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Performance analysis of low-complexity dual-cell random beamforming transmission with user scheduling

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Abstract

In this paper, we study three low-complexity random beamforming transmission schemes for dual-cell multiuser multi-input-single-output systems. Among them, selfish random beamforming and interference-aware random beamforming need no information exchange between cells, while random beamforming with limited coordination (LC-RB) selects the beamforming vector and users with the help of a small amount of overhead signaling. We develop the exact analytical expressions of the ergodic sum-rate for the resulting systems, based on which, we compare the performance of the proposed schemes in dual-cell environment. We show through selected numerical examples that LC-RB achieves tremendous performance gain over the other schemes, especially when users are populated along the cell boundary, while only requiring beam index sharing between base stations. Furthermore, we propose an adaptive implementation strategy for the more general scenario, where users are arbitrarily distributed within the cell.

Keywords: network-MIMO, coordinated beamforming, codebook, sum-rate analysis and wireless communications

1. Introduction

In order to meet the increasing demand for high data rate multimedia wireless services, future wireless systems are evolving toward universal frequency reuse, where neighboring cells may utilize the same radio spectrum. Such scenario also applies to the emerging femtocell systems. As such, the performance of future wireless systems will be mainly limited by intercell interference [1]. In parallel, multiple antenna techniques (MIMO) can improve the spectral efficiency of wireless communication systems and provide significant throughput gains. In addition, multiple antennas can also be exploited to suppress intercell interference through coordination among multiple base stations (BSs) [2]. Therefore, the resulting coordinated multicell transmission, also known as network-MIMO, has drawn significant research attention recently.

With conventional network-MIMO approach, multiple coordinated BSs effectively constitute a 'super-BS',

which transforms several interfering channels into a MIMO broadcast channel [3-5]. The optimal dirty paper coding (DPC) [6] and sub-optimal linear precoders have been studied for network-MIMO scenario [7-12]. With some simplified network models, analytical results have appeared in [13-15]. These coordination strategies require, however, the complete channel state information and sometimes, even the user data to be shared among coordinating BSs, which introduce huge load of overhead signaling [16]. Note that, although BSs are usually connected with wired connections with each other through the switching center, these connections are already fully loaded with the increasing amount of multimedia data traffics. Recently, an adaptive strategy was proposed which cancels intercell interference between scheduled user using joint beamforming only when the interference was significant [17]. But user selection was not considered there.

Unlike previous works in the literature, we focus on more practical coordinated beamforming transmission schemes for dual-cell MIMO systems based on random beamforming in this paper. For MIMO systems with random beamforming, in order to achieve good

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performance, proper user selection is essential. That also applies to coordinated beamforming transmission. To limit the amount of overhead signal between BSs and minimize the additional burden to the back haul connections, we consider the user selection schemes that exchange no or limited amount of control information to achieve coordinated beamforming. Specifically, we present and study selfish random beamforming (SRB), interference-aware random beamforming (IA-RB), both of which require no information exchange between cells, and random beamforming with limited coordination (LC-RB), where only the selected beam index is shared among BSs. We would like to point out that some of these schemes have already been discussed in certain standard activities, such as 3GPP framework [18]. Instead of asymptotic analysis according to most of the literature, which assumes that the number of users is very large, our contribution is to accurately quantify their performance through statistical analysis. We firstly derive the exact analytical expression for the sum-rate of the resulting systems assuming that all the users are located along the cell boundary and average intercell interference power at mobile users can be considered approximately identical. Selected numerical examples show that LC-RB can offer significant sum-rate capacity gain with low system complexity. During the sum-rate performance analysis, we develop the exact statistics of users' SINRs based on some new statistic results of projection norm squares, which can be broadly applied into the performance analysis of other related systems.

We then extend the study to the more practical scenario, where the users are randomly distributed within the whole cell, and average intercell interference power can no longer be regarded as identical, due to the different distances from the neighboring BS to the users. In this case, we propose an adaptive coordinated beamforming scheme and evaluate its performance and complexity. Specifically, the BS can decide whether to perform LC-RB to mitigate the intercell interference, or just to perform SRB, based on the distance information gathered from the mobile users. Note that our scheme differs from the adaptive scheme in [17] in that we consider user selection in each cell. Selected numerical examples show that LC-RB can offer significant sum-rate capacity gain with low system complexity.

The rest of the paper is organized as follows. In the next section, the system and channel models are introduced. Section 3 presents the proposed transmission strategies. The sum-rate performance analysis of the proposed systems is given in Section 4 (for identical interference power case) and in Section 5 (for non-identical interference power case). In Section 6, we investigate the adaptive implementation strategy for the general case. The paper concludes in Section 7. This

paper generalizes the conference version in [19] by considering the analysis of all three schemes and extending the design to non-identical interference power case.

2. System and channel models

The system under consideration as shown in Figure 1 consists two base stations, utilizing the same radio spectrum to serve their selected users. Both base stations are equipped with N antennas, which facilitates beamforming transmission, whereas each user has only a single receive antenna due to its size or complexity constraint. The work in this paper relies on the fundamental assumption that only one user in each cell is served in each time slot. The general multi-user scheduling per cell case will be included in the future work. The user set in cell 1 is denoted by $\mathcal{I} = \{1, 2, \ldots, i, \ldots, K_1\}$, and that in cell 2 by $\mathcal{J} = \{1, 2, \ldots, j, \ldots, K_2\}$. The channel vectors are defined as following,

$\mathbf{h}_{1i}$ is the $N \times 1$ channel vector from the base station 1 to the ith user in cell 1, i.e. $i \in \mathcal{I}$.

$\mathbf{h}_{2i}$ is the $N \times 1$ channel vector from the base station 2 to the ith user in cell 1, i.e. $i \in \mathcal{I}$.

$\mathbf{h}_{1j}$ is the $N \times 1$ channel vector from the base station 1 to the jth user in cell 2, i.e. $j \in \mathcal{J}$.

$\mathbf{h}_{2j}$ is the $N \times 1$ channel vector from the base station 2 to the jth user in cell 2, i.e. $j \in \mathcal{J}$.

