Hindawi Publishing Corporation
EURASIP Journal on Wireless Communications and Networking
Volume 2006, Article ID 56914, Pages 1–11
DOI 10.1155/WCN/2006/56914

Routing and Power Allocation in Asynchronous Gaussian Multiple-Relay Channels

Zigui Yang and Anders Høst-Madsen

Department of Electrical Engineering, University of Hawaii at Manoa, Honolulu, HI 96822, USA

Received 31 October 2005; Revised 28 April 2006; Accepted 2 May 2006

We investigate the cooperation efficiency of the multiple-relay channel when carrier-level synchronization is not available and all nodes use a decode-forward scheme. We show that by using decode-forward relay signaling, the transmission is effectively interference-free even when all communications share one common physical medium. Furthermore, for any channel realization, we show that there always exist a sequential path and a corresponding simple power allocation policy, which are optimal. Although this does not naturally lead to a polynomial algorithm for the optimization problem, it greatly reduces the search space and makes finding heuristic algorithms easier. To illustrate the efficiency of cooperation and provide prototypes for practical implementation of relay-channel signaling, we propose two heuristic algorithms. The numerical results show that in the low-rate regime, the gain from cooperation is limited, while the gain is considerable in the high-rate regime.

1. INTRODUCTION

A wireless ad hoc network is an infrastructureless network, in which the communications between two nodes are typically maintained by the cooperation of other nodes. The traditional multihopping operation lets each intermediate node receive information only from its immediate predecessor and then send it to its immediate successor. A more advanced operation is to use relay-channel signaling. The essential difference between the traditional multihopping and the relay-channel signaling is that in the latter, a node uses the information from all its upstream nodes instead of the information from the closest one.

The relay channel was first introduced by van der Meulen [1, 2]. In a simplest case, a relay channel has only one relay to assist the transmission between the source and the destination. The relay channel can be denoted by $(\mathcal{X}_1, \mathcal{X}_2, p(y_2, y_3 \mid x_1, x_2), \mathcal{Y}_2, \mathcal{Y}_3)$, where $\mathcal{X}_1$, $\mathcal{X}_2$ are the transmitter alphabets of the source and the relay, respectively, $\mathcal{Y}_2$, $\mathcal{Y}_3$ are the receiver alphabets of the relay and the destination, respectively, and a collection of probability $p(\cdot, \cdot \mid x_1, x_2)$ on $\mathcal{Y}_2$, $\mathcal{Y}_3$, one for each $(x_1, x_2) \in \mathcal{X}_1, \mathcal{X}_2$. Here x_1, x_2 are the channel inputs by the source and the relay and y_2, y_3 are the outputs of the relay and the destination, respectively. The relay channel was extensively studied in [3], where two cooperation schemes, decode-forward and compress-forward, were proposed. Inspired by a renewed interest in ad hoc networks and network information theory, much research has been done recently on relay channels and cooperative diversity [4–14].

We assume that every node uses a decode-forward scheme. Although the other two relaying schemes, amplify-forward and compress-forward, can achieve higher rates under certain channel realizations [4, 6, 7], they are difficult to scale to large networks. In an amplify-forward scheme, the relays essentially act as analog repeaters, and therefore enhance the system noise. Another challenge in using the amplify-forward scheme in large networks is the difficulty of implementing routing algorithms. Compress-forward requires complex Wyner-Ziv coding, which is difficult to be implemented in practice [15], especially when scaled to large networks. Decode-forward has its own drawback in that it requires full decoding at each relay, and therefore may cause error propagation. However, this can be compensated by strong channel coding.

The achievable rate of a one-relay channel using a decode-forward scheme is [3]

$$R \leq \max_{P(x_1,x_2)} \min \{ I(X_1; Y_2 \mid X_2), I(X_1, X_2; Y_3) \}. \quad (1)$$

The interpretation is that the relay first fully decodes the message from the inputs of the source, which results in the first term in the $\min\{\cdot\}$ function, and then the destination decodes the messages from the inputs of both the source and the relay, and thus gives the second term in the $\min\{\cdot\}$

function. An adaptive transmission scheme will allow the source to communicate directly with the destination if the relay has a poor link to the source—one form of routing. It then gives the following achievable rate:

$$R \le \max_{P(x_1,x_2)} \max \{ \min \{ I(X_1; Y_2 \mid X_2), I(X_1, X_2; Y_3) \}, \\ I(X_1; Y_3 \mid X_2) \}. \tag{2}$$

For a physically degraded channel, that is, when $X_1 \to (X_2, Y_2) \to Y_3$ forms a Markov chain, (1) achieves the capacity. However, for a general relay channel, the capacity is unknown even for one-relay case. Therefore, most of the research on multiple-relay channels concentrated on achievable rates and capacity bounds [5, 6, 8, 16] or on the capacity for some special type of multiple-relay channels such as the degraded multiple-relay channel [17]. A multiple-relay channel is generally a multilevel structure, in which each level contains one or more nodes and the nodes in the same level decode a message at the same time.

The wireless communication broadcast property is referred to as "wireless multicast advantage" (WMA) or "wireless broadcast advantage" (WBA) and may be used in the routing algorithm in wireless networks to reduce power consumption and improve reliability [18, 19]. If different transmitters can be synchronized at carrier level and thus are able to coordinate to use beamforming techniques, it is shown that cooperation achieves significant gain in reducing total power consumption [20]. Relay-channel signaling further exploits the broadcast transmission and multiaccess reception properties by allowing a node to accumulate the soft information of all its received signals, that is, a node's decoding may depend on multiple received signals. Although it is obvious that relay-channel signaling can further improve the performance, it is at the cost of higher complexity. One fundamental question is whether it pays off using relay-channel signaling or not. In this paper, we will investigate the cooperation efficiency in the multiple-relay-channel setting. Specifically, we consider the quasistatic Gaussian wireless multiple-relay channel.

A quasistatic channel here means that the channel realization remains unchanged during the transmission of one message and goes to another independent realization in the transmission period of the next message. One useful measure of the performance in this scenario is outage probability, which is the probability that the channel cannot support a particular communication rate under certain constraints. The quasistatic model is suitable for delay-sensitive services that have strict delay requirements. For delay-insensitive services, the source and the relay may choose to adjust their transmission rate according to the channel condition [7]. In many applications such as sensor networks, the nodes typically operate on limited-energy batteries, which are usually not rechargeable or replaceable, and thus results in severe energy constraints. A main concern is therefore optimizing energy consumption in the network.

Consider at first a simple point-to-point channel in Rayleigh fading with a channel gain h. If it is desired to transmit at a certain constant rate R, the required power is proportional to h^{-2} and the average power is proportional to $E[h^{-2}]$, which can be shown to be infinite. Thus, it is impossible to transmit in all channel conditions, and a threshold h_0 has to be chosen so that if $h < h_0$, no transmission is done and an outage is declared. Equivalently, a threshold power P_0 can be set so that if the required power P for transmission at rate R is larger than P_0, transmission is given up and an outage is declared. The average power consumption is an increasing function of P_0, while the outage probability is a decreasing function of P_0, which should therefore be chosen as a compromise between power consumption and acceptable outage probability. Notice that P_0 is not related to the physical power constraint of the transmission circuit of the terminal, although of course P_0 must be chosen less than this.

