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Enhancement of wireless positioning in outdoor suburban NLOS environment using hybrid-network-GPS systems

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Abstract

This article will introduce a method for locating mobile stations (MSs) in outdoor suburban non-line-of-sight (NLOS) environment. The measurements used to locate the MS are taken from three base stations and a satellite. Such a setup of measurements is named the hybrid-network-GPS system. The proposed method uses constraint nonlinear optimization to minimize the NLOS error. The problem is simplified to three independent nonlinear equations of three unknowns, then it is solved to find the MS location. Numerical simulations are introduced to assess the performance of the proposed method compared with other positioning algorithms.

Keywords: NLOS, GPS, TOA, constrained minimization

1. Introduction

The well-known non-line-of-sight (NLOS) problem in wireless location has gained a great attention in the last decade. An important aspect of this problem is the huge error it induces in locating wireless devices. Thus, it is of interest to develop wireless positioning algorithms that will minimize the NLOS error.

In the literature, many articles addressed the NLOS problem. These articles differed in their approaches to solve the problem. The articles in [1-6] used nonstatistical methods to locate the mobile station (MS) using network based measurements. In [1], the authors propose using linear-programming method to locate the MS in NLOS environments. The authors in [2], linearize the inequality of range models corrupted with NLOS errors for wireless positioning. In [3], the authors propose a constrained-optimization algorithm to locate the MS using the sequential quadratic programming algorithm. The authors in [4] propose a geometry-assisted location estimation algorithm utilizing the different geometric layouts between the MS and the base stations (BSs). In [5], the authors propose a constrained optimization technique to locate the MS. The method in [6] is a constrained nonlinear optimization approach, with constraints derived from the

geometry of the cell layout and range measurements. Satellite assisted techniques were also proposed in [7,8] for wireless positioning but they did not assume NLOS environment and measurements were taken at more than one satellite.

This article uses the hybrid-network-GPS system to minimize the NLOS error when locating the MS. The proposed method is named the hybrid-network-GPS constrained algorithm (HNGPS-CA) which depends on minimizing a constraint objective function to locate the MS. The proposed objective function used in the HNGPS-CA method has the advantage of guaranteed convexity. This advantage is not guaranteed in the regularly used least square (LS) objective function which might be non-convex in some cases as will be clarified in Section 4. The environment considered in this article is the outdoor suburban NLOS environment. We will consider that there are three time of arrival (TOA) measurements available from three BSs and only one TOA measurement from the satellite.

The rest of the article is organized as follows: Section 2 presents the problem formulation. Section 3 presents the HNGPS-CA algorithm. In Section 4, some insight on the convexity of the objective function will be provided. Section 5 shows simulation results for the proposed method. Conclusions are presented in Section 6.

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2. Problem formulation

Assume that there is a transmitting MS (location $\psi_m = (x_{ms}, y_{ms}, 0)$) with the TOA measurements taken at the receiving BS_i (location $\psi_i = (x_i, y_i, 0)$, where $i = 1, 2, 3$ which is an index for the BS) and at one GPS satellite (location $\psi_s = (x_s, y_s, z_s)$). Then, the true distance between the MS and BS_i is

$$R_i = ||\psi_m - \psi_i|| = \sqrt{(x_{ms} - x_i)^2 + (y_{ms} - y_i)^2} \quad \text{for } i = 1, 2, 3 \quad (1)$$

The true distance between the MS and the GPS satellite is

$$R_s = ||\psi_m - \psi_s|| = \sqrt{(x_{ms} - x_s)^2 + (y_{ms} - y_s)^2 + (0 - z_s)^2}. \quad (2)$$

A layout of the MS, multiple BSs and satellite geometry is shown in Figure 1.

Since the consideration for this article is the outdoor suburban environment, then it is reasonable to assume that it is highly unlikely that there will be obstacles between the MS and the GPS satellite in such an environment. Thus, the measured distance between the MS and the GPS satellite ($\tilde{R}_s$) will be assumed to be the same as the true distance (R_s) added to it the measurement noise (μ_s), i.e., $\tilde{R}_s = R_s + \mu_s$. The measurement

noise (μ_s) is assumed to be additive white Gaussian noise (AWGN) of zero mean and variance of σ_s^2.