We assume that, with proper power control mechanism, the users experience homogeneous Rayleigh fading with respect to their target BS. Thus, each component of h_{1i} and h_{2j} is modeled as independent and identically distributed (i.i.d) complex Gaussian random variables with zero mean and unit variance. When mobile users are randomly populated in their specific cell coverage area, the average received interference power is dynamic, due to the various distances from the neighboring BS to the users. Each component of the interference channel vector h_{1j} and h_{2i} is modeled as independent complex Gaussian random variables with zero mean and variance δ_j(resp. δ_i) with respect to user j (resp. i). As will be seen in later section, we will focus mostly on the interference channel from BS_2 to the selected user in cell 1, denoted by $\mathbf{h}_{2i^*}$. We assume that each component of $\mathbf{h}_{2i^*}$ is modeled as i.i.d. complex Gaussian random variables with zero mean and a common variance δ_{i^*}. For the special case that the mobile users are distributed along the cell boundary, and thus all the users have approximately the same distance with the neighboring BS, we can assume each component of $\mathbf{h}_{1j}$ and $\mathbf{h}_{2i}$ is modeled as i.i.d. complex Gaussian random variable with zero mean and variance δ, i.e. $\delta_i = \delta_j = \delta$ for all i and j.

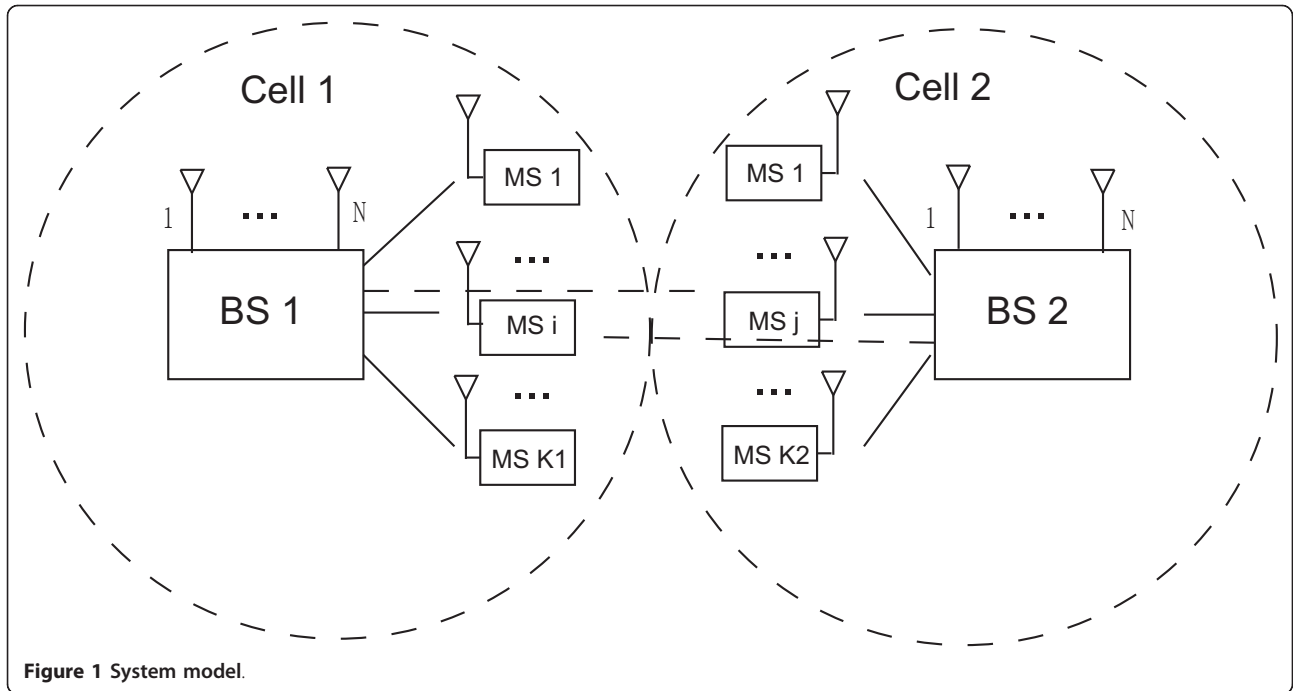We assume that each base station employs a codebook-based random beamforming strategy to serve one selected

Figure 1 System model.

user in its coverage area.[1] The codebook is assumed to consist of B unit-norm vectors of length N, randomly generated from an isotropic distribution [20]. With their wired connection to the switching center, the BSs can exchange a limited amount of control information for coordinated beamforming transmission. Specifically, the BS can communicate the utilized beamforming vectors to each other and to the users using the index of the codebook. Assume that the channel keeps constant in a feedback cycle. With the proper design of the beamforming vectors and user selection, the inter-cell interference can be controlled. The specific design and selection scheme proposed in this work will be discussed in the following secions. For the multitransmit antenna case under consideration, the received signal at the ith user in cell 1 and jth user in cell 2 can be written as

$$
\begin{aligned}
y_i &= \sqrt{P_1}\mathbf{h}_{1i}^T\mathbf{w}_1 s_1 + \sqrt{P_2}\mathbf{h}_{2i}^T\mathbf{w}_2 s_2 + n_i, i \in \mathcal{I}, \\
y_j &= \sqrt{P_2}\mathbf{h}_{2j}^T\mathbf{w}_2 s_2 + \sqrt{P_1}\mathbf{h}_{1j}^T\mathbf{w}_1 s_1 + n_j, j \in \mathcal{J}.
\end{aligned}
\tag{1}
$$

respectively, where $s_i(i = 1, 2)$ are data symbols to selected users and $\mathbf{w}_i(i = 1, 2)$ are the corresponding beamforming vectors. We generally have $||\mathbf{w}_i||^2 = 1$, $i = 1, 2$. P_1 and P_2 are the corresponding transmit powers for cell 1 and 2, n_i and n_j are the additive Gaussian noise.

3. Transmission strategies
In this section, we present the fundamental principles and the mode of operations of several reduced-

complexity dual-cell beamforming transmission strategies. For analytical tractability, we focus on dual-cell scenario.

