Hindawi Publishing Corporation
EURASIP Journal on Wireless Communications and Networking
Volume 2006, Article ID 86026, Pages 1–7
DOI 10.1155/WCN/2006/86026

A Conjugate-Cyclic-Autocorrelation Projection-Based Algorithm for Signal Parameter Estimation

Valentina De Angelis,[1] Luciano Izzo,[1] Antonio Napolitano,[2] and Mario Tanda[1]

[1] *Dipartimento di Ingegneria Elettronica e delle Telecomunicazioni, Università di Napoli "Federico II," Via Claudio 21, 80125 Napoli, Italy*
[2] *Dipartimento per le Tecnologie, Università di Napoli "Parthenope," Via Acton 38, 80133 Napoli, Italy*

Received 1 March 2005; Revised 8 March 2006; Accepted 13 March 2006

Recommended for Publication by Alex Gershman

A new algorithm to estimate amplitude, delay, phase, and frequency offset of a received signal is presented. The frequency-offset estimation is performed by maximizing, with respect to the conjugate cycle frequency, the projection of the measured conjugate-cyclic-autocorrelation function of the received signal over the true conjugate second-order cyclic autocorrelation. It is shown that this estimator is mean-square consistent, for moderate values of the data-record length, outperforms a previously proposed frequency-offset estimator, and leads to mean-square consistent estimators of the remaining parameters.

1. INTRODUCTION

Demodulation in digital communication systems requires knowledge of symbol timing, frequency offset, and phase shift of the received signal. Moreover, in several applications (e.g., power control) the knowledge of the amplitude of the received signal is also required.

Several blind (i.e., non data-aided) algorithms for estimating some of the parameters of interest have been proposed in the literature. In particular, some of them exploit the cyclostationarity properties exhibited by almost all modulated signals [1]. Cyclostationary signals have statistical functions such as the autocorrelation function, moments, and cumulants that are almost-periodic functions of time. The frequencies of the Fourier series expansion of such almost-periodic functions are called cycle frequencies and are related to parameters such as the carrier frequency and the baud rate. Unlike second-order stationary statistics, second-order cyclic statistics (e.g., the cyclic autocorrelation function and the conjugate-cyclic-autocorrelation function [1]) preserve phase information and, hence, are suitable for developing blind estimation algorithms.

Cyclostationarity-exploiting blind estimation algorithms for synchronization parameters have been proposed and analyzed in [2–10]. In particular, the carrier-frequency-offset (CFO) estimator proposed in [3, 5, 9], termed conjugate-cyclic-autocorrelation norm (CCAN), performs the maximization, with respect to the conjugate cycle frequency, of the L^2-norm of the conjugate-cyclic-autocorrelation function. In [3], it is shown that such an estimator is asymptotically Gaussian and mean-square consistent (i.e., the mean-square error approaches zero) with asymptotic variance $\mathcal{O}(N^{-3})$, where N is the sample size.

The technique proposed in [5, 9] for the multiuser scenario, exploits the estimated frequency shifts to obtain the unknown conjugate cycle frequencies of the received signal. These conjugate cycle frequencies are then filled in cyclic statistic estimators that are used to estimate the remaining parameters (amplitudes, delays, and phases). Since it is well known that cyclic statistic estimators are very sensitive to errors in the cycle frequency values [1], a new CFO estimator, termed conjugate-cyclic-autocorrelation projection (CCAP) is proposed here for the single-user case. It is based on the maximization, with respect to the conjugate cycle frequency, of the projection of the measured conjugate-cyclic-autocorrelation function of the received signal over the true conjugate-cyclic autocorrelation. The amplitude, delay, and phase estimates are then obtained by exploiting the single-user version of the algorithm proposed in [5, 9]. This algorithm, for small or moderate values of the data-record length, outperforms the previously proposed CCAN method where the CFO estimation is obtained by maximizing with respect to the conjugate cycle frequency the L^2-norm of the conjugate-cyclic-autocorrelation function, that is, the projection of the measured conjugate-cyclic-autocorrelation function over itself (i.e., over a noisy reference). In the paper, the asymptotic performance analysis of the CCAP method is also derived. Specifically, it is shown that the CCAP CFO estimator is asymptotically Gaussian and mean-square consistent with asymptotic variance $\mathcal{O}(N^{-3})$. Consequently, the estimators of amplitude, delay, and phase are proved to be in turn consistent. Moreover, simulations are carried out to show that, for finite N, the CCAP CFO estimator variance can be smaller than that of the CCAN estimator. It is worthwhile to emphasize that the considered algorithm is not based on the usual assumption of white and/or Gaussian ambient noise, and it exhibits the typical interference and noise immunity of the algorithms based on the cyclostationarity properties of the involved signals.

2. THE ESTIMATION ALGORITHM

In this section the estimation algorithm is presented. First partial results were presented in [7].

Let us consider the complex envelope of the continuous-time received signal

$$y_a(t) = Ae^{j\varphi}x_a(t-d_a)e^{j2\pi\nu_a t} + w_a(t), \qquad (1)$$

where $w_a(t)$ is additive noise, $x_a(t)$ is the transmitted signal, and A, φ, d_a, and ν_a are the scaling amplitude, phase shift, time delay, and frequency shift, respectively. If $y_a(t)$ is uniformly sampled with sampling period $T_s = 1/f_s$, we obtain the discrete-time signal

$$y(n) \triangleq y_a(t)\big|_{t=nT_s} = Ae^{j\varphi}x_d(n)e^{j2\pi\nu n} + w(n), \qquad (2)$$

where $x_d(n) \triangleq x_a(t-d_a)|_{t=nT_s}$, $w(n) \triangleq w_a(t)|_{t=nT_s}$, and $\nu \triangleq \nu_a T_s$.

