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Blotto game-based low-complexity fair multiuser subcarrier allocation for uplink OFDMA networks

Chee Keong Tan[*], Moh Lim Sim, Teong Chee Chuah and Su Wei Tan

Abstract

This article presents a subcarrier allocation scheme based on a Blotto game (SABG) for orthogonal frequency-division multiple access (OFDMA) networks where correlation between adjacent subcarriers is considered. In the proposed game, users simultaneously compete for subcarriers using a limited budget. In order to win as many good subcarriers as possible in this game, users are required to wisely allocate their budget. Efficient power and budget allocation strategies are derived for users for obtaining optimal throughput. By manipulating the total budget available for each user, competitive fairness can be enforced for the SABG. In addition, the conditions to ensure the existence and uniqueness of Nash equilibrium (NE) for the SABG are also established. An low-complexity algorithm that ensures convergence to NE is proposed. Simulation results show that the proposed low-complexity SABG can allocate resources fairly and efficiently for both uncorrelated and correlated fading channels.

Keywords: OFDMA, subcarrier allocation, Blotto game, fairness, efficiency, complexity; correlated fading

1. Introduction

The need for more efficient spectrum utilization techniques in wireless networks is becoming increasingly important as the number of users continues to grow rapidly while the radio spectrum remains a scarce resource. Among the emerging broadband wireless access technologies, orthogonal frequency-division multiple access (OFDMA) is a promising technique for efficient utilization of the available spectrum. Owing to its several attractive features, OFDMA has been envisaged for possible implementation in future-generation wireless networks, e.g., 3GPP-long term evolution [1], cognitive radio networks [2], and IEEE 802.20 mobile broadband wireless access [3].

By exploiting multiuser diversity, efficient subcarrier allocation (SA) mechanism can be designed to ensure quality of service provisioning of multimedia services in OFDMA systems. Some recent studies on SA schemes have been reported in [4-7]. Nevertheless, most of the aforementioned SA schemes focus on throughput maximization and/or transmit power reduction subject to some constraints. The fairness issue, however, has largely been ignored. The deployment of such schemes naturally gives rise to the following problem: users with better

channel conditions always dominate usage of resources, causing low throughput for users with poor channel conditions. Thus, the max-min fairness criterion is used in [8] to maximize the throughput of users with poor channel conditions, but this approach results in severe spectrum inefficiency. To resolve this conflict, Nash bargaining solution (NBS) [9] is applied where the system performance is maximized subject to the constraint that each user is guaranteed a portion of resources. Although the fairness and efficiency issues have been considered in the NBS scheme, however, the computational complexity remains prohibitive for practical mobile applications.

According to [10], the aforementioned types of fairness that are artificially decided by the system are not truly fair from the users' perspective. Hence, competitive fairness is introduced in [10] using an auction method where each user competes for the resources and is responsible for its own action and resulting throughput. Nevertheless, auction-based spectrum sharing is not appropriate for multicarrier systems. Han et al. [11] proposed an enhanced auction-based SA method for OFDMA systems for achieving high spectrum utilization and ensure competitive fairness. However, previous studies on fairness [8-11] do not take into account the correlation between adjacent subcarriers. In fact, spectrum unfairness is more significant in correlated OFDM channels because the subcarriers

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utilized by a user may experience deep fades simultaneously and the use of techniques proposed in [8-11] may not lead to desired fairness in such scenarios.

In this study, we aim to develop an SA technique which can achieve a good trade-off performance among fairness, efficiency, and complexity for both correlated and uncorrelated OFDMA channels. Unlike the conventional auction method, we propose a multi-dimensional auction-based subcarrier allocation scheme based on a Blotto game (SABG) [12] where users simultaneously compete for subcarriers using a limited budget. In this game, users need to allocate power and budget wisely across the available subcarriers to win as many good subcarriers as possible. Subject to power and budget constraints, efficient power allocation (PA) and budget allocation (BA) strategies are derived. Next, we propose a low-complexity algorithm to guide the SABG to reach the unique Nash equilibrium (NE) where all the users can obtain optimal throughput fairly. Besides, by manipulating the amount of budget available for each user, the SABG can enjoy a good trade-off between throughput and spectrum fairness. It will be shown via simulation results that the SABG, which is a low-complexity SA scheme, can allocate more resources fairly and efficiently in both uncorrelated and correlated OFDM channels as compared to some existing SA techniques.

The rest of this article is organized as follows. In Section 2, we outline the system model. Section 3 formulates the SABG and investigates the conditions to ensure the existence and uniqueness of NE in SABG. An algorithm that guides the SABG to the unique NE is proposed in Section 4 and its complexity is analyzed. Simulation results and performance analysis are presented in Section 5. We end the article with some concluding remarks in Section 6.

2. System model

We consider the uplink of a single-cell OFDMA system with one base station (BS) accommodating K users (transmitters). The available spectrum is divided into N subchannels each with a bandwidth w which is less than the coherence bandwidth of the channel such that each subcarrier experiences flat fading. Besides, the orthogonality between subcarriers is assumed to be preserved perfectly so that there is no intersymbol interference between adjacent symbols. In addition, perfect synchronization is assumed so that no intercarrier interference occurs. We denote the user and subcarrier sets as $\mathcal{K} = \{1, 2, \ldots, K\}$ and $\mathcal{N} = \{1, 2, \ldots, N\}$, respectively.

In a typical single-cell OFDMA network, N is always much larger than K and no subcarrier can simultaneously support transmission for more than one user. Hence, all the users can simultaneously transmit data to the BS on one or more subcarriers without interfering

each other. In this context, we assume that the BS periodically estimates the uplink channel gains on all subcarriers for all the users through pilot signals. The channel variation on each subcarrier is assumed to be relatively slow as compared to the channel estimation rate performed by the BS. With this assumption, the BS can accurately track the channel state information (CSI) for all the users on different subcarriers. Furthermore, it is also assumed that the OFDMA network is geographically static in the sense that the time scale of algorithm convergence is shorter than the channel's coherence time. Thus, the channel gains on subcarriers remain unchanged in one implementation of the algorithm.

The available channel is assumed to exhibit frequency-selective Rayleigh fading where noticeable correlation between the channel gains of adjacent subcarriers exists. The subcarrier correlation coefficient between the mth and the nth subcarriers for the kth user is defined as [13]

$$\chi_k^{m,n} = \frac{\left(1 + \delta_k^{m,n}\right) \widetilde{\mathbf{E}} \left\{ \dfrac{2\sqrt{\delta_k^{m,n}}}{1 + \delta_k^{m,n}} \right\} - \dfrac{\pi}{2}}{2 - \dfrac{\pi}{2}} \tag{1}$$

where $\widetilde{\mathbf{E}}$ is the complete elliptic integral of the second kind and $\delta_k^{m,n}$ is

$$\delta_k^{m,n} = \frac{1}{\sqrt{1 + \left[2\pi \left(j - k\right) \sigma_\tau^{\mathrm{rms}}\right]}} \tag{2}$$

In Equation 2, $\sigma_\tau^{\mathrm{rms}}$ denotes the root-mean-square channel delay spread normalized by the number of subcarriers and is expressed in [9] as

$$\sigma_\tau^{\mathrm{rms}} = \sqrt{\frac{\sum\limits_{l=1}^{L} A_l^2 \tau_l^2 - \sum\limits_{l=1}^{L} A_l^2 \tau_l}{\sum\limits_{l=1}^{L} A_l^2}} \tag{3}$$

where A_l and τ_l are the amplitude and time delay for the lth ray, respectively while L is the total number of rays for the Rayleigh model.

In the OFDMA system, different subcarriers experience different channel gains and we denote the unique channel gain for the kth user on the nth subcarrier as g_k^n. If the kth user transmits with power p_k^n on the nth subcarrier, then its received signal-to-noise ratio is given by

$$\gamma_k^n = \frac{p_k^n g_k^n}{\eta_k^n} \tag{4}$$

where η_k^n is the background noise power on the signal received from the kth user using the nth subcarrier. The maximal transmit power of the kth user is $P_k^{\max} \geq \sum_{n=1}^N a_k^n p_k^n$ where a_k^n is a binary decision variable defined as

$$a_k^n = \begin{cases} 1, & \text{if the } n\text{th subcarrier is allocated to the } k\text{th user} \\ 0, & \text{otherwise.} \end{cases} \quad (5)$$

Since each subcarrier is independent of each other and is exclusively assigned to only one user at one time, we have

$$\sum_{k=1}^K a_k^n \leq 1, \ \forall n \in \mathcal{N} \text{ and } \sum_{k=1}^K \sum_{n=1}^N a_k^n \leq N \quad (6)$$

We assume that M-array quadrature amplitude modulation is adopted on the subcarriers and no channel coding is employed. For a fixed desired bit-error rate performance, the achievable throughput for the kth user on the nth subcarrier is given by [9]

$$r_k^n = w \log_2 \left(1 + \frac{\gamma_k^n}{\Theta} \right) \quad (7)$$

where $\Theta = (\ln(0.2/\text{BER}))/1.5$ [9]. The achievable throughput for the kth user is $R_k = \sum_{n=1}^N a_k^n r_k^n$. Accordingly, the total throughput R_T of all the users over all subcarriers can be shown to be

$$R_\text{T} = w \sum_{k=1}^K \sum_{n=1}^N a_k^n \log_2 \left(1 + \frac{p_k^n g_k^n}{\Theta \eta_k^n} \right) \quad (8)$$

To date, Equation 8 has been the most common objective function adopted in the literature [4-7] to quantify the overall system efficiency. The main focus in these studies is to maximize the total throughput subject to some constraints. These proposed SA schemes, however, suffer from a fairness problem whereby users with good channel conditions dominate the usage of subcarriers and deprive users with poor channel conditions of the opportunity to utilize spectrum.