This assumption is not valid for the connection between the MS and the BSs where it is very likely that obstacles exist in the MS-BSs connections. Thus, we will consider that the distance measurements at the BSs will have NLOS error (unlike the measurement made at the MS-GPS connection).

The measured distance at BS_i (between BS_i and MS) would be

$$l_i = R_i + \eta_i + \mu_i \quad (3)$$

where η_i is the effective NLOS component and μ_i is the measurement noise, which can be considered as AWGN of zero mean and variance of σ_i^2. Since the NLOS causes the signal to arrive from a path which is longer than the true distance, then $\eta_i \geq 0$. It is generally considered that the NLOS error (η_i) is usually much more severe than the measurement noise (μ_i).

Another distance measurement is obtained by calculating the time the wave takes to go from the GPS satellite-to-MS-to-BS_i which we will call ($\bar{l}_i$), as follows:

$$\bar{l}_i = l_i + \tilde{R}_s. \quad (4)$$

This distance can be practically obtained by assuming the MS will receive the signal from the satellite and then transmits the TOA measurement to BS_i or retransmits the same signal and the BS_i computes the total distance ($\bar{l}_i$).

Since the GPS satellite is located far from the earth surface (i.e., R_s is a huge distance when compared to cells dimensions), then ($\bar{l}_i$) is much greater than l_i (i.e., $\bar{l}_i >> l_i$).

Next, the HNGPS-CA method is presented to locate the MS accurately in the presence of the NLOS error.

3. Hybrid-network-GPS constrained algorithm

The HNGPS-CA method is based on minimizing a constraint objective function to estimate the MS location. The objective function is defined as the squared summation of the differences between ($\bar{l}_i$) and R_i (for $i = 1, 2, 3$). In other words, the objective function that will be minimized is

$$F_{HNGPS-CA}(x_{ms}, y_{ms}) = \sum_{i=1}^{3} \alpha_i(\bar{l}_i - R_i)^2 \quad (5)$$

where α_i (for $i = 1; 2; 3$) are positive weighting factors for the objective function (i.e., $\alpha_i > 0$). Each positive weighting factor (α_i) can be chosen separately, depending on the measurement accuracy [5] or the

Figure 1 A geometry of the satellite, MS and BSs layout.
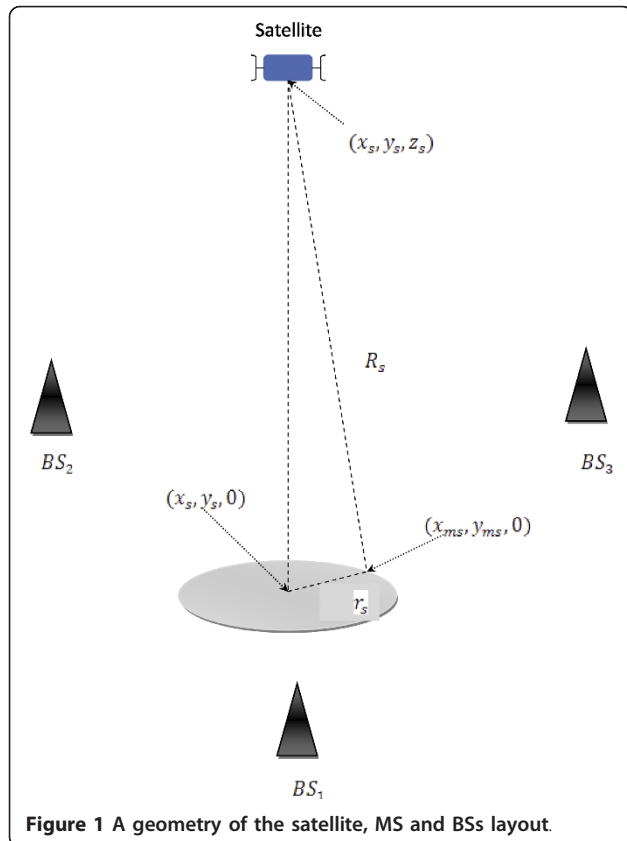
measurement geometry [6]. Recall that R_i is function of x_{ms} and y_{ms} as shown in (1), and they are the variables over which $F_{HNGPS-CA}(x_{ms}, y_{ms})$ in (5) is minimized.

