

Spreading Sequence Design for Multiple-Cell Synchronous DS-CDMA Systems under Total Weighted Squared Correlation Criterion

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An algorithm for designing spreading sequences for an overloaded multicellular synchronous DS-CDMA system on uplink is introduced. The criterion used to measure the optimality of the design is the total weighted square correlation (TWSC) assuming the channel state information known perfectly at both transmitter and receiver. By using this algorithm it is possible to obtain orthogonal generalized WBE sequences sets for any processing gain. The bandwidth of initial generalized WBE signals of each cell is preserved in the extended signal space associated to multicellular system. Mathematical formalism is illustrated by selected numerical examples.

Keywords and phrases: total (weighted) squared correlation, Welch bound equality sequences, CDMA codeword optimization, and multiple-access design.

1. INTRODUCTION

Due to the emerging demand on new multimedia applications, next generation wireless systems are expected to support higher data rates. This goal is particularly challenging for the systems that are power, bandwidth, and complexity limited. Many users typically share wireless channels, so that multiple-access interference (MAI) is one of the main problems that information transmission through such channels faces. CDMA systems are interference limited and several techniques have been proposed for dealing with the interference either to the transmitter or to the receiver side.

The recent literature on designing codes for uplink overloaded CDMA systems can be divided into two general categories: those that assume random signature sequences and those that target the optimization of given criterion. Some of the optimization criteria in the literature are the signal-to-interference ratio [1, 2, 3], the required signal bandwidth [4, 5, 6], the sum capacity [7, 8, 9], the user capacity [10, 11], the total squared correlation (TSC) [12, 13, 14, 15, 16, 17, 18, 19, 20], the generalized total squared correlation (GTSC) [21], total weighted square correlation (TWSC) [22, 23, 24, 25, 26, 27], or the extended total (weighted) squared correlation (ET(W)SC) [28, 29, 30].

We consider multicellular DS-CDMA systems and focus on the uplink by considering the overloaded DS-CDMA

systems where the number of users in each cell is greater than the processing gain. While the extension of same optimal results from one cell to multiple cells is straightforward [31, 32, 33, 34, 35], the design of spreading sequences under TWSC criterion is a more challenging task due to the amount of interference constraints that are considerably stricter than in the single-cell case. In the case of multicellular systems, we assume the cooperation of multiple-base stations by *sharing the same extended signal space* and requiring each base station to have its own power constraint.

The algorithms given in [1, 8, 9, 10] may fail to generate valid sequences for overloaded multicellular DS-CDMA systems. For example, the so-called *T*-transform method presented [36] and used in [8] generates sequences such that the cells are not orthogonal to each other and cannot be used for structured block matrices. The method presented in [1, 2] does not guarantee the preserving of the bandwidth of initial generalized Welch bound equality (WBE) sequences in the multicellular context. Extending the previous algorithms to multiple cells is not a trivial task due to the amount of the intercell interference.

By solving an inverse eigenvalue problem of the structured block matrices, an algorithm that overcomes the limitations of previous algorithms is derived. It is based on majorization theory and it allows obtaining generalized WBE sequences for any signal space dimensionality of each cell

under TWSC criterion. The proposed algorithm has the same performance with respect to the cost of computation when it is particularized to single-cell case and compared with the previous algorithms.

This paper is organized as follows. In Section 2, we derive the multicellular model of DS-CDMA systems. In Section 3, based on single-cell model, the TWSC criterion in the multicellular scenario is introduced. The main features of the proposed algorithm are presented in Section 4. Numerical examples are given in Section 5 and the conclusions are drawn in Section 6.

2. SYSTEM MODEL

Consider the uplink of a multicell synchronous CDMA system. Denote by M , K_i , and N_i the number of cells, the number of independent active users for each cell i and the processing gain, respectively. Since we are dealing with deterministic signature sequences, K_i and N_i will be fixed throughout this paper.

In the presence of a flat fading channel and assuming the noise additive and white with a Gaussian-type distribution having zero mean and power spectral density σ^2 , the received signal vector for the i th cell considering one symbol interval is

$$\mathbf{r}_i = \mathbf{S}_i \mathbf{P}_i^{1/2} \mathbf{H}_i \mathbf{b}_i + \mathbf{n}, \quad (1)$$

where $\mathbf{S}_i = [\mathbf{s}_{i1}, \mathbf{s}_{i2}, \dots, \mathbf{s}_{iK_i}]$ is the spreading signature sequence matrix (with signature waveform as columns), \mathbf{P}_i is the power matrix, and \mathbf{H}_i is the channel gain matrix associated with each cell. Specifically we have

$$\begin{aligned} \mathbf{P}_i^{1/2} &= \text{diag}(\sqrt{P_{i1}}, \dots, \sqrt{P_{iK_i}}),^1 \\ \mathbf{H}_i &= \text{diag}(h_{i1}, \dots, h_{iK_i}). \end{aligned} \quad (2)$$

We use snapshot analysis when an immobile system is assumed, that is, in each cell i , $1 \leq i \leq M$, the number of mobiles K_i and path gain matrices \mathbf{H}_i of mobile are fixed.

