Hindawi Publishing Corporation
EURASIP Journal on Wireless Communications and Networking
Volume 2006, Article ID 43275, Pages 1–10
DOI 10.1155/WCN/2006/43275

Decision-Directed Recursive Least Squares MIMO Channels Tracking

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Received 14 June 2005; Revised 22 November 2005; Accepted 22 December 2005

Recommended for Publication by Jonathon Chambers

A new approach for joint data estimation and channel tracking for multiple-input multiple-output (MIMO) channels is proposed based on the decision-directed recursive least squares (DD-RLS) algorithm. RLS algorithm is commonly used for equalization and its application in channel estimation is a novel idea. In this paper, after defining the weighted least squares cost function it is minimized and eventually the RLS MIMO channel estimation algorithm is derived. The proposed algorithm combined with the decision-directed algorithm (DDA) is then extended for the blind mode operation. From the computational complexity point of view being $O(3)$ versus the number of transmitter and receiver antennas, the proposed algorithm is very efficient. Through various simulations, the mean square error (MSE) of the tracking of the proposed algorithm for different joint detection algorithms is compared with Kalman filtering approach which is one of the most well-known channel tracking algorithms. It is shown that the performance of the proposed algorithm is very close to Kalman estimator and that in the blind mode operation it presents a better performance with much lower complexity irrespective of the need to know the channel model.

1. INTRODUCTION

In recent years, MIMO communications are introduced as an emerging technology to offer significant promise for high data rates and mobility required by the next generation wireless communication systems [1]. Use of MIMO channels, when bandwidth is limited, has much higher spectral efficiency versus single-input single-output (SISO), single-input multiple-output (SIMO), and multiple-input single-output (MISO) channels [2]. It should be noted that the maximum achievable diversity gain of MIMO channels is the product of the number of transmitter and receiver antennas. Therefore, by employing MIMO channels not only the mobility of wireless communications can be increased, but also its robustness against fading that makes it efficient for the requirements of the next generation wireless services.

To achieve maximum capacity and diversity gain in MIMO channels, some optimization problems should be considered. Joint detection [3, 4], channel estimation [5, 6], and tracking [7, 8] are the most important issues in MIMO communications. Without joint detection, inter substream interference occurs. Joint detection algorithms used in MIMO channels are developed based on multiuser detection (MUD) algorithms in CDMA systems. Maximum likelihood (ML) is the optimum joint detection algorithm [9]. The computational complexity of the optimum receiver is impracticable if the number of transmitting substreams is large [10]. On the other hand, with inaccurate channel information which occurs when channel estimator tracking speed is insufficient for accurate tracking of the channel variations, the implementation of the optimum receiver is more complex. Therefore, suboptimum joint detection algorithms seem to be more efficient solutions. In this paper, the minimum mean square error (MMSE) detector which is the best linear joint detection algorithm is used as the joint detector [11, 12] due to its reasonable complexity and the fact that it provides soft output.

In SISO channels, especially in the flat fading case, channel estimation and its precision does not have a drastic impact on the performance of the receiver. Whereas, in MIMO channels, especially in outdoor MIMO channels where channel is under fast fading, the precision and convergence speed of the channel estimator has a critical effect on the performance of the receiver [13, 14]. In SISO communications, channel estimators can either use the training sequence or not. Although the distribution of training symbols in a block of data affects the performance of systems [15], but due to simplicity, it is conventional to use the training symbols in the first part of each block. In case the training sequence is not used, the estimator is called blind channel estimator. A blind channel estimator uses information latent in statistical properties of the transmitting data [16]. The statistical properties of data can be derived as directly or indirectly. The scope of indirect blind methods are based on soft [17] or hard [18] decision-directed algorithms using the previous estimation of the channel for detection of data and applying it for estimation of the channel in the present snapshot. Therefore, with decision directing, most of the nonblind algorithms can be implemented as blind. In full rank MIMO channels, use of initial training data is mandatory and without it channel estimator does not converge. In most of the previous works, block fading channels are assumed, that is, assumption of a nearly constant channel state in the length of a block of data [19, 20]. In these works, the MIMO channel state is estimated by the use of the training data in the beginning of the block that is applied for detection of data in its remaining part. With the nonblock fading assumption, the channel tracking must be performed in the nontraining part of the data. These algorithms are called semi-blind algorithms. One of the most well-known tracking algorithms is the Kalman filtering estimation algorithm proposed by Komninakis et al. [7, 8]. In these papers, a Kalman filter is used as a MIMO channel tracker. The performance of this algorithm is shown to be relatively acceptable for Rice channels where a part of the channel, due to line-of-sight components, is deterministic. But this algorithm has high complexity in the order of 5. In [21], maximum likelihood estimator is proposed for tracking of MIMO channels. This algorithm extracts equations for maximum likelihood estimation of a time-invariant channel and extends it to a time-variant channel. Therefore, this algorithm does not have a desirable performance for time-varying channels. In [22], maximum likelihood algorithm with an efficient tracking performance is derived for time-varying MIMO channels. But this algorithm, like the Kalman filtering is dependent on the channel model.

RLS algorithm is a low complexity iterative algorithm commonly used in equalization and filtering applications which is independent on the channel model [23]. The only parameter in the RLS algorithm that depends on the channel variation speed is the forgetting factor that can be empirically set to its optimum value. In this paper, the RLS algorithm is used as a channel estimator whose complexity is in the order of 3 and is then extended as a MIMO channel estimator. To derive the RLS-based MIMO channel estimator, first, cost function is defined as the weighted sum of error squares; and then this cost function is optimized versus the channel matrix. In the next step, the derived equation is implemented iteratively by applying the matrix inversion lemma. Finally, the derived iterative algorithm is combined with DDA to be implemented as a blind MIMO channel tracking algorithm.

The rest of this paper is organized as follows. In Section 2, the signal transmission model and the channel model are introduced. In Section 3, the least squares algorithm is derived and extended as a joint blind channel detection and estimation algorithm. In Section 4, simulation results of the proposed receiver are presented and compared with the Kalman filtering approach. Concluding remarks are presented in Section 5. Note that in Appendix A, the derivation of the required equation in recursive least squares MIMO channel tracking algorithm is presented.

