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Relay Techniques for MIMO Wireless Networks with Multiple Source and Destination Pairs

Tetsushi Abe,[1] Hui Shi,[2] Takahiro Asai,[2] and Hitoshi Yoshino[2]

[1] *DoCoMo Communications Laboratories Europe GmbH, 312 Landsbergerstreet, Munich 80687, Germany*
[2] *NTT DoCoMo, Inc., Japan*

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A multiple-input multiple-output (MIMO) relay network comprises source, relay, and destination nodes, each of which is equipped with multiple antennas. In a previous work, we proposed a MIMO relay scheme for a relay network with a single source and destination pair in which each of the multiple relay nodes performs QR decompositions of the backward and forward channel matrices in conjunction with phase control (QR-P-QR). In this paper, we extend this scheme to a MIMO relay network employing multiple source and destination pairs. Towards this goal, we use a *group nulling* approach to decompose a multiple S-D MIMO relay channel into parallel independent S-D MIMO relay channels, and then apply the QR-P-QR scheme to each of the decomposed MIMO relay links. We analytically show the logarithmic capacity scaling of the proposed relay scheme. Numerical examples confirm that the proposed relay scheme offers higher capacity than existing relay schemes.

1. INTRODUCTION

A wireless network comprises a number of nodes connected by wireless channels. Using internode transmission (relaying) is an important technique to widen network coverage. Network information theory has shown that the use of multiple relay nodes in source and destination (S-D) communications increases the capacity of the S-D system logarithmically with the number of relay nodes [1].

The use of multiple antennas at each node provides additional degrees of freedom to improve further the capacity per S-D pair in the relay network. A significant capacity improvement achieved with multiple-input multiple-output (MIMO) transmission was revealed in [2–5] for a point-to-point wireless link, and in [6–9] for multiple-access and broadcast channels. The capacity bounds of the MIMO relay network have recently been derived in [10, 11] where the capacity of the MIMO relay network was analyzed in terms of *distributed array gain*, which offers logarithmic capacity scaling, *spatial multiplexing gain*, and *receive array gain*. In [12], we proposed a MIMO relay scheme for a relay network comprising a single S-D pair and multiple relay nodes. The relay technique in [12], called QR-P-QR, performs the QR decomposition (QRD) in the backward and forward channels in conjunction with employing phase control at each relay node, and successive interference cancellation (SIC) at the destination node to detect multiple data streams. This architecture achieves both *distributed array gain* and *receive array gain* while maintaining the maximum *spatial multiplexing gain*, which leads to higher capacity than the existing zero-forcing (ZF) and amplify and forward (AF) relaying techniques [11].

In this paper, we consider a relay network of multiple S-D pairs and multiple relay nodes, and provide a new relaying technique. The proposed relay architecture employs (1) a *group nulling* (GN) technique, which is applied to the backward and forward MIMO relay channels to decompose the multiple S-D MIMO relay channel into parallel independent S-D MIMO relay channels, and (2) the QR-P-QR scheme, which is applied to each of the decomposed S-D relay links. The *group nulling* technique separates multiple S-D pairs via unitary transforms that project both received and transmitted signal vectors at a relay node onto the null space of the signals of nondesired S-D pairs. Thus, the *group nulling* technique retains a higher degree of freedom than the ZF-based stream-wise nulling in MIMO relay channels. Furthermore, the QR-P-QR scheme achieves both *distributed array gain* and *receive array gain* while maintaining the maximum *spatial multiplexing gain* at each of the decomposed MIMO relay links. We analyze the asymptotic capacity of the proposed relay technique and through numerical examples show that the proposed relay

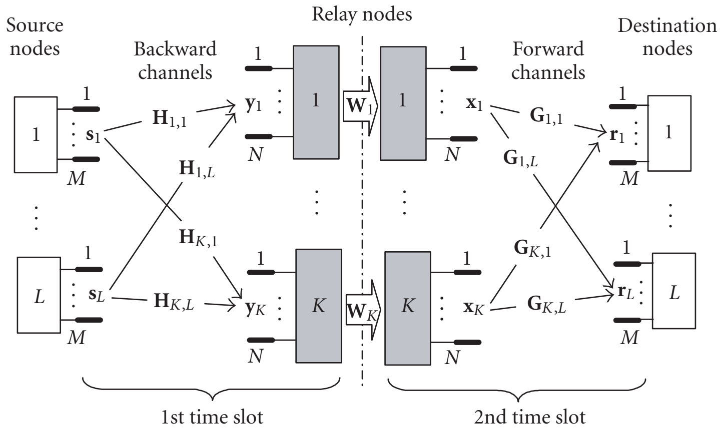

FIGURE 1: MIMO relay network with multiple source and destination pairs.

technique achieves higher capacity than other existing relay schemes.

The rest of this paper is organized as follows. Section 2 shows a system model and the upper bound for the capacity of the MIMO relay network. We describe the proposed and existing relay schemes in Section 3. Numerical examples are given in Section 4. Finally, Section 5 concludes this paper.

Notation

E{•} and tr{•} denote the expectation and trace operation, respectively. $\|\mathbf{a}\|$ stands for the norm of vector $\mathbf{a}$, and superscripts T, H, and $*$ represent the transpose, the conjugate transpose, and the conjugate operation, respectively. $(\mathbf{A})_i$ and $(\mathbf{A})_{i,j}$ denote the ith row and (i, j)th entry of matrix $\mathbf{A}$, respectively. $\mathbf{I}_i$ is the $i \times i$ identity matrix.

2. MIMO RELAY NETWORK

The MIMO relay network used in this paper is illustrated in Figure 1. This paper assumes a one-hop relay network comprising L source and destination nodes, each of which has M antennas, and K relay nodes, each of which has N antennas. In addition, we assume that the relay nodes do not transmit and receive simultaneously. In other words, two time slots are required to send a message from the source to the destination as shown in Figure 1.

