

# Time-Slotted Multiuser MIMO Systems: Beamforming and Scheduling Strategies

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We investigate the problem of scheduling for the uplink of a time-slotted multiuser multiple-input multiple-output (MIMO) system with sum capacity as the performance metric. We first consider scheduling users' transmissions for fixed transmit beamformers. Upper bounds for the sum capacity of the time-slotted system with the optimum scheduler are found via relaxing a set of structural constraints. Next, we present a low-complexity scheduling algorithm that aims to approach the capacity's upper bound. The performance of the multislot multiuser MIMO system is a function of the users' transmit beamformers. In turn, the transmit beamformers can be shaped depending on the channel state information available at the transmitter. We investigate how the transmit beamformers should be chosen with different levels of feedback, and combine the proposed scheduling algorithm with antenna selection, eigen transmit beamforming, and perfectly controlled transmit beamforming models. We observe that as the available feedback level is increased, the performance of the scheduling algorithm approaches the upper bounds developed. In particular, a substantial sum capacity gain is attained when the individual channel state information is available at the transmitter side.

**Keywords and phrases:** multiuser MIMO systems, sum capacity, scheduling, transmit beamforming.

## 1. INTRODUCTION

In recent years, there have been intensive efforts in developing spectrally efficient multiuser transmission schemes for wireless communications. Given the scarcity of available bandwidth, a significant part of these efforts involved multiple-antenna systems that promise substantial capacity gain over single-antenna systems [1, 2]. The use of multiple antennas at the receiver provides the system with increased reliability by means of spatial diversity [3]. The transmitter side can exploit the spatial dimensions by coding and multiplexing for single-user systems [4, 5, 6, 7, 8].

The performance of a MIMO system is very much dependent on the channel state information (CSI) available at the transmitter side as the transmitters can be adopted to exploit this information. Recently, there have been several transmission schemes proposed with different levels of available feedback. In the absence of feedback, layered architectures as in BLAST and space-time coding are used, see for example [4, 5, 6]. Antenna selection can be employed in the case of limited feedback at the transmitter side. The capacity of MIMO systems with antenna selection has been stud-

ied recently [9, 10]. Optimum MIMO transmission schemes with antenna selection are analyzed in [11].

The substantial capacity offered by the single-user MIMO systems motivates the use of multiple antennas in shared multiaccess channels, that is, *multiuser MIMO systems*. Transceiver design for such systems has been studied and performance enhancing transmission schemes are proposed in [12, 13]. The capacity of multiuser MIMO systems is investigated for flat-fading channels in [14]. The optimum transmission schemes require vector-coding strategies that may increase the complexity of both the transmitter and the receiver. Hence, suboptimum, but less complex transmission schemes, that enable scalar coding, such as transmit and receive beamforming, have attracted some attention. Receiver beamforming has been shown to be effective in interference suppression in multiuser systems [3, 15]. Jointly optimum transmit powers and receiver beamformers are found in [16]. Reference [17] proposes an iterative algorithm for determining the downlink powers and transmit beamformers given a signal-to-interference ratio (SIR) target at the single-antenna receiver of each user. The optimality of a similar algorithm is shown in [18]. Algorithms that identify

transmit and receive beamforming strategies and the corresponding transmit power assignments are proposed in [19] with the aim of maximizing the minimum achievable SIR or providing each user with its SIR target. The algorithms suggested are numerically shown to enhance system performance, but observed to converge to local optima [19].

In addition to the fact that using multiple antennas forms a stand-alone multiaccess scheme, they typically also provide a substantial performance gain when incorporated into existing multiaccess schemes such as time-division multiple-access (TDMA). The user capacity of TDMA is limited to the number of time slots. Using multiple antennas provides TDMA the opportunity to allocate the same channel (time slot) to different users, and hence increases the user capacity of the system. The channel allocation problem for SDMA/TDMA has been studied in the context of handover management and dynamic slot allocation problems, and heuristic channel allocation algorithms are proposed by [20, 21].

Our aim in this work is to design scheduling algorithms for multiuser MIMO systems that achieve near optimum sum capacities. It is important to note that scheduling, where each user is constrained to transmit in a single slot, is in general suboptimum. The maximum sum rate of a multiuser MIMO system is achieved in the absence of this constraint, and the optimum transmitter covariance matrices of the users can be found by the iterative waterfilling [14]. Besides the fact that the optimum transmission scheme achieving the maximum sum capacity of the multiuser MIMO systems requires vector coding and decoding techniques, the perfect feedback of the optimum transmitter covariance matrices to the transmitter is needed. These requirements increase the complexity of both the transmitter and receiver structures. Motivated by simpler transceiver structures and milder feedback requirements, in this paper, we consider a time-slotted multiuser MIMO system where transmit and receive beamforming that enables scalar coding is used, and each user is assigned to only one time slot. We emphasize that this constraint naturally brings a complexity versus achievable rate trade-off for the system design. In practice, one might opt for a simpler design. In this context, SDMA/TDMA has attracted considerable attention up to date [22, 23]. Reference [24] studies the sum capacity optimization of SDMA/TDMA, and points out that the slot allocation problem is NP-complete. Since exponential complexity is unacceptable for practical systems, scheduling algorithms that require less complexity but achieve near maximum sum capacity are needed, the design of which we will address in this paper.

Our approach relies on viewing the multiuser MIMO scheduler as a special case of a multiuser MIMO system with constrained transmit beamforming vectors. Using this observation and by relaxing these constraints to varying degrees, we can develop simple upper bounds on the sum capacity of the system under any given scheduling algorithm, including the optimum scheduler. We can then devise scheduling algorithms that aim to approach the upper bounds on the capacity. The motivation behind this is that we would attain near-optimum scheduler capacity, if the scheduler we design

results in a sum capacity near the upper bound of the optimum scheduler sum capacity.

First, we consider the case of a multiuser MIMO system with fixed transmit beamforming vectors, and devise a scheduling algorithm. Since the performance of the scheduler is a function of the spatial transmitters of the users, we next tackle the problem of designing the transmit beamforming vectors for different levels of feedback available at the transmit side of each user. We propose several methods for selecting the spatial transmit beamforming vectors in accordance with different levels of feedback available at the transmit side. Our results suggest that the joint design of transmit beamformers and the scheduler improves the performance, and better sum capacity is attained with increased level of feedback at the transmit side.

The organization of the paper is as follows. In Section 2, the system model is introduced and the multiuser MIMO approach is presented. The sum capacity of the multiuser MIMO scheduler is formulated, and the performance analysis method that will be used throughout the paper is developed in Section 3. The scheduling algorithm is proposed for given spatial transmit beamforming vectors in Section 4. In Section 5, the impact of spatial transmit beamforming vectors is investigated and the transmit beamforming selection methods are proposed for different available feedback levels. The performance of the algorithms under different system assumptions is investigated, and numerical results are presented in Section 6. Section 7 concludes the paper.