By assuming $x_a(t)$ and $w_a(t)$ zero mean and statistically independent, the cyclic autocorrelation and conjugate-cyclic-autocorrelation functions of $y(n)$ are

$$\begin{aligned} r^{\alpha}_{yy^*}(m) &\triangleq \lim_{N\to\infty}\frac{1}{2N+1}\sum_{n=-N}^{N}E\{y(n+m)y^*(n)\}e^{-j2\pi\alpha n} \\ &= A^2e^{-j2\pi\alpha d}r^{\alpha}_{xx^*}(m)e^{j2\pi\nu m} + r^{\alpha}_{ww^*}(m), \end{aligned} \qquad (3)$$

$$\begin{aligned} r^{\beta}_{yy}(m) &\triangleq \lim_{N\to\infty}\frac{1}{2N+1}\sum_{n=-N}^{N}E\{y(n+m)y(n)\}e^{-j2\pi\beta n} \\ &= A^2e^{-j2\pi(\beta-2\nu)d}e^{j2\varphi}r^{\beta-2\nu}_{xx}(m)e^{j2\pi\nu m} + r^{\beta}_{ww}(m), \end{aligned} \qquad (4)$$

respectively, provided that there are no cycle frequencies or conjugate cycle frequencies of $x_a(t)$ whose magnitude exceeds $f_s/2$ (see [11]). In equations (3) and (4), $d \triangleq d_a/T_s$ is not necessarily an integer number, and $r^{\alpha}_{xx^*}(m)$ and $r^{\beta}_{xx}(m)$ are the cyclic-autocorrelation and the conjugate-cyclic-autocorrelation function, respectively, of $x(n) \triangleq x_a(t)|_{t=nT_s}$.

Under the assumption that the disturbance signal $w(n)$ does not exhibit neither cyclostationarity with cycle frequency α, nor conjugate cyclostationarity with conjugate cycle frequency β, that is,

$$r^{\alpha}_{ww^*}(m) \equiv r^{\beta}_{ww}(m) \equiv 0, \qquad (5)$$

(3) and (4) provide useful relationships to derive algorithms highly immune against noise and interference, regardless of the extent of the temporal and spectral overlap of the signals $x(n)$ and $w(n)$. Note that, even if the disturbance term $w(n)$ can contain, in general, both stationary noise and non-stationary interference, the assumption (5) on $w(n)$ is mild. In fact, it is verified provided that there is at least one (conjugate) cycle frequency of the user signal and its frequency-shifted version that is different from the interference (conjugate) cycle frequencies. Moreover, the stationary component of the noise term never gives contribution to the cyclic statistics of $w(n)$.

Let $w_a(t)$ be circular (i.e., with zero conjugate correlation function) and $x_a(t)$ noncircular and with conjugate cyclostationarity with period QT_s. Thus, $w(n)$ is circular (i.e., its conjugate correlation function $r_{ww}(n,m) \triangleq E\{w(n+m)w(n)\}$ is identically zero), and, moreover, $x(n)$ is noncircular and exhibits conjugate cyclostationarity with period Q. Consequently, $y(n)$ exhibits a conjugate correlation

$$r_{yy}(n,m) = \sum_{k=0}^{Q-1} r^{\beta_k}_{xx}(m)A^2e^{j2\varphi}e^{-j2\pi\beta_k d}e^{j2\pi\nu m}e^{j2\pi(\beta_k+2\nu)n}, \qquad (6)$$

where $\beta_k \triangleq k/Q$.

Let $\mathbf{y}_2(n) \triangleq [y(n-M)y(n),\ldots,y(n+M)y(n)]^T$ be the second-order lag product vector. The conjugate-cyclic-correlogram vector

$$\mathbf{r}^{\beta}_{yy,N} \triangleq \frac{1}{2N+1}\sum_{n=-N}^{N}\mathbf{y}_2(n)e^{-j2\pi\beta n} \qquad (7)$$

is an estimate of the conjugate-cyclic-autocorrelation vector $\mathbf{r}^{\beta}_{yy} \triangleq [r^{\beta}_{yy}(-M),\ldots,r^{\beta}_{yy}(M)]^T$ at conjugate cycle frequency β, evaluated on the basis of the received signal observed over a finite interval of length $2N+1$.

The proposed CCAP CFO estimator is

$$\widehat{\omega}_N \triangleq \arg\max_{\omega\in I_0}\big|f_N(\omega)\big|^2 \qquad (8)$$

with

$$\begin{aligned} f_N(\omega) &\triangleq \sum_{m=-M}^{M} r^{\beta_k+2\omega}_{yy,N}(m)e^{-j2\pi\omega m}r^{\beta_k}_{xx}(m)^* \\ &= \left[\mathbf{r}^{\beta_k+2\omega}_{yy,N}\odot\mathbf{a}(\omega)^*\right]^T(\mathbf{r}^{\beta_k}_{xx})^*, \end{aligned} \qquad (9)$$

where $\odot$ denotes the Hadamard matrix product, $\mathbf{a}(\omega) \triangleq [e^{-j2\pi\omega M},\ldots,e^{j2\pi\omega M}]^T$, and β_k is a (possibly zero) conjugate cycle frequency of $x(n)$. In (8), $I_0 \triangleq [\beta_k - \Delta\beta/2, \beta_k + \Delta\beta/2]$ with $\Delta\beta$ and the frequency shift satisfying the conditions $|\nu| \le \Delta\beta/4$ and $\Delta\beta < 1/Q$.