First, $M \times 1$ vector $\mathbf{s}_l (l = 1, \dots, L)$, destined for the lth destination node, is sent to all relay nodes from the lth source node without using any channel state information (CSI). The $N \times 1$ vector received at the kth relay node is expressed as $\mathbf{y}_k = \sum_{l=1}^{L} \mathbf{H}_{k,l}\mathbf{s}_l + \mathbf{n}_k$, where $\mathbf{H}_{k,l}$ $(k = 1, \dots, K)$ is the $N \times M$ MIMO channel matrix between the lth source node and the kth relay node (backward channel), and $\mathbf{n}_k$ refers to the $N \times 1$ noise vector at the kth relay node with zero mean and covariance matrix E $\{\mathbf{n}_k\mathbf{n}_k^H\} = \sigma_r^2\mathbf{I}_N$. We constrain the transmitted signal power at the source node to $\text{E}\{\mathbf{s}_l\mathbf{s}_l^H\} = (P/M)\mathbf{I}_M$, where P is the total transmit power. A relay operation is performed at the kth relay node by using $N \times N$ relay matrix $\mathbf{W}_k$ to obtain $N \times 1$ transmitted signal vector $\mathbf{x}_k = E_k\mathbf{W}_k\mathbf{y}_k$, where E_k is a power coefficient resulting from total power constraint $\text{E}\{\mathbf{x}_k{}^H\mathbf{x}_k\} = P$. This can be expressed as

$$E_k = \sqrt{\frac{PM}{P\,\text{tr}\left\{(\mathbf{W}_k\mathbf{H}_k)^H(\mathbf{W}_k\mathbf{H}_k)\right\} + M\sigma_r^2\,\text{tr}\left\{(\mathbf{W}_k)^H(\mathbf{W}_k)\right\}}}, \quad (1)$$

where $N \times LM$ matrix $\mathbf{H}_k = [\mathbf{H}_{k,1}, \dots, \mathbf{H}_{k,L}]$. Finally, the $M \times 1$ receive vector given by

$$\mathbf{r}_l = \sum_{k=1}^{K} \mathbf{G}_{k,l}\,\mathbf{x}_k + \mathbf{z}_l \quad (2)$$

is obtained at the lth destination node, where $\mathbf{G}_{k,l}$ and $\mathbf{z}_l$ are the $M \times N$ channel matrix between the kth relay node and the lth destination node (forward channel), and the $M \times 1$ noise vector added at the lth destination node with zero mean and covariance matrix $\text{E}\{\mathbf{z}_l\mathbf{z}_l^H\} = \sigma_d^2\mathbf{I}_M$, respectively.

Using the cut-set theorem [13], the upper bound for the capacity of the MIMO relay network is derived in [10] as

$$C_{\text{upper}} = \mathbf{E}_{\{\mathbf{H}k\}}\left\{\frac{1}{2}\log\left[\det\left(\mathbf{I}_{LM} + \frac{P}{M\sigma_r^2}\sum_{k=1}^{K}\mathbf{H}_k^H\mathbf{H}_k\right)\right]\right\}. \quad (3)$$

3. MIMO RELAY TECHNIQUES

In this paper, we assume that each relay node knows the CSI of its own backward and forward channels. However, we do not allow source nodes, relay nodes, and destination nodes to exchange their CSI with other nodes.

3.1. ZF relaying scheme [11]

The ZF relaying scheme computes backward and forward ZF matrices $\mathbf{H}_k^+$ and $\mathbf{G}_k^+$ that satisfy $\mathbf{H}_k^+\mathbf{H}_k = \mathbf{I}_{LM}$ and $\mathbf{G}_k\mathbf{G}_k^+ = \mathbf{I}_{LM}$ with $LM \times N$ matrix $\mathbf{G}_k = [\mathbf{G}_{k,1}^T, \dots, \mathbf{G}_{k,L}^T]^T$. Relay matrix $\mathbf{W}_k$ for the ZF scheme is then written as$\mathbf{W}_k = \mathbf{G}_k^+\mathbf{H}_k^+$.

Note here that the ZF scheme requires that $N \geq LM$. In this case, the effective signal-to-noise ratio (SNR) for the mth data stream, $\lambda_{l,m}^{ZF}(m = 1,\ldots,M)$, at the lth destination node is

$$\lambda_{l,m}^{ZF} = \frac{(P/M)\left(\sum_{k=1}^{K} E_k\right)^2}{\sigma_r^2\left(\sum_{k=1}^{K} E_k^2 \|(\mathbf{H}_k^{+})_m\|^2\right) + \sigma_d^2}. \tag{4}$$

From (4), we find that due to the transmit and receive ZF operations the signals from K relay nodes are coherently combined at the destination node, which leads to *distributed array gain* [11].

3.2. GN/QR-P-QR relaying scheme

The first step of the GN/QR-P-QR scheme is to compute a pre-*group nulling* filter at a relay node to suppress the signal component from all source nodes except from the lth source node. To accomplish this, we define $N \times M(L-1)$ matrix $\mathbf{H}_k^{(l)} \equiv [\mathbf{H}_{k,1},\ldots,\mathbf{H}_{k,l-1},\mathbf{H}_{k,l+1},\ldots,\mathbf{H}_{k,L}]$. Note that the channel matrix between the lth source and kth relay node, $\mathbf{H}_{k,l}$, is removed. Next, we perform the singular value decomposition (SVD) of $\mathbf{H}_k^{(l)}$ as

$$\mathbf{H}_k^{(l)} = \begin{bmatrix}\mathbf{U}_{k,1}^{(l)} & \cdots & \mathbf{U}_{k,L-1}^{(l)} & \mathbf{U}_{k,L}^{(l)}\end{bmatrix}\begin{bmatrix}\Lambda_{k,1}^{(l)} & & \mathbf{O}\\ & \ddots & \\ & & \Lambda_{k,L-1}^{(l)}\\ \mathbf{O} & & \mathbf{O}\end{bmatrix}\begin{bmatrix}\mathbf{V}_{k,1}^{(l)H}\\ \vdots \\ \mathbf{V}_{k,L-1}^{(l)H}\end{bmatrix}, \tag{5}$$

where $M \times M$ matrices $\Lambda_{k,1}^{(l)},\ldots,\Lambda_{k,L-1}^{(l)}$ are diagonal matrices, and $N \times M$ matrices $\mathbf{U}_{k,1}^{(l)},\ldots,\mathbf{U}_{k,L-1}^{(l)}$ and $M(L-1)\times M$ matrices $\mathbf{V}_{k,1}^{(l)},\ldots,\mathbf{V}_{k,L-1}^{(l)}$ have orthonormal columns. $N\times N-M(L-1)$ matrix $\mathbf{U}_{k,L}^{(l)}$ spans the null space of $\mathbf{H}_k^{(l)}$. Matrix $\mathbf{U}_{k,L}^{(l)}$ is then multiplied to $\mathbf{y}_k$ to obtain $N-M(L-1)\times 1$ vector $\mathbf{y}_{k,l}$ as

$$\mathbf{y}_{k,l} = \mathbf{U}_{k,L}^{(l)H}\mathbf{y}_k = \mathbf{U}_{k,L}^{(l)H}\mathbf{H}_{k,l}\,\mathbf{s}_l + \mathbf{U}_{k,L}^{(l)H}\mathbf{n}_k. \tag{6}$$

