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Energy efficiency in multiaccess fading channels under QoS constraints

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Abstract

In this article, transmission over multiaccess fading channels under quality-of-service (QoS) constraints is studied in the low-power and wideband regimes. QoS constraints are imposed as limitations on the buffer violation probability. The effective capacity, which characterizes the maximum constant arrival rates in the presence of such statistical QoS constraints, is employed as the performance metric. A two-user multiaccess channel model is considered, and the minimum bit energy levels and wideband slope regions are characterized for different transmission and reception strategies, namely time-division multiple-access (TDMA), superposition coding with fixed decoding order, and superposition coding with variable decoding order. It is shown that the minimum received bit energies achieved by these different strategies are the same and independent of the QoS constraints in the low-power regime, while they vary with the QoS constraints in the wideband regime. When wideband slope regions are considered, the suboptimality of TDMA with respect to superposition coding is proven in the low-power regime. On the other hand, it is shown that TDMA in the wideband regime can interestingly outperform superposition coding with fixed decoding order. The impact of varying the decoding order at the receiver under certain assumptions is also investigated. Overall, energy efficiency of different transmission strategies under QoS constraints are analyzed and quantified.

Keywords: effective capacity, energy efficiency, energy per information bit, low-power regime, multiple-access fading channels, quality of service, superposition coding, time-division multiple access, wideband regime, wideband slope

1. Introduction

Energy efficiency is an important consideration in wireless systems and has been rigorously analyzed from an information-theoretic perspective. In [1], Verdú has extensively studied the spectral efficiency-bit energy tradeoff in the wideband regime, considering the Shannon capacity as the performance metric. For the Gaussian multiaccess channel, Caire et al. [2] have shown that time division multiple-access (TDMA) is in general a suboptimal transmission scheme in the low-power regime unless one considers the asymptotic scenario in which the power vanishes. It is also shown that fading channel makes the superposition strategy more favorable. In this analysis, Shannon capacity formulation is again adopted as the main performance metric. However, Shannon capacity does not quantify the performance in the presence of quality-of-service (QoS) limitations in the form of constraints on queueing delays or queue lengths. Indeed, most communication- and information-theoretic studies, while providing powerful results, do not generally concentrate on delay and QoS constraints [3].

At the same time, providing QoS guarantees is one of the key requirements in the development of next generation wireless communication networks since data traffic with multimedia content is expected to grow significantly and in real-time multimedia applications, such as voice over IP (VoIP) and interactive-video (e.g., videoconferencing), latency is a key QoS metric. Many efforts have been made to incorporate the delay constraints in the performance analysis [4-7]. In [4], delay limited capacity has been proposed as a performance metric, which is defined as the rate that can be attained regardless of the values of the fading states. In [6], the tradeoff between the average transmission power and average delay has been analyzed

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by considering an optimization problem in which the weighted combination of the average power and average delay is minimized over transmission policies that determine the transmission rate by taking into account the arrival state, buffer occupancy, channel state jointly together. In [7], the long-term average capacity has been proposed to study the fading multiaccess channel in the wideband regime and the suboptimality of TDMA has been shown again.

In this article, we follow a different approach. We consider statistical QoS constraints and study the energy efficiency under such limitations. We adopt the notion of effective capacity [8], which can be seen as the maximum constant arrival rate that a given time-varying service process can support while providing statistical QoS guarantees. The analysis and application of effective capacity in various settings have attracted much interest recently (see e.g. [9-16] and references therein). For instance, related to this study, in [13,14], energy efficiency is addressed in a *single-user setting* when the wireless systems operate under buffer constraints and employ either adaptive or fixed transmission schemes for point-to-point links. The effective capacity regions for multiaccess channel with different scheduling policies have been characterized in [16]. In that work, it has been found that TDMA and superposition coding with variable decoding with respect to channel states can outperform superposition strategy with fixed decoding. In this article, we consider the performance of TDMA and superposition strategy in the presence of statistical QoS constraints but concentrate on the low-SNR regime. More specifically, we employ the tools provided in [1,2] to investigate the bit energy and wideband slope regions under QoS constraints in the low-power and wideband regimes. The main contributions of this article are summarized in the following:

(1) We show that different transmission and reception strategies do not affect the minimum bit energy levels required by each user. Additionally, we prove that while the minimum bit energies are independent of the QoS constraints in the low power regime, they vary with the QoS constraints in the wideband regime.

(2) We determine that superposition coding with variable decoding order does not improve the performance in terms of slope region with respect to fixed decoding order in the low power regime, while it can achieve larger slope region in the wideband regime.

(3) When wideband slope regions are considered, we show that TDMA is always suboptimal in the low power regime except the special case in which fading states are linearly dependent. On the other hand, TDMA in certain cases is demonstrated to perform better than superposition coding with fixed decoding order in the wideband regime. We also identify the condition for TDMA to be suboptimal in this regime.

The remainder of the article is organized as follows. In Section 2, the system model is briefly discussed. Section 3 presents some preliminaries on the analysis tools and effective capacity. The results in the low-power regime are provided in Section 4. Section 5 presents the results in the wideband regime. Finally, Section 6 concludes this article.

2. System model

As shown in Figure 1, we consider an uplink scenario where M users with individual power and buffer constraints (i.e., QoS constraints) communicate with a single receiver. It is assumed that the transmitters generate data sequences which are divided into frames of duration T. These data frames are initially stored in the buffers before they are transmitted over the wireless channel. The discrete-time signal at the receiver in the ith symbol duration is given by

$$Y[i] = \sum_{j=1}^{M} h_j[i] X_j[i] + n[i], \; i = 1, 2, \ldots \tag{1}$$

where M is the number of users, $X_j[i]$ and $h_j[i]$ denote the complex-valued channel input and the fading coefficient of the jth user, respectively. We assume that $\{h_j[i]\}$'s are jointly stationary and ergodic discrete-time processes, and we denote the magnitude-square of the fading coefficients by $z_j[i] = |h_j[i]|^2$. Let $\mathbf{z} = (z_1, z_2, ..., z_m)$ be the channel state vector. Above, $n[i]$ is a zero-mean, circularly symmetric, complex Gaussian random variable with variance $\mathbb{E}\left\{|n[i]|^2\right\} = N_0$. The additive Gaussian noise samples $\{n[i]\}$ are assumed to form an independent and identically distributed (i.i.d.) sequence. Finally, $Y[i]$ denotes the received signal.

The channel input of user j is subject to an average energy constraint $\mathbb{E}\left\{|x_j[i]|^2\right\} \leq \overline{P}_j/B$ for all j, where B is the bandwidth available in the system. Assuming that the symbol rate is B complex symbols per second, we can see that this formulation indicates that user j is subject to an average power constraint of $\bar{P}_j$. With these

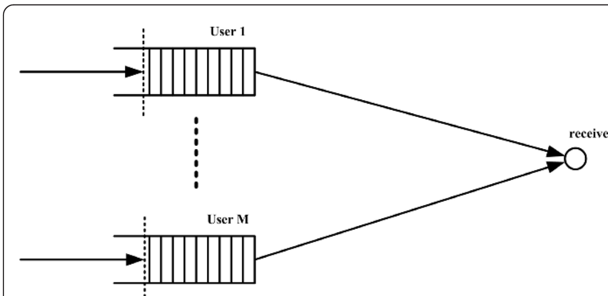

Figure 1 The system model.

definitions, the average transmitted signal to noise ratio of user j is $\mathrm{SNR}j = \frac{\bar{P}_j}{N_0 B}$.

3. Preliminaries

3.1. Effective capacity region of the MAC channel

In [8], effective capacity is defined as the maximum constant arrival rate that a given service process can support in order to guarantee a statistical QoS requirement specified by the QoS exponent θ. The effective capacity is formulated as

$$C(\theta) = -\lim_{t\to\infty} \frac{1}{\theta t}\log_e \mathbb{E}\left\{e^{-\theta S[t]}\right\} \quad \text{bits/s}, \tag{2}$$

where the expectation is with respect to $S[t] = \sum_{i=1}^{t} s[i]$, which is the time-accumulated service process, and $\{s[i], i = 1, 2, ...\}$ denotes the discrete-time stochastic service process. Effective capacity can be regarded as the maximum throughput of the system while the buffer violation probability is guaranteed to decay exponential fast with decay rate controlled by θ, i.e., the buffer violation probability behaves as $\Pr\{Q > Q_{\max}\} \approx e^{-\theta Q_{\max}}$ for large $Q_{\max}$, where Q is stationary queue length.

We assume that the fading coefficients stay constant over the frame duration T and vary independently from one frame to another for each user. Hence, we basically consider a block-fading model. In this scenario, $s[i] = TR[i]$, where $R[i]$ is the instantaneous service rate in the ith frame duration $[iT; (i+1)T)$. Then, the effective capacity in (2) can be expressed as

$$C(\theta) = -\frac{1}{\theta T}\log_e \mathbb{E}_{\mathbf{z}}\left\{e^{-\theta T R[i]}\right\} \quad \text{bits/s}, \tag{3}$$

where $R[i]$ is in general a function of the fading state $\mathbf{z}$. (3) is obtained using the fact that instantaneous rates $\{R[i]\}$ vary independently from one frame to another. It is interesting to note that as $\theta \to 0$ and hence QoS constraints relax, effective capacity approaches the ergodic capacity, i.e., $C(\theta) \to \mathbb{E}_{\mathbf{z}}\{R[i]\}$. On the other hand, as shown in [13], $C(\theta)$ converges to the delay limited capacity as θ grows without limit (i.e., $\theta \to \infty$) and QoS constraints become increasingly more strict. Therefore, effective capacity enables us to study the performance levels between the two extreme cases of delay limited capacity, which can be seen as a deterministic service guarantee or equivalently as a performance level attained under hard QoS limitations, and ergodic capacity, which is achieved in the absence of any QoS considerations.

Suppose that $\Theta = (\theta_1, ..., \theta_M)$ is a vector composed of the QoS constraints of M users. Let $\beta_j = \dfrac{\theta_j TB}{\log_e 2}$, $j = 1, 2, ..., M$ be the associated normalized QoS constraints. Also, let $\mathrm{C}(\Theta) = (\mathrm{C}_1(\theta_1), ..., \mathrm{C}_M(\theta_M))$ denote the vector of the normalized effective capacities.

In [16], the effective capacity regions of the multi-access channel for different scheduling policies have been characterized. The effective capacity region achieved by TDMA is

$$\bigcup_{\{\delta_j\}}\left\{\mathrm{C}(\Theta) \geq \mathbf{0} : \mathrm{C}_j(\theta_j) \leq -\frac{1}{\theta_j TB}\log_e \mathbb{E}\left\{e^{-\delta_j\theta_j TB\log_2\left(1+\frac{\mathrm{SNR}_j}{\delta_j}z_j\right)}\right\}\right\} \tag{4}$$

where δ_j is the fraction of time allocated to user j.

The effective capacity region achieved by superposition coding with fixed decoding order is given by

$$\bigcup_{\{\tau_m\}}\left\{\mathrm{C}(\Theta) \geq \mathbf{0} : \mathrm{C}_j(\theta_j) \leq -\frac{1}{\theta_j TB}\log_e \mathbb{E}_{\mathbf{z}}\left\{e^{-\theta_j T\sum_{m=1}^{M!}\tau_m R_{\pi_m^{-1}(j)}}\right\}\right\} \tag{5}$$

where τ_m is the fraction of time allocated to a specific decoding order π_m, $R_{\pi_m^{-1}(j)}$ represents the maximal instantaneous service rate of user j at a given decoding order π_m, which is given by

$$R_{\pi_m^{-1}(j)} = B\log_2\left(1 + \frac{\mathrm{SNR}_j z_j}{1 + \sum_{\pi_m^{-1}(i) > \pi_m^{-1}(j)} \mathrm{SNR}_i z_i}\right) \tag{6}$$

where π_m^{-1} is the inverse trace function of π_m.

Decoding orders can be varied for each channel fading state $\mathbf{z}$. Suppose the vector space $\Re_+^M$ of the possible values for $\mathbf{z}$ is partitioned into $M!$ disjoint regions $\{\mathcal{Z}_m\}_{m=1}^{M!}$ with respect to decoding orders $\{\pi_m\}_{m=1}^{M!}$. Then, the maximum effective capacity that can be achieved by the jth user is

$$\begin{aligned}\mathrm{C}_j(\theta_j) &= -\frac{1}{\theta_j TB}\log_e \mathbb{E}_{\mathbf{z}}\left\{e^{-\theta_j T R_j}\right\} \\ &= -\frac{1}{\theta_j TB}\log_e\left(\sum_{m=1}^{M!}\int_{\mathbf{z}\in\mathcal{Z}_m} e^{-\theta_j T R_{\pi_m^{-1}(j)}} p_{\mathbf{z}}(\mathbf{z})\, d\mathbf{z}\right)\end{aligned} \tag{7}$$

for $j = 1, ..., M$, where $p_{\mathbf{z}}$ is the distribution function of $\mathbf{z}$ and $R_{\pi_m^{-1}(j)}$ is given in (6).