Generalizing this to networks, we consider a total power constraint, that is, at any time, the overall power consumption cannot exceed a particular amount of power P_0. This seems the most reasonable point of view: if the total power (energy) needed in the network exceeds a certain threshold, transmission is given up. A precise statement of this is as follows. Assuming that the source-destination pair in the multiple-relay channel wants to maintain a constant communication rate R, we define an outage event for a given transmission scheme $\mathcal{T}$ and the channel realization $\mathcal{H}$ as

$$E_1 : R_{\mathcal{T}}(P_0, \mathcal{H}) \le R, \tag{3}$$

where $R_{\mathcal{T}}(P, \mathcal{H})$ is the maximal rate that the transmission scheme $\mathcal{T}$ can achieve for the channel realization $\mathcal{H}$ with a total power consumption of at most P_0. For all reasonable transmission schemes $\mathcal{T}$, $R_{\mathcal{T}}(P, \mathcal{H})$ is a nondecreasing function of P. We define $P_{\mathcal{T}}(R, \mathcal{H})$ as the minimum total power required by transmission scheme $\mathcal{T}$ to achieve the rate R for the channel realization $\mathcal{H}$. Then the outage event is equivalent to the event

$$E_2 : P_{\mathcal{T}}(R, \mathcal{H}) > P_0. \tag{4}$$

Therefore, we can minimize outage probability by minimizing the total power needed to achieve the target rate R for each channel realization. This problem was investigated for parallel (two-hop) relay channels in [21, 22]. Here we generalize this to multihop channels where arbitrary interrelay communication is allowed. The problem then becomes more complicated, requiring finding both an optimal arrangement of nodes and a corresponding power allocation policy.

Apart from the above overall power constraint, individual node power constraints may also be relevant. Firstly, the power allocation can result in uneven power consumption among the nodes. However, with channel variations, this is averaged out; furthermore, if all nodes at sometime or other act as source-destination pairs, the power consumption can be expected to be fairly distributed. Secondly, the power allocation could result in a solution where an individual node power consumption is above what the node is physically capable of. However, taking this into account would just complicate the solution without giving further insight.

The rest of the paper is organized as follows. In Section 2, we give the model for the Gaussian multiple-relay channel, for which we will find an optimum arrangement of nodes, shown to be a sequential path, and its corresponding optimal power allocation policy is given in Section 3. To investigate the performance of the relay-channel signaling and to provide some prototype algorithms for practical implementation of relay-channel signaling, we provide two heuristic algorithms for the cooperative relay-channel signaling problem in Section 4. In Section 5, we extend our discussion to the case when nodes have only limited signal processing capability. The numerical results are provided in Section 6 and a brief summarization is given in Section 7.

2. CHANNEL MODEL

In this paper, we consider a quasistatic multiple-relay channel with N nodes, numbered from 1 to N. Without loss of generality, we assume that 1 and N is the source-destination pair and that the other nodes act as relays. We assume that all nodes operate in full-duplex mode, and thus they can receive and transmit in the same frequency band at the same time. Full-duplex communication is generally regarded as difficult to achieve in practice, but there are techniques that make it possible [23].

Another important assumption is on synchronization among nodes. There are three levels of synchronization: frame, symbol, and carrier. We assume that the receivers are completely synchronized at all levels. For transmitters, it is realistic to assume that frame- and symbol-level synchronizations are available. The contentious point is on carrier-level synchronization, which requires that separate microwave oscillators at different nodes are synchronized. This seems highly unrealistic. Left by themselves, the drift of the oscillators makes synchronization impossible. It might be possible to couple oscillators, and very closely spaced nodes could even autocouple, but this requires nontrivial microwave innovation, and in general this seems quite improbable especially for sensor networks with simple nodes. We will therefore assume that there is no carrier synchronization. The link between any pair of nodes (i, j) can be parameterized by a complex channel gain h_{ij}, which is assumed to be symmetric, that is, $h_{ij} = h_{ji}$. The channel gains h_{ij} are independent random variables as a result of the random movement of nodes and (or) fading. They are assumed to be fixed during one-message transmission period and go to another independent realization in the next-message transmission period.

The source wants to send a message w to the destination during the duration of each channel realization $\mathcal{H} = \{h_{ij} : i, j \in \{1,\dots,N\},\ i \neq j\}$. Let $X_i(k)$, $i \in \{1,\dots,N-1\}$, be the channel input of node i at time k and let $Y_j(k)$, $j \in \{2,\dots,N\}$, be the channel output of node j at time k, we have

$$Y_i(k) = \sum_{j\in\{1,\dots,N-1\},\, j\neq i} h_{ij}X_j(k) + Z_i(k), \quad i \in \{2,\dots,N\}, \tag{5}$$

where $Z_i(k) \sim \mathcal{CN}(0,1)$ are i.i.d. unit power white Gaussian noises for all i,k. We assume that full channel state information is available noncausally to all nodes. While this may not be realistic in fast-changing channels, it is possible if the channel is not varying too quickly. Furthermore, this gives a bound on performance as for the case when less knowledge is available.

3. ACHIEVABLE RATES OF THE GAUSSIAN MULTIPLE-RELAY CHANNEL

In [5], Gupta and Kumar demonstrated an achievable region for a multiple-relay channel, and later Xie and Kumar [16] established an explicit formula for the achievable rate, which, in general, exceeds the rate in [5]. Here we restate the theorem in [16] as follows.

Theorem 1 (see [16, Theorem 3.4]). *For a discrete memoryless multiple-relay channel with source node 1, destination node N, and the other nodes arranged into $L-1$ levels with each level k consisting of a set of nodes Γ_k, $k = 1,\dots,L-1$, the following rate is achievable:*

$$R < \max_{P(\mathbf{X}_0,\dots,\mathbf{X}_{L-1})} \min_{1\le k\le L} \min_{i:i\in\Gamma_k} I(\mathbf{X}_0,\dots,\mathbf{X}_{k-1}; Y_i \mid \mathbf{X}_k,\dots,\mathbf{X}_L), \tag{6}$$

where boldface characters denote vectors for inputs of the nodes in each group. Here $\Gamma_0 := \{1\}$ and $\Gamma_L := \{N\}$.

For an asynchronous Gaussian multiple-relay channel, we have the following corollary of Theorem 1.

Proposition 1. *Assume that node j uses transmission power P_j. For an asynchronous Gaussian multiple-relay channel with $L-1$ levels of relay nodes, the following rate is achievable:*

$$R \le \min_{1\le l\le L} \min_{i\in\Gamma_l} \frac{1}{2}\log\left(1 + \sum_{m=0}^{l-1}\sum_{j\in\Gamma_m} P_j \left|h_{ij}\right|^2\right). \tag{7}$$

Proof. The message w is first split into B blocks $w_1,\dots,w_B$ of nR bits each. Each node i generates a codebook with 2^{nR} i.i.d. n-sequences with i.i.d. Gaussian components and index them as $\underline{x}_i(w_j)$, $w_j \in \{1,\dots,2^{nR}\}$. The whole transmission is performed in $B+L-1$ time slots, and thus the overall rate is $R\cdot B/(B+L-1)$ bits per channel use. By making B large, we can get the rate arbitrarily close to R. In each of the first B time slots, the source node 1 transmits the codeword $\underline{x}_1(w_i)$ for each w_i, $i \in \{1,\dots,B\}$, and in the remaining time slots, it transmits constant signals $\underline{x}_1(1)$. A node i in level k, $1 \le k \le L$, starts the decoding of w_1 at the end of kth time slot and sends out $\underline{x}_i(w_1)$ in time slot $k+1$. It continues the same decoding and encoding procedure in each time slot thereafter until it has decoded and sent out all the messages. It transmits some constant signals $\underline{x}_i(1)$ in the remaining slots. To illustrate the encoding scheme, we give an example of a relay channel of 5 nodes, in which $(1,5)$ is the source-destination pair. Nodes 2 and 3 are assigned to level 1 and node 4 is in level 2. The message w is split into 6 message blocks. The encoding scheme is shown in Figure 1.

The relays and the destination decode each w_i, $i \in \{1,\dots,B\}$, using similar sliding-*window decoding* technique

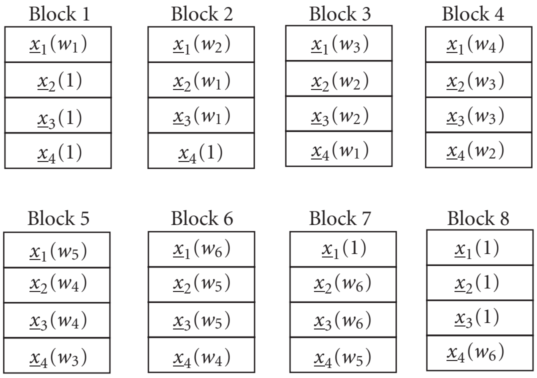

Figure 1: Encoding scheme.