3.1. Selfish random beamforming (SRB)
This scheme assumes that the system is completely unaware of the intercell interference. BS_1 and BS_2 just perform the conventional random beamforming separately. BS_1 (resp. BS_2) randomly selects a vector, denoted by $\mathbf{w}_1$ (resp. $\mathbf{w}_2$) from its codebook as beamforming vector and transmits a pilot symbol with this vector. Every user in the coverage area of BS_1 (resp. BS_2) will estimate and feed back its received signal-to-noise ratio (SNR), which will be proportional the projection power of users channel vector on to the beamforming direction, i.e. $|\mathbf{h}_{1i}^T\mathbf{w}_1|^2(\text{resp.}|\mathbf{h}_{2j}^T\mathbf{w}_2|^2)$. Note that users will not need to estimate its channel vector in this process and each only needs to feed back a real number for user selection. BS_1 (resp. BS_2) will select the user achieving the largest SNR among all users, i.e. user i^* (resp. j^*), where $i^* = \arg\max_i|\mathbf{h}_{1i}^T\mathbf{w}_1|^2(\text{resp. } j^* = \arg\max_j|\mathbf{h}_{2j}^T\mathbf{w}_2|^2)$. With conventional random beamforming strategy, transmission will then start without any mechanism for controlling the interference from the other base station. Therefore, we can determine the SINRs of the selected users as

$$
\gamma_1 = \frac{P_1\max_i|\mathbf{h}_{1i}^T\mathbf{w}_1|^2}{P_2|\mathbf{h}_{2i^*}^T\mathbf{w}_2|^2 + N_0}, \gamma_2 = \frac{P_2\max_j|\mathbf{h}_{2j}^T\mathbf{w}_2|^2}{P_1|\mathbf{h}_{1j^*}^T\mathbf{w}_1|^2 + N_0},
\tag{2}
$$

where N_0 denotes the Gaussian noise power.

3.2. Interference-aware random beamforming (IA-RB)

The operations of this scheme shares a lot in common with the SRB scheme. The only difference is that every user in the coverage area of BS_2 (BS_1 would follow exactly the same operations) will estimate and feed back its received signal-to-noise and interference ratio (SINR), with signal power proportional to $|\mathbf{h}_{2j}^T\mathbf{w}_2|^2$ and interference power to $|\mathbf{h}_{1j}^T\mathbf{w}_1|^2$. Specifically, the SINR of the jth user in the BS_2's coverage is given by

$$\gamma_{2,j} = \frac{P_2|\mathbf{h}_{2j}^T\mathbf{w}_2|^2}{P_1|\mathbf{h}_{1j}^T\mathbf{w}_1|^2 + N_0}. \quad (3)$$

Then, the BSs will select the user that achieves the largest SINR. Note that as long as the two BSs do not transmit their pilot symbols simultaneously, the users will not need to estimate their channel vectors to determine SINR. Again each user will only feed back a real number for user selection. And the achieved SINRs of the two selected users can be presented as

$$\gamma_1 = \max_i\left(\frac{P_1\left|\mathbf{h}_{1i}^T\mathbf{w}_1\right|^2}{P_2\left|\mathbf{h}_{2i}^T\mathbf{w}_2\right|^2 + N_0}\right), \ \gamma_2 = \max_j\left(\frac{P_2\left|\mathbf{h}_{2j}^T\mathbf{w}_2\right|^2}{P_1\left|\mathbf{h}_{1j}^T\mathbf{w}_1\right|^2 + N_0}\right). \quad (4)$$

3.3. Random beamforming with limited coordination (LC-RB)

The scheme differs from SRB and IA-RB as it achieves coordinated beamforming transmission with limited overhead signaling. Without loss of generality, we assume that BS_1 starts its user selection for beamforming transmission first. In particular, BS_1 performs exactly the same as SRB to complete the beam and user selection for first cell.

The selected user by BS_1, referred as user i^*, will estimate its MISO channel from the interfering base station BS_2, denoted by $\mathbf{h}_{2i^*}$. With this channel state information, user i^* will determine the beamforming vector that leads to the smallest amount of interference to itself and should be used by BS_2, and feed its index back. Mathematically speaking, the beamforming vector $\mathbf{w}_2$ should satisfy $|\mathbf{h}_{2i^*}^T\mathbf{w}_2|^2 = \min_l|\mathbf{h}_{2i^*}^T\mathbf{w}_l|^2$.

BS_1 will inform BS_2 the desired beamforming vector to use through the wired backhaul connection. BS_2 will broadcast training symbol using the selected beamforming vector for its own user selection. Every user in the coverage area of BS_2 will estimate and feedback its received SINR. BS_2 will select the user that achieves the maximum SINR among all users to serve, i.e. user j^* where $j^* = \arg\max_j \gamma_{2,j}$.

Based on the above mode of operation, we can determine the SINRs of the selected users with LC-RB, as

$$\gamma_1 = \frac{P_1\max_i|\mathbf{h}_{1i}^T\mathbf{w}_1|^2}{P_2\min_l|\mathbf{h}_{2i^*}^T\mathbf{w}_l|^2 + N_0}, \gamma_2 = \max_j\left(\frac{P_2|\mathbf{h}_{2j}^T\mathbf{w}_2|^2}{P_1|\mathbf{h}_{1j}^T\mathbf{w}_1|^2 + N_0}\right). \quad (5)$$

According to the transmission schemes described above, it has been observed that all the strategies can be smoothly extended into the general multi-cell cases. For instance, SRB and IA-RB can be directly applied in multi-cells, due to its non-coordination between BSs. And for LC-RB, all the participating cells can work sequentially in order to complete the scheduling. Indeed, a 3-cell or 7-cell setup will be more practical, however, despite the number of interfering sources and geometric distributions, the strategies keep similar from the interference control perspective.

It is also worth noting that the similar design have been considered in the standard activities for LTE Advanced and IEEE 802.16 m [21]. In this work, we complement those simulation studies of such designs with the exact sum-rate capacity analysis.

4. Sum-rate analysis for identical average interference power case

This section provides the sum-rate analysis assuming that the average interference power is identically distributed, i.e. $\delta_i = \delta_i = \delta$. Essentially, we consider the scenario that mobile users are distributed along the cell boundary. Meanwhile, the transmitted signal energies for both cells have been defined as $E_{s1} = P_1T$, $E_{s2} = P_2T$, in which T denotes the transmit time duration. Notice that in our later work, without loss of generality, we all assume $P_1 = P_2 = 1$. That is why all the power term has been dismissed in the later derivations.

4.1. Common analysis

We first present some statistical results on the ordered projection norm squares, which will be broadly applied in the later analysis. Noting that each component of vectors discussed in this section is modeled as i.i.d. complex Gaussian random variable with zero mean and unit variance.