3.2. Spectral efficiency vs. bit energy

If we denote the effective capacity normalized by bandwidth or equivalently the spectral efficiency in bits per second per Hertz by

$$\mathrm{C}_E(\mathrm{SNR}, \theta) = \frac{C_E(\mathrm{SNR}, \theta)}{B} = -\frac{1}{\theta TB}\log_e \mathbb{E}\left\{e^{-\theta T R[i]}\right\}, \tag{8}$$

then it can be easily seen that $\frac{E_b}{N_0}_{\min}$ under QoS constraints can be obtained from [1]

$$\frac{E_b}{N_0}_{\min} = \lim_{\mathrm{SNR}\to 0} \frac{\mathrm{SNR}}{\mathrm{C}_E(\mathrm{SNR})} = \frac{1}{\dot{\mathrm{C}}_E(0)}. \tag{9}$$

Hence, energy efficiency improves as SNR diminishes and the minimum bit energy is attained as SNR vanishes. At $\frac{E_b}{N_0}_{\min}$, the slope $\mathcal{S}_0$ of the spectral efficiency versus E_b/N_0 (in dB) curve is is called the wideband slope, and is defined as [1]

$$\mathcal{S}_0 = \lim_{\frac{E_b}{N_0}\downarrow\frac{E_b}{N_0}_{\min}} \frac{\mathrm{C}_E\left(\frac{E_b}{N_0}\right)}{10\log_{10}\frac{E_b}{N_0} - 10\log_{10}\frac{E_b}{N_0}_{\min}} 10\log_{10}2. \tag{10}$$

Considering the expression for normalized effective capacity, the wideband slope can be found from[a]

$$\mathcal{S}_0 = -\frac{2\left(\dot{\mathrm{C}}_E(0)\right)^2}{\ddot{\mathrm{C}}_E(0)}\log_e 2 \tag{11}$$

where $\dot{\mathrm{C}}_E(0)$ and $\ddot{\mathrm{C}}_E(0)$ are the first and second derivatives, respectively, of the function $\mathrm{C}_E(\mathrm{SNR})$ in bits/s/Hz at zero SNR [1]. The minimum bit energy $\frac{E_b}{N_0}_{\min}$ and the wideband slope provide a linear approximation of the spectral efficiency-bit energy curve at low SNR levels and enables us to characterize and quantify the energy efficiency in the low-SNR regime.

4. Energy efficiency in the low-power regime

As described above, in order to transmit energy efficiently and achieve bit energy levels close to the minimum level, one needs to operate in the low-SNR regime in which either the power is low or bandwidth is large. In this section, we consider the low-power regime. We concentrate on the two-user multiaccess channel. Below, we first note the maximum effective capacities attained through different transmission strategies described in in Section 1. Subsequently, we identify the corresponding minimum bit energies and the wideband slopes.

Now, for the two-user TDMA, if we fix the fraction of time allocated to user 1 as $\delta \in [0, 1]$, the maximum effective capacities of the two-users in the TDMA region given by (4) become

$$\mathrm{C}_1(\mathrm{SNR}_1) = -\frac{1}{\theta_1 TB}\log_e \mathbb{E}_{\mathbf{z}}\left\{e^{-\delta\theta_1 TB\log_2\left(1+\frac{\mathrm{SNR}_1 z_1}{\delta}\right)}\right\} \tag{12}$$

and

$$\mathrm{C}_2(\mathrm{SNR}_2) = -\frac{1}{\theta_2 TB}\log_e \mathbb{E}_{\mathbf{z}}\left\{e^{-(1-\delta)\theta_2 TB\log_2\left(1+\frac{\mathrm{SNR}_2 z_2}{1-\delta}\right)}\right\}, \tag{13}$$

respectively,

Next, consider superposition coding with fixed decoding order. We denote the ratio of the transmitter-side signal-to-noise ratios as $\lambda = \frac{\mathrm{SNR}_1}{\mathrm{SNR}_2} = \left(\frac{\overline{P}_1}{N_0 B}\right)/\left(\frac{\overline{P}_2}{N_0 B}\right)$. We assume that the value of this ratio is arbitrary but is kept fixed as SNR_1 and SNR_2 diminish in the low-SNR regime. Additionally, we let τ denote the fraction of time in which the decoding order (2, 1) is employed. Note that if the decoding order is (2, 1), the receiver first decodes the second user's signal in the presence of interference from first user's signal, and subsequently decodes the first user's signal with no interference. Note that the symmetric case occurs when the decoding order is (1, 2) in the remaining (1 -τ) fraction of the time. When this strategy is used, the maximum effective capacities in the region described in (5) can now be expressed as

$$\mathrm{C}_1(\mathrm{SNR}_1) = -\frac{1}{\theta_1 TB}\log_e \mathbb{E}_{\mathbf{z}}\left\{e^{-\theta_1 TB\left(\tau\log_2(1+\mathrm{SNR}_1 z_1)+(1-\tau)\log_2\left(1+\frac{\mathrm{SNR}_1 z_1}{1+\mathrm{SNR}_1 z_2/\lambda}\right)\right)}\right\}, \tag{14}$$

$$\mathrm{C}_2(\mathrm{SNR}_2) = -\frac{1}{\theta_2 TB}\log_e \mathbb{E}_{\mathbf{z}}\left\{e^{-\theta_2 TB\left(\tau\log_2\left(1+\frac{\mathrm{SNR}_2 z_2}{1+\lambda\mathrm{SNR}_2 z_1}\right)+(1-\tau)\log_2(1+\mathrm{SNR}_2 z_2)\right)}\right\}. \tag{15}$$

Finally, we turn our attention to superposition coding with variable decoding order. In this case, the decoding order depends on the fading coefficients (z_1, z_2). We define $z_2 = g(\mathrm{SNR}_1) = g(\lambda\mathrm{SNR}_2)$ as the partition function in the z_1 - z_2 space.[b] Depending on which decoding order is employed in each region, we have different effective capacity expressions. If users are decoded in the order (1,2) when $z_2 < g(\mathrm{SNR}_1)$ and are decoded in the order (2,1) when $z_2 > g(\mathrm{SNR}_1)$, the effective capacities are given by

$$\mathrm{C}_1(\mathrm{SNR}_1) = -\frac{1}{\theta_1 TB}\log_e\left(\int_0^\infty\int_{g(\mathrm{SNR}_1)}^\infty e^{-\theta_1 TB\log_2(1+\mathrm{SNR}_1 z_1)}p_{\mathbf{z}}(z_1,z_2)\,dz_2dz_1 + \int_0^\infty\int_0^{g(\mathrm{SNR}_1)} e^{-\theta_1 TB\log_2\left(1+\frac{\mathrm{SNR}_1 z_1}{1+\mathrm{SNR}_1 z_2/\lambda}\right)}p_{\mathbf{z}}(z_1,z_2)\,dz_2dz_1\right), \tag{16}$$

$$\mathrm{C}_2(\mathrm{SNR}_2) = -\frac{1}{\theta_2 TB}\log_e\left(\int_0^\infty\int_0^{g(\lambda\mathrm{SNR}_2)} e^{-\theta_2 TB\log_2(1+\mathrm{SNR}_2 z_2)}p_{\mathbf{z}}(z_1,z_2)\,dz_2dz_1 + \int_0^\infty\int_{g(\lambda\mathrm{SNR}_2)}^\infty e^{-\theta_2 TB\log_2\left(1+\frac{\mathrm{SNR}_2 z_2}{1+\lambda\mathrm{SNR}_2 z_1}\right)}p_{\mathbf{z}}(z_1,z_2)\,dz_2dz_1\right). \tag{17}$$

Similar effective capacity expressions can be derived if users are decoded in the order (2,1) if $z_2 < g(\text{SNR}_1)$ and decoded in the order (1,2) if $z_2 > g(\text{SNR}_1)$.

Assumption 1: Throughout the article, we consider the partition functions $g(\text{SNR}_1)$ that satisfy the following properties:

(1) $g(0)$ is finite.

(2) The first and second derivatives of g with respect to SNR_1, $\dot{g}(\text{SNR}_1)$ and $\ddot{g}(\text{SNR}_1)$, exist. Moreover, $\dot{g}(0)$ and $\ddot{g}(0)$ are finite.

As will be seen in the ensuing analysis, the finiteness assumptions above will serve as sufficient conditions to ensure that the derivatives of effective capacity in the limit as SNR vanishes are finite.

Denote $\frac{E_{b,i}}{N_0} = \frac{\text{SNR}_i}{\text{C}_i}$ as the bit energy of user $i = 1, 2$. The received bit energy is

$$\frac{E^r_{b,i}}{N_0} = \frac{E_{b,i}}{N_0}\mathbb{E}\{z_i\}. \tag{18}$$

As the following result shows, the minimum received bit energies for the different strategies are the same.

Theorem 1: For all $\lambda = \frac{\text{SNR}_1}{\text{SNR}_2}$ and all $g(z_1, \text{SNR}_1)$ satisfying the properties in Assumption 1, the minimum received bit energy for the multiaccess fading channel attained through TDMA, superposition coding with fixed decoding order, or superposition decoding with varying decoding order, is the same and is given by

$$\frac{E^r_{b,1}}{N_0}_{\min} = \frac{E^r_{b,2}}{N_0}_{\min} = \log_e 2 = -1.59 \text{ dB}. \tag{19}$$

Proof: See Appendix 1.

Remark 1: The result of Theorem 1 shows that different transmission strategies (e.g., TDMA or superposition coding) and different reception schemes (e.g., fixed or variable decoding orders) lead to the same fundamental limit on the minimum bit energy. Similarly as in [2], TDMA is optimally efficient in the asymptotic regime in which the signal-to-noise ratio vanishes. More interestingly, we note that this result is obtained in the presence of QoS constraints. Additionally, the minimum bit energy is clearly independent of the QoS limitations parametrized by the QoS exponents θ_1 and θ_2. Hence, the energy efficiency is not adversely affected by the buffer constraints in this asymptotic regime in which $\text{SNR} \to 0$.

Remark 2: It can be easily shown using the effective capacity expressions provided in (4), (5), and (7) that the characterization in Theorem 1, i.e., the result that the minimum received energy per bit requirement for each user is -1.59 dB under QoS constraints, holds in a more general setting in which the number of users $M \geq 2$.

Having shown that the minimum bit energies achieved by different transmission and reception strategies are the same for each user, we note that the wideband slope regions have become more interesting since they quantify the performance in the non-asymptotic regime in which SNRs are small but nonzero. With the analysis approach introduced in [2], we have the following results.

Theorem 2: The multiaccess slope region achieved by TDMA is given by

$$\begin{aligned}\mathcal{S} = \Big\{(\mathcal{S}_1, \mathcal{S}_2) : 0 \leq \mathcal{S}_1 \leq \mathcal{S}_1^{\text{up}}, \quad 0 \leq \mathcal{S}_2 \leq \mathcal{S}_2^{\text{up}},\\ \frac{\kappa_{11}\kappa_{12}}{\kappa_{11} - \mathcal{S}_1} + \frac{\kappa_{21}\kappa_{22}}{\kappa_{21} - \mathcal{S}_2} \leq 1 + \kappa_{12} + \kappa_{22}\Big\}\end{aligned} \tag{20}$$

where

$$\begin{aligned}
\mathcal{S}_1^{\text{up}} &= \frac{2(\mathbb{E}\{z_1\})^2}{\beta_1\left(\mathbb{E}\{z_1^2\} - (\mathbb{E}\{z_1\})^2\right) + \mathbb{E}\{z_1^2\}},\\
\mathcal{S}_2^{\text{up}} &= \frac{2(\mathbb{E}\{z_2\})^2}{\beta_2\left(\mathbb{E}\{z_2^2\} - (\mathbb{E}\{z_2\})^2\right) + \mathbb{E}\{z_2^2\}},\\
\kappa_{11} &= \frac{2(\mathbb{E}\{z_1\})^2}{\beta_1\left(\mathbb{E}\{z_1^2\} - (\mathbb{E}\{z_1\})^2\right)},\\
\kappa_{12} &= \frac{\mathbb{E}\{z_1^2\}}{\beta_1\left(\mathbb{E}\{z_1^2\} - (\mathbb{E}\{z_1\})^2\right)},\\
\kappa_{21} &= \frac{2(\mathbb{E}\{z_2\})^2}{\beta_2\left(\mathbb{E}\{z_2^2\} - (\mathbb{E}\{z_2\})^2\right)},\\
\kappa_{22} &= \frac{\mathbb{E}\{z_2^2\}}{\beta_2\left(\mathbb{E}\{z_2^2\} - (\mathbb{E}\{z_2\})^2\right)},
\end{aligned}$$

$\beta_1 = \theta_1 T B \log_2 e$ and $\beta_2 = \theta_2 T B \log_2 e$.

Proof: See Appendix 2.

The following results provide the wideband slope expressions when superposition transmission is employed.