[6, 24]. A node i in level l can decode w_1 at the end of lth time slot using a window of the first l received blocks. After decoding the first message, the window is shifted by one and the part due to the transmission of the first message is subtracted from the received signals in the new window and then the second message is decoded. It continues until all messages are decoded. For each message, node i is actually receiving information from l independent parallel channels [25]. Thus for node i to successfully decode the message, we have

$$R < \max_{P(\mathbf{X}_0,\ldots,\mathbf{X}_{L-1})} I(\mathbf{X}_0, \mathbf{X}_1, \ldots, \mathbf{X}_{l-1}; Y_i \mid \mathbf{X}_l, \ldots, \mathbf{X}_{L-1}) \tag{8}$$

$$\leq \frac{1}{2}\log\left(1 + \sum_{m=0}^{l-1}\sum_{j\in\Gamma_m} P_j \left|h_{ij}\right|^2\right), \tag{9}$$

where P_j is the power assigned to node j. Since each node except for the source needs to fully decode each message block, we have

$$R \leq \min_{1\leq l\leq L}\min_{i\in\Gamma_l}\frac{1}{2}\log\left(1 + \sum_{m=0}^{l-1}\sum_{j\in\Gamma_m} P_j \left|h_{ij}\right|^2\right). \tag{10}$$

Here for simplification of notation, we assume that

$$\sum_{k=0}^{-1}\sum_{t\in\Gamma_k} P_t |h_{it}|^2 = 0, \quad \forall i. \tag{11}$$

□

Remark 1. Note that we do not introduce any correlation between the inputs of the nodes as it will not produce any gain if no carrier-level synchronization between transmitters is available [6, 7].

Remark 2. To achieve the rate in (7), all X_i's are Gaussian distributed and mutually independent.

Remark 3. As can be seen from (9), the interference from other nodes is effectively cancelled out after a node subtracts from its received signals the part contributed from the messages it knows. From (9), we obtain an equivalent form of (7).

Corollary 1. *Fix a rate R and define $d_{ij} = (2^{2R} - 1)/|h_{ij}|^2$. For the rate R to be achievable, the powers $P_1, P_2, \ldots, P_N$ have to satisfy*

$$\sum_{m=0}^{l-1}\sum_{j\in\Gamma_m}\frac{P_j}{d_{ij}} \geq 1 \tag{12}$$

for all l.

3.1. The optimal multilevel structure and power allocation policy

As we have shown, in order to minimize the outage probability in a quasistatic channel, we need to find a multilevel structure $\mathbf{S}$ and a corresponding power allocation policy $\mathbf{T}(\mathbf{S})$ such that the total power to achieve the rate requirement R is minimized. Assuming that a multilevel structure $\mathbf{S}$ has $L+1$ levels, we denote the nodes in each level $0 \leq l \leq L$ by Γ_l and the size of Γ_l by $|\Gamma_l|$. We have $\Gamma_0 = \{1\}$ and $\Gamma_L = \{N\}$. Denote the level of a node i as $\ell(i)$. Note that $\mathbf{S}$ may not include all the nodes, that is, some nodes may be chosen not to participate in the transmission. Denote the power assigned to node $i \in \mathbf{S}$ by a power allocation policy $\mathbf{T}(\mathbf{S})$ for $\mathbf{S}$ as $P_i(\mathbf{T}, \mathbf{S})$. We then need to solve the following optimization problem:

$$\begin{aligned}&\min_{\mathbf{T},\mathbf{S}}\sum_{i\in\mathbf{S}} P_i(\mathbf{T},\mathbf{S}) \text{ such that } R\\ &\quad \leq \min_{1\leq l\leq L}\min_{i\in\Gamma_l}\frac{1}{2}\log\left(1 + \sum_{m=0}^{l-1}\sum_{j\in\Gamma_m} P_j(\mathbf{T},\mathbf{S})\left|h_{ij}\right|^2\right).\end{aligned} \tag{13}$$

Since a Gaussian multiple-relay channel in general is not a degraded channel as the one studied in [17], it does not have a natural arrangement of nodes that is optimal. However, it does have some special properties for an optimal multilevel structure $\mathbf{S}$ and its corresponding optimal power allocation policy $\mathbf{T}(\mathbf{S})$ as stated in the next two theorems.

Theorem 2. *For any channel realization $\mathcal{H}$ and rate requirement R, the overall power allocation is minimized by a sequential-path multilevel structure S, that is, one with $|\Gamma_l| = 1$ for all l.*

Proof. We need to show the existence of a sequential path $\mathbf{P}$ that is optimal. For any channel realization $\mathcal{H}$, there always exist a multilevel structure $\mathbf{S}$ and a corresponding power allocation policy $\mathbf{T}(\mathbf{S})$ that are optimal. Assuming that $\mathbf{S}$ has $L+1$ levels, we prove by induction that it can always be converted to an equivalent path $\mathbf{P}$ without increasing total power consumption by properly removing some nodes in $\mathbf{S}$ and adjusting transmission power of the remaining nodes.

First, for level L, $\Gamma_L = \{N\}$, thus $|\Gamma_L| = 1$. Suppose that for decoding orders $l \geq T+1$ ($T < L$), we have $|\Gamma_l| = 1$. We will then show that we can always make $|\Gamma_T| = 1$ without violating the constraints. For convenience of presentation, we denote the only node in Γ_l, $l \geq T+1$, by ζ_l.

If $|\Gamma_T| = 1$, we are done. Otherwise, assume that $|\Gamma_T| = M$ ($M \geq 2$) and $\Gamma_T = \{t_m : 1 \leq m \leq M\}$. Without risk of confusion, we simplify the notation of $P_i(\mathbf{T}, \mathbf{S})$ to P_i and we have $P_i > 0$, for all $i \in \mathbf{S}$.

We consider two cases.

Case 1. There exists $\{t_1, t_2\} \in \Gamma_T$ such that $d_{t_i\zeta_{T+1}} > 0$, $i = 1, 2$.

We perform the following recursive power updating procedure.

(1) Fix the transmission power of all nodes that reside in level T or higher except for t_1 and t_2. Adjust the transmission power P_{t_1} of t_1 to $P_{t_1}^{\text{new}} = P_{t_1} + \delta$, where δ is a small value.

(2) Adjust the transmission power P_{t_2} of t_2 such that the left-hand side of the constraint (12) for ζ_{T+1} remains unchanged. Therefore, we have $P_{t_2}^{\text{new}} = P_{t_2} - (d_{t_2\zeta_{T+1}}/d_{t_1\zeta_{T+1}})\delta$.

(3) Adjust the transmission power of ζ_{T+1} such that the left-hand side of constraint (12) is kept the same for ζ_{T+2} to get

$$P_{\zeta_{T+1}}^{\text{new}} = P_{\zeta_{T+1}} + \left(\frac{d_{t_2\zeta_{T+1}} d_{\zeta_{T+1}\zeta_{T+2}}}{d_{t_1\zeta_{T+1}} d_{t_2\zeta_{T+2}}} - \frac{d_{\zeta_{T+1}\zeta_{T+2}}}{d_{t_1\zeta_{T+2}}}\right)\delta. \tag{14}$$

(4) Recursively update the transmission power of node $i, i = \zeta_{T+2}, \ldots, \zeta_{L-1}$, such that the left-hand side of the constraint (12) is kept the same for the node right behind it.