2. THE SYSTEM MODEL

Block diagram of the transmitter in a spatial multiplexed MIMO system with M antennas is shown in Figure 1.

The input main block is coded and demultiplexed to M sub-blocks. Then, after space time coding, which is optional, all M sub-blocks are transmitted separately via transmitters. In the receiver, linear combinations of all transmitted sub-blocks are distorted by time-varying Rayleigh or Ricean fading, and the intersymbol interference (ISI) is observed under the additive white Gaussian noise. In this paper, without loss of generality, flat fading MIMO channel with Rayleigh distribution under first-order Markov model variation is assumed. The observable signal r_k^i from receiver i (with $i = 1, \dots, N$) at discrete time index k is

$$r_k^i = \sum_{j=1}^{M} h_k^{i,j} s_k^j + w_k^i, \tag{1}$$

where s_k^j is the transmitted symbol in time index k, w_k^i is the additive white Gaussian noise in the ith received element, and $h_k^{i,j}$ is the propagation attenuation between jth input and the ith output of the MIMO channel which is a complex number with Rayleigh distributed envelope. Therefore, in each time instance, the MN channel parameters must be estimated which greatly vary in the duration of data block transmission with the following autocorrelation [24]:

$$E\left\{h_k^{i,j}\left[h_l^{i,j}\right]^*\right\} \cong J_0\left(2\pi f_D^{i,j} T |k-l|\right), \tag{2}$$

where $J_0(\cdot)$ is the zero-order Bessel function of the first kind, superscript $*$ denotes the complex conjugate, $f_D^{i,j}$ is the Doppler frequency shift for path between the jth transmitter and ith receiver, and T is the duration of each symbol. According to the wide sense stationary uncorrelated scattering (WSSUS) model of Bello [25], all the channel taps are independent, namely, all $h_k^{i,j}$s vary independently according to the autocorrelation model of (2). The normalized spectrum for each tap $h_k^{i,j}$ is [25],

$$S_k(f) = \begin{cases} \dfrac{1}{\pi f_D^{i,j} T \sqrt{1-\left(f/f_D^{i,j}\right)^2}}, & |f| < f_D^{i,j} T, \\ 0, & \text{otherwise}. \end{cases} \tag{3}$$

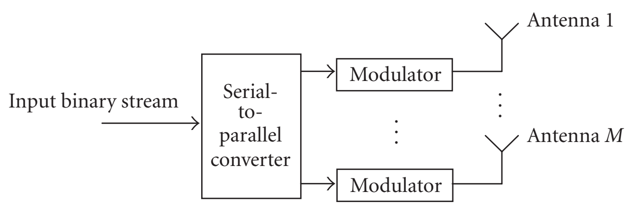

Figure 1: Block diagram of a simple spatial multiplexed MIMO transmitter.

The exact modeling of the process $h_k^{i,j}$ with a finite length autoregressive (AR) model is impossible. For implementation of a channel estimator, $h_k^{i,j}$ can be approximated by the following AR process of order L:

$$h_k^{i,j} = \sum_{l=1}^{L} \alpha_{i,j,l} h_{k-l}^{i,j} + v_{i,j,k}, \tag{4}$$

where $\alpha_{i,j,l}$ is lth coefficient between jth transmitter and ith receiver and $v_{i,j,k}$s are zero-mean i.i.d. complex Gaussian processes with variances given by

$$E\left(v_{i,j,k}[v_{i,j,k}]^*\right) = \sigma^2_{v_{i,j,k}}. \tag{5}$$

Optimum selection of channel AR model parameters from correlation functions can be derived by solving the L following Wiener equations,

$$J_0(2\pi f_D^{i,j} T|k-t|) = \sum_{l=1}^{L} J_0(2\pi f_D^{i,j} T|k-l-t|)\alpha_{i,j,l}, \quad t = k-L, k-L+1, \dots, k-1. \tag{6}$$

The length of the channel model must be chosen to a minimum of 90% of the energy spectrum of each channel coefficient which is contained in the frequency range of $|f| < f_D^{i,j} T$.

Equation (1) can be written in a matrix form as

$$\mathbf{r}_k = \mathbf{H}_k \mathbf{s}_k + \mathbf{w}_k, \tag{7}$$

where $\mathbf{r}_k$ is the received vector, $\mathbf{H}_k$ is the channel matrix, and $\mathbf{s}_k$ is the transmitted symbol all in time index k, and $\mathbf{w}_k$ is the vector with i.i.d. AWGN elements with variance $\sigma^2_{\mathbf{w}}$.

The speed of channel variations is dependent on the Doppler shift, or equivalently on the relative velocity between the transmitter's and the receiver's elements. A reasonable assumption, conventional in most scenarios, is the equal Doppler shifts, i.e., $f_D^{i,j} = f_D$, which does not make any changes in the derived algorithm. With this assumption, the matrix coefficients of the AR model can be replaced by scalar coefficients. The time-varying behavior of the channel matrix can be described as

$$\mathbf{H}_k = \alpha \mathbf{H}_{k-1} + \mathbf{V}_k, \tag{8}$$

where $\mathbf{V}_k$ is a matrix with i.i.d. Rayleigh elements with variance $\sigma^2_{\mathbf{V}}$, and α is a constant parameter that can be calculated by solving Wiener equation as follows:

$$\alpha = J_0(2\pi f_D T). \tag{9}$$

It is obvious that the larger Doppler rates lead to smaller α, and therefore faster channel variations. Because of the orthogonality between the channel state and the additive random part in the first-order AR channel model, the power of time-varying part of each tap is as follows:

$$P_k = E|h_{m,k}^{i,j}|^2 = \frac{\sigma^2_{\mathbf{V}}}{1-\alpha^2}. \tag{10}$$