The information symbol vector $\mathbf{b}_i = [b_{i1}, b_{i2}, \dots, b_{iK_i}]^T$ (by T we mean transpose) has the components with zero mean and variance $E[b_{ik_i}^2] = 1$. The noise vector \mathbf{n} with zero mean is the projection of the additive noise onto the basis of the N_i -dimensional signal space and $E[\mathbf{nn}^T] = \sigma^2 \mathbf{I}_{N_i}$, where \mathbf{I}_{N_i} denotes the $N_i \times N_i$ identity matrix.

In order to have a suitable multicellular model for TWSC (introduced in the next section where the user signatures across cells are to be designed) we need a model similar to (1) by allowing users with different overloading ($K_i > N_i$) in each cell. In this context, the presented model of uplink

multicellular synchronous CDMA systems is more general² than (1), where the long received vector (of length $\sum_{i=1}^M N_i$) is given by

$$\mathbf{r} = [\mathbf{r}_1, \dots, \mathbf{r}_i, \dots, \mathbf{r}_M]^T = \mathbf{S} \mathbf{P}^{1/2} \mathbf{H} \mathbf{b} + \mathbf{n}. \quad (3)$$

The composite spreading signature matrix \mathbf{S} (associated with all base stations) is of the form

$$\mathbf{S} = \mathbf{S}_1 \oplus \mathbf{S}_2 \oplus \dots \oplus \mathbf{S}_M \quad (4)$$

based on the definition of direct sum of two matrices \mathbf{A} and \mathbf{B} , that is, $\mathbf{A} \oplus \mathbf{B} = \begin{pmatrix} \mathbf{A} & \mathbf{0} \\ \mathbf{0} & \mathbf{B} \end{pmatrix}$ [37].

By using (4), the signature waveforms of each cell lie in the orthogonal subspaces (of the extended signal space with dimensionality $\sum_{i=1}^M K_i$), as they are required by the optimality of TWSC criterion. The composite power matrix $\mathbf{P}^{1/2}$ associated with the uplink multicellular system is

$$\mathbf{P}^{1/2} = \text{diag}(\sqrt{P_{11}}, \dots, \sqrt{P_{1K_1}}, \dots, \sqrt{P_{i1}}, \dots, \sqrt{P_{iK_i}}, \dots, \sqrt{P_{M1}}, \dots, \sqrt{P_{MK_M}}), \quad (5)$$

where on the main diagonal are given the power constraints in each cell. With the model (3), the information bits associated with each cell are included in the following composite vector of whole system $\mathbf{b} = [b_{11}, \dots, b_{1K_1}, b_{21}, \dots, b_{2K_2}, \dots, b_{M1}, \dots, b_{MK_M}]$ and the composite channel matrix \mathbf{H}^3 is given by

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_1 & \mathbf{H}_{K_1 \times K_2} & \dots & \mathbf{H}_{K_1 \times K_M} \\ \mathbf{H}_{K_2 \times K_1} & \mathbf{H}_2 & \dots & \mathbf{H}_{K_2 \times K_M} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{H}_{K_M \times K_1} & \mathbf{H}_{K_M \times K_2} & \dots & \mathbf{H}_M \end{bmatrix}. \quad (6)$$

3. TOTAL WEIGHTED SQUARE CORRELATION CRITERION

3.1. Review of single-cell results

Definition 1. Given a signature sequence set $\mathbf{S} = [\mathbf{s}_1, \mathbf{s}_2, \dots, \mathbf{s}_k, \dots, \mathbf{s}_K]$, then the TSC of \mathbf{S} is the Frobenius

²When model (3) is particularized to noncollaborative scenario (1), it allows analyzing the case where in each cell the users have different spreading gain N_i as in future wireless networks. The extension of the previous results by considering the spreading matrix of the form $\mathbf{S} = [\mathbf{S}_1, \mathbf{S}_2, \dots, \mathbf{S}_M]$ is possible only in the case of equal spreading gain N for each cell. This brings us back to single-cell model in N -dimensional signal space.

³The generalized model of (1) must involve the entire channel gains $h(k_i, j)$, where $1 \leq i \leq M$, $1 \leq j \leq M$, and $1 \leq k_i \leq K_i$. By using model (3), the intercellular interference is treated no different than the intracellular interference and the model under consideration does not involve multipath signal propagation. Extension of the results of this paper to the generalized model, which accounts for the geography of the cell, is an open problem and it is left for future research. I thank the anonymous reviewer for this perspective.