Thus, an alternative performance metric is needed to address the unfairness issue. For this reason, we use the Jain's fairness index [14], which is a widely used fairness indicator. In this context, the throughput of all the users in the OFDMA network is used to compute the Jain's fairness index, ρ as follows

$$\rho = \frac{\left(\sum_{k=1}^K R_k \right)^2}{K \sum_{k=1}^K (R_k)^2} \quad (9)$$

where ρ can range from $1/K$ (the most unfair case) to 1 (the fairest case).

3. Problem formulation

In game theory, a Blotto game [12] is a two-person zero-sum game where the players are tasked to simultaneously distribute their limited resources over several objects, and the player allocating the most resources to an object wins the object. Intuitively, the goal of the players is to win the highest number of objects and their resource allocation strategies are crucial in determining the outcome. The payoff of the game is then equal to the total number of objects won. In fact, this game has widely been used to characterize some competitive real environments: allocating campaign budgets for elections, distributing soldiers for battlefields, etc. In wireless communications, Blotto game has been first used to model the PA problem under malicious jamming attacks for cognitive radio networks [15]. In the last few decades, intensive research effort has been devoted to study the equilibrium properties of Blotto games. In [16], a pure-strategy symmetric monotonic Bayesian equilibrium is found for a Blotto game with incomplete information. The idea of deriving the equilibrium point is very constructive and some of the equilibrium properties are useful in solving the fairness, efficiency, and complexity problems in the SABG to be proposed in this article.

3.1 Blotto game formulation

In this study, we extend the two-person Blotto game into a stochastic K-player Blotto game [16] which is more appropriate to model the SA problem in the OFDMA system described in Section 2. In this game, each user is allocated a total budget of $B_k^{\max}$ which is to be spent on N subcarriers subject to the constraint $\sum_{n=1}^N b_k^n \leq B_k^{\max}$ where b_k^n is the BA strategy of the kth user on the nth subcarrier. The budget could be in the form of fictitious credit [10] issued by the BS for bidding purposes.

Let SABG = $\langle \mathcal{K}, \{\mathcal{P}_k, \mathcal{B}_k\}, \{u_k(\bullet)\} \rangle$ denote the Blotto game where $\mathcal{K}$ is the index set for the bidders (users), $\mathcal{P}_k = [0, P_k^{\max}]$ and $\mathcal{B}_k = [0, B_k^{\max}]$ are the PA and BA strategy sets, respectively, while $u_k(\bullet)$ is the utility function for the kth user. In this game, every user tries to allocate their power and budget to maximize their utility functions (or win as many subcarriers as possible). To preserve high efficiency, the users should be encouraged to win the "good" subcarriers. (*Note*: "good" subcarriers refer to the subcarriers with good channel conditions for a particular user.) In a fair auction, the user who allocates the highest budget on a subcarrier has the highest chance to win that subcarrier. Therefore, a

rational user will always allocate more budget to the good subcarriers. To achieve competitive fairness and high efficiency, the utility function can be formulated as

$$u_k\left(\mathbf{b}_k, \mathbf{p}_k\right) = \sum_{n=1}^{N} w \log_2\left(1 + \frac{p_k^n g_k^n}{\Theta \eta_k^n}\right)\left(\frac{b_k^n}{\sum_{i=1}^{K} b_i^n}\right) \qquad (10)$$

where $b_k^n \big/ \sum_{i=1}^{K} b_i^n$ is the probability of the kth user winning the nth subcarrier. From Equation 10, $\mathbf{p}_k = \left(p_k^1, p_k^2, \ldots, p_k^N\right)$ and $\mathbf{b}_k = \left(b_k^1, b_k^2, \ldots, b_k^N\right)$ are the power and budget vectors for the kth user, respectively, where $p_k^n \in \mathcal{P}_k$ and $b_k^n \in \mathcal{B}_k$. The utility function in Equation 10 combines PA and BA where $\mathbf{p}_k$ and $\mathbf{b}_k$ are both coupled on each subcarrier and under the budget and power constraints of each user. Unlike other game theoretic approaches, the utility function in Equation 10 does not represent the actual throughput that a user can achieve at the end of the game. Instead, it provides a mechanism for users to allocate budget to different subcarriers based on their expected throughput on the subcarriers. The payoff of a user is equal to the number of subcarriers won, which is then quantified in the (actual) throughput.

In the SABG, users compete with each others in an auction market. Unlike the conventional auctions, the proposed SABG allows users to simultaneously bid for all subcarriers. Owing to limited budget, the users need to spend it wisely to win as many good subcarriers as possible to maximize their own throughput. To this end, optimal PA and BA strategies are required for the SABG such that Equation 10 can be maximized subject to the power and budget constraints, i.e.,

$$(\text{SABG}) \max_{\mathbf{b}_k, \mathbf{p}_k} u_k\left(\mathbf{b}_k, \mathbf{p}_k\right), \text{ s.t. } \begin{cases} \sum_{n=1}^{N} p_k^n \leq P_k^{\max} \\ \sum_{n=1}^{N} b_k^n \leq B_k^{\max} \end{cases} \qquad (11)$$

The approach used in [16] to derive the equilibrium state in Blotto games is adopted in this article to analyze the proposed PA and BA strategies for the SABG as given in the following proposition.

Proposition 1: If all the users maximize their utility function according to Equation 11, then the optimal PA strategy for the kth user on all subcarriers is

$$p_k^n = \left(\Delta_k - \frac{\Theta \eta_k^n}{g_k^n}\right)^+, \quad \forall n \in \mathcal{N} \qquad (12)$$

where Δ_k is the waterfilling level of the kth user and $x^+ = \max\{x, 0\}$. The optimal BA strategy for the kth user on all subcarriers is

$$b_k^n = \frac{B_k^{\max} \sqrt{r_k^n \sum_{i=1, i \neq k}^{K} b_i^n}}{\sum_{m=1}^{N} \sqrt{r_k^m \sum_{i=1, i \neq k}^{K} b_i^m}}, \quad \forall n \in \mathcal{N} \qquad (13)$$

Proof: See Appendix.

It is worth mentioning that the PA strategy given in Equation 12 is found to correspond to the traditional waterfilling PA techniques which can provide high spectral efficiency [3]. Since the PA strategy is independent of the BA strategy, the transmit power can be waterfilled onto different subcarriers before implementing BA.

After BA, the BS which acts as an auctioneer will assign the available subcarriers to the users based on their biddings. In general, SA is a combinatorial problem which requires complex algorithms to obtain the optimal solution [5,9]. In this study, we modify the binary constraint in Equation 5 into a probabilistic SA decision to facilitate design of a low-complexity SA algorithm. Thus, Equation 5 becomes

$$a_k^n = \begin{cases} 1, \text{ if } k = \arg\max_{i \in \mathcal{K}}\left\{b_i^n\right\} \\ 0, \text{ otherwise.} \end{cases} \qquad (14)$$

Next, we define the set $\mathcal{S}_k$ to include all the subcarriers that have been allocated to the kth user, i.e.,

$$\mathcal{S}_k = \left\{n \,\big|\, a_k^n = 1, \quad \forall n \in \mathcal{N}\right\} \qquad (15)$$

In this section, a fair, efficient, and low-complexity SA scheme has been proposed. Note, however, that the BA strategy proposed in Proposition 1 is not symmetrical as what has been derived in [16]. Therefore, the symmetric monotonic Bayesian equilibrium in [16] is not applicable in the SABG. In the following section, we will study the equilibrium of the SABG.