Let us firstly consider the projection norm squares of K independent vectors $\mathbf{h}_i$, $i = 1,..., K$ to a normalized vector $\mathbf{w}$, i.e. $a_i \doteq |\mathbf{h}_i^T\mathbf{w}|^2$, $i = 1, 2,..., K$. Since $\mathbf{h}_i$ are independent, and a_i are i.i.d. chi-square random variable with two degrees of freedom [22]. It follows that the probability density function (PDF) of the lth largest among totally K projection norm square $a_{l:K} = \text{rank}_l\{a_i\}$, $i = 1, 2,..., K$ is given, after applying the basic ordered statistic result, by:

$$f_{a_{l:K}}(x) = \frac{K!}{(K-l)!(l-1)!}(1 - e^{-x})^{K-l}e^{-lx}, x \geq 0. \quad (6)$$

We now consider the project norm square of vector **h** onto B normalized vectors, $\mathbf{w}_j$, $j = 1,..., B$, i.e. $b_j = |\mathbf{h}^T\mathbf{w}_j|^2$, $j = 1, 2,..., B$, and focus again on the lth largest one among totally B projection norm square, i.e. $b_{l:B}$. Since $\mathbf{w}_j$ are not necessarily orthogonal with one another, the projection norm squares no longer constitute a set of independent random variables. To overcome such difficulty, we rewrite $b_{l:B}$ as

$$b_{l:B} = \text{rank}_l\{|\frac{\mathbf{h}^T}{||\mathbf{h}^T||}\mathbf{w}_j|^2\} \cdot ||\mathbf{h}^T||^2 = u \cdot v. \quad (7)$$

It can be shown that $|\frac{\mathbf{h}^T}{||\mathbf{h}^T||}\mathbf{w}_j|^2$ follows i.i.d. beta distribution with parameters 1 and $N - 1$ [23], with PDF given by:

$$f_\beta(x) = (N-1)(1-x)^{N-2}, x \in (0, 1). \quad (8)$$

Now u becomes lth largest one of B i.i.d. beta random variables, whose PDF can be obtained as

$$f_u(x) = \frac{B!(N-1)}{(B-l)!(l-1)!} \cdot \sum_{i=0}^{B-l}\binom{B-l}{i}(-1)^{B-l-i} \cdot \sum_{j=0}^{A}\binom{A}{j}(-x)^{A-j}, x \in (0, 1), \quad (9)$$

where $A = (N - 1)(B - 1 - i) + N - 2$.

Noting that $v = ||\mathbf{h}||^2$ follows a modified $\chi^2_{(2N)}$ distribution, with PDF given by:

$$f_v(x) = \frac{1}{(N-1)!}x^{N-1}e^{-x}, x \geq 0, \quad (10)$$

the PDF of $b_{l:B}$ could be obtained as the product of two random variables [24]. After several steps of computation, we have

$$f_{b_{l:B}}(z) = \frac{B!}{(B-l)!(l-1)!(N-2)!} \cdot \sum_{i=0}^{B-l}\binom{B-l}{i}(-1)^{B-l-i} \cdot \sum_{j=0}^{A}\binom{A}{j} \quad (11)$$
$$z^{N-1}(-1)^{A-j}I(A - j - N_m; -z), x \geq 0,$$

where

$$I(a;b) = \int_0^1 x^a e^{b/x}dx = (-b)^a \left(-\frac{\pi \csc(\pi a)b}{\Gamma(2+a)} - \frac{b\Gamma(-a)}{1+a} + \frac{(-b)^{-a}e^b}{1+a} + \frac{\Gamma(-a,-b)b}{1+a} \right). \quad (12)$$

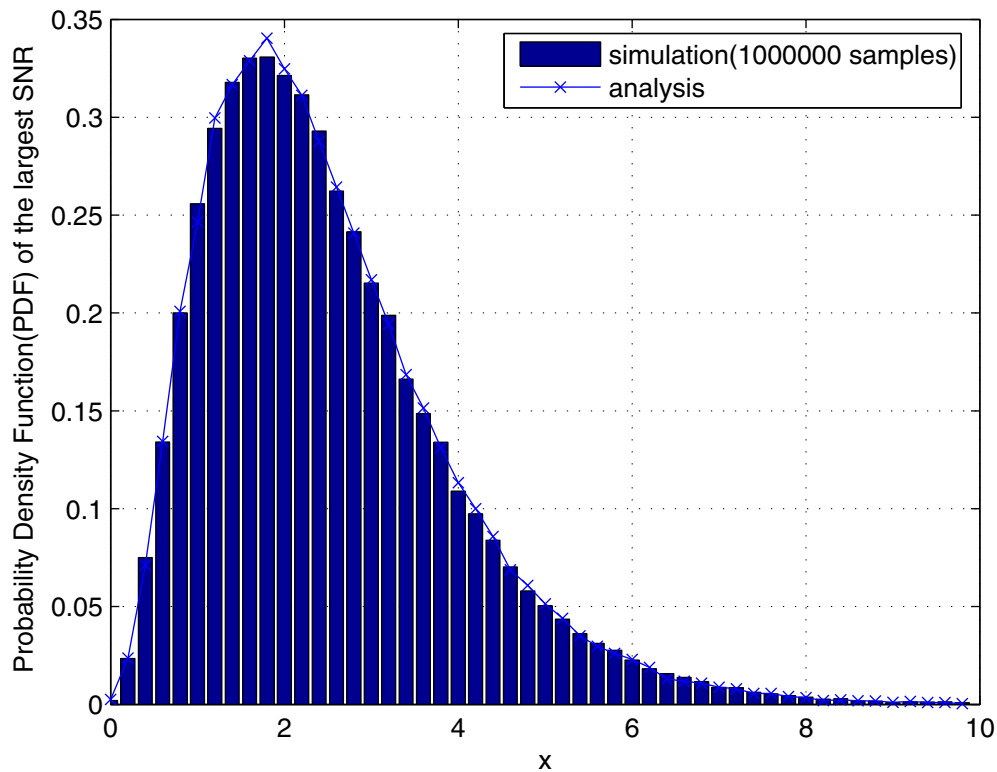Note that this result can be broadly applied in other related analysis. In Figure 2, we plot the PDF of $b_{1:B}$,

Figure 2 PDF of maximum projection power of a channel vector onto $B = 16$ beamforming directions $b_{1:B}(N = 4)$.

and find that it matches perfectly with the simulation results.

4.2. Sum-rate analysis

In this part, we analyze the ergodic sum-rate performance of the beamforming transmission schemes under consideration. The sum-rate of the proposed dual-cell random beamforming system can be calculated as

$$R = \int_0^\infty \log_2(1 + \gamma)(f_{\gamma_1}(\gamma) + f_{\gamma_2}(\gamma))d\gamma. \tag{13}$$

where $f_{\gamma_1}(\gamma)$ and $f_{\gamma_2}(\gamma)$ are the PDF of received SINR of the selected users in cell 1 and 2, respectively. We now derive the exact statistics of the selected users' SINRs.