## 2. SYSTEM MODEL

We consider the uplink of a single-cell synchronous multiuser MIMO system with  $K$  users and  $N$  time slots. The common receiver is equipped with  $N_R$  receive antennas and user  $j$  has  $N_{T_j}$  transmit antennas. We assume that the  $j$ th user transmits its symbol by precoding it with an  $N_{T_j} \times 1$  unit norm spatial transmit beamforming vector  $\mathbf{f}_j$  in the time slot allocated. We assume that the spatial transmit beamforming vector for each user is given and fixed first. We will relax this assumption and investigate the impact of spatial transmit beamforming on the system in Section 5. Similar to the notation in [7, 24], the received vector in each time slot  $i$  is

$$\mathbf{r}_i = \sum_{j \in \mathcal{K}_i} \sqrt{P_j} \mathbf{H}_j \mathbf{f}_j s_j + \mathbf{n}_i, \quad i = 1, \dots, N, \quad (1)$$

where  $P_j$ ,  $s_j$ , and  $\mathbf{H}_j$  are the transmit power, symbol, and the  $N_R \times N_{T_j}$  complex MIMO channel matrix of user  $j$ , respectively, and  $\mathbf{n}_i$  is the zero-mean complex Gaussian noise vector in the  $i$ th time slot with  $E[\mathbf{n}_i \mathbf{n}_i^\dagger] = \sigma^2 \mathbf{I}$ , where  $(\cdot)^\dagger$  denotes the Hermitian of a vector or matrix.  $\mathcal{K}_i$  denotes the set of users that are assigned to the  $i$ th time slot, with each set satisfying  $\mathcal{K}_i \cap \mathcal{K}_l = \emptyset$ , for all  $i \neq l$  and  $\bigcup_{i=1}^N \mathcal{K}_i = \{1, 2, \dots, K\}$ . We assume that the channels are flat fading and the channel realizations are constant over a frame of coded symbols, and perfectly known at the receiver side. For clarity of exposition, we denote the joint effect of the transmit power, channel matrix, and spatial transmit beamforming vector of user  $j$  as

$\mathbf{a}_j = \sqrt{P_j} \mathbf{H}_j \mathbf{f}_j$  which forms a spatial signature for user  $j$ . The received vector in each time slot can be represented as

$$\mathbf{r}_i = \sum_{j \in \mathcal{K}_i} \mathbf{a}_j s_j + \mathbf{n}_i, \quad i = 1, \dots, N. \quad (2)$$

Stacking all the received signals at each time slot, the received signal can be represented in a long vector form of

$$\mathbf{r} = \begin{bmatrix} \mathbf{r}_1 \\ \mathbf{r}_2 \\ \vdots \\ \mathbf{r}_N \end{bmatrix} = \begin{bmatrix} \sum_{j \in \mathcal{K}_1} \mathbf{a}_j s_j \\ \sum_{j \in \mathcal{K}_2} \mathbf{a}_j s_j \\ \vdots \\ \sum_{j \in \mathcal{K}_N} \mathbf{a}_j s_j \end{bmatrix} + \begin{bmatrix} \mathbf{n}_1 \\ \mathbf{n}_2 \\ \vdots \\ \mathbf{n}_N \end{bmatrix}. \quad (3)$$

Defining  $(NN_R) \times N$  block diagonal matrices

$$\mathbf{A}_j = \begin{bmatrix} \mathbf{a}_j & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{a}_j & \vdots & \vdots \\ \vdots & \vdots & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{a}_j \end{bmatrix}, \quad \forall j = 1, \dots, K, \quad (4)$$

and setting  $\mathbf{e}_i$  as an  $N \times 1$  vector with  $N - 1$  zeros and 1 in the  $i$ th entry, the received signal can be represented as

$$\mathbf{r} = \sum_{i=1}^N \sum_{j \in \mathcal{K}_i} \mathbf{A}_j \mathbf{e}_i s_j + \mathbf{n} \quad (5)$$

or

$$\mathbf{r} = \sum_{j=1}^K \mathbf{A}_j \mathbf{t}_j s_j + \mathbf{n}, \quad (6)$$

where  $\mathbf{t}_j = \mathbf{e}_i$  if  $j \in \mathcal{K}_i$ . Observe that (6) has the same form as the received signal of a multiuser MIMO system where users have channel matrices of  $\{\mathbf{A}_j\}$ , and transmit beamforming vectors of  $\mathbf{t}_j = \mathbf{e}_i$  if the user  $j$  is a member of  $\mathcal{K}_i$ . Thus, the multiuser MIMO scheduling problem can be viewed as a special case of a multiuser MIMO system with transmit beamforming vectors given in (6). Throughout the paper, we consider multiuser MIMO systems with  $K \leq NN_R$  users and develop scheduling algorithms for such systems.

### 3. SUM CAPACITY AND UPPER BOUNDS FOR MULTIUSER MIMO SCHEDULING

Our aim in this section is to investigate the effect of scheduling on the sum capacity of multiuser MIMO systems and to describe our approach for developing near-optimum scheduling algorithms.

Previous work showed that the information theoretic sum capacity of a multiuser MIMO scheduler in the time slot  $i$  with effective signatures  $\{\mathbf{a}_j = \sqrt{P_j} \mathbf{H}_j \mathbf{f}_j\}$  is given by [25]

$$C_{\mathcal{K}_i} = \frac{1}{2N} \log \left[ \det \left( \mathbf{I}_{N_R} + \sigma^{-2} \sum_{j \in \mathcal{K}_i} \mathbf{a}_j \mathbf{a}_j^\dagger \right) \right]. \quad (7)$$

The total sum capacity of all time slots is

$$\begin{aligned} C_{\text{sum}} &= \sum_{i=1}^N C_{\mathcal{K}_i} \\ &= \sum_{i=1}^N \frac{1}{2N} \log \left[ \det \left( \mathbf{I}_{N_R} + \sigma^{-2} \sum_{j \in \mathcal{K}_i} \mathbf{a}_j \mathbf{a}_j^\dagger \right) \right]. \end{aligned} \quad (8)$$

Formally, the sum capacity optimization problem for multiuser MIMO scheduling is given by

$$\begin{aligned} \max_{\mathcal{K}_i} C_{\text{sum}} &= \sum_{i=1}^N C_{\mathcal{K}_i} \\ \text{s.t. } \bigcup_{i=1}^N \mathcal{K}_i &= \{1, 2, \dots, K\}, \quad \mathcal{K}_i \cap \mathcal{K}_l = \emptyset, \quad \forall i \neq l. \end{aligned} \quad (9)$$

The sum capacity maximization for multiuser MIMO scheduling is a combinatorial optimization problem and has been recently studied in the context of SDMA/TDMA systems where it has been pointed out to be NP-complete [24]. As expected, the globally optimum schedule improves the sum capacity of the SDMA/TDMA systems significantly [24]. However, obviously, the associated complexity would render the optimum scheduler impractical even for a moderate number of users: simpler scheduling algorithms that achieve near maximum sum capacity are needed.

