tag{25}$$

Tang-Fan bound (2001) [57]:

$$\delta_m^2 \geq \frac{ML_o + M - 2N + 1}{(ML_o + M - 1)(2N - 1)}. \tag{26}$$

When $L_o = N$, the new bounds for GQO sequences become normal sequence bounds as follows.

Peng-Fan bound (2002) [14]:

$$\begin{aligned}
&\frac{3(N-1)}{2MN^2 - 3N^2 + M}\delta_a^2 + \frac{3N(M-1)}{2MN^2 - 3N^2 + M}\delta_c^2 \geq 1, \\
&\frac{\sqrt{3}N - \sqrt{M}}{(\sqrt{3M} - 2\sqrt{M})N^2}\delta_a^2 + \frac{\sqrt{3}(N-1)}{(\sqrt{3M} - 2\sqrt{M})N}\delta_c^2 \geq 1, \\
&\left\{1 - \frac{(32 - 3\pi^2)(N^2 - M)^2(2N^2 - \sqrt{2MN^2 - M^2})}{128N^2M(2N^2 - M)\sqrt{2MN^2 - M^2}}\right\}\delta_m^2 \\
&\qquad \geq N - \left\lceil \frac{\pi N}{\sqrt{8M}} \right\rceil, \quad M \leq N^2.
\end{aligned} \tag{27}$$

It should be noted that all the previous known aperiodic bounds for normal spreading sequences can be considered as special cases of the new bounds for generalized quasiorthogonal sequences, and in fact, weaker than the new bounds. These previous known bounds are as follows.

Levenshtein bound (1999) [13]:

$$\begin{aligned}
\delta_m^2 &\geq \frac{3LMN - 3N^2 - M - ML^2}{3(ML - 1)}, \quad 1 \leq L \leq N, \\
\delta_m^2 &\geq N - \left\lceil \frac{\pi N}{\sqrt{8M}} \right\rceil, \quad M \leq N^2.
\end{aligned} \tag{28}$$

Sarwate bound (1979) [11]:

$$\frac{2(N-1)}{(M-1)N^2} \times \delta_a^2 + \frac{2N-1}{N^2} \times \delta_c^2 \geq 1. \tag{29}$$

Welch bound (1974) [9]:

$$\delta_m^2 \geq \frac{(M-1)N^2}{2MN - M - 1}. \tag{30}$$

6. THEORETICAL LIMITS FOR GO/GQO HOPPING SEQUENCES

Early in 1974, Lempel and Greenberger established some bounds on the periodic Hamming correlation of FH sequences for $M = 1$ or 2 [15]. Let $M = q^{k+1}$, where k denotes the maximum number of coincidences between any pair of hopping sequences S, Seay derived a different bound in 1982 [16].

Given a set of FH sequences with family size M and length N over a given frequency slot set F with size q, GQO zone L_o, and $I = \lfloor NM/q \rfloor$, we have the following results for GQO hopping sequences,

$$\begin{aligned}
&qL_oH_a + q(M-1)(L_o+1)H_c \geq N(ML_o + M - q), \\
&L_oH_a + (M-1)(L_o+1)H_c \geq (L_o+1)MN/q - N, \\
&L_oH_a + (M-1)(L_o+1)H_c \\
&\qquad \geq (L_o+1) \times [2I + 1 - (I+1)Iq/MN] - N.
\end{aligned} \tag{31}$$

As a special case, when $H_m = \max\{H_a, H_c\} = 0$, that is, $L_o = Z_o$, we have the following periodic GO hopping bound obtained by Ye and Fan [54],

$$M(Z_o + 1) \leq q, \quad \text{when } N = kZ_o,\ k = 1, 2, \ldots. \tag{32}$$

When $L_o = N - 1$, we have the following normal hopping sequence bound (only one is given here for simplicity) [17],

$$q(N-1)H_a + q(M-1)NH_c \geq N(NM - q). \tag{33}$$

Note that $H_m = \max\{H_a, H_c\}$, we have

$$\begin{aligned}
H_m &\geq \frac{(NM - q)N}{(NM - 1)q}, \\
H_m &\geq \frac{2INM - (I+1)Iq}{(NM - 1)M}.
\end{aligned} \tag{34}$$

In particular, if $M = 1$, then $N = Iq + r$, where $0 \leq r < q$, we have

$$H_a \geq \frac{(N - r)(N + r - q)}{(N - 1)q}. \tag{35}$$

This result was derived firstly by Lempel and Greenberger in 1974 [15].