This recursive updating procedure guarantees that the constraint (12) is still satisfied at all relay nodes and at the destination. Since we vary the transmission power of only one node at each step, the total amount of power change is proportional to δ. Denote the total transmission power for the multilevel structure $\mathbf{S}$ and the corresponding power allocation policy $\mathbf{T}(\mathbf{S})$ as $\xi(\mathbf{S}, \mathbf{T}(\mathbf{S}))$, that is, $\xi(\mathbf{S}, \mathbf{T}(\mathbf{S})) = \sum_{i \in \mathbf{S}} P_i(\mathbf{T}, \mathbf{S})$. Then

$$\xi(\mathbf{S}, \mathbf{T}^{\text{new}}(\mathbf{S})) = \xi(\mathbf{S}, \mathbf{T}(\mathbf{S})) + f(\mathbf{S})\delta, \tag{15}$$

where $\mathbf{T}^{\text{new}}$ is the new power allocation policy after the power updating procedure and $f(\mathbf{S})$ is a constant that does not depend on δ but only on the multilevel structure $\mathbf{S}$ if $|\delta|$ is small enough. Obviously, δ is allowed to be either positive or negative, that is, we can either increase or decrease the transmission power of t_1. Thus, if $f(\mathbf{S}) \neq 0$, we can always choose the sign of δ such that the total amount of power change $f(\mathbf{S})\delta < 0$, and hence

$$\xi(\mathbf{S}, \mathbf{T}^{\text{new}}(\mathbf{S})) < \xi(\mathbf{S}, \mathbf{T}(\mathbf{S})). \tag{16}$$

This contradicts the fact that the original multilevel structure and power allocation policy pair $(\mathbf{S}, \mathbf{T}(\mathbf{S}))$ is optimal. Therefore we must have $f(\mathbf{S}) = 0$, and thus $(\mathbf{S}, \mathbf{T}^{\text{new}}(\mathbf{S}))$ is also optimal. In this case, we can repeatedly perform the same updating procedure by decreasing the transmission power of t_1 (or t_2) and increasing the transmission power of t_2 (or t_1) until either the transmission power of node i, $P_i^{\text{new}} = 0$, $i \in \{\zeta_{T+1}, \ldots, \zeta_{L-1}\}$ or $P_{t_i}^{\text{new}} = 0$, $i = 1, 2$. If $P_i^{\text{new}} = 0$, $i \in \{\zeta_{T+1}, \ldots, \zeta_{L-1}\}$, then node i can be removed from the relaying structure and we can continue the updating procedure above. If $P_{t_i} = 0$, $i = 1, 2$, it means that we can remove t_i from the structure $\mathbf{S}$.

If there still exist two or more nodes with decoding order T, we can always take out two of them and repeat the same procedure above to remove one node each time until only one node is kept.

Case 2. $d_{t_i\zeta_{T+1}} = 0$, for all $t_i \in \Gamma_T \setminus \{t_1\}$, and $d_{t_1\zeta_{T+1}} > 0$.

In this case, there is only one node t_1 in level T that has finite-length link to node ζ_{T+1}. This case is actually essentially the same as in Case 1. Pick a node t_2 in Γ_T, $t_2 \neq t_1$, and a node $\zeta_{T+i} \in \{\zeta_{T+2}, \ldots, \zeta_L\}$ such that $\ell(\zeta_{T+i}) < \ell(k)$, for all $k \in \{\zeta_{T+2}, \ldots, \zeta_L\}$, $k \neq \zeta_{T+i}$, and $d_{t_2\zeta_{T+i}} > 0$. Then we can perform the same recursive power updating algorithm as in Case 1. The only difference is that node ζ_{T+i-1} takes the place of t_1 in Case 1. Thus we can always reduce the power of t_2 to 0 and thus remove it from the multilevel relaying structure.

Combining our discussions of Cases 1 and 2, we can conclude that we are always able to keep only one node at decoding order T without increasing the total power consumption.

By induction l, for all $1 \leq l \leq L$, we may have $|\Gamma_l| = 1$ and this establishes the proof.

Note that the new relaying path $\mathbf{P}$ does not necessary have the same number of levels as $\mathbf{S}$. □

The implication of Theorem 2 is that we can restrict our search to sequential paths without loss of optimality. In doing so, we greatly reduce the search space. The following theorem shows how power is optimally allocated given a sequential relaying path.

Theorem 3. *For a sequential relaying path* $\mathbf{P}$*, the optimal power allocation policy* $\mathbf{T}(\mathbf{P})$ *can be implemented by a recursive power-filling procedure, that is, along path* $\mathbf{P}$*, starting from the source, each node i adjusts its transmission power such that the constraint* (12) *is satisfied with equality sign at its immediate successor j,* $\ell(j) = \ell(i) + 1$.

Proof. Let the relaying path be $\mathbf{P} = (\zeta_0, \zeta_1, \ldots, \zeta_L)$, where $\zeta_0 = 1$, $\zeta_L = N$. Initially we set the power of all nodes to 0. Since node ζ_1 only receives information from the source ζ_0, we must let the source transmit at a power level such that constraint (12) is exactly satisfied at node ζ_1. Now the message is known to ζ_0 and ζ_1 and only they are eligible to transmit. With the objective to save transmission power, at any time we always let the node whose transmission is most efficient (results in less total transmission power) increase its transmission power. Now the transmission of node ζ_1 will be more efficient. Otherwise, if the transmission of ζ_0 is more efficient, it will increase its transmission power until a node other than node ζ_1 satisfies constraint (12). That node can then decode in the same decoding order as node ζ_1 and it contradicts the fact that there is only one node in each level. Thus the source has to stop increasing its transmission power as long as node ζ_1 satisfies (12). Node ζ_1 then adjusts its power level such that ζ_2 satisfies constraint (12) with equality sign. This procedure proceeds until the destination meets condition (12) exactly. □

Here we do not need to know how to exactly determine the efficiency of the transmission of a particular node. What we only need to know is that it depends on the structure of the relaying path and the state of the relaying path, that is, whether constraint (12) is satisfied at the nodes in the path or not. Therefore, before the state of the relaying path changes, the transmission efficiency of any node that has satisfied (12) remains unchanged. Theorem 3 implies that every node except for the destination transmits with certain level of positive power and every node except for the source receives exactly enough information from its upstream nodes.

3.2. Example

Now we give a simple example to illustrate the benefit of cooperative relay signaling. Figure 2 shows a multiple-relay-channel network with 4 nodes in which $(1, 4)$ is the source-destination pair. The label attached to the link (i, j) is the value d_{ij} as defined before. All 4 possible sequential relaying paths and their corresponding total power consumption are presented in Table 1. The path $1 \rightarrow 3 \rightarrow 2 \rightarrow 4$ is not an eligible relaying path as by the power allocation policy, node 2 cannot decode after node 3. The total power consumption is calculated using the recursive power-filling procedure. For example, for the path $1 \rightarrow 2 \rightarrow 4$, in order to make node 2 able to decode, we have $P_1 = 10$. To make node 4 able to decode, we have $P_1/42 + P_2/30 = 1$, and thus $P_2 \approx 22.86$. The overall power consumption is then $P_1 + P_2 = 32.86$. A traditional multihop operation that uses the shortest path algorithms will find $1 \rightarrow 2 \rightarrow 4$ as the optimal path with overall power consumption 40. However, the transmission from node 1 to node 2 will give rise to interference to the communications between node 2 and node 4. Therefore, the actual power consumption will be larger than 40. From Table 1, it is interesting to see that the best relaying path $1 \rightarrow 2 \rightarrow 3 \rightarrow 4$ is the worst one from the point of view of traditional multihopping algorithms.

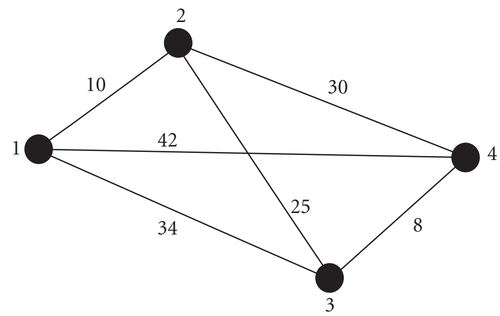

Figure 2: A multiple-relay channel with 4 nodes.