Theorem 3: For any $\lambda = \frac{\text{SNR}_1}{\text{SNR}_2}$, the multiaccess slope region achieved by the superposition coding with fixed decoding order is

$$\begin{aligned}\mathcal{S} = \Big\{(\mathcal{S}_1, \mathcal{S}_2) : 0 \leq \mathcal{S}_1 \leq \mathcal{S}_1^{\text{up}}, \quad 0 \leq \mathcal{S}_2 \leq \mathcal{S}_2^{\text{up}},\\ \frac{\lambda(\mathbb{E}\{z_1\})^2}{\mathbb{E}\{z_1 z_2\}}\left(\frac{1}{\mathcal{S}_1} - \frac{1}{\mathcal{S}_1^{\text{up}}}\right) + \frac{(\mathbb{E}\{z_2\})^2}{\lambda\mathbb{E}\{z_1 z_2\}}\left(\frac{1}{\mathcal{S}_2} - \frac{1}{\mathcal{S}_2^{\text{up}}}\right) = 1\Big\},\end{aligned} \tag{21}$$

where $\mathcal{S}_1^{\text{up}}$ and $\mathcal{S}_2^{\text{up}}$ are the same as defined in Theorem 2.

Proof: See Appendix 3.

Theorem 4: For any $\lambda = \frac{\text{SNR}_1}{\text{SNR}_2}$, and any $g(\text{SNR}_1)$ satisfying the properties in Assumption 1, the multiaccess slope region achieved by superposition coding with variable decoding order is

$$\mathcal{S} = \Big\{(\mathcal{S}_1, \mathcal{S}_2) : 0 \le \mathcal{S}_1 \le \mathcal{S}_1^{\text{up}}, \quad 0 \le \mathcal{S}_2 \le \mathcal{S}_2^{\text{up}}, \\ \frac{\lambda(\mathbb{E}\{z_1\})^2}{\mathbb{E}\{z_1 z_2\}}\left(\frac{1}{\mathcal{S}_1} - \frac{1}{\mathcal{S}_1^{\text{up}}}\right) + \frac{(\mathbb{E}\{z_2\})^2}{\lambda \mathbb{E}\{z_1 z_2\}}\left(\frac{1}{\mathcal{S}_2} - \frac{1}{\mathcal{S}_2^{\text{up}}}\right) = 1\Big\}, \tag{22}$$

where $\mathcal{S}_1^{\text{up}}$ and $\mathcal{S}_2^{\text{up}}$ are the same as defined in Theorem 2.

Proof: See Appendix 4.

Remark 3: Comparing (63) with (65) or (64) with (66) in the proof of Theorem 4 in Appendix 4, we see that different decoding orders do not change the wideband slope values for given user only if $g(0) = z_1$, i.e., the z_1-z_2 space is equally divided. One more interesting remark is that if we compare the third conditions in (21) and (22), we notice that fixed decoding order achieves the same performance as variable decoding order.

Remark 4: It is interesting to note in the above results that, unlike the minimum bit energy levels, the wideband slopes depend on the QoS exponents θ_1 and θ_2 through β_1 and β_2. Indeed, as can be seen from the expressions of the upper bounds $\mathcal{S}_1^{\text{up}}$ and $\mathcal{S}_2^{\text{up}}$, the wideband slopes tend to diminish as QoS constraints become more stringent and θ_1 and θ_2 increase. Smaller slopes indicate that at a given energy per bit level greater than $\frac{E_b^r}{N_0}_{\text{min}}$, a smaller spectral efficiency is attained. Therefore, spectral efficiency degrades under more strict QoS constraints. Equivalently, to achieve the same level of spectral efficiency, higher energy per bit is required. Hence, from this perspective, a penalty in energy efficiency is experienced as buffer limitations become more stringent.

In the following result, we establish the suboptimality of TDMA.

Theorem 5: The wideband slope region of TDMA is inside the one attained with superposition coding.

Proof: We only need to consider the third conditions of (20) and (21). Substituting (58) and (59) into the left-hand side (LHS) of the third constraint in (20), we obtain

$$\kappa_{12} + \kappa_{22} + \frac{\mathbb{E}\{z_1^2\}}{\mathbb{E}\{z_1^2\} + \frac{2(1-\tau)}{\lambda}\mathbb{E}\{z_1 z_2\}} + \frac{\mathbb{E}\{z_2^2\}}{\mathbb{E}\{z_2^2\} + 2\lambda\tau\mathbb{E}\{z_1 z_2\}}. \tag{23}$$

Comparing the sum of the last two terms with 1 (or more precisely subtracting 1 from the sum), we can write

$$\frac{\mathbb{E}\{z_1^2\}}{\mathbb{E}\{z_1^2\} + \frac{2(1-\tau)}{\lambda}\mathbb{E}\{z_1 z_2\}} + \frac{\mathbb{E}\{z_2^2\}}{\mathbb{E}\{z_2^2\} + 2\lambda\tau\mathbb{E}\{z_1 z_2\}} - 1 \\ = \frac{\mathbb{E}\{z_1^2\}\mathbb{E}\{z_2^2\} - 4\tau(\mathbb{E}\{z_1 z_2\})^2 + 4(\mathbb{E}\{z_1 z_2\})^2\tau^2}{\left(\mathbb{E}\{z_1^2\} + \frac{2(1-\tau)}{\lambda}\mathbb{E}\{z_1 z_2\}\right)\left(\mathbb{E}\{z_2^2\} + 2\lambda\tau\mathbb{E}\{z_1 z_2\}\right)}. \tag{24}$$

We are interested in the numerator which is a quadratic function of the parameter τ . We note that the discriminant of this quadratic function satisfies

$$\Delta = 16(\mathbb{E}\{z_1 z_2\})^4 - 16(\mathbb{E}\{z_1 z_2\})^2\mathbb{E}\{z_1^2\}\mathbb{E}\{z_2^2\} \\ = 16(\mathbb{E}\{z_1 z_2\})^2\left((\mathbb{E}\{z_1 z_2\})^2 - \mathbb{E}\{z_1^2\}\mathbb{E}\{z_2^2\}\right) \le 0 \tag{25}$$

where the Cauchy-Schwarz inequality $(\mathbb{E}\{z_1 z_2\})^2 \le \mathbb{E}\{z_1^2\}\mathbb{E}\{z_2^2\}$ is used. Thus, the numerator of (24) is always nonnegative, i.e., the slope region achieved by TDMA is inside the one achieved by superposition coding. The equality holds only if z_1 and z_2 are linearly dependent. □

In Figure 2, we plot the slope regions in independent Rayleigh fading channels with variances $\mathbb{E}\{z_1\} = \mathbb{E}\{z_2\} = 1$. We assume $\beta_1 = 1$ and $\beta_2 = 2$. From the figure, we immediately observe the suboptimality of TDMA compared with superposition coding.

5. Energy efficiency in the wideband regime

In this section, we consider the wideband regime in which the overall bandwidth of the system B is large. Let $\zeta = \frac{1}{B}$. Similar as in [13], we know that the minimum bit energy achieved in sparse multipath fading channels[c] as $B \to \infty$ (or equivalently $\zeta \to 0$) can be expressed as

$$\frac{E_{b,i}}{N_0}_{\text{min}} = \lim_{\zeta \to 0} \frac{\overline{P}_i \zeta / N_0}{\mathrm{C}_i(\zeta)} = \frac{\overline{P}_i / N_0}{\dot{\mathrm{C}}_i(0)}, \quad i = 1, 2. \tag{26}$$

To make the analysis more clear, below we first express the capacity expressions in (12)-(17) as functions of ζ. (12) and (13) can be rewritten as

$$\mathrm{C}_1(\zeta) = -\frac{\zeta}{\theta_1 T}\log_e \mathbb{E}_z\left\{ e^{-\frac{\delta\theta_1 T}{\zeta}\log_2\left(1 + \frac{\overline{P}_1 z_1 \zeta}{\delta N_0}\right)}\right\}, \tag{27}$$

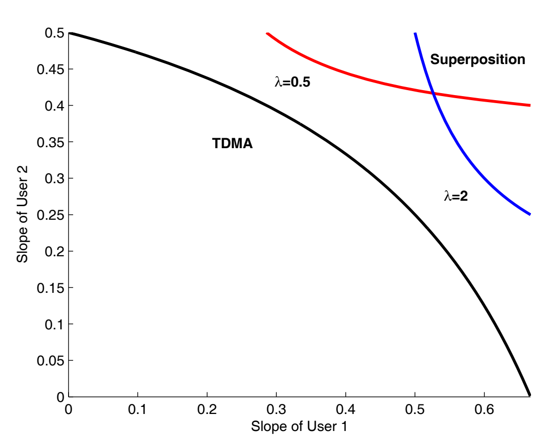

Figure 2 The slope regions for independent Rayleigh fading channels.

and

$$C_2(\zeta) = -\frac{\zeta}{\theta_2 T}\log_e \mathbb{E}_z\left\{e^{-\frac{(1-\delta)\theta_2 T}{\zeta}\log_2\left(1+\frac{\overline{P}_2 z_2 \zeta}{(1-\delta)N_0}\right)}\right\}, \quad (28)$$

respectively.

For superposition coding with fixed decoding order, and fixed $\lambda = \frac{\text{SNR}_1}{\text{SNR}_2} = \frac{\overline{P}_1\zeta/N_0}{\overline{P}_2\zeta/N_0} = \frac{\overline{P}_1}{\overline{P}_2}$, (14) and (15) now become

$$C_1(\zeta) = -\frac{\zeta}{\theta_1 T}\log_e \mathbb{E}_z\left\{e^{-\frac{\theta_1 T}{\zeta}\left(\tau\log_2\left(1+\frac{\bar{P}_1 z_1\zeta}{N_0}\right)+(1-\tau)\log_2\left(1+\frac{\frac{\bar{P}_1 z_1\zeta}{N_0}}{1+\frac{\bar{P}_2 z_2\zeta}{N_0}}\right)\right)}\right\}, \quad (29)$$

$$C_2(\zeta) = -\frac{\zeta}{\theta_2 T}\log_e \mathbb{E}_z\left\{e^{-\frac{\theta_2 T}{\zeta}\left(\tau\log_2\left(1+\frac{\frac{\bar{P}_2 z_2\zeta}{N_0}}{1+\frac{\bar{P}_1 z_1\zeta}{N_0}}\right)+(1-\tau)\log_2\left(1+\frac{\bar{P}_2 z_2\zeta}{N_0}\right)\right)}\right\}. \quad (30)$$

Note that we can write $g(\text{SNR}_1)$ as $g\left(\frac{\overline{P}_1\zeta}{N_0}\right)$, so similarly we can write (16) and (17) as functions of ζ

$$C_1(\zeta) = -\frac{\zeta}{\theta_1 T}\log_e\left(\int_0^\infty \int_{g\left(\frac{\overline{P}_1\zeta}{N_0}\right)}^\infty e^{-\frac{\theta_1 T}{\zeta}\log_2\left(1+\frac{\overline{P}_1 z_1\zeta}{N_0}\right)} p_z(z_1,z_2)\,dz_2dz_1 + \int_0^\infty \int_0^{g\left(\frac{\overline{P}_1\zeta}{N_0}\right)} e^{-\frac{\theta_1 T}{\zeta}\log_2\left(1+\frac{\frac{\overline{P}_1 z_1\zeta}{N_0}}{1+\frac{\overline{P}_2 z_2\zeta}{N_0}}\right)} p_z(z_1,z_2)\,dz_2dz_1\right) \quad (31)$$

$$C_2(\zeta) = -\frac{\zeta}{\theta_2 T}\log_e\left(\int_0^\infty \int_0^{g\left(\frac{\overline{P}_2\zeta}{N_0}\right)} e^{-\frac{\theta_2 T}{\zeta}\log_2\left(1+\frac{\overline{P}_2 z_2\zeta}{N_0}\right)} p_z(z_1,z_2)\,dz_2dz_1 + \int_0^\infty \int_{g\left(\frac{\overline{P}_2\zeta}{N_0}\right)}^\infty e^{-\frac{\theta_2 T}{\zeta}\log_2\left(1+\frac{\frac{\overline{P}_2 z_2\zeta}{N_0}}{1+\frac{\overline{P}_1 z_1\zeta}{N_0}}\right)} p_z(z_1,z_2)\,dz_2dz_1\right). \quad (32)$$

Then we immediately have the following result.

Theorem 6: For all g(SNR1) satisfying the properties in Assumption 1, the minimum bit energies for the two-user multiaccess fading channel in the wideband regime attained through TDMA, superposition coding with fixed decoding order, and superposition decoding with varying decoding order, depend on the individual QoS constraints at the users and are given by

$$\left.\frac{E_{b,1}}{N_0}\right._{\min} = \frac{-\frac{\theta_1 T\overline{P}_1}{N_0}}{\log_e \mathbb{E}_{z_1}\left\{e^{-\frac{\theta_1 T\overline{P}_1}{N_0\log_e 2}z_1}\right\}}, \quad (33)$$

$$\left.\frac{E_{b,2}}{N_0}\right._{\min} = \frac{-\frac{\theta_2 T\overline{P}_2}{N_0}}{\log_e \mathbb{E}_{z_2}\left\{e^{-\frac{\theta_2 T\overline{P}_2}{N_0\log_e 2}z_2}\right\}}, \quad (34)$$

respectively.

Proof: See Appendix 5.