From (6), we see that $\mathbf{U}_{k,L}^{(l)}$ removes the signal contribution from all source nodes except that from the lth source node due to the projection of the received signal vector onto the null space of nondesired source nodes. A null space-based method was also employed in [14] for the precoding in a MIMO down link transmission.

The second step of the GN/QR-P-QR scheme is the transformation of $\mathbf{y}_{k,l}$ using $N-M(L-1)\times N-M(L-1)$ matrix $\Phi_{k,l}$ to obtain vector $\mathbf{y}'_{k,l} = \Phi_{k,l}\mathbf{U}_{k,L}^{(l)H}\mathbf{y}_k$. The computation of $\Phi_{k,l}$ will be described later in this section.

The third step is to compute the post-*group nulling* filter to suppress the transmitted signal to all destination nodes except that to the lth destination node. Toward this goal, we define $N \times M(L-1)$ matrix $\mathbf{G}_k^{(l)} \equiv [\mathbf{G}_{k,1}^H,\ldots,\mathbf{G}_{k,l-1}^H,\mathbf{G}_{k,l+1}^H,\ldots,\mathbf{G}_{k,L}^H]$. Next, we perform the SVD of $\mathbf{G}_k^{(l)}$ as

$$\mathbf{G}_k^{(l)} = \begin{bmatrix}\mathbf{A}_{k,1}^{(l)} & \cdots & \mathbf{A}_{k,L-1}^{(l)} & \mathbf{A}_{k,L}^{(l)}\end{bmatrix} \times \begin{bmatrix}\Omega_{k,1}^{(l)} & & \mathbf{O}\\ & \ddots & \\ & & \Omega_{k,L-1}^{(l)}\\ \mathbf{O} & \cdots & \mathbf{O}\end{bmatrix}\begin{bmatrix}\mathbf{B}_{k,1}^{(l)H}\\ \vdots \\ \mathbf{B}_{k,L-1}^{(l)H}\end{bmatrix}, \tag{7}$$

where $M\times M$ matrices $\Omega_{k,1}^{(l)},\ldots,\Omega_{k,L-1}^{(l)}$ are diagonal matrices, and $N \times M$ matrices $\mathbf{A}_{k,1}^{(l)},\ldots,\mathbf{A}_{k,L-1}^{(l)}$ and $M(L-1)\times M$ matrices $\mathbf{B}_{k,1}^{(l)},\ldots,\mathbf{B}_{k,L-1}^{(l)}$ have orthonormal columns. $N \times N - M(L-1)$ matrix $\mathbf{A}_{k,L}^{(l)}$ spans the null space of $\mathbf{G}_k^{(l)}$. Matrix $\mathbf{A}_{k,L}^{(l)}$ is then multiplied to $\mathbf{y}'_{k,l}$ to obtain $N \times 1$ vector $\mathbf{y}''_{k,l} = \mathbf{A}_{k,L}^{(l)}\Phi_{k,l}\mathbf{U}_{k,L}^{(l)H}\mathbf{y}_k$. Note here that similar to the ZF scheme, the *group nulling* scheme also requires that $N \geq LM$ in order to obtain null space matrices $\mathbf{U}_{k,L}^{(l)}$ and $\mathbf{A}_{k,L}^{(l)}$.

The above three-step procedure is performed for all L source and destination pairs ($l = 1,\ldots,L$) at the kth relay node. Finally, the $N \times 1$ signal vector transmitted from the kth relay node is $\mathbf{x}_k = E_k\sum_{l=1}^{L}\mathbf{y}''_{k,l} = E_k\sum_{l=1}^{L}\mathbf{A}_{k,L}^{(l)}\Phi_{k,l}\mathbf{U}_{k,L}^{(l)H}\mathbf{y}_k$. In this case, the relaying matrix is written as $\mathbf{W}_k = \sum_{l=1}^{L}\mathbf{A}_{k,L}^{(l)}\Phi_{k,l}\mathbf{U}_{k,L}^{(l)H}$, and the received signal vector at the lth destination is written from (2) as

$$\begin{aligned}\mathbf{r}_l &= \sum_{k=1}^{K} E_k\mathbf{G}_{k,l}\mathbf{A}_{k,L}^{(l)}\Phi_{k,l}\mathbf{U}_{k,L}^{(l)H}\mathbf{H}_{k,l}\mathbf{s}_l \\ &\quad + \sum_{k=1}^{K} E_k\mathbf{G}_{k,l}\mathbf{A}_{k,L}^{(l)}\Phi_{k,l}\mathbf{U}_{k,L}^{(l)H}\mathbf{n}_k + \mathbf{z}_l.\end{aligned} \tag{8}$$

Equation (8) shows that at the lth destination node, the signal contribution from all source nodes is removed except that from the lth source node. Namely, we can establish an independent MIMO relay link between the lth source and destination nodes that is characterized by $M \times M$ MIMO channel matrix $E_k\mathbf{G}_{k,l}\mathbf{A}_{k,L}^{(l)}\Phi_{k,l}\mathbf{U}_{k,L}^{(l)H}\mathbf{H}_{k,l}$.