1) SRB: Due to the symmetry, let us consider the received SINR of selected user by BS_1, as given in (2), which can be rewritten as

$$\gamma_1 = \frac{\max_i |\mathbf{h}_{1i}^T \mathbf{w}_1|^2}{|\mathbf{h}_{2i}^T \mathbf{w}_2|^2 + \rho} = \frac{a_{1:K_1}}{n_i + \rho}, \tag{14}$$

where ρ is the normalized noise power, equal to N_0/E_s. n_i follows the chi-square distribution with 2 degrees of freedom, whose common PDF is represented as

$$f_{n_i}(x) = \frac{1}{\delta^2} e^{-1/\delta^2 x}. \tag{15}$$

And the PDF of $a_{1:K_1}$ was given in (6), with K changed to K_1. Noting the independence of n_i and $a_{1:K_1}$, the PDF of γ_1 can be calculated using the PDFs of n_i and $a_{1:K_1}$, as [22],

$$f_{\gamma_1}(x) = \int_0^\infty (z + \rho) f_{a_{1:K_1}}(x(z + \rho)) f_{n_i}(z) dz. \tag{16}$$

After carrying out the integration with proper substitutions, we have

$$f_{\gamma_1}(x) = \frac{1}{\delta^2} \sum_{i=0}^{K_1-1} (-1)^{K_1-1-i} K_1 \times e^{-\rho(K_1-i)x} \cdot \left(\frac{\rho}{K_1 x - ix + \frac{1}{\delta^2}} + \frac{1}{(K_1 x - ix + \frac{1}{\delta^2})^2} \right). \tag{17}$$

2) IA-RB: Again due to symmetry, we consider PDF of the received SNR at the selected user by BS_2, which was given in (4) as the maximum of K_2 independent random variables, defined as

$$\gamma_j' = \frac{|\mathbf{h}_{2j}^T \mathbf{w}_2|^2}{|\mathbf{h}_{1j}^T \mathbf{w}_1|^2 + \rho} = \frac{p}{q_j + \rho}. \tag{18}$$

Note that p term follows i.i.d. $\chi_{(2)}^2$ distribution over $\mathcal{J}$, with PDF

$$f_p(x) = e^{-x}, \tag{19}$$

and q_j term are i.i.d. with $\chi_{(2)}^2$ distribution over $\mathcal{J}$, but with variance δ^2, whose PDF is the same as (15).

Following the similar steps as for SRB, we can obtain the PDF of γ_j', $f_{\gamma_j'}(\cdot)$, as

$$f_{\gamma_j'}(x) = \frac{1}{\delta^2} e^{-\rho x} \left(\frac{\rho}{x + 1/\delta^2} + \frac{1}{(x + 1/\delta^2)^2} \right). \tag{20}$$

It follows the CDF of γ_j', denoted by $F_{\gamma_j'}(\cdot)$ is given by

$$F_{\gamma_j'}(x) = \int_0^x f_{\gamma_j'}(\gamma) d\gamma = \frac{1}{\delta^2} \left(\frac{e^{-\rho x}}{-x - 1/\delta^2} + \delta^2 \right). \tag{21}$$

Finally, the PDF of γ_2 is obtained as,

$$f_{\gamma_2}(x) = K_2 [F_{\gamma_j'}(x)]^{K_2-1} f_{\gamma_j'}(x). \tag{22}$$

3) LC-RB: Based on the notation introduced in previous subsection, the first user's SINR, can be written as,

$$\gamma_1 = \frac{\max_i |\mathbf{h}_{1i}^T \mathbf{w}_1|^2}{\min_l |\mathbf{h}_{2i*}^T \mathbf{w}_l|^2 + \rho} = \frac{a_{1:K_1}}{b_{B:B} + \rho}. \tag{23}$$

The PDF of both $a_{1:K_1}$ and $b_{B:B}$ can be obtained as the special case of the general result in (6) and (11), as

$$f_{a_{1:K_1}}(x) = K_1 (1 - e^{-x})^{K_1-1} e^{-x}, \tag{24}$$

and

$$f_{b_{B:B}}(x) = \frac{B}{(N-2)!\delta^{2N}} \cdot \sum_{j=0}^{A} \binom{A}{j} x^{N-1} (-1)^{A-j} I(A - j - N_m; -z/\delta^2), \tag{25}$$

respectively. Note that the element of vector $\mathbf{h}_{2i}^*$ has variance δ here.

Consequently, the PDF of γ_1 can be calculated in terms of PDFs of $a_{1:K_1}$ and $b_{B:B}$, as

$$f_{\gamma_1}(x) = \int_0^\infty (z + \rho) f_{a_{1:K_1}}(x(z + \rho)) f_{b_{B:B}}(z) dz. \tag{26}$$

The statistics of the received SINR at the selected user by BS_2 is exactly the same as that of IA-RB scheme presented previously, with PDF given in (22).

4.3. Numerical examples

In this section, we present and discuss selected numerical examples to illustrate the mathematical formalism on the sum-rate analysis of the proposed coordinated beamforming schemes. Noting that all the analytical

results in this paper have been verified through Monte-Carlo simulation.

For comparison purpose, we also provide the simulation results of one of the popular conventional coordinated beamforming techniques with user selection, with CSI exchange between cells, which is called coordinated zero-forcing beamforming (CZF). Specifically, the CZF option relies on a simple multiuser scheduling method, i.e. to select the user with the largest channel vector norm square. After the full CSI sharing between two cells, the new 'super-BS' uses zero-forcing method to transform the interference channel into a MIMO broadcast channel [3-5]. Suppose that $\mathbf{h}_{1i^*}$ and $\mathbf{h}_{2j^*}$ are the two selected user's channel vectors respectively in cell 1 and 2. Then, the beamforming vector $\mathbf{w}_1$ needs to satisfy the orthogonality condition $\mathbf{h}_{1j^*}^H \mathbf{w}_1 = 0$ to cancel its interference for cell 2.

In Figure 3, we compare the single-cell achieved rates for three schemes under consideration as functions of the common number of users $K = K_1 = K_2$. The radius of each cell R is 1 km, the path loss exponent is 3.7, and both BSs are equipped with $N = 4$ antennas. Both analytical-and simulation-based curves have been provided. It

will be firstly observed that at high SNR regime (20 dB), CZF provides the best rate performance, due to its interference cancelation. And the cell 1 for LC-RB outperforms all the others, and can nearly approach CZF, especially when the volume of users is large enough. More specifically, under LC-RB, rate for cell 1 performs better than that for cell 2, owing to the effective interference control from BS_2 to cell 1. When the channel SNR is low (5 dB) and the system is noise limited, LC-RB is still the best, while CZF performs the worst, since interference effect is trivial now.