For any given prime number p and positive integers k, n, $0 \leq k < n$, let $N = p^n - 1$, $q = p^k$. If $0 < M < q$, we have

$$\begin{aligned}
&(p^n - 2)H_a + (M-1)(p^n - 1)H_c \\
&\qquad \geq Mp^{2n-k} - 2Mp^{n-k} - p^n + 2, \\
&H_m \geq \frac{Mp^{2n-k} - 2Mp^{n-k} - p^n + 2}{Mp^n - M - 1}.
\end{aligned} \tag{36}$$

When $M = 1$, we have

$$H_a \geq p^{n-k} - 1, \tag{37}$$

which is also a Lempel-Greenberger bound [15].

Let $k = H_m$, if $M = q^{k+1}$, then

$$k \geq \frac{Nq^k - 1}{Nq^{k+1} - 1} \times N, \tag{38}$$

which is tighter than the following Seay bound [16],

$$k \geq \frac{q^k - 1}{q^{k+1} - 1} \times N. \tag{39}$$

Similarly, one can also investigate aperiodic hopping bounds [17]. However, unlike the normal correlations, the periodic Hamming correlation is generally worse (bigger) than the aperiodic one, therefore, it is normally enough to consider the periodic hopping case.

7. APPLICATIONS OF GO/GQO SPREADING SEQUENCES TO QS -CDMA SYSTEMS

In practice, for a multipath fading channel, the synchronization would be very difficult to achieve between different users, because very accurate timing synchronization at network level must be achieved, which is in general not easy. Further, to hold a perfect orthogonality between different codes at the receiver is a highly challenging task. Traditional CDMA systems employ almost exclusively Walsh-Hadamard or OVSF orthogonal codes, jointly with m-sequences, and Gold/Kasami sequences, and so forth. In these systems, due to the difficulty in timing synchronization and the large crosscorrelation values around the origin, there exists a "near far" effect, such that fast power control has normally to be employed in order to keep a uniform received signal level at the base station. On the other hand, in forward channel all the signals' power must be kept at a uniform level. Since the transmitting power of a user would interfere with others and even itself, if one of the users in the system increases its power unilaterally, all other users power should be simultaneously increased.

In a QS-CDMA system, it is normally assumed that each user experienced an independent delay of τ_k, which obeys $|t'_k| \leq \tau_{\max} = Z_o T_c$, where τ'_k is relative delay of the kth signal, T_C is the chip period, and Z_o is the predefined orthogonal zone. This maximum quasisynchronous access delay $\tau_{\max} = Z_o T_c$ can be achieved in several ways, such as by invoking a global positioning system (GPS) assisted synchronization protocol. If multipath effect exists, however, the following condition should be maintained, that is,

$$\max\{\tau', \tau''\} < \tau_{\max} = Z_o T_c, \tag{40}$$

where τ'_k is relative delay due to quasisynchronous access, and τ''_k is the delay due to multipath transmission, as shown by the received QS-CDMA signal $r(t)$ in a 2-path channel in

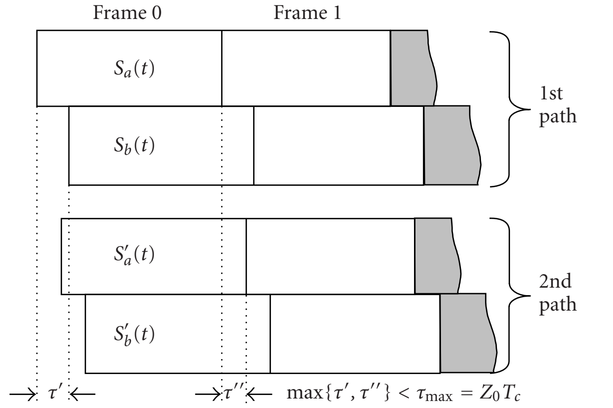

Figure 3: Received OS-CDMA signal $r(t)$ in a 2-path channel, $r(t) = s_a(t) + s_b(t) + s'_a(t) + s'_b(t) + n(t)$.