The reason for choosing such an objective function rather than the regular LS objective function given by

$$F_{LS}(x_{ms}, y_{ms}) = \sum_{i=1}^{3} \alpha_i (l_i - R_i)^2 \qquad (6)$$

is the fact that using the objective function in (5) is assured to be convex. Whereas, the one in (6) might be non-convex in some cases as will be explained in Section 4.

Now, let us consider the distance R_s which represents the distance between the MS and the GPS satellite (which is very close to its measured value $\tilde{R}_s$ if we neglect the measured noise μ_s remembering that $R_s >> \mu_s$). The locus of R_s will be a circle on the earth plane (the x-y plane) as shown in Figure 1. This circle is centered at (x_s, y_s) with radius $r_s = \sqrt{(R_s^2 - z_s^2)}$. Thus, the MS estimated coordinates must be located on this circle, i.e.,

$$(x_{ms} - x_s)^2 + (y_{ms} - y_s)^2 = r_s^2. \qquad (7)$$

Thus, (7) provides an equality constraint on finding (x_{ms}, y_{ms}) when applying the HNGPS-CA.

As mentioned earlier, we intend to estimate the MS location by minimizing the objective function in (5) taking into consideration the equality constraint in (7). This constraint minimization can be done using the Lagrange multipliers as follows:

$$\zeta(x_{ms}, y_{ms}, \lambda) = \sum_{i=1}^{3} \alpha_i \left(\bar{l}_i - R_i\right)^2 + \lambda\left((x_{ms} - x_s)^2 + (y_{ms} - y_s)^2 - r_s^2\right). \qquad (8)$$

where λ is the Lagrange multiplier.

To find the minimum of $\zeta(x_{ms}, y_{ms}, \lambda)$, we take its derivative with respect to the variables upon which $\zeta(x_{ms}, y_{ms}, \lambda)$ depends, i.e., $(x_{ms}, y_{ms}, \lambda)$ separately and equate each result to zero,

$$\frac{\partial \zeta(x_{ms}, y_{ms}, \lambda)}{\partial x_{ms}} = -\sum_{i=1}^{3} \alpha_i \frac{\left(\bar{l}_i - R_i\right)(x_{ms} - x_s)}{R_i} + \lambda(x_{ms} - x_s) = 0, \qquad (9)$$

$$\frac{\partial \zeta(x_{ms}, y_{ms}, \lambda)}{\partial y_{ms}} = -\sum_{i=1}^{3} \alpha_i \frac{\left(\bar{l}_i - R_i\right)(y_{ms} - y_s)}{R_i} + \lambda(y_{ms} - y_s) = 0 \qquad (10)$$

and

$$\frac{\partial \zeta(x_{ms}, y_{ms}, \lambda)}{\partial \lambda} = (x_{ms} - x_s)^2 + (y_{ms} - y_s)^2 - r_s^2 = 0. \qquad (11)$$

Thus, (9), (10), and (11) provide three independent nonlinear equations with three unknowns $(x_{ms}, y_{ms}, \lambda)$. These equations can be solved to find the unknowns especially the MS location coordinates (x_{ms}, y_{ms}). We will use numerical techniques to solve these equations. Here, we use fsolve.m in MATLAB to solve the set of nonlinear equations in (9), (10), and (11).

4. Convexity of the objective functions

As shown in Section 3, we use the objective function $F_{HNGPS-CA}(x_{ms}, y_{ms})$ in (5) rather than the LS obcetive function $(F_{LS}(x_{ms}, y_{ms}))$ shown in (6). To verify this choice, let us look at the objective functions $F_{HNGPS-CA}(x_{ms}, y_{ms})$ and $F_{LS}(x_{ms}, y_{ms})$.

