

Extended Lock Range Zero-Crossing Digital Phase-Locked Loop with Time Delay

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The input frequency limit of the conventional zero-crossing digital phase-locked loop (ZCDPLL) is due to the operating time of the digital circuitry inside the feedback loop. A solution that has been previously suggested is the introduction of a time delay in the feedback path of the loop to allow the digital circuits to complete their sample processing before the next sample is received. However, this added delay will limit the stable operation range and hence lock range of the loop. The objective of this work is to extend the lock range of ZCDPLL with time delay by using a chaos control. The tendency of the loop to diverge is measured and fed back as a form of linear stabilization. The lock range extension has been confirmed through the use of a bifurcation diagram, and Lyapunov exponent.

Keywords and phrases: nonuniform sampling, digital phase locked loops, chaos control.

1. INTRODUCTION

Digital phase locked loops (DPLLs) were introduced to minimize some of the problems associated with the analogue loops such as sensitivity to DC drift and the need for periodic adjustments [1, 2]. The most commonly used DPLL is the zero-crossing digital phase-locked loop (ZCDPLL). The ZCDPLL operation is based on nonuniform sampling techniques. The loop is simple to implement and easy to model. The ZCDPLL consists of a sampler that acts as phase detector, digital filter, and digital-controlled oscillator (DCO). In the ZCDPLL, there is a limit on the frequency of the incoming signal beyond which the loop ceases to function properly any longer. This limit is reached when the period of the incoming signal becomes equal to the total operating time of the digital circuits in the loop. One way to increase this upper limit of the input frequency is by the introduction of a time delay in the loop. In this case the sampling instances controlled by the DCO are determined by the sample of the input which was taken two sampling intervals earlier. Therefore, the upper limit of the operating frequency of the ZCDPLL can be increased. The introduction of the delay, however, will limit the loop stability range or the lock range of the loop as will be seen later.

The objective of this work is to increase the stability and lock range of ZCDPLL with time delay by incorporat-

ing a chaos control technique known as “time-delayed feedback stabilization.” The ZCDPLL has been shown to exhibit chaotic behaviour in the unstable region of operation [3]. Time-delayed feedback stabilization introduced by Pyragas consists of a continuous linear feedback applied at each computation time step which stabilizes unstable periodic orbits (UPO) [4]. Pyragas’s method is used to broaden the tracking range by extending the stable operation behaviour of the first-order ZCDPLL to a larger control parameter (K_1), which leads to larger input frequency ω . Our results are based on bifurcation theory and numerical simulation. Chaos control technique is used to overcome the problem of limited operating range when a time delay is added to the feedback path of the loop. The paper analyzes the steady-state loop operation ZCDPLL and chaos-controlled ZCDPLL. The pull-in behaviour, higher-order loops will be considered in future work.

In Section 2, the ZCDPLL with time delay model is described. Section 3 presents the chaos control technique used to broaden the lock range. In Section 4 simulation results are presented, and finally conclusions are given in Section 5.

2. ZCDPLL WITH TIME DELAY

The structure of ZCDPLL with time delay is shown in Figure 1. The first register simply serves to store incoming data temporarily until the filter portion finishes its operation on previous data. As soon as the filter finishes its operation, the stored data are transferred to the second register and the first register is cleared to be ready for taking in new data.

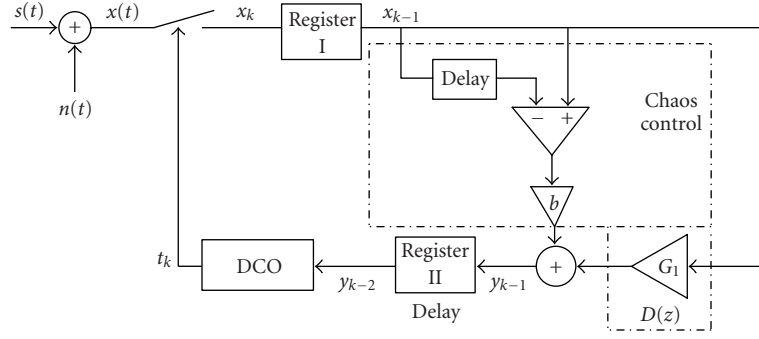


FIGURE 1: Block diagram of chaos-controlled ZCDPLL with time delay.