Recall that the multiuser MIMO scheduler is a special case of a multiuser MIMO system with users transmitting with constrained transmit beamforming vectors given in (6). The sum capacity of a multiuser MIMO system with exact CSI at the transmitter and receiver side is studied and the optimum transmission schemes are identified for deterministic channel models [14]. Since this case corresponds to relaxation of transmit beamformer constraints, it constitutes an upper bound on the sum capacity of the optimum multiuser MIMO scheduler. Given this observation, and the fact that the sum capacity of the optimum scheduler does not have a closed form, we may try to design scheduling strategies to approach the bound instead. The rationale is that if the schedulers we design yield performance near the upper bound, their performance must be closer to that of the optimum scheduler. To this end, we define the upper bound for the sum capacity of multiuser MIMO scheduler, the actual sum capacity of multiuser MIMO scheduler, and the achieved sum capacity of a given scheduling algorithm for the multiuser MIMO system as  $C_{\text{upper}}$ ,  $C_{\text{actual}}$ , and  $C_{\text{achieved}}$ , respectively. It is evident that

$$C_{\text{achieved}} \leq C_{\text{actual}} \leq C_{\text{upper}} \quad (10)$$

and that as the achieved sum capacity approaches the upper bound, the actual sum capacity of multiuser MIMO scheduler is approached as well. Hence, without computing the exact maximum sum capacity of multiuser MIMO scheduler, one can investigate the performance of a given scheduling algorithm by comparing with the upper bound.

### 3.1. Multiuser MIMO capacity upper bound

The sum capacity of the multiuser MIMO scheduler can be reformulated as

$$C_{\text{sum}} = \frac{1}{2N} \log \left[ \det \left( \mathbf{I}_{NN_R} + \sigma^{-2} \sum_{j=1}^K \mathbf{A}_j \mathbf{t}_j \mathbf{t}_j^\dagger \mathbf{A}_j^\dagger \right) \right] \quad (11)$$

with  $\mathbf{t}_j \in \{\mathbf{e}_1, \mathbf{e}_2, \dots, \mathbf{e}_N\}$  defined in the previous section. Defining the transmitter vector covariance matrix as  $\mathbf{R}_j = \mathbf{t}_j \mathbf{t}_j^\dagger$ , the optimization problem for the sum capacity of a multiuser MIMO scheduler is

$$\begin{aligned} \max_{\{\mathbf{R}_j\}_{j=1,\dots,K}} C_{\text{sum}} &= \frac{1}{2N} \log \left[ \det \left( \mathbf{I}_{NN_R} + \sigma^{-2} \sum_{j=1}^K \mathbf{A}_j \mathbf{R}_j \mathbf{A}_j^\dagger \right) \right] \\ \text{s.t. } \mathbf{R}_j &\in \{\mathbf{e}_1 \mathbf{e}_1^\dagger, \mathbf{e}_2 \mathbf{e}_2^\dagger, \dots, \mathbf{e}_N \mathbf{e}_N^\dagger\}, \quad j = 1, 2, \dots, K. \end{aligned} \quad (12)$$

Observe that  $\text{tr}\{\mathbf{R}_j\} = \{\mathbf{e}_k \mathbf{e}_k^\dagger\} = 1$ . Hence, by relaxing the  $\mathbf{R}_j \in \{\mathbf{e}_1 \mathbf{e}_1^\dagger, \mathbf{e}_2 \mathbf{e}_2^\dagger, \dots, \mathbf{e}_N \mathbf{e}_N^\dagger\}$  constraint to a power constraint,  $\text{tr}\{\mathbf{R}_j\} \leq 1$ , one can easily obtain an upper bound for the sum capacity of the multiuser MIMO scheduler, that is,

$$\begin{aligned} C_{\text{actual}} &= \max_{\{\mathbf{R}_j \in \{\mathbf{e}_1 \mathbf{e}_1^\dagger, \mathbf{e}_2 \mathbf{e}_2^\dagger, \dots, \mathbf{e}_N \mathbf{e}_N^\dagger\}_{j=1,\dots,K}} C_{\text{sum}} \\ &\leq C_{\text{upper1}} = \max_{\{\mathbf{R}_j | \text{tr}\{\mathbf{R}_j\} = 1\}_{j=1,\dots,K}} C_{\text{sum}}. \end{aligned} \quad (13)$$

The upper bound for the sum capacity of multiuser MIMO scheduler found by this relaxation is the sum capacity of the multiuser MIMO system with users having the channel matrices  $\{\mathbf{A}_j\}$  and unit power constraints. For a given multiuser MIMO system, the sum capacity maximizing transmit covariance matrices,  $\{\mathbf{R}_j\}$ , can be found easily by the iterative waterfilling procedure as defined in [14]. At each step of the waterfilling procedure, the signals of other users are viewed as noise and the individual transmit covariance matrix of a user is optimized for maximizing the sum capacity. Assuming that all users start with zero transmit covariance matrices,  $\mathbf{R}_j = \mathbf{0}$ , iterative waterfilling converges to  $\mathbf{R}_j = (1/N)\mathbf{I}$  for all users consequently terminating in exactly  $K$  steps. This structure of transmitter covariance matrix corresponds to sending independent data streams with equal power  $1/N$  at each time slot. Thus, the sum capacity of such MIMO systems, and consequently, an upper bound for the sum capacity of multiuser MIMO scheduler, is

$$\begin{aligned} C_{\text{actual}} &\leq C_{\text{upper1}} \\ &= \frac{1}{2} \log \left[ \det \left( \mathbf{I}_{NN_R} + \sigma^{-2} \sum_{j=1}^K \frac{1}{N} \mathbf{a}_j \mathbf{a}_j^\dagger \right) \right]. \end{aligned} \quad (14)$$

### 3.2. Unconstrained effective signature upper bound

The information theoretic sum capacity of the multiuser system is a function of the effective signatures  $\{\mathbf{b}_j = \mathbf{A}_j \mathbf{t}_j\}$  and the subspace of the effective signature is defined by the span of the channel matrices  $\{\mathbf{A}_j\}$ . The received signal power of

user  $j$  is  $\mathbf{t}_j^\dagger \mathbf{A}_j^\dagger \mathbf{A}_j \mathbf{t}_j$  which is simply  $\|\mathbf{a}_j\|^2$  for the multiuser MIMO scheduler. From a system point of view, a multiuser MIMO scheduler can be viewed as a CDMA system with a processing gain of  $NN_R$  and received powers  $\{\|\mathbf{a}_j\|^2\}$ . However, the multiuser MIMO scheduler brings additional subspace constraints on the effective signatures which may decrease the sum capacity below the sum capacity of such a CDMA system. Thus, the sum capacity of a CDMA system with a processing gain of  $NN_R$  and received powers  $\{\|\mathbf{a}_j\|^2\}$  forms an upper bound for the sum capacity of multiuser MIMO scheduling. For a multiuser MIMO scheduler with  $K \leq NN_R$ , the upper bound is the sum capacity of underloaded CDMA system which assigns orthogonal effective signature sequences as follows:

$$C_{\text{actual}} \leq C_{\text{upper2}} = \frac{1}{2N} \sum_{j=1}^K \log \left[ 1 + \sigma^{-2} \|\mathbf{a}_j\|^2 \right]. \quad (15)$$

Both of the upper bounds for the sum capacity of multiuser MIMO scheduler are obtained by relaxing different constraints of the actual optimization problem. A tighter upper bound can be obtained by simply taking  $C_{\text{upper}} = \min(C_{\text{upper1}}, C_{\text{upper2}})$ , which is what we will use as a performance benchmark.

