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Impact of the Gaussian Approximation on the Performance of the Probabilistic Data Association MIMO Decoder

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The probabilistic data association (PDA) decoder is investigated for use in coded multiple-input multiple-output (MIMO) systems and its strengths and weaknesses are determined. The conventional PDA decoder includes two approximations. The received symbols are assumed to be statistically independent and a Gaussian approximation is applied for the interference and noise term. We provide an analytical formula for the exact probability density function (PDF) of the interference and noise term, which is used to discuss the impact of the Gaussian approximation in the presence of a soft-input soft-output channel decoder. The results obtained resemble those obtained for the well-known PDA multiuser detector in coded CDMA systems for which similar investigations have been done before.

Keywords and phrases: probabilistic data association, MIMO systems, stochastic approximation, iterative methods, interference.

1. INTRODUCTION AND BACKGROUND

Probabilistic data association (PDA) has originally been developed for target tracking by Yaakov Bar-Shalom in the 1970s. Since then, it has been applied in many different areas, including digital communications. In the area of digital communications, the PDA algorithm is a reduced complexity alternative to the *a posteriori* probability (APP) decoder/detector/equalizer. Near-optimal results were demonstrated for a PDA-based multiuser decoder (MUD) for code division multiple access (CDMA) systems [1, 2]. Recently, probabilistic data association has been shown to achieve good results in multiple-input multiple-output (MIMO) systems [3, 4]. In [5], a PDA was presented for turbo equalization of a single antenna system. It should also be noted that the Gaussian assumption made in the PDA decoder is used in several other MUD detection schemes, especially when applying iterative detection and decoding schemes, for example, [6, 7, 8]. In [9], it was shown that the performance of a coded CDMA system with PDA decoder degrades if the number of users is not large enough.

In this paper, results for a PDA MIMO decoder in combination with a soft-input soft-output channel decoder are presented, where both decoders are not forming an iterative detection and decoding scheme (see Figure 1). This is done in order to demonstrate the impact of the unreliable

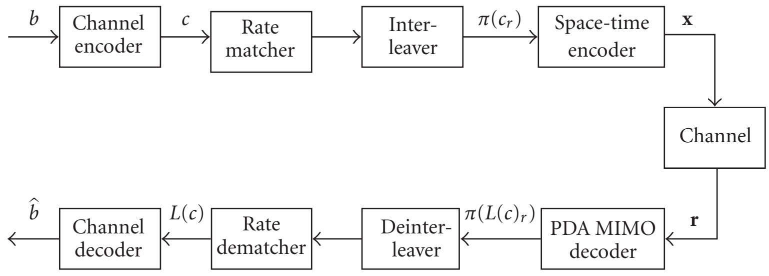

Figure 1: Communication system investigated throughout this paper.

soft outputs which is far less obvious when using an iterative decoding scheme. Because the PDA decoder inherently provides estimates of the *a posteriori* probabilities of the transmitted data symbols, it seems to be well suited for the use in conjunction with a soft-input channel decoder. However, the results presented in the following show that the PDA MIMO decoder does not always work as well as expected. We provide an exact formula for the probability density function (PDF) of the interference and noise term to calculate the exact symbol probabilities for the symbol-by-symbol detection done in the PDA. Simulations based on these probabilities show that the Gaussian approximation made in the PDA decoder has a large impact on the quality of the soft outputs provided to the channel decoder, and therefore on the channel decoding itself. It can be concluded that the quality of the Gaussian approximation, and therefore of the soft outputs, depends on the number of transmit antennas and on the cardinality of the symbol alphabet. To our best knowledge, such an analysis of the PDA MIMO decoder has not been presented before.

The remainder of this paper is organized as follows. We first introduce the system model under investigation in Section 2. In Section 3, a PDA decoder for use in a coded MIMO system is presented, followed by an analysis of the Gaussian approximation and its impact on the decoding process. A confirmation of the analytical results in form of simulations is given in Section 4. Conclusions are drawn in Section 5.