5. Extension to non-identical average interference power case

As stated before, the identical average intercell interference power assumption only applies to the case that users are distributed along the cell boundary. In this section, we extend to the more general scenario, where users are randomly distributed in the cell coverage.

5.1. SINR analysis

1) SRB: For the non-identical interference case, the first selected user's SINR can still be as given in (17). On the other hand, n_is are no longer identically distributed. The
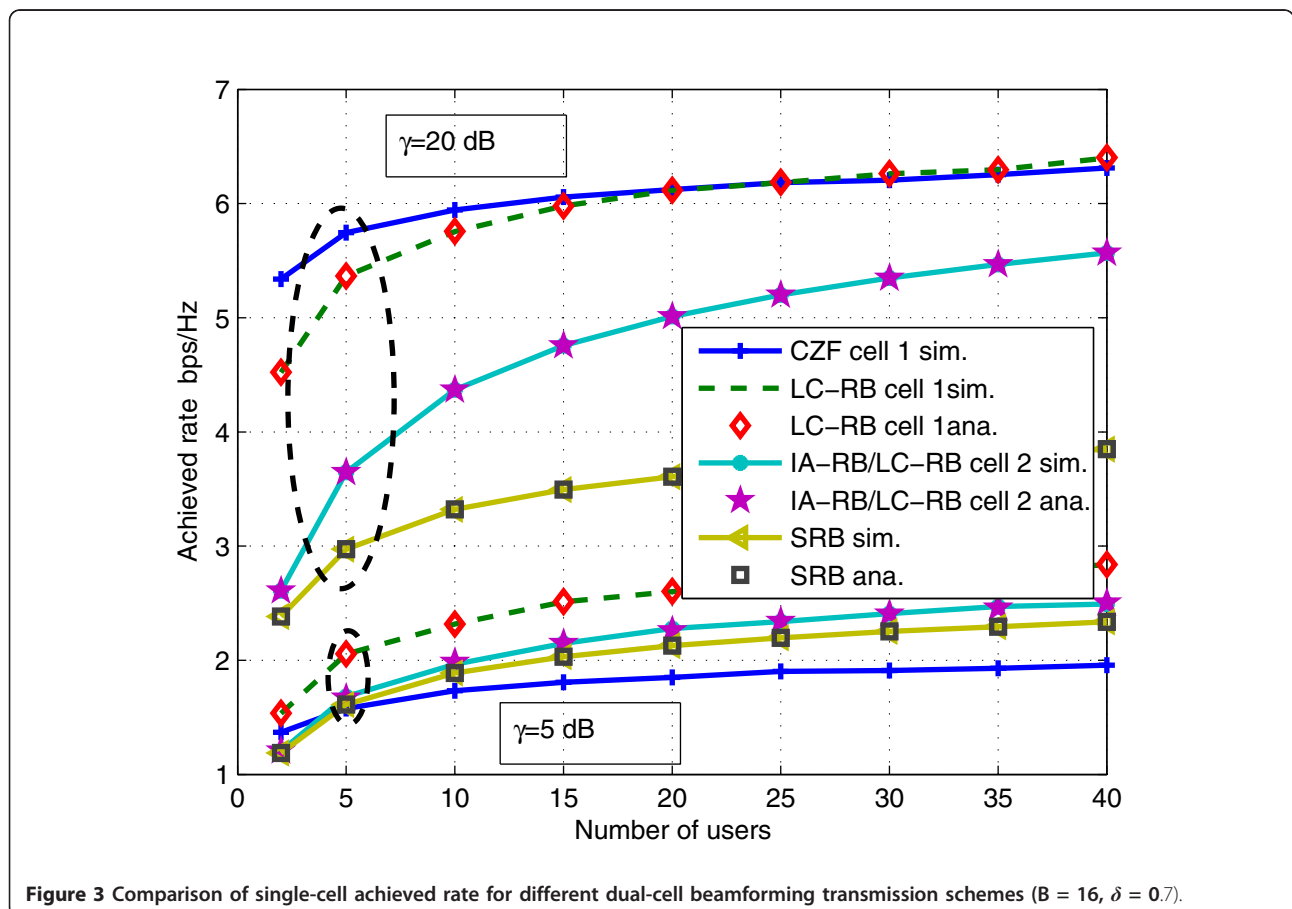

Figure 3 Comparison of single-cell achieved rate for different dual-cell beamforming transmission schemes ($B = 16$, $\delta = 0.7$).

PDF of n_i becomes

$$f_{n_i}(x) = \frac{1}{\delta_i^2} e^{-1/\delta_i^2 x}. \tag{27}$$

After applying (27) into (17), we can obtain the PDF of γ_1 as

$$f_{\gamma_1}(x) = \frac{1}{\delta_i^2} \sum_{l=0}^{K_1-1} (-1)^{K_1-1-l} K_1 \times e^{-\rho(K_1-l)x} \cdot \left(\frac{\rho}{K_1 x - lx + \frac{1}{\delta_i^2}} + \frac{1}{(K_1 x - lx + \frac{1}{\delta_i^2})^2} \right). \tag{28}$$

2) IA-RB: For non-identical interference case, γ_j' in (18) are independent but not identically distributed. Specifically, the PDF of q_j becomes (15), but with parameter δ_j instead of common δ. Applying similar strategies as in (20) and (21), we can obtain the PDF and CDF of γ_j' for non-identical interference case as

$$f_{\gamma_j'}(x) = \frac{1}{\delta_j^2} e^{-\rho x} \left(\frac{\rho}{x + 1/\delta_j^2} + \frac{1}{(x + 1/\delta_j^2)^2} \right), \tag{29}$$

and

$$F_{\gamma_j'}(x) = \frac{1}{\delta_j^2} \left(\frac{e^{-\rho x}}{-x - 1/\delta_j^2} + \delta_j^2 \right), \tag{30}$$

respectively. Therefore, we can obtain the PDF of γ_2, as,

$$f_{\gamma_2}(x) = \sum_{k=1}^{K_2} \left(\prod_{j=1, j \neq k}^{K_2} F_{\gamma_j'}(x) \right) f_{\gamma_k'}(x). \tag{31}$$

3) LC-RB: In this case, the entries of $\mathbf{h}_{2i*}$ are i.i.d. with variance δ_{i*}. It follows that the PDF of $b_{B:B}$ in (23) can be obtained as

$$f_{b_{B:B}}(x) = \frac{B}{(N-2)! \delta_i^{2N}} \cdot \sum_{j=0}^{A} \binom{A}{j} x^{N-1} (-1)^{A-j} I(A - j - N_m; -z/\delta_i^2), \tag{32}$$

where $I(\cdot; \cdot)$ was defined in (12). Applying (24) and (32) into (26), we can obtain the PDF of first selected user's SINR. Give the expression of f_{γ_1} after the substitution. That would make it easier to follow. Similar to identical interference case, the SINR PDF of the second selected user shares exactly the same expression as that of IA-RB scheme as given in (31).