¹We will denote the diagonal matrix whose main diagonal entries are the same as those of the matrix \mathbf{A} as $\text{diag}(\mathbf{A})$ and the diagonal matrix whose diagonal entries are formed from vector $\mathbf{a} = [a_1, \dots, a_K]$ as $\text{diag}(\mathbf{a})$.

norm of the Gram matrix associated to \mathbf{S} ,

$$\text{TSC}(\mathbf{S}) = \|\mathbf{G}\|_F = \sum_{i=1}^K \sum_{j=1}^K |\langle \mathbf{s}_i, \mathbf{s}_j \rangle|^2 = \sum_{i=1}^K \lambda_i^2, \quad (7)$$

where λ_i are the eigenvalues of the Gram matrix $\mathbf{G} = \mathbf{S}^* \mathbf{S}$. (* denotes the complex conjugate transpose operation.)

Definition 2. Given a signature sequence set

$$\mathbf{S} = [\mathbf{s}_1, \mathbf{s}_2, \dots, \mathbf{s}_k, \dots, \mathbf{s}_K] \quad (8)$$

and the power matrix $\mathbf{P} = \text{diag}(\mathbf{p})$ defined by (1), then the TWSC is the weighted Frobenius norm of Gram matrix associated to \mathbf{S}

$$\begin{aligned} \text{TWSC}(\mathbf{S}) &= \|\mathbf{G}\|_W = \sum_{i=1}^K \sum_{j=1}^K p_i p_j |\langle \mathbf{s}_i, \mathbf{s}_j \rangle|^2 \\ &= \|\mathbf{P}^{1/2} \mathbf{G} \mathbf{P}^{1/2}\|_F = \sum_{i=1}^K \mu_i^2, \end{aligned} \quad (9)$$

where μ_i are the eigenvalues of the matrix $\mathbf{P}^{1/2} \mathbf{G} \mathbf{P}^{1/2}$.

TSC measure is a particular case of TWSC criterion for the same particular received power of all users. Real sequences, which meet the lower bound of TSC and TWSC, are called WBE and generalized WBE sequences, respectively. These sequences satisfy the properties

- (a) $\mathbf{S} \mathbf{S}^T = (K/N) \mathbf{I}_N$,
- (b) $\mathbf{S} \mathbf{P} \mathbf{S}^T = (\sum_{i=1}^K p_i/N) \mathbf{I}_N$.

Minimizing TSC in a single cell is equivalent to maximizing sum capacity for real WBE sequences [2]. This is no longer true for binary sequences as it was pointed out recently in [19].⁴

3.2. Uplink multi-cell DS-CDMA

Using the same Gram matrix approach as in the single-cell case [21], we obtain for composite spreading Gram matrix the following expression:

$$\mathbf{G} = \mathbf{S}^* \mathbf{S} = \mathbf{S}_1^* \mathbf{S}_1 \oplus \mathbf{S}_2^* \mathbf{S}_2 \oplus \dots \oplus \mathbf{S}_M^* \mathbf{S}_M. \quad (10)$$

If we introduce the power for each base station \mathbf{P}_i and channel gain matrix \mathbf{H}_i , then the weighted Gram matrix \mathbf{W} associated to the multicellular system satisfies the following condition:

$$\begin{aligned} \text{diag}(\mathbf{W}) &= \text{diag}(\mathbf{H}_1^* \mathbf{P}_1^{1/2} \mathbf{S}_1^* \mathbf{S}_1 \mathbf{P}_1^{1/2} \mathbf{H}_1, \dots, \mathbf{H}_i^* \mathbf{P}_i^{1/2} \mathbf{S}_i^* \mathbf{S}_i \mathbf{P}_i^{1/2} \mathbf{H}_i, \dots, \\ &\quad \mathbf{H}_M^* \mathbf{P}_M^{1/2} \mathbf{S}_M^* \mathbf{S}_M \mathbf{P}_M^{1/2} \mathbf{H}_M). \end{aligned} \quad (11)$$

Definition 3. Given for each base station i the signature sequence matrix \mathbf{S}_i , the power matrix \mathbf{P}_i , and channel gain matrix \mathbf{H}_i as in model (3), then the TWSC of uplink multicellular DS-CDMA system in the presence of flat fading is the Frobenius norm of the weighted Gram matrix (10)

$$\begin{aligned} \text{TWSC}(\mathbf{S}) &= \|\mathbf{W}\|_F \\ &= \|\mathbf{H}^* \mathbf{P}^{1/2} \mathbf{G} \mathbf{P}^{1/2} \mathbf{H}\|_F \\ &= \sum_{i=1}^M \sum_{j=1}^{K_i} \sum_{k=1}^{K_i} p_{ij} p_{ik} |\langle \mathbf{s}_{ij}, \mathbf{s}_{ik} \rangle|^2 \langle \mathbf{H}_i^*, \mathbf{H}_j \rangle \\ &= \sum_{i=1}^M \sum_{j=1}^{K_i} \mu_{ij}^2, \end{aligned} \quad (12)$$

where μ_{ij} are the eigenvalues of the matrix $\mathbf{H}^* \mathbf{P}^{1/2} \mathbf{G} \mathbf{P}^{1/2} \mathbf{H}$. The inner product $\langle \mathbf{H}_i^*, \mathbf{H}_j \rangle$ is defined in the Frobenius sense [37].