To compute the intermediate filter $\Phi_{k,l}$, we use the QR-P-QR scheme [12]. The QR-P-QR relaying scheme first performs the QRD of $N-M(L-1)\times M$ matrices $\mathbf{U}_{k,L}^{(l)H}\mathbf{H}_{k,l}$ and $(\mathbf{G}_{k,l}\mathbf{A}_{k,L}^{(l)})^H$ as $\mathbf{U}_{k,L}^{(l)H}\mathbf{H}_{k,l} = \mathbf{Q}_{1k,l}\mathbf{R}_{1k,l}$ and $(\mathbf{G}_{k,l}\mathbf{A}_{k,L}^{(l)})^H = \mathbf{Q}_{2k,l}\mathbf{R}_{2k,l}$, where $N-M(L-1)\times M$ matrices $\mathbf{Q}_{1k,l}$ and $\mathbf{Q}_{2k,l}$ have orthonormal columns, and $M \times M$ matrices $\mathbf{R}_{1k,l}$ and $\mathbf{R}_{2k,l}$ are upper triangular matrices. By using these results, the intermediate filter is computed as $\Phi_{k,l} = \mathbf{Q}_{2k,l}\mathbf{D}_{k,l}\prod\mathbf{Q}_{1k,l}^H$, where the $M \times M$ matrix, $\mathbf{D}_{k,l}$, is a diagonal matrix whose mth diagonal entry is $d_{k,l,m} = (\mathbf{R}_{2k,l}^H\Pi\mathbf{R}_{1k,l})_{m,M-m+1}/\|(\mathbf{R}_{2k,l}^H\Pi\mathbf{R}_{1k,l})_{m,M-m+1}\|$ and Π is an $M\times M$ exchange matrix (see [12] for details). We can see that $\Phi_{k,l}$ consists of two orthogonal matrices, $\mathbf{Q}_{1k,l}$ and $\mathbf{Q}_{2k,l}$, obtained by the QRD in the backward and forward channels with phase control matrix $\mathbf{D}_{k,l}$ in between (for this reason this scheme is called QR-P(Phase)-QR). Finally, by using the computed $\Phi_{k,l}$, (8) is rewritten as

$$\mathbf{r}_l = \sum_{k=1}^{K} E_k \mathbf{R}_{2k,l}^H \mathbf{D}_{k,l} \mathbf{R}_{1k,l} \mathbf{s}_l + \sum_{k=1}^{K} E_k \mathbf{R}_{2k,l}^H \mathbf{D}_{k,l} \mathbf{Q}_{1k,l}^H \mathbf{U}_{k,L}^{(l)H} \mathbf{n}_k + \mathbf{z}_l. \tag{9}$$

An important note here is that $E_k \mathbf{R}_{2k,l}^H \mathbf{D}_{k,l} \mathbf{R}_{1k,l}$ takes the lower triangular form with positive scalars in diagonal entries. The triangular structure provides the *receive array gain* by using the SIC at the destination node to detect each data stream. The positive diagonal entries achieved by the phase control matrix enable the diagonal elements transmitted from K relay nodes to be coherently combined at the destination node, which obtains the *distributed array gain*.

The lth destination node simply performs SIC by using the CSI of compound triangular channel $\sum_{k=1}^{K} E_k \mathbf{R}_{2k,l}^H \mathbf{D}_{k,l} \mathbf{R}_{1k,l}$ to detect each of the multiple streams. The effective signal-to-interference-plus-noise ratio (SINR) for the mth data stream at the lth destination node can be expressed as

$$\lambda_{l,m}^{QR} = \frac{(P/M)\left(\sum_{k=1}^{K} (E_k \mathbf{R}_{2k,l}^H \mathbf{D}_{k,l} \mathbf{R}_{1k,l})_{m,M-m+1}\right)^2}{\sigma_r^2 \left(\sum_{k=1}^{K} E_k^2 \left\| \left(\mathbf{R}_{2k,l}^H \mathbf{D}_{k,l}\right)_m \right\|^2\right) + \sigma_d^2}. \tag{10}$$

Consequently, the ergodic capacity of the relay network with total L S-D pairs is

$$C_{QR} = \mathbf{E}_{\{\mathbf{H}_k, \mathbf{G}_k\}} \left\{ \frac{1}{2} \sum_{l=1}^{L} \sum_{m=1}^{M} \log_2 \left(1 + \lambda_{l,m}^{QR}\right) \right\}. \tag{11}$$

3.3. Achievable gains in the relay schemes

To evaluate the achievable gains of the GN/QR-P-QR relay technique, we investigate its asymptotic capacity when K approaches infinity. From (10) and (11), when K approaches infinity, the capacity becomes

$$\begin{aligned} C_{QR} &= \frac{1}{2} \sum_{l=1}^{L} \sum_{m=1}^{M} \log_2 \Bigg(1 \\ &\quad + \frac{(PK/M)\left(\sum_{k=1}^{K} (1/K) E_k (\mathbf{R}_{2k,l}^H)_{m,m} (\mathbf{R}_{1k,l})_{M-m+1,M-m+1}\right)^2}{\sigma_r^2 (1/K) \left(\sum_{k=1}^{K} E_k^2 \left\| (\mathbf{R}_{2k,l}^H \mathbf{D}_{k,l})_m \right\|^2\right) + (1/K)\sigma_d^2} \Bigg) \\ &\xrightarrow{K\to\infty} \frac{ML}{2} \log_2(K) \\ &\quad + \frac{1}{2} \sum_{l=1}^{L} \sum_{m=1}^{M} \log_2 \left(\frac{(P/M)\mathbf{E}\left\{E_k (\mathbf{R}_{2k,l}^H)_{m,m} (\mathbf{R}_{1k,l})_{M-m+1,M-m+1}\right\}^2}{\sigma_r^2 \mathbf{E}\left\{E_k^2 \left\| \left(\mathbf{R}_{2k,l}^H \mathbf{D}_{k,l}\right)_m \right\|^2\right\}} \right), \end{aligned} \tag{12}$$

where we use the approximation $\log_2(1+x) \approx \log_2 x (x \gg 1)$. From (12), we see that the capacity of the GN/QR-P-QR scheme scales with $(LM/2)\log_2(K)$ asymptotically in K. The term $\log_2(K)$ indicates that the *distributed array gain* of the GN/QR-P-QR scheme is K. In addition, the prelog term $LM/2$ implies that the *multiplexing gain* is $LM/2$, where 1/2 represents the loss when using two time slots in each transmission. Furthermore, it was shown in [11] that the upper bound of the capacity in (3) and the capacity of the ZF scheme asymptotically scale with $(LM/2)\log_2(K)$. Thus, we see that the GN/QR-P-QR scheme as well as the ZF scheme exhibit the optimum capacity scaling for a large K value.