Extending (7) and (8) to frequency selective channels is very simple as the following:

$$\mathbf{r}_k = \widetilde{\mathbf{H}}_k \widetilde{\mathbf{s}}_k + \mathbf{w}_k, \tag{11}$$

where

$$\begin{aligned} \widetilde{\mathbf{H}}_k &= \left[\mathbf{H}_{k,0} \;\; \mathbf{H}_{k,1} \;\; \cdots \;\; \mathbf{H}_{k,P-1} \;\; \mathbf{H}_{k,P}\right], \\ \widetilde{\mathbf{s}}_k &= \left[\mathbf{s}_k^H \;\; \mathbf{s}_{k-1}^H \;\; \cdots \;\; \mathbf{s}_{k-P+1}^H \;\; \mathbf{s}_{k-P}^H\right]^H, \end{aligned} \tag{12}$$

where $\widetilde{\mathbf{H}}_k$ and $\widetilde{\mathbf{s}}_k$ are the extended channel matrix and the data vector, respectively, P is the length of the impulse response of the channel, and $\mathbf{H}_{k,p}$ is the pth path channel matrix that varies according to the following model:

$$\mathbf{H}_{k,p} = \alpha_p \mathbf{H}_{k-1,p} + \mathbf{V}_{k,p}, \tag{13}$$

where $\mathbf{V}_{k,p}$ is a matrix with i.i.d. Rayleigh elements with variance $\sigma^2_{\mathbf{V}_p}$, and α_p is a constant parameter that can be calculated by solving Wiener equation as follows:

$$\alpha_p = J_0(2\pi f_{D,p} T), \tag{14}$$

where $f_{D,p}$ is the Doppler frequency of the pth path channel.

3. THE BLIND RECURSIVE LEAST SQUARES JOINT DETECTION AND ESTIMATION

In Appendix A, recursive least squares algorithm is derived for the estimation of the MIMO channel matrices. In training-based mode of the operation, this algorithm can be summarized as follows:

(A) initializing the parameters,

$$\mathbf{R}_0 = \mathbf{0}_{N\times M}, \quad \mathbf{Q}_0 = \delta \mathbf{I}_M,$$

where δ is an arbitrary very large number and $\mathbf{I}_M$ is the $M \times M$ identity matrix,

(B) updating $\mathbf{R}_n$ and $\mathbf{Q}_n$ in each snapshot using iterative equations, (A.7) and (A.8),

(C) calculating the channel matrix estimation from the following equation and returning to (B) in the next snapshot,

$$\widehat{\mathbf{H}}_n = \mathbf{R}_n \mathbf{Q}_n. \tag{15}$$

The derived algorithm can be extended for frequency selective channels only with replacing $\mathbf{s}_k$ with $\widetilde{\mathbf{s}}_k$ in (A.7) and (A.8) and $\widehat{\mathbf{H}}_k$ in (A.9) with $\widehat{\widetilde{\mathbf{H}}}_k$ which is the estimated value of $\widetilde{\mathbf{H}}_k$.

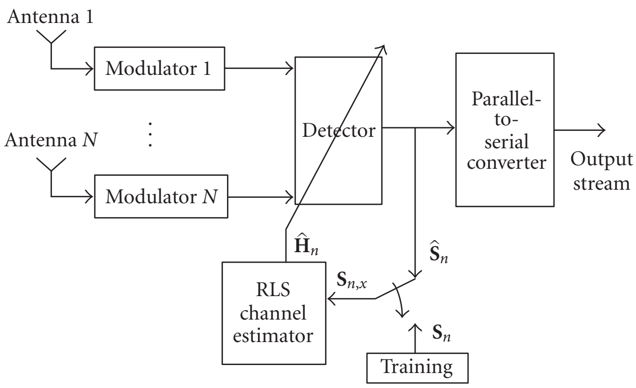

Figure 2: Block diagram of the receiver.

Hereafter, the proposed algorithm is extended to be used as blind joint channel estimator and data detector by employing DDA. In DDA-based blind channel tracking, as shown in Figure 2, in each snapshot the transmitted vector is estimated by assuming that the channel matrix is equal to the previous snapshot. This assumption is valid when the tracking error is acceptable. Then by using the estimated transmitted vector $\widehat{\mathbf{s}}_n$, just like the training vector $\mathbf{s}_n$ in (A.9), channel tracking is constructed. In other words, (A.9) is changed as follows:

$$\mathbf{Q}_n = \lambda^{-1}\mathbf{Q}_{n-1} - \frac{\lambda^{-2}\mathbf{Q}_{n-1}\mathbf{s}_{n,x}\mathbf{s}_{n,x}^H\mathbf{Q}_{n-1}}{1+\lambda^{-1}\mathbf{s}_{n,x}^H\mathbf{Q}_{n-1}\mathbf{s}_{n,x}}, \tag{16}$$

where $\mathbf{s}_{n,x} = \mathbf{s}_n$ in the training mode, and $\mathbf{s}_{n,x} = \widehat{\mathbf{s}}_n$ in the blind mode of operation. The performance of a DDA blind channel estimator is highly dependent on the performance of the joint detection algorithm. In this paper, two joint detectors are proposed to combine with the RLS-based channel estimator. The first detector, based on ML algorithm, is derived as follows:

$$\widehat{\mathbf{s}}_{k,\mathrm{ML}} = \mathrm{Arg\,Max}_{\mathbf{s}_k}\left\{\mathrm{Log}\,P\left(\mathbf{s}_k \mid \mathbf{H}_k = \widehat{\mathbf{H}}_{k-1}, \mathbf{r}_k\right)\right\}, \tag{17}$$

where $\widehat{\mathbf{s}}_{k,\mathrm{ML}}$ is the data estimated by the ML algorithm which is fed to (16) in the blind mode of operation. With the assumption of AWGN noise, (17) can be rewritten as

$$\widehat{\mathbf{s}}_{k,\mathrm{ML}} = \mathrm{Arg\,Min}_{\mathbf{s}_k}\left\{\left[\left(\mathbf{r}_k - \widehat{\mathbf{H}}_{k-1}\mathbf{s}_k\right)^H\left(\mathbf{r}_k - \widehat{\mathbf{H}}_{k-1}\mathbf{s}_k\right)\right]\right\}. \tag{18}$$