TWSC given by (12) is minimized when for each cell the spreading sequences are real generalized WBE sequences. The proof is following the same lines as in single-cell case [2, 15] due to orthogonal nature of matrix (4). Minimizing TWSC is not equivalently maximizing sum capacity [38]. However, we are interested in designing spreading sequences, which meet minimum TWSC since this criterion characterizes total amount of interferences in multibase scenario [38]. An algorithm to construct such sequences is given in the next section.

4. THE ALGORITHM

In order to design spreading sequences for each cell, it is necessary to use an algorithm for construction of the weighted Gram matrix given by (11) with imposed eigenvalues and a structured block pattern. Only the case related to inverse eigenvalue problem for a single cell was solved in [8]. The problem of constructing a complex symmetric matrix with prescribed singular values and eigenvalues with specified entries on main diagonal and a given structured block pattern is under research right now. For a single cell, similar algorithms obtaining WBE or generalized WBE sequences were developed in [1, 8, 9, 10, 13, 17] for the real case and in [18] for binary WBE sequences case, respectively.

The extension of the previous algorithms to multiple cells is not a trivial task due to inherent bandwidth extension of spreading signature in multidimensional signal space corresponding to composite signature matrix given by (4). We need to preserve the bandwidth of generalized WBE sequences for each cell in the context of cooperative multibase DS-CDMA systems and, in the same time, we need to control all the eigenvalues of the composite weighted Gram matrix given by (11). The application of T -transform as in [8, 36] in extended signal space does not guarantee the pattern required for composite Gram matrix given in (10) as proven

⁴Remarkably for underloaded binary signature sets in [20] besides TSC and sum capacity were analyzed other metrics such as maximum squared correlation (MSC) and total asymptotic efficiency (TAE). For overloaded multicellular CDMA systems TAE(S) = 0 since the Gram matrix $\mathbf{S}^* \mathbf{S}$ is rank deficient.

by the numerical Example 2 in Section 5.⁵ We need the following theorem.

Theorem 1. *Given the vector \mathbf{x} majorized by the vector \mathbf{y} (of length n), then there exists an orthogonal matrix \mathbf{U} such that the diagonal entries of $\mathbf{U}^T \text{diag}(\mathbf{y})\mathbf{U}$ are the components of \mathbf{x} and \mathbf{U} can be written as the product of at most $n - 1$ orthogonal rotations.*

Proof. We use induction on n . Without loss of generality we can assume that the eigenvalues λ_i and diagonal elements a_i are arranged in decreased order $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_n$ and $a_1 \geq a_2 \geq \dots \geq a_n$. \mathbf{a} is said to be majorized by $\boldsymbol{\lambda}$, denoted by $\mathbf{a} < \boldsymbol{\lambda}$ if [36]

$$\sum_{i=1}^k a_i \leq \sum_{i=1}^k \lambda_i, \quad k = 1, 2, \dots, n-1, \quad (13a)$$

$$\sum_{i=1}^n a_i = \sum_{i=1}^n \lambda_i. \quad (13b)$$

(i) Verify for $n = 2$. Condition (13) is equivalent to $a_1 \leq \lambda_1$, $a_1 + a_2 = \lambda_1 + \lambda_2$ and becomes $\lambda_1 \geq a_1 \geq a_2 \geq \lambda_2$ since

$$a_1 = \lambda_1 + \lambda_2 - a_2 \geq \underbrace{\lambda_1 + \lambda_2 - a_1}_{a_2} \geq \lambda_2. \quad (14)$$

If $\lambda_1 = \lambda_2$, the proof is trivial. Otherwise $\lambda_1 > \lambda_2$ and it follows from (13) that $\lambda_1 \geq a_1 \geq a_2 > \lambda_2$. For the matrix $\text{diag}(\lambda_1, \lambda_2)$ there exists a rotation matrix \mathbf{V} such that the (1,1) and (2,2) elements of $\mathbf{V}^T \text{diag}(\lambda_1, \lambda_2)\mathbf{V}$ are a_1 and $\lambda_1 + \lambda_2 - a_1$, respectively [27]. Hence the theorem holds for $n = 2$.

