Phan et al. *EURASIP Journal on Wireless Communications and Networking* 2012, **2012**:11
http://jwcn.eurasipjournals.com/content/2012/1/11

EURASIP Journal on Wireless Communications and Networking
a SpringerOpen Journal

RESEARCH **Open Access**

Adaptive transmission in MIMO AF relay networks with orthogonal space-time block codes over Nakagami-*m* fading

Hoc Phan[1*], Trung Q Duong[1], Hans-Jürgen Zepernick[1] and Lei Shu[2]

Abstract

In this article, we apply different adaptive transmission techniques to dual-hop multiple-input multiple-output amplify-and-forward relay networks using orthogonal space-time block coding over independent Nakagami-*m* fading channels. The adaptive techniques investigated are optimal simultaneous power and rate (OSPR), optimal rate with constant power (ORCP), and truncated channel inversion with fixed rate (TCIFR). The expressions for the channel capacity of OSPR, ORCP, and TCIFR, and the outage probability of OSPR, and TCIFR are derived based on the characteristic function of the reciprocal of the instantaneous signal-to-noise ratio (SNR) at the destination. For sufficiently high SNR, the channel capacity of ORCP asymptotically converges to OSPR while OSPR and ORCP achieve higher channel capacity compared to TCIFR. Although TCIFR suffers from an increase in the outage probability relative to OSPR, it provides the lowest implementation complexity among the considered schemes. Along with analytical results, we further adopt Monte Carlo simulations to validate the theoretical analysis.

Keywords: adaptive transmission; amplify-and-forward; orthogonal space-time block coding; optimal simultaneous power and rate; optimal power with constant rate; truncated channel inversion with fixed rate.

1 Introduction

During the last decade, multiple-input multiple-output (MIMO) techniques have attracted great attention as a way of improving spectral efficiency and reliability in wireless communications. MIMO systems with orthogonal space-time block coding (OSTBC) transmission are considered as a means of providing full diversity gain and linear decoding complexity [1–3]. In recent years, the combination of MIMO systems using OSTBC transmission with relay networks has been significantly considered (see, e.g., [4–9] and the references therein). In [4] and [5], the outage probability and symbol error rate (SER) performance of MIMO decode-and-forward (DF) relay networks with OSTBC transmission over Rayleigh fading channels were investigated, respectively. In [6], the SER of MIMO systems in which the source employs OSTBC transmission to transmit the signal to the destination through the help of semi-blind amplify-and-forward (AF) relays over Rayleigh fading channels was derived. In [7], the authors investigated the bit error rate (BER) performance of MIMO channel state information (CSI)-assisted AF relay networks with OSTBC transmission over Rayleigh fading channels. More recently, by taking the direct link between source and destination into account, the SER and outage probability of MIMO CSI-assisted AF relay cooperative networks with OSTBC transmission over Rayleigh fading channels were investigated [8]. Furthermore, closed-form expressions for the outage probability and the SER of dual-hop MIMO CSI-assisted AF relay networks with OSTBC transmission have been derived for independent and correlated Nakagami-*m* fading channels in [9, 10], respectively.

Although MIMO relay networks with OSTBC transmission have received a lot of research efforts, all of the above mentioned contributions concentrate on cooperative communications with constant transmission rate and power. For adaptive transmission, depending on the link quality provided by the fading channels, the system will adapt its transmission power, transmission rate, coding rate/scheme, modulation scheme, or the arbitrary com-

*Correspondence: hoc.phan@bth.se
[1]Blekinge Institute of Technology, SE-371 79 Karlskrona, Sweden
Full list of author information is available at the end of the article

bination of these techniques to the fluctuations induced by the fading channels in order to enhance the spectral efficiency [11–17]. In particular, a cooperative relay network where the source employs constant transmission power and adapts transmission rate through M-ary quadrature amplitude modulation (M-QAM) has been studied in terms of outage probability, SER, and spectral efficiency for Rayleigh fading channels [13]. The combination of opportunistic incremental relaying with adaptive modulation deployed in cooperative relay networks was analyzed in [14]. This scheme has been shown to guarantee a specific BER performance level and improve both spectral efficiency and outage probability. In [15], approximations for the channel capacity of opportunistic cooperative multiple relay networks over Rayleigh fading channels under optimal simultaneous power and rate (OSPR), optimal rate with constant power (ORCP), and truncated channel inversion with fixed rate (TCIFR) were investigated. The upper bounds of channel capacity for AF cooperative systems over Rayleigh fading channels under adaptive transmission were derived in [16]. Furthermore, the use of different adaptive schemes in AF multi-hop relaying networks over Nakagami-m fading environments was studied in [17] wherein the achievable channel capacity was evaluated by using the characteristic function (CHF) of the reciprocal of the instantaneous SNR at the destination. In [18], the Shannon channel capacity of the maximum ratio combining (MRC) receiver over η-μ fading channels and adaptive transmission has been investigated. Upper bounds for the capacity of different adaptive transmission techniques for an AF system with best relay selection over Rayleigh fading channels have been reported in [19]. The study of [20] has presented a framework for practical application of adaptive transmission to MIMO systems to enhance the transmission rate of broadband wireless systems.

Most efforts that have been made for the utilization of adaptive transmission focus on traditional cooperative relay networks over Rayleigh or Nakagami-m fading channels. To the best of the authors' knowledge, there is no previous study considering adaptive transmission for MIMO AF relay networks with OSTBC transmission. Although OSTBC transmission provides diversity gain and decoding simplicity, it decreases transmission rate as compared to other kinds of space-time block codes (STBC). Therefore, it is beneficial to employ adaptive transmission schemes for MIMO AF relay networks with OSTBC transmission in order to enhance its spectral efficiency. In this article, we therefore analyze the performance of these systems over independent, identically distributed (i.i.d.) and independent, non-identically distributed (i.n.i.d.) Nakagami-m fading channels. Our key contributions can be summarized as follows. We investigate the performance of MIMO CSI-assisted AF relay networks with OSTBC transmission for the adaptive schemes of OSPR, ORCP, and TCIFR over Nakagami-m fading channels. Specifically, we derive analytical expressions for the channel capacity of the three adaptive MIMO AF relay networks with OSTBC based on the CHF of the reciprocal of the instantaneous SNR at the destination. Furthermore, we present expressions of the outage probability for the considered cooperative relay networks with OSPR and TCIFR wherein transmission will be suspended as long as the instantaneous SNR falls below an optimal value. For the ORCP scheme, however, there is no need to evaluate the outage probability as the source constantly keeps transmission regardless of the value of the instantaneous SNR.

The remaining parts of this article are organized as follows. In Section 2, we present the system and channel model, describing fundamental concepts of MIMO CSI-assisted AF relay networks with OSTBC transmission. In Section 3, we derive the CHF of the reciprocal of the instantaneous SNR for the considered cooperative relay networks with OSTBC and adpative transmission. The derivation of the channel capacity and the outage probability of these cooperative relay networks are presented in Section 4. We then derive the channel capacity of the investigated systems with ORCP in Section 5. In Section 6, the channel capacity and the outage probability of the cooperative relay networks using TCIFR are derived. Analytical results and Monte Carlo simulations along with further discussions are presented in Section 7. Finally, in Section 8, conclusions of this study are given.

Notation: Throughout this article, we will use the following notations. The Frobenius norm of a vector or matrix is denoted as $\|\cdot\|_F$. The probability density function (PDF) and the cumulative distribution function (CDF) of a random variable (RV) X are denoted as $f_X(\cdot)$ and $F_X(\cdot)$, respectively. Then, $\Phi_X(\cdot)$ and $\Psi_X(\cdot)$ stand for the moment generating function (MGF) and the CHF of an RV X, respectively. In addition, $\mathcal{CN}(0, N_0)$ represents an additive white Gaussian noise (AWGN) RV with zero mean and variance N_0. We denote $\Gamma(n)$ as the gamma function [21, eq. (8.310.1)] and $\Gamma(n,x)$ as the incomplete gamma function [21, eq. (8.350.2)]. Furthermore, $Ei(\cdot)$ indicates the exponential integral function [22, eq. (6.9.2.25)], and $\mathcal{K}_n(\cdot)$ is the nth order modified Bessel function of the second kind [21, eq. (8.432.1)]. Finally, $\mathcal{R}\{\cdot\}$ is the real part of a complex expression.