For sake of simplicity, from now on we will drop the argument (x_{ms}, y_{ms}) in $F_{LS}(x_{ms}, y_{ms})$ and $F_{HNGPS-CA}(x_{ms}, y_{ms})$. Thus, we can write (6) as follows

$$F_{LS} = \sum_{i=1}^{3} \alpha_i f_i \qquad (12)$$

where

$$f_i = (l_i - R_i)^2.$$

From the definition of l_i in (3), the function f_i can be simplified to

$$f_i = (\eta_i + \mu_i)^2. \qquad (13)$$

Let us recall that we intend to minimize η_i which always has a nonnegative value, i.e., $\eta_i \geq 0$. Also, η_i is actually a distance and function of the estimated MS coordinates $(\hat{x}_{ms}, \hat{y}_{ms})$, where {^} represents the estimated value of the variable, i.e., for example $\hat{v}$ is the estimated value of v. Since η_i is a positive error which increases as the estimated MS coordinates $(\hat{x}_{ms}, \hat{y}_{ms})$ get further from the actual MS coordinates (x_{ms}, y_{ms}), then η_i is a convex surface which has a minima (zero value) at (x_{ms}, y_{ms}). Now, because μ_i is a noise signal it can have both negative and positive values. If it has a positive value then the surface $\eta_i + \mu_i$ has a value which is greater than zero, i.e., $\eta_i + \mu_i > 0$ and has a minima at (x_{ms}, y_{ms}). Thus, $f_i = (\eta_i + \mu_i)^2$ will be convex in this case and F_{LS} which is the summation of positively weighted convex functions (f_i) for $i = 1, 2, 3$ will be convex in this case as well [9]. Whereas, if μ_i is negative, i.e., $\mu_i < 0$, then parts of the surface $\eta_i + \mu_i$ will be below zero and other parts will be above zero with the minimum located at (x_{ms}, y_{ms}) to be below zero. When taking the square of $\eta_i + \mu_i$, i.e., $(\eta_i + \mu_i)^2$, then, the point (x_{ms}, y_{ms}) will have a local maxima and the surface surrounding it will be decreasing until it hits the zero level. Then, the surface $(\eta_i + \mu_i)^2$ is reflected back to the increasing mode as we get further from the point (x_{ms}, y_{ms}), causing $(\eta_i + \mu_i)^2$ to be a non-convex surface, i.e., f_i will be non-convex in this case, causing F_{LS} which is the summation of positively

weighted non-convex functions (f_i) for i = 1, 2, 3 will be non-convex in this case [9]. This is illustrated in Figure 2. Thus, when minimizing F_{LS} we will reach an unwanted local minimum which is not located at (x_{ms}, y_{ms}). So, there will be an error in locating the MS.

Similarly, $F_{HNGPS-CA}$ in (5) can be written as follows

$$F_{HNGPS-CA} = \sum_{i=1}^{3} \alpha_i g_i \qquad (14)$$

where

$$g_i = (\bar{l}_i - R_i)^2.$$

From the definition of ($\bar{l}_i$) in (4), the function g_i can be simplified to