The input signal to the loop is taken as $x(t) = s(t) + n(t)$, where $s(t) = A \sin(w_0 t + \theta(t))$, $n(t)$ is additive white Gaussian noise (AWGN); $\theta(t) = \theta_0 + \Omega_0 t$ from which the signal dynamics are modeled; θ_0 is the initial phase which we will assume to be zero; Ω_0 is the frequency offset from the nominal value w_0 . The input signal is sampled at time instances t_k determined by the digital-controlled oscillator (DCO). The DCO period control algorithm as given by [5] is

$$T_k = T_0 - y_{k-2} = t_k - t_{k-1}, \quad (1)$$

where $T_0 = (2\pi/w_0)$ is the nominal period, y_{k-2} is the delayed output of the loop digital filter $D(z)$. The sample value of the incoming signal $x(t)$ at t_k is

$$x(t_k) = s(t_k) + n(t_k) \quad (2)$$

or

$$x_k = s_k + n_k, \quad (3)$$

where $s_k = A \sin[w_0 t_k + \theta(t_k)]$. The sequence x_k is passed through a digital filter $D(z)$ whose output y_k is used to control the period of the DCO. The time instances t_k can be rewritten as

$$t_k = \sum_{i=1}^k T_i = kT_0 - \sum_{i=0}^{k-2} y_i, \quad k = 1, 2, 3, \dots \quad (4)$$

Thus

$$x_k = A \sin \left[w_0 \left(kT_0 - \sum_{i=0}^{k-2} y_i \right) + \theta_k \right] + n_k. \quad (5)$$

For noise-free analysis $n_k = 0$, then

$$x_k = A \sin \left[w_0 \left(kT_0 - \sum_{i=0}^{k-2} y_i \right) + \theta_k \right]. \quad (6)$$

The phase error is defined to be

$$\phi_k = \theta_k - w_0 \sum_{i=0}^{k-2} y_i. \quad (7)$$

Also

$$\phi_{k-1} = \theta_{k-1} - w_0 \sum_{i=0}^{k-3} y_i. \quad (8)$$

Taking the difference of (7) and (8) results in

$$\phi_k - \phi_{k-1} = \theta_k - \theta_{k-1} - w_0 y_{k-2}. \quad (9)$$

The z transform of the output of the digital filter is

$$Y(z) = D(z)X(z), \quad (10)$$

where $X(z)$ is the z transform of $x(t)$. If the digital filter used is a gain block only, then $D(z) = G_1$, where G_1 is the block gain. In this case $Y(z) = G_1 X(z)$, and the time domain equivalent will be $y_k = G_1 x_k = A G_1 \sin[\phi_k]$.

If a frequency step $\Omega_0 = (w - w_0)$ is applied, then $\theta_k = (w - w_0)t_k$. Using (4), then

$$\theta_k - \theta_{k-1} = (w - w_0)T_0 - (w - w_0)y_{k-2}. \quad (11)$$

Equation (9) can also be rewritten as

$$\phi_k = \phi_{k-1} - K_1 \sin[\phi_{k-2}] + \Lambda_0 = f(\phi), \quad (12)$$

where $\Lambda_0 = (w - w_0)T_0 = 2\pi((w - w_0)/w_0)$, $K_1 = wG_1A$. It can be easily shown that this system has equilibrium state at $\sin^{-1}(\Lambda_0/K_1)$, not at $\phi = 0$. This implies that $|\Lambda_0/K_1| < 1$, or $K_1 > |\Lambda_0|$. The following transformation makes the equilibrium at $\phi = 0$:

$$\psi_k = \phi_k - \sin^{-1} \frac{\Lambda_0}{K_1}, \quad (13)$$

then

$$\psi_k = \psi_{k-1} - K_2 \sin[\psi_{k-2}] - \Lambda_0 \cos(\psi_{k-2}) + \Lambda_0, \quad (14)$$

where $K_2 = \sqrt{K_1^2 - \Lambda_0^2}$. Define the system state vector $\zeta_k = \psi_{k-2}$, $\xi_k = \psi_{k-1}$, $\mathbf{x} = (\zeta, \xi)^T$, then (14) can be rewritten as

$$\begin{pmatrix} \zeta_{k+1} \\ \xi_{k+1} \end{pmatrix} = \begin{pmatrix} \xi_k \\ \xi_k - K_2 \sin(\zeta_k) - \Lambda_0 \cos(\zeta_k) + \Lambda_0 \end{pmatrix} \triangleq \begin{pmatrix} g_1(\mathbf{x}_k) \\ g_2(\mathbf{x}_k) \end{pmatrix}. \quad (15)$$