2 System and channel model

In this article, we consider a dual-hop relay network with OSTBC transmission consisting of an n_1-antenna source S, a single-antenna relay R, and an n_2-antenna destination D (see Figure 1). Suppose that the direct communication link between S and D is not applicable due to severe shadowing. To achieve spatial diversity, the source utilizes OSTBC consisting of M symbols, $s_1, \ldots, s_M$, which are chosen from a particular modulation constellation. An OSTBC matrix

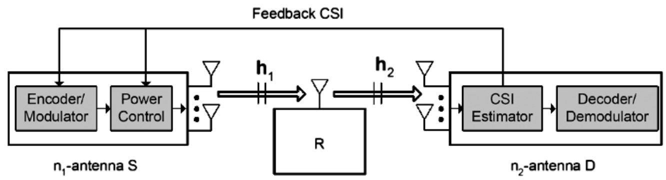

Figure 1 System model of the considered dual-hop MIMO AF relay network with OSTBC and adaptive transmission.

$\mathbf{C}$ has the size of $n_1 \times N$, where N is the block-length. The transmit power of each symbol is denoted as P_s and the code rate of an OSTBC is defined as $R_c = M/N$. In addition, all channels are assumed to experience mutually i.i.d. or i.n.i.d. quasi-static Nakagami-m fading. Accordingly, the fading coefficients are constant during a transmission block and change independently for every block. We also assume that the relay utilizes CSI-assisted AF mode to forward the signal to the destination. The transmission from source S to destination D stretches over two hops. During the first hop, source S transmits an OSTBC matrix $\mathbf{C}$ to relay R. The signal received at R is therefore given by

$$\mathbf{r} = \mathbf{h}_1\mathbf{C} + \mathbf{n}_R \tag{1}$$

where $\mathbf{h}_1 = [h_{1,1}, \ldots, h_{1,n_1}]$ is the row vector of the fading channel coefficients of the link from S to R and $\mathbf{n}_R = [n_{R,1}, \ldots, n_{R,N}]$ denotes the row vector of the complex AWGN at relay R. Furthermore, $h_{1,l}$, $l = 1, 2, \ldots, n_1$, represents the fading channel coefficient of the link from the lth antenna of S to R and $n_{R,t}$, $t = 1, 2, \ldots, N$, denotes the AWGN with zero mean and variance N_0 at R during the tth symbol period.

In the second hop, relay R first amplifies the signal received from source S with an amplifying gain β and then forwards the amplified signal to destination D. Accordingly, the signal received at destination D can be expressed as

$$\mathbf{Y} = \beta\mathbf{h}_2^T\mathbf{r} + \mathbf{W}_D \tag{2}$$

where $\mathbf{h}_2 = [h_{2,1}, \ldots, h_{2,n_2}]$ is the row vector of the fading channel coefficients from R to D and $\mathbf{W}_D$ is the $n_2 \times N$ AWGN matrix at D whose entries represent complex Gaussian RVs $\mathcal{CN}(0, N_0)$. In particular, $h_{2,l}$, $l = 1, 2, \ldots, n_2$, denotes the fading channel coefficient of the link from R to the lth antenna of D. The channel power gain $g_{i,l} = |h_{i,l}|^2$ follows the gamma distribution of which the PDF and the CDF, respectively, are given by

$$f_{g_{i,l}}(x) = \frac{\alpha_{i,l}^{m_{i,l}}}{\Gamma(m_{i,l})} x^{m_{i,l}-1} \exp(-\alpha_{i,l}x) \tag{3}$$

$$F_{g_{i,l}}(x) = 1 - \frac{\Gamma(m_{i,l}, \alpha_{i,l}x)}{\Gamma(m_{i,l})} \tag{4}$$

where $\alpha_{i,l} = \frac{m_{i,l}}{\Omega_{i,l}}$, $m_{i,l}$ denotes the fading severity parameter of the Nakagami-m channel, and $\Omega_{i,l} = E\{g_{i,l}\}$ is the average channel power, $i \in \{1, 2\}$, $l \in \{1, 2, \ldots, n_i\}$.

Assuming that CSI in terms of the channel coefficient vector $\mathbf{h}_1$ is perfectly known to relay R, the amplifying gain β can be derived based on the constraint that the transmit power of R for the second hop is set to be equal to the transmit power of S used in the first hop. In case of high SNR, β can be approximated as $\beta^2 = (\|\mathbf{h}_1\|_F^2)^{-1}$. Due to the orthogonality associated with OSTBCs, maximum-likelihood (ML) decoding at the destination reduces to symbol-wise decoding. This means that decoding can be performed on a symbol-by-symbol basis with the related instantaneous SNR per symbol at destination D given as [7]

$$\gamma = \rho \frac{\|\mathbf{h}_1\|_F^2 \|\mathbf{h}_2\|_F^2}{\|\mathbf{h}_1\|_F^2 + \|\mathbf{h}_2\|_F^2} \tag{5}$$

where $\rho = \frac{\rho_0}{n_1 R_c}$, and $\rho_0 = \frac{P_s}{N_0}$ represents the average SNR of the source. For the sake of brevity, let us denote $\gamma_1 = \|\mathbf{h}_1\|_F^2$, and $\gamma_2 = \|\mathbf{h}_2\|_F^2$. The instantaneous SNR in (5) can then be rewritten as

$$\gamma = \rho \frac{\gamma_1\gamma_2}{\gamma_1 + \gamma_2} \tag{6}$$

It is noted that the tractable form of the instantaneous SNR in (6), which is expressed as the harmonic mean of γ_1 and γ_2, is utilized for deriving the performance of dual-hop MIMO AF relay networks with OSTBC and adaptive transmission in the sequel.

3 Derivation of the CHF of the reciprocal of the instantaneous SNR

Generally, the PDF of the instantaneous SNR, γ, is required to quantify the channel capacity of the considered relay networks with adaptive transmission [12, 16]. However, due to the complex mathematical expression for the PDF, $f_\gamma(\gamma)$, rather intractable analytical expressions are obtained for the channel capacity with this approach. Hence, in this article, we utilize a treatment deployed in [17] that takes advantage of the harmonic mean form of the instantaneous SNR by using the CHF of its reciprocal. In view of

(6), to derive the CHF of $\gamma^{-1} = \rho^{-1}(\gamma_1^{-1} + \gamma_2^{-1})$, let us denote

$$X_i = \gamma_i^{-1}, \quad i = 1, 2 \tag{7}$$

$$Y = \gamma^{-1} = \rho^{-1}(X_1 + X_2) \tag{8}$$

The CHF of Y, $\Psi_Y(s)$, can be obtained from its corresponding MGF, $\Phi_Y(s)$, by substituting $s = -\jmath\omega$ where $\jmath = \sqrt{-1}$. Due to the fact that γ_i, $i = 1, 2$, are mutually independent, the CHF of Y can be obtained as

$$\Psi_Y(\omega) = \Phi_{X_1}\left(\frac{-\jmath\omega}{\rho}\right)\Phi_{X_2}\left(\frac{-\jmath\omega}{\rho}\right) \tag{9}$$

In addition, the PDF of Y can be represented via its corresponding CHF as follows:

$$f_Y(y) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\Psi_Y(\omega)\exp\left(-\jmath\omega y\right)d\omega \tag{10}$$

Depending on whether the fading channels are identically distributed, the obtained CHF of γ^{-1} has different forms, resulting in different performance expressions for each case. We now continue to derive the MGF and CHF of Y based on its respective PDF for i.i.d. and i.n.i.d. Nakagami-*m* fading channels.