Table 1: Relaying paths and overall power consumptions.

Path	Overall power
$1 \rightarrow 4$	42
$1 \rightarrow 2 \rightarrow 4$	32.86
$1 \rightarrow 3 \rightarrow 4$	35.52
$1 \rightarrow 2 \rightarrow 3 \rightarrow 4$	28.99

4. HEURISTIC ALGORITHMS

From Theorems 2 and 3, we have shown that for any channel realization $\mathcal{H}$, there exist an optimal relaying path $\mathbf{P}$ and a corresponding simple power allocation policy $\mathbf{T}(\mathbf{P})$. Thus limiting our search to sequential paths can greatly reduce the search space for optimal solutions. There have been some elegant shortest path algorithms to find a shortest path in a network [26]. However, the Bellman principle used in these traditional shortest path algorithms is not satisfied here. For example, consider a relay network with 4 nodes $V = \{1, 2, 3, 4\}$ and costs $d_{21} = 3$, $d_{32} = 4$, $d_{31} = 6$, $d_{41} = 7$, $d_{42} = 12$, $d_{43} = 0.1$. We may verify that the optimal relaying path is $1 \rightarrow 3 \rightarrow 4$. By the Bellman principle, the optimal cooperative relaying path from 1 to 3 should be the direct link from 1 to 3, which requires a total power consumption of 34. However, from 1 to 3 we can find that the path $1 \rightarrow 2 \rightarrow 3$ actually requires a smaller total power consumption of $(10 + (34 - 10)/34 \times 25) \approx 27.65$. This shows that the Bellman principle does not apply to the cooperative routing problem.

Another difference between the optimal relaying path problem in this paper and the traditional shortest path problem is that in the former we have to use a node-based metric instead of a link-based metric since we want to minimize the total power consumption of all nodes. Therefore, we cannot expect using standard shortest path algorithms to find an optimal relaying path. An exhaustive search algorithm that searches through all multilevel structures has a complexity of $O((N-2)^{(N-2)})$. Theorem 2 reduces this complexity to $O((N-1)!)$. We may improve on this using the property of an optimal relaying path in Theorem 3 to remove many unqualified candidates. As implied in Theorem 3, when selecting the node for a particular level, it is not necessary to consider those nodes that have already satisfied condition (12). Otherwise, they will receive more information than necessary. This reduces the worst case complexity to 2^{N-2} candidate paths, which makes it possible to find the optimum solution for small networks (i.e., less than 20 nodes). Still, for larger networks, the complexity is too high. We therefore consider heuristic algorithms for finding relaying paths and the corresponding power allocation policies for general multiple-relay channels. The algorithms provide achievable rates which might not be optimal for the given coding scheme, but simulation results show that one of the heuristic algorithms is essentially equal to the optimum solutions for small networks where the optimal solution can be found. Furthermore, the heuristic algorithms provide prototype algorithms for practical (central) implementation of relay-channel signaling.

The following heuristic algorithms are based on Theorems 2 and 3. From Theorem 2, although it is still difficult to find an optimal path, we may try to search for a path that is close to optimum. We then enforce the optimal power allocation policy in Theorem 3 on the path selected.

4.1. Heuristic algorithm 1: CTNCR

A traditional noncooperative multihopping algorithm finds a shortest path assuming no interference from upstream nodes and, in general, it generates a suboptimal path. However, it might be a starting point for finding a good relay-signaling cooperative path. In this heuristic, we first find a shortest noncooperative path using standard Dijkstra's algorithm based on the link-based metric and then use the power allocation policy in Theorem 3 to determine the overall power consumption and possibly remove some nodes from the path. The algorithm works as follows.

Step 1 (initialization). Find a noncooperative path $\mathbf{P}$ using Dijkstra's algorithm. Set the transmission power of all nodes in $\mathbf{P}$ to 0. Set the source as the active node, which is the only one that can adjust transmission power.

Step 2. Among the active nodes' downstream nodes that have not satisfied (12), find node K such that it requires the least transmission power of the active node to decode the message (satisfying condition (12)). Remove the nodes between the active node and K from $\mathbf{P}$. Set K as the active node.

Step 3 (stop criterion). Stop if K is the destination and the new $\mathbf{P}$ is the final path with the transmission power of nodes as determined in Step 2; otherwise go to Step 2.

The computational complexity of Dijkstra's algorithm is $O(N^2)$ [26]. In Step 2, we note that there are $|\mathbf{P}| - 1$ iterations and the number of operations in each iteration is proportional to $|\mathbf{P}|$. Therefore in the worst case, the computation in Step 2 is $O(|\mathbf{P}|^2)$. Thus the computation of CTNCR is $O(N^2 + |\mathbf{P}|^2)$. Since $|\mathbf{P}| \le N$, in the worst case, the computation of CTNCR is $O(N^2)$.

4.2. Heuristic algorithm 2: SNER

This heuristic algorithm is essentially a greedy algorithm similar to the Prim-Dijkstra spanning-tree algorithm but it stops whenever the destination is included in the tree. The algorithm works as follows.

Step 4 (initialization). Form a set of nodes Ξ_d, which is called the decoded set, with only the source node included and a nondecoded set $\Xi_n = V - \Xi_d$, where V is the set of all nodes.

Step 5. For each node $K \in \Xi_n$, find a node T in Ξ_d as its predecessor that requires the least total power consumption for K to satisfy (12) using the recursive power-filling procedure. Record the path and the corresponding overall power allocation for K to satisfy (12). Among all $K \in \Xi_n$, find the node that requires the least overall power, denote it by $K_{\min}$. Add $K_{\min}$ to Ξ_d and remove it from Ξ_n.

Step 6 (stop criterion). If $K_{\min}$ is the destination, stop; otherwise, go to Step 5.

To estimate the computation required by SNER algorithm, we note that in the worst case there are $N - 1$ iterations. In each iteration, for each node $K \in \Xi_n$, we need to do $N - |\Xi_n|$ comparisons. Hence in each iteration, the computation is $|\Xi_n|(N - |\Xi_n|)$. In the worst case, the computation of SNER is $\sum_{i=1}^{N-1} i(N-i) = N^3/6 - N^2 + N/6$. Thus the computation complexity of SNER is $O(N^3)$. However, since we only need $|\mathbf{P}| - 1$ iterations, the actual computation of SNER is then $\sum_{i=1}^{|\mathbf{P}|-1} i(N-i) = (N|\mathbf{P}|^2 - N|\mathbf{P}|)/2 - (2|\mathbf{P}|^3 - 3|\mathbf{P}|^2 + |\mathbf{P}|)/6$. Since $N > |\mathbf{P}|$, the computational complexity of SNER algorithm is $O(N|\mathbf{P}|^2)$.

5. COMPLEXITY-CONSTRAINED NETWORKS

In our previous discussion, every node is assumed to be able to store and process all related received signals to decode a message. In some applications, the relays may have only limited memory and signal processing capability, and thus cannot combine all these signals, especially if the path is long. On the other hand, the signals received from remote upstream nodes bring insignificant information or interference to the decoding of the message and it may not pay off to include these signals in the decoding of the message. Therefore we may treat them as pure noise with possibly only a slight increase of the overall power consumption. We hence consider a variation of decode-forward relaying path problem by adding a constraint that the relays and the destination decode each message only based on the most current F received signals. The encoding scheme is the same as in Section 2. The difference lies in the decoding of the relays and the destination in that the sizes of their decoding windows are at most F. Note that the relays in level i, $i \le F$, can use all the related received signals. We still assume that a node can subtract all interferences from downstream nodes. Since a node has already decoded the message downstream nodes are transmitting, it also knows precisely what signal downstream nodes are transmitting. This interference subtraction is much less complex than the joint decoding required to handle the signal transmitted by upstream nodes, so the algorithm is complexity constrained. However, in practice, the complexity could be reduced more by only subtracting the signal from the first few nodes downstream.