If the system equation is linearised around the equilibrium $\mathbf{x} = 0$, so that $\sin(\zeta_k) \approx \zeta_k$, $\sin(\xi_k) \approx \xi_k$, $\cos(\zeta_k) \approx 1$, $\cos(\xi_k) \approx 1$, then (15) becomes

$$\begin{pmatrix} \zeta_{k+1} \\ \xi_{k+1} \end{pmatrix} = \begin{pmatrix} \zeta_k \\ \xi_k - K_2 \zeta_k \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -K_2 & 1 \end{pmatrix} \begin{pmatrix} \zeta_k \\ \xi_k \end{pmatrix} = \mathbf{B}\mathbf{x}_k. \quad (16)$$

To satisfy Lyapunov stability criterion for the above system, the matrix $\mathbf{B}^T\mathbf{B} - \mathbf{I}$ must be negative definite. This implies that the eigenvalues of $\mathbf{B}^T\mathbf{B}$ must be less than one [6]. This in turn results that $K_2^2 < 1$ or $K_1 < \sqrt{1 + \Lambda_0^2}$, which is less than that of conventional ZCDPLL range of operation ($K_1 < \sqrt{4 + \Lambda_0^2}$) [5].

We re-examine the loop stability in terms of the variable $K_{10} = (G_1 w_0 A)$, which is directly related to the filter gain and free running frequency of the DCO. In this case,

$$2\pi \left| \frac{w}{w_0} - 1 \right| < K_{10} \frac{w}{w_0} < \sqrt{1 + (2\pi)^2 \left(\frac{w}{w_0} - 1 \right)^2}. \quad (17)$$

The condition of the convergence becomes

$$2\pi \left| 1 - \frac{w_0}{w} \right| < K_{10} < \sqrt{1 + (2\pi)^2 \left(\frac{w_0}{w} \right)^2 - 2(2\pi)^2 \left(\frac{w_0}{w} \right) + (2\pi)^2}. \quad (18)$$

3. EXTENDED LOCK RANGE ZCDPLL WITH TIME DELAY

The time-delayed feedback stabilization introduced by Pyragas is incorporated and used to extend the stable behaviour of ZCDPLL with time delay to larger values of K_1 . Just beyond the critical values of K_1 , the tendency of the loop to converge to UPO is measured by the value $\Delta x_{k-1} = A(\sin(\phi_{k-1}) - \sin(\phi_{k-2}))$. A multiple of these differences $b\Delta x_{k-1}$, where b is an empirical adjustable weight, is fed back as a form of non-linear stabilization [7]. The resultant system dynamics will be

$$\phi_k = \phi_{k-1} - (K_1 + b) \sin[\phi_{k-2}] + b \sin(\phi_{k-1}) + \Lambda_0. \quad (19)$$

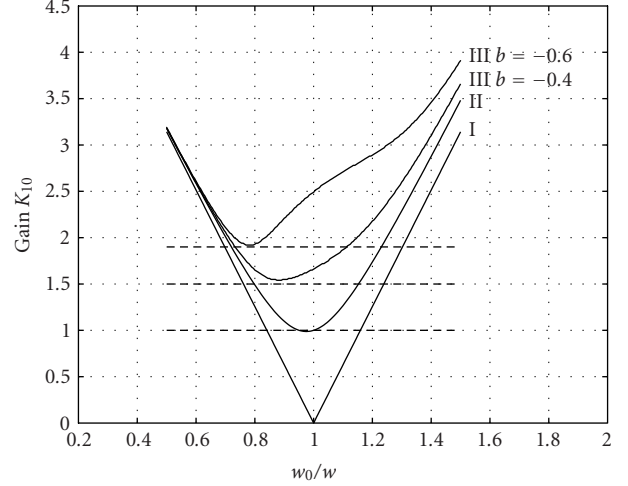


FIGURE 2: Frequency lock region as a function of K_{10} and w_0/w .