2. SYSTEM MODEL

Consider a MIMO system with M transmit and N receive antennas. Like in V-BLAST [10], a single data stream is multiplexed into M parallel data streams and then mapped onto complex modulation symbols. The M symbols are transmitted simultaneously by the corresponding antennas. Before the multiplexing is done, the data stream is encoded by a channel encoder and interleaved by a channel interleaver. Assuming flat fading, the equivalent discrete-time channel model can be written in complex baseband notation as

$$\mathbf{r} = \mathbf{H}\mathbf{x} + \mathbf{v}, \tag{1}$$

where baud-rate sampling is assumed. The channel matrix $\mathbf{H} \in \mathbb{C}^{N\times M}$ is assumed to be constant during one data block (block fading assumption) and perfectly known at the receiver. The channel matrix coefficients $h_{n,m}$ represent the gain between transmit antenna m $(1 \le m \le M)$ and receive antenna n $(1 \le n \le N)$. The vector $\mathbf{x} \in \mathbb{Q}^{M\times 1}$ consists of the complex-valued transmitted modulation symbols taken from a symbol alphabet $\mathbb{Q}$ with cardinality Q, while the vector $\mathbf{r} \in \mathbb{C}^{N\times 1}$ contains the received samples. Additive noise is given by $\mathbf{v} \in \mathbb{C}^{N\times 1}$, with complex elements that are independent and identically distributed white Gaussian noise samples with zero mean and variance $\sigma_v^2 = \mathrm{E}\{|v_n|^2\}$. At the receiver, the demultiplexing, or MIMO decoding, operation is performed by the PDA followed by a deinterleaver and a soft-input channel decoder. An overview of the system is given in Figure 1. Please note that no turbo equalization as in [4, 5] or feedback from the channel decoder to the PDA as in [11] is used.

3. PDA DECODER

3.1. Basic algorithm

The conventional PDA decoder[1] uses two approximations. Firstly, the PDA decoder looks only at one transmitted symbol at a time, treating the received symbols as statistically independent. A second approximation is the Gaussian approximation ("Gaussian forcing") of the PDF of the interference and noise. The PDA decoder approximates *a posteriori* probabilities $\Pr(x_m \mid \mathbf{r})$ for every element x_m of $\mathbf{x}$. All symbols interfering with x_m and the noise are modeled as a single vector

$$\mathbf{w} = \sum_{k\neq m} x_k \mathbf{h}_k + \mathbf{v}, \tag{2}$$

where $\mathbf{h}_k$ denotes the kth column of $\mathbf{H}$, and x_k the kth element of $\mathbf{x}$. The interference and noise term in (2) is assumed to be an n-variate Gaussian distributed random variable with mean

$$\boldsymbol{\mu}_w = \mathrm{E}\Big\{\sum_{k\neq m} x_k \mathbf{h}_k + \mathbf{v}\Big\} = \sum_{k\neq m} \mathrm{E}\{x_k\}\mathbf{h}_k, \tag{3}$$

[1] The conventional PDA decoder uses the non-decoupled system model (which means that the received signal $\mathbf{r}$ is not multiplied with the inverse of the channel matrix $\mathbf{H}^{-1}$). Because we are interested in a fundamental property of the PDA decoder rather than complexity reduction, no complexity reduction techniques as proposed in [1] are applied. Hence, the PDA decoder presented here suffers from higher complexity, but achieves nearly the same performance and gives most of the equations a more comprehensive look.

and covariance

$$\mathbf{R}_{ww} = \mathrm{E}\Big\{\left(\mathbf{w} - \boldsymbol{\mu}_w\right)\left(\mathbf{w} - \boldsymbol{\mu}_w\right)^H\Big\} = \sum_{k \neq m} \mathrm{Var}\left\{x_k\right\}\mathbf{h}_k\mathbf{h}_k^H + \sigma_v^2\mathbf{I}. \tag{4}$$

If no *a priori* information is available, the PDA decoder initializes the symbol probabilities as a uniform distribution. Assuming the Gaussian distribution of the noise and interference term, the *a posteriori* probabilities for the possible symbols x_m can be computed using (3) and (4):

$$\begin{aligned}\Pr\left(x_m \mid \mathbf{r}\right) &= \frac{p\left(\mathbf{r} \mid x_m\right)\Pr\left(x_m\right)}{p(\mathbf{r})} \\ &= c\exp\Big(-\left(\mathbf{r} - x_m\mathbf{h}_m - \boldsymbol{\mu}_w\right)^H\mathbf{R}_{ww}^{-1}\left(\mathbf{r} - x_m\mathbf{h}_m - \boldsymbol{\mu}_w\right)\Big).\end{aligned} \tag{5}$$