Figure 3. Therefore, in designing a QS-CDMA system, in order to reduce or eliminate the multiple access interference and multipath interference, it is generally required to design a set of spreading sequences having an orthogonal zone Z_o satisfying (40).

For a typical LAS-CDMA2000 system [19], the key design parameters are frame length: 20 ms, chip rate: 1.2288 Mcps, channel spacing: 1.25 MHz, LA code number: 8, LA "pulse" number/LA code: 16, LS code number/LA "pulse": 32 × 2 ($Z_o = 4$), modulation: 16 QAM (high mobility up to 500 km/h), 32 QAM (medium mobility up to 100 km/h), duplex: 2 × 1.25 MHz frequency-division duplex (FDD) or time-division duplex (TDD), maximum apparent data rate 1634.4 kpbs (high mobility) and 2048 kbps (medium mobility). By excluding the encoding rate and other costs, such as pilot symbols and frame overheads, the spectral efficiency of LAS-CDMA2000 can be obtained as 1.31072 bps/Hz (high mobility) and 1.6384 bps/Hz (medium mobility), which is higher than the spectral efficiency of cdma2000-1x by about 0.6144 bps/Hz in medium mobility environment under the same assumptions. This advantage is due to the employment of the GO sequences, that is, the LA/LS codes.

Another good application example of QS-CDMA by employing GO/GQO spreading sequences is multicarrier and OFDM CDMA which is generally believed to be a promising technology due to its inherent bandwidth efficiency and frequency diversity in wireless environment [60, 62]. OFDM can also overcome multipath problem by using cyclic prefix, added to each OFDM symbol, which insures the orthogonality between the main path component and the multipath components, provided that the length of the cyclic-prefix is larger than the maximum multipath delay. By employing GO/GQO spreading sequences appropriately in time and frequency domain, one can eliminate or reduce interference even further due to the inherent multipath interference immunity possessed by the GO/GQO codes [25]. Different from Rake receiver, it would be more advantageous to have all the multipath components combined with the main one by an orthogonal multipath combiner. Here, the key to

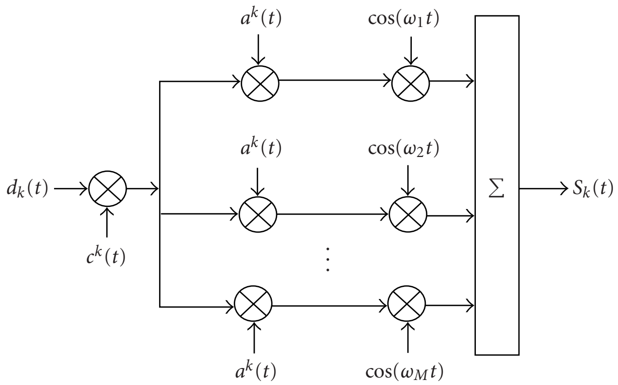

FIGURE 4: QS-MC-CDMA transmitter.

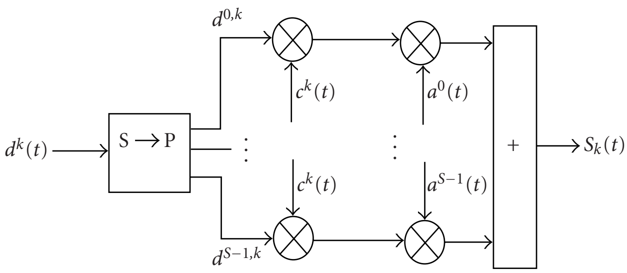

FIGURE 5: Two-level spreading QS-CDMA transmitter.

a proper system operation is how to keep the orthogonality between subcarriers of MC/OFDM signals. For the multicarrier CDMA, instead of the time-domain correlation which is not a proper interference measure, one may use spectral correlation, together with crest factor and the dynamic range of the corresponding multicarrier waveforms [61]. Therefore, in order to make full use of the nice time-domain correlation properties of GO/GQO, one may consider the hybrid time/frequency spreading multicarrier CDMA systems, as shown in Figure 4 (only transmitter is drawn for simplicity), where $c^k(t)$ of size M_1, and $a^k(t)$ of size M_2 are the time and frequency domain spreading sequences, respectively, where $c^k(t)$ is chosen from a set of GO/GQO sequences, and $a^k(t)$ is chosen from a set of sequences with good spectral correlation and crest factor properties, such as multilevel Huffman sequences, Zadoff-Chu sequences, Legendre sequences, or another set of GO/GQ sequences. Here, it is clear that the total number of users supported would be $M = M_1M_2$.