Remark 5: As Theorem 6 shows, the same minimum bit energy is achieved through different transmission strategies (e.g., TDMA or superposition coding) and different reception schemes (e.g., fixed or variable decoding orders), and therefore TDMA is optimally energy efficient in the wideband regime as $B \to \infty$. As before, Theorem 6 can be readily extended and similar expressions for the minimum energy per bit can be easily obtained for cases in which there are more than 2 users, i.e., $M \geq 2$.

Remark 6: A stark difference from the result in Theorem 1 is that the minimum bit energy now varies with the specific QoS constraints at the users. When $\theta = 0$, we can immediately show that the right-hand sides of (33) and (34) become $\frac{\log_e 2}{\mathbb{E}\{z_1\}}$ and $\frac{\log_e 2}{\mathbb{E}\{z_2\}}$, respectively, which is equivalent to (19). For $\theta > 0$, the energy efficiency is now adversely affected by the buffer constraints in the wideband regime.

Similarly as in Section 4, we next investigate the wideband slopes in order to quantify the performances and energy efficiencies of different transmission and reception methods in the non-asymptotic regime in which the bandwidth B is large but finite. We have the following results.

Theorem 7: In the wideband regime, the multiaccess slope region achieved by TDMA is given by

$$\mathcal{S} = \left\{(\mathcal{S}_1,\mathcal{S}_2): 0 \leq \mathcal{S}_1 \leq \mathcal{S}_1^{\text{up}},\quad 0 \leq \mathcal{S}_2 \leq \mathcal{S}_2^{\text{up}},\quad \frac{\mathcal{S}_1}{\mathcal{S}_1^{\text{up}}} + \frac{\mathcal{S}_2}{\mathcal{S}_2^{\text{up}}} \leq 1\right\} \quad (35)$$

where

$$\mathcal{S}_1^{\text{up}} = 2\left(\frac{N_0\log_e 2}{\theta_1 T\overline{P}_1}\right)^2 \frac{\mathbb{E}_{z_1}\left\{e^{-\frac{\theta_1 T\overline{P}_1}{N_0\log_e 2}z_1}\right\}\left(\log_e \mathbb{E}_{z_1}\left\{e^{-\frac{\theta_1 T\overline{P}_1}{N_0\log_e 2}z_1}\right\}\right)^2}{\mathbb{E}_{z_1}\left\{e^{-\frac{\theta_1 T\overline{P}_1}{N_0\log_e 2}z_1} z_1^2\right\}},$$

$$\mathcal{S}_2^{\text{up}} = 2\left(\frac{N_0\log_e 2}{\theta_2 T\overline{P}_2}\right)^2 \frac{\mathbb{E}_{z_2}\left\{e^{-\frac{\theta_2 T\overline{P}_2}{N_0\log_e 2}z_2}\right\}\left(\log_e \mathbb{E}_{z_2}\left\{e^{-\frac{\theta_2 T\overline{P}_2}{N_0\log_e 2}z_2}\right\}\right)^2}{\mathbb{E}_{z_2}\left\{e^{-\frac{\theta_2 T\overline{P}_2}{N_0\log_e 2}z_2} z_2^2\right\}}.$$

Proof: See Appendix 6.

Theorem 8: In the wideband regime, the multiaccess slope region achieved by superposition coding with fixed decoding order is

$$\mathcal{S} = \left\{(\mathcal{S}_1,\mathcal{S}_2) : 0 \le \mathcal{S}_1 \le \mathcal{S}_1^{\text{up}},\quad 0 \le \mathcal{S}_2 \le \mathcal{S}_2^{\text{up}},\quad \left(\frac{N_0\log_e 2}{\theta_1 T}\right)^2 \frac{\left(\log_e \mathbb{E}_{z_1}\left\{e^{-\frac{\theta_1 T\bar{P}_1}{N_0\log_e 2}z_1}\right\}\right)^2 \mathbb{E}_{z_1}\left\{e^{-\frac{\theta_1 T\bar{P}_1}{N_0\log_e 2}z_1}\right\}}{\bar{P}_1\bar{P}_2\mathbb{E}_{\mathbf{z}}\left\{e^{-\frac{\theta_1 T\bar{P}_1}{N_0\log_e 2}z_1} z_1 z_2\right\}} \left(\frac{1}{\mathcal{S}_1} - \frac{1}{\mathcal{S}_1^{\text{up}}}\right) + \left(\frac{N_0\log_e 2}{\theta_2 T}\right)^2 \frac{\left(\log_e \mathbb{E}_{z_2}\left\{e^{-\frac{\theta_2 T\bar{P}_2}{N_0\log_e 2}z_2}\right\}\right)^2 \mathbb{E}_{z_2}\left\{e^{-\frac{\theta_2 T\bar{P}_2}{N_0\log_e 2}z_2}\right\}}{\bar{P}_1\bar{P}_2\mathbb{E}_{\mathbf{z}}\left\{e^{-\frac{\theta_2 T\bar{P}_2}{N_0\log_e 2}z_2} z_1 z_2\right\}} \left(\frac{1}{\mathcal{S}_2} - \frac{1}{\mathcal{S}_2^{\text{up}}}\right) = 1\right\} \tag{36}$$

where $\mathcal{S}_1^{\text{up}}$ and $\mathcal{S}_2^{\text{up}}$ are defined in Theorem 7.

Proof: See Appendix 7.

Theorem 9: For any $g(\text{SNR1})$ satisfying the properties in Assumption 1, the multiaccess slope regions achieved by superposition coding with variable decoding order in the wideband regime are different for different decoding orders. The slope region is

$$\mathcal{S} = \bigcup_{\{g(0)\}} \left\{(\mathcal{S}_1,\mathcal{S}_2) : 0 \le \mathcal{S}_1 \le 2\left(\frac{N_0\log_e 2}{\theta_1 T}\right)^2 \frac{\left(\log_e \mathbb{E}_{z_1}\left\{e^{-\frac{\theta_1 T\overline{P}_1}{N_0\log_e 2}z_1}\right\}\right)^2 \mathbb{E}_{z_1}\left\{e^{-\frac{\theta_1 T\overline{P}_1}{N_0\log_e 2}z_1}\right\}}{\overline{P}_1^2\mathbb{E}_{z_1}\left\{e^{-\frac{\theta_1 T\overline{P}_1}{N_0\log_e 2}z_1} z_1^2\right\} + 2\overline{P}_1\overline{P}_2\int_0^\infty\int_0^{g(0)} e^{-\frac{\theta_1 T\overline{P}_1}{N_0\log_e 2}z_1} z_1 z_2 p(z_1,z_2)\,dz_2 dz_1},\right.$$
$$\left. 0 \le \mathcal{S}_2 \le 2\left(\frac{N_0\log_e 2}{\theta_2 T}\right)^2 \frac{\left(\log_e \mathbb{E}_{z_2}\left\{e^{-\frac{\theta_2 T\overline{P}_2}{N_0\log_e 2}z_2}\right\}\right)^2 \mathbb{E}_{z_2}\left\{e^{-\frac{\theta_2 T\overline{P}_2}{N_0\log_e 2}z_2}\right\}}{\overline{P}_2^2\mathbb{E}_{z_2}\left\{e^{-\frac{\theta_2 T\overline{P}_2}{N_0\log_e 2}z_2} z_2^2\right\} + 2\overline{P}_1\overline{P}_2\int_0^\infty\int_{g(0)}^\infty e^{-\frac{\theta_2 T\overline{P}_2}{N_0\log_e 2}z_2} z_1 z_2 p(z_1,z_2)\,dz_2 dz_1}\right\} \tag{37}$$

if the decoding order is (1,2) when $z_2 < g(z_1, \text{SNR}_1)$, and the decoding order is (2,1) when $z_2 > g(z_1, \text{SNR}_1)$.

The slope region is

$$\mathcal{S} = \bigcup_{\{g(0)\}} \left\{(\mathcal{S}_1,\mathcal{S}_2) : 0 \le \mathcal{S}_1 \le 2\left(\frac{N_0\log_e 2}{\theta_1 T}\right)^2 \frac{\left(\log_e \mathbb{E}_{z_1}\left\{e^{-\frac{\theta_1 T\overline{P}_1}{N_0\log_e 2}z_1}\right\}\right)^2 \mathbb{E}_{z_1}\left\{e^{-\frac{\theta_1 T\overline{P}_1}{N_0\log_e 2}z_1}\right\}}{\overline{P}_1^2\mathbb{E}_{z_1}\left\{e^{-\frac{\theta_1 T\overline{P}_1}{N_0\log_e 2}z_1} z_1^2\right\} + 2\overline{P}_1\overline{P}_2\int_0^\infty\int_{g(0)}^\infty e^{-\frac{\theta_1 T\overline{P}_1}{N_0\log_e 2}z_1} z_1 z_2 p(z_1,z_2)\,dz_2 dz_1},\right.$$
$$\left. 0 \le \mathcal{S}_2 \le 2\left(\frac{N_0\log_e 2}{\theta_2 T}\right)^2 \frac{\left(\log_e \mathbb{E}_{z_2}\left\{e^{-\frac{\theta_2 T\overline{P}_2}{N_0\log_e 2}z_2}\right\}\right)^2 \mathbb{E}_{z_2}\left\{e^{-\frac{\theta_2 T\overline{P}_2}{N_0\log_e 2}z_2}\right\}}{\overline{P}_2^2\mathbb{E}_{z_2}\left\{e^{-\frac{\theta_2 T\overline{P}_2}{N_0\log_e 2}z_2} z_2^2\right\} + 2\overline{P}_1\overline{P}_2\int_0^\infty\int_0^{g(0)} e^{-\frac{\theta_2 T\overline{P}_2}{N_0\log_e 2}z_2} z_1 z_2 p(z_1,z_2)\,dz_2 dz_1}\right\} \tag{38}$$

if the decoding order is (2,1) when $z_2 < g(z_1, \text{SNR}_1)$, and the decoding order is (1,2) when $z_2 > g(z_1, \text{SNR}_1)$.

Proof: See Appendix 8.

Remark 7: Unlike previous discussions, we have no closed form expression for the wideband slope region achieved by superposition coding with variable decoding order in the wideband regime. Another observation in the above result is that different decoding orders can result in different wideband slope regions.

Below we show the superiority of superposition coding with variable decoding compared with fixed decoding order.

Theorem 10: Superposition coding with variable decoding order achieves better performance in terms of wideband slope region with respect to superposition coding with fixed decoding order.

Proof: See Appendix 9.

In the following, we present the condition under which the suboptimality of TDMA compared with superposition coding with fixed decoding order can be established.

Theorem 11: If the following is satisfied

$$\mathbb{E}_{\mathbf{z}}\left\{e^{-\frac{\theta_1 T\bar{P}_1}{N_0\log_e 2}z_1} z_1 z_2\right\}\mathbb{E}_{\mathbf{z}}\left\{e^{-\frac{\theta_2 T\bar{P}_2}{N_0\log_e 2}z_2} z_1 z_2\right\} \le \mathbb{E}_{z_1}\left\{e^{-\frac{\theta_1 T\bar{P}_1}{N_0\log_e 2}z_1} z_2^2\right\}\mathbb{E}_{z_2}\left\{e^{-\frac{\theta_2 T\bar{P}_2}{N_0\log_e 2}z_2} z_2^2\right\}, \tag{39}$$

then the wideband slope region of TDMA is inside the one attained with superposition coding with fixed decoding order.