For an estimate of the symbol x_m, no information on symbols x_k, $k \geq m$, is available. In order to provide information on these symbols, the PDA decoder may use multiple iterations, in each iteration using the symbol probabilities obtained by the previous iteration. As in [1], the mean (3) and the variance (4) are updated for every symbol probability estimate, incorporating the new information gained from symbol probabilities already computed in the current or previous iterations. Given the PDF in (5), log-likelihood ratios (LLRs) can be computed to serve as soft-input for the channel decoder after the last iteration of the PDA decoder:

$$L(c_\kappa) = \log\frac{\sum_{x_m \in \mathbb{Q}^+}\Pr\left(x_m \mid \mathbf{r}\right)}{\sum_{x_m \in \mathbb{Q}^-}\Pr\left(x_m \mid \mathbf{r}\right)}, \tag{6}$$

where

$$\mathbb{Q}^+ := \left\{x_m : c_\kappa = \mathrm{bit}_\kappa(x_m) = +1\right\}, \tag{7}$$

$$\mathbb{Q}^- := \left\{x_m : c_\kappa = \mathrm{bit}_\kappa(x_m) = -1\right\}. \tag{8}$$

3.2. Actual PDF of interference and noise term

The actual PDF of the interference and noise term is a sum of Q^{M-1} Gaussian distributions, each of them caused by one possible interfering symbol constellation as a convolution of the discrete symbol probabilities and the PDF of the Gaussian noise vector $\mathbf{v}$. Let $\mathbb{X}^s$ be the set of all possible symbol vector combinations causing interference for a fixed x_m. It can be easily shown that the actual PDF of the interference and noise term is

$$p_{\mathrm{W}}\left(\mathbf{w}(\mathbf{v})\right) = \sum_{\mathbf{x} \in \mathbb{X}^s}\Pr(\mathbf{x})\left(\frac{1}{\pi\sigma_v^2}\right)^N\exp\left(-\frac{\left\|\mathbf{v} - \sum_{k \neq m} x_k\mathbf{h}_k\right\|^2}{\sigma_v^2}\right). \tag{9}$$

The PDF in (9) is a summation of $Q^{M-1} = |\mathbb{X}^s|$ single Gaussian distributions with means depending on the channel as well as the interfering modulation symbols. It is *not* the PDF used for optimal (APP) detection; being the exact PDF of the interference for one of the detected symbols, it is not employing the Gaussian approximation but still treating the symbols as statistically independent. A derivation for the CDMA case can be found in [12, Chapter 3.1] and was also published in [6].

According to the central limit theorem, the quality of the Gaussian approximation used in the PDA decoder improves by increasing the number of transmit antennas. On the other hand, the approximation becomes worse when modulation schemes with more constellation points are used. With an increasing number of constellation points, a soft bit according to (6) is calculated by a larger number of (approximated) probabilities, and is therefore more likely to be unreliable. It should also be noted that the approximation is better in the presence of strong noise. As can be seen in (9), the variance of the single Gaussian distribution is larger for a larger σ_v^2, which makes the sum more likely to be Gaussian-like.

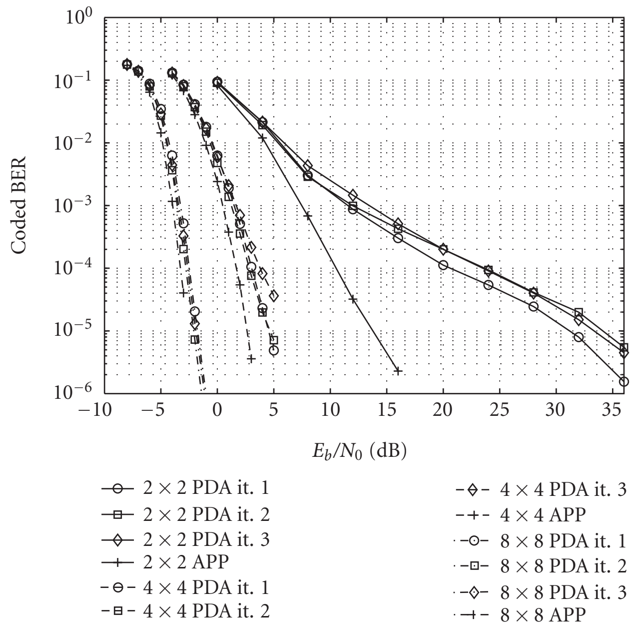

FIGURE 2: BER performance of a turbo-coded $M \times N$ MIMO system with PDA decoder. As a benchmark, the BER performance for an APP decoder is shown as well.

3.3. Consequences for soft-input channel decoder

Soft-input channel decoders use reliability information on the input in form of LLRs. The reliability of the LLRs is essential for channel decoding; unreliable soft inputs cause wrong estimates of the information bits. The LLRs delivered by the PDA decoder are calculated from the symbol probabilities which are based on the approximated PDF of the interference and noise term. As shown above, the Gaussian approximation, and therefore the soft inputs of the channel decoder, can be quite poor and thus inhibits the channel decoder from achieving good performance. Similar results were obtained for a coded CDMA system in [9].