Using the same procedure of previous section, (17) can be written as

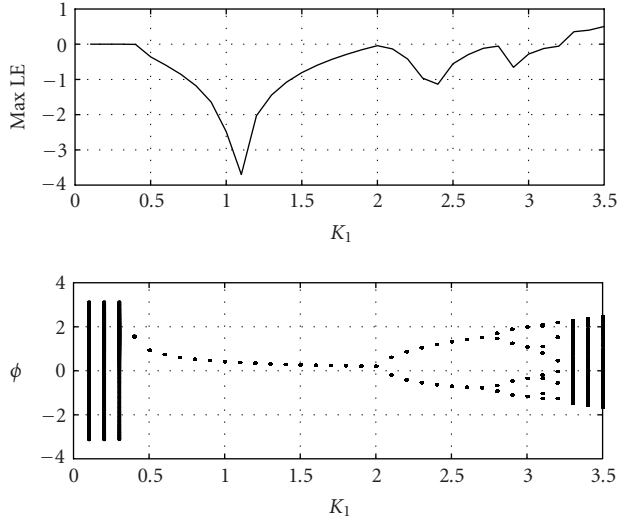
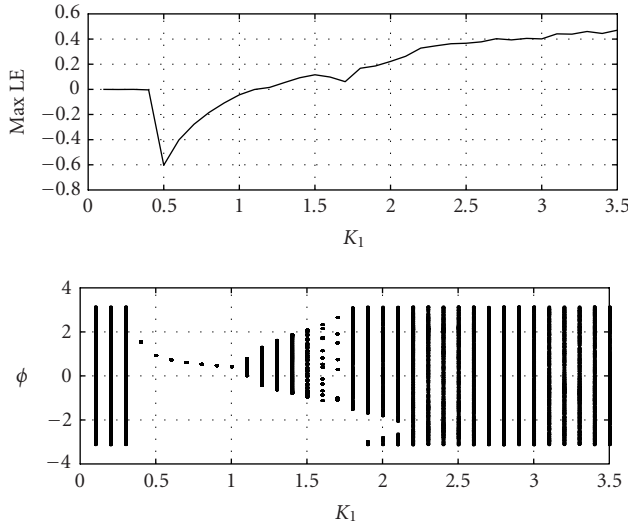
$$\begin{pmatrix} \zeta_{k+1} \\ \xi_{k+1} \end{pmatrix} = \begin{pmatrix} \xi_k \\ \xi_k \left(1 + b \frac{K_2}{K_1}\right) - K_2 \left(1 + b \frac{K_2}{K_1}\right) \zeta_k \end{pmatrix} \quad (20)$$

$$\triangleq \begin{pmatrix} g_1(\mathbf{x}_k) \\ g_2(\mathbf{x}_k) \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -K_2 r & r \end{pmatrix} \begin{pmatrix} \zeta_k \\ \xi_k \end{pmatrix},$$

where $r = 1 + b(K_2/K_1)$. Following the Lyapunov stability criteria, K_2^2 should be less than $1/r^2$ in order to guarantee that the eigenvalues of $\mathbf{B}^T\mathbf{B}$ are less than one. The condition for the loop locking is

$$(K_1^2 - \Lambda_0^2) \left(1 + b \frac{\sqrt{K_1^2 - \Lambda_0^2}}{K_1}\right)^2 < 1. \quad (21)$$

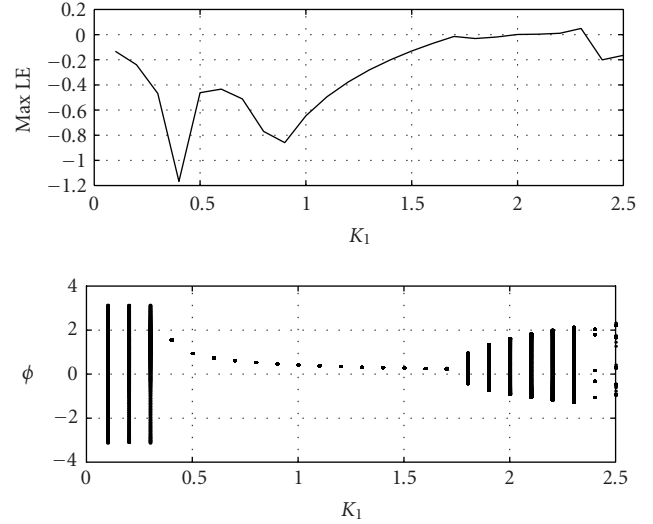
Stabilization corresponds to negative values of (b), in which the feedback term corrects the tendency to UPO [7]. Since b is negative, then r will be less than one and this will ensure that the loop can have K_1 higher than that for ZCDPLL with time delay derived in previous section. The convergence region of the conventional ZCDPLL with time delay described by (18) and the proposed chaos-controlled ZCDPLL described by (21) for different input frequency offsets have been plotted in Figure 2. The region between the curves I and II indicates the region of stable operation for the conventional ZCDPLL, while the region between the curves I and III represents that for the proposed loop plotted for different values of b . It is clear that the widest lock range occurs at $K_{10} = 1.0$ for the conventional ZCDPLL, while it will be at $K_{10} = 1.5$ when the value of b is -0.4 for the proposed chaos-controlled ZCDPLL. Figure 2 also shows that when the value of b is increased to -0.6 , the proposed chaos-controlled loop has widest lock range occurring at $K_{10} \approx 2.0$. So the greater the absolute value of b , the wider the lock range. Thus, the K_{10} , w_0/w plane gives a realistic indication of the loop's ability to track frequency offsets.