With m-ary signaling this minimization is done with search in m^M existing data vector. Therefore, the computational complexity of ML detection is increased exponentially with the number of the transmitted sub-blocks. MMSE detector which is the optimum linear detector is a linear filter maximizing the signal to noise plus interference ratio or, in other words, minimizing the mean square of the error which is the sum of the remained noise and interference power. In a DDA channel estimator, the MMSE detection is implemented as follows:

$$\widehat{\mathbf{s}}_{k,\mathrm{MMSE}} = g\left[\left(\widehat{\mathbf{H}}_{k-1}^H\widehat{\mathbf{H}}_{k-1} + \sigma_w^2 I_M\right)^{-1}\widehat{\mathbf{H}}_{k-1}^H\mathbf{r}_k\right], \tag{19}$$

where $\widehat{\mathbf{s}}_{k,\mathrm{MMSE}}$ is the data estimated by the MMSE algorithm fed to (16) in the blind mode of operation and $g(\cdot)$ is a function modeling the decision device that can be hard or soft. In the special case of BPSK signaling, with hard detection considered in the paper, $g(\cdot)$ is a signum function. It is interesting to note that the MMSE detector is a special case of ML in which the Gaussian distribution is considered for data symbols.

4. SIMULATION RESULTS

The proposed algorithm is simulated for flat fading MIMO channels with first-order Markov model channel variation using Monte Carlo simulation technique. In Section 2, it is shown that the model of a frequency selective channel can be considered as a flat fading channel and, therefore, the simulations presented for flat fading MIMO channels can also cover the frequency selective channels. The Kalman algorithm which has the best performance among symbol by symbol MIMO channel tracking algorithms is considered for comparison, and the MSE of tracking is considered as the criterion for comparison. Blocks of data are assumed to be as 100 BPSK modulated symbols and E_b/N_0 (of course average E_b/N_0) is assumed to be 10 dB. The training symbols are assigned in the first part of blocks where the equal normalized power for training and data bits are assumed. In all simulations, 4 receiver antennas and 2 and 4 transmitter antennas are considered that correspond to 2×4 (half rank) and 4×4 (full rank) MIMO channels. The channel paths' strength is normalized to one. In blind modes, the DDA algorithm with MMSE and ML detectors are considered. ML is applicable when the number of transmitter antennas is low. In this section, the performance of the ML-based DDA algorithm is presented along with MMSE-based algorithm for comparison. The values for α are considered as 0.9998 and 0.999 which correspond to $f_DT = 0.004$ and 0.01, respectively. The optimum values of the forgetting factor for $f_DT = 0.004$ and 0.01 obtained through various simulations are 0.953 and 0.9, respectively.

In the first part of simulations, the BER of the proposed algorithm for different values of f_DT and channel ranks with 10 percent training are presented in Figures 3 and 4 that correspond to MMSE and ML detection cases, respectively. All simulation results are averaged over 10000 different runs. While using the MMSE detector, as in Figure 3, for all cases the proposed algorithm presents a BER that is very close to the Kalman filtering approach. Of course, while using the ML detector, the BER presented by the Kalman filtering algorithm is sensibly better than the proposed algorithm. The high values of the observed BERs are due to the nature of the uncoded MIMO Rayleigh channels. It should be noted that the proposed architecture can be directly coupled to an error correction code to improve the BER. Therefore, BER cannot usually provide a proper tool to evaluate the MIMO channel tracking algorithm. Thus, another part of this section the MSE of tracking is considered as the criterion for comparison. The MSE of tracking is added to the AWGN noise that makes an equivalent remained noise in the system.

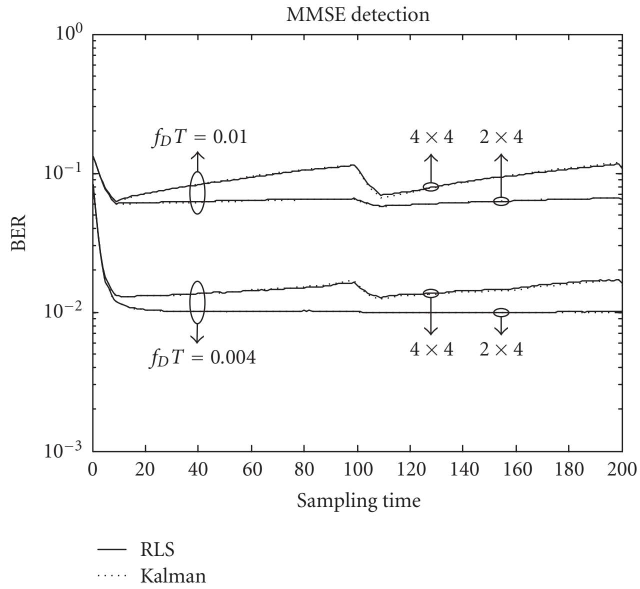

Figure 3: BER of the proposed algorithm with MMSE detection.

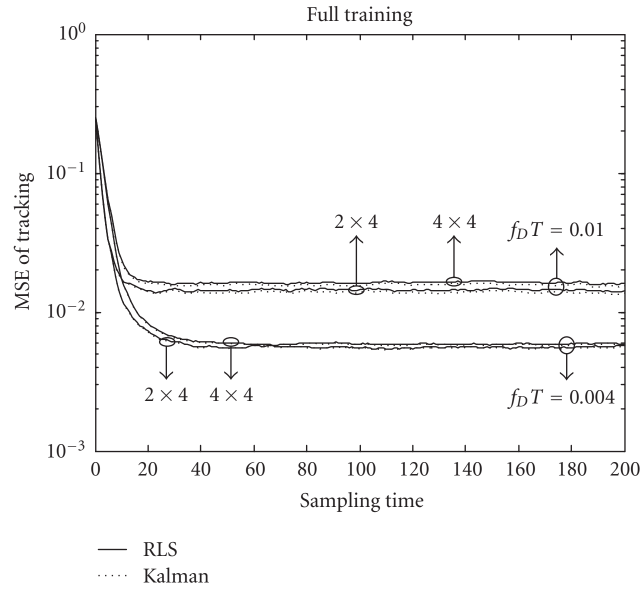

Figure 5: MSE of the proposed algorithm and Kalman filter while 100% data is training.