The difference between the GN/QR-P-QR scheme and the ZF scheme is the available degrees of freedom remaining after interference suppression among multiple S-D pairs. The ZF scheme performs complete stream-wise nulling in both the backward and forward channels. At each channel the ZF scheme separates LM streams, which requires $LM - 1$ degrees of freedom. Thus, the degrees of freedom that remain after the ZF relaying are $N - (LM - 1)$. On the other hand, since the proposed scheme performs group-wise nulling, it preserves a higher degree of freedom than the ZF scheme. To be more specific, we define the $N - M(L-1) \times M$ decomposed forward MIMO channel for the lth S-D pair from (6) as $\widetilde{\mathbf{H}}_{k,l} \equiv \mathbf{U}_{k,L}^{(l)H} \mathbf{H}_{k,l}$. Assuming $(\mathbf{H}_{k,l})_{i,j}$ are i.i.d. complex random variables with zero mean and unit variance, $(\widetilde{\mathbf{H}}_{k,l})_{i,j}$ has the following statistical property:

$$\mathrm{E}\left\{ (\widetilde{\mathbf{H}}_{k,l})_{i,j}^* (\widetilde{\mathbf{H}}_{k,l})_{i',j'} \right\} = \begin{cases} 1, & i = i',\ j = j', \\ 0, & \text{otherwise}. \end{cases} \tag{13}$$

Proof. When $i = i'$ and $j = j'$, $\mathrm{E}\{(\widetilde{\mathbf{H}}_{k,l})_{i,j}^* (\widetilde{\mathbf{H}}_{k,l})_{i',j'}\} = 1$ because $\mathrm{E}\{\mathbf{H}_{k,l}^H \mathbf{H}_{k,l}\} = \mathbf{I}_M$ and the norm of each column in $\mathbf{U}_{k,l}^{(l)}$ is one. When $i = i'$ and $j \neq j'$, $\mathrm{E}\{(\widetilde{\mathbf{H}}_{k,l})_{i,j}^* (\widetilde{\mathbf{H}}_{k,l})_{i',j'}\} = 0$ because $(\mathbf{H}_{k,l})_{i,j}$ are mutually uncorrelated. When $i \neq i'$ and $j = j'$, $\mathrm{E}\{(\widetilde{\mathbf{H}}_{k,l})_{i,j}^* (\widetilde{\mathbf{H}}_{k,l})_{i',j'}\} = 0$ because the columns of $\mathbf{U}_{k,l}^{(l)}$ are mutually orthogonal. Equation (13) is then proven. □

We can see from (6) and (13) that the *group nulling* transforms $N \times M$ i.i.d. matrix $\mathbf{H}_{k,l}$ to an $N - M(L-1) \times M$ i.i.d. matrix $\widetilde{\mathbf{H}}_{k,l}$. This shows that due to the *group nulling*, $M(L-1)$ degrees of freedom are lost for the lth S-D pair, but $\widetilde{\mathbf{H}}_{k,l}$ still holds $N - M(L-1)$ degrees of freedom. Furthermore, it is straightforward that the same discussion holds for the backward decomposed channel $\mathbf{G}_{k,l} \mathbf{A}_{k,L}^{(l)}$. Thus, after the *group nulling* operations, the proposed scheme holds $N - M(L-1)$ degrees of freedom, which are higher than that of ZF by $M - 1$. This additional degree of freedom is converted as the *receive array gain* through the channel triangulation in (9) using the QR-P-QR technique and the following SIC at the destination node.

3.4. Other simple schemes

For GN-based relaying, we could simply employ an AF relay scheme instead of the QR-P-QR scheme, which gives the intermediate filter $\Phi_{k,l} = \mathbf{I}_{N-M(L-1)}$. In this case, however, we cannot obtain the *distributed array gain* because signals from K relay nodes are randomly combined at the destination node. In addition, [10, 15] describe another simple matched filter (MF) relaying scheme in which each relay node performs receive and transmit MF operations. For the MF relaying, the relay matrix is expressed as $\mathbf{W}_k = \mathbf{G}_k^H \mathbf{H}_k^H$. Unlike the ZF and the proposed schemes, this scheme does not require that $N \geq LM$, and the capacity still scales logarithmically with the number of relay nodes [10].

4. NUMERICAL RESULTS

The ergodic capacities of the relaying schemes presented in the previous section were evaluated. We obtained the capacity plots of the upper bound, ZF, GN/QR-P-QR, GN/AF, and MF. In addition, we evaluated as a reference the capacity of QR-P-QR when all relay and destination nodes fully cooperate. To be more specific, we calculated the capacity of the QR-P-QR scheme in a network comprising a source node with LM transmit antennas, a relay node with KN antennas, and a destination node with LM antennas. In this case, the power constraints at the source and relay are LP and KP, respectively. We assumed a flat fading channel in which each component of $\mathbf{H}_k$ and $\mathbf{G}_k$ is an i.i.d. complex random variable with zero mean and unit variance. We set $\sigma_r^2 = \sigma_d^2$ and identical transmit power P for all source and relay nodes. We did not take into account path loss.

4.1. Capacity versus the number of relay nodes

Figure 2 shows the capacity versus the number of relay nodes K for $L = 2$, $M = 4$, and $N = 8$. The total transmit power-to-noise ratio (PNR $= P/\sigma_r^2$) was set to 20 dB. The graph shows that the capacity of the GN/AF scheme is saturated when K becomes large. This is because although the separation of multiple S-D pairs is accomplished by the *group nulling*, the signals relayed from multiple relay nodes are randomly combined at each destination node due to the simple AF relay operation, and thus the *distributed array gain* is not obtained. On the other hand, we can see that the GN/QR-P-QR scheme, ZF scheme, and MF scheme exhibit logarithmic capacity scaling as does the upper bound of the capacity. This is due to the fact that signal components from multiple relay nodes are coherently combined at the destination node. Furthermore, the GN/QR-P-QR scheme offers higher capacity than the ZF scheme due to the higher degree of freedom converted to the *receive array gain* at the destination node as described in Section 3.3. The capacity of the MF scheme is lower than that of the others due to its inability to suppress actively the interference among S-D pairs. The capacity gap between GN/QR-P-QR and the upper bound is due to the imperfect cooperation among nodes. As mentioned in [10], the capacity upper bound in (3) can be achieved if all the relay nodes perform joint decoding and encoding. To examine this, we obtained the capacity of QR-P-QR when all the relay nodes and all destination nodes cooperate. Note that in this case, there is no need for GN. We can see that the capacity of the QR-P-QR scheme with perfect node cooperation approaches the upper bound. Furthermore, when K becomes larger the gap between the two becomes narrower.