The function $|f_N(\omega)|$ represents the magnitude of the projection of the conjugate-cyclic-autocorrelation function estimate $r_{yy,N}^{\beta_k+2\omega}(m)$ over its asymptotic ($N \to \infty$) expression obtained by setting $\beta = \beta_k + 2\omega = \beta_k + 2\nu$ into (4) with $r_{ww}^{\beta}(m) \equiv 0$. Thus, in the limit for $N \to \infty$, $|f_N(\omega)|$ is nonzero only in correspondence of the discrete set of values of ω such that $\beta = \beta_k + 2\omega$ are conjugate cycle frequencies of the signal $y(n)$. Consequently, it is nonzero only for $\omega = \nu$, provided that $r_{ww}^{\beta}(\tau) \equiv 0$ for $\beta \in I_0$. Thus, in the limit for $N \to \infty$, $|f_N(\omega)|$ exhibits a peak at $\omega = \nu$, and, for finite N, an estimate $\widehat{\omega}_N$ of the frequency shift ν can be obtained by locating the maximum of the function $|f_N(\omega)|$ for $\omega \in I_0$.

Note that, for finite observation interval, the CCAP CFO estimator is expected to outperform the CCAN estimator. In fact, in [3, 9], the CCAN CFO estimate is obtained by maximizing the function $\omega \to \|\mathbf{r}_{yy,N}^{\beta_k+2\omega}\|^2$ which is the projection of the conjugate-cyclic-autocorrelation function estimate over itself. That is, for finite observation interval, the reference signal for the inner product (projection) in $\|\mathbf{r}_{yy,N}^{\beta_k+2\omega}\|^2$ is a noisy version of that adopted in (9).

Once the frequency-shift estimate $\widehat{\omega}_N$ has been obtained, the estimation of amplitude, delay, and phase can be performed by considering the single-user version of the algorithm proposed in [5, 9] for the multiuser scenario.

Let us assume now that α_x is a known nonzero cycle frequency of $x(n)$. Equation (3) (with $r_{ww^*}^{\alpha_x}(m) \equiv 0$) suggests that the estimation of amplitude and time-delay parameters can be performed by minimizing with respect to γ the function

$$g(\gamma, \gamma^*) \triangleq \left\| \mathbf{r}_{yy^*,N}^{\alpha_x} - \gamma \mathbf{r}_{xx^*}^{\alpha_x} \odot \mathbf{a}(\widehat{\omega}_N) \right\|^2. \tag{10}$$

In fact, in the limit for $N \to \infty$ and for $\widehat{\omega}_N = \nu$, it results that $g(\gamma, \gamma^*) = 0$ for

$$\gamma = A^2 e^{-j2\pi\alpha_x d}. \tag{11}$$

For finite N, the value of γ that minimizes $g(\gamma, \gamma^*)$ is given by

$$\gamma_{\text{opt}} = \left[\mathbf{r}_{yy^*,N}^{\alpha_x}\right]^T \left[\mathbf{r}_{xx^*}^{\alpha_x} \odot \mathbf{a}(\widehat{\omega}_N)\right]^* \|\mathbf{r}_{xx^*}^{\alpha_x}\|^{-2}. \tag{12}$$

Thus, accounting for (11), the estimates of the amplitude A and the arrival time d are

$$\widehat{A} = \sqrt{|\gamma_{\text{opt}}|}, \tag{13}$$

$$\widehat{d} = -\frac{\angle[\gamma_{\text{opt}}]}{2\pi\alpha_x}, \tag{14}$$

respectively, where $\angle[\cdot]$ is the angle of a complex number.

Let us assume now that β_x is a known conjugate cycle frequency of $x(n)$. Equation (4) (with $r_{ww}^{\beta}(\tau) \equiv 0$ for $\beta \in [\beta_x - \Delta\beta/2, \beta_x + \Delta\beta/2]$) suggests that the estimation of the phase φ can be performed by minimizing with respect to $\bar{\gamma}$ the function

$$h(\bar{\gamma}, \bar{\gamma}^*) \triangleq \left\| \mathbf{r}_{yy,N}^{\beta_x+2\widehat{\omega}_N} - \bar{\gamma} \mathbf{r}_{xx}^{\beta_x} \odot \mathbf{a}(\widehat{\omega}_N) \right\|^2. \tag{15}$$

In fact, in the limit for $N \to \infty$ and for $\widehat{\omega}_N = \nu$, it results that $h(\bar{\gamma}, \bar{\gamma}^*) = 0$ for

$$\bar{\gamma} = A^2 e^{-j2\pi\beta_x d} e^{j2\varphi}. \tag{16}$$

For finite N, the value of $\bar{\gamma}$ that minimizes $h(\bar{\gamma}, \bar{\gamma}^*)$ is given by

$$\bar{\gamma}_{\text{opt}} = \left[\mathbf{r}_{yy,N}^{\beta_x+2\widehat{\omega}_N}\right]^T \left[\mathbf{r}_{xx}^{\beta_x} \odot \mathbf{a}(\widehat{\omega}_N)\right]^* \|\mathbf{r}_{xx}^{\beta_x}\|^{-2}. \tag{17}$$

Thus, accounting for (11) and (16), it follows that the estimate of the phase φ is given by

$$\widehat{\varphi} = \frac{1}{2} \angle \left[\frac{\bar{\gamma}_{\text{opt}}}{\gamma_{\text{opt}}} e^{j2\pi(\beta_x - \alpha_x)\widehat{d}} \right]. \tag{18}$$

It can be straightforwardly verified that the stationary points so determined for both the functions (10) and (15) are points of minimum.

Note that, in order to avoid ambiguities in the estimates (14) and (18), the following relationships must hold: $|d| \le 1/2|\alpha_x|$ and $|\varphi| \le \pi/2$. In [7] it is shown that, for an appropriate choice of the cycle frequency α_x, the condition on the delay is not a restriction for the synchronization purpose. On the contrary, the condition on the phase leads to a phase ambiguity that can be resolved by using differential encoding.

3. ASYMPTOTIC PERFORMANCE ANALYSIS OF THE CCAP CFO ESTIMATOR

In this section, the asymptotic performance analysis of the considered estimation algorithm is carried out. First partial results were presented in [2]. First, by following the guidelines given in [3], the CFO estimator is shown to be mean-square consistent with variance $\mathcal{O}(N^{-3})$. Then, it is shown that such an asymptotic behavior allows to prove the consistency of the estimators of the remaining parameters.