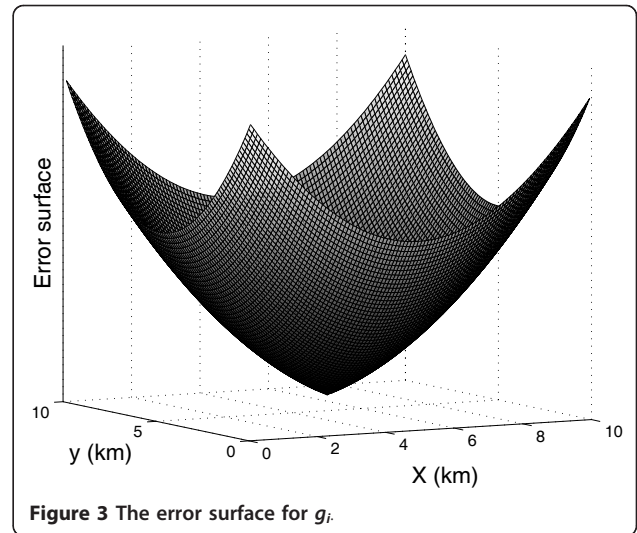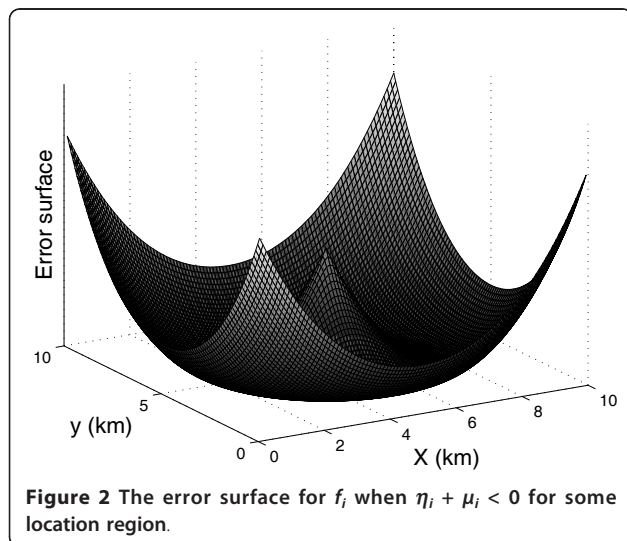$$g_i = (\eta_i + \mu_i + R_s + \mu_s)^2. \qquad (15)$$

Let us introduce the variable h_i which is defined as

$$h_i = \mu_i + R_s + \mu_s.$$

Thus,

$$g_i = (\eta_i + h_i)^2.$$

Because R_s is the distance between the MS and the satellite which is much greater than zero, i.e., $R_s \gg 0$, and because $R_s \gg \mu_i$ and $R_s \gg \mu_s$, then $h_i \gg 0$ for all realistic values of μ_i and μ_s. Thus, the function g_i is the square of the summation of a positive surface (η_i) which has a minimum value of zero at (x_{ms}, y_{ms}) and a positive variable (h_i). So, g_i is always a convex function and has a minimum value at (x_{ms}, y_{ms}). This is illustrated in Figure 3. Thus, $F_{HNGPS-CA}$ is convex as well because the positively weighted sum of convex functions is also convex
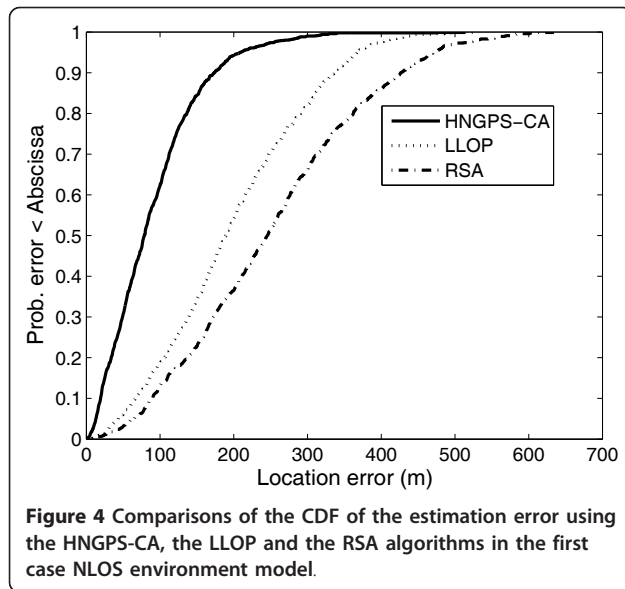

Figure 2 The error surface for f_i when $\eta_i + \mu_i$ < 0 for some location region.

Figure 3 The error surface for g_i.