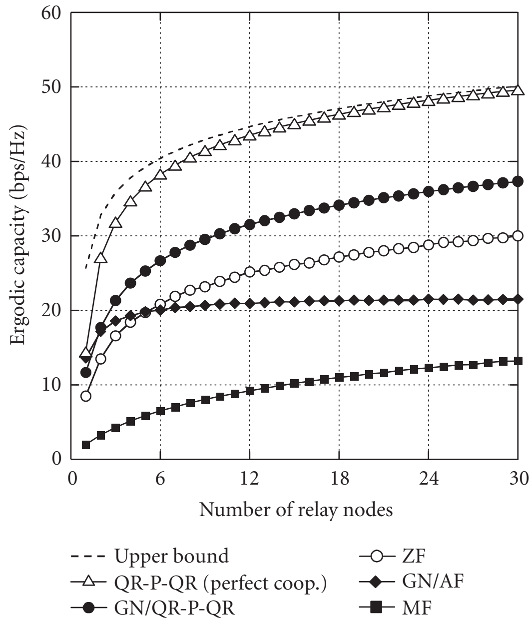

Figure 2: Capacity versus the number of relay nodes ($L = 2$, $M = 4$, $N = 8$).

This can be briefly explained as follows. The capacity upper bound in (3) only depends on the backward channel. On the other hand, the capacity expressions of QR-P-QR in (10) with (11) show that the noise power at destination node σ_d^2 becomes less significant when K becomes large. Thus, the capacity depends more on the backward channel and thus approaches closer to the upper bound. Therefore, if we allow relay nodes to perform the joint relay operation, we could approach closer to the bound. However, this requires all relay nodes and all the destination nodes to exchange their CSI. In addition, the joint relay operation requires the QRD of $KN \times LM$ matrix, which might be practically demanding in terms of complexity. Figure 3 shows capacity plots for $L = 2$, $M = 2$, and $N = 4$. A similar tendency is observed, but the gap between GN/QR-P-QR and ZF is decreased. This is because the number of antennas at each node is reduced by half, and thus the *receive array gain* obtained in the GN/QR-P-QR scheme is decreased. Figure 4 shows capacity plots for $L = 4$, $M = 2$, and $N = 8$. In this case, the total number of antennas in the network is the same as in the case in Figure 2, but the capacity obtained by each relay scheme is higher than that in Figure 2 except for MF. This is because the total transmit power in the network is increased due to the increased number of the S-D pairs.

4.2. Capacity versus PNR

Figures 5 and 6 show the capacity versus the PNR for $L = 2$, $M = 4$, and $N = 8$ for $K = 2$ and 8, respectively. The figures show that the GN/QR-P-QR and the GN/AF schemes offer similar capacity for $K = 2$. However, Figure 6 shows that when $K = 8$, GN/QR-P-QR outperforms GN/AF due to the *distributed array gain*. In both figures, the capacity of the

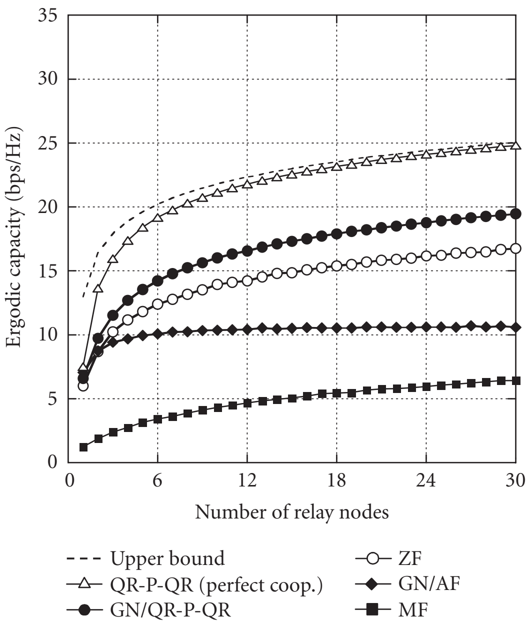

Figure 3: Capacity versus the number of relay nodes ($L = 2$, $M = 2$, $N = 4$).

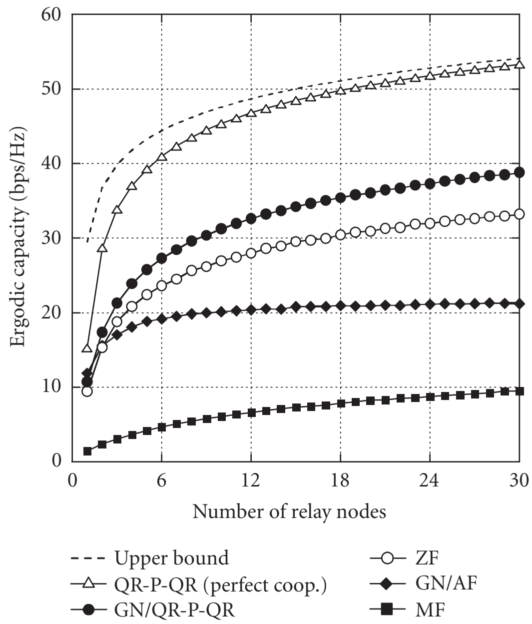

Figure 4: Capacity versus the number of relay nodes ($L = 4$, $M = 2$, $N = 8$).

MF scheme is better than the other schemes in a low PNR region due to the SNR gain of the matched filtering. However, the capacity saturates in a high PNR region due to the interference among S-D pairs.