3.1 MGF and CHF for i.i.d. Nakagami-*m* fading channels

In this case, the statistics of all channels within each hop are identical, given as $m_{i,1} = \cdots = m_{i,n_i} = m_i$ and $\alpha_{i,1} = \cdots = \alpha_{i,n_i} = \alpha_i$. To obtain the MGF and CHF of γ^{-1}, we require the PDF of γ_i, $i = 1, 2$, which is given by [23]

$$f_{\gamma_i}(\gamma) = \frac{\alpha_i^{n_i m_i}}{\Gamma(n_i m_i)}\gamma^{n_i m_i - 1}\exp(-\alpha_i\gamma) \tag{11}$$

Given the relationship, $f_{T^{-1}}(t) = t^{-2}f_T(t^{-1})$, for an RV T, the PDF of X_i is obtained as

$$f_{X_i}(x) = \frac{\alpha_1^{n_i m_i}}{\Gamma(n_i m_i)}\left(\frac{1}{x}\right)^{n_i m_i + 1}\exp\left(-\frac{\alpha_i}{x}\right) \tag{12}$$

The MGF of X_i, $i = 1, 2$, can be derived from the PDF given in (12) by using the transformation

$$\Phi_{X_i}(s) = \int_{-\infty}^{\infty} f_{X_i}(x)\exp(-sx)dx \tag{13}$$

and is obtained, in view of [21, eq. (3.471.9)], as

$$\Phi_{X_i}(s) = \frac{(2\alpha_i s)^{n_i m_i/2}}{\Gamma(n_i m_i)}\mathcal{K}_{n_i m_i}(2\sqrt{\alpha_i s}) \tag{14}$$

From (9) and (14), we finally obtain the CHF of the reciprocal of the instantaneous SNR for the considered relay networks in the presence of i.i.d. Nakagami-*m* fading channels as

$$\begin{aligned}\Psi_Y(\omega) = {} & \frac{4\alpha_1^{n_1 m_1/2}\alpha_2^{n_2 m_2/2}}{\Gamma(n_1 m_1)\Gamma(n_2 m_2)}\left(-\frac{\jmath\omega}{\rho}\right)^{(n_1 m_1 + n_2 m_2)/2} \\ & \times \mathcal{K}_{n_1 m_1}\left(2\sqrt{-\frac{\jmath\alpha_1\omega}{\rho}}\right) \\ & \times \mathcal{K}_{n_2 m_2}\left(2\sqrt{-\frac{\jmath\alpha_2\omega}{\rho}}\right)\end{aligned} \tag{15}$$

3.2 MGF and CHF for i.n.i.d. Nakagami-*m* fading channels

In practical systems, the antennas of the source and the destination are often located asymmetrically. This leads to non-identical fading statistics among the source-relay hop and the relay-destination hop. Therefore, in this article, we also consider i.n.i.d. Nakagami-*m* fading channels. For this case, we utilize [24, eq. (6)] in order to derive the PDF of γ_i and eventually obtain

$$f_{\gamma_i}(\gamma) = \sum_{l=1}^{n_i}\sum_{k=1}^{m_{i,l}}\frac{\eta_{i,l}(k)\alpha_{i,l}^k}{\Gamma(k)}\gamma^{k-1}\exp\left(-\alpha_{i,l}\gamma\right) \tag{16}$$

where the weighting coefficients $\eta_{i,l}(k)$ are determined from [24, eq. (7)] as

$$\begin{aligned}\eta_{i,l}(k) = {} & \sum_{t_1=k}^{m_{i,l}}\sum_{t_2=k}^{t_1}\cdots\sum_{t_{n_i-2}=k}^{t_{n_i-3}}\left[\frac{(-1)^{m_i - m_{i,l}}\prod_{r=1}^{n_i}(\alpha_{i,r})^{m_{i,r}}}{(\alpha_{i,l})^k}\right. \\ & \times \frac{(m_{i,l} + m_{i,1+U(1-l)} - t_1 - 1)!}{(m_{i,1+U(1-l)} - 1)!(m_{i,l} - t_1)!} \\ & \times (\alpha_{i,l} - \alpha_{i,1+U(1-l)})^{t_1 - m_{i,l} - m_{i,1+U(1-l)}} \\ & \times \frac{(t_{n_i-2} + m_{i,n_i-1+U(n_i-1-l)} - k - 1)!}{(m_{i,n_i-1+U(n_i-1-l)} - 1)!(t_{n_i-2} - k)!} \\ & \times (\alpha_{i,l} - \alpha_{i,n_i-1+U(n_i-1-l)})^{k - t_{n_i-2} - m_{i,n_i-1+U(n_i-1-l)}} \\ & \times \prod_{w=1}^{n_i-3}\frac{(t_w + m_{i,w+1+U(w+1-l)} - t_{w+1} - 1)!}{(m_{i,w+1+U(w+1-l)} - 1)!(t_w - t_{w+1})!} \\ & \left.\times (\alpha_{i,l} - \alpha_{i,w+1+U(w+1-l)})^{t_{w+1} - t_w - m_{i,w+1+U(w+1-l)}}\right]\end{aligned} \tag{17}$$

with $m_i = \sum_{l=1}^{n_i} m_{i,l}$ and $U(x)$ being the unit step function. Similarly to obtaining (12), the PDF of X_i can be written as

$$f_{X_i}(x) = \sum_{l=1}^{n_i}\sum_{k=1}^{m_{i,l}}\frac{\eta_{i,l}(k)\alpha_{i,l}^k}{\Gamma(k)}x^{-k-1}\exp\left(-\frac{\alpha_{i,l}}{x}\right) \tag{18}$$

Again, using [21, eq. (3.471.9)] with several manipulations, the MGF of X_i can be derived as

$$\Phi_{X_i}(s) = 2\sum_{l=1}^{n_i}\sum_{k=1}^{m_{i,l}} \frac{\eta_{i,l}(k)\alpha_{i,l}^{k/2}}{\Gamma(k)} s^{k/2}\mathcal{K}_k\left(2\sqrt{\alpha_{i,l}s}\right) \tag{19}$$

By substituting (19) into (9), the CHF of the reciprocal of the instantaneous SNR, $Y = \gamma^{-1}$, for i.n.i.d. Nakagami-m fading channels is written as

$$\begin{aligned}\Psi_Y(\omega) &= 4\sum_{l=1}^{n_1}\sum_{k=1}^{m_{1,l}}\sum_{p=1}^{n_2}\sum_{q=1}^{m_{2,l}} \frac{\eta_{1,l}(k)\eta_{2,p}(q)\alpha_{1,l}^{k/2}\alpha_{2,p}^{q/2}}{\Gamma(k)\Gamma(q)} \\ &\quad\times\left(\frac{-\jmath\omega}{\rho}\right)^{(k+q)/2}\mathcal{K}_k\left(2\sqrt{-\frac{\jmath\alpha_{1,l}\omega}{\rho}}\right) \\ &\quad\times\mathcal{K}_q\left(2\sqrt{-\frac{\jmath\alpha_{2,p}\omega}{\rho}}\right)\end{aligned} \tag{20}$$

In the following sections, we utilize the CHF of the reciprocal of the instantaneous SNR to analyze the performance of dual-hop MIMO AF relay networks using OSTBC with different adaptive transmission schemes. Specifically, the source will adapt its transmission rate and/or power to the variations of the channel coefficients. Moreover, for implementing adaptive transmission, it is required that the instantaneous SNR is perfectly measured at the destination and is then sent back to the source through a feedback channel. This feedback channel is assumed to be error free with negligible delay and therefore enables the source to timely perform the transmission rate and/or power adaptation.

Recall that the Shannon capacity of a fading channel determines the theoretical upper bound on the maximum transmission rate with an arbitrarily small error probability. Adaptive transmission has been known as a means of achieving this bound [12]. For dual-hop MIMO AF relay networks using OSTBC where adaptation is only implemented at the source, the channel capacity of different adaptive schemes is analyzed in the sequel.

4 Optimal simultaneous power and rate adaptation

In this section, we investigate the channel capacity and the outage probability of the considered MIMO AF relay network with OSTBC for the case of adaptive transmission with OSPR. Accordingly, in response to the instantaneous SNR fed back from the destination, the source will adapt its transmission power and transmission rate in an optimal way, i.e., maximizing the channel capacity subject to the average transmit power constraint. In order to conserve transmission power during deep fades, the transmission will be suspended under such channel conditions. This mode of operation remains as long as the instantaneous SNR, γ, that is fed back from the destination to the source, is below the optimal cutoff SNR, γ_0, for OSPR.