## 4. THE SUM CAPACITY BASED MULTIUSER MIMO SCHEDULER

Our aim in this section is to design an algorithm that schedules each user to a time slot for fixed beamformers and achieves sum capacity near the upper bounds developed in the previous section.

Recall that the sum capacity at time slot  $i$  is represented in (7). Assume that user  $k$  is transmitting in the  $i$ th time slot. We define the set of users in slot  $i$  excluding user  $k$  as  $\mathcal{K}_i - \{k\} = \bar{\mathcal{K}}_i^{(k)}$ . The contribution of user  $k$  on the sum capacity of  $C_{\mathcal{K}_i}$  is  $C_{\mathcal{K}_i} - C_{\bar{\mathcal{K}}_i^{(k)}}$  which is simply

$$\begin{aligned} C_{\mathcal{K}_i} - C_{\bar{\mathcal{K}}_i^{(k)}} &= \frac{1}{2N} \log \left[ \det \left( \left( \mathbf{I}_{N_R} + \sigma^{-2} \sum_{j \in \mathcal{K}_i} \mathbf{a}_j \mathbf{a}_j^\dagger \right) \right. \right. \\ &\quad \left. \left. \times \left( \mathbf{I}_{N_R} + \sigma^{-2} \sum_{j \in \bar{\mathcal{K}}_i^{(k)}} \mathbf{a}_j \mathbf{a}_j^\dagger \right)^{-1} \right) \right] \\ &= \frac{1}{2N} \log \left[ 1 + \mathbf{a}_k^\dagger \left( \sigma^2 \mathbf{I}_{N_R} + \sum_{j \in \bar{\mathcal{K}}_i^{(k)}} \mathbf{a}_j \mathbf{a}_j^\dagger \right)^{-1} \mathbf{a}_k \right]. \end{aligned} \quad (16)$$

As pointed out in the previous section, the optimum unconstrained effective signatures for  $NN_R \geq K$  are the orthogonal signature sequences. However, the predefined spatial signatures of the users may not always be orthogonal to each other. Thus, the near-optimum scheduling strategy should

try to assign the slots to users such that the effective signatures  $\{\mathbf{A}_j \mathbf{t}_j\}$  are as close to being orthogonal as they can. Observe that assigning more than  $N_R$  users to the same time slot is likely to cause high correlation among the users. In addition to the fact that each time slot should not be assigned more than  $N_R$  users, intuitively, it is logical to assign no more than  $\lceil \text{Number of users} / \text{Number of time slots} \rceil \leq N_R$  users to each time slot for the sake of fairness.

The above observations suggest that an  $N$  step sequential user assignment algorithm that tries to select the spatially less correlated users for each time slot is a good candidate for near-optimum performance. Specifically, at each step, the number of users that will be assigned for time slot is  $\lceil \text{Number of available users} / \text{Number of available time slots} \rceil$  which is guaranteed not to exceed  $N_R$  users for  $NN_R \geq K$ .

Recall that the contribution of a user to the sum capacity of a time slot is given by (16). Also, the capacity of user  $k$  with unconstrained effective signature orthogonal to the other users, that is, the single-user capacity, is

$$C_{\text{user}(k),\text{opt}} = \frac{1}{2N} \log \left( 1 + \sigma^{-2} \|\mathbf{a}_k\|^2 \right). \quad (17)$$

Observe that the assignment of user  $k$  to time slot  $i$  will result in a difference of  $C_{\text{user}(k),\text{opt}} - (C_{\mathcal{K}_i} - C_{\bar{\mathcal{K}}_i^{(k)}})$  between the unconstrained effective signature sum capacity upper bound, and the achieved sum capacity of the multiuser MIMO scheduler from user  $k$ 's perspective. This difference can be expressed as

$$\begin{aligned} C_{\text{user}(k),\text{opt}} - (C_{\mathcal{K}_i} - C_{\bar{\mathcal{K}}_i^{(k)}}) \\ = \frac{1}{2N} \log \frac{\left( 1 + \sigma^{-2} \|\mathbf{a}_k\|^2 \right)}{\left( 1 + \mathbf{a}_k^\dagger \left( \sigma^2 \mathbf{I}_{N_R} + \sum_{j \in \bar{\mathcal{K}}_i^{(k)}} \mathbf{a}_j \mathbf{a}_j^\dagger \right)^{-1} \mathbf{a}_k \right)}. \end{aligned} \quad (18)$$

Thus, the user with the highest

$$\begin{aligned} \frac{\left( 1 + \mathbf{a}_k^\dagger \left( \sigma^2 \mathbf{I}_{N_R} + \sum_{j \in \bar{\mathcal{K}}_i^{(k)}} \mathbf{a}_j \mathbf{a}_j^\dagger \right)^{-1} \mathbf{a}_k \right)}{\left( 1 + \sigma^{-2} \|\mathbf{a}_k\|^2 \right)} \\ \approx \frac{\sigma^2 \mathbf{a}_k^\dagger \left( \sigma^2 \mathbf{I}_{N_R} + \sum_{j \in \bar{\mathcal{K}}_i^{(k)}} \mathbf{a}_j \mathbf{a}_j^\dagger \right)^{-1} \mathbf{a}_k}{\left( \|\mathbf{a}_k\|^2 \right)} = z_{ik} \end{aligned} \quad (19)$$

will result in the smallest difference from the upper bound from a single user's perspective. Approaching the sum capacity maximization problem for multiuser MIMO scheduler as a sequential user/slot assignment problem, we choose to minimize the difference between the sum capacity of the unconstrained effective signature upper bound and the achieved sum capacity of the multiuser MIMO scheduler from a single user's perspective, that is, at each user/slot assignment step, we choose the user that have the highest  $z_{ik}$  to assign to time slot  $i$ .

This algorithm obviously favors the *earlier* time slots, since these time slots will have more users to select from the available users' set. However, by limiting the number of users assigned to each time slot by  $\lceil \text{Number of available}$

users/Number of available time slots], the *earlier* time slots may have 1 user more than the later ones and the unfairness is somewhat decreased. Notice that the algorithm has no preference for the first user to be assigned to a time slot. Thus, an arbitrary user can be chosen from the available users. The algorithm proposed is summarized in Algorithm 1.