Again using the parallel channels argument [25], for node i with decoding order $l \ge 1$ in a path $\mathbf{P}$ to decode a message at rate R, we have

$$\begin{aligned} R &\le \sum_{m=\lceil l-F \rceil}^{l-1} \log\left(1 + \frac{P_m |h_{im}|^2}{1 + \sum_{k=0}^{m-1} P_k |h_{ik}|^2}\right) \\ &= \log\left(1 + \frac{\sum_{m=\lceil l-F \rceil}^{l-1} P_m |h_{im}|^2}{1 + \sum_{k=0}^{\lceil l-F \rceil - 1} P_k |h_{ik}|^2}\right), \end{aligned} \tag{17}$$

where $\lceil x \rceil = \max(0, x)$ and again for notation simplification, we assume that $\sum_{k=0}^{-1} P_k |h_{ik}|^2 = 0$. Notice that there is no interference from downstream nodes in (17) in accordance with the assumption of interference subtraction for downstream nodes.

To reduce the complexity of signal processing at the relays and the destination, it is always desirable to keep F small. On the other hand, to be more power efficient, it is desirable to choose a larger F. Therefore, there is a tradeoff in properly selecting the value of F. Again, as in the unlimited signal processing case, any optimal multilevel relaying structure can be converted to a relaying path without increasing power consumption.

Theorem 4. *For any channel realization $\mathcal{H}$, rate requirement R, and signal processing length F, there always exist a sequential path $\mathbf{P}$ and a corresponding power allocation policy $\mathbf{T}(\mathbf{P})$ that minimize overall power consumption.*

Proof. The proof is essentially the same as for Theorem 2. The only difference is that $f(\mathbf{S})$ is changed to $f(\mathbf{S}, \mathbf{T}(\mathbf{S}))$, that is, it also depends on the original power allocation policy. □

Similarly, the optimal power allocation policy $T(\mathbf{P})$ for any limited data processing path $\mathbf{P}$ is still the recursive power-filling procedure as before.

Theorem 5. *For a sequential relaying path $\mathbf{P}$ with limited signal processing capability, the optimal power allocation policy can be implemented by a recursive power-filling procedure as stated in Theorem 3.*

The proof is similar to the proof of Theorem 3.

The two heuristic algorithms CTNCR and SNER can be easily adapted to the limited signal processing capability case. Here we only consider the variation of SNER algorithm and we denote the SNER algorithm with signal processing length L as SNERvL.

6. NUMERICAL RESULTS

In this section, we illustrate the performance of the relay-channel signaling by simulation. Since our results only depend on the amplitude of channel gains h_{ij}, we consider only the theoretical model of $|h_{ij}|$, the model of which we use in our simulations is

$$|h_{ij}| = \frac{\alpha_{ij}}{(\ell_{ij})^{n/2}}, \tag{18}$$

where ℓ_{ij} is the distance between i and j, n is the path loss exponent, and α_{ij} is a constant or a random variable. We consider two cases.

(1) $\alpha_{ij} = 1$, for all i, j. In this case, a signal is attenuated only by path loss. The randomness of the channel realization comes from the random movement of the nodes.

(2) α_{ij} is a unit-variance Rayleigh distributed random variable. A signal is then attenuated not only by path loss but also by small scale fading characterized by the parameters α_{ij}. All α_{ij}'s are assumed to be mutually independent.

A typical value of the path loss exponent n is between 2 and 5. In our simulations, we consider the cases when $n = 2$, a low attenuation regime; and $n = 4$, a high attenuation regime. To simulate the random movement of nodes, for each channel realization we randomly place all the nodes, in our simulations 20 or 50 nodes, in a 100×100 grid and randomly pick two of them as the source-destination pair. For $n = 2$, we consider a desired rate of either $R = 0.5$ or $R = 1$; and for $n = 4$ a desired rate of either $R = 0.5$ or $R = 2$. The results are based on 100 000 simulation runs for each case. The noncooperative multihopping routes are found by the Bellman-Ford algorithm using the link-based metric. As in Section 5, we assume that nodes can subtract interference from all downstream nodes. Traditional multihopping systems most likely do not have this ability, and the curves for performance of noncooperative multihopping should therefore be seen as a lower bound for the performance of practical multihopping. Multihopping is therefore identical to SNERv1, except that SNERv1 uses an interference sensitive routing. The optimal solution for network size 20 is found by exhaustive search over all paths according to Theorems 2 and 3. The simulation results are presented in Figures 3, 4, 5, 6, 7, and 8, which show the outage performance of various algorithms under different total power constraints.

The first that can be noticed is that in all the 20 node cases, the heuristic optimization algorithm SNER gives a performance which is essentially identical to the optimal performance, while the less complex CTNCR has a performance slightly worse. We do not present the optimal solutions for network size 50 due to the overwhelming computational task, but based on the results for network size 50 we can expect SNER to be representative also of the optimal solution.

The second remarkable result is the qualitative difference between the low-rate case ($R = 0.5$) and the high-rate case ($R = 1$ or $R = 2$). In the low-rate case, the gain from cooperation is limited—at most 5 dB[1] for $n = 2$ and network size 50, and for the high-attenuation case $n = 4$, no gain at all. On the other hand, for high rate, the gain from cooperation is very large, up to 18 dB in Figure 5. Recall that the noncooperation curve is actually a lower bound for practical multihopping, so the gain could very well be even larger. This indicates that a main advantage of cooperation is interference avoidance, as interference increases with rate for traditional multihopping, while relay-channel signaling completely avoids interference. The results for $n = 4$ confirm the results in [16, 27, 28] that multihopping is a reasonable choice, but only in the high-attenuation/low-rate regime.

The results for SNERvL show that it is not necessary to use the full relay-channel signaling to get significant gains. In all cases considered, SNERv4 gets very close to the optimal relay-channel signaling, so that it would be enough to decode the transmission of the 4 "nearest" neighbors upstream.

7. CONCLUSIONS

In this paper, we show that the optimal operation of an asynchronous Gaussian multiple-relay channel with decode-forward signaling is given by a path with a corresponding simple power allocation policy. This reduces the complexity

[1] All dB gains discussed are for outage probability 10^{-3}.

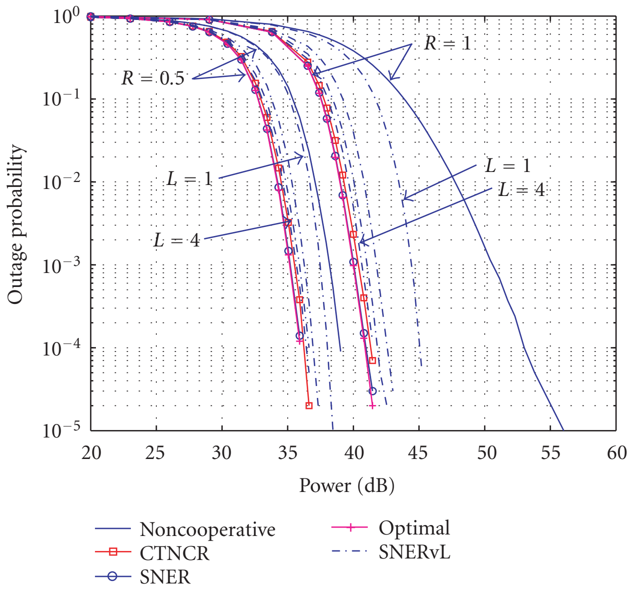

FIGURE 3: Outage probability versus total power consumption for path loss exponent 2 and network size 20 with pure path loss.