Proof: We consider the third conditions in (35) and (36). Substituting (86) and (87) into the LHS of the third condition in (35), we have

$$1-\frac{2(1-\tau)\bar{P}_2\mathbb{E}_z\left\{e^{-\frac{\theta_1 T\bar{P}_1}{N_0\log_e 2}z_1}z_1z_2\right\}}{\bar{P}_1\mathbb{E}_{z_1}\left\{e^{-\frac{\theta_1 T\bar{P}_1}{N_0\log_e 2}z_1}z_1^2\right\}+2(1-\tau)\mathbb{E}_z\left\{e^{-\frac{\theta_1 T\bar{P}_1}{N_0\log_e 2}z_1}z_1z_2\right\}}+\frac{\bar{P}_2\mathbb{E}_{z_2}\left\{e^{-\frac{\theta_2 T\bar{P}_2}{N_0\log_e 2}z_2}z_2^2\right\}}{\bar{P}_2\mathbb{E}_{z_2}\left\{e^{-\frac{\theta_2 T\bar{P}_2}{N_0\log_e 2}z_2}z_2^2\right\}+2\tau\bar{P}_1\mathbb{E}_z\left\{e^{-\frac{\theta_2 T\bar{P}_2}{N_0\log_e 2}z_2}z_1z_2\right\}} \tag{40}$$

So if the wideband slope region is inside the one attained with superposition coding with fixed decoding order, we must have the above value to be greater than 1 for all $0 \le \tau \le 1$. After subtracting 1 from (40), we can obtain

$$\begin{aligned}&\frac{\bar{P}_1}{\bar{P}_1\mathbb{E}_{z_1}\left\{e^{-\frac{\theta_1 T\bar{P}_1}{N_0\log_e 2}z_1}z_1^2\right\}+2(1-\tau)\mathbb{E}_z\left\{e^{-\frac{\theta_1 T\bar{P}_1}{N_0\log_e 2}z_1}z_1z_2\right\}}\\ &\frac{\bar{P}_2}{\bar{P}_2\mathbb{E}_{z_2}\left\{e^{-\frac{\theta_2 T\bar{P}_2}{N_0\log_e 2}z_2}z_2^2\right\}+2\tau\bar{P}_1\mathbb{E}_z\left\{e^{-\frac{\theta_2 T\bar{P}_2}{N_0\log_e 2}z_2}z_1z_2\right\}}\\ &\times\Bigg(4\mathbb{E}_z\left\{e^{-\frac{\theta_1 T\bar{P}_1}{N_0\log_e 2}z_1}z_1z_2\right\}\mathbb{E}_z\left\{e^{-\frac{\theta_2 T\bar{P}_2}{N_0\log_e 2}z_2}z_1z_2\right\}\tau^2\\ &\quad-4\mathbb{E}_z\left\{e^{-\frac{\theta_1 T\bar{P}_1}{N_0\log_e 2}z_1}z_1z_2\right\}\mathbb{E}_z\left\{e^{-\frac{\theta_2 T\bar{P}_2}{N_0\log_e 2}z_2}z_1z_2\right\}\tau\\ &\quad+\mathbb{E}_{z_1}\left\{e^{-\frac{\theta_1 T\bar{P}_1}{N_0\log_e 2}z_1}z_1^2\right\}\mathbb{E}_{z_2}\left\{e^{-\frac{\theta_2 T\bar{P}_2}{N_0\log_e 2}z_2}z_2^2\right\}\Bigg)\end{aligned} \tag{41}$$

The first two terms of the multiplication are positive values. The minimum value of the third term which is a quadratic function of τ is achieved at $\tau = \frac{1}{2}$, and the minimum value is

$$\begin{aligned}&\mathbb{E}_{z_1}\left\{e^{-\frac{\theta_1 T\bar{P}_1}{N_0\log_e 2}z_1}z_1^2\right\}\mathbb{E}_{z_2}\left\{e^{-\frac{\theta_2 T\bar{P}_2}{N_0\log_e 2}z_2}z_2^2\right\}\\ &-\mathbb{E}_z\left\{e^{-\frac{\theta_1 T\bar{P}_1}{N_0\log_e 2}z_1}z_1z_2\right\}\mathbb{E}_z\left\{e^{-\frac{\theta_2 T\bar{P}_2}{N_0\log_e 2}z_2}z_1z_2\right\}\end{aligned} \tag{42}$$

Thus, we obtain the condition stated in (39) for TDMA to be suboptimal. □

Remark 8: It is interesting that if the condition (39) is not satisfied, TDMA can achieve some points outside the wideband slope region attained with superposition coding with fixed decoding order. This tells us that TDMA can be a better choice compared with superposition coding with fixed decoding order in some cases. As an additional point, we note that if, on the other hand, the condition in (39) is satisfied, TDMA performs worse than superposition coding with variable decoding order as well due to the characterization in Theorem 10.

In the numerical results, we plot the wideband slope regions for independent Rayleigh fading channels with variances $\mathbb{E}\{z_1\} = \mathbb{E}\{z_2\} = 1$. We assume $\theta_1 = 0.01$, $\theta_2 = 0.1$, $T = 2$ ms. In Figure 3, we assume $\frac{\bar{P}_1}{N_0} = 2\frac{\bar{P}_2}{N_0} = 10^4$. The LHS of (39) is 0.1009, while the right-hand side is 0.1283. Hence, the inequality is satisfied. From the figure, we can see that TDMA is suboptimal compared with superposition coding. In Figure 4, we assume $\frac{\bar{P}_1}{N_0} = \frac{1}{2}\frac{\bar{P}_2}{N_0} = 10^4$. The LHS of (39) is 0.0131, while the right-hand side is 0.006. Hence, the inequality is not satisfied. Confirming the above discussion, we can observe in the figure that TDMA indeed achieves points outside the slope region attained with superposition coding with fixed decoding order.

6. Conclusion

In this article, we have analyzed the energy efficiency of two-user multiaccess fading channels under QoS constraints by employing the effective capacity as a measure of the maximal throughput. We have characterized the minimum bit energy and the wideband slope regions for different transmission strategies. We have conducted our analysis in two regimes: low-power regime and wideband regime. Through this analysis, we have shown the impact of QoS constraints on the energy efficiency of multiaccess

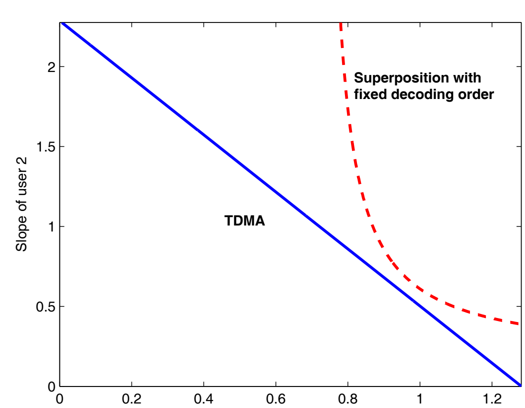

Figure 3 The slope regions for independent Rayleigh fading channels.

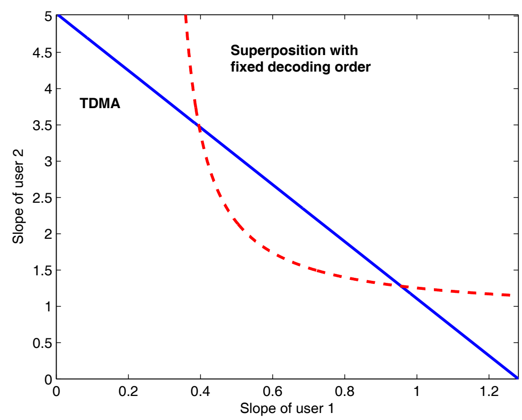

Figure 4 The slope regions for independent Rayleigh fading channels.

fading channels. More specifically, we have found that the minimum bit energies are the same for each user when different transmission and reception techniques are employed. While these minimum values are equal those that can be attained in the absence of QoS constraints in the low-power regime, we have shown that strictly higher bit energy values, which depend on the QoS constraints, are needed in the wideband regime. We have also seen that while TDMA is suboptimal in the low-power regime when wideband slope regions are considered, it can outperform superposition coding with fixed decoding order in the wideband regime. Moreover, we have proven in the wideband regime that varying the decoding order can achieve larger slope region when compared with fixed decoding order for superposition coding. Numerical results validating our results are provided as well.

Appendix

1. Proof of Theorem 1

Consider the TDMA strategy. Taking the first derivative of the functions in (12) and (13) and letting $\text{SNR}_1 = 0$, $\text{SNR}_2 = 0$, we obtain

$$\dot{C}_1(0) = \frac{\mathbb{E}\{z_1\}}{\log_e 2}, \tag{43}$$

$$\dot{C}_2(0) = \frac{\mathbb{E}\{z_2\}}{\log_e 2}. \tag{44}$$

Substituting (43) and (44) into (9), we have

$$\frac{E_{b,1}}{N_0}_{\min} = \frac{\log_e 2}{\mathbb{E}\{z_1\}}, \tag{45}$$

$$\frac{E_{b,2}}{N_0}_{\min} = \frac{\log_e 2}{\mathbb{E}\{z_2\}} \tag{46}$$

which imply (19) according to (18).

For the superposition coding with fixed decoding, evaluating the first derivative of (14) and (15) at $\text{SNR}_1 = 0$ and $\text{SNR}_2 = 0$, we immediately obtain

$$\dot{C}_1(0) = \frac{\mathbb{E}\{z_1\}}{\log_e 2} \tag{47}$$

$$\dot{C}_2(0) = \frac{\mathbb{E}\{z_2\}}{\log_e 2} \tag{48}$$

which again imply (19) taking into consideration (9) and (18).

Next, we prove the result for the variable decoding case. First, we consider (16) and (17) with the associated decoding order assignment. The first derivative of (16) can be expressed as

$$\begin{aligned}
\dot{C}_1(\text{SNR}_1) &= -\frac{\dot{\phi}_1}{\beta_1\phi_1\log_e 2} \\
&= -\frac{1}{\beta_1\phi_1\log_e 2}\Bigg(-\int_0^\infty (1+\text{SNR}_1 z_1)^{-\beta_1} \\
&\qquad p\left(z_1, g(\text{SNR}_1)\right)\dot{g}(\text{SNR})\,dz_1 \\
&\quad -\beta_1\int_0^\infty\int_{g(\text{SNR}_1)}^\infty (1+\text{SNR}_1 z_1)^{-\beta_1-1} z_1 p(z_1,z_2)\,dz_2 z_1 \\
&\quad +\int_0^\infty\left(1+\frac{\text{SNR}_1 z_1}{1+\text{SNR}_1 g(\text{SNR}_1)/\lambda}\right)^{-\beta_1} \\
&\qquad p\left(z_1, g(\text{SNR}_1)\right)\dot{g}(\text{SNR}_1)\,dz_1 \\
&\quad -\beta_1\int_0^\infty\int_0^{g(\text{SNR}_1)}\left(1+\frac{\text{SNR}_1 z_1}{1+\text{SNR}_1 z_2/\lambda}\right)^{-\beta_1-1} \\
&\qquad \frac{z_1}{(1+\text{SNR}_1 z_2/\lambda)^2}p(z_1,z_2)\,dz_2 dz_1\Bigg)
\end{aligned} \tag{49}$$

where $\dot{\phi}_1$ is the first derivative of φ_1, which is defined as

$$\phi_1 = \int_0^\infty \int_{g(z_1,\mathrm{SNR}_1)}^\infty e^{-\theta_1 TB \log_2(1+\mathrm{SNR}_1 z_1)} p_{\mathbf{z}}(z_1,z_2)\, dz_2 dz_1 + \int_0^\infty \int_0^{g(z_1,\mathrm{SNR}_1)} e^{-\theta_1 TB \log_2\left(1+\frac{\mathrm{SNR}_1 z_1}{1+\mathrm{SNR}_1 z_2/\lambda}\right)} p_{\mathbf{z}}(z_1,z_2)\, dz_2 dz_1. \tag{50}$$

Under the assumptions that $g(0)$ and $\dot{g}(0)$ are finite, we can easily see from (49) that letting $\mathrm{SNR}_1 = 0$ leads to

$$\dot{\mathrm{C}}_1(0) = \frac{\mathbb{E}\{z_1\}}{\log_e 2}. \tag{51}$$

Similarly, taking the first derivative of (17) and letting $\mathrm{SNR}_2 = 0$, we obtain

$$\dot{\mathrm{C}}_2(0) = \frac{\mathbb{E}\{z_2\}}{\log_e 2}. \tag{52}$$

Applying the definitions (9) and (18), we prove (19) for this decoding order assignment. For the reverse decoding order assignment (i.e., users are decoded in the order (2,1) if $z_2 < g(\mathrm{SNR}_1)$ and decoded in the order (1,2) if $z_2 > g(\mathrm{SNR}_1)$), following similar steps, we again obtain the result in (19). □

2. Proof of Theorem 2

Taking the second derivatives of the functions in (12) and (13) and letting $\mathrm{SNR}_1 = 0$, $\mathrm{SNR}_2 = 0$, we obtain

$$\ddot{\mathrm{C}}_1(0) = \frac{1}{\log_e 2}\left(\beta_1\left((\mathbb{E}\{z_1\})^2 - \mathbb{E}\{z_1^2\}\right) - \frac{1}{\delta}\mathbb{E}\{z_1^2\}\right) \tag{53}$$

and

$$\ddot{\mathrm{C}}_2(0) = \frac{1}{\log_e 2}\left(\beta_2\left((\mathbb{E}\{z_2\})^2 - \mathbb{E}\{z_2^2\}\right) - \frac{1}{1-\delta}\mathbb{E}\{z_2^2\}\right). \tag{54}$$

Combining (43), (44), (53), and (54) with (11), we now get

$$\mathcal{S}_1 = \frac{2(\mathbb{E}\{z_1\})^2}{\beta_1\left(\mathbb{E}\{z_1^2\} - (\mathbb{E}\{z_1\})^2\right) + \frac{1}{\delta}\mathbb{E}\{z_1^2\}} \tag{55}$$

$$\mathcal{S}_2 = \frac{2(\mathbb{E}\{z_2\})^2}{\beta_2\left(\mathbb{E}\{z_2^2\} - (\mathbb{E}\{z_2\})^2\right) + \frac{1}{1-\delta}\mathbb{E}\{z_2^2\}} \tag{56}$$

which, after eliminating δ, provide us the third condition in (20). □

3. Proof of Theorem 3

The second derivatives of the functions (14) and (15) at zero signal-to-noise ratio are

$$\begin{aligned} \ddot{\mathrm{C}}_1(0) &= \frac{1}{\log_e 2}\left(\beta_1(\mathbb{E}\{z_1\})^2 - (\beta_1+1)\,\mathbb{E}\{z_1^2\} - \frac{2(1-\tau)}{\lambda}\mathbb{E}\{z_1 z_2\}\right) \\ \ddot{\mathrm{C}}_2(0) &= \frac{1}{\log_e 2}\left(\beta_2(\mathbb{E}\{z_2\})^2 - (\beta_2+1)\,\mathbb{E}\{z_2^2\} - 2\lambda\tau\mathbb{E}\{z_1 z_2\}\right). \end{aligned} \tag{57}$$