## 5. SPATIAL TRANSMIT BEAMFORMING WITH DIFFERENT LEVELS OF FEEDBACK

The previous section considered the case where the spatial transmit beamforming vectors are fixed inputs for the multiuser MIMO scheduler. The performance of the multiuser MIMO systems with the near-optimum scheduler is clearly a function of the choice of the transmit beamformers. In turn, the choice of the transmit beamformers is highly dependent on the feedback level at the transmitter side. In this section, we consider different levels of feedback at the transmitter side, and determine the corresponding transmit beamformers to be employed. In this context, antenna selection feedback, individual CSI feedback, and the perfect transmit beamforming feedback cases will be investigated.

### 5.1. Antenna selection

In the case of limited channel state feedback, a popular approach is antenna selection, where the only required feedback is which antenna(s) should be used [11]. Consider the case where one transmitter antenna will be selected. The sum capacity of the multiuser MIMO scheduler and the unconstrained effective signature upper bound are highly dependent on the received powers of users. Thus, intuitively, an effective antenna selection method for the multiuser MIMO scheduler is to choose the transmitter antenna with the highest received power. The approach is expected to perform well especially when the received power of one transmitter antenna is significantly larger than the received powers of other transmitter antennas, and the MIMO channels of the users are independent of each other. If this is not the case, we have to consider the performance of all transmitter antennas on the sum capacity of time slot  $i$ :

$$\begin{aligned} C_{\mathcal{K}_i} - C_{\bar{\mathcal{K}}_i^{(k)}} \\ = \frac{1}{2N} \log \left[ 1 + P_k \mathbf{h}_{k,m}^\dagger \left( \sigma^2 \mathbf{I}_{N_R} + \sum_{j \in \bar{\mathcal{K}}_i^{(k)}} \mathbf{a}_j \mathbf{a}_j^\dagger \right)^{-1} \mathbf{h}_{k,m} \right]. \end{aligned} \quad (20)$$

Here,  $\mathbf{h}_{k,m}$  is the  $m$ th column vector of the channel matrix  $\mathbf{H}_k$  and the spatial signature of the transmitter antenna  $m$  of user  $k$ . The contribution of each transmitter antenna can be evaluated by (20). Note that the capacity upper bound with unconstrained effective signatures for each user is defined by the maximum received power of the transmitter antennas,

$$C_{\text{user}(k),\text{opt}} = \frac{1}{2N} \log \left( 1 + \sigma^{-2} P_k \max_{m=1, \dots, N_{T_k}} \|\mathbf{h}_{k,m}\|^2 \right). \quad (21)$$

System	Parameters
$\mathcal{K}_a$	: Available users that are not assigned to a time slot.
$\mathcal{K}_i$	: The users that are assigned to the time slot $i = 1, \dots, N$ .
$\{\mathbf{a}_j\}$	: Effective spatial signatures of users.
Av. user	: Number of users that will be assigned to the time slot.
<i>Scheduling Algorithm</i>	
$\mathcal{K}_a = \{1, 2, \dots, K\}$	
For $i = 1, \dots, N$	
	User selection for time slot $i$
	Av. user = $\left\lceil \frac{n(\mathcal{K}_a)}{N - i + 1} \right\rceil$
	For $j = 1 : \text{Av. user}$
	$k^* = \arg \max_{k \in \mathcal{K}_a} \frac{\mathbf{a}_k^\dagger (\sigma^2 \mathbf{I}_{N_R} + \sum_{j \in \mathcal{K}_i} \mathbf{a}_j \mathbf{a}_j^\dagger)^{-1} \mathbf{a}_k}{\mathbf{a}_k^\dagger \mathbf{a}_k}$
	$\mathcal{K}_i = \mathcal{K}_i \cup \{k^*\}$
	$\mathcal{K}_a = \mathcal{K}_a \setminus \{k^*\}$ .
	End
End	

ALGORITHM 1: Sum capacity based sequential scheduling algorithm.

System	Parameters
$\mathcal{K}_a$	: Available users that are not assigned to a time slot.
$\mathcal{K}_i$	: The users that are assigned to the time slot $i = 1, \dots, N$ .
$\{\mathbf{a}_j\}$	: Effective spatial signatures of users.
Av. user	: Number of users that will be assigned to the time slot.
<i>Scheduling Algorithm</i>	
$\mathcal{K}_a = \{1, 2, \dots, K\}$	
For $i = 1, \dots, N$	
	User and antenna selection for time slot $i$
	Av. user = $\left\lceil \frac{n(\mathcal{K}_a)}{N - i + 1} \right\rceil$
	For $j = 1 : \text{Av. user}$
	$(k^*, m^*) = \arg \max_{k \in \mathcal{K}_a, m \in \{1, 2, \dots, N_{T_k}\}} \frac{\mathbf{h}_{k,m}^\dagger (\sigma^2 \mathbf{I}_{N_R} + \sum_{j \in \mathcal{K}_i} \mathbf{a}_j \mathbf{a}_j^\dagger)^{-1} \mathbf{h}_{k,m}}{\max_m \mathbf{h}_{k,m}^\dagger \mathbf{h}_{k,m}}$
	$\mathbf{a}_{k^*} = \sqrt{P_{k^*}} \mathbf{h}_{k^*, m^*}$
	$\mathcal{K}_i = \mathcal{K}_i \cup \{k^*\}$
	$\mathcal{K}_a = \mathcal{K}_a \setminus \{k^*\}$ .
	End
End	

ALGORITHM 2: Sum capacity based scheduling with generalized antenna selection.

The user selection algorithm considers each transmitter and chooses the transmitter antenna of the user that has the best performance in the sense of minimizing the gap between the sum capacity of multiuser MIMO scheduler and the unconstrained effective signature upper bound from a single user's perspective. We term this algorithm *the multiuser MIMO scheduling algorithm with generalized antenna selection*.

The algorithm with maximum received power antenna selection is a simple extension of the algorithm presented in Algorithm 1 with the effective spatial signatures replaced by the spatial signatures of the transmitter antennas with maximum received powers. The scheduling algorithm with generalized antenna selection is presented in Algorithm 2.

## 5.2. Eigenmode selection

Another scenario with limited feedback is when each user has its own CSI at the transmitter side. This is a reasonable assumption when the system is operated in time-division duplex mode. The eigenmodes of the channel matrices can be viewed as the transmitter antennas with different received powers. Similar to the antenna selection case described above, the simplest form of spatial transmit beamformer selection is choosing the eigenmode of the channel with the highest eigenvalue. This approach requires no feedback to the transmitter side and is expected to perform well especially if one eigenvalue is significantly larger than the others, and the MIMO channels of the users are independent

of each other. If this is not the case, one may choose to consider an approach similar to generalized antenna selection, considering all eigenmodes and comparing the performance:

$$\begin{aligned} C_{\mathcal{K}_i} - C_{\bar{\mathcal{K}}_i^{(k)}} \\ = \frac{1}{2N} \log \left[ 1 + P_k \mathbf{u}_{k,m}^\dagger \mathbf{H}_k^\dagger \left( \sigma^2 \mathbf{I}_{N_R} + \sum_{j \in \bar{\mathcal{K}}_i^{(k)}} \mathbf{a}_j \mathbf{a}_j^\dagger \right)^{-1} \mathbf{H}_k \mathbf{u}_{k,m} \right], \end{aligned} \quad (22)$$

where  $\mathbf{u}_{k,m}$  is the  $m$ th eigenvector of  $\mathbf{H}_k^\dagger \mathbf{H}_k$  and the effective spatial signature of user  $k$  is  $\mathbf{a}_k = \sqrt{P_k} \mathbf{H}_k \mathbf{u}_{k,m^*}$  when  $m^*$ th eigenmode is selected.