Specifically, given an average transmit power constraint, the channel capacity of a system with OSPR, C_{OSPR} (bits/second), is defined as [12]

$$C_{\mathrm{OSPR}} = \frac{B}{2}\int_{\gamma_0}^{\infty}\log_2\left(\frac{\gamma}{\gamma_0}\right)f_\gamma(\gamma)d\gamma \tag{21}$$

where B (Hz) denotes the channel bandwidth. Note that the factor 1/2 in (21) accounts for the fact that communication from source S to destination D through the help of relay R occupies two time slots. In order to solve the integral given in (21), we apply a change of variable as $t = \gamma^{-1}$ together with (10) and [21, eq. (8.212.1)], yielding

$$\begin{aligned}C_{\mathrm{OSPR}} &= \frac{B}{2}\int_0^{1/\gamma_0}\log_2\left(\frac{1}{\gamma_0 t}\right)f_Y(t)dt \\ &= \frac{B}{4\pi\ln 2}\int_{-\infty}^{\infty}\frac{\Psi_Y(\omega)}{\jmath\omega} \\ &\quad\times\left[C+\ln(\jmath\omega)-\ln(\gamma_0)+Ei\left(\frac{\jmath\omega}{\gamma_0}\right)\right]d\omega\end{aligned} \tag{22}$$

where $C = 0.577215$ is Euler's constant [21, p. xxxii]. Furthermore, making use of [17, eq. (9)], (22) can be refined as

$$\begin{aligned}C_{\mathrm{OSPR}} &= \frac{B}{2\pi\ln 2}\int_0^{\pi/2}\Re\left\{\frac{\Psi_Y(\tan(\theta))}{\jmath\tan(\theta)}\left[C+\ln(\jmath\tan(\theta))\right.\right. \\ &\quad\left.\left.-\ln(\gamma_0)+Ei\left(\frac{\jmath\tan(\theta)}{\gamma_0}\right)\right]\right\} \\ &\quad\times\sec^2(\theta)\,d\theta\end{aligned} \tag{23}$$

where $\sec(\cdot)$ denotes the secant function, i.e., $\sec(\cdot) = 1/\cos(\cdot)$. To the best of the authors' knowledge, no closed-form solution is available for this integration. However, we now can readily evaluate the performance of the considered relay network by simply using the numerical integration method as in [17].

As far as the optimal cutoff SNR, γ_0, for OSPR is concerned, it must satisfy the average transmit power constraint

$$J = \int_{\gamma_0}^{\infty}\left(\frac{1}{\gamma_0}-\frac{1}{\gamma}\right)f_\gamma(\gamma)d\gamma = 1 \tag{24}$$

The optimal cutoff SNR, γ_0, can be found by numerically solving (24) for γ_0. Again, utilizing the change of variable $t = \gamma^{-1}$ for the integral in (24) along with (10) and [17, eq. (9)], the average transmit power constraint is obtained as

$$\begin{aligned} J &= \int_0^{1/\gamma_0} \left(\frac{1}{\gamma_0} - t\right) f_Y(t)dt \\ &= \frac{1}{2\pi}\int_{-\infty}^{\infty} \Psi_Y(\omega)\left[\frac{\gamma_0 - \jmath\omega - \gamma_0 \exp(-\jmath\omega/\gamma_0)}{\gamma_0\omega^2}\right]d\omega \\ &= \frac{1}{\pi}\int_0^{\pi/2} \Re\left\{\Psi_Y(\tan(\theta))\right. \\ &\quad \times \left.\frac{\gamma_0 - \jmath\tan(\theta) - \gamma_0\exp(-\jmath\tan(\theta)/\gamma_0)}{\gamma_0\tan(\theta)^2}\right\} \\ &\quad \times \sec^2(\theta)d\theta = 1 \end{aligned} \tag{25}$$

As the transmission in OSPR is suspended for the duration of the instantaneous SNR, γ, being below the optimal cutoff SNR, γ_0, the related outage probability, $P_{\text{o,OSPR}}$, can be defined as the probability of the event $\gamma < \gamma_0$ and may hence be formulated as

$$P_{\text{o,OSPR}} = \int_0^{\gamma_0} f_\gamma(\gamma)d\gamma = 1 - \int_{\gamma_0}^{\infty} f_\gamma(\gamma)d\gamma \tag{26}$$

Also, by using [17, eq. (9)], the outage probability can be given by

$$\begin{aligned} P_{\text{o,OSPR}} = 1 - \frac{1}{\pi}\int_0^{\pi/2} \Re\left\{\frac{\Psi_Y(\tan(\theta))}{\jmath\tan(\theta)}\right. \\ \times \left.\left[1 - \exp\left(\frac{-\jmath\tan(\theta)}{\gamma_0}\right)\right]\right\}\sec^2(\theta)d\theta \end{aligned} \tag{27}$$

4.1 Channel capacity and outage probability for i.i.d. Nakagami-*m* fading channels

By substituting the CHF expression of Y from (15) into the general formulae of $C_{\text{o,OSPR}}$ given in (23), the analytical expression for the channel capacity of dual-hop MIMO AF relay networks with OSTBC transmission employing OSPR over i.i.d. Nakagami-m fading channels is obtained as

$$\begin{aligned} C_{\text{OSPR}} &= \frac{2B\alpha_1^{n_1m_1/2}\alpha_2^{n_2m_2/2}}{\pi\ln 2\,\Gamma(n_1m_1)\Gamma(n_2m_2)} \\ &\quad \times \int_0^{\pi/2}\Re\left\{\frac{1}{\jmath\tan(\theta)}\left[C + \ln(\jmath\tan(\theta))\right.\right. \\ &\quad - \left.\ln(\gamma_0) + Ei\left(\frac{\jmath\tan(\theta)}{\gamma_0}\right)\right] \\ &\quad \times \left(-\frac{\jmath\tan(\theta)}{\rho}\right)^{(n_1m_1+n_2m_2)/2} \\ &\quad \times \left.\prod_{i=1}^{2}\mathcal{K}_{n_im_i}\left(2\sqrt{-\frac{\jmath\alpha_i\tan(\theta)}{\rho}}\right)\right\}\sec^2(\theta)d\theta \end{aligned} \tag{28}$$

The average transmit power constraint for solving the optimal cutoff SNR, γ_0, in (28) is found by substituting (15) into (25) as

$$\begin{aligned} J &= \frac{4\alpha_1^{n_1m_1/2}\alpha_2^{n_2m_2/2}}{\pi\,\Gamma(n_1m_1)\Gamma(n_2m_2)} \\ &\quad \times \int_0^{\pi/2}\Re\left\{\frac{\gamma_0 - \jmath\tan(\theta) - \gamma_0\exp(-\jmath\tan(\theta)/\gamma_0)}{\gamma_0\tan(\theta)^2}\right. \\ &\quad \times \left(-\frac{\jmath\tan(\theta)}{\rho}\right)^{(n_1m_1+n_2m_2)/2} \\ &\quad \times \left.\prod_{i=1}^{2}\mathcal{K}_{n_im_i}\left(2\sqrt{-\frac{\jmath\alpha_i\tan(\theta)}{\rho}}\right)\right\} \\ &\quad \times \sec^2(\theta)d\theta = 1 \end{aligned} \tag{29}$$

Substituting (15) in (27), the expression for the outage probability is given by

$$\begin{aligned} P_{\text{o,OSPR}} &= 1 - \frac{4\alpha_1^{n_1m_1/2}\alpha_2^{n_2m_2/2}}{\pi\,\Gamma(n_1m_1)\Gamma(n_2m_2)} \\ &\quad \times \int_0^{\pi/2}\Re\left\{\frac{1 - \exp(-\jmath\tan(\theta)/\gamma_0)}{\jmath\tan(\theta)}\right. \\ &\quad \times \left(-\frac{\jmath\tan(\theta)}{\rho}\right)^{(n_1m_1+n_2m_2)/2} \\ &\quad \times \left.\prod_{i=1}^{2}\mathcal{K}_{n_im_i}\left(2\sqrt{-\frac{\jmath\alpha_i\tan(\theta)}{\rho}}\right)\right\} \\ &\quad \times \sec^2(\theta)d\theta \end{aligned} \tag{30}$$

4.2 Channel capacity and outage probability for i.n.i.d. Nakagami-*m* fading channels

By substituting (20) into (23), the expression for the channel capacity of dual-hop MIMO AF relay systems with OSTBC transmission using OSPR over i.n.i.d. Nakagami-m fading channels is given by

$$\begin{aligned} C_{\text{OSPR}} &= \frac{2B}{\pi\ln 2}\sum_{l=1}^{n_1}\sum_{k=1}^{m_{1,l}}\sum_{p=1}^{n_2}\sum_{q=1}^{m_{2,l}}\frac{\eta_{1,l}(k)\eta_{2,p}(q)\alpha_{1,l}^{k/2}\alpha_{2,p}^{q/2}}{\Gamma(k)\Gamma(q)} \\ &\quad \times \int_0^{\pi/2}\Re\left\{\frac{1}{\jmath\tan(\theta)}\left[C + \ln(\jmath\tan(\theta))\right.\right. \\ &\quad - \left.\ln(\gamma_0) + Ei\left(\frac{\jmath\tan(\theta)}{\gamma_0}\right)\right] \\ &\quad \times \left(\frac{-\jmath\tan(\theta)}{\rho}\right)^{(k+q)/2}\mathcal{K}_k\left(2\sqrt{-\frac{\jmath\alpha_{1,l}\tan(\theta)}{\rho}}\right) \end{aligned} \tag{31}$$

$$\times \mathcal{K}_q\left(2\sqrt{-\frac{\jmath\alpha_{2,p}\tan(\theta)}{\rho}}\right)\Bigg\}\sec^2(\theta)d\theta$$