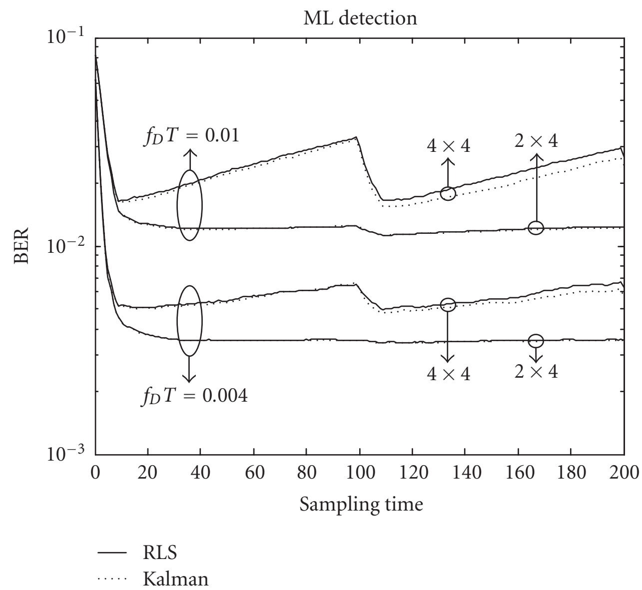

Figure 4: BER of the proposed algorithm with ML detection.

The MSE of tracking of the proposed algorithm and the Kalman algorithm for different values of $f_D T$, channel ranks, detection algorithms, and training percents are shown in Figures 5–9. In all cases, initial channel estimates are zero matrices and, therefore, due to normalized channel coefficients assumption the starting point of all curves is 1. In Figure 5, the tracking behavior of both algorithms when all transmitted data is known at the receiver, or in other words, the full training case, is presented. Although this case is virtual, it provides not only useful insights on the performance of channel tracking algorithms especially when compared with the simulations that follow, but also provides a lower bound on the performance in semi-blind and full-blind operations. As it can be seen, in this case both the proposed algorithm and the Kalman algorithm present a very close performance. An interesting point that is obvious for both algorithms is the very close tracking behavior of half and full load channels although the MSE of tracking for full load channel is a little worse than the half load channel. Also, in all curves the settling points, that is, the points where the curves are very close to final values, are about 10. Consequently, the proper choice for the training length seems to be about 10 symbols. Therefore, in each block of data after using 10 training symbols the blind mode of operation can be started.

In Figures 6 and 7, the tracking behavior, where 10 percent of each block is considered as training, is presented that corresponds to 2×4 and 4×4 MIMO channels, respectively. All curves show a saw-tooth behavior, that is, MSE of tracking is increased when the algorithm operates in the blind mode. In 2×4 MIMO channel for $f_D T = 0.004$, the performance of both algorithms for ML and MMSE detection is completely similar and overlap. In this case, the slope of curves in the blind mode of operation is negligible and, therefore, the observed performance is very close to full training. But in $f_D T = 0.01$ the ML-based DDA presents a slightly better performance. Also, in the training-based and blind modes the better slopes of curves are for the Kalman and the proposed algorithm, respectively. In 4×4 MIMO channel case, the difference between curves is more resolvable. In this case, the performance of the ML-based DDA algorithms for both $f_D T$ values is much better than the MMSE-based algorithms. Of course, in this case too, the better slopes of curves are for the Kalman and the proposed algorithm, respectively.

In all the figures presented so far, the MMSE of tracking is very efficient with only 10 symbols training in a 100

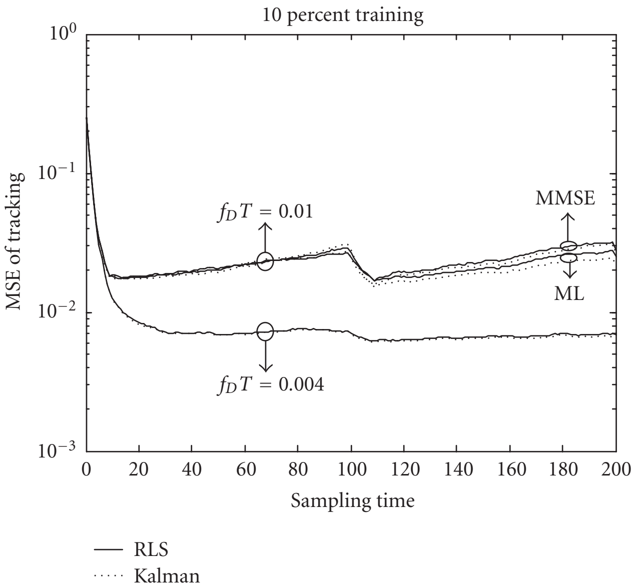

Figure 6: MSE of the proposed algorithm and Kalman filter for 2×4 MIMO channel while 10% data is training.

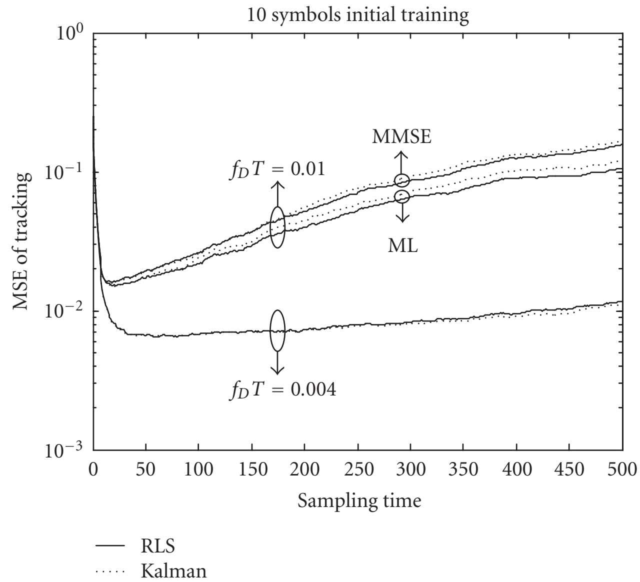

Figure 8: MSE of the proposed algorithm and Kalman filter for 2×4 MIMO channel while the 10 first symbols are used as training.