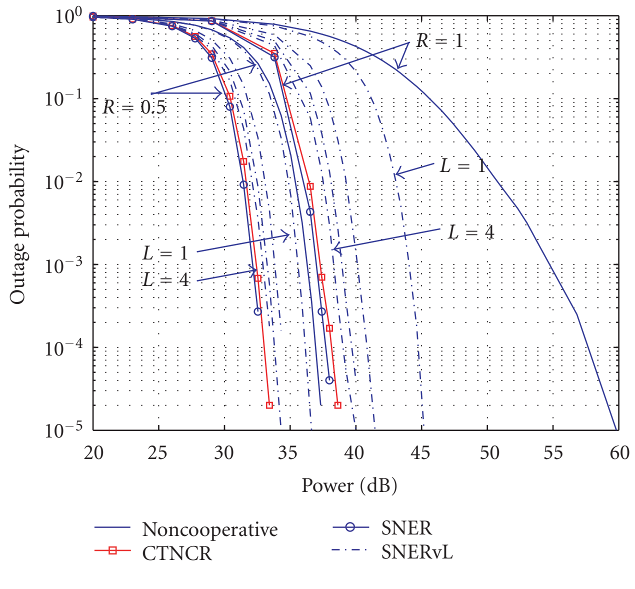

FIGURE 5: Outage probability versus total power consumption for path loss exponent 2 and network size 50 with pure path loss.

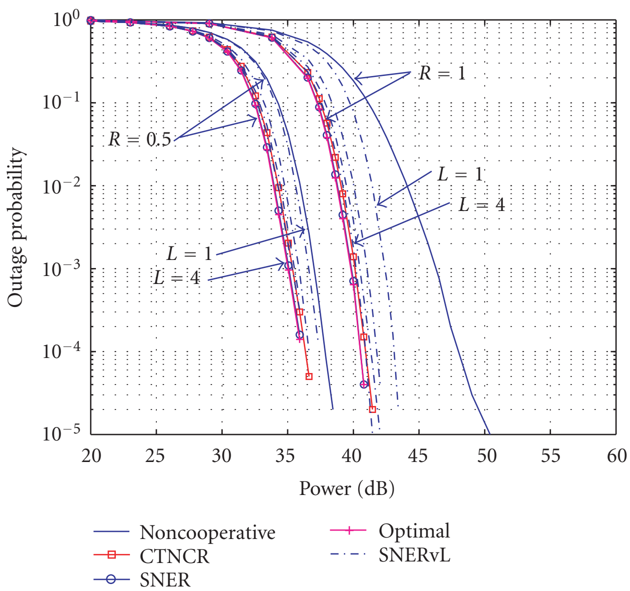

FIGURE 4: Outage probability versus total power consumption for path loss exponent 2 and network size 20 with path loss and Rayleigh fading.

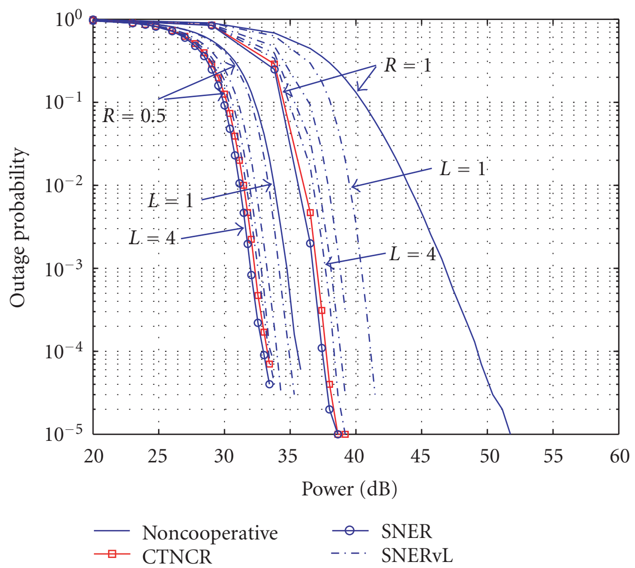

FIGURE 6: Outage probability versus total power consumption for path loss exponent 2 and network size 50 with path loss and Rayleigh fading.

of finding the optimal solution, although the complexity is still exponential. We therefore propose heuristic polynomial-time algorithms for path finding, and numerical results show that these heuristic algorithms give solutions very close to the optimal solution.

Our numerical results show that in the low-attenuation regime, both with and without Rayleigh fading, cooperation through relay-channel signaling shows significant gains over traditional noncooperative operation. The gains increase as the rate increases because of the interference explosion for a noncooperative algorithm. In the high-attenuation regime, however, for low rate, more traditional multihopping operation that uses single-signal-based decoding can be a quite reasonable choice as cooperation brings little gain. For high rate, the cooperative algorithms still show significant gain because of the poor performance of the traditional multihopping algorithm, which, however, may be greatly improved by carefully choosing paths to try to avoid heavy interference.

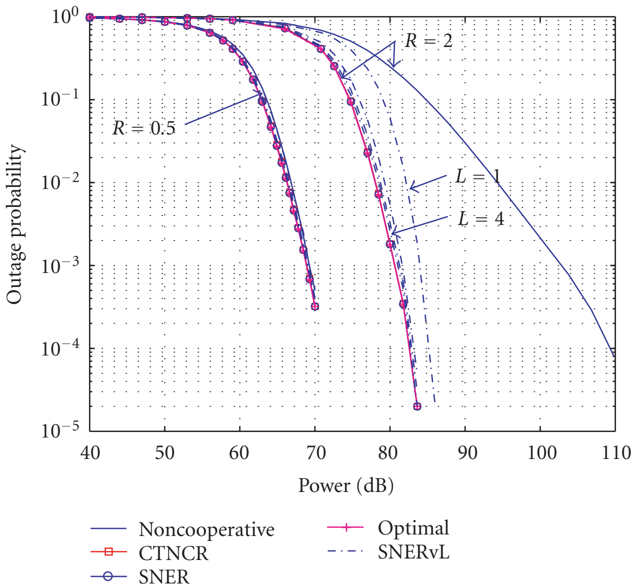

FIGURE 7: Outage probability versus total power consumption for path loss exponent 4 and network size 20 with pure path loss.

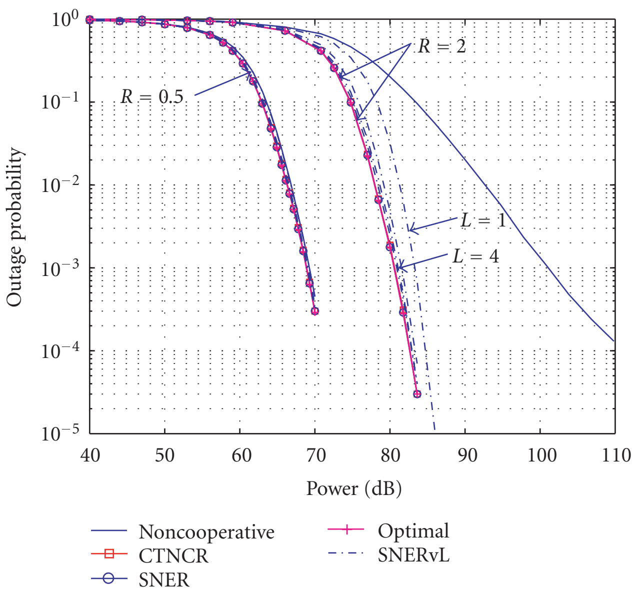

FIGURE 8: Outage probability versus total power consumption for path loss exponent 4 and network size 20 with path loss and Rayleigh fading.

The heuristic algorithms developed here for calculating rate can be used as a starting point for developing practical routing algorithms for relay channels. In challenge, however, is the assumption of full network information at each node. This requirement can be mitigated considering further simplification to the proposed heuristic algorithm. For example, we may consider further simplification of SNERvL by finding the path using some rough channel state information, for example, the positions of nodes, and cancelling only the interference from the transmissions of the most immediate L downstream nodes. In this case, a node only needs to know the positions of other nodes and the perfect channel gains between itself and its $2L$ closest nodes in the path selected. The heuristic algorithms can also be adapted to distributed (distance-vector or link-state-based) versions.

Another basic assumption is that nodes use full duplex. It will be interesting to extend the results to half-duplex case, which, however, is not trivial as it involves an additional complicated scheduling problem of time slots or frequency bands. Another interesting problem that we may consider in future work is the optimization problem when nodes have individual power constraints in addition to a global power constraint.