The optimal cutoff SNR, γ_0, in (31) is the numerical solution of the respective average transmit power constraint, which is achieved by substituting (20) into (25), as

$$\begin{aligned} J &= \frac{4}{\pi}\sum_{l=1}^{n_1}\sum_{k=1}^{m_{1,l}}\sum_{p=1}^{n_2}\sum_{q=1}^{m_{2,l}}\frac{\eta_{1,l}(k)\eta_{2,p}(q)\alpha_{1,l}^{k/2}\alpha_{2,p}^{q/2}}{\Gamma(k)\Gamma(q)} \\ &\times \int_0^{\pi/2}\Re\left\{\left(\frac{-\jmath\tan(\theta)}{\rho}\right)^{(k+q)/2}\right. \\ &\times \frac{\gamma_0-\jmath\tan(\theta)-\gamma_0\exp\left(-\jmath\tan(\theta)/\gamma_0\right)}{\gamma_0\tan(\theta)^2} \\ &\times \mathcal{K}_k\left(2\sqrt{-\frac{\jmath\alpha_{1,l}\tan(\theta)}{\rho}}\right) \\ &\times \mathcal{K}_q\left(2\sqrt{-\frac{\jmath\alpha_{2,p}\tan(\theta)}{\rho}}\right)\Bigg\}\sec^2(\theta)d\theta = 1 \end{aligned} \quad (32)$$

Similarly to the case of i.i.d. Nakagami-m fading channels, by substituting (20) into (27), the expression for the outage probability is found as

$$\begin{aligned} P_{\mathrm{o,OSPR}} &= 1-\frac{4}{\pi}\sum_{l=1}^{n_1}\sum_{k=1}^{m_{1,l}}\sum_{p=1}^{n_2}\sum_{q=1}^{m_{2,l}}\frac{\eta_{1,l}(k)\eta_{2,p}(q)\alpha_{1,l}^{k/2}\alpha_{2,p}^{q/2}}{\Gamma(k)\Gamma(q)} \\ &\times \int_0^{\pi/2}\Re\left\{\frac{1-\exp\left(-\jmath\tan(\theta)/\gamma_0\right)}{\jmath\tan(\theta)}\right. \\ &\times \left(\frac{-\jmath\tan(\theta)}{\rho}\right)^{(k+q)/2} \\ &\times \mathcal{K}_k\left(2\sqrt{-\frac{\jmath\alpha_{1,l}\tan(\theta)}{\rho}}\right) \\ &\times \mathcal{K}_q\left(2\sqrt{-\frac{\jmath\alpha_{2,p}\tan(\theta)}{\rho}}\right)\Bigg\}\sec^2(\theta)d\theta \end{aligned} \quad (33)$$

5 Optimal rate adaptation with constant transmit power

In this section, we derive an analytical expression for the channel capacity of ORCP in the context of the considered relay networks. For the case of OSPR, the source only adapts its transmission rate in response to the channel states as $\frac{B}{2}\log_2(1+\gamma)$. The instantaneous SNR at the destination is also provided to the source through a feedback channel. Compared to the fixed rate systems, wherein the transmission rate of the source is designed in advance to operate efficiently in specific level of channel quality, the ORCP scheme can take advantage of the variations of fading channels because it constantly adapts the transmission rate to the channel condition [25]. In addition, as the source adapts only its transmission rate but not the power, the implementation of ORCP is less complex as compared to OSPR. Since the transmission remains at arbitrary value of the instantaneous SNR, no outage event occurs for ORCP scheme.

The channel capacity, C_{ORCP}, of ORCP can be calculated as [12]

$$C_{\mathrm{ORCP}} = \frac{B}{2}\int_0^\infty \log_2(1+\gamma)f_\gamma(\gamma)d\gamma \quad (34)$$

which basically represents the average channel capacity of a flat-fading channel with respect to the distribution of the instantaneous SNR at the destination.

As with OSPR, using the change of variable, $t=\gamma^{-1}$, the integral of C_{ORCP} can be expressed as

$$\begin{aligned} C_{\mathrm{ORCP}} &= \frac{B}{2}\int_0^\infty \log_2\left(1+\frac{1}{t}\right)\frac{1}{t^2}f_\gamma\left(\frac{1}{t}\right)dt \\ &= \frac{B}{2}\int_0^\infty \log_2\left(1+\frac{1}{t}\right)f_Y(t)dt \end{aligned} \quad (35)$$

From (10) and [17, eq. (9)], we have

$$\begin{aligned} C_{\mathrm{ORCP}} &= \frac{B}{4\pi\ln 2}\int_{-\infty}^{\infty}\Psi_Y(\omega) \\ &\quad\times\frac{\ln(\jmath\omega)+\exp(\jmath\omega)Ei(\jmath\omega)+C}{\jmath\omega}d\omega \\ &= \frac{B}{2\pi\ln 2}\int_0^{\pi/2}\Re\left\{\Psi_Y(\tan(\theta))\big(\ln(\jmath\tan(\theta))\right. \\ &\quad\left.+\exp(\jmath\tan(\theta))Ei(\jmath\tan(\theta))+C\big)/(\jmath\tan(\theta))\right\} \\ &\quad\times\sec^2(\theta)d\theta \end{aligned} \quad (36)$$

where $\sec(\cdot)=1/\cos(\cdot)$, and $\omega=\tan(\theta)$. In fact, the integral of the real part of a complex expression given in (36) can be numerically solved by mathematics software packages.

5.1 Channel capacity for i.i.d. Nakagami-*m* fading channels

The channel capacity of ORCP for i.i.d. Nakagami-m fading channels can be calculated by substituting the CHF of the reciprocal of the instantaneous SNR given in (15) into (36), yielding

$$C_{\mathrm{ORCP}} = \frac{2B\alpha_1^{n_1m_1/2}\alpha_2^{n_2m_2/2}}{\pi\ln 2\,\Gamma(n_1m_1)\Gamma(n_2m_2)}$$

$$\times \int_0^{\pi/2} \Re \Big\{ \big(\ln(\jmath \tan(\theta)) + \exp(\jmath \tan(\theta)) \times Ei(\jmath \tan(\theta)) + C\big)/(\jmath \tan(\theta)) \times \left(-\frac{\jmath \tan(\theta)}{\rho}\right)^{(n_1 m_1 + n_2 m_2)/2} \times \prod_{i=1}^{2} \mathcal{K}_{n_i m_i}\left(2\sqrt{-\frac{\jmath \alpha_i \tan(\theta)}{\rho}}\right)\Big\} \sec^2(\theta) d\theta \tag{37}$$

5.2 Channel capacity for i.n.i.d. Nakagami-*m* fading channels

Similarly, the channel capacity for i.n.i.d. Nakagami-*m* fading channels is obtained by substituting (20) into (36)

$$C_{\mathrm{ORCP}} = \frac{2B}{\pi \ln 2} \sum_{l=1}^{n_1} \sum_{k=1}^{m_{1,l}} \sum_{p=1}^{n_2} \sum_{q=1}^{m_{2,l}} \frac{\eta_{1,l}(k)\eta_{2,p}(q)\alpha_{1,l}^{k/2}\alpha_{2,p}^{q/2}}{\Gamma(k)\Gamma(q)} \times \int_0^{\pi/2} \Re \Big\{ \big(\ln(\jmath \tan(\theta)) + \exp(\jmath \tan(\theta)) \times Ei(\jmath \tan(\theta)) + C\big)/(\jmath \tan(\theta)) \times \left(\frac{-\jmath \tan(\theta)}{\rho}\right)^{(k+q)/2} \mathcal{K}_k\left(2\sqrt{-\frac{\jmath \alpha_{1,l} \tan(\theta)}{\rho}}\right) \times \mathcal{K}_q\left(2\sqrt{-\frac{\jmath \alpha_{2,p} \tan(\theta)}{\rho}}\right)\Big\} \sec^2(\theta) d\theta \tag{38}$$

These above integrals given in (37) and (38) can be numerically solved with the help of standard mathematics software packages.