3.2 Existence and uniqueness of NE in the SABG
In the SABG, the individual user maximizes its own utility function in a distributed fashion. Since the proposed PA strategy is independent of the proposed BA strategy and does not require knowledge of other users' action for implementation, the PA is an iterative waterfilling algorithm without divergence problem. However, the BA strategy proposed in Equation 13 demonstrates some strategic interdependence among the users. The utility function of each user is governed by its own strategy as well as those of other users. Hence, the BA which optimizes individual utility also depends on the BA of other users in the system. Thus, it is necessary to characterize a set of BA strategies whereby all the users are satisfied with the utility attained given the BA

strategies of other users. Such an equilibrium operating point is called a NE in game theory [17]. The NE concept offers a predictable and stable outcome for a game where multiple agents with conflicting interests compete through self-optimization and reach a state from which no player wishes to deviate [17]. In other words, at a NE, given the BA of other users, no user can improve its subcarrier utility level by making individual changes in the BA. Nevertheless, such a state may not necessarily exist. Thus, we first need to investigate the existence of NE in the SABG.

Theorem 1: NE exists in the SABG.

Proof: According to the implicit function theorem [18], a Jacobian matrix must be non-singular at the point of existence. By using Equation 13, we define

$$F_k = -b_k^n + \frac{B_k^{\max}\sqrt{r_k^n \sum\limits_{i=1,i\neq k}^{K} b_i^n}}{\sum\limits_{m=1}^{N}\sqrt{r_k^m \sum\limits_{i=1,i\neq k}^{K} b_i^m}} = 0, \quad \forall k \in \mathcal{K}, \quad n \in \mathcal{N} \quad (16)$$

where F_k, $\forall k \in \mathcal{K}$ are differentiable functions. Taking the partial derivative $\partial F_k / \partial b_k^n$, $\forall k \in \mathcal{K}$ results in -1 on the main diagonal of the Jacobian matrix while the terms outside the main diagonal can be expressed as

$$\frac{\partial F_k}{\partial b_i^n} = \frac{r_k^n}{\sqrt{r_k^n \sum\limits_{i=1,i\neq k}^{K} b_i^n}} \frac{B_k^{\max}\sum\limits_{m=1,m\neq n}^{N}\sqrt{r_k^m \sum\limits_{i=1,i\neq k}^{K} b_i^m}}{\left(\sum\limits_{m=1}^{N}\sqrt{r_k^m \sum\limits_{i=1,i\neq k}^{K} b_i^m}\right)^2}, \quad \forall i \in \mathcal{K}, \quad i \neq k \quad (17)$$

Since $r_k^n \gg 1$, $\forall k \in \mathcal{K}$, $n \in \mathcal{N}$ (normally in Mbps), $\forall i \in \mathcal{K} \forall i \in \mathcal{K}$, $i \neq k$ is always of the order of 10^{-3} or smaller. Therefore, the values of the terms $\partial F_k / \partial b_i^n$, $\forall i \in \mathcal{K}$, $i \neq k$ are extremely small and will only have a negligible impact on the non-singularity of the corresponding Jacobian matrix. Furthermore, since the Jacobian matrix is a continuous function with respect to r_k^n, solutions exist for the entire range of large values of r_k^n. In conclusion, by assuming that r_k^n, $\forall k \in \mathcal{K}$, $n \in \mathcal{N}$ are large enough, the solution for Equation 16 exists and hence the existence of NE can be ensured.

We next prove the uniqueness of this NE, which ensures convergence of the algorithm to be proposed in Section 4. For this purpose, we use a discrete-time model where time is divided into iterations and we assume that all the users only act once in one iteration and remain static during that iteration. Let $b_k^n(t+1)$ and $b_k^n(t)$ be the BA strategies of the kth user on the nth subcarrier at the next and current iterations, respectively, where t is the iteration number. We can rewrite Equation 16 as

$$f\left(b_k^n(t)\right) := b_k^n(t+1) = \frac{B_k^{\max}\sqrt{r_k^n \sum\limits_{i=1,i\neq k}^{K} b_i^n(t)}}{\sum\limits_{m=1}^{N}\sqrt{r_k^m \sum\limits_{i=1,i\neq k}^{K} b_i^m(t)}}, \quad \forall k \in \mathcal{K}, \quad n \in \mathcal{N} \quad (18)$$

Using Equaiton 18, the uniqueness of the NE which exists in the proposed games can be shown next.

Theorem 2: The SABG have the unique NE.

Proof: In [18], it is shown that if a fixed point $b_k^n(t+1) = f\left(b_k^n(t)\right)$ exists and if the function f satisfies three properties: positivity $f\left(b_k^n\right) > 0$, monotonicity $b_k^n > \left(b_k^n\right)' \Rightarrow f\left(b_k^n\right) > f\left(\left(b_k^n\right)'\right)$, and scalability $\left(\varepsilon f\left(b_k^n\right) > f\left(\varepsilon b_k^n\right), \quad \forall \varepsilon > 1\right)$, convergence to a fixed and unique point is guaranteed. For brevity, we drop the iteration index t in the following proof. Since all the elements on the right-hand side (RHS) of Equation 18 are positive, $b_k^n > 0$ and hence $f\left(b_k^n\right) > 0$, which ensure positivity of $f\left(b_k^n\right)$. Next, to prove the monotonicity property, we modify Equation 18 and obtain

$$f\left(b_k^n\right) - f\left(\left(b_k^n\right)'\right) = B_k^{\max}\left(\frac{\sqrt{r_k^n \sum\limits_{i=1,i\neq k}^{K} b_i^n}}{\sum\limits_{m=1}^{N}\sqrt{r_k^m \sum\limits_{i=1,i\neq k}^{K} b_i^m}} - \frac{\sqrt{r_k^n \sum\limits_{i=1,i\neq k}^{K} \left(b_i^n\right)'}}{\sum\limits_{m=1}^{N}\sqrt{r_k^m \sum\limits_{i=1,i\neq k}^{K} \left(b_i^m\right)'}}\right) \quad (19)$$

It is observed from Equation 19 that a decrease in b_k^n to $\left(b_k^n\right)' < b_k^n$ results in a decrease in b_i^n to $\left(b_i^n\right)' < b_i^n$, $\forall i \in \mathcal{K}$, $i \neq k$ as b_k^n is a continuous function of b_i^n, $\forall i \in \mathcal{K}$, $i \neq k$ as shown in Equation 13. However, this decrease results in an increase in b_k^m, $\forall m \in \mathcal{N}$, $m \neq n$ because more budget is available to be spent on other subcarriers and users tend to spend all additional budget to increase the chance of winning more subcarriers. Therefore, in the second term on the RHS of Equation 19, the denominator remains the same but the numerator decreases. Hence, on the RHS of Equation 19, the second term is always smaller than the first term. This ensures that $f\left(b_k^n\right) > f\left(\left(b_k^n\right)'\right)$ and thus the monotonicity condition is satisfied. Similarly, to prove the scalability property, Equation 18 can be rewritten as

$$\varepsilon f\left(b_k^n\right) - f\left(\varepsilon b_k^n\right) = B_k^{\max}\left(\frac{\varepsilon\sqrt{r_k^n \sum\limits_{i=1,i\neq k}^{K} b_i^n}}{\sum\limits_{m=1}^{N}\sqrt{r_k^m \sum\limits_{i=1,i\neq k}^{K} b_i^m}} - \frac{\sqrt{r_k^n \sum\limits_{i=1,i\neq k}^{K} \varepsilon b_i^n}}{\sum\limits_{m=1}^{N}\sqrt{r_k^m \sum\limits_{i=1,i\neq k}^{K} \varepsilon b_i^m}}\right) \quad (20)$$

Since ε is a scalar multiplication constant for the summation in the second term on the RHS of Equation 20, the term remains the same. Hence, the first term on the RHS of Equation 20 which is multiplied with $\varepsilon > 1$ is always larger than the second term. This indicates that $\varepsilon f\left(b_k^n\right) > f\left(\varepsilon b_k^n\right)$ and thus the scalability condition is fulfilled. Since all the three properties are satisfied, the

solution which exists in the SABG, and hence the corresponding NE, is unique.

3.3 Properties of the SABG

Some of the useful equilibrium properties in [16] can be adopted in the SABG and are summarized in the following theorem.

Theorem 3: The SABG has the following properties:

1. All users in the SABG will adhere to the BA strategy in Equation 13 if the knowledge of the BA strategies of other users is not available, but it is believed that the available budget will be allocated according to Equation 13.
2. The NE which exists in the SABG has a monotonic property where the user with the highest budget has the highest chance of winning more subcarriers.
3. All the users in the SABG compete for subcarriers. Even a small amount of additional budget improves the chance of winning more subcarriers.
4. All the users in the SABG tend to fully spend their budget to increase the chance of winning more subcarriers.

Proof: Since a unique monotonic equilibrium exists in the SABG, the latter inherits all the properties of a Blotto game listed in [16].