Analytical nonasymptotic results of CFO estimators based on cyclic statistics are difficult to obtain due to the difficulty of obtaining analytic nonasymptotic results for the cyclic statistic estimators. In fact, even if analytical expressions for the bias and variance can be obtained for finite data-record-length estimators of cyclic temporal and spectral moments and cumulants, these expressions are extremely complicated. Moreover, only asymptotic results for the distribution function of the cyclic statistic estimators have been derived in the literature (see, e.g., [12] and references therein).

Let us consider the Taylor series expansion of the derivative of $|f_N(\omega)|^2$ with Lagrange residual term:

$$\frac{\mathrm{d}}{\mathrm{d}\omega} |f_N(\omega)|^2 \Big|_{\omega=\widehat{\omega}_N} = \frac{\mathrm{d}}{\mathrm{d}\omega} |f_N(\omega)|^2 \Big|_{\omega=\nu} + \frac{\mathrm{d}^2}{\mathrm{d}\omega^2} |f_N(\omega)|^2 \Big|_{\omega=\widetilde{\omega}_N} (\widehat{\omega}_N - \nu), \tag{19}$$

where $\widetilde{\omega}_N = \nu + \eta_N(\widehat{\omega}_N - \nu)$ and $\eta_N \in [0, 1]$. By following the guidelines in [3, 13], it can be shown that

$$\lim_{N\to\infty} N(\widehat{\omega}_N - \nu) = 0 \quad \text{a.s.}, \tag{20}$$

and, hence,

$$\lim_{N\to\infty} \widetilde{\omega}_N = \nu \quad \text{a.s.} \tag{21}$$

By setting $[\mathrm{d}|f_N(\omega)|^2/\mathrm{d}\omega]_{\omega=\widehat{\omega}_N} = 0$, it follows that

$$(2N+1)^{3/2}(\widehat{\omega}_N - \nu) = -\mathcal{A}_N^{-1}\mathcal{B}_N, \tag{22}$$

where

$$\begin{aligned}\mathcal{A}_N &\triangleq (2N+1)^{-2}\frac{\mathrm{d}^2}{\mathrm{d}\omega^2}|f_N(\omega)|^2\Big|_{\omega=\widetilde{\omega}_N}\\ &= 2(2N+1)^{-2}\,\mathrm{Re}\left\{f_N''(\widetilde{\omega}_N)f_N(\widetilde{\omega}_N)^*\right\}\\ &\quad + 2(2N+1)^{-2}|f_N'(\widetilde{\omega}_N)|^2,\end{aligned} \tag{23}$$

$$\begin{aligned}\mathcal{B}_N &\triangleq (2N+1)^{-1/2}\frac{\mathrm{d}}{\mathrm{d}\omega}|f_N(\omega)|^2\Big|_{\omega=\nu}\\ &= 2(2N+1)^{-1/2}\,\mathrm{Re}\left\{f_N'(\nu)f_N(\nu)^*\right\}\end{aligned} \tag{24}$$

with $f_N'(\omega)$ and $f_N''(\omega)$ denoting the first-and the second-order derivative, respectively, of $f_N(\omega)$.

As regards the computation of the term $\mathcal{A}_N$, let us observe that the second-order lag product vector $\mathbf{y}_2(n)$ can be decomposed into the sum of a periodic term (the conjugate correlation vector) and a residual term $\mathbf{e}(n)$ not containing any finite-strength additive sine wave component and generally satisfying some mixing conditions expressed in terms of the summability of its cumulants [3]:

$$\mathbf{y}_2(n) = \sum_{h=0}^{Q-1}\mathbf{r}_{xx}^{\beta_h}\odot\mathbf{a}(\nu)e^{j2\pi(\beta_h+2\nu)n} + \mathbf{e}(n), \tag{25}$$

where, for the purpose of CFO estimation error asymptotic analysis, without lack of generality, $A = 1$, $\varphi = 0$, and $d = 0$ have been assumed.

By substituting (25) into (7) one has

$$\mathbf{r}_{yy,N}^{\beta_k+2\omega} = \sum_{h=0}^{Q-1}\mathbf{r}_{xx}^{\beta_h}\odot\mathbf{a}(\nu)\mathcal{D}_N(\beta_k+2\omega-\beta_h-2\nu) + \mathbf{s}_N^{(0)}(\beta_k+2\omega), \tag{26}$$

where

$$\mathbf{s}_N^{(K)}(\alpha) \triangleq \frac{1}{(2N+1)^{K+1}}\sum_{n=-N}^{N}\mathbf{e}(n)n^K e^{-j2\pi\alpha n}, \tag{27}$$

$$\mathcal{D}_N(\xi) \triangleq \frac{1}{2N+1}\sum_{n=-N}^{N}e^{-j2\pi\xi n} = \frac{\sin(\pi\xi(2N+1))}{(2N+1)\sin(\pi\xi)}. \tag{28}$$

Moreover, by substituting (26) into (9), and accounting for (20), (21), and the results of Appendices A and B, it can be shown that

$$\begin{aligned}&\lim_{N\to\infty} f_N(\widetilde{\omega}_N) = ||\mathbf{r}_{xx}^{\beta_k}||^2,\\ &\lim_{N\to\infty}(2N+1)^{-1}f_N'(\widetilde{\omega}_N) = 0,\\ &\lim_{N\to\infty}(2N+1)^{-2}f_N''(\widetilde{\omega}_N) = -\frac{4\pi^2}{3}||\mathbf{r}_{xx}^{\beta_k}||^2.\end{aligned} \tag{29}$$