4.3. Effectiveness of spatially multiplexing multiple S-D pairs

Figure 7 shows the capacity curves of the GN/QR-P-QR scheme for $L = 2$, $M = 4$, and $N = 8$ with $K = 2$ and 8. Here, we measured the capacity for two cases: time-division multiplexing (TDM) and spatial-division multiplexing (SDM) for the two S-D pairs. Note that in the former case, only one S-D pair is active at any instant, and thus *group nulling* is not needed. Figure 7 shows that in a low PNR region, TDM provides higher capacity, but in higher PNR regions, SDM offers significantly higher capacity, which matches results of conventional studies on the trade-off between spatial multiplexing and beam-forming. Furthermore, the figure shows that when K increases, the crosspoint of SDM and TDM is shifted to lower PNR regions. This is because the effective SNR at the destination node increases as K increases. Thus, it is clear that it is more advantageous to multiplex spatially multiple S-D pairs in a situation, where the PNR is relatively high or the number of relay nodes is relatively large.

4.4. Capacity versus the number of antennas at the relay node

Figure 8 shows the capacity of the GN/QR-P-QR and the ZF schemes with various N for $L = 2$ and $M = 4$. K is set to 2 and 8. We can see that when the number of antennas per relay node, N, increases, the capacity gap between the GN/QR-P-QR and the ZF schemes becomes smaller. This is because as N becomes larger, both the GN and the ZF operations retain enough degrees of freedom after the interference suppression as shown in Section 3.3.

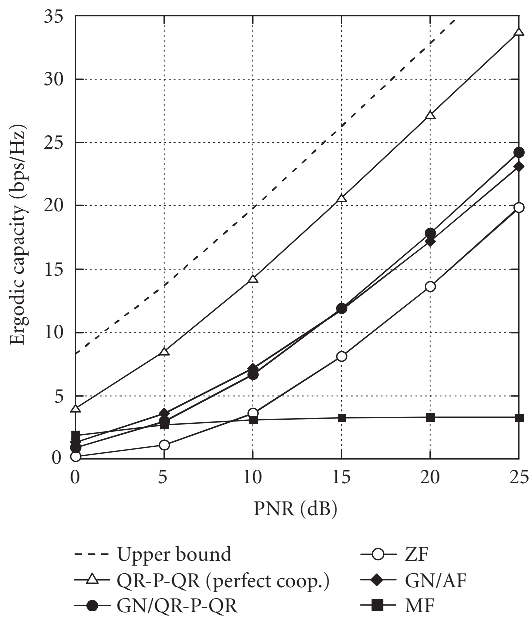

Figure 5: Capacity versus PNR ($L = 2$, $M = 4$, $N = 8$, $K = 2$).

4.5. Complexity

Finally, Table 1 shows the computational complexity of the relaying schemes. The complexities were measured as the

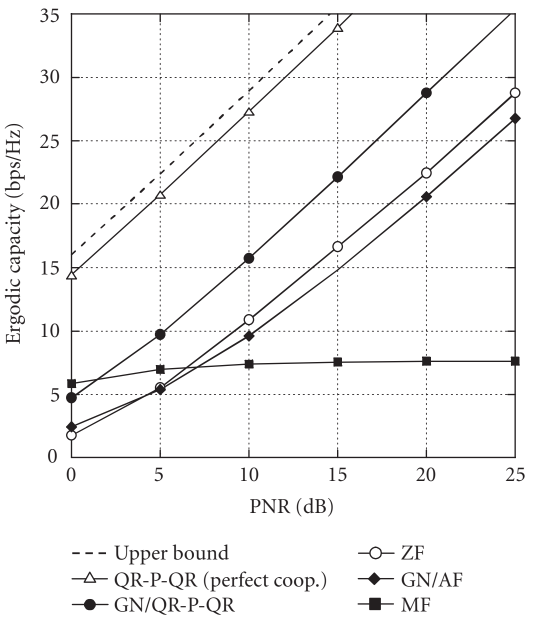

Figure 6: Capacity versus PNR ($L = 2, M = 4, N = 8, K = 8$).

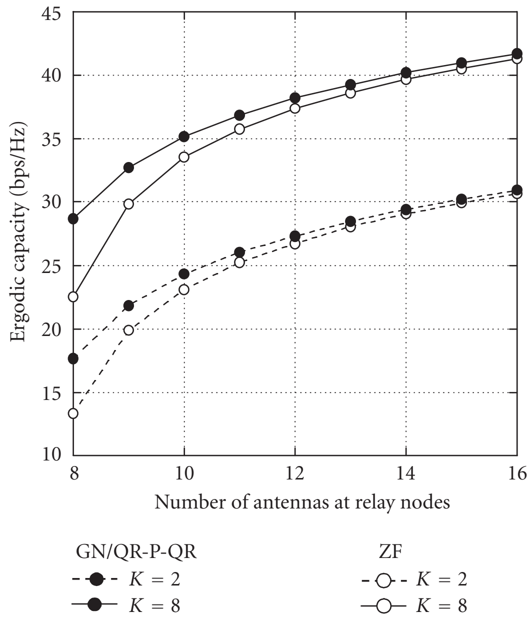

Figure 8: Capacity of GN/QR-P-QR versus ZF for various $N(L = 2, M = 4)$.

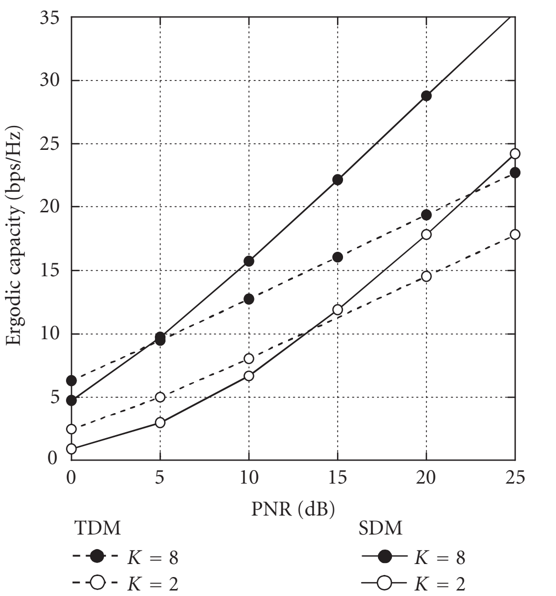

Figure 7: Capacity of GN/QR-P-QR: SDM versus TDM ($L = 2, M = 4, N = 8$).