ACKNOWLEDGMENT

This work was supported in part by NSF Grant CCR03-29908.

REFERENCES

[1] E. C. van der Meulen, "Three- terminal communication channels," *Advances in Applied Probability*, vol. 3, no. 1, pp. 120–154, 1971.

[2] E. C. van der Meulen, *Transmission of information in a T-terminal discrete memoryless channel*, Ph.D. thesis, Department of Statistics, University of California, Berkeley, Calif, USA, 1968.

[3] T. M. Cover and A. A. El Gamal, "Capacity theorems for the relay channel," *IEEE Transactions on Information Theory*, vol. 25, no. 5, pp. 572–584, 1979.

[4] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, "Cooperative diversity in wireless networks: efficient protocols and outage behavior," *IEEE Transactions on Information Theory*, vol. 50, no. 12, pp. 3062–3080, 2004.

[5] P. Gupta and P. R. Kumar, "Towards an information theory of large networks: an achievable rate region," *IEEE Transactions on Information Theory*, vol. 49, no. 8, pp. 1877–1894, 2003.

[6] G. Kramer, M. Gastpar, and P. Gupta, "Cooperative strategies and capacity theorems for relay networks," *IEEE Transactions on Information Theory*, vol. 51, no. 9, pp. 3037–3063, 2005.

[7] A. Høst-Madsen and J. Zhang, "Capacity bounds and power allocation for the wireless relay channel," *IEEE Transactions on Information Theory*, vol. 51, no. 6, pp. 2020–2040, 2005.

[8] A. Høst-Madsen, "Capacity bounds for cooperative diversity," *IEEE Transactions on Information Theory*, vol. 52, no. 4, pp. 1522–1544, 2006.

[9] B. Schein and R. Gallager, "The Gaussian parallel relay network," in *Proceedings of IEEE International Symposium on Information Theory*, p. 22, Sorrento, Italy, June 2000.

[10] G. K. Karagiannidis, T. A. Tsiftsis, and R. K. Mallik, "Bounds for multihop relayed communications in Nakagami-m fading," *IEEE Transactions on Communications*, vol. 54, no. 1, pp. 18–22, 2006.

[11] G. K. Karagiannidis, "Performance bounds of multihop wireless communications with blind relays over generalized fading channels," *IEEE Transactions on Wireless Communications*, vol. 5, no. 2, pp. 498–503, 2006.

[12] K. Azarian, H. El Gamal, and P. Schniter, "On the achievable diversity-multiplexing tradeoff in half-duplex cooperative channels," *IEEE Transactions on Information Theory*, vol. 51, no. 12, pp. 4152–4172, 2005.

[13] A. Bletsas, A. Khisti, D. P. Reed, and A. Lippman, "A simple cooperative diversity method based on network path selection," *IEEE Journal on Selected Areas in Communications*, vol. 24, no. 3, pp. 659–672, 2006, special issue on 4G.
[14] E. Perevalov and R. S. Blum, "Delay limited capacity of ad hoc networks: asymptotically optimal transmission and relaying strategy," *IEEE Transactions on Communications*, vol. 52, no. 11, pp. 1957–1968, 2004.
[15] S. Cheng and Z. Xiong, "Successive refinement for the Wyner-Ziv problem and layered code design," in *Proceedings of Data Compression Conference (DCC '04)*, p. 531, Snowbird, Utah, USA, March 2004.
[16] L.-L. Xie and P. R. Kumar, "A network information theory for wireless communication: scaling laws and optimal operation," *IEEE Transactions on Information Theory*, vol. 50, no. 5, pp. 748–767, 2004.
[17] A. Reznik, S. R. Kulkarni, and S. Verdú, "Degraded Gaussian multirelay channel: capacity and optimal power allocation," *IEEE Transactions on Information Theory*, vol. 50, no. 12, pp. 3037–3046, 2004.
[18] J. E. Wieselthier, G. D. Nguyen, and A. Ephremides, "On the construction of energy-efficient broadcast and multicast trees in wireless networks," in *Proceedings of 19th Annual Joint Conference of the IEEE Computer and Communications Societies (INFOCOM '00)*, vol. 2, pp. 585–594, Tel Aviv, Israel, March 2000.
[19] A. Srinivas and E. Modiano, "Minimum energy disjoint path routing in wireless ad hoc networks," in *Proceedings of the Annual International Conference on Mobile Computing and Networking (MOBICOM '03)*, pp. 122–133, San Diego, Calif, USA, September 2003.
[20] A. Khandani, J. Abounadi, E. Modiano, and L. Zheng, "Cooperative routing in wireless networks," in *Proceedings of 41st Annual Allerton Conference on Communications, Control and Computing*, pp. 1270–1279, Monticello, Ill, USA, October 2003.
[21] I. Maric and R. D. Yates, "Bandwidth and power allocation for cooperative strategies in Gaussian relay networks," in *Proceedings of Asilomar Conference on Signals, Systems and Computers*, vol. 2, pp. 1907–1911, Monterey, Calif, USA, November 2004.
[22] I. Maric and R. Yates, "Forwarding strategies for parallel-relay networks," in *Proceedings of Information Sciences and Systems (CISS '04)*, Princeton, NJ, USA, March 2004.
[23] S. Chen, M. A. Beach, and J. P. McGeehan, "Division-free duplex for wireless applications," *Electronics Letters*, vol. 34, no. 2, pp. 147–148, 1998.
[24] A. B. Carleial, "Multiple-access channels with different generalized feedback signals," *IEEE Transactions on Information Theory*, vol. 28, no. 6, pp. 841–850, 1982.
[25] M. C. Valenti and B. Zhao, "Capacity approaching distributed turbo codes for the relay channel," in *Proceedings of the 57th IEEE Semiannual Vehicular Technology Conference*, Jeju, Korea, April 2003.
[26] D. Bertsekas and R. Gallager, *Data Networks*, Prentice-Hall, Englewood Cliffs, NJ, USA, 1987.
[27] F. Xue, L.-L. Xie, and P. R. Kumar, "The transport capacity of wireless networks over fading channels," *IEEE Transactions on Information Theory*, vol. 51, no. 3, pp. 834–847, 2005.
[28] A. Chakrabarti, A. Sabharwal, and B. Aazhang, "Multi-hop communication is order-optimal for homogeneous sensor networks," in *Proceedings of the 3rd International Symposium on Information Processing in Sensor Networks (IPSN '04)*, pp. 178–185, Berkeley, Calif, USA, April 2004.

Zigui Yang received his B.S. degree in electronics science from Nankai University, Tianjin, China, in 1998, and his M.Eng. degree in telecommunications engineering from Beijing University of Posts and Telecommunications, Beijing, China, in 2001. He is currently a Ph.D. student in Electrical Engineering Department at the University of Hawaii at Manoa. His research interests include wireless communications and multiuser information theory.

Anders Høst-Madsen was born in Denmark in 1966. He received the M.S. degree in electrical engineering in 1990 and the Ph.D. degree in mathematics in 1993, both from the Technical University of Denmark. From 1993 to 1996, he was with Dantec Measurement Technology A/S, Copenhagen, Denmark, from 1996 to 1998, he was an Assistant Professor at Kwangju Institute of Science and Technology, Kwangju, Korea, and from 1998 to 2000, he was an Assistant Professor at Department of Electrical and Computer Engineering, University of Calgary, Calgary, AB, Canada, and a Staff Scientist at TRLabs, Calgary. Since 2001, he has been with the Department of Electrical Engineering, University of Hawaii at Manoa, Honolulu, currently as an Associate Professor. He has also been a visitor at Department of Mathematics, University of California, Berkeley, through 1992. His research interests are in statistical signal processing, information theory, and wireless communications, including multiuser detection, equalization, ad hoc networks, and wireless sensor networks. He currently serves as the Editor for Multiuser Communications for the IEEE Transactions on Communications and as an Associate Editor for Detection and Estimation for the IEEE Transactions on Information Theory.