FIGURE 3: Bifurcation of conventional ZCDPLL with $\Lambda_0 = 0.4$.FIGURE 4: Bifurcation of ZCDPLL with delay with $\Lambda_0 = 0.4$.

The nonlinear feedback procedure is applied without a priori knowledge of the location of the periodic orbits. A disadvantage of the method is that it achieves control over a limited range of the parameter space (b values). A given orbit will become eventually unstable if the feedback parameter is varied beyond that range.

4. SYSTEM PERFORMANCE

Consider a modulation-free input signal $y(t) = \sin(\omega t)$, where the center frequency of the DCO is $\omega_0 = 1$. After discarding the first 1000 points, the next 100 000 points are collected and recorded to produce a bifurcation plot and maximum Lyapunov exponent. The numeric bifurcation diagrams will be used to study the operation range of the ZCDPLL with time delay. Along the horizontal axis of the bifurcation diagram, the parameter K_1 of the system is varied, while the successive values of the phase error ϕ_k are plotted.

FIGURE 5: Bifurcation of chaos-controlled ZCDPLL with delay with $\Lambda_0 = 0.4$ and $b = -0.7$.

Lyapunov exponent can be used to measure the exponential divergence of trajectory in a dynamical system. The exponent measures the average rate of separation of two nearby trajectories coming from different initial conditions. A positive Lyapunov exponent indicates chaos and the system will be very sensitive to initial conditions. The largest Lyapunov exponent (LE) for the two-dimensional dynamical systems is defined as [8]

$$LE = \lim_{N \rightarrow \infty} \frac{1}{2N} \sum_{n=0}^{N-1} \ln \left| \frac{(a + bY'_n)^2 + (c + dY'_n)^2}{(1 + Y_n'^2)} \right|, \quad (22)$$

where Y' is the tangent of the direction of maximum growth which evolves according to

$$Y'_{n+1} = \frac{(c + dY'_n)}{(a + bY'_n)}, \quad (23)$$

and $a = \partial g_1 / \partial x_k$, $b = \partial g_1 / \partial y_k$, $c = \partial g_2 / \partial x_k$, and $d = \partial g_2 / \partial y_k$ are members of the Jacobian matrix of (15), (16), and (20).

In order to study the behaviour of the ZCDPLL with time delay, it is advantageous to use bifurcation diagrams and maximum LE. Figure 3 shows the bifurcation diagrams and maximum Lyapunov exponent of conventional first-order loop ZCDPLL as the controlled parameter K_1 is varied from 0 up to 3.5. It shows that the loop has stable operation for $\Lambda_0 < K_1 < \sqrt{4 + \Lambda_0^2}$ and this agrees with the normal loop operation found earlier [5]. Also it shows that LE will be positive in the chaotic operation range. When a time delay is added to conventional ZCDPLL, the range of K_1 which offers stable loop operation is reduced as shown in Figure 4 and the new value is only $\Lambda_0 < K_1 < \sqrt{1 + \Lambda_0^2}$. This agrees with the range derived in Section 2. This reduced operation range will affect the lock range of the loop. The bifurcation plot and LE for ZCDPLL with time delay is provided in Figure 5

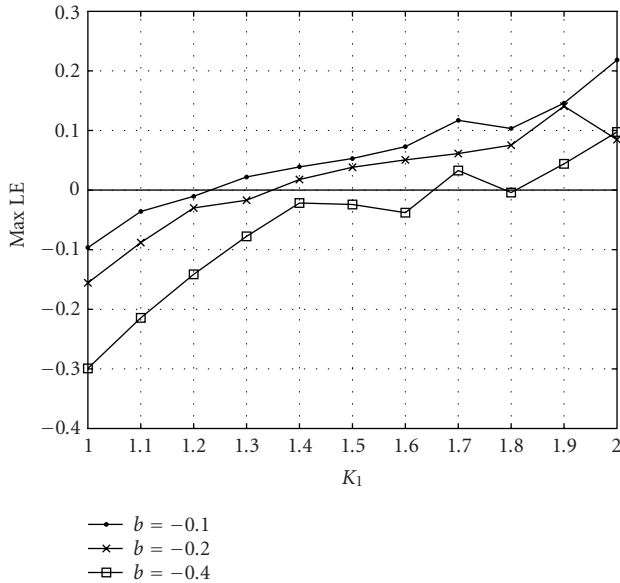


FIGURE 6: Maximum Lyapunov of chaos-controlled ZCDPLL with delay for different values of b when $\Lambda_0 = 0.4$.