From the second property in Theorem 3, the BS can manipulate the amount of budget available to each user to achieve different objectives. In this study, we aim to enforce competitive fairness into the SABG while incurring a minimal loss in total throughput particularly in correlated fading channels. The following proposition gives the condition to achieve fairness in the SABG.
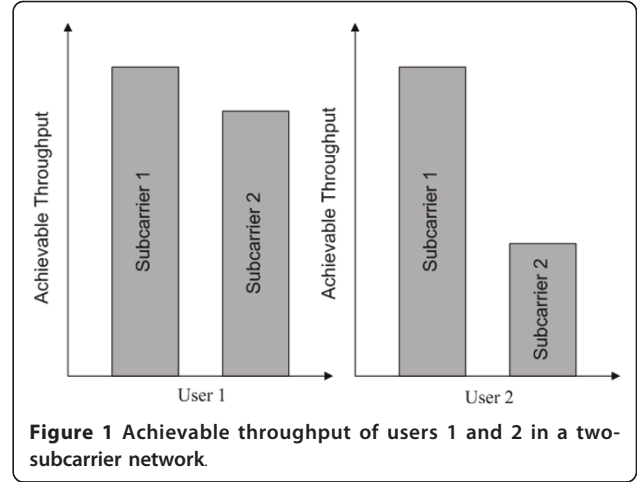
Proposition 2: The SABG can achieve competitive fairness if each user is allocated an equal amount of budget.

Proof: Consider two users (users 1 and 2) whose achievable throughputs on two subcarriers are shown in Figure 1 where $r_1^1 - r_1^2 < r_2^1 - r_2^2$. According to [11], competitive fairness can be achieved if their BA strategies are $b_2^1 > b_1^1 > b_1^2 > b_2^2$. Using Equation 13, we have

$$b_1^1 = \frac{B_1^{\max}\sqrt{r_1^1 b_2^1}}{\sqrt{r_1^1 b_2^1} + \sqrt{r_1^2 b_2^2}}, \quad b_1^2 = \frac{B_1^{\max}\sqrt{r_1^2 b_2^2}}{\sqrt{r_1^1 b_2^1} + \sqrt{r_1^2 b_2^2}}, \quad b_2^1 = \frac{B_2^{\max}\sqrt{r_2^1 b_1^1}}{\sqrt{r_2^1 b_1^1} + \sqrt{r_2^2 b_1^2}}, \quad b_2^2 = \frac{B_2^{\max}\sqrt{r_2^2 b_1^2}}{\sqrt{r_2^1 b_1^1} + \sqrt{r_2^2 b_1^2}} \quad (21)$$

The following inequalities can be formulated using the competitive fairness conditions

$$\frac{B_1^{\max}\sqrt{r_1^1 b_2^1}}{\sqrt{r_1^1 b_2^1} + \sqrt{r_1^2 b_2^2}} - \frac{B_2^{\max}\sqrt{r_2^1 b_1^1}}{\sqrt{r_2^1 b_1^1} + \sqrt{r_2^2 b_1^2}} < 0, \quad \frac{B_2^{\max}\sqrt{r_2^1 b_1^1}}{\sqrt{r_2^1 b_1^1} + \sqrt{r_2^2 b_1^2}} - \frac{B_1^{\max}\sqrt{r_1^1 b_2^1}}{\sqrt{r_1^1 b_2^1} + \sqrt{r_1^2 b_2^2}} < 0,$$
$$\frac{B_1^{\max}\sqrt{r_1^2 b_2^2}}{\sqrt{r_1^1 b_2^1} + \sqrt{r_1^2 b_2^2}} - \frac{B_1^{\max}\sqrt{r_1^1 b_2^1}}{\sqrt{r_1^1 b_2^1} + \sqrt{r_1^2 b_2^2}} < 0, \quad \frac{B_2^{\max}\sqrt{r_2^2 b_1^2}}{\sqrt{r_2^1 b_1^1} + \sqrt{r_2^2 b_1^2}} - \frac{B_2^{\max}\sqrt{r_2^1 b_1^1}}{\sqrt{r_2^1 b_1^1} + \sqrt{r_2^2 b_1^2}} < 0 \quad (22)$$

Figure 1 Achievable throughput of users 1 and 2 in a two-subcarrier network.

The inequalities in Equation 22 can be simplified as

$$B_1^{\max}\left(\sqrt{r_1^1 r_2^1 b_1^1 b_2^1} + \sqrt{r_1^1 r_2^2 b_1^2 b_2^1}\right) < B_2^{\max}\left(\sqrt{r_1^1 r_2^1 b_1^1 b_2^1} + \sqrt{r_1^2 r_2^1 b_1^1 b_2^2}\right)$$
$$B_2^{\max}\left(\sqrt{r_1^2 r_2^2 b_1^2 b_2^2} + \sqrt{r_1^1 r_2^2 b_1^2 b_2^1}\right) < B_1^{\max}\left(\sqrt{r_1^2 r_2^2 b_1^2 b_2^2} + \sqrt{r_1^2 r_2^1 b_1^1 b_2^2}\right) \quad (23)$$
$$r_1^1 b_2^1 > r_1^2 b_2^2$$
$$r_2^1 b_1^1 > r_2^2 b_1^2$$

Given that $b_1^1 + b_1^2 + b_2^1 + b_2^2 = B_1^{\max} + B_2^{\max}$, it is noted from the inequalities (23) that $B_2^{\max} < (1 + \varepsilon)B_1^{\max}$ and $B_2^{\max} > (1 - \varepsilon)B_1^{\max}$ such that $\varepsilon \approx 0$ as r_k^n (Mbps) are large enough. Therefore, the only solution that satisfies the above inequalities is $B_1^{\max} \approx B_2^{\max}$. This condition is both necessary and sufficient to achieve competitive fairness in the SABG. According to [11], the proof for the above two-user case can be extended to the K-user general case ($K > 2$) by introducing multivariable inequalities. Using the same approach above, the solution can be obtained as $B_1^{\max} \approx B_2^{\max} \approx \cdots \approx B_K^{\max}$, as proved in [11].

In general, the SABG scheme is adaptive and can be adjusted to reach two extreme states (i.e., maximal-rate and max-min fairness) by manipulating the budget allocated to each user. The adaptive SABG can be achieved using

$$B_k^{\max} = \frac{B\sum_{n=1}^{N}\frac{\beta r_k^n + 1}{(1 - \beta) r_k^n + 1}}{\sum_{n=1}^{N}\sum_{k=1}^{K}\frac{\beta r_k^n + 1}{(1 - \beta) r_k^n + 1}}, \quad \forall k \in \mathcal{K} \quad (24)$$

where B is the amount of budget allocated by the BS and $0 \le \beta \le 1$ is an adjustable coefficient controlled by the BS. When $\beta = 0.5$, it is shown in Equation 24 that every user is allocated an equal amount of budget and competitive fairness is attained.

Proposition 3: The throughput achieved by the SABG approximates maximal-rate throughput if $\beta = 1$ while max-min fairness is achieved by the SABG if $\beta = 0$.

Proof: Let $\beta = 1$ and Equation 24 indicate that the amount of budget allocated to every user is proportional to the achievable throughput where users with better channel conditions obtain more budget. As shown in Equation 14, having more budget will increase the probability of acquiring subcarriers. In other words, subcarriers are always allocated to users with the best channel gains. Instead of assigning subcarriers based on the budget allocated, the SA decision can be modified as

$$a_k^n = \begin{cases} 1, & \text{if } k = \arg\max_{i \in \mathcal{K}} \{r_i^n\} \\ 0, & \text{otherwise.} \end{cases} \quad (25)$$

The binary decision in Equation 25 corresponds to the maximal-rate scheme. On the other hand, let $\beta = 0$ and Equation 24 indicate that the amount of budget allocated to every user is inversely proportional to the achievable throughput where users with worse channel conditions obtain more budget. In this scenario, users with poor channel conditions are assigned more subcarriers to improve their throughput. Therefore, Equation 14 can be modified as

$$a_k^n = \begin{cases} 1, & \text{if } k = \arg\min_{i \in \mathcal{K}} \{r_i^n\} \\ 0, & \text{otherwise.} \end{cases} \quad (26)$$

where Equation 26 corresponds to the max-min fairness scheme.

4. Proposed algorithm for the SABG
4.1 Algorithms for the SABG
It is noted from Equation 13 that the BA strategies of all the users are public information, so that the SABG can be analyzed as a simultaneous-move game with complete information [16]. Given this information, the users can cheat in the games by purposely allocating budget slightly higher than the maximum one across all subcarriers to win all the subcarriers. To overcome this problem, we propose a distributed and iterative BA strategy with incomplete information by rewriting Equation 13 as

$$b_k^n(t+1) = \frac{B_k^{\max}\sqrt{r_k^n\left(B^n(t) - b_k^n(t)\right)}}{\sum_{m=1}^{N}\sqrt{r_k^m\left(B^m(t) - b_k^m(t)\right)}}, \quad \forall n \in \mathcal{N} \quad (27)$$

where $B^n(t)$ is the total budget allocated by all the users on the nth subcarrier. To update its budget iteratively according to Equation 27, the kth user requires to know its achievable throughput on every subcarrier (r_k^n, $\forall n$) and the sum of all users' allocated budget on every

subcarrier (B^n, $\forall n$). In this study, we adopt the assumption made in [9] such that a reliable feedback channel from the BS to every user is available to share this information. Using Equation 27, any cheating among the users can be avoided because each user is unable to predict others' strategies by just having the information of B^n, $\forall n$.