5.2. Numerical examples
Figure 4 plots the sum-rate of three dual-cell transmission schemes for non-identical interference power case. It can be observed that LC-RB and IA-RB offer comparable performance gain over SRB at high SNR regime (20 dB), and the three share almost the same performance at low SNR regime (5 dB), owing to the tremendous noise effects. Moreover, the analysis results match

perfectly with simulation results, which verifies our analytical approach. We also find the sum-rate gaps between LC-RB and the other two schemes are smaller than those for the identical interference power case. It attributes to the fact that the intercell interference for the user can be ignorable when the distance between the selected user and its neighboring BS is large,

The observation is further confirmed in Figure 5, where we examine the effect of interference strength, characterized by the distance from the selected user of cell 1 to BS$_2$. At high SNR regime (SNR = 15 dB) and the system performance is interference limited. The smaller the distance is, the larger the gap between LC-RB and SRB gets, which shows the effectiveness of LC-RB on intercell interference control. And at medium and low SNR regime (SNR = 5, 0 dB) when the overall system suffers from severe Gaussian noise, LC-RB and SRB share almost the same performance. The fact leads to the idea of adaptive implementation, to further reduce the coordination overhead while maintaining the same sum-rate performance, which will be presented in the next section.

6. Adaptive implementations
As stated above, there is a tradeoff between sum-rate performance versus coordination overhead between LC-RB and SRB scheme, especially when the interference is severe, i.e. the selected user is close to the neighboring BS. Specifically, if the neighboring BS is far away from the selected user, the BS may decide only to perform SRB without coordination. Later simulation results will address that through adaptive implementation, we have managed to further reduce the coordination load with only little rate performance loss for compensation. Noting that the decision-making process only depends on the distance information from the BS to the selected mobile user, the adaptive scheme is easy to implement. Note that the distance information is assumed to be shared between BSs during scheduling.

6.1. Mode of operations
With adaptive implementation, the selected user of cell 1 will firstly estimate its distance to the neighboring BS based on the average interference power. If the distance is larger than a threshold, denoted by d_{TH}, and as such, the interference can be viewed as negligible, the user will suggest BS$_1$ to perform SRB. Otherwise, BS$_1$ will perform LC-RB so as to control the intercell interference. Note that only in the later case, the selected user of BS$_1$ needs to estimate the channel from the neighboring BS. Also, with the adaptive implementation, the coordination overhead is reduced and only used if necessary.
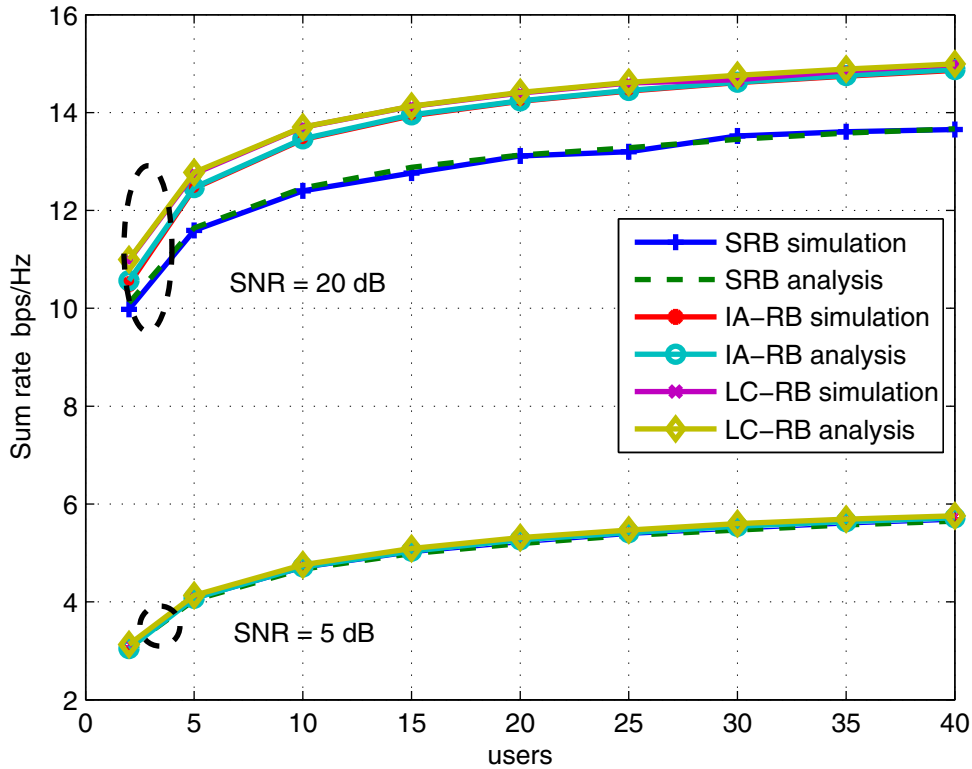
Figure 4 Sum-rate comparison for non-identical average interference power case.

6.2. Coordination overload

We now quantify the average signaling overhead for coordinated beamforming with adaptive implementation. For the adaptive implementation, there are two styles of signaling message between the two BSs, depending coordination is needed or not. Specifically, if $d > d_{TH}$, BS_1 sends one bit of information to BS2 to indicate no coordination is needed, else if $d < d_{TH}$, BS_1 sends the index of $\mathbf{w}_2$ in the codebook, plus one bit of coordination indicator, which leads to $1 + \log_2 B$ bits of overhead signaling.