Then, the wideband slopes are given by

$$\mathcal{S}_1 = \frac{2(\mathbb{E}\{z_1\})^2}{\beta_1\left(\mathbb{E}\{z_1^2\} - (\mathbb{E}\{z_1\})^2\right) + \mathbb{E}\{z_1^2\} + \frac{2(1-\tau)}{\lambda}\mathbb{E}\{z_1 z_2\}} \tag{58}$$

$$\mathcal{S}_2 = \frac{2(\mathbb{E}\{z_2\})^2}{\beta_2\left(\mathbb{E}\{z_2^2\} - (\mathbb{E}\{z_2\})^2\right) + \mathbb{E}\{z_2^2\} + 2\lambda\tau\mathbb{E}\{z_1 z_2\}}. \tag{59}$$

After solving for τ in (58) and (59) and subtracting the resulting equations, we obtain the third condition in (21) □

4. Proof of Theorem 4

We need to consider the wideband slopes for different decoding order assignments. Due to the complex expressions involved, we here state the derivation for $\mathcal{S}_1$ for the case in which the decoding order is (1,2) when $z_2 < g(z_1, \mathrm{SNR}_1)$, and the decoding order is (2,1) when $z_2 > g(z_1, \mathrm{SNR}_1)$. Taking the second derivative of (16), we have

$$\ddot{\mathrm{C}}_1(\mathrm{SNR}_1) = -\frac{\ddot{\phi}_1\phi_1 - (\dot{\phi}_1)^2}{\beta_1\phi_1^2\log_e 2} \tag{60}$$

where $\dot{\phi}_1$ is provided in (49) and $\ddot{\phi}_1$ is given by

$$\begin{aligned} \ddot{\phi}_1 = &\int_0^\infty \left(1 + \frac{\mathrm{SNR}_1 z_1}{1+\mathrm{SNR}_1 g(\mathrm{SNR}_1)/\lambda}\right)^{-\beta_1} p\left(z_1, g(\mathrm{SNR}_1)\right)\ddot{g}(\mathrm{SNR}_1)\, dz_1 \\ &- 2\beta_1 \int_0^\infty \left(1 + \frac{\mathrm{SNR}_1 z_1}{1+\mathrm{SNR}_1 g(\mathrm{SNR}_1)/\lambda}\right)^{-\beta_1-1} \\ &\frac{z_1}{\left(1+\mathrm{SNR}_1 g(\mathrm{SNR}_1)/\lambda\right)^2} p\left(z_1, g(\mathrm{SNR}_1)\right)\dot{g}(\mathrm{SNR}_1)\, dz_1 \\ &+ \int_0^\infty \left(1 + \frac{\mathrm{SNR}_1 z_1}{1+\mathrm{SNR}_1 g(\mathrm{SNR}_1)/\lambda}\right)^{-\beta_1} \dot{p}\left(z_1, g(\mathrm{SNR}_1)\right)\left(\dot{g}(\mathrm{SNR}_1)\right)^2 dz_1 \\ &+ \beta_1(\beta_1+1)\int_0^\infty\int_0^\infty \left(1 + \frac{\mathrm{SNR}_1 z_1}{1+\mathrm{SNR}_1 z_1/\lambda}\right)^{-\beta_1-2} \\ &\qquad \frac{z_1^2}{\left(1+\mathrm{SNR}_1 g(\mathrm{SNR}_1)/\lambda\right)^4} p(z_1,z_2)\, dz_2 dz_1 \\ &+ \frac{2\beta_1}{\lambda}\int_0^\infty\int_0^{g(\mathrm{SNR}_1)} \left(1 + \frac{\mathrm{SNR}_1 z_1}{1+\mathrm{SNR}_1 z_1/\lambda}\right)^{-\beta_1-1} \\ &\qquad \frac{z_1 z_2}{\left(1+\mathrm{SNR}_1 g(\mathrm{SNR}_1)/\lambda\right)^3} p(z_1,z_2)\, dz_2 dz_1 \\ &- \int_0^\infty (1+\mathrm{SNR}_1 z_1)^{-\beta_1} p\left(z_1, g(\mathrm{SNR}_1)\right)\ddot{g}(\mathrm{SNR}_1)\, dz_1 \\ &+ 2\beta_1\int_0^\infty (1+\mathrm{SNR}_1 z_1)^{-\beta_1-1} z_1 p\left(z_1, g(\mathrm{SNR}_1)\right)\dot{g}(\mathrm{SNR}_1)\, dz_1 \\ &- \int_0^\infty (1+\mathrm{SNR}_1 z_1)^{-\beta_1} \dot{p}\left(z_1, g(\mathrm{SNR}_1)\right)\left(\dot{g}(\mathrm{SNR}_1)\right)^2 dz_1 \\ &+ \beta_1(\beta_1+1)\int_0^\infty\int_{g(\mathrm{SNR}_1)}^\infty (1+\mathrm{SNR}_1 z_1)^{-\beta_1-2} z_1^2 p(z_1,z_2)\, dz_2 dz_1. \end{aligned} \tag{61}$$

Letting $\mathrm{SNR}_1 = 0$ and supposing that $g(0)$, $\dot{g}(0)$, and $\ddot{g}(0)$ are finite, we have

$$\ddot{C}_1(0) = -\frac{1}{\log_e 2}\left(\beta_1\left(\mathbb{E}\{z_1^2\} - (\mathbb{E}\{z_1\})^2\right) + \mathbb{E}\{z_1^2\} + \frac{2}{\lambda}\int_0^{\infty}\int_0^{g(0)} z_1 z_2 p(z_1, z_2)\, dz_2 dz_1\right) \tag{62}$$

Substituting (62) and (51) into (11), we obtain

$$\mathcal{S}_1 = \frac{2(\mathbb{E}\{z_1\})^2}{\beta_1\left(\mathbb{E}\{z_1^2\} - (\mathbb{E}\{z_1\})^2\right) + \mathbb{E}\{z_1^2\} + \frac{2}{\lambda}\int_0^{\infty}\int_0^{g(0)} z_1 z_2 p(z_1, z_2)\, dz_2 dz_1}. \tag{63}$$

Similarly, we can derive

$$\mathcal{S}_2 = \frac{2(\mathbb{E}\{z_2\})^2}{\beta_2\left(\mathbb{E}\{z_2^2\} - (\mathbb{E}\{z_2\})^2\right) + \mathbb{E}\{z_2^2\} + 2\lambda\int_0^{\infty}\int_{g(0)}^{\infty} z_1 z_2 p(z_1, z_2)\, dz_2 dz_1}. \tag{64}$$

If the decoding order is (2,1) when $z_2 < g(z_1, \mathrm{SNR}_1)$, and is (1,2) when $z_2 > g(z_1, \mathrm{SNR}_1)$, following the steps described above, we can obtain

$$\mathcal{S}_1 = \frac{2(\mathbb{E}\{z_1\})^2}{\beta_1\left(\mathbb{E}\{z_1^2\} - (\mathbb{E}\{z_1\})^2\right) + \mathbb{E}\{z_1^2\} + \frac{2}{\lambda}\int_0^{\infty}\int_{g(0)}^{\infty} z_1 z_2 p(z_1, z_2)\, dz_2 dz_1} \tag{65}$$

$$\mathcal{S}_2 = \frac{2(\mathbb{E}\{z_2\})^2}{\beta_2\left(\mathbb{E}\{z_2^2\} - (\mathbb{E}\{z_2\})^2\right) + \mathbb{E}\{z_2^2\} + 2\lambda\int_0^{\infty}\int_0^{g(0)} z_1 z_2 p(z_1, z_2)\, dz_2 dz_1}. \tag{66}$$

Combining (63) and (64) and eliminating $g(0)$, we can obtain the third condition in (22). It is interesting that combining (65) and (66) and eliminating $g(0)$, we still get the same third condition stated in (22). This shows us that the slope regions for different decoding order assignments overlap. □

5. Proof of Theorem 6

Taking the first derivatives of (27) and (28) and letting $\zeta = 0$, we obtain

$$\dot{C}_1(0) = -\frac{1}{\theta_1 T}\log_e \mathbb{E}_{z_1}\left\{e^{-\frac{\theta_1 T \overline{P}_1}{N_0 \log_e 2} z_1}\right\} \tag{67}$$

$$\dot{C}_2(0) = -\frac{1}{\theta_2 T}\log_e \mathbb{E}_{z_2}\left\{e^{-\frac{\theta_2 T \overline{P}_2}{N_0 \log_e 2} z_2}\right\}. \tag{68}$$

Substituting (67) and (68) into (26), we get the results in (33) and (34).

Next, we consider the superposition coding with fixed decoding. Evaluating the first derivative of (29) and (30) at $\zeta = 0$, we again get

$$\dot{C}_1(0) = -\frac{1}{\theta_1 T}\log_e \mathbb{E}_{z_1}\left\{e^{-\frac{\theta_1 T \overline{P}_1}{N_0 \log_e 2} z_1}\right\} \tag{69}$$

$$\dot{C}_2(0) = -\frac{1}{\theta_2 T}\log_e \mathbb{E}_{z_2}\left\{e^{-\frac{\theta_2 T \overline{P}_2}{N_0 \log_e 2} z_2}\right\}. \tag{70}$$

which imply the results in (33) and (34).

We can also prove the results for the variable decoding case similarly as in the proof of Theorem 1.

Consider (31) and (32) with the associated decoding order. The first derivative of (31) can be expressed as

$$\dot{C}_1(\zeta) = -\frac{1}{\theta_1 T}\log_e \phi_1 - \frac{\zeta \dot{\phi}_1}{\theta_1 T \phi_1} \tag{71}$$

where φ_1 is

$$\begin{aligned}\phi_1 = &\int_0^{\infty}\int_{g\left(\frac{\overline{P}_1\zeta}{N_0}\right)}^{\infty} e^{-\frac{\theta_1 T}{\zeta}\log_2\left(1+\frac{\overline{P}_1 z_1 \zeta}{N_0}\right)} p_{\mathbf{z}}(z_1, z_2)\, dz_2 dz_1 \\ &+ \int_0^{\infty}\int_0^{g\left(\frac{\overline{P}_1\zeta}{N_0}\right)} e^{-\frac{\theta_1 T}{\zeta}\log_2\left(1+\frac{\frac{\overline{P}_1 z_1\zeta}{N_0}}{1+\frac{\overline{P}_2 z_2 \zeta}{N_0}}\right)} p_{\mathbf{z}}(z_1, z_2)\, dz_2 dz_1\end{aligned} \tag{72}$$

and $\dot{\phi}_1$ is

$$\begin{aligned}\dot{\phi}_1 = &-\int_0^{\infty}\dot{g}\left(\frac{\overline{P}\zeta}{N_0}\right)\frac{\overline{P}_1}{N_0} e^{-\frac{\theta_1 T}{\zeta}\log_2\left(1+\frac{\overline{P}_1 z_1\zeta}{N_0}\right)} p_{\mathbf{z}}\left(z_1, g\left(\overline{P}_1\zeta/N_0\right)\right) dz_1 \\ &+ \int_0^{\infty}\int_{g\left(\frac{\overline{P}_1\zeta}{N_0}\right)}^{\infty} e^{-\frac{\theta_1 T}{\zeta}\log_2\left(1+\frac{\overline{P}_1 z_1\zeta}{N_0}\right)} \\ &\left(\frac{\theta_1 T}{\zeta^2}\log_2\left(1+\frac{\overline{P}_1 z_1\zeta}{N_0}\right) - \frac{\theta_1 T}{\zeta}\frac{\frac{\overline{P}_1 z_1}{N_0\log_e 2}}{1+\frac{\overline{P}_1 z_1\zeta}{N_0}}\right) p_{\mathbf{z}}(z_1, z_2)\, dz_2 dz_1 \\ &+ \int_0^{\infty}\dot{g}\left(\frac{\overline{P}_1\zeta}{N_0}\right)\frac{\overline{P}_1}{N_0} e^{-\frac{\theta_1 T}{\zeta}\log_2\left(1+\frac{\frac{\overline{P}_1 z_1\zeta}{N_0}}{1+\frac{\overline{P}_2 g\left(\overline{P}_1\zeta/N_0\right)\zeta}{N_0}}\right)} p_{\mathbf{z}}\left(z_1, g\left(\overline{P}_1\zeta/N_0\right)\right) dz_1 \\ &+ \int_0^{\infty}\int_0^{g\left(\frac{\overline{P}_1\zeta}{N_0}\right)} e^{-\frac{\theta_1 T}{\zeta}\log_2\left(1+\frac{\frac{\overline{P}_1 z_1\zeta}{N_0}}{1+\frac{\overline{P}_2 z_2\zeta}{N_0}}\right)} \left(\frac{\theta_1 T}{\zeta^2}\log_2\left(1+\frac{\frac{\overline{P}_1 z_1\zeta}{N_0}}{1+\frac{\overline{P}_2 z_2\zeta}{N_0}}\right)\right. \\ &\left. - \frac{\theta_1 T}{\zeta}\frac{\frac{\overline{P}_1 z_1}{N_0\log_e 2}}{\left(1+\frac{\overline{P}_2 z_2\zeta}{N_0}\right)\left(1+\frac{\overline{P}_1 z_1\zeta}{N_0}+\frac{\overline{P}_2 z_2\zeta}{N_0}\right)}\right) p_{\mathbf{z}}(z_1, z_2)\, dz_2 dz_1.\end{aligned} \tag{73}$$