The SABG algorithm is summarized in Table 1. First, users allocate budget equally to every subcarrier and choose the initial powers randomly. The random initial PA strategies will not cause the algorithm to diverge because convergence of the SABG algorithm is independent of the PA strategy (as shown in Theorem 1). All the necessary information is obtained from the BS via the feedback channels. Given all the necessary information, the users iteratively update their BA strategies as in Equation 27 after PA. At each iteration, users need to submit their biddings to the BS to update $B_n(t)$, $\forall n$. This can be done by allowing users asynchronously and iteratively broadcast a beacon at certain power level on a subcarrier which corresponds to their budget on that subcarrier. Using the available CSI, the BS can estimate the budget allocated by each user on each subcarrier. Furthermore, it is shown in Table 1 that the condition $\forall i \in \mathcal{K}$, $\forall i \in \mathcal{K}$, $m \in \mathcal{N}$ must be fulfilled before NE is declared where ψ is a convergence threshold. Upon reaching NE, the BS assigns the subcarriers to the users paying the highest budget and the users can waterfill their power to the assigned subcarriers.

4.2 Signaling and computational complexity of the SABG
While allocating budget in the SABG, information exchange between the BS and users is unavoidable. The signaling complexity of the SABG is similar to that of the iterative auction-based spectrum sharing [19], which has been proven to be feasible for practical implementation. In addition, our algorithm is more favorable than that of [19] becaue of rapid convergence (to be proven in Section 5).

In [9], the NBS approach adopts cooperative game theory that requires users to negotiate and form coalitions to obtain the optimal solution. However, this results in high computational complexity. The Hungarian method is therefore introduced to the NBS scheme for reducing the overall complexity to $O((K^2N\,log_2N + K^4)T_{\text{NBS}})$ where T_{NBS} is the number of iterations required for the NBS algorithm to converge. Nevertheless, the NBS approach is still not feasible for mobile applications particularly in the network with large number of users and subcarriers because its complexity is exponential to the number of users and subcarriers. In this study, the SABG algorithm is proposed to solve the complexity problem. Based on the algorithm summarized in Table 1, the overall complexity of the SABG is O

Table 1 The SABG algorithm

1. Initialization

   User k obtains $B_k^{\max}$ from the BS and randomly select $p_k^n(0)$, $\forall n \in \mathcal{N}$. Let $b_k^n(0) = B_k^{\max}/N$, $\forall n \in \mathcal{N}$ and $t = 1$

2. PA strategy

   Based on the CSI obtained from the BS, user k waterfills its power to all subcarriers according to Equation 12

3. BA strategy

   User k obtains the information $(B^n(t-1), \quad \forall n \in \mathcal{N})$ from the BS

   For $m = 1,...,N$

User k allocates its budget to subcarrier m according to Equation 27 and updates $b_k^m(t) = b_k^m$

   End

   User k sends the bidding information to the BS

4. Convergence

   For $i = 1,...,K$

      For $m = 1,...,N$

         The BS checks if $\left| b_i^m(t) - b_i^m(t-1) \right| < \psi$, NE is declared, otherwise let $t = t+1$ and repeat step 3

      End

   End

5. Subcarrier allocation

   For $m = 1,...,N$

      The BS assigns subcarrier m to user k according to Equation 14.

   End

6. Power allocation

   User k waterfills its power to subcarrier n using Equation 12 for $n \in \mathcal{S}_k$.

$((2T_{\mathrm{WF}} + 2T_{\mathrm{BA}} + 1)KN)$ where T_{WF} and T_{BA} are the numbers of iterations required for the waterfilling PA and BA algorithms to converge, respectively. Unlike the NBS scheme, the complexity of the SABG increases linearly with the number of users and subcarriers.

5. Simulation results

In the simulation setup for the uplink of a single-cell OFDMA system, the cell radius is assumed to be 1 km and the BS is placed at the center of the cell within which the users are uniformly distributed. To model a urban non-line-of-sight propagation environment, we use the path loss exponent, $v \sim \mathcal{U}(3, 5)$. The shadowing effect is assumed to be $10\log_{10} S_k \sim \mathcal{N}(0, \sigma_s^2)$ where $\sigma_s \sim \mathcal{U}(4, 10)$[20]. We simulate a correlated frequency-selective Rayleigh fading channel using an exponential power-delay profile which has $\sigma_\tau^{\mathrm{rms}} = 0.1\ \mu\mathrm{s}$[12]. Each subchannel has a bandwidth $w = 25$ kHz and the symbol duration is 40 μs [9]. Each user deploys an isotropic transmitter with a maximum power of $P_k^{\max} = 50$ mW and the desired error probability at the output of the 16-QAM demodulator is BER = 10^{-4}. The background noise is assumed to be additive white Gaussian noise of $\eta_k^n \sim \mathcal{N}(4 \times 10^{-12})$. For convergence, $\psi = 10^{-5}$ is used. To enforce competitive fairness, we assume that the BS always allocates an equal amount of budget to every user, $B_k^{\max} = 1$, $\forall k \in \mathcal{K}$. Each curve is obtained as the average of 1,000 channel realizations. For each realization, the SABG algorithm is implemented and convergence is achieved within one realization.

We first show the convergence curves of the proposed SABG algorithm for uncorrelated and correlated OFDM channels in Figure 2a,b, respectively, where an OFDMA system with $K = 10$ and $N = 32$ is considered. Each curve in these subfigures corresponds to the total budget allocated by all the users on each subcarrier. Initially, each user allocates equal amount of budget to every subcarrier and the budget allocated on each subcarrier is seen to monotonically converge to the unique NE. After the tenth iteration, optimal BA is achieved by the SABG and this verifies Theorems 1 and 2 by showing that the proposed algorithm always converges to the unique NE. Next, we investigate the convergence speed of the SABG for different network scales (with different K and N values) in uncorrelated and correlated OFDM channels. For both channels, it is observed in Figure 2c, d that the number of iterations needed for convergence increases linearly with K and N. An important advantage of the proposed algorithms is that when K and N are large (typically $K > 25$ and $N > 128$), the convergence speed becomes independent of network scales and less than or equal to 13 iterations are required to achieve convergence.

As proved in Proposition 2, the BS needs to allocate an equal amount of budget for every user to achieve competitive fairness. To emphasize the importance of
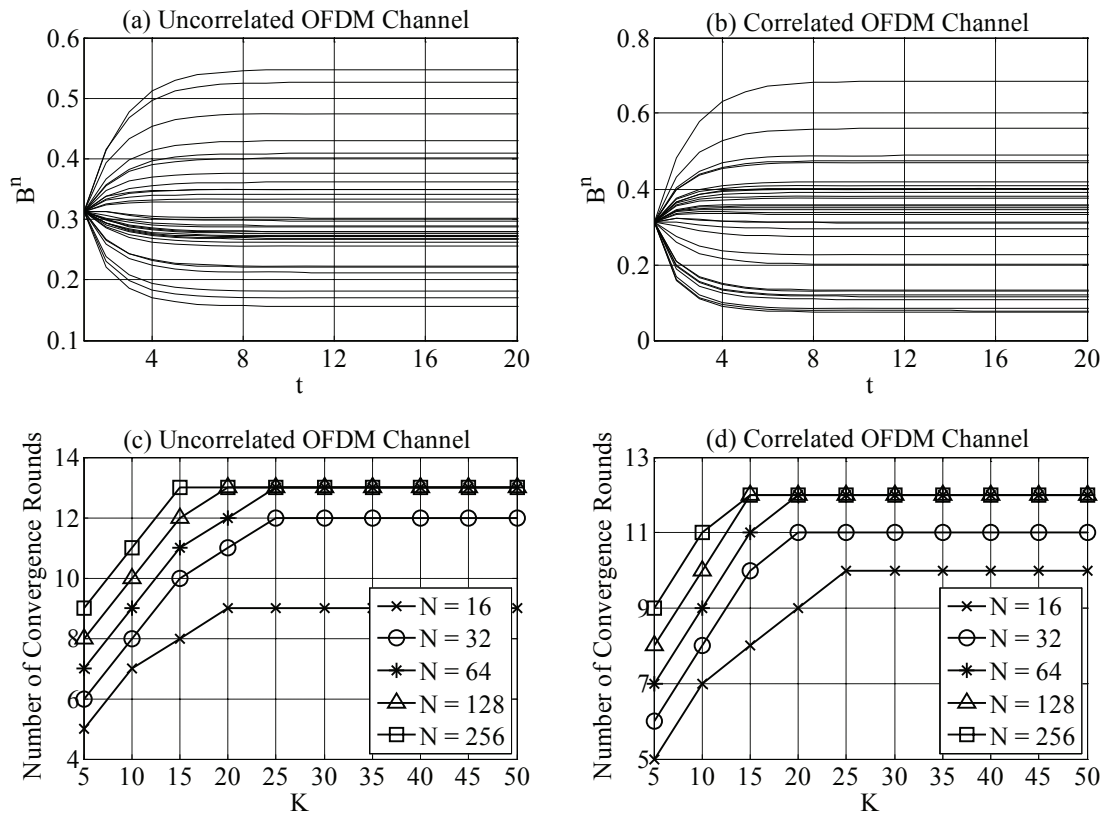
Figure 2 Convergence curves of the SABG for (a) uncorrelated and (b) correlated OFDM channels; convergence speed of the SABG algorithms for (c) uncorrelated and (d) correlated OFDM channels.