The user selection algorithm considers each eigenvector as a virtual user and chooses the eigenmode of a user that has the best performance in terms of minimizing the gap between the sum capacity of multiuser MIMO scheduler and the unconstrained effective signature upper bound from a single user's perspective. The algorithm is the same as the one described in Algorithm 2 with the transmitter antennas and received powers replaced by the eigenmodes and eigenvalues, respectively, and the maximum received power argument replaced by the largest eigenvalue.

### 5.3. Spatial transmit beamforming feedback to the transmitter side

In this section, we assume that we have a reliable and an error-free feedback channel to the transmitter side and that we will design beamformers for this system. The motivation for this design is to obtain a performance upper bound for that of any beamformer design with limited feedback. The spatial transmit beamforming vector of each user affects both the received powers and effective spatial signatures of users that determine the performance and the scheduling strategy. By choosing the right spatial transmit beamforming vectors, the effective spatial signatures of users can be made less correlated to the other users in turn improving the performance of the scheduler. One must also account for the fact that the choice of the transmit beamformers affects the received powers of the users. Thus, there is a trade-off between the spatial correlation and received powers of the users.

In the development of the scheduling algorithm, the time-slot allocation is done by considering the performance in terms of minimizing the gap between the sum capacity of multiuser MIMO scheduler and the unconstrained effective signature upper bound from a single user's perspective. Recall that the contribution of a user to the sum capacity of a time slot is (18) with effective signatures of  $\{\mathbf{a}_j = \sqrt{P_j} \mathbf{H}_j \mathbf{f}_j\}_{j=1}^K$ . The effect of the spatial transmit beamforming vectors can be expressed as

$$\begin{aligned} C_{\mathcal{K}_i} - C_{\bar{\mathcal{K}}_i^{(k)}} \\ = \frac{1}{2N} \log \left[ 1 + P_k \mathbf{f}_k^\dagger \mathbf{H}_k^\dagger \left( \sigma^2 \mathbf{I}_{N_R} + \sum_{j \in \bar{\mathcal{K}}_i^{(k)}} \mathbf{a}_j \mathbf{a}_j^\dagger \right)^{-1} \mathbf{H}_k \mathbf{f}_k \right]. \end{aligned} \quad (23)$$

Observe that the best transmit beamforming vector one can choose is the maximum eigenvalue eigenvector of  $\mathbf{H}_k^\dagger (\sigma^2 \mathbf{I}_{N_R} + \sum_{j \in \bar{\mathcal{K}}_i^{(k)}} \mathbf{a}_j \mathbf{a}_j^\dagger)^{-1} \mathbf{H}_k$  that will maximize the sum capacity of the system in terms of user  $k$ 's transmit beamforming vector, and the capacity of user  $k$  with unconstrained effective signatures is determined by the largest eigenvalue of  $\mathbf{H}_k^\dagger \mathbf{H}_k$ . The modified version of the scheduling algorithm with transmit beamformer selection is presented in Algorithm 3.

## 6. NUMERICAL RESULTS

In this section, we present numerical results related to the performance of the scheduling algorithms. We also compare the performance of the scheduling strategies for different levels of feedback to investigate the benefit gained by exploiting the CSI. The simulations are performed for a multiuser MIMO scheduler with  $N = 8$  time slots with  $K = 16$  users. First, the systems with multiple antennas only at the receiver with  $N_R = 2, 4$ , and 6 are considered. Then, the effect of multiple transmitter antennas is investigated with different levels of feedback for  $2 \times 2$ ,  $4 \times 4$ , and  $6 \times 6$  MIMO systems. The channels are realizations of a flat-fading channel model where all links are assumed to be independent and identically distributed complex Gaussian random variables. The received SNR of each user is 7 dB. CDF curves for sum capacity obtained by simulating 10 000 channel realizations are presented.

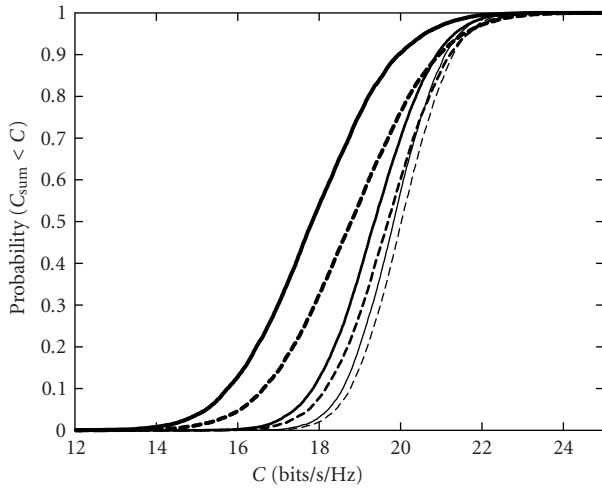
First, we consider a  $K = 16$  multiuser MIMO scheduler with single transmit antenna per user and  $N_R = 2, 4$ , and 6. The upper bounds, and the performance of the proposed scheduling schemes presented in Algorithm 1, are given in Figure 1. We observe that the proposed time-slot allocation algorithms achieve sum capacities very close to the upper bounds. As we increase the number of the receiver antennas, the performance of the scheduling algorithm is enhanced. This is expected since for each added receiver antenna, the spatial diversity increases, and the users become less correlated.

For the multiuser MIMO scheduler with multiple transmitters, the effect of spatial transmit beamforming vectors and the feedback level is given for  $K = 16$  user  $2 \times 2$ ,  $4 \times 4$ , and  $6 \times 6$  MIMO systems. First, the performance of the algorithm with selecting the transmitter antenna with maximum received power is given in Figure 2. Next, generalized antenna selection approach as outlined in Algorithm 2 is explored in Figure 3. As the dimensions of the MIMO channel is increased, the proposed algorithm achieves sum capacities closer to the upper bounds.