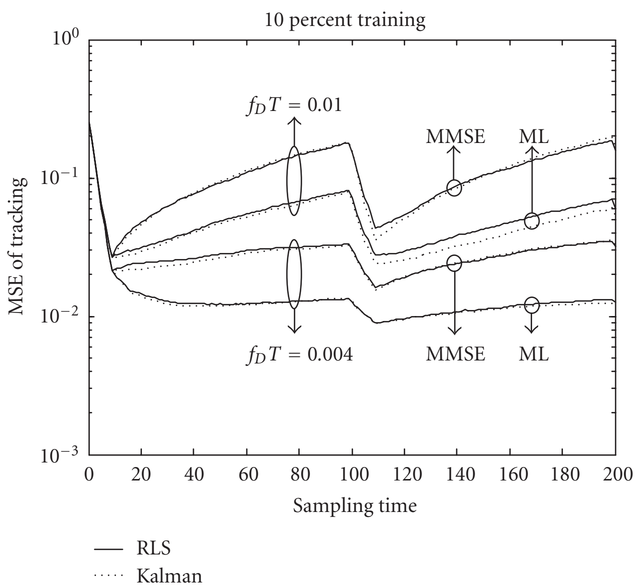

Figure 7: MSE of the proposed algorithm and Kalman filter for 4×4 MIMO channel while 10% data is training.

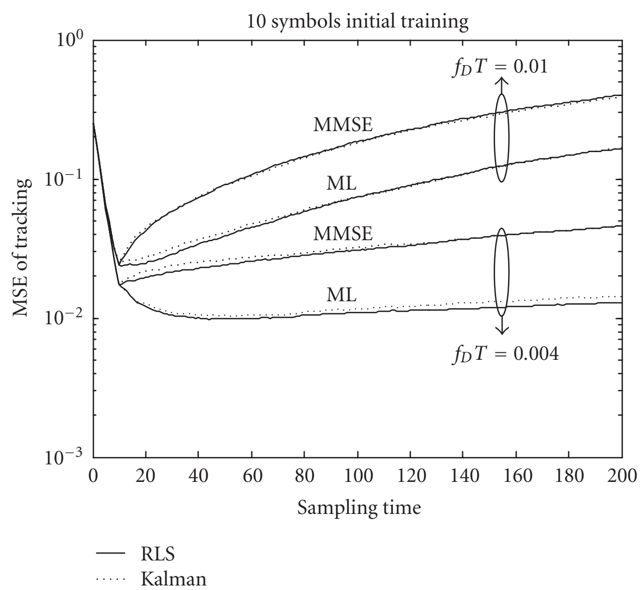

Figure 9: MSE of the proposed algorithm and Kalman filter for 4×4 MIMO channel while the 10 first symbols are used as training.

symbols block. In all cases, the performance loss (loss in effective average E_b/N_0) is negligible. Only in 4×4 MIMO channel with $f_D T = 0.01$ and MMSE detection, the maximum MSE of tracking is comparable to noise power. In other cases, especially in 2×4 MIMO channel with $f_D T = 0.004$, the maximum MSE of tracking is much lower than the power of the noise. But to what length of the block can this efficiency continue? In Figures 8 and 9, the tracking behaviors when only a 10 symbol initial training is used are presented corresponding to 2×4 and 4×4 MIMO channels, respectively. In 2×4 MIMO channel when $f_D T = 0.004$ for both ML- and MMSE-based algorithms, the MSE of tracking after about 500 symbols in the blind mode operation is much lower than the power of the noise; and with regard to its very small slope in this case, both algorithms can support the blind mode operation in much larger block lengths. In this channel, when $f_D T = 0.01$, the maximum MSE of tracking after 500 symbols is about the same as the power of noise and, therefore, this block length seems to be appropriate. In 4×4 MIMO channel, the slope of curves is higher and, therefore, this case is simulated for 200 symbols block length.

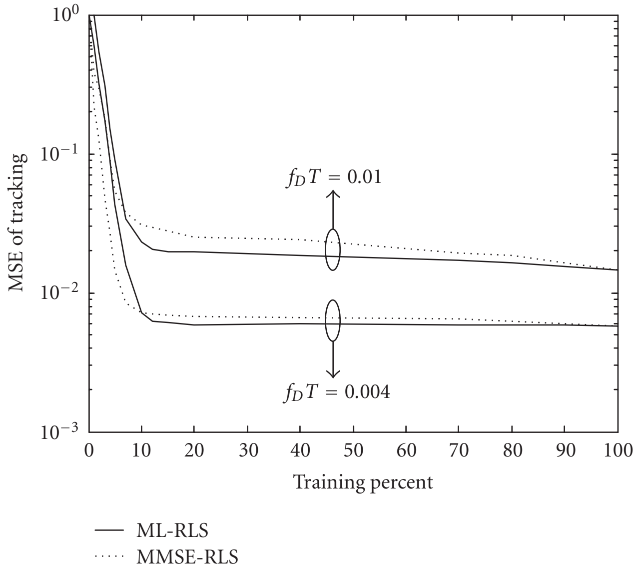

Figure 10: MSE of the proposed algorithm versus the training percent for 2×4 MIMO channel.