6 Truncated channel inversion with fixed rate

With TCIFR, the source only adapts its transmit power to provide a constant instantaneous SNR at the destination while keeping the transmission rate fixed. In other words, it inverts the channel response as long as channel conditions are better than a specific level. Hence, the fading channels appear as time-invariant and a fixed transmission rate can be maintained by changing the transmit power properly regardless of the channel conditions. Therefore, TCIFR incurs the least implementation complexity among OSPR and ORCP schemes. For TCIFR, transmission will be suspended during periods of deep fading. Specifically, as long as the instantaneous SNR is greater than the optimal cutoff SNR, γ_c, TCIFR is active.

The channel capacity, C_{TCIFR}, of TCIFR can be determined as follows [12]:

$$C_{\mathrm{TCIFR}} = \frac{B}{2} \log_2\left(1 + K^{-1}\right)\left(1 - P_{o,\mathrm{TCIFR}}\right) \tag{39}$$

where the outage probability of TCIFR, $P_{o,\mathrm{TCIFR}} = \Pr\{\gamma < \gamma_c\}$, and the average of the reciprocal of the instantaneous SNR, K, are given by, respectively

$$P_{o,\mathrm{TCIFR}} = 1 - \int_{\gamma_c}^{\infty} f_\gamma(\gamma) d\gamma \tag{40}$$

$$K = \int_{\gamma_c}^{\infty} \frac{1}{\gamma} f_\gamma(\gamma) d\gamma \tag{41}$$

and γ_c denotes the optimal cutoff SNR for TCIFR. The optimal cutoff SNR, γ_c, in (41) is chosen such that a particular value of the outage probability is provided while channel capacity is maximized. It can be obtained by numerically solving $\partial C_{\mathrm{TCIFR}}/\partial \gamma_c = 0$. That is,

$$\partial C_{\mathrm{TCIFR}}/\partial \gamma_c = \frac{1}{\gamma_c K(1+K)}\left(1 - P_{o,\mathrm{TCIFR}}\right) - \ln\left(1 + \frac{1}{K}\right) = 0 \tag{42}$$

The expression for the outage probability of TCIFR may be reformulated in terms of the CHF of Y by replacing γ_0 given in (27) with γ_c and can then be written as

$$P_{o,\mathrm{TCIFR}} = 1 - \frac{1}{\pi} \int_0^{\pi/2} \Re\left\{\frac{\Psi_Y(\tan(\theta))}{\jmath \tan(\theta)} \times \left[1 - \exp\left(\frac{-\jmath \tan(\theta)}{\gamma_c}\right)\right]\right\} \sec^2(\theta) d\theta \tag{43}$$

Again, using the change of variable, $t = \gamma^{-1}$, together with (10) and [17, eq. (9)], term K can be obtained as

$$K = \int_0^{1/\gamma_c} t f_Y(t) dt = \frac{1}{\pi} \int_0^{\pi/2} \Re\left\{\frac{\Psi_Y(\tan(\theta))}{\tan(\theta)^2 \gamma_c}\left[(\jmath \tan(\theta) + \gamma_c) \times \exp\left(\frac{-\jmath \tan(\theta)}{\gamma_c}\right) - \gamma_c\right]\right\} \sec^2(\theta) d\theta \tag{44}$$

6.1 Channel capacity and outage probability for i.i.d. Nakagami-*m* fading channels

By substituting (15) into (44), term K in (39) and (42) can be determined as

$$K = \frac{4\alpha_1^{n_1 m_1/2}\alpha_2^{n_2 m_2/2}}{\pi \Gamma(n_1 m_1)\Gamma(n_2 m_2)} \times \int_0^{\pi/2} \Re\Big\{\big((\jmath \tan(\theta) + \gamma_c) \times \exp\left(-\jmath \tan(\theta)/\gamma_c\right) - \gamma_c\big)/(\tan(\theta)^2 \gamma_c) \tag{45}$$

$$\times\left(-\frac{\jmath\tan(\theta)}{\rho}\right)^{(n_1m_1+n_2m_2)/2}$$
$$\times\prod_{i=1}^{2}\mathcal{K}_{n_im_i}\left(2\sqrt{-\frac{\jmath\alpha_i\tan(\theta)}{\rho}}\right)\Bigg\}\sec^2(\theta)d\theta$$

Similarly, substituting (15) in (43), the outage probability, $P_{o,\mathrm{TCIFR}}$, is

$$\begin{aligned}P_{o,\mathrm{TCIFR}}=1-&\frac{4\alpha_1^{n_1m_1/2}\alpha_2^{n_2m_2/2}}{\pi\Gamma(n_1m_1)\Gamma(n_2m_2)}\\&\times\int_0^{\pi/2}\Re\left\{\frac{1-\exp\left(-\jmath\tan(\theta)/\gamma_c\right)}{\jmath\tan(\theta)}\right.\\&\times\left(-\frac{\jmath\tan(\theta)}{\rho}\right)^{(n_1m_1+n_2m_2)/2}\\&\times\mathcal{K}_{n_1m_1}\left(2\sqrt{-\frac{\jmath\alpha_1\tan(\theta)}{\rho}}\right)\\&\times\left.\mathcal{K}_{n_2m_2}\left(2\sqrt{-\frac{\jmath\alpha_2\tan(\theta)}{\rho}}\right)\right\}\sec^2(\theta)d\theta\end{aligned}\tag{46}$$

Both integral expressions given in (45) and (46) can be numerically solved by mathematics software packages. The respective channel capacity, C_{TCIFR}, is obtained by substituting (45) and (46) in (39).

6.2 Channel capacity and outage probability for i.n.i.d. Nakagami-*m* fading channels

The expression for K in (39) and (42) is obtained, by substituting (20) into (44), as

$$\begin{aligned}K=&\frac{4}{\pi}\sum_{l=1}^{n_1}\sum_{k=1}^{m_{1,l}}\sum_{p=1}^{n_2}\sum_{q=1}^{m_{2,l}}\frac{\eta_{1,l}(k)\eta_{2,p}(q)\alpha_{1,l}^{k/2}\alpha_{2,p}^{q/2}}{\Gamma(k)\Gamma(q)}\\&\times\int_0^{\pi/2}\Re\left\{\frac{(\jmath\tan(\theta)+\gamma_c)\exp\left(-\jmath\tan(\theta)/\gamma_c\right)-\gamma_c}{\tan(\theta)^2\gamma_c}\right.\\&\times\left(\frac{-\jmath\tan(\theta)}{\rho}\right)^{(k+q)/2}\\&\times\mathcal{K}_k\left(2\sqrt{-\frac{\jmath\alpha_{1,l}\tan(\theta)}{\rho}}\right)\\&\times\left.\mathcal{K}_q\left(2\sqrt{-\frac{\jmath\alpha_{2,p}\tan(\theta)}{\rho}}\right)\right\}\sec^2(\theta)d\theta\end{aligned}\tag{47}$$

Substituting (20) in (43), the outage probability, $P_{o,\mathrm{TCIFR}}$, is determined as

$$\begin{aligned}P_{o,\mathrm{TCIFR}}=1-&\frac{4}{\pi}\sum_{l=1}^{n_1}\sum_{k=1}^{m_{1,l}}\sum_{p=1}^{n_2}\sum_{q=1}^{m_{2,l}}\frac{\eta_{1,l}(k)\eta_{2,p}(q)\alpha_{1,l}^{k/2}\alpha_{2,p}^{q/2}}{\Gamma(k)\Gamma(q)}\\&\times\int_0^{\pi/2}\Re\left\{\frac{\left[1-\exp\left(-\jmath\tan(\theta)/\gamma_c\right)\right]}{\jmath\tan(\theta)}\right.\\&\times\left(\frac{-\jmath\tan(\theta)}{\rho}\right)^{(k+q)/2}\\&\times\mathcal{K}_k\left(2\sqrt{-\frac{\jmath\alpha_{1,l}\tan(\theta)}{\rho}}\right)\\&\times\left.\mathcal{K}_q\left(2\sqrt{-\frac{\jmath\alpha_{2,p}\tan(\theta)}{\rho}}\right)\right\}\sec^2(\theta)d\theta\end{aligned}\tag{48}$$

The above expressions in (47) and (48) can be solved numerically. The respective channel capacity, C_{TCIFR}, is obtained by substituting (47) and (48) in (39).