If we define $f(\zeta) = \frac{\theta_1 T}{\zeta^2}\log_2\left(1+\frac{\overline{P}_1 z_1\zeta}{N_0}\right) - \frac{\theta_1 T}{\zeta}\frac{\frac{\overline{P}_1 z_1}{N_0\log_e 2}}{1+\frac{\overline{P}_1 z_1\zeta}{N_0}}$

we can show that

$$\lim_{\zeta\to 0} f(\zeta) = \theta_1 T \lim_{\zeta\to 0} \frac{\dfrac{\log_2\left(1+\dfrac{\overline{P}_1 z_1 \zeta}{N_0}\right)}{\zeta} - \dfrac{\dfrac{\overline{P}_1 z_1}{N_0 \log_e 2}}{1+\dfrac{\overline{P}_1 z_1 \zeta}{N_0}}}{\zeta}$$
$$= \theta_1 T \lim_{\zeta\to 0}\left(-\frac{1}{\zeta^2}\log_2\left(1+\frac{\overline{P}_1 z_1\zeta}{N_0}\right) + \frac{1}{\zeta}\frac{\dfrac{\overline{P}_1 z_1}{N_0\log_e 2}}{1+\dfrac{\overline{P}_1 z_1\zeta}{N_0}} + \frac{\left(\dfrac{\overline{P}_1 z_1}{N_0}\right)^2}{\left(1+\dfrac{\overline{P}_1 z_1\zeta}{N_0}\right)^2\log_e 2}\right) \tag{74}$$
$$= -\lim_{\zeta\to 0} f(\zeta) + \frac{\theta_1 T}{\log_e 2}\left(\frac{\overline{P}_1 z_1}{N_0}\right)^2$$

which gives us that

$$\lim_{\zeta\to 0} f(\zeta) = \frac{\theta_1 T}{2\log_e 2}\left(\frac{\overline{P}_1 z_1}{N_0}\right)^2. \tag{75}$$

Similarly, we can show that

$$\lim_{\zeta\to 0}\left(\frac{\theta_1 T}{\zeta^2}\log_2\left(1+\frac{\dfrac{\bar{P}_1 z_1\zeta}{N_0}}{1+\dfrac{\bar{P}_2 z_2\zeta}{N_0}}\right) - \frac{\theta_1 T}{\zeta}\frac{\dfrac{\bar{P}_1 z_1}{N_0\log_e 2}}{\left(1+\dfrac{\bar{P}_2 z_2\zeta}{N_0}\right)\left(1+\dfrac{\bar{P}_1 z_1\zeta}{N_0}+\dfrac{\bar{P}_2 z_2\zeta}{N_0}\right)}\right)$$
$$= \frac{\theta_1 T}{2\log_e 2}\left(\frac{\bar{P}_1 z_1}{N_0}\right)^2 + \frac{\theta_1 T\bar{P}_1\bar{P}_2 z_1 z_2}{N_0^2\log_e 2}. \tag{76}$$

With (75) and (76) in mind, we can obtain

$$\lim_{\zeta\to 0}\dot{\phi}_1 = \frac{\theta_1 T}{2\log_e 2}\mathbb{E}_{\mathbf{z}}\left\{e^{-\frac{\theta_1 T\bar{P}_1}{N_0\log_e 2}z_1}\left(\frac{\bar{P}_1 z_1}{N_0}\right)^2\right\}$$
$$+\frac{\theta_1 T}{\log_e 2}\int_0^{\infty}\int_0^{g(0)} e^{-\frac{\theta_1 T\bar{P}_1}{N_0\log_e 2}z_1}\frac{\bar{P}_1\bar{P}_2 z_1 z_2}{N_0^2}p(z_1,z_2)\,dz_2 dz_1 \tag{77}$$

and hence

$$\dot{C}_1(0) = -\frac{1}{\theta_1 T}\log_e \mathbb{E}_{z_1}\left\{e^{-\frac{\theta_1 T\overline{P}_1}{N_0\log_e 2}z_1}\right\}. \tag{78}$$

Similarly, taking the derivative of (32) and letting $\zeta = 0$, we have

$$\dot{C}_2(0) = -\frac{1}{\theta_2 T}\log_e \mathbb{E}_{z_2}\left\{e^{-\frac{\theta_2 T\overline{P}_2}{N_0\log_e 2}z_2}\right\}. \tag{79}$$

which, after incorporating (26), again gives us the results in (33) and (34). For the reverse decoding order assignment, following similar steps, we still get the results in (33) and (34). □

6. Proof of Theorem 7

The second derivatives of (27) and (28) at $\zeta = 0$ are

$$\ddot{C}_1(0) = -\frac{1}{\delta\log_e 2}\left(\frac{\overline{P}_1}{N_0}\right)^2\frac{\mathbb{E}_{z_1}\left\{e^{-\frac{\theta_1 T\overline{P}_1}{N_0\log_e 2}z_1}z_1^2\right\}}{\mathbb{E}_{z_1}\left\{e^{-\frac{\theta_1 T\overline{P}_1}{N_0\log_e 2}z_1}\right\}} \tag{80}$$

$$\ddot{C}_2(0) = -\frac{1}{(1-\delta)\log_e 2}\left(\frac{\overline{P}_2}{N_0}\right)^2\frac{\mathbb{E}_{z_2}\left\{e^{-\frac{\theta_2 T\overline{P}_2}{N_0\log_e 2}z_2}z_2^2\right\}}{\mathbb{E}_{z_2}\left\{e^{-\frac{\theta_1 T\overline{P}_1}{N_0\log_e 2}z_2}\right\}} \tag{81}$$

Using the definition in (11), we can express the wideband slopes as

$$\mathcal{S}_1 = 2\delta\left(\frac{N_0\log_e 2}{\theta_1 T\overline{P}_1}\right)^2\frac{\mathbb{E}_{z_1}\left\{e^{-\frac{\theta_1 T\overline{P}_1}{N_0\log_e 2}z_1}\right\}\left(\log_e\mathbb{E}_{z_1}\left\{e^{-\frac{\theta_1 T\overline{P}_1}{N_0\log_e 2}z_1}\right\}\right)^2}{\mathbb{E}_{z_1}\left\{e^{-\frac{\theta_1 T\overline{P}_1}{N_0\log_e 2}z_1}z_1^2\right\}} \tag{82}$$

$$\mathcal{S}_2 = 2(1-\delta)\left(\frac{N_0\log_e 2}{\theta_2 T\overline{P}_2}\right)^2\frac{\mathbb{E}_{z_2}\left\{e^{-\frac{\theta_2 T\overline{P}_2}{N_0\log_e 2}z_2}\right\}\left(\log_e\mathbb{E}_{z_2}\left\{e^{-\frac{\theta_2 T\overline{P}_2}{N_0\log_e 2}z_2}\right\}\right)^2}{\mathbb{E}_{z_2}\left\{e^{-\frac{\theta_2 T\overline{P}_2}{N_0\log_e 2}z_2}z_2^2\right\}} \tag{83}$$

which after simple computation give us the third condition in (35). □

7. Proof of Theorem 8

Evaluating the second derivatives of (29) and (30) at $\zeta = 0$ yields

$$\ddot{C}_1(0) = -\frac{1}{\log_e 2}\frac{\left(\frac{\bar{P}_1}{N_0}\right)^2 \mathbb{E}_{z_1}\left\{e^{-\frac{\theta_1 T\bar{P}_1}{N_0\log_e 2}z_1} z_1^2\right\} + \frac{2(1-\tau)\bar{P}_1\bar{P}_2}{N_0^2}\mathbb{E}_z\left\{e^{-\frac{\theta_1 T\bar{P}_1}{N_0\log_e 2}z_1} z_1 z_2\right\}}{\mathbb{E}_{z_1}\left\{e^{-\frac{\theta_1 T\bar{P}_1}{N_0\log_e 2}z_1}\right\}} \tag{84}$$

$$\ddot{C}_2(0) = -\frac{1}{\log_e 2}\frac{\left(\frac{\bar{P}_2}{N_0}\right)^2 \mathbb{E}_{z_2}\left\{e^{-\frac{\theta_2 T\bar{P}_2}{N_0\log_e 2}z_2} z_2^2\right\} + \frac{2\tau\bar{P}_1\bar{P}_2}{N_0^2}\mathbb{E}_z\left\{e^{-\frac{\theta_2 T\bar{P}_2}{N_0\log_e 2}z_2} z_1 z_2\right\}}{\mathbb{E}_{z_2}\left\{e^{-\frac{\theta_2 T\bar{P}_2}{N_0\log_e 2}z_2}\right\}} \tag{85}$$

and as a result, the wideband slopes are given by

$$\mathcal{S}_1 = 2\left(\frac{N_0\log_e 2}{\theta_1 T}\right)^2\frac{\left(\log_e \mathbb{E}_{z_1}\left\{e^{-\frac{\theta_1 T\bar{P}_1}{N_0\log_e 2}z_1}\right\}\right)^2 \mathbb{E}_{z_1}\left\{e^{-\frac{\theta_1 T\bar{P}_1}{N_0\log_e 2}z_1}\right\}}{\bar{P}_1^2\mathbb{E}_{z_1}\left\{e^{-\frac{\theta_1 T\bar{P}_1}{N_0\log_e 2}z_1} z_1^2\right\} + 2(1-\tau)\bar{P}_1\bar{P}_2\mathbb{E}_z\left\{e^{-\frac{\theta_1 T\bar{P}_1}{N_0\log_e 2}z_1} z_1 z_2\right\}} \tag{86}$$

$$\mathcal{S}_2 = 2\left(\frac{N_0\log_e 2}{\theta_2 T}\right)^2\frac{\left(\log_e \mathbb{E}_{z_2}\left\{e^{-\frac{\theta_2 T\bar{P}_2}{N_0\log_e 2}z_2}\right\}\right)^2 \mathbb{E}_{z_2}\left\{e^{-\frac{\theta_2 T\bar{P}_2}{N_0\log_e 2}z_2}\right\}}{\bar{P}_2^2\mathbb{E}_{z_2}\left\{e^{-\frac{\theta_2 T\bar{P}_2}{N_0\log_e 2}z_2} z_2^2\right\} + 2\tau\bar{P}_1\bar{P}_2\mathbb{E}_z\left\{e^{-\frac{\theta_2 T\bar{P}_2}{N_0\log_e 2}z_2} z_1 z_2\right\}} \tag{87}$$

After solving for τ in (86) and (87) and subtracting the resulting equations, we have the third condition in (36). □