Therefore, by substituting (29) into (23), this results in

$$\lim_{N\to\infty}\mathcal{A}_N = -\frac{8\pi^2}{3}||\mathbf{r}_{xx}^{\beta_k}||^4. \tag{30}$$

As regards the term $\mathcal{B}_N$, accounting for (B.1) and the results of Appendix A, we have

$$\begin{aligned}&\lim_{N\to\infty} f_N(\nu) = ||\mathbf{r}_{xx}^{\beta_k}||^2,\\ &\lim_{N\to\infty}(2N+1)^{-1/2}f_N'(\nu) = -j4\pi[\boldsymbol{\zeta}\odot\mathbf{a}^*(\nu)]^T(\mathbf{r}_{xx}^{\beta_k})^*,\end{aligned} \tag{31}$$

where

$$\boldsymbol{\zeta} \triangleq \lim_{N\to\infty}(2N+1)^{1/2}\mathbf{s}_N^{(1)}(\beta_k+2\nu) \tag{32}$$

is a zero-mean complex Gaussian vector whose covariance matrix can be determined accounting for the results of [3].

Therefore, by substituting (31) and (32) into (24), this results in

$$\lim_{N\to\infty}\mathcal{B}_N = -8\pi||\mathbf{r}_{xx}^{\beta_k}||^2\,\mathrm{Re}\left\{j[\boldsymbol{\zeta}\odot\mathbf{a}^*(\nu)]^T(\mathbf{r}_{xx}^{\beta_k})^*\right\}. \tag{33}$$

Finally, by substituting (30) and (33) into (22) this results in

$$\begin{aligned}&\lim_{N\to\infty}(2N+1)^{3/2}(\widehat{\omega}_N-\nu)\\ &= -\lim_{N\to\infty}\mathcal{A}_N^{-1}\mathcal{B}_N = -\frac{3}{\pi}||\mathbf{r}_{xx}^{\beta_k}||^{-2}\,\mathrm{Re}\left\{j[\boldsymbol{\zeta}\odot\mathbf{a}^*(\nu)]^T(\mathbf{r}_{xx}^{\beta_k})^*\right\}.\end{aligned} \tag{34}$$

That is, the CFO estimation error is asymptotically Gaussian with zero mean and variance $\mathcal{O}(N^{-3})$. In [8, 9], it is shown that such an asymptotic behavior assures that the (conjugate-) cyclic-correlogram at the estimated (conjugate) cycle frequency $\beta_k+2\widehat{\omega}_N$ is a mean-square consistent estimate of the (conjugate-) cyclic-autocorrelation function at the actual cycle frequency $\beta_k+2\nu$. Consequently, since the parameters γ_{opt} and $\bar{\gamma}_{\mathrm{opt}}$ are finite linear combinations of elements of the cyclic correlogram and the conjugate-cyclic-correlogram vectors, it follows that amplitude, delay, and phase estimators are in turn consistent.

Let us consider now the two-sided-mean counterparts of the quantities defined in [3, (11) and (12)], that is,

$$\begin{aligned}\bar{\mathcal{A}}_N &\triangleq (2N+1)^{-2}\frac{\mathrm{d}^2}{\mathrm{d}\beta^2}\left|\left|\mathbf{r}_{yy,N}^{\beta}\right|\right|^2\Big|_{\beta=\widetilde{\beta}_N},\\ \bar{\mathcal{B}}_N &\triangleq (2N+1)^{-1/2}\frac{\mathrm{d}}{\mathrm{d}\beta}\left|\left|\mathbf{r}_{yy,N}^{\beta}\right|\right|^2\Big|_{\beta=\beta_k+2\nu},\end{aligned} \tag{35}$$

where $\widetilde{\beta}_N = \beta_k + \eta_N(\widehat{\beta}_N - \beta_k)$, $\eta_N \in [0,1]$, and

$$\widehat{\beta}_N \triangleq \arg\max_{\beta\in J_0}\left|\left|\mathbf{r}_{yy,N}^{\beta}\right|\right|^2 \tag{36}$$

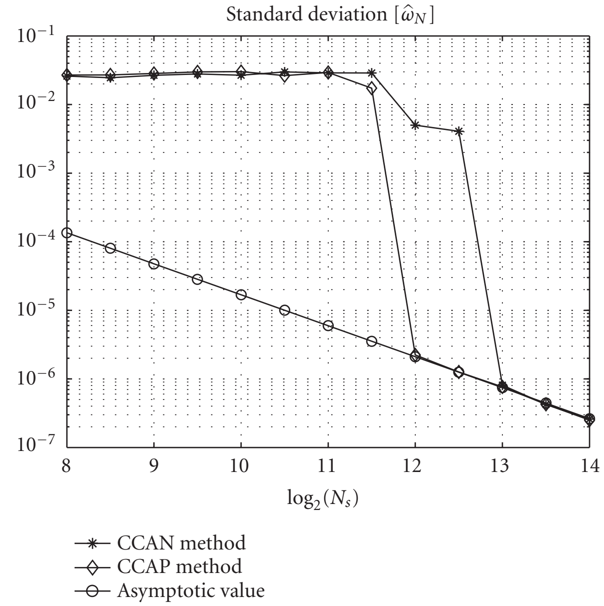

Figure 1: Standard deviation of the CFO estimators with $\beta_k = 1/Q$.