this proposition, we illustrate the impact on spectrum fairness due to asymmetrical BA by the BS in a two-user network with 64 subcarriers. From Figure 3a, we note that when User 1's available budget is less than that of User 2, the spectrum is dominated by User 2. This scenario is conjectured in the second property of Theorem 3 that a user with a higher budget has a higher chance to win every subcarrier. Therefore, a low Jain's fairness index is obtained in this scenario as shown in Figure 3c. By gradually increasing the budget for User 1, the fairness index increases as User 1 has more budget to compete for subcarriers. The fairness index approaches 1 as the amount of budget available for User 1 is equal to that of User 2. A further increase in the budget of User 1 will cause unfairness to User 2 because the spectrum would then be dominated by User 1. In Figure 3b, note that the highest total throughput can be acquired when the BS allocates budget equally to the two users. This further encourages the BS to implement symmetrical BA to every user, not only for achieving competitive fairness, but also for attaining high spectral efficiency.

Next, we evaluate the performance of the SABG in both uncorrelated and correlated OFDM channels. To achieve competitive fairness, $\beta = 0.5$ is used. The BA strategy adopted by the users are based on Equation 27. Figure 4 shows the total throughput and fairness index versus the number of users in a system with 128 subcarriers for the maximal-rate, max-min, NBS, and SABG schemes. Owing to multiuser diversity, it is observed in Figure 4a,b that the performance of these four schemes gradually improves when the number of users increases. In an uncorrelated OFDM channel, it is shown in Figure 4a,c that the SABG has a similar performance to that of the NBS scheme in terms of efficiency and fairness. On the other hand, Figure 4b,d shows that the SABG is more spectrally efficient and fair than the NBS approach in a correlated OFDM channel. In this channel, a scenario where all the subcarriers of a user are in deep fade may arise. The NBS scheme fails to attain a good trade-off between fairness and efficiency because of the requirement to fulfill minimum rate for every user. In SABG, however, the manipulation of the budget available for each user allows them to compete fairly and the
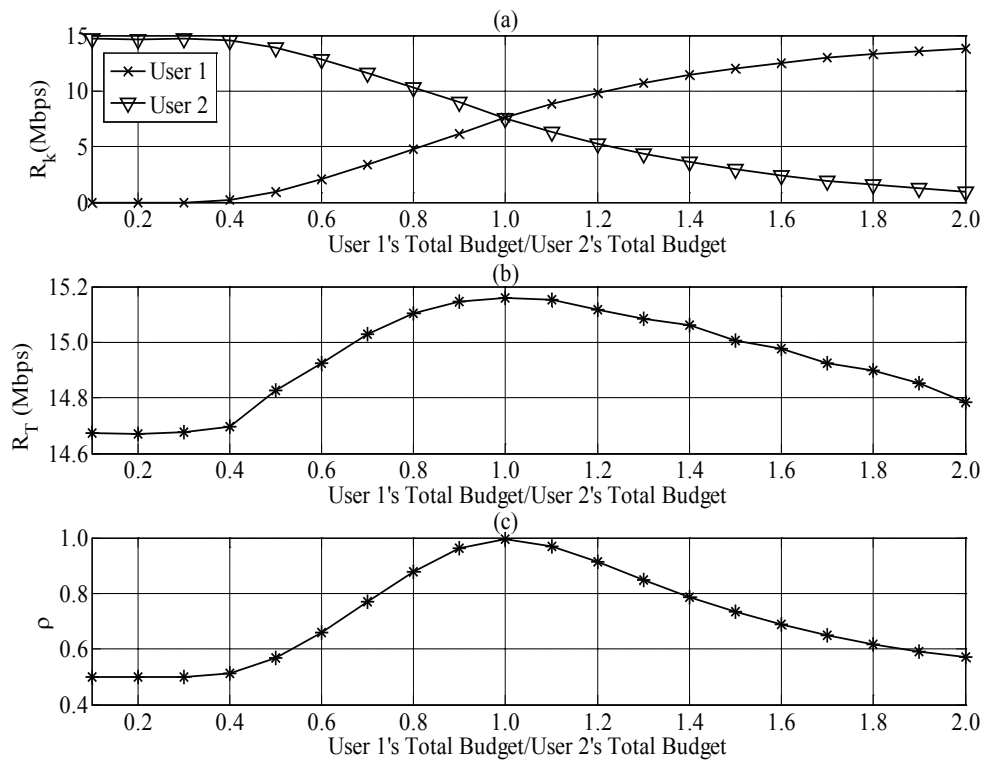
Figure 3 Impact of different budgets on (a) individual throughput, (b) total throughput, and (c) fairness in a two-user OFDMA system.
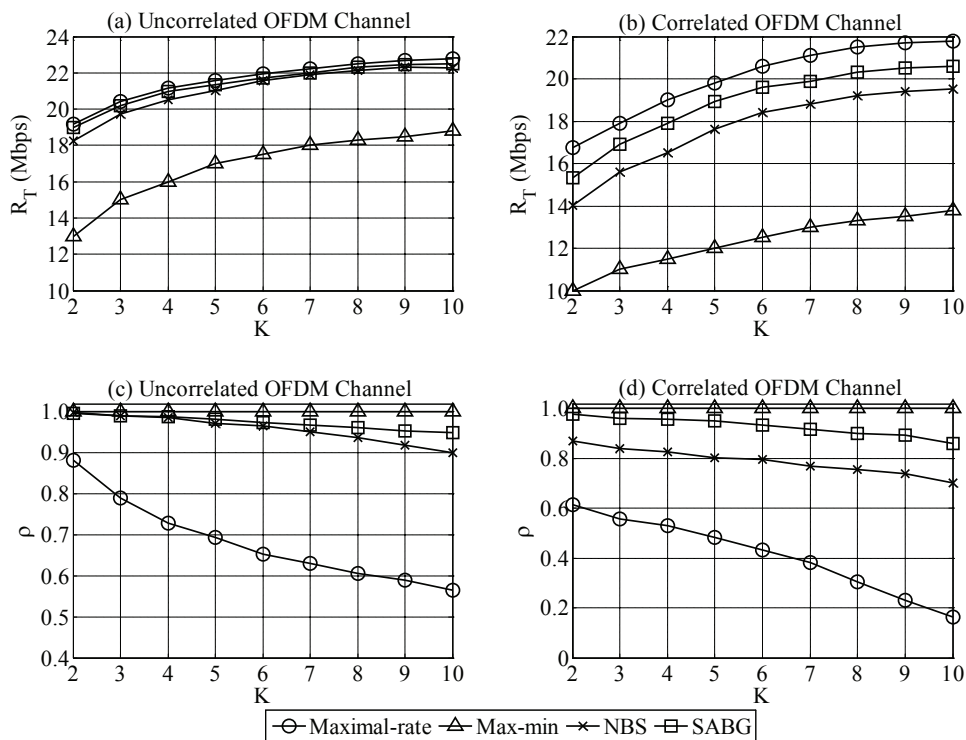
Figure 4 Comparison of total throughput and fairness index for the maximal-rate, max-min, NBS and SABG schemes in uncorrelated and correlated OFDM channels.

willingness to pay in this auction environment guarantees the best set of subcarriers for every user.

In Figure 5, the overall complexities of the NBS and SABG schemes are compared. The NBS and SABG algorithms are implemented in the similar correlated OFDM channel and their complexities are computed upon their convergence. From Figure 5, it is observed that the overall complexity of the SABG increases linearly with K and N. Though the order of complexity of the SABG is similar to that of the maximal-rate and max-min fairness schemes, the overall complexity of the SABG is slightly higher because of the iterative PA and BA algorithms. On the other hand, the overall complexity of the NBS scheme increases exponentially with K and N. Even though the convergence speed of the SABG is slower than that of the NBS scheme, the SABG attains a much lower overall complexity than that of the NBS scheme. Hence, the SABG is more favorable and feasible for practical implementation, particularly in a network with large K and N.