In the case where each user knows its own channel, the eigenvectors of the channel matrix  $\mathbf{H}_k^\dagger \mathbf{H}_k$  can be used as spatial transmit beamforming vectors. The performance of the scheduling algorithm with transmit beamforming using the maximum eigenvalued eigenvector of each user's channel matrix is presented in Figure 4. The performance achieved by transmit beamforming with the maximum eigenvalued eigenvector shows significant improvement as compared to antenna selection. This is due the fact that eigenvector

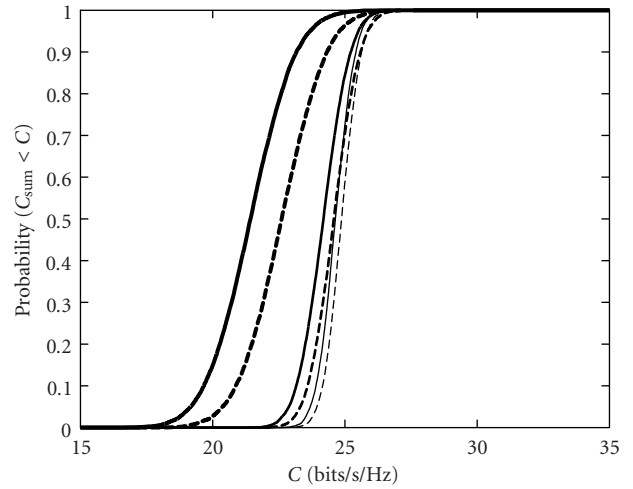
System	Parameters
$\mathcal{K}_a$	: Available users that are not assigned to a time slot
$\mathcal{K}_i$	: The users that are assigned to the time slot $i = 1, \dots, N$
$\{\mathbf{a}_j\}$	: Effective spatial signatures of users
Av. user	: Number of users that will be assigned to the time slot
<b>Scheduling Algorithm</b>	
$\mathcal{K}_a = \{1, 2, \dots, K\}$	
For $i = 1, \dots, N$	
	User selection for time slot $i$
	Av. user = $\left\lfloor \frac{n(\mathcal{K}_a)}{N - i + 1} \right\rfloor$
	For $j = 1 : \text{Av. user}$
	$(k^*, \mathbf{f}_{k^*}) = \arg \max_{k \in \mathcal{K}_a, \mathbf{f}_k \in \mathbb{C}^{N_T k} \text{ s.t. } \mathbf{f}_k^\dagger \mathbf{f}_k = 1} \frac{\mathbf{f}_k^\dagger \mathbf{H}_k^\dagger (\sigma^2 \mathbf{I}_{N_R} + \sum_{j \in \mathcal{K}_i} \mathbf{a}_j \mathbf{a}_j^\dagger)^{-1} \mathbf{H}_k \mathbf{f}_k}{\max_{\mathbf{f}_k \in \mathbb{C}^{N_T k} \text{ s.t. } \mathbf{f}_k^\dagger \mathbf{f}_k = 1} \mathbf{f}_k^\dagger \mathbf{H}_k^\dagger \mathbf{H}_k \mathbf{f}_k}$
	$\mathbf{a}_{k^*} = \sqrt{P_{k^*}} \mathbf{H}_{k^*} \mathbf{f}_{k^*}$
	$\mathcal{K}_i = \mathcal{K}_i \cup \{k^*\}$
	$\mathcal{K}_a = \mathcal{K}_a \setminus \{k^*\}$ .
	End
End	

ALGORITHM 3: Sum capacity based scheduling and spatial transmit beamformer design.



- $N_R = 2$
- - -  $N_R = 2$ , upper bound
- $N_R = 4$
- - -  $N_R = 4$ , upper bound
- $N_R = 6$
- - -  $N_R = 6$ , upper bound

FIGURE 1:  $K = 16$ -user MIMO system with single transmitter and  $N_R = 2, 4, 6$ . Comparison of the CDF curves of the upper bound and the multiuser MIMO scheduling algorithm.



- $2 \times 2$  MIMO
- - -  $2 \times 2$  MIMO, upper bound
- $4 \times 4$  MIMO
- - -  $4 \times 4$  MIMO, upper bound
- $6 \times 6$  MIMO
- - -  $6 \times 6$  MIMO, upper bound

FIGURE 2:  $K = 16$ -user systems with  $2 \times 2, 4 \times 4,$  and  $6 \times 6$  MIMO models. Comparison of the CDF curves of the upper bound and the multiuser MIMO scheduling algorithm with maximum received power antenna selection.

beamforming exploits the individual CSI to maximize the received power of the user. Next, generalized eigenmode selection is explored in Figure 5 where similar performance is observed to the scheduling algorithm with transmit beamformers using the maximum eigenvalued eigenvector.

In the case of perfect feedback, that is, the case developed in Section 5.3, the users have the opportunity to adapt their transmit beamforming vectors in response to the interference from existing users. The performance of the algorithm is presented in Figure 6.



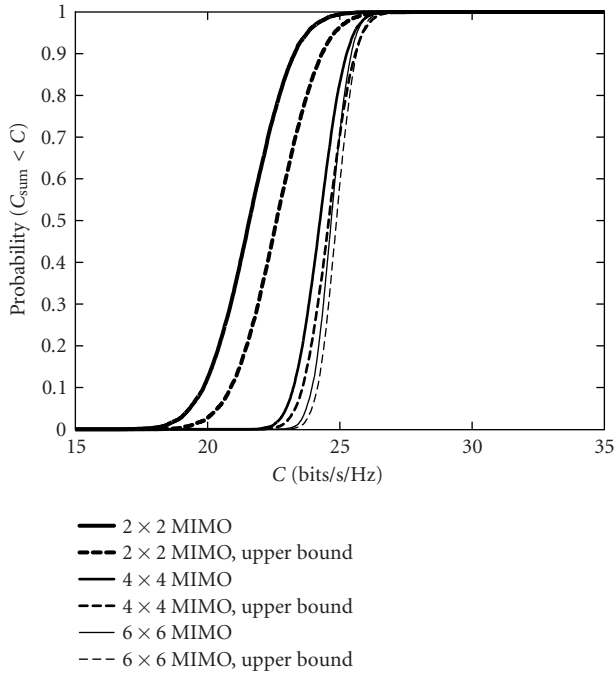


FIGURE 3:  $K = 16$ -user systems with  $2 \times 2$ ,  $4 \times 4$ , and  $6 \times 6$  MIMO models. Comparison of the CDF curves of the upper bound and the multiuser MIMO scheduling algorithm with generalized antenna selection.

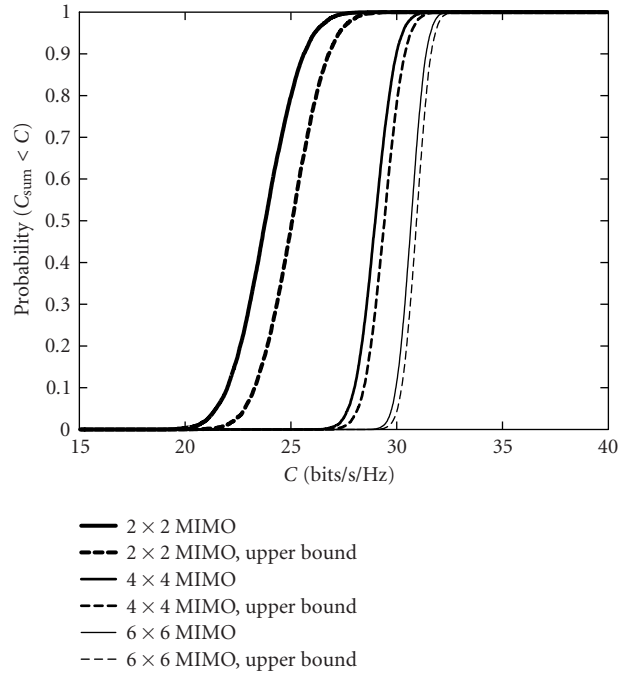


FIGURE 5:  $K = 16$ -user systems with  $2 \times 2$ ,  $4 \times 4$ , and  $6 \times 6$  MIMO models. Comparison of the CDF curves of the upper bound and the multiuser MIMO scheduling algorithm with generalized eigenmode selection.