number of required complex multiplications at each relay node. We approximated the complexity by computing only matrix inversion, multiplication, SVD, and QRD parts and evaluated only terms with the highest order (cubic) in terms of matrix size. First, we observe that the complexity of the MF scheme is much lower than that of others due to its simple operations. The ZF scheme needs only one matrix inversion for both the backward and forward channel matrices ($\mathbf{H}_k$ and $\mathbf{G}_k^T$), but the matrix size $N \times LM$ is the largest. The GN/AF scheme requires SVD for every S-D pair of both equivalent backward and forward channel matrices ($\mathbf{H}_k^{(l)}$ and $\mathbf{G}_k^{(l)}$), but the matrix size $N \times M(L-1)$ is smaller than that in ZF. The GN/QR-P-QR scheme further requires QRD for every S-D pair of both equivalent backward and forward channels $\mathbf{U}_{k,L}^{(l)H}\mathbf{H}_{k,l}$ and $(\mathbf{G}_{k,l}\mathbf{A}_{k,L}^{(l)})^H$, and their matrix size, $N - M(L - 1) \times M$, is smaller than that in ZF. Thus, when the number of S-D pairs is small, such as when $(L, M, N) = (2, 2, 4)$ and $(2, 4, 8)$, the GN-based relay schemes offer lower complexity than the ZF due to the matrix size reduction. On the other hand, when the number of S-D pairs becomes larger, such as when $(L, M, N) = (4, 2, 8)$, the ZF scheme offers lower complexity due to fewer matrix operations. Therefore, when the number of S-D pairs is small, the GN/QR-P-QR scheme achieves higher capacity with lower complexity than the ZF scheme.

5. CONCLUDING REMARKS

In this paper, we proposed a relay technique for a MIMO relay network with multiple S-D pairs. The *group nulling* technique projects the receive and transmitted signal vectors at the relay node onto the null space of the signals of nondesired S-D pairs, so the multiple S-D MIMO relay channel is decomposed into parallel independent MIMO channels. To each decomposed MIMO relay link, the QR-P-QR technique is applied. This relaying architecture preserves a higher degree of freedom in the MIMO relay channel than the ZF scheme and enables coherent combination of the signals at the destination to achieve *distributed array gain*. We analyzed the asymptotic capacity of the proposed relay technique and clarified its achievable gains. Numerical examples confirmed that the proposed relay scheme achieves higher capacity than other existing relay schemes. It should be mentioned,

Table 1: Computational complexity per relay node (number of complex multiplications), ($A = M(L-1)$, $B = N - M(L-1)$).

	Complexity	$(L, M, N) = (2, 2, 4)$	$(L, M, N) = (2, 4, 8)$	$(L, M, N) = (4, 2, 8)$
MF	$N(ML)^2$	64	512	512
ZF	$(3N^2(ML) + 2(ML)^3 + N^3) \times 2 + N^3$	832	6656	6656
GN/AF	$(3N^2A + N^3) \times L \times 2 + N^2A \times L$	704	5632	14048
GN/QR-P-QR	$(3N^2A + N^3) \times L \times 2$ $+(3B^2M - 3/2BM^2 + M^3) \times L \times 2$ $+(2MB^2 + N^2B) \times L$	816	6528	14848

however, that the requirement for the number of antennas, $N \geq LN$, in the proposed scheme as well as in the ZF relay scheme could still be a limiting factor in some application scenarios. In addition, since the relay techniques described in this paper assume perfect CSI knowledge for both the backward and forward MIMO channels at each relay terminal, investigation of their capacity with imperfect CSI is an important future research topic.

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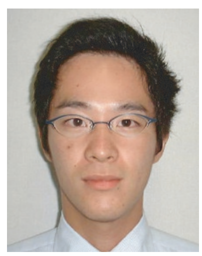

Tetsushi Abe received his B.S. degree and M.S. degree in electrical and electronic engineering from Tokyo Institute of Technology, Tokyo, Japan, in 1998 and 2000, respectively. During 1998-1999, he studied in the Department of Electrical and Computer Engineering in University of Wisconsin, Madison, USA, under the scholarship exchange student program offered by the Japanese Ministry of Education. He joined NTT DoCoMo, Inc., in 2000. Since 2005, he has been with DoCoMo Euro-Labs. He has conducted researches on signal processing for wireless communications: multiple-input and multiple-output (MIMO) transmission, space-time turbo equalization, relay transmission, and OFDM transmission. He is a Member of IEEE and IEICE.

Hui Shi received his B.S. degree in mechanic engineering from Dalian University of Technology, Dalian, China, in 1998 and M.S. degree in electrical and electronic engineering from Nagoya University, Nagoya, Japan, in 2002. Since 2002, he has been with the Research Laboratories at NTT DoCoMo, Inc. His research interests cover the wireless network systems, relay networks, multiple-input and multiple-output (MIMO) transmission, and information theory issues. He is a Member of IEEE and IEICE.

Takahiro Asai received the B.E. and M.E. degrees from Kyoto University, Kyoto, Japan, in 1995 and 1997, respectively. In 1997, he joined NTT Mobile Communications Network, Inc. (now NTT DoCoMo, Inc.). Since joining NTT Mobile Communications Network, Inc., he has been engaged in the research of signal processing for mobile radio communication. He is a Member of IEEE.

Hitoshi Yoshino received the B.S. and M.S. degrees in electrical engineering from the Science University of Tokyo, Tokyo, Japan, in 1986 and 1988, respectively, and the Dr.Eng. degree in communications and integrated systems from the Tokyo Institute of Technology, Tokyo, Japan, in 2003. From 1988 to 1992, he was with Radio Communication Systems Laboratories, Nippon Telegraph and Telephone Corporation (NTT), Japan. Since 1992, he has been with NTT Mobile Communications Network, Inc. (currently, NTT DoCoMo, Inc.), Japan. Since joining NTT DoCoMo, he has been engaged in the areas of mobile radio communication systems and digital signal processing. From 1998 to 1999, he was at the Deutsche Telekom Technologiezentrum, Darmstadt, Germany, as a Visiting Researcher. He is currently an Executive Research Engineer in Wireless Laboratories, NTT DoCoMo, Inc. He received the Young Engineer Award and the Excellent Paper Award from the Institute of Electronics, Information, and Communication Engineers (IEICE) of Japan both in 1995. He is a Member of IEEE.