(ii) Now suppose that the theorem holds for $n \geq 2$ and we will prove for $n + 1$. The main idea is to use two block orthogonal rotation matrices of order $n + 1$,

$$\mathbf{U}_1 = \begin{bmatrix} \mathbf{I}_{n-1} & \mathbf{0}_{(n-1) \times 2} \\ \mathbf{0}_{2 \times (n-1)} & \mathbf{V} \end{bmatrix}, \quad \mathbf{U}_2 = \begin{bmatrix} \mathbf{U}_n & \mathbf{0}_{n \times 1} \\ \mathbf{0}_{1 \times n} & 1 \end{bmatrix}. \quad (15)$$

The required orthogonal matrix is $\mathbf{U} = \mathbf{U}_2\mathbf{U}_1$ which can be written as the product of $(n-1)+1 = n$ orthogonal rotations.

Let $\boldsymbol{\Lambda} = \text{diag}(\lambda_1, \lambda_2, \dots, \lambda_n, \lambda_{n+1})$ be a diagonal matrix of order $n + 1$. By Schur theorem [36, 37], there exists a Hermitian matrix with the eigenvalues $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_n \geq \lambda_{n+1}$ and diagonal elements $a_1 \geq a_2 \geq \dots \geq a_n \geq a_{n+1}$ satisfying (13). This implies also $\lambda_1 \geq a_1 \geq a_{n+1} \geq \lambda_{n+1}$ and we can find the least integer $k > 1$ such that $\lambda_k \geq a_{n+1} \geq \lambda_{k+1}$. Taking into account the eigenvalue λ_k , we can permute the elements of $\boldsymbol{\Lambda}$ to obtain the matrix $\boldsymbol{\Lambda}_1 =$

$\text{diag}(\lambda_1, \lambda_2, \dots, \lambda_{k-1}, \lambda_{k+1}, \dots, \lambda_n, \lambda_k, \lambda_{n+1})$. Using the matrix \mathbf{U}_1 , we obtain

$$\mathbf{U}_1^T \boldsymbol{\Lambda}_1 \mathbf{U}_1 = \begin{bmatrix} \boldsymbol{\Lambda}_2 & \mathbf{y} \\ \mathbf{y}^T & a_{n+1} \end{bmatrix}, \quad (16)$$

where $\boldsymbol{\Lambda}_2 = \text{diag}(\lambda_1, \lambda_2, \dots, \lambda_{k-1}, \lambda_{k+1}, \dots, \lambda_n, \lambda_k + \lambda_{n+1} - a_{n+1})$ and \mathbf{y} is an appropriate column vector. Let $\boldsymbol{\lambda}_2 = (\lambda_1, \lambda_2, \dots, \lambda_{k-1}, \lambda_{k+1}, \dots, \lambda_n, \lambda_k + \lambda_{n+1} - a_{n+1})$ and $\mathbf{a}_2 = (a_1, a_2, \dots, a_n)$, respectively. Now, all we need is to prove that $\mathbf{a}_2 < \boldsymbol{\lambda}_2$. If this is true under the induction assumption, there exists an orthogonal matrix \mathbf{U}_n (included in \mathbf{U}_2) such that the diagonal elements of $\mathbf{U}_n^T \boldsymbol{\Lambda}_2 \mathbf{U}_n$ are precisely (a_1, a_2, \dots, a_n) . In order to verify that $\mathbf{a}_2 < \boldsymbol{\lambda}_2$, we need to check conditions (13a) and (13b) again. For condition (13a), it is enough to show that

$$\sum_{i=k}^j a_i \leq \sum_{i=k+1}^j \lambda_i + (\lambda_k + \lambda_{n+1} - a_{n+1}), \quad j = k, k+1, \dots, n-1. \quad (17)$$

By using (13b) for the matrix of order $n + 1$, we have

$$\sum_{i=1}^n a_i = \sum_{i=1}^{n+1} a_i - a_{n+1} = \sum_{i=1}^{n+1} \lambda_i - a_{n+1} = \sum_{i=1}^n \lambda_i + \lambda_{n+1} - a_{n+1} \quad (18)$$

which can be written using (13a) for the matrix of order $n + 1$ as

$$\sum_{i=1}^j a_i \leq \sum_{i=1}^j \lambda_i + \lambda_{n+1} - a_{n+1}, \quad j = 1, 2, \dots, n, \quad (19)$$

or equivalently

$$\sum_{i=1}^{k-1} a_i + \sum_{i=k}^j a_i \leq \sum_{i=1}^{k-1} \lambda_i + \sum_{i=k+1}^j \lambda_i + (\lambda_k + \lambda_{n+1} - a_{n+1}). \quad (20)$$

Since already $\sum_{i=1}^{k-1} a_i \leq \sum_{i=1}^{k-1} \lambda_i$, the desired results follows. Condition (13b) is easy to verify,

$$\sum_{i=1}^{n+1} \lambda_i - a_{n+1} = \sum_{i=1}^{n+1} a_i - a_{n+1} = \sum_{i=1}^n a_i, \quad (21)$$

which completes the proof of the theorem for $n + 1$. \square

By sharing the same expanded signal space, targeting the optimization of TWSC (12), running offline the proposed algorithm at each user in multicellular system, then (in contrast to single-cell case) a single orthogonal matrix for all users is necessary and the next algorithm provides it.