Figure 5 describes a two-level scheme [31], where concatenated WH/m-sequences $c^k(t)$ are used as the first-level (FL) codes to provide the user and cell division, and a class of GO sequences $a^k(t)$ are employed as the second-level (SL) sequences to distinguish channels belonging to the same user. The data bit of the sth channel $d^{s,k}$ is first spread by the FL code $c^k(t)$ to L_1 chips with the chip duration of $T_1 = T_b/L_1$, where T_b is the bit duration. Then each FL chip $d^{s,k}c_n^k$, $n = 0, 1, \ldots, L_1 - 1$, is further spread by the SL code $a'^{(s)}$ of length L_2 and the resultant chip duration $T_c = T_b/(L_1 \times L_2)$. It can be shown that, compared with that of the conventional single-level spreading system, the two-level QS-CDMA system employing GO sequences and partial interference cancellation exhibits better system performance.

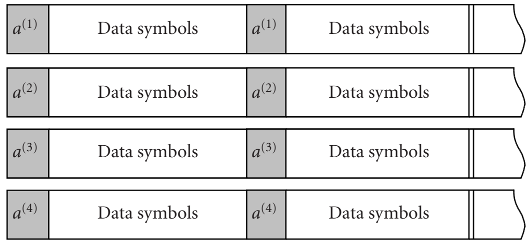

FIGURE 6: MIMO channel estimation with GO sequences, $a^{(1)}$, $a^{(2)}$, $a^{(3)}$, and $a^{(4)}$.

In order to accurately and efficiently perform channel estimation in single- and multiple-antenna communication systems, single GO sequence [34, 35] and set of GO/GQO sequences [36, 37] can be used. In particular, for a multiple-input multiple-output (MIMO) channel estimation system shown in Figure 6, if the training sequence $a^{(i)}$, $i = 1, 2, 3, 4$, allocated to each antenna is not only orthogonal to its shifts within Z_o taps but also orthogonal to the training sequences in other antennas and their shifts within Z_o taps, then the mutual interference among different antennas will be kept minimum, which makes the GO sequence set an excellent candidate. In fact, for MIMO channel estimation, it is shown in [36, 37] that the use of the GO sequences, *or* (P, V, M) sequences as named by Yang and Wu [36], can effectively reduce the mutual interference among different transmitting antennas, compared with the pseudorandom binary sequences and arbitrarily chosen sequences.

8. CONCLUDING REMARKS

It is clear that the new GO/GQO concepts have opened a new direction for the spreading sequence design, and a potential promising application for the new GO/GQO spreading sequences is the quasisynchronous CDMA systems, in particular the quasisynchronous multicarrier CDMA systems and LAS-CDMA systems. In addition, other suitable application areas are still under investigation by many researchers.

It is noted in this paper that the new theoretical bounds for the GO/GQO sequences include the bounds for conventional spreading sequences as special cases. Furthermore, stronger bounds can be obtained for conventional sequences in certain cases. However, for specific GO/GQO sequence design, such as ternary LA/LS codes and binary GO/GQO sequences, there are still many theoretical limit issues that need further attention and investigation. Besides, the relationship between the GO/GQO theoretical limits and other research fields such as error correction coding, combinatorics, algebraic theory, and so forth is not yet clear.

As for the task of GO/GQO sequence design, it is by no means completed. Instead, it is still a long way to construct the desirable optimal GO/GQO spreading sequences satisfying the theoretical bounds for different lengths, such as binary and nonbinary GO/GQO sequences, two- or higher-dimensional GO/GQO sequences, GO/GQO hopping sequences, and so on. Even for the construction of single binary sequence case ($M = 1$), it is still an open problem for finding a GO sequence with $Z_o = N/2$ for arbitrary length N, since only partial solutions have been found for short and specific lengths.

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