for the control parameter $b = -0.7$, where $b = 0$ corresponds to the conventional ZCDPLL. It is clear that chaos-controlled ZCDPLL will start period doubling process when $K_1 = 1.7$ compared to $K_1 = 1.1$ for conventional ZCDPLL with time delay. The proposed chaos-controlled ZCDPLL with time delay extends the stable region of operation to higher values of K_1 which leads to higher operating frequency (higher tracking range). Figure 6 shows the variation of largest Lyapunov exponent versus the value of K_1 for different values of the feedback control parameter (b). A positive largest Lyapunov exponent corresponds to chaotic operation. It can be seen that chaos-controlled ZCDPLL offers convergent-to-fixed point operation at higher K_1 values compared to conventional ZCDPLL. Figure 6 indicates that for $b = -0.4$, the highest value of K_1 will be around 1.65 and this corresponds to 1.5 times the highest input frequency of conventional loop. If the absolute value of b is increased further to about $b = -2.0$, the loop can no longer be controlled and it will exhibit chaotic behaviour as shown in Figure 7. So it is desirable to select the value of b carefully to avoid such chaotic loop operation.

5. CONCLUSIONS

The limit on the incoming signal frequency beyond which the zero-crossing digital phase-locked loop (ZCDPLL) does not function properly can be extended by the addition of a time delay in the feedback path of the loop. This paper has proposed and described a chaos control technique to broaden the tracking range of ZCDPLL with time delay. The delayed feedback control method of chaos control proposed by Pyragas is used to stabilize the ZCDPLL chaotic operation. A feedback loop which measures loop tendency to chaos is used to bring the ZCDPLL from chaotic operation region back to its stable orbit. The bifurcation plot and largest Lyapunov

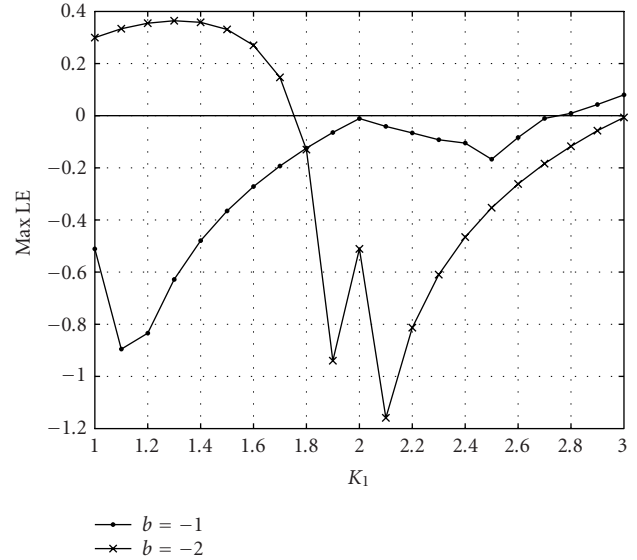


FIGURE 7: Maximum Lyapunov of chaos-controlled ZCDPLL with delay for different values of b when $\Lambda_0 = 0.4$.

exponent shown in Figures 5 and 6 display the dependence of chaos-controlled ZCDPLL convergent operation upon the feedback parameter b . As the feedback control parameter b is varied, the loop will remain in stable orbit for larger values of K_1 . This will extend the range of incoming signal frequency or expand the tracking range. While the conventional ZCDPLL bifurcated when $K_1 \approx 1.1$, Figure 5 shows that the chaos-controlled ZCDPLL bifurcates when $K_1 \approx 1.7$. The same effect can be seen when maximum Lyapunov exponent is determined instead of the bifurcation diagram as shown in Figure 6. The Lyapunov exponent will be positive at higher value of K_1 as the absolute value of the control parameter b is increased and hence wider lock range. Figure 7 indicates that if the absolute value of b is increased such that $b = -2$, the chaos-controlled ZCDPLL with time delay is completely unstable for the range of K_1 used. So it is desirable to select the value of b carefully to avoid such uncontrolled chaotic behaviour of the loop.

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