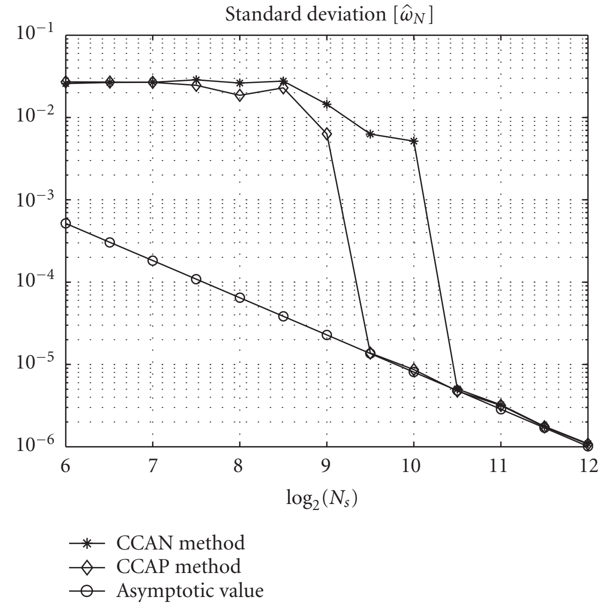

Figure 2: Standard deviation of the CFO estimators with $\beta_k = 0$.

with $J_0 \triangleq (\beta_k - 1/2Q, \beta_k + 1/2Q)$. By using definition (32) in the results of [3] we get

$$\begin{aligned} \lim_{N\to\infty} \bar{\mathcal{A}}_N &= -\frac{2\pi^2}{3}||\mathbf{r}_{xx}^{\beta_k}||^2, \\ \lim_{N\to\infty} \bar{\mathcal{B}}_N &= -4\pi \operatorname{Re}\left\{ j[\boldsymbol{\zeta} \odot \mathbf{a}^*(\nu)]^T (\mathbf{r}_{xx}^{\beta_k})^* \right\}. \end{aligned} \tag{37}$$

Thus, the asymptotic errors of the CCAP CFO estimator $\hat{\omega}_N$ and of the CCAN CFO estimator $\hat{\theta}_N \triangleq (\hat{\beta}_N - \beta_k)/2$ have the same statistical characterization. In fact,

$$\begin{aligned} &\lim_{N\to\infty} (2N+1)^{3/2} (\hat{\theta}_N - \nu) \\ &\quad = \lim_{N\to\infty} (2N+1)^{3/2} \frac{1}{2} (\hat{\beta}_N - (\beta_k + 2\nu)) \\ &\quad = -\frac{1}{2} \lim_{N\to\infty} \bar{\mathcal{A}}_N^{-1} \bar{\mathcal{B}}_N \\ &\quad = -\frac{3}{\pi} ||\mathbf{r}_{xx}^{\beta_k}||^{-2} \operatorname{Re}\left\{ j[\boldsymbol{\zeta} \odot \mathbf{a}^*(\nu)]^T (\mathbf{r}_{xx}^{\beta_k})^* \right\} \\ &\quad = \lim_{N\to\infty} (2N+1)^{3/2} (\hat{\omega}_N - \nu). \end{aligned} \tag{38}$$

In particular, the errors have the same asymptotic variance.

In the following section, however, simulation results are reported showing that for moderate values of N the CCAP CFO estimator can outperform the CCAN estimator. Note that, since the (conjugate-) cyclic-autocorrelation estimate is highly sensitive to the errors in the cycle frequency knowledge [1], even a slight performance improvement in the frequency-shift estimate can lead to a significant performance enhancement of the (conjugate-) cyclic-autocorrelation estimate and, hence, of the remaining parameters.

4. SIMULATION RESULTS

In this section, simulation results are reported to corroborate the effectiveness of the theoretical results of Section 3.

In the experiments, the useful signal $x(n)$ is a binary pulse-amplitude-modulated (PAM) signal with full-duty cycle rectangular pulse with oversampling factor $Q = 4$ and $w(n)$ is complex circular stationary Gaussian noise.

In the first experiment, the sample standard deviation of the considered CFO estimators, evaluated on the basis of 500 Monte Carlo trials, is reported as a function of the number of processed symbols $N_s = (2N+1)/Q$, with signal-to-noise ratio (SNR) fixed at -10 dB, where SNR is the ratio between the signal and noise powers. Thus, SNR $= \mathcal{E}_b/(N_0 Q)$, where $\mathcal{E}_b$ is the per-bit energy and N_0 is the spectral density of the bandpass white noise. The two cases $\beta_k = 1/Q$ (Figure 1) and $\beta_k = 0$ (Figure 2) have been analyzed. In both cases it is evident that for N sufficiently large both the CFO estimators exhibit a variance $\mathcal{O}(N^{-3})$ and, moreover, their asymptotic variance is the same and approaches the theoretical value given in [3]. The CCAP CFO estimator, however, outperforms the CCAN estimator for moderate values of N, especially in correspondence with the threshold values $N_s = 2^{12}$ (for $\beta_k = 1/Q$) and $N_s = 2^{10}$ (for $\beta_k = 0$). Such a result is in accordance with the fact that both methods perform the CFO estimation by maximizing a cost function which is the magnitude of the inner product of the vector $\mathbf{r}_{yy,N}^{\beta_k+2\omega}$ over a reference vector. In the CCAN method, however, the reference vector is a noisy version of that of CCAP.

In the second experiment, the sample root-mean-squared error (RMSE) of the considered CFO estimators, evaluated on the basis of 500 Monte Carlo trials, is reported as a function of SNR, with $N_s = 2^{12}$ for $\beta_k = 1/Q$ (Figure 3) and $N_s = 2^{10}$ for $\beta_k = 0$ (Figure 4). Also this experiment

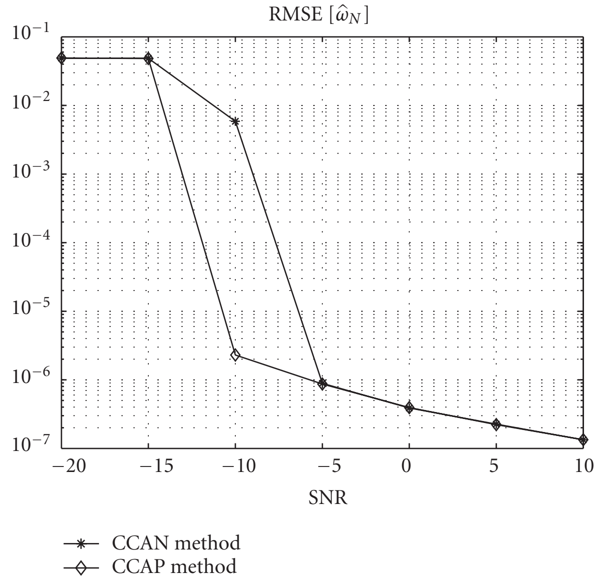

FIGURE 3: RMSE of the CFO estimators with $\beta_k = 1/Q$.