Based on these observations, we can easily calculate the coordination overload c for the adaptive implementation, as

$$c = \frac{P_R}{\pi R^2}(\log_2 B + 1) + (1 - \frac{P_R}{\pi R^2}), \qquad (33)$$

where P_R is the area in BS_1 that the coordination is needed, which can be calculated using some geometric analysis as

$$P_R = \pi R^2 \frac{\theta_1}{\pi} + \pi d_{TH}^2 \frac{\theta_2}{\pi} - \frac{1}{2}(R^2 \sin(2\theta_1) + d_{TH}^2 \sin(2\theta_2)), \quad (34)$$

and

$$\theta_1 = \arccos\left(\frac{5R^2 - d_{TH}^2}{4R^2}\right), \theta_2 = \arccos\left(\frac{d_{TH}^2 + 3R^2}{2R^2}\right). \quad (35)$$

6.3. Numerical examples

Figure 6 presents the throughput of cell 1 and coordination overhead with the adaptive implementation, as the function of the normalized threshold d_{TH}, for various channel conditions. From Figure 6a, we can see that as d_{TH} increases, the throughput of cell increases as the system will invoke more coordination. Note that if $d_{TH} = 3R$, the adaptive implementation is equivalent to the conventional LC-RB, and it reduces to SRB when $d_{TH} = R$. We also notice that the performance improvement with larger threshold is more significant for high SNR range when the system is more interference limited. From Figure 6b, we can see that the coordination overload is also increasing as the threshold increases. Therefore, the threshold d_{TH} can be used to balance the tradeoff of throughput gain versus overhead signaling, especially when the system is within high SNR regime (e.g., 20 dB). And in medium and low SNR regime (e.g.,
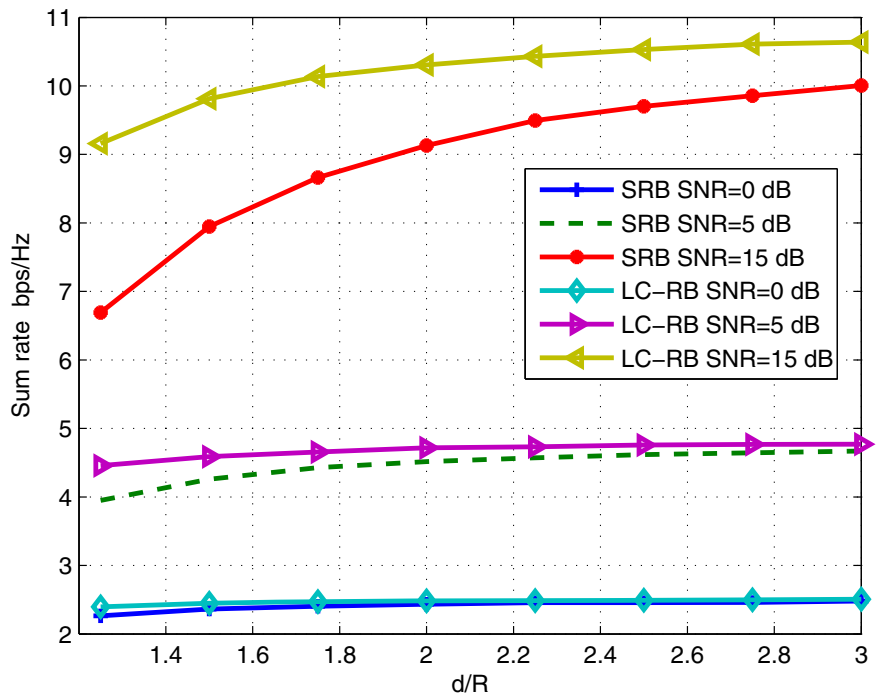
Figure 5 Sum-rate comparison as function of normalized distance threshold *d*/*R* (K = 10).
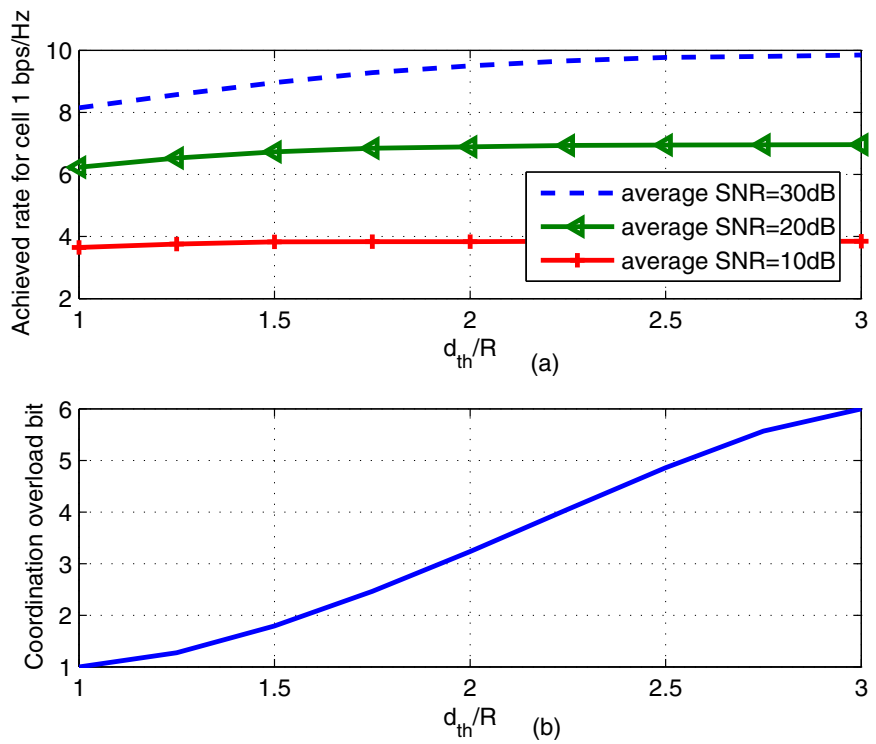

Figure 6 Throughput performance with adaptive implementation.

10 dB and below), we incline to only apply SRB, since the rate will not enhance much, even when the coordination overload is getting higher.

7. Conclusion

In this paper, we studied the ergodic capacity of dual-cell MISO broadcast channels with low-complexity random beamforming. In particular, we derived the exact analytical expressions of the ergodic sum-rate for three schemes with the help of some new statistical results and compared their performance in dual-cell environment. We showed through selected numerical examples that the LC-RB scheme achieves tremendous performance over SRB and IA-RB for any volume of active users, with only a beam index sharing between cells. Moreover, we have extended the scenario to the more practical case, where users are arbitrarily distributed within the overall cell coverage and proposed an adaptive coordination scheme. The generalization of the proposed schemes for multiuser parallel transmission is under investigation.

Endnotes

[1]Other codebook such as Grassmannian codebook may lead to better performance. But for analytical tractability, we limit ourself to random codebook.

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Competing interests
The authors declare that they have no competing interests.

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