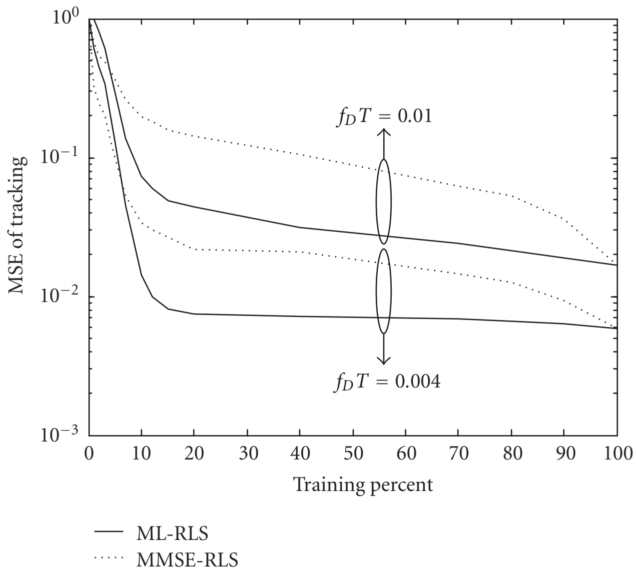

Figure 11: MSE of the proposed algorithm versus the training percent for 4×4 MIMO channel.

In Figures 10 and 11, the MSE of tracking of the proposed algorithm at the end of each 100 symbols block which is the maximum MSE of tracking is presented versus training percent for 2×4 and 4×4 MIMO channels, respectively. An interesting point that can be seen from these figures is the better performance of the MMSE-based than the ML-based DDA algorithms in low training percents. In other words, in low training percents, the MMSE-based DDA algorithm presents a sharp slope. These figures confirm the efficiency of using 10 symbols as training concluded in the previous simulations. The training length for very close to optimum operation of the proposed algorithm is the settling point of these curves. Proper selection of this point seems to be 8 and 12 percents for 2×4 and 4×4 MIMO channels, respectively. In training lengths higher than this point, the difference between the lower bound of tracking and the MSE presented by the proposed algorithm with DDA is very small and presents a very close to optimum performance.

5. CONCLUSION

In this paper, a new approach in estimation and tracking of spatial multiplexed MIMO channels based on the RLS algorithm is presented and then combined with the DDA algorithm with ML and MMSE detection to operate in the blind mode operation as well. The output of DDA is considered as virtual training symbols in the blind mode operation. The proposed algorithm is simulated for half and full rank flat Rayleigh fading MIMO channels under first-order Markov model channel variations with $f_DT = 0.004$ and 0.01 via Monte Carlo simulation technique and is compared with Kalman filtering approach which is one of the most well-known channel tracking algorithms. It is assumed that 100 symbols block of BPSK signals are transmitted independently on each transmitter antenna, and training symbols with equal power to data are located in the first part of each block. Through various simulations, the forgetting factor for $f_DT = 0.004$ and 0.01 is optimized to their optimum values, that is, 0.953 and 0.9, respectively.

The proposed algorithm presents a very close to Kalman estimator performance with a slightly better performance for Kalman estimator in the training mode and a better performance for the proposed algorithm in the blind mode of operation, whereas the computational complexity of the proposed algorithm is much lower than the Kalman estimator. It is shown that in 2×4 MIMO channel when $f_DT = 0.004$ for both the ML- and the MMSE-based algorithms, the MSE of tracking after about 500 symbols is much smaller than the power of the noise. In other words, the performance loss due to channel tracking error is shown to be negligible and, hence, the proposed algorithm with only 10 symbols initial training can support the blind mode of operation in much larger block lengths. Also, the performance of the proposed algorithm is simulated versus the training percents. It is observed that the MSE of tracking settles to the neighborhood of the performance of full training case which is the lower bound of the blind operation performance in about 8 and 12 training percents for 2×4 and 4×4 MIMO channels, respectively. Therefore, these two values seem to be the optimum training lengths.

APPENDICES

A. THE DERIVATION OF THE RECURSIVE LEAST SQUARES CHANNEL ESTIMATION ALGORITHM

In this section, the proposed channel estimation algorithm is presented. Without loss of generality in this section the flat fading model is considered. At first, cost function is defined as a weighted average of error squares. Because of the additive

Gaussian noise assumption for channel estimation this cost function must be minimized. The cost function in time instant n is defined by the following:

$$\begin{aligned} C_n &= \sum_{k=1}^{n} \lambda^{n-k} \left\| (\mathbf{r}_k - \mathbf{H}_n \mathbf{s}_k) \right\|^2 \\ &= \sum_{k=1}^{n} \lambda^{n-k} \Big[(\mathbf{r}_k - \mathbf{H}_n \mathbf{s}_k)^H (\mathbf{r}_k - \mathbf{H}_n \mathbf{s}_k) \Big], \end{aligned} \tag{A.1}$$

where superscript H presents the conjugate transpose operator and λ is the forgetting factor which is $0 < \lambda \leq 1$, the optimum value of which is dependent on the Doppler frequency shift and is chosen empirically.

This cost function is convex and, therefore, has a global minimum point found by forcing the gradient of the cost function versus channel matrix to zero. The gradient of the above-mentioned cost: function is as follows:

$$\frac{1}{2} \nabla_{\mathbf{H}_n} C_n = \sum_{k=1}^{n} \lambda^{n-k} \Big[(\mathbf{r}_k - \mathbf{H}_n \mathbf{s}_k) \mathbf{s}_k^H \Big], \tag{A.2}$$

where $\nabla_{\mathbf{H}_n}$ is the gradient operator versus $\mathbf{H}_n$. Consequently, channel estimate in time index n can be obtained by solving the following:

$$\sum_{k=1}^{n} \lambda^{n-k} \Big[(\mathbf{r}_k - \hat{\mathbf{H}}_n \mathbf{s}_k) \mathbf{s}_k^H \Big] = \mathbf{0}_{N \times M}, \tag{A.3}$$

where $\hat{\mathbf{H}}_n$ is estimation of $\mathbf{H}_n$ and $\mathbf{0}_{N\times M}$ is a $N \times M$ zero matrix. Equation (A.3) can be solved for $\hat{\mathbf{H}}_n$ as follows:

$$\hat{\mathbf{H}}_n = \left(\sum_{k=1}^{n} \lambda^{n-k} \mathbf{r}_k \mathbf{s}_k^H \right) \left(\sum_{k=1}^{n} \lambda^{n-k} \mathbf{s}_k \mathbf{s}_k^H \right)^{-1}. \tag{A.4}$$