It is noted that the obtained expressions for the outage probability and channel capacity given in (28), (30), (31), (33), (37), (38), and (45)-(48) are represented in terms of one-dimensional integrals with definite limits. These expressions can readily be solved by standard mathematics software packages such as Mathematica. It is noted that for single antenna relay networks, the obtained expressions for the system performance are given in integral forms in [17].

7 Numerical results

In this section, we present numerical examples for the performance metrics derived above. Monte Carlo simulations are provided together with analytical results in order to verify our analysis. Importantly, in all tested scenarios, there is very close agreement between the analytical and simulated curves. This confirms the correctness of the analysis presented in this article.

Firstly, let us examine the channel capacity of the considered relay networks with adaptive transmission undergoing i.i.d. Nakagami-*m* fading channels. The number of antennas at source S and destination D are selected as $n_1 = n_2 = 1, 2, 3$, and the fading severity parameter is set as $m = 2$. The optimal cutoff SNR for OSPR and TCIFR can be found by numerically solving (25) and (42), respectively, that is presented in Table 1.

Figure 2 depicts the channel capacity per unit bandwidth, i.e., C_{OSPR}/B, C_{ORCP}/B, and C_{TCIFR}/B versus average SNR. It can be seen from Figure 2 that OSPR achieves a slight improvement in capacity as compared to ORCP, and this improvement decreases as the average SNR, ρ_0, increases. The reason for this is that, for both OSPR and

Table 1 Optimal cutoff SNR γ_0 and γ_c for i.i.d. Nakagami-*m* fading channels and different number of antennas.

SNR	$n_1 = n_2 = 1$		$n_1 = n_2 = 2$		$n_1 = n_2 = 3$	
	γ_0	γ_c	γ_0	γ_c	γ_0	γ_c
0 dB	0.1069	0.1338	0.1086	0.1293	0.1090	0.1267
4 dB	0.2010	0.2706	0.2140	0.2715	0.2196	0.2713
8 dB	0.3489	0.5267	0.3852	0.5475	0.4007	0.5569
12 dB	0.5408	0.7237	0.6018	1.0557	0.6227	1.0898
16 dB	0.7302	1.7913	0.7888	1.9736	0.8058	2.0639
20 dB	0.8663	3.2490	0.9035	3.6980	0.9125	3.9181

ORCP, the source tends to transmit with constant power in the high SNR regime. More importantly, as the number of antennas increases, the channel capacity improvement of OSPR relative to ORCP tends to be negligible. This is due to the fact that the instantaneous SNR approaches infinity more rapidly for larger number of antennas when the average SNR converges to infinity. However, a system with OSPR adapts both its transmission rate and power simultaneously, thereby incurring higher implementation complexity as compared to the one using ORCP. Furthermore, TCIFR suffers the largest reduction in channel capacity in comparison with OSPR and ORCP. In fact, with specific average transmit power constraint, TCIFR must consume more power to compensate for bad channel conditions in order to maintain a constant value of instantaneous SNR at the destination, causing considerable reduction in channel capacity relative to other schemes. Unlikely, OSPR and ORCP take advantage of favorable fading channel conditions by deploying higher transmission rate and/or consuming more transmit power. As expected, when the number of antennas increases, the effects of severe fading can be reduced, and thus this capacity reduction diminishes significantly. In practice, due to the limitation of the number of antennas for each terminal, perfectly eliminating the impact of fading by exploiting spatial diversity is almost impossible. Since TCIFR provides the lowest implementation complexity, it is needed to consider a tradeoff between channel capacity and complexity.

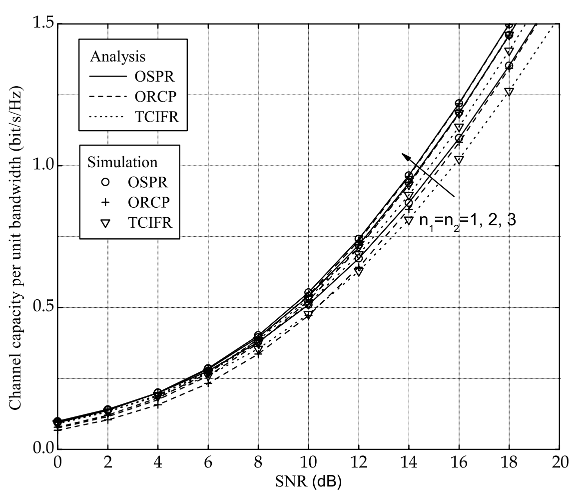

Figure 2 Channel capacity per unit bandwidth versus average SNR of the source, ρ_0, of MIMO AF relay networks with OSTBC transmission over i.i.d. Nakagami-*m* fading channels using different adaptive schemes. (Fading severity parameter $m = 2$, average channel power gain $\Omega = 0.25$.)

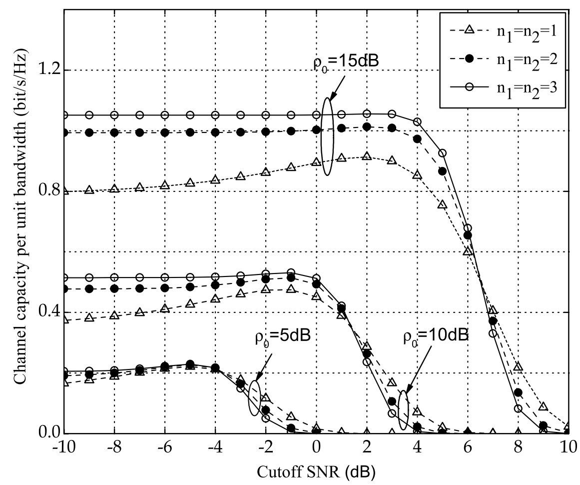

Figure 3 Channel capacity per unit bandwidth versus γ_c of MIMO AF relay networks with OSTBC transmission over i.i.d. Nakagami-*m* fading channels using TCIFR. (Fading severity parameter $m = 2$, average channel power gain $\Omega = 0.25$.)

Figure 3 shows the channel capacity per unit bandwidth given in (39), C_{TCIFR}/B, versus γ_c, with constant values for $\rho_0 = 5, 10, 15$ dB. It can be seen that there exists a specific optimal cutoff SNR of γ_c at which the channel capacity of the considered networks with TCIFR is maximized. Also, as expected from Table 1, the values of the optimal cutoff SNR, γ_0, are smaller than those for the optimal cutoff SNR, γ_c, of TCIFR at the same average SNR. This observation implies that maintaining a constant value of the instantaneous SNR in TCIFR consumes more power than maximizing channel capacity in OSPR with the same fading condition. In other words, fading conditions for activating OSPR is likely less favorable than TCIFR, causing considerable reduction in channel capacity and outage probability of TCIFR scheme.

In addition, Figure 4 plots the outage probability of the considered relay networks with OSPR and TCIFR. Note that for these schemes, the transmission process will be suspended once the instantaneous SNR falls below the optimal cutoff SNR γ_0 and γ_c, respectively. The achievable values of these optimal metrics impose a substantial impact on the channel capacity and the outage probability. As expected, one can see from Figure 4 that the outage probability of OSPR outperforms that of TCIFR considerably. Especially, when the number of antennas increases

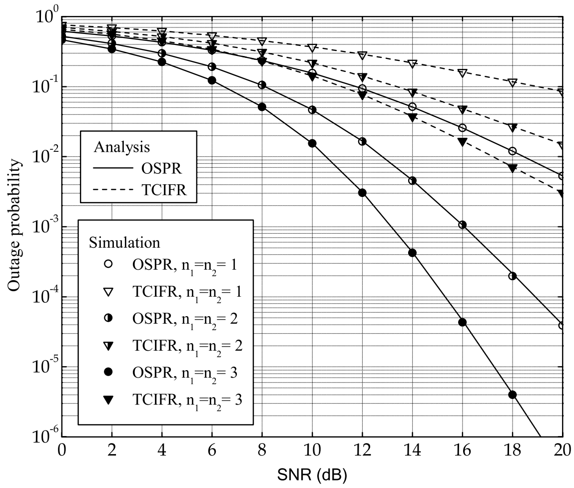

Figure 4 Outage probability versus average SNR of the source, ρ_0, of MIMO AF relay networks with OSTBC transmission over i.i.d. Nakagami-*m* fading channels using either OSPR or TCIFR. (Fading severity parameter $m = 2$, average channel power gain $\Omega = 0.25$.)

this outage probability improvement becomes more significant.