4. NUMERICAL RESULTS

In order to illustrate the influence of the Gaussian assumption on the performance of the PDA decoder, an $M \times N$

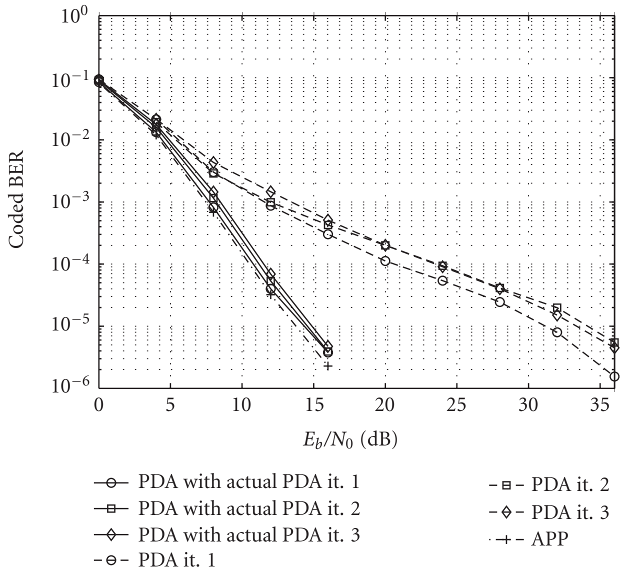

Figure 3: BER performance of a turbo-coded 2×2 MIMO system with conventional PDA decoder and PDA decoder using the actual PDF of the interference and noise term. As a benchmark, the BER performance for an APP decoder is shown as well.

MIMO system in conjunction with a turbo code has been investigated. As a benchmark, the BER performance for an APP decoder has been simulated as well. A block length of 2304 information bits is used. The bit energy to noise ratio is defined as $E_b/N_0 = \sigma_x^2\sigma_h^2/qR\sigma_v^2$, with q being the number of bits per modulation symbol and R denoting the code rate. The average power per symbol constellation point is denoted by σ_x^2. The elements $h_{n,m}$ of $\mathbf{H}$ are statistically independent random variables (each component being complex Gaussian distributed with zero mean and variance $\sigma_h^2 = \mathrm{E}\{|h_{n,m}|^2\}$). A rate 1/2 turbo code with polynomials (5,7) and 4 iterations in the turbo decoder is applied. The rate matcher ensures that the coded block length is a multiple of qM, and therefore can be multiplexed to the M transmit antennas.

The number of iterations given in Figures 2 and 3 are the iterations done in the PDA algorithm before the soft estimates of the bits are given to the channel decoder. While the PDA achieves good results when using no channel code [3], the results of the coded system can be far from the optimum. In Figure 2, it can be seen that the difference between the APP and the PDA decoder is the largest for the 2×2 system and improves with an increasing number of antennas. Especially for the 2×2 system, the gap between the APP and the PDA decoder is getting larger with an increase in E_b/N_0. Furthermore, the third iteration is not, as it should be, the best one. This is explained by the quality of the soft-output generated by the PDA decoder, which degrades with every iteration as (unreliable) probabilities computed by the previous iteration are used.

To demonstrate the impact of the Gaussian approximation on the performance of the coded PDA system, in Figure 3, the results for the 2×2 system are shown for the PDA decoder using the Gaussian approximation compared to the decoder using the actual PDF of the interference and noise. It is clearly seen that the problems arise from the Gaussian approximation made in the PDA, as the PDA decoder using the nonapproximated PDF achieves near-optimal results. We have found similar results for convolutional codes and different code rates.

5. CONCLUSION

The impact of the Gaussian approximation in the conventional PDA MIMO decoder on the performance of a MIMO system using a soft-input channel decoder was shown. It was shown that the Gaussian approximation is the best for a large number of transmitting antennas and a small number of constellation points in the modulation scheme. Its influence on the quality of the soft outputs, and therefore the channel decoder has been investigated. Furthermore, it has been illustrated that the main degradation of the performance of the PDA decoder is the Gaussian approximation and not the symbol-by-symbol decoding. The results of this paper hold, in principle, also for a multiuser detection scenario where the usually large number of interferers results in a good approximation. The PDA decoder was applied in iterative decoding schemes for CDMA [2] and MIMO [11] systems. In iterative schemes, the PDA decoder may achieve a performance close to optimum. A formula for the actual PDF of interference and noise for CDMA MUD can be found in [12]. A way to improve the performance when using the PDA MIMO decoder with a soft input channel decoder might be importance sampling as proposed in [11] or the combination with sphere decoding [13].

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