The main problem of (A.4) is the need for matrix inversion, therefore, it is reformed to be solved iteratively as follows. By assuming,

$$\begin{aligned} \mathbf{P}_n &= \left(\sum_{k=1}^{n} \lambda^{n-k} \mathbf{s}_k \mathbf{s}_k^H \right), \\ \mathbf{Q}_n &= \mathbf{P}_n^{-1}, \\ \mathbf{R}_n &= \left(\sum_{k=1}^{n} \lambda^{n-k} \mathbf{r}_k \mathbf{s}_k^H \right). \end{aligned} \tag{A.5}$$

$\mathbf{P}_n$ and $\mathbf{R}_n$ can be calculated using the following iterative equations:

$$\mathbf{P}_n = \lambda \mathbf{P}_{n-1} + \mathbf{s}_n \mathbf{s}_n^H, \tag{A.6}$$

$$\mathbf{R}_n = \lambda \mathbf{R}_{n-1} + \mathbf{r}_n \mathbf{s}_n^H, \tag{A.7}$$

and also $\mathbf{Q}_n$ can be calculated iteratively by using the matrix inversion lemma as follows:

$$\mathbf{Q}_n = \lambda^{-1} \mathbf{Q}_{n-1} - \frac{\lambda^{-2} \mathbf{Q}_{n-1} \mathbf{s}_n \mathbf{s}_n^H \mathbf{Q}_{n-1}}{1 + \lambda^{-1} \mathbf{s}_n^H \mathbf{Q}_{n-1} \mathbf{s}_n}. \tag{A.8}$$

Finally, after recursive calculation of $\mathbf{R}_n$ and $\mathbf{Q}_n$, the channel matrix is estimated by the following:

$$\hat{\mathbf{H}}_n = \mathbf{R}_n \mathbf{Q}_n. \tag{A.9}$$

Table 1: Complexity of the RLS and the Kalman MIMO channel tracking algorithms.

Complexity components	Algorithms: RLS algorithm	Algorithms: Kalman algorithm
Number of sum operations	$M^2(N+2)$	$2M^2N^3 + 2MN^3 - MN^2$ $+2MN - N^2 + N$
Number of product operations	$M^2N + 3M^2$ $+2MN + M$	$3M^2N^3 + M^2N^2 + MN^3$ $+2MN^2 + 2MN$

B. THE KALMAN MIMO CHANNEL TRACKING ALGORITHM [7]

In order to derive the Kalman MIMO channel tracking algorithm, first input-output equation, (7), must be reformed as follows:

$$\mathbf{r}_k = \mathbf{S}_k \cdot \mathbf{h}_k + \mathbf{w}_k, \tag{B.1}$$

where $\mathbf{h}_k$ is the vectorized form of the channel matrix which is an $MN \times 1$ vector derived by concatenation of columns of the channel matrix and $\mathbf{S}_k$ is $N \times MN$ data matrix defined as Kronecker product of data vector in identity matrix as follows:

$$\mathbf{S}_k = \mathbf{s}_k \otimes \mathbf{I}_N. \tag{B.2}$$

Therefore, using Kalman equations, channel vector $\mathbf{h}_k$ can be recursively estimated by the following procedure:

(A) initialization step,

$$\mathbf{P}_0 = \delta \mathbf{I}_{MN}, \tag{B.3}$$

where δ is an arbitrary very large number,

(B) channel tracking step,

$$\mathbf{R}_{e,k} = \sigma_w^2 \mathbf{I}_N + \mathbf{S}_k \mathbf{P}_{k-1} \mathbf{S}_k^H, \tag{B.4}$$

$$\mathbf{K}_k = (\alpha \mathbf{P}_{k-1} \mathbf{S}_k^H) \mathbf{R}_{e,k}^{-1}, \tag{B.5}$$

$$\mathbf{e}_k = \mathbf{r}_k - \mathbf{S}_k \cdot \hat{\mathbf{h}}_{k-1}, \tag{B.6}$$

$$\mathbf{P}_k = \alpha^2 \mathbf{P}_{k-1} + \sigma_v^2 \mathbf{I}_{MN} - \mathbf{K}_{k-1} \mathbf{R}_{e,k-1} \mathbf{K}_{k-1}^H, \tag{B.7}$$

$$\hat{\mathbf{h}}_k = \alpha \hat{\mathbf{h}}_{k-1} + \mathbf{K}_k \mathbf{e}_k. \tag{B.8}$$

C. COMPLEXITY COMPARISON

Here, the complexity of the Kalman filtering approach and the proposed algorithm is evaluated and compared. Complexity is considered as the number of sum and product operations.

In each iteration of the RLS MIMO channel tracking algorithm equations (A.7), (A.8), and (A.9) are calculated. But in each iteration of the Kalman filtering approach, (B.4) to (B.8) must be computed. The total required sum and product operations for the RLS and the Kalman algorithms are presented in Table 1.

Table 2: Complexity of the RLS and the Kalman MIMO channel tracking algorithms.

Complexity components	Algorithms and channel orders			
	RLS algorithm $M = 2$ and $N = 4$	RLS algorithm $M = 4$ and $N = 4$	Kalman algorithm $M = 2$ and $N = 4$	Kalman algorithm $M = 4$ and $N = 4$
Number of sum operations	24	96	740	2516
Number of product operations	46	148	1040	3744

Numerical comparison of the complexity of the RLS and the Kalman MIMO channel tracking algorithms is presented in Table 2. As it can be seen, the number of the required sum and product operation in the Kalman MIMO channel tracking algorithm is much higher than the RLS algorithm.

ACKNOWLEDGMENT

The authors would like to express their sincere thanks to the Center of Excellence on Electromagnetic Systems, Department of Electrical and Computer Engineering, University of Tehran, for supporting this work.

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