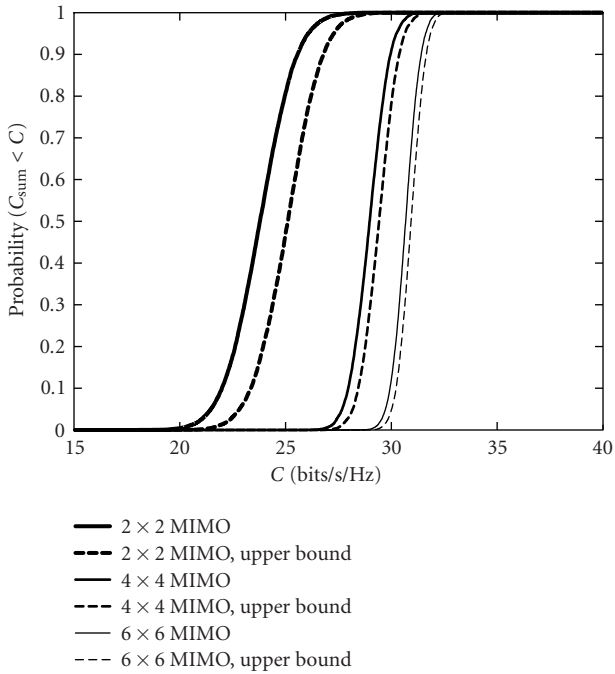


FIGURE 4:  $K = 16$ -user systems with  $2 \times 2$ ,  $4 \times 4$ , and  $6 \times 6$  MIMO models. Comparison of the CDF curves of the upper bound and the multiuser MIMO scheduling algorithm using maximum eigenmode selection.

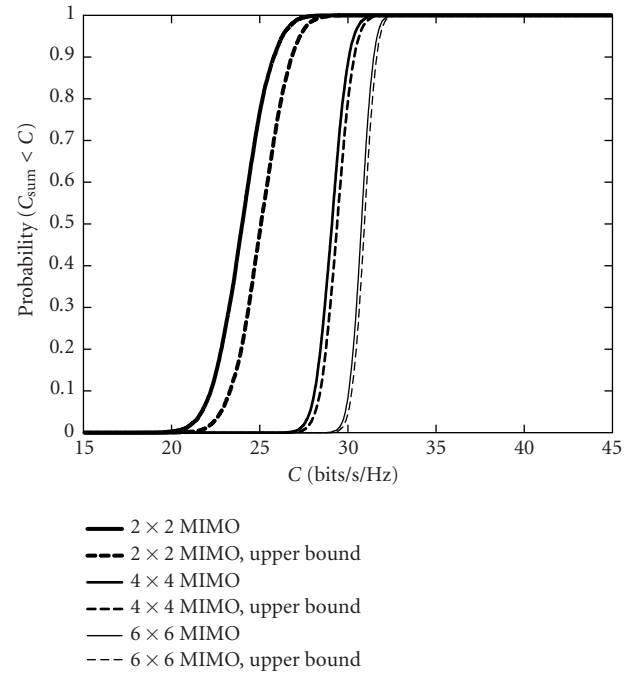


FIGURE 6:  $K = 16$ -user systems with  $2 \times 2$ ,  $4 \times 4$ , and  $6 \times 6$  MIMO models. Comparison of the CDF curves of the upper bound and the multiuser MIMO scheduling algorithm with transmit beamformer design.

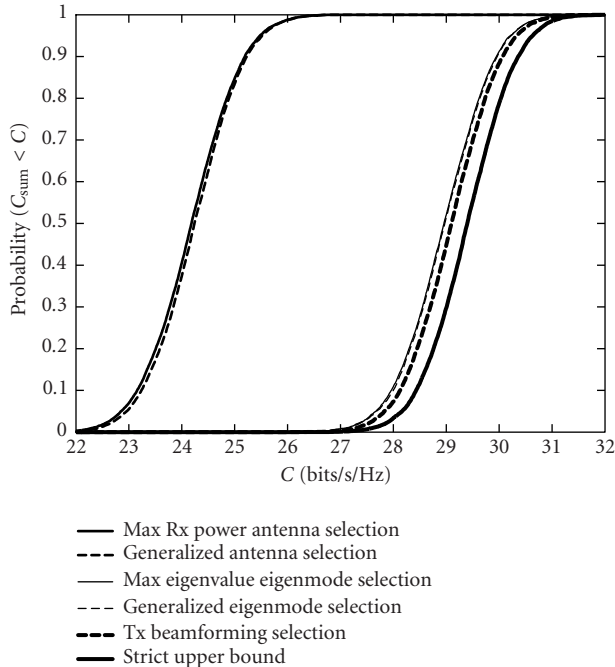


FIGURE 7:  $K = 16$ -user system with  $4 \times 4$  MIMO model. Comparison of the CDF curves of the upper bound and the multiuser MIMO scheduling algorithm with different levels of feedback.

We compare the performance of the scheduling strategies for  $K = 16$ -user  $4 \times 4$  MIMO system with different levels of feedback to investigate the benefit gained by exploiting the CSI in Figure 7. As expected, the performance of the algorithms is improved as the level of feedback is increased and the scheduling algorithm with transmit beamformer selection performs the best. However, we note that the largest relative gain is obtained when the feedback related to each user's own CSI is available, which enables each user to select its transmit beamforming vector so that its received power is maximized.

Throughout the simulations, the performance results show that as the level of feedback increases, the performance of the proposed scheduling scheme gets better and comes close to the upper bounds. The individual CSI provides a substantial capacity gain due to the fact that the individual received powers can be maximized by channel information at the transmitter. As expected, the gap between the performance of the proposed algorithm and the upper bounds decreases as the dimension of the MIMO system is increased.

## 7. CONCLUDING REMARKS

In this paper, we considered the problem of designing scheduling algorithms for multiuser MIMO systems with low complexity that achieve near-optimum sum capacities. As the problem of finding the optimum scheduler is NP-complete, we have taken the approach of designing sched-

ulers with good heuristics, that perform close to sum capacity upper bounds we developed for this system. Since the performance of the scheduler depends on the transmit beamformers, we have considered transmit beamformer selection next, and proposed several methods with different levels of feedback at the transmitter side. We have observed that as the feedback level at the transmitter is increased, the performance of the proposed algorithms approaches the capacity upper bounds, and consequently it approaches the capacity of the optimum scheduler. Notably, the individual CSI feedback facilitates a substantial gain.

## 8. ACKNOWLEDGMENT

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