Algorithm

Input

K_i , N_i , and $\mathbf{x} = (\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_M)$ and $\mathbf{y} = (\mathbf{y}_1, \mathbf{y}_2, \dots, \mathbf{y}_M)$ such that \mathbf{x}_i is majorized by \mathbf{y}_i , $\text{diag}(\mathbf{P})$, and $\text{eig}(\mathbf{H}^* \mathbf{P}^{1/2} \mathbf{G} \mathbf{P}^{1/2} \mathbf{H})$.

⁵Due to the orthogonal nature of the spreading matrix \mathbf{S} (4) or Gram matrix \mathbf{G} (10) one might think that the optimization of the spreading codes in such a case results in the optimization of K_i independent spreading codes for M cells and the problem can be simplified to spreading code optimization for a single-cell case. This is true only if assuming noncollaborative scenario. T -transform works fine in this case and Theorem 1 can independently provide an orthogonal matrix for each base station.

Output

\mathbf{G} matrix is such that $\text{diag}(\mathbf{G}) = (\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_M)$ and $\text{eig}(\mathbf{G}) = (\mathbf{y}_1, \mathbf{y}_2, \dots, \mathbf{y}_M)$ and \mathbf{W} is matrix such that $\text{eig}(\mathbf{W}) = \text{eig}(\mathbf{H}^* \mathbf{P}^{1/2} \mathbf{G} \mathbf{P}^{1/2} \mathbf{H})$.

Update

- (1) Use Theorem 1 to construct matrix \mathbf{U} .
- (2) For each i check \mathbf{S}_i such that $\text{trace}(\mathbf{S}_i^T \mathbf{S}_i) \leq \text{trace}(\mathbf{P}_i)$.
- (3) Construct \mathbf{W} such that $\text{trace}(\mathbf{H}^* \mathbf{P}^{1/2} \mathbf{G} \mathbf{P}^{1/2} \mathbf{H}) \leq \text{trace}(\mathbf{P})$.

The algorithm is convergent in $K - 1$ steps for one cell with K users and the total computational cost is $O(K^2)$ as in [8]. It is easy to check that for the model (3) the algorithm is convergent in $\sum_{i=1}^M K_i - M$ steps and the complexity is $O(\sum_{i=1}^M K_i^2)$.

5. NUMERICAL RESULTS

Example 1. In this experiment, we will consider unequal power design with two oversized users [8] in the first cell. Unequal power and no oversized users are assumed in the second cell. The extended signal space is of dimension 5. The weighted composite Gram matrix is of order 7. The input is $K_1 = 4, N_1 = 3, K_2 = 3, N_2 = 2, \mathbf{x} = (\mathbf{x}_1, \mathbf{x}_2), \mathbf{y} = (\mathbf{y}_1, \mathbf{y}_2), \mathbf{x}_1 = (1, 1, 1, 1), \mathbf{y}_1 = (4/3, 4/3, 4/3, 0), \mathbf{x}_2 = (1, 1, 1), \mathbf{y}_2 = (3/2, 3/2, 0), \text{diag}(\mathbf{P}_1) = (8, 7, 1, 1), \text{eig}(\mathbf{S}_1 \mathbf{P}_1 \mathbf{S}_1^T) = (9, 7, 1, 0), \text{diag}(\mathbf{P}_2) = (1.2, 1.1, 1), \text{eig}(\mathbf{S}_2 \mathbf{P}_2 \mathbf{S}_2^T) = (2.2, 1.1, 0), \text{diag}(\mathbf{H}_1) = (-0.2517 + 0.7565i, 1.7573 - 0.6483i, 0.1695 - 0.3972i, -0.7809 - 0.3602i), \text{diag}(\mathbf{H}_2) = (0.8983 + 0.5839i, -1.6711 - 0.1807i, 0.1172 - 0.1628i)$.