Figure 6 shows the trade-off performance between spectrum fairness and spectral efficiency in a system with 128 subcarriers. Let $\beta = 1$, the BS allocates budget

to every user as in Equation 24. Now, the SABG acts like the maximal-rate scheme by assigning subcarriers to users with good channel gains without considering fairness. Thus, it is observed in Figure 6a,c that the total throughput achieved by the SABG is comparable to that of the maximal rate scheme, but the former has poor performance in preserving spectrum fairness. To approximate max-min fairness, let $\beta = 0$ and the BS allocates budget to every users as in Equation 24. As shown in Figure 6b,d, the fairness achieved by the SABG approximates max-min fairness but it suffers from a large performance loss in total throughput. This is because more subcarriers are assigned to users with poor channel conditions to improve their throughput, and hence the performance gap among the users is reduced. In conclusion, by manipulating β, the SABG can enjoy a flexible trade-off between spectral efficiency and spectrum fairness.

6. Conclusion

In this article, the SA problem in OFDMA networks has been studied in a realistic channel model which considers correlation between adjacent subcarriers. We have
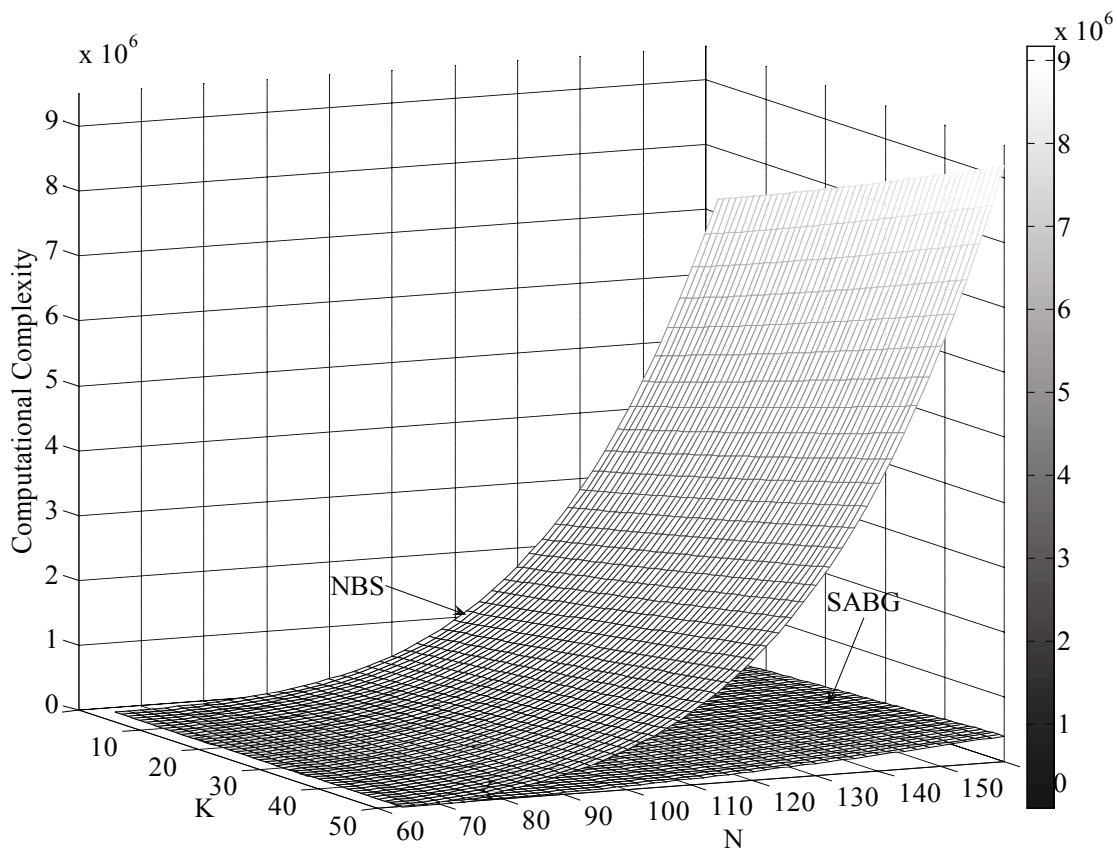
Figure 5 Computational complexity of the NBS and SABG algorithms under different number of users and subcarriers.
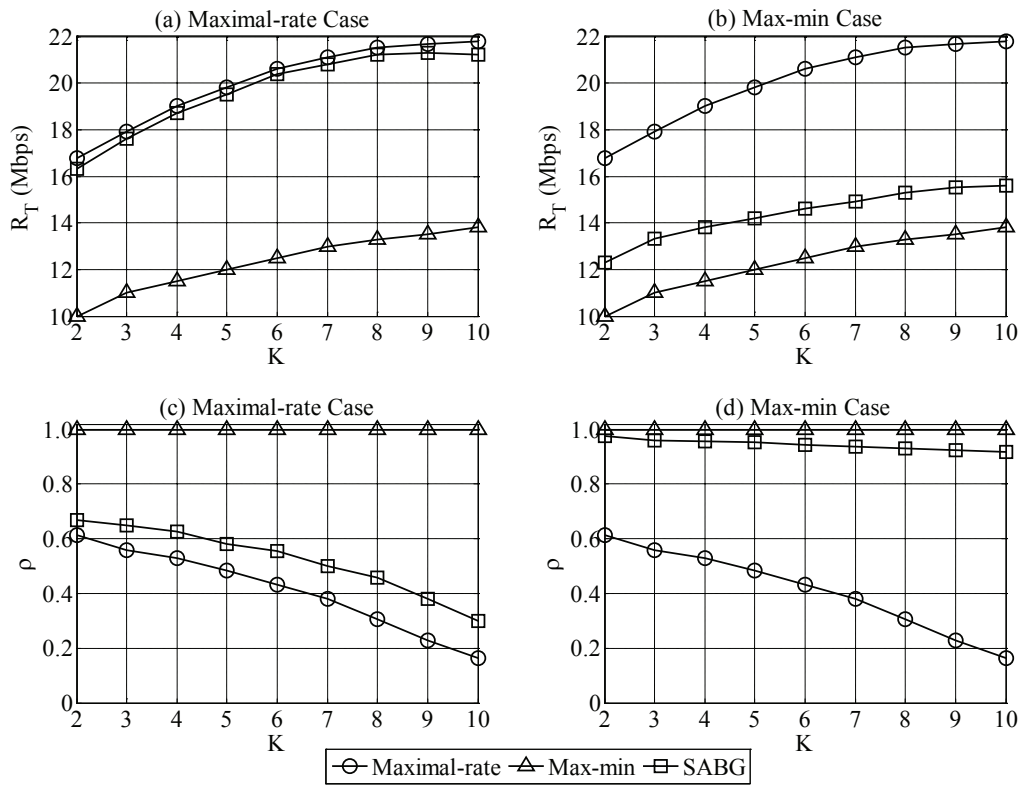
Figure 6 Approximation of the SABG to the maximal-rate (β = 1) and max-min (β = 0) schemes in a correlated OFDM channel.

shown that spectrum unfairness is more severe in correlated fading channels than in uncorrelated ones. To enforce competitive fairness in correlated fading channels while attaining high spectral efficiency and maintaining low complexity, we have proposed the SABG in which all the users simultaneously compete for the available subcarriers using a limited budget. Efficient PA and BA strategies have been derived for users for obtaining optimal throughput fairly. By manipulating the amount of budget available for each user, the SABG can enjoy a good trade-off between spectral efficiency and spectrum fairness. If the BS allocates equal amounts of budget to all users, then the fairest spectrum sharing among users can be enforced at a minimal loss in total throughput. The SABG has been shown to be more favorable than the existing SA schemes because of its high efficiency, fairness assurance, and low complexity. Simulation results demonstrate that the SABG algorithm can converge rapidly to the unique NE where upon reaching this point, subcarriers can be allocated fairly and efficiently to every user.