Figure 5 shows the channel capacity per unit bandwidth versus average SNR of the considered relay networks using OSPR, ORCP, and TCIFR in the presence of i.n.i.d. Nakagami-m fading channels. The number of antennas at source S and destination D are selected as $n_1 = n_2 = 1, 2, 3$ while the fading severity parameters m and average channel power gains Ω are as follows:

$$
\begin{aligned}
n_1 = n_2 = 1: \quad & \{(m_{1,1}, \Omega_{1,1})\} = \{(2, 0.2)\} \\
& \{(m_{2,1}, \Omega_{2,1})\} = \{(3, 0.3)\} \\
n_1 = n_2 = 2: \quad & \{(m_{1,i}, \Omega_{1,i})\}_{i=1}^{2} = \{(1, 0.25); (2, 0.35)\} \\
& \{(m_{2,i}, \Omega_{2,i})\}_{i=1}^{2} = \{(2, 0.3); (1, 0.4)\} \\
n_1 = n_2 = 3 \quad & \{(m_{1,i}, \Omega_{1,i})\}_{i=1}^{3} \\
& \quad = \{(2, 0.25); (3, 0.3); \{2, 0.4\}\} \\
& \{(m_{2,i}, \Omega_{2,i})\}_{i=1}^{3} \\
& \quad = \{(3, 0.4); (2, 0.2); (4, 0.35)\}
\end{aligned}
$$

Table 2 Optimal cutoff SNR γ_0 and γ_c for i.n.i.d. Nakagami-*m* fading channels and different number of antennas.

SNR	$n_1 = n_2 = 1$		$n_1 = n_2 = 2$		$n_1 = n_2 = 3$	
	γ_0	γ_c	γ_0	γ_c	γ_0	γ_c
0 dB	0.1054	0.1301	0.1297	0.1609	0.1318	0.1535
4 dB	0.2011	0.2658	0.2450	0.3278	0.2620	0.3281
8 dB	0.3527	0.5226	0.4210	0.6416	0.4648	0.6710
12 dB	0.5492	0.9860	0.6306	1.2065	0.6850	1.3071
16 dB	0.7397	1.8056	0.8069	2.2186	0.8450	2.4823
20 dB	0.8731	3.2986	0.9125	4.1195	0.9319	4.7573

The optimal cutoff SNR for OSPR and TCIFR is presented in Table 2. Again, it can be seen from Figure 5 that the channel capacity of OSPR and ORCP outperforms that of TCIFR for sufficiently large values of the average SNR. More importantly, OSPR obtains the best performance at the expense of high implementation complexity. Figure 6 depicts the channel capacity per unit bandwidth

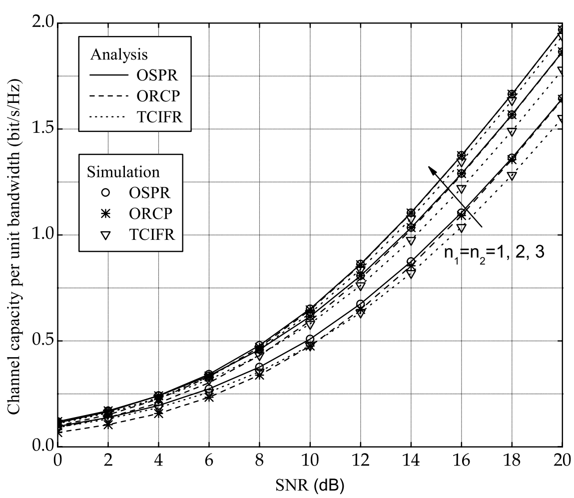

Figure 5 Channel capacity per unit bandwidth versus average SNR of the source, ρ_0, of MIMO AF relay networks with OSTBC transmission over i.n.i.d. Nakagami-*m* fading channels using different adaptive schemes.

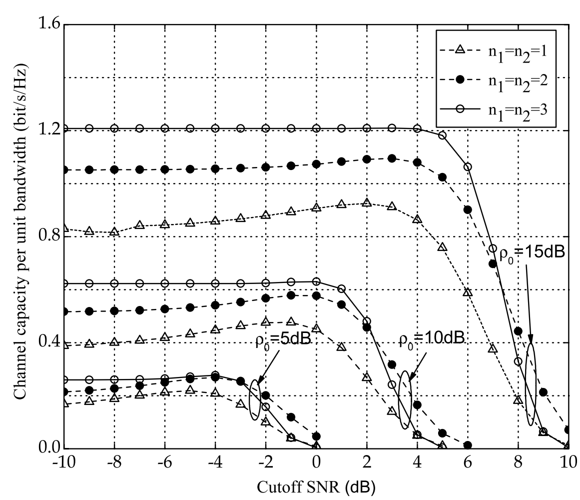

Figure 6 Channel capacity per unit bandwidth versus γ_c of MIMO AF relay networks with OSTBC transmission over i.n.i.d. Nakagami-*m* fading channels using TCIFR.

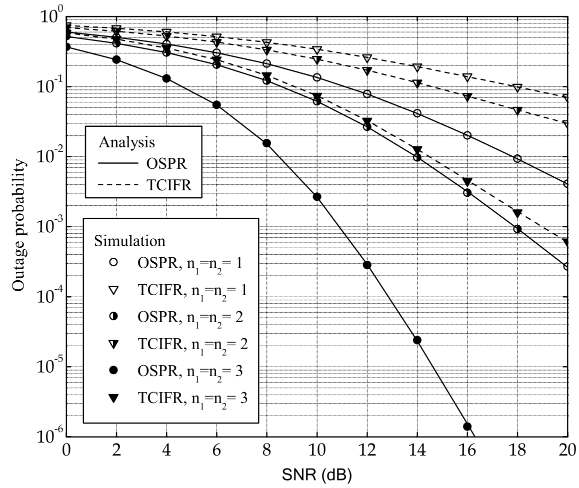

Figure 7 Outage probability versus average SNR of the source, ρ_0, of MIMO AF relay networks with OSTBC transmission over i.n.i.d. Nakagami-*m* fading channels using either OSPR or TCIFR.

given in (39), $\frac{C_{\text{TCIFR}}}{B}$, versus γ_c, at constant values of $\rho_0 = 5, 10, 15$ dB. It can be also seen that the channel capacity obtains the maximum value at one optimal value of γ_c. The outage probability versus average SNR for OSPR and TCIFR is shown in Figure 7. As can be seen, OSPR outperforms TCIFR in terms of outage probability. The best outage performance can be obtained for the case of $n_1 = n_2 = 3$.

8 Conclusions

We have analyzed the performance of the three adaptive transmission schemes of OSPR, ORCP, and TCIFR applied to MIMO CSI-assisted AF cooperative relay networks with OSTBC. Our analysis is based on both the i.i.d. and i.n.i.d. Nakagami-*m* fading channels, which generalizes a wide class of multi-path fading environments. Closed-form expressions for the MGFs of the reciprocal of the instantaneous SNR are derived and then utilized to evaluate the performance metrics. We present Monte Carlo simulations, which are in a very close agreement with analytical results, to validate our analysis. We also show that for sufficiently high SNR the channel capacity of OSPR and ORCP are almost the same and better than that of TCIFR for the considered scenarios. Regarding the practical implementation, ORCP is less complex to set up as compared to OSPR. It is also seen that for low SNR, the channel capacity of TCIFR outperforms that of the ORCP scheme. Moreover, it can be seen that the outage probability of OSPR is significantly lower than that of TCIFR; however, TCIFR offers the least complexity for practical implementation.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
HP conceived and designed the presented research work, carried out the mathematical analysis and numerical experiments, drafted and amended the article. TQD conceived the work, helped with the literature review and result discussion, contributed to the system model, and helped with revising the article. HJZ helped with refining and structuring the presented approach, participated in drafting the article, corrected and revised it significantly, helped with interpreting the results, and provided the supervision for this research. LS helped with the literature review, aided the result discussion, and participated in correcting and revising the manuscript. All authors read and approved the final manuscript.

Author details
[1]Blekinge Institute of Technology, SE-371 79 Karlskrona, Sweden. [2]Graduate School of Information Science and Technology, Osaka University, Osaka, Japan.

Received: 13 August 2011 Accepted: 11 November 2011
Published: 12 January 2012

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doi:10.1186/1687-1499-2012-11
Cite this article as: Phan et al.: **Adaptive transmission in MIMO AF relay networks with orthogonal space-time block codes over Nakagami-*m* fading.** *EURASIP Journal on Wireless Communications and Networking* 2012 **2012**:11.