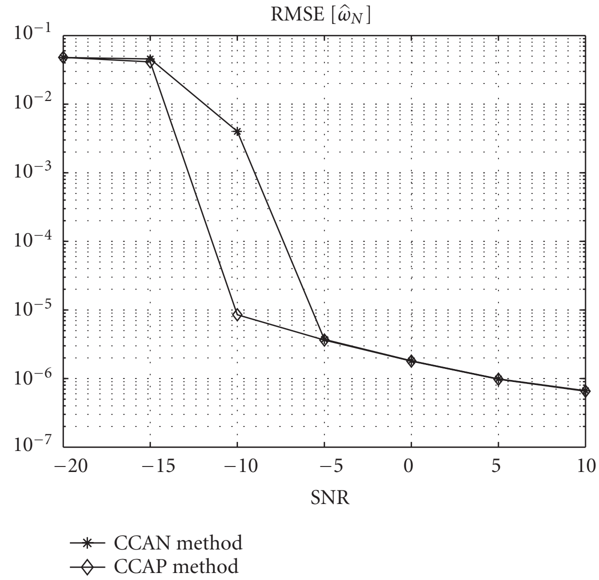

FIGURE 4: RMSE of the CFO estimators with $\beta_k = 0$.

corroborates the usefulness of the proposed CCAP CFO estimator for moderate values of N and low SNR values.

APPENDICES

A. RESULTS ON $\mathbf{s}_N^{(K)}(\alpha)$

Let us consider the vector function $\mathbf{s}_N^{(K)}(\alpha)$ defined in (27). It can be easily shown that

$$\frac{\mathrm{d}\mathbf{s}_N^{(K)}(\alpha)}{\mathrm{d}\alpha} = -j2\pi(2N+1)\mathbf{s}_N^{(K+1)}(\alpha). \quad \text{(A.1)}$$

Under appropriate mixing conditions expressed in terms of the summability of the cumulant of the vector process $\mathbf{e}(n)$ this results in (see [3, Lemma 1])

$$\lim_{N\to\infty} \sup_{\alpha\in[-1/2,1/2[} ||\mathbf{s}_N^{(K)}(\alpha)|| = 0 \quad \text{a.s. } \forall K. \quad \text{(A.2)}$$

Moreover, let $\{\xi_N\}_{N\in\mathbb{N}}$ be a real-valued sequence such that $\xi_N \in X$ with X compact set contained in $[-1/2, 1/2[$ and $\lim_{N\to\infty}\xi_N$ exists. Then

$$\lim_{N\to\infty} ||\mathbf{s}_N^{(K)}(\xi_N)|| = 0 \quad \text{a.s. } \forall K. \quad \text{(A.3)}$$

B. RESULTS ON $\mathcal{D}_N(\xi)$

Let us consider the function $\mathcal{D}_N(\xi)$ defined in (28) and denote by $\mathcal{D}'_N(\xi)$ and $\mathcal{D}''_N(\xi)$ its first- and second-order derivatives, respectively.

This results in

$$\lim_{N\to\infty} (2N+1)^{-1/2}\mathcal{D}'_N(\xi) = 0 \quad \forall\xi. \quad \text{(B.1)}$$

Let $\{\xi_N\}_{N\in\mathbb{N}}$ be a real-valued sequence such that $\xi_N \in X$ with X compact set contained in $[-1/2, 1/2[$.

If $\lim_{N\to\infty}\xi_N = 0$, and $\lim_{N\to\infty} N\xi_N = 0$, then

$$\begin{aligned} &\lim_{N\to\infty} \mathcal{D}_N(\xi_N) = 1, \\ &\lim_{N\to\infty} (2N+1)^{-1}\mathcal{D}'_N(\xi_N) = 0, \\ &\lim_{N\to\infty} (2N+1)^{-2}\mathcal{D}''_N(\xi_N) = -\frac{\pi^2}{3}, \end{aligned} \quad \text{(B.2)}$$

otherwise if $\lim_{N\to\infty}\xi_N \neq 0$, then

$$\begin{aligned} &\lim_{N\to\infty} \mathcal{D}_N(\xi_N) = 0, \\ &\lim_{N\to\infty} (2N+1)^{-1}\mathcal{D}'_N(\xi_N) = 0. \end{aligned} \quad \text{(B.3)}$$

REFERENCES

[1] W. A. Gardner, *Statistical Spectral Analysis: A Nonprobabilistic Theory*, Prentice-Hall, Englewood Cliffs, NJ, USA, 1988.

[2] V. De Angelis, L. Izzo, A. Napolitano, and M. Tanda, "Performance analysis of a conjugate-cyclic-autocorrelation projection-based algorithm for signal parameter estimation," in *Proceedings of 6th International Symposium on Wireless Personal Multimedia Communications (WPMC '03)*, Yokosuka, Kanagawa, Japan, October 2003.

[3] P. Ciblat, P. Loubaton, E. Serpedin, and G. B. Giannakis, "Performance analysis of blind carrier frequency offset estimators for noncircular transmissions through frequency-selective channels," *IEEE Transactions on Signal Processing*, vol. 50, no. 1, pp. 130–140, 2002.