Running the *algorithm* (offline), the generalized WBE spreading sequences for each cell (before including channel matrix gain \mathbf{H}_1 and \mathbf{H}_2) are obtained as

$$\mathbf{S}_{W1}(4, 3) = \begin{bmatrix} 2.8284 & 0 & 0 & 1.0000 \\ 0 & 2.6458 & 0 & 0 \\ 0 & 0 & 1.0000 & 0 \end{bmatrix},$$

$$\mathbf{S}_{W2}(3, 2) = \begin{bmatrix} 1.0954 & 0 & 1.0000 \\ 0 & 1.0488 & 0 \end{bmatrix},$$

and the weighted Gram matrix obtained before to include the channel matrix gain is

$\mathbf{W}(7, 7)$

$$= \begin{bmatrix} 8.0000 & 0.0000 & 0.0000 & 2.8284 & 0 & 0 & 0 \\ 0.0000 & 7.0000 & 0.0000 & 0.0000 & 0 & 0 & 0 \\ 0.0000 & 0.0000 & 1.0000 & 0.0000 & 0 & 0 & 0 \\ 2.8284 & 0.0000 & 0.0000 & 1.0000 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.2000 & 0.0000 & 1.0954 \\ 0 & 0 & 0 & 0 & 0.0000 & 1.0000 & 0.5000 \\ 0 & 0 & 0 & 0 & 1.0954 & 0.0000 & 1.0000 \end{bmatrix}. \quad (22)$$

After including channel matrix gain \mathbf{H}_1 and \mathbf{H}_2 , the generalized WBE sequences are

$$\mathbf{S}_{W1}(4, 3) = \begin{bmatrix} 4.9557 & 0 & 0 & 0.4319 \\ 0 & 2.2550 & 0 & 0 \\ 0 & 0 & 0.8599 & 0 \end{bmatrix},$$

$$\mathbf{S}_{W2}(3, 2) = \begin{bmatrix} 1.6807 & 0 & 0.2014 \\ 0 & 1.1737 & 0 \end{bmatrix}.$$

In collaborative scenario, the first base station will choose the spreading codes corresponding to weighted Gram matrix of order 5 (the corresponding signal space has dimension 4), while the second base station will choose the spreading codes corresponding to weighted Gram matrix of order 3 (the signal space is of dimension 2). The corresponding weighted Gram matrix is given below. We can check that $\text{TWSC}(\mathbf{W}) = \text{TWSC}(\mathbf{S}_{W1}) + \text{TWSC}(\mathbf{S}_{W2})$,

$\mathbf{W}(7, 7)$

$$= \begin{bmatrix} 24.5588 & 0.0000 & 0.0000 & 2.1401 & 0 & 0 & 0 \\ 0.0000 & 5.0852 & 0.0000 & 0.0000 & 0 & 0 & 0 \\ 0.0000 & 0.0000 & 0.7395 & 0.0000 & 0 & 0 & 0 \\ 2.1401 & 0.0000 & 0.0000 & 0.1865 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2.8252 & 0.0000 & 0.3338 \\ 0 & 0 & 0 & 0 & 0.0000 & 1.3775 & 0.0000 \\ 0 & 0 & 0 & 0 & 0.3338 & 0.0000 & 0.0802 \end{bmatrix}. \quad (23)$$

Example 2. In this experiment, we choose $K_1 = 5, N_1 = 4, K_2 = 5, N_2 = 3$ (equal power case). We select the eigenvalues of Gram matrix $\text{eig}(\mathbf{G}) = (35/12, 35/12, 35/12, 5/4, 0, 0, 0, 0, 0)$ in decreased order, as they are required to apply T -transform [8, 9, 10]. The obtained WBE spreading sequences are given below. The bandwidth of these sequences is represented by signal space dimension, that is, $N_1 = 4$ and $N_2 = 3$, respectively.

$$\mathbf{S}_1(5, 4) = \begin{bmatrix} 0.7906 & 0 & 0 & 0 & 0.7906 \\ -0.4564 & 0.9129 & 0 & 0 & 0.4564 \\ -0.3227 & -0.3227 & 0.9682 & 0 & 0.3227 \\ -0.2500 & -0.2500 & -0.2500 & 1.0000 & 0.2500 \end{bmatrix},$$

$$\mathbf{S}_2(5, 3) = \begin{bmatrix} 0.9129 & 0 & 0 & 0 & 0.9129 \\ -0.3727 & 0.7454 & 0 & 0.9129 & 0.3727 \\ -0.1667 & -0.6667 & 1.0000 & 0.4082 & 0.1667 \end{bmatrix}.$$