Appendices

A. Proof of Proposition 1

Using Lagrangian relaxation, Equation 11 can be rewritten as

$$
\mathcal{L}\left(\boldsymbol{b}_k, \boldsymbol{p}_k, \alpha_k, \mu_k\right) = \sum_{n=1}^{N} w\log_2\left(1 + \frac{p_k^n g_k^n}{\Theta \eta_k^n}\right)\left(\frac{b_k^n}{\sum_{i=1}^{K} b_i^n}\right) \\
- \alpha_k\left(\sum_{n=1}^{N} b_k^n - B_k^{\max}\right) - \mu_k\left(\sum_{n=1}^{N} p_k^n - P_k^{\max}\right) \tag{A1}
$$

where α_k and μ_k are the positive Lagrangian multipliers. Taking the partial derivative of (A1) with respect to $p_k^n, \mu_k, \forall n \in \mathcal{N}$ gives

$$
\frac{\partial \mathcal{L}}{\partial p_k^n} = \left(\frac{wg_k^n}{p_k^n g_k^n + \Theta \eta_k^n}\right)\left(\frac{b_k^n}{\sum_{i=1}^{K} b_i^n}\right) - \mu_k, \quad \forall n \in \mathcal{N} \tag{A2}
$$

$$
\frac{\partial \mathcal{L}}{\partial \mu_k} = P_k^{\max} - \sum_{n=1}^{N} p_k^n \tag{A3}
$$

From Equation A3, note that p_k^n is constrained by $P_k^{\max}$. Thus, we can rearrange Equation A2 and obtain

$$
p_k^n = \frac{b_k^n w}{\mu_k \sum_{i=1}^{K} b_i^n} - \frac{\Theta \eta_k^n}{g_k^n} \tag{A4}
$$

From Equations A3 and A4, we notice that the PA strategy can be modeled using the waterfilling algorithm [3]. Let $\mu_k(t+1) = (\mu_k(t) - \xi(t)D_k(t))^+$ be the subgradient update methods where $\xi(t) > 0$ are the update steps while $D_k(t)$ is

$$D_k(t) = P_k^{\max} - \sum_{n=1}^{N} p_k^n \qquad (A5)$$

We can decouple the PA from BA by fixing $\mathbf{b}_k$ while updating $\mathbf{p}_k$. Therefore, optimal PA can be obtained as

$$p_k^n = \left(\Delta_k - \frac{\Theta \eta_k^n}{g_k^n} \right)^+ \qquad (A6)$$

where $\Delta_k = b_k^n w \big/ \mu_k \sum_{i=1}^{K} b_i^n$ is the waterfilling level of the kth user. Specifically, for N number of subcarriers, Δ_k can be obtained as [21]

$$\Delta_n = \frac{1}{N} \left(P_k^{\max} - \sum_{n=1}^{N} \frac{\eta_k^n}{g_k^n} \right) \qquad (A7)$$

From Equation A6, it is noticed that the PA and BA can be decoupled and thus the BA strategy can be viewed as a one-dimensional maximization problem with $\mathbf{p}_k$ fixed. Taking the partial derivatives of Equation A1 with respect to b_k^n, α_k, $\forall n \in \mathcal{N}$ gives

$$\frac{\partial \mathcal{L}}{\partial b_n^k} = \left(\frac{\sum_{i=1,i\neq n}^{N} b_i^k}{\left(\sum_{i=1}^{M} b_i^k \right)^2} \right) w \log_2 \left(1 + \frac{p_n^k g_n^k}{\Theta I_n^k} \right) - \alpha_n, \quad \forall k \in \mathcal{K} \ (A8)$$

$$\frac{\partial \mathcal{L}}{\partial \alpha_k} = B_k^{\max} - \sum_{n=1}^{N} b_k^n \qquad (A9)$$

For simplicity, let $r_k^n = w \log_2 \left(1 + p_k^n g_k^n / \Theta \eta_k^n \right)$ and $\partial \mathcal{L} / \partial b_k^n = 0$, we have

$$\left(\frac{\sum_{i=1,i\neq k}^{K} b_i^1}{\left(\sum_{i=1}^{K} b_i^1 \right)^2} \right) r_k^1 = \left(\frac{\sum_{i=1,i\neq k}^{K} b_i^2}{\left(\sum_{i=1}^{K} b_i^2 \right)^2} \right) r_k^2 = \cdots = \left(\frac{\sum_{i=1,i\neq k}^{K} b_i^N}{\left(\sum_{i=1}^{K} b_i^N \right)^2} \right) r_k^N = \alpha_k (A10)$$

Without loss of generality, Equation A10 can be rearranged and represented using a general expression, i.e.,

$$\left(B^n \right)^2 r_k^m \sum_{i=1,i\neq k}^{K} b_i^m = \left(B^m \right)^2 r_k^n \sum_{i=1,i\neq k}^{K} b_i^n, \quad \forall m, n \in \mathcal{N} \ (A11)$$

where $B^n = \sum_{i=1}^{K} b_i^n$ is the total budget allocated by all users on the nth subcarrier. By rearranging Equation A11, B^m is denoted as

$$B^m = \frac{B^n \sqrt{r_k^m \sum_{i=1,i\neq k}^{K} b_i^m}}{\sqrt{r_k^n \sum_{i=1,i\neq k}^{K} b_i^n}}, \quad \forall m, \quad n \in \mathcal{N} \qquad (A12)$$

Note that

$$B = \sum_{m=1}^{N} B^m = \sum_{i=1}^{K} B_i \qquad (A13)$$

where B_i is the total budget spent by the ith user and B is the total budget spent by all users on all subcarriers. Substituting Equation A12 into Equation A13 yields

$$B = \sum_{m=1}^{N} B^m = \frac{B^n \sum_{m=1}^{N} \sqrt{r_k^m \sum_{i=1,i\neq k}^{K} b_i^m}}{\sqrt{r_k^n \sum_{i=1,i\neq k}^{K} b_i^n}} \qquad (A14)$$

By rearranging Equation A14, we have

$$B^n = \frac{\sqrt{r_k^n \sum_{i=1,i\neq k}^{K} b_i^n}}{\sum_{m=1}^{N} \sqrt{r_k^m \sum_{i=1,i\neq k}^{K} b_i^m}} B = C_k^n B \qquad (A15)$$

The ratio of Equation A10 for the jth user over that of the kth user gives

$$\frac{r_j^1 \sum_{i=1,i\neq j}^{K} b_i^1}{r_k^1 \sum_{i=1,i\neq k}^{K} b_i^1} = \frac{r_j^2 \sum_{i=1,i\neq j}^{K} b_i^2}{r_k^2 \sum_{i=1,i\neq k}^{K} b_i^2} = \cdots = \frac{r_j^N \sum_{i=1,i\neq j}^{K} b_i^N}{r_k^N \sum_{i=1,i\neq k}^{K} b_i^N} = \frac{\alpha_j}{\alpha_k} (A16)$$

Equation A16 can be rewritten in a general expression as follows

$$\left(B^n - b_j^n \right) r_j^n = \frac{\alpha_j}{\alpha_k} \left(B^n - b_k^n \right) r_k^n, \ \forall n \in \mathcal{N} \qquad (A17)$$

Note that

$$\sum_{i=1,i\neq j}^{K} B_i = \sum_{n=1}^{N} \sum_{i=1,i\neq j}^{K} b_i^n = \frac{\alpha_j r_k^n}{\alpha_k r_j^n} \sum_{i=1,i\neq k}^{K} B_i \qquad (A18)$$

Therefore,

$$\frac{\alpha_j}{\alpha_k} = \frac{r_j^n \sum_{i=1,i\neq j}^{K} B_i}{r_k^n \sum_{i=1,i\neq k}^{K} B_i} = \frac{r_j^n \left(B - B_j \right)}{r_k^n \left(B - B_k \right)} \qquad (A19)$$

Substituting Equations A15 and A19 into Equation A17 produces

$$\left(C_k^n B - b_j^n \right) = \frac{(B - B_j)}{(B - B_k)} \left(C_k^n B - b_k^n \right) \qquad (A20)$$

Rearranging Equation A20 gives

$$b_j^n = \left(\frac{B_j - B_k}{B - B_k} \right) C_k^n B + \left(\frac{B - B_j}{B - B_k} \right) b_k^n \qquad (A21)$$

Note that

$$B^n = \sum_{j=1}^{K} b_j^n = \sum_{j=1}^{K} \left(\left(\frac{B_j - B_k}{B - B_k} \right) C_k^n B + \left(\frac{B - B_j}{B - B_k} \right) b_k^n \right)$$
$$= \frac{1}{B - B_k} \left(C_k^n B \sum_{j=1}^{K} (B_j - B_k) + b_k^n B (K - 1) \right) \qquad (A22)$$

Rearranging Equation A20, we obtain b_k^n as

$$b_k^n = \frac{B^n}{(K-1)B} \left((B - B_k) - \sum_{j=1}^{K} (B_j - B_k) \right)$$
$$= \frac{B^n B_k}{B} = C_k^n B_k \qquad (A23)$$

If we assume that the total available budget will be fully spent, then $B_k = B_k^{\max}$ and Equation A23 becomes

$$b_k^n = \frac{B_k^{\max} \sqrt{r_k^n \sum_{i=1, i \neq k}^{K} b_i^n}}{\sum_{m=1}^{N} \sqrt{r_k^m \sum_{i=1, i \neq k}^{K} b_i^m}}, \quad \forall n \in \mathcal{N} \qquad (A24)$$

The proposed BA strategy is thus obtained.

Abbreviations

BA: budget allocation; BS: base station; CSI: channel state information; NBS: Nash bargaining solution; NE: Nash equilibrium; OFDMA: rthogonal frequency-division multiple access; PA: power allocation; SA,subcarrier allocation; SABG: subcarrier allocation scheme based on a Blotto game.

Competing interests

The authors declare that they have no competing interests.

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