[9] (recall that $F_{HNGPS-CA} = \Sigma_{i=1}^{3} \alpha_i g_i$). This will assure that when minimizing $F_{HNGPS-CA}$ we will not reach an unwanted local minimum as the case for minimizing F_{LS}, but rather we will reach the true MS location (x_{ms}, y_{ms}).
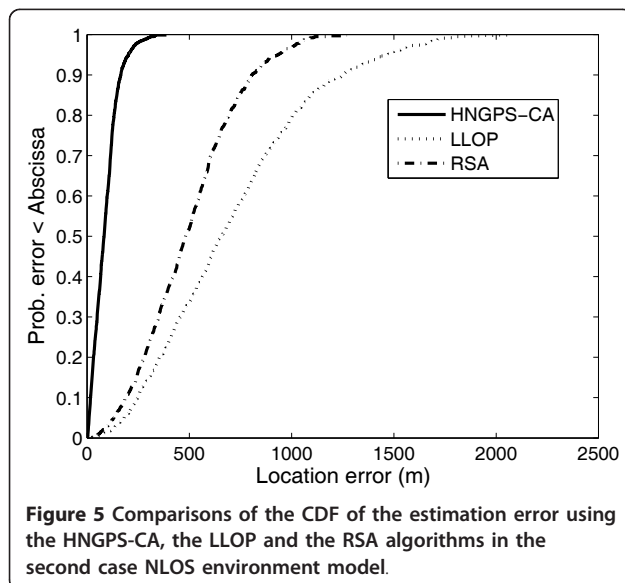
5. Simulation results

The results in this section are averaged over 2,000 ensemble runs. The BSs locations are (0, 0), (8.66, 0) and (4.33, 7.5) with all units in km. The MS location is (x_{ms}, y_{ms}) where x_{ms} = 4.33 · (u + 0.5) in km, y_{ms} = 0.5 · (7.5 + u) in km and u is a random variable uniformly distributed in the region [0,1]. Two NLOS environment models are assumed. The first case considered to model the NLOS effect is the disk of scatterers (DOS) environments presented in [10]. In the DOS model, the scatterers are located on a solid circular disk of fixed radius R_d with the MS at the center. The distance to a scatterer from the MS, r_{DOS}, is uniformly distributed over [0, R_d], and the angle is uniformly distributed over [0, 2π]. The radius R_d is set to 300 m. In the second case we assume that the NLOS error is a ratio of the measured distance. The reasoning behind this assumption is the fact that as the wave travels through a larger distance there will be a higher probability that the NLOS error will be larger. Thus, in this case $\eta_i = \beta l_i$ where β is the NLOS error factor and it is assumed to be 0.3 in the simulations. The range measurement error is assumed to have 50 m^2 variance, i.e, σ_s^2 = 50 m^2. The weight factors (α_1, α_2, α_3) in (9) and (10) are all set to 1.

The HNGPS-CA algorithm is compared with the range scaling algorithm (RSA) in [6] and the linear line of positions (LLOP) algorithm in [11]. Figures 4 and 5 show the error cumulative distribution function (CDF) of the different algorithms for the first and second NLOS environment cases, respectively. To show what

Figure 4 Comparisons of the CDF of the estimation error using the HNGPS-CA, the LLOP and the RSA algorithms in the first case NLOS environment model.

these CDFs mean in both figures, let us consider Figure 4 for example. The CDF curve in Figure 4 for the HNGPS-CA at the 100 m location error at the x-axis gives 0.59 at the y-axis which means that 59% of the time the error is below 100 m, whereas, it was 18 and 12% for the RSA and LLOP algorithms, respectively, for the same location error (100 m). As for the location error of 200 m, the HNGPS-CA, RSA and LLOP algorithms had an error of less than 200 m with a probability of 95, 50 and 34%, respectively. Thus, clearly the HNGPS-CA is giving better performance than the RSA and LLOP algorithms. The same conclusion can be achieved by looking at Figure 5. Thus, both figures indicate that the HNGPS-CA algorithm performs considerably better than both the LLOP and RSA algorithms.

Figure 5 Comparisons of the CDF of the estimation error using the HNGPS-CA, the LLOP and the RSA algorithms in the second case NLOS environment model.

6. Conclusion

An algorithm, named HNGPS-CA, is proposed and investigated via simulations for mitigating the effect of NLOS error in outdoor suburban NLOS environment. The HNGPS-CA algorithm locates the MS by minimizing an objective function formed from TOA measurements at three BSs and utilizes the equality constraint provided by the GPS TOA measurement. This proposed objective function of the HNGP-CA algorithm has the guaranteed convexity advantage over the regularly used LS objective function. Simulations results showed that the proposed HNGPS-CA algorithm gave better performance than other positioning algorithms.

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