[4] F. Gini and G. B. Giannakis, "Frequency offset and symbol timing recovery in flat-fading channels: a cyclostationary approach," *IEEE Transactions on Communications*, vol. 46, no. 3, pp. 400–411, 1998.

[5] A. Napolitano and M. Tanda, "Blind estimation of amplitudes, phases, time delays, and frequency shifts in multiuser communication systems," in *Proceedings of IEEE 51st Vehicular Technology Conference (VTC '00)*, vol. 2, pp. 844–848, Tokyo, Japan, May 2000.

[6] A. Napolitano and M. Tanda, “Blind parameter estimation in multiple-access systems,” *IEEE Transactions on Communications*, vol. 49, no. 4, pp. 688–698, 2001.
[7] A. Napolitano and M. Tanda, “A non-data-aided cyclic-autocorrelation-based algorithm for signal parameter estimation,” in *Proceedings of 4th International Symposium on Wireless Personal Multimedia Communications (WPMC ’01)*, Aalborg, Denmark, September 2001.
[8] A. Napolitano and M. Tanda, “Performance analysis of a Doppler-channel blind identification algorithm for noncircular transmissions in multiple-access systems,” in *Proceedings of 7th IEEE International Symposium on Signal Processing and Its Applications (ISSPA ’03)*, vol. 2, pp. 307–310, Paris, France, July 2003.
[9] A. Napolitano and M. Tanda, “Doppler-channel blind identification for noncircular transmissions in multiple-access systems,” *IEEE Transactions on Communications*, vol. 52, no. 12, pp. 2073–2078, 2004.
[10] D. A. Streight, G. K. Lott, and W. A. Brown, “Maximum likelihood estimates of the time and frequency differences of arrival of weak cyclostationary digital communications signals,” in *Proceedings of 21st Century Military Communications Conference (MILCOM ’00)*, vol. 2, pp. 957–961, Los Angeles, Calif, USA, October 2000.
[11] A. Napolitano, “Cyclic higher-order statistics: input/output relations for discrete- and continuous-time MIMO linear almost-periodically time-variant systems,” *Signal Processing*, vol. 42, no. 2, pp. 147–166, 1995.
[12] A. V. Dandawaté and G. B. Giannakis, “Asymptotic theory of mixed time averages and *k*th-order cyclic-moment and cumulant statistics,” *IEEE Transactions on Information Theory*, vol. 41, no. 1, pp. 216–232, 1995.
[13] T. Hasan, “Nonlinear time series regression for a class of amplitude modulated cosinusoids,” *Journal of Time Series Analysis*, vol. 3, no. 2, pp. 109–122, 1982.

Valentina De Angelis received her Dr.Eng. degree in electronic engineering (summa cum laude) in 2002 and the Ph.D. degree in electronic and telecommunication engineering in 2006, both from the University of Naples Federico II. Her main research interests are in the field of signal processing, with particular emphasis on the blind estimation of synchronization parameters.

Luciano Izzo was born in Napoli, Italy, on September 17, 1946. He received the Dr. Eng. degree in electronic engineering from the University of Naples in 1971. Since 1973, he has been with the University of Naples. Specifically, from 1973 to 1983, he was with the Electrical Engineering Institute, and since January 1984, he has been with the Department of Electronic and Telecommunication Engineering. From 1977 to 2000, he was an Associate Professor of electrical communications (until 1985), radio engineering (until 1993), and again of electrical communications with the University Naples Federico II. From 1984 to 1998, he was an Appointed Professor of radio engineering (until 1992) and telecommunication systems (since 1992) at the University of Salerno, Salerno, Italy. Since November 1998, he has been an Appointed Professor of electrical communications at the Second University of Naples. Since November 2000, he has been a Full Professor with the University of Naples Federico II, where, from November 2002 to October 2005, he was the Chair of the Department of Electronic and Telecommunication Engineering. He is the author of numerous research journal and conference papers in the fields of digital communication systems, detection, estimation, statistical signal processing, and the theory of higher-order statistics of nonstationary signals.

Antonio Napolitano was born in Naples, Italy, on February 7, 1964. He received the Dr.Eng. degree (summa cum laude) in electronic engineering in 1990 and the Ph.D. degree in electronic and computer engineering in 1994, both from the University of Naples Federico II. From 1994 to 1995, he was an Appointed Professor at the University of Salerno, Italy. From 1995 to 2005 he was Assistant Professor and then Associate Professor at the University of Naples Federico II. From 2005 he has been Full Professor of Telecommunications at the University of Naples “Parthenope.” He held visting positions in 1997 at the Department of Electrical and Computer Engineering at the University of California, Davis; from 2000 to 2002 at the Centro de Investigacion en Matematicas (CIMAT), Guanajuato, Gto, Mexico; from 2002 to 2005 at the Econometric Department, Wyzsza Szkola Biznesu, WSB-NLU, Nowy Sacz, Poland; and in 2005 at the Institute de Recherche Mathematique de Rennes (IRMAR), University of Rennes 2, Haute Bretagne, France. His research interests include statistical signal processing, the theory of higher order statistics of nonstationary signals, and wireless systems. Dr. Napolitano received the Best Paper of the Year Award from the European Association for Signal Processing (EURASIP) in 1995 for his paper on higher-order cyclostationarity.

Mario Tanda was born in Aversa, Italy, on July 15, 1963. He received the Dr.Eng. degree (summa cum laude) in electronic engineering in 1987 and the Ph.D. degree in electronic and computer engineering in 1992, both from the University of Naples Federico II. Since 1995, he has been an Appointed Professor of signal theory at the University of Naples Federico II. Moreover, he has been an Appointed Professor of electrical communications (from 1996 until 1997) and telecommunication systems (from 1997) at the Second University of Naples. He is currently Associate Professor of signal theory at the University of Naples Federico II. His research activity is in the area of signal detection and estimation, multicarrier, and multiple access communication systems.