In the extended signal space (of dimension 10), the bandwidth of WBE spreading sequences is not preserved. Nine out of ten sequences occupy all signal space dimensions. It is easy to check that $\text{TSC}(\mathbf{G}) > \text{TSC}(\mathbf{S}_1) + \text{TSC}(\mathbf{S}_2)$. The equality holds only for binary sequences [7, 18],

$$\mathbf{G} = \begin{bmatrix} 1.0000 & -0.2500 & -0.2500 & -0.2500 & 0.2500 & 0.9456 & -0.1250 & -0.3227 & -0.5484 & 0.4978 \\ -0.2500 & 1.0000 & -0.2500 & -0.2500 & 0.2500 & -0.2864 & 0.8956 & -0.3227 & 0.7016 & 0.2864 \\ -0.2500 & -0.2500 & 1.0000 & -0.2500 & 0.2500 & -0.1614 & 0.6455 & 0.9682 & 0.3953 & 0.1614 \\ -0.2500 & -0.2500 & -0.2500 & 1.0000 & 0.2500 & 0 & 0 & 0 & 0 & 0 \\ 0.2500 & 0.2500 & 0.2500 & 0.2500 & 1.0000 & 0.4978 & 0.1250 & 0.3227 & 0.5484 & 0.9456 \\ 0.9456 & -0.2864 & -0.1614 & 0 & 0.4978 & 1.0000 & -0.1667 & -0.1667 & -0.4082 & 0.6667 \\ -0.1250 & 0.8956 & -0.6455 & 0 & 0.1250 & -0.1667 & 1.0000 & -0.6667 & 0.4082 & 0.1667 \\ -0.3227 & -0.3227 & 0.9682 & 0 & 0.3227 & -0.1667 & -0.6667 & 1.0000 & 0.4082 & 0.1667 \\ -0.5484 & 0.7016 & 0.3953 & 0 & 0.5484 & -0.4082 & 0.4082 & 0.4082 & 1.0000 & 0.4082 \\ 0.4978 & 0.2864 & 0.1614 & 0 & 0.9456 & 0.6667 & 0.1667 & 0.1667 & 0.4082 & 1.0000 \end{bmatrix}. \quad (24)$$

Example 3. With the same parameters as in previous example, in the first cell, we select users with different users power $\text{diag}(\mathbf{P}_1) = (1.5, 1.4, 1.3, 1.2, 1.1)$ and unit power for the users in the second cell. Again, using T -transform we get the following generalized WBE sequences in the first cell and WBE sequences in the second cell,

$$\mathbf{S}_{W1}(5,4) = \begin{bmatrix} 1.2247 & 0 & 0 & 0 & 1.0488 \\ 0 & 1.1832 & 0 & 0 & 0 \\ 0 & 0 & 1.1402 & 0 & 0 \\ 0 & 0 & 0 & 1.0954 & 0 \end{bmatrix},$$

and

$$\mathbf{S}_2(5,3) = \begin{bmatrix} 1.0000 & 0 & 0 & 1.0000 & 1.0000 \\ 0 & 1.0000 & 0 & 0 & 0 \\ 0 & 0 & 1.0000 & 0 & 0 \end{bmatrix}.$$

The obtained weighted Gram matrix in the absence of flat fading is

$$\mathbf{W} = \begin{bmatrix} 1.5000 & -0.3963 & -0.3818 & 0 & -0.9293 & 0.7014 & 0.7014 & 0.7014 & 0.7014 & 0.7014 \\ -0.3963 & 1.4000 & -0.2500 & 0 & 0.4690 & 0.0678 & 0.0678 & 0.0678 & 0.0678 & 0.0678 \\ -0.3818 & -0.9636 & 1.3000 & 0 & 0.4520 & 0.0653 & 0.0653 & 0.0653 & 0.0653 & 0.0653 \\ 0 & 0 & 0 & 1.2000 & 0 & 0 & 0 & 0 & 0 & 0 \\ -0.9293 & 0.4690 & 0.4520 & 0 & 1.1000 & 0.1590 & 0.1590 & 0.1590 & 0.1590 & 0.1590 \\ 0.7014 & 0.0678 & 0.0653 & 0 & 0.1590 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 \\ 0.7014 & 0.0678 & 0.0653 & 0 & 0.1590 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 \\ 0.7014 & 0.0678 & 0.0653 & 0 & 0.1590 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 \\ 0.7014 & 0.0678 & 0.0653 & 0 & 0.1590 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 \\ 0.7014 & 0.0678 & 0.0653 & 0 & 0.1590 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 \end{bmatrix}. \quad (25)$$

Again, the restrictive condition on the required bandwidth is violated. It is easy to check that $\text{TWSC}(\mathbf{W}) > \text{TWSC}(\mathbf{S}_{W1}) + \text{TSC}(\mathbf{S}_2)$. The optimality of TWSC criterion is assured when in the extended signal space there exists an orthogonal matrix common to all users.

6. CONCLUSIONS

The problem of designing real multicellular CDMA spreading design was addressed. Assuming that all base stations are sharing the same extended signal space, the model of up-link was introduced. A composite Gram matrix approach has been used in order to characterize the generalized WBE sequences in the extended signal space. A specific algorithm under TWSC criterion was derived. Numerical examples for two cells are given. The extension of the results obtained in

this paper to the case of colored noise is an open and challenging problem.

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