8. Proof of Theorem 9

Similar to Theorem 4, we here present the derivation for $S1$ for the case when the decoding order is (1,2) when $z_2 < g(z1, \mathrm{SNR}_1)$, and the decoding order is (2,1) when $z_2 > g(z_1, \mathrm{SNR}_1)$. The second derivative of (31) is

$$\ddot{C}_1(\zeta) = -\frac{2\dot{\phi}_1}{\theta_1 T\phi_1} - \frac{\zeta\left(\ddot{\phi}_1\phi_1 - \dot{\phi}_1^2\right)}{\theta_1 T\phi_1^2} \tag{88}$$

where φ_1 and $\dot{\phi}_1$ are (72) and (73), respectively, and $\ddot{\phi}_1$ is given by

$$\begin{aligned}
\ddot{\phi}_1 = &-\int_0^\infty \ddot{g}\left(\frac{\bar{P}_1\zeta}{N_0}\right)\left(\frac{\bar{P}_1}{N_0}\right)^2 e^{-\frac{\theta_1 T}{\zeta}\log_2\left(1+\frac{\bar{P}_1 z_1\zeta}{N_0}\right)} p\left(z_1, g\left(\bar{P}_1\zeta/N_0\right)\right) dz_1 \\
&-2\int_0^\infty \dot{g}\left(\frac{\bar{P}_1\zeta}{N_0}\right)\frac{\bar{P}_1}{N_0} e^{-\frac{\theta_1 T}{\zeta}\log_2\left(1+\frac{\bar{P}_1 z_1\zeta}{N_0}\right)} \\
&\left(\frac{\theta_1 T}{\zeta^2}\log_2\left(1+\frac{\bar{P}_1 z_1\zeta}{N_0}\right) - \frac{\theta_1 T}{\zeta}\frac{\frac{\bar{P}_1 z_1}{N_0\log_e 2}}{1+\frac{\bar{P}_1 z_1\zeta}{N_0}}\right) p\left(z_1, g\left(\bar{P}_1\zeta/N_0\right)\right) dz_1 \\
&-\int_0^\infty \left(\dot{g}\left(\frac{\bar{P}_1\zeta}{N_0}\right)\frac{\bar{P}_1}{N_0}\right)^2 e^{-\frac{\theta_1 T}{\zeta}\log_2\left(1+\frac{\bar{P}_1 z_1\zeta}{N_0}\right)} \\
&\left(\frac{\theta_1 T}{\zeta^2}\log_2\left(1+\frac{\bar{P}_1 z_1\zeta}{N_0}\right) - \frac{\theta_1 T}{\zeta}\frac{\frac{\bar{P}_1 z_1}{N_0\log_e 2}}{1+\frac{\bar{P}_1 z_1\zeta}{N_0}}\right) \dot{p}\left(z_1, g\left(\bar{P}_1\zeta/N_0\right)\right) dz_1 \\
&+\int_0^\infty \int_{g\left(\frac{\bar{P}_1\zeta}{N_0}\right)}^\infty e^{-\frac{\theta_1 T}{\zeta}\log_2\left(1+\frac{\bar{P}_1 z_1\zeta}{N_0}\right)} \left(\left(\frac{\theta_1 T}{\zeta^2}\log_2\left(1+\frac{\bar{P}_1 z_1\zeta}{N_0}\right) - \frac{\theta_1 T}{\zeta}\frac{\frac{\bar{P}_1 z_1}{N_0\log_e 2}}{1+\frac{\bar{P}_1 z_1\zeta}{N_0}}\right)^2\right. \\
&-\frac{2}{\zeta}\left(\frac{\theta_1 T}{\zeta^2}\log_2\left(1+\frac{\bar{P}_1 z_1\zeta}{N_0}\right) - \frac{\theta_1 T}{\zeta}\frac{\frac{\bar{P}_1 z_1}{N_0\log_e 2}}{1+\frac{\bar{P}_1 z_1\zeta}{N_0}}\right) \\
&\left.+\frac{\theta_1 T}{\zeta\log_e 2}\left(\frac{\frac{\bar{P}_1 z_1}{N_0}}{1+\frac{\bar{P}_1 z_1\zeta}{N_0}}\right)^2\right) p(z_1, z_2)\, dz_2 dz_1 \\
&+\int_0^\infty \ddot{g}\left(\frac{\bar{P}_1\zeta}{N_0}\right)\left(\frac{\bar{P}_1}{N_0}\right)^2 e^{-\frac{\theta_1 T}{\zeta}\log_2\left(1+\frac{\frac{\bar{P}_1 z_1\zeta}{N_0}}{1+\frac{\bar{P}_2 g\left(\bar{P}_1\zeta/N_0\right)\zeta}{N_0}}\right)} p\left(z_1, g\left(\bar{P}_1\zeta/N_0\right)\right) dz_1 \\
&+2\int_0^\infty \dot{g}\left(\frac{\bar{P}_1\zeta}{N_0}\right)\frac{\bar{P}_1}{N_0} e^{-\frac{\theta_1 T}{\zeta}\log_2\left(1+\frac{\frac{\bar{P}_1 z_1\zeta}{N_0}}{1+\frac{\bar{P}_2 g\left(\bar{P}_1\zeta/N_0\right)\zeta}{N_0}}\right)} \left(\frac{\theta_1 T}{\zeta^2}\log_2\left(1+\frac{\frac{\bar{P}_1 z_1\zeta}{N_0}}{1+\frac{\bar{P}_2 g(\bar{P}_1\zeta/N_0)\zeta}{N_0}}\right)\right. \\
&\left.-\frac{\theta_1 T}{\zeta}\frac{\frac{\bar{P}_1 z_1}{N_0\log_e 2}}{\left(1+\frac{\bar{P}_2 g(\bar{P}_1\zeta/N_0)\zeta}{N_0}\right)\left(1+\frac{\bar{P}_1 z_1\zeta}{N_0}+\frac{\bar{P}_2 g(\bar{P}_1\zeta/N_0)\zeta}{N_0}\right)}\right) p\left(z_1, g\left(\bar{P}_1\zeta/N_0\right)\right) dz_1 \\
&+\int_0^\infty \left(\dot{g}\left(\frac{\bar{P}_1\zeta}{N_0}\right)\frac{\bar{P}_1}{N_0}\right)^2 e^{-\frac{\theta_1 T}{\zeta}\log_2\left(1+\frac{\frac{\bar{P}_1 z_1\zeta}{N_0}}{1+\frac{\bar{P}_2 g\left(\bar{P}_1\zeta/N_0\right)\zeta}{N_0}}\right)} \dot{p}\left(z_1, g\left(\bar{P}_1\zeta/N_0\right)\right) dz_1 \\
&+\int_0^\infty \int_0^{g\left(\frac{\bar{P}_1\zeta}{N_0}\right)} e^{-\frac{\theta_1 T}{\zeta}\log_2\left(1+\frac{\frac{\bar{P}_1 z_1\zeta}{N_0}}{1+\frac{\bar{P}_2 z_2\zeta}{N_0}}\right)} \left(\left(\frac{\theta_1 T}{\zeta^2}\log_2\left(1+\frac{\frac{\bar{P}_1 z_1\zeta}{N_0}}{1+\frac{\bar{P}_2 z_2\zeta}{N_0}}\right)\right.\right. \\
&\left.-\frac{\theta_1 T}{\zeta}\frac{\frac{\bar{P}_1 z_1}{N_0\log_e 2}}{\left(1+\frac{\bar{P}_2 z_2\zeta}{N_0}\right)\left(1+\frac{\bar{P}_1 z_1\zeta}{N_0}+\frac{\bar{P}_2 z_2\zeta}{N_0}\right)}\right)^2 \\
&-\frac{2}{\zeta}\left(\frac{\theta_1 T}{\zeta^2}\log_2\left(1+\frac{\frac{\bar{P}_1 z_1\zeta}{N_0}}{1+\frac{\bar{P}_2 z_2\zeta}{N_0}}\right) - \frac{\theta_1 T}{\zeta}\frac{\frac{\bar{P}_1 z_1}{N_0\log_e 2}}{\left(1+\frac{\bar{P}_2 z_2\zeta}{N_0}\right)\left(1+\frac{\bar{P}_1 z_1\zeta}{N_0}+\frac{\bar{P}_2 z_2\zeta}{N_0}\right)}\right) \\
&\left.+\frac{\theta_1 T}{\zeta}\frac{\frac{\bar{P}_1 z_1}{N_0\log_e 2}\left(\frac{\bar{P}_2 z_2}{N_0}\left(1+\frac{\bar{P}_1 z_1\zeta}{N_0}+\frac{\bar{P}_2 z_2\zeta}{N_0}\right)+\left(\frac{\bar{P}_1 z_1}{N_0}+\frac{\bar{P}_2 z_2}{N_0}\right)\left(1+\frac{\bar{P}_2 z_2\zeta}{N_0}\right)\right)}{\left(1+\frac{\bar{P}_2 z_2\zeta}{N_0}\right)^2\left(1+\frac{\bar{P}_1 z_1\zeta}{N_0}+\frac{\bar{P}_2 z_2\zeta}{N_0}\right)}\right) \\
&p(z_1, z_2)\, dz_2 dz_1
\end{aligned} \tag{89}$$

By letting $\zeta = 0$ and recalling (75) and (76), we can show that

$$\ddot{C}_1(0) = -\frac{1}{\log_e 2}\frac{\left(\frac{\bar{P}_1}{N_0}\right)^2 \mathbb{E}_{z_1}\left\{e^{-\frac{\theta_1 T\bar{P}_1}{N_0\log_e 2}z_1} z_1^2\right\} + \frac{2\bar{P}_1\bar{P}_2}{N_0^2}\int_0^\infty\int_0^{g(0)} e^{-\frac{\theta_1 T\bar{P}_1}{N_0\log_e 2}z_1} z_1 z_2\, p(z_1, z_2)\, dz_2 dz_1}{\mathbb{E}_{z_1}\left\{e^{-\frac{\theta_1 T\bar{P}_1}{N_0\log_e 2}z_1}\right\}} \tag{90}$$

Combining (78) and (90) with (11), we have

$$\mathcal{S}_1 = 2\left(\frac{N_0\log_e 2}{\theta_1 T}\right)^2 \frac{\left(\log_e \mathbb{E}_{z_1}\left\{e^{-\frac{\theta_1 T\bar{P}_1}{N_0\log_e 2}z_1}\right\}\right)^2 \mathbb{E}_{z_1}\left\{e^{-\frac{\theta_1 T\bar{P}_1}{N_0\log_e 2}z_1}\right\}}{\bar{P}_1^2\mathbb{E}_{z_1}\left\{e^{-\frac{\theta_1 T\bar{P}_1}{N_0\log_e 2}z_1} z_1^2\right\} + 2\bar{P}_1\bar{P}_2\int_0^\infty\int_0^{g(0)} e^{-\frac{\theta_1 T\bar{P}_1}{N_0\log_e 2}z_1} z_1 z_2 p(z_1, z_2)dz_2 dz_1}. \tag{91}$$

Following similar steps, we can derive that

$$\mathcal{S}_2 = 2\left(\frac{N_0\log_e 2}{\theta_2 T}\right)^2 \frac{\left(\log_e \mathbb{E}_{z_2}\left\{e^{-\frac{\theta_2 T\bar{P}_2}{N_0\log_e 2}z_2}\right\}\right)^2 \mathbb{E}_{z_2}\left\{e^{-\frac{\theta_2 T\bar{P}_2}{N_0\log_e 2}z_2}\right\}}{\bar{P}_2^2\mathbb{E}_{z_2}\left\{e^{-\frac{\theta_2 T\bar{P}_2}{N_0\log_e 2}z_2} z_2^2\right\} + 2\bar{P}_1\bar{P}_2\int_0^\infty\int_{g(0)}^{0} e^{-\frac{\theta_2 T\bar{P}_2}{N_0\log_e 2}z_2} z_1 z_2 p(z_1, z_2)dz_2 dz_1}. \tag{92}$$

If the decoding order is (2,1) when $z_2 < g(z_1, \mathrm{SNR}_1)$, and is (1,2) when $z_2 > g(z_1, \mathrm{SNR}_1)$, following the steps described above, we can obtain

$$\mathcal{S}_1 = 2\left(\frac{N_0\log_e 2}{\theta_1 T}\right)^2 \frac{\left(\log_e \mathbb{E}_{z_1}\left\{e^{-\frac{\theta_1 T\bar{P}_1}{N_0\log_e 2}z_1}\right\}\right)^2 \mathbb{E}_{z_1}\left\{e^{-\frac{\theta_1 T\bar{P}_1}{N_0\log_e 2}z_1}\right\}}{\bar{P}_1^2\mathbb{E}_{z_1}\left\{e^{-\frac{\theta_1 T\bar{P}_1}{N_0\log_e 2}z_1} z_1^2\right\} + 2\bar{P}_1\bar{P}_2\int_0^\infty\int_{g(0)}^{\infty} e^{-\frac{\theta_1 T\bar{P}_1}{N_0\log_e 2}z_1} z_1 z_2 p(z_1, z_2)dz_2 dz_1}, \tag{93}$$

$$\mathcal{S}_2 = 2\left(\frac{N_0\log_e 2}{\theta_2 T}\right)^2 \frac{\left(\log_e \mathbb{E}_{z_2}\left\{e^{-\frac{\theta_2 T\bar{P}_2}{N_0\log_e 2}z_2}\right\}\right)^2 \mathbb{E}_{z_2}\left\{e^{-\frac{\theta_2 T\bar{P}_2}{N_0\log_e 2}z_2}\right\}}{\bar{P}_2^2\mathbb{E}_{z_2}\left\{e^{-\frac{\theta_2 T\bar{P}_2}{N_0\log_e 2}z_2} z_2^2\right\} + 2\bar{P}_1\bar{P}_2\int_0^\infty\int_0^{g(0)} e^{-\frac{\theta_2 T\bar{P}_2}{N_0\log_e 2}z_2} z_1 z_2 p(z_1, z_2)dz_2 dz_1}. \tag{94}$$

Also note that the wideband slopes have non-negative values and we have the inequalities in (37) and (38). □

9. Proof of Theorem 10

We need to compare the upper bound of the slope region in (36) with the upper bounds of both (37) and (38).

By moving the term with $g(0)$ to the LHS of the equation, we can rewrite (91) and (92) as

$$\begin{aligned}&\frac{\int_0^\infty\int_0^{g(0)} e^{-\frac{\theta_1 T\overline{P}_1}{N_0\log_e 2}z_1} z_1 z_2 p(z_1, z_2)dz_2 dz_1}{\mathbb{E}_{\mathbf{z}}\left\{e^{-\frac{\theta_1 T\overline{P}_1}{N_0\log_e 2}z_1} z_1 z_2\right\}} \\ &= \left(\frac{N_0\log_e 2}{\theta_1 T}\right)^2 \frac{\left(\log_e \mathbb{E}_{z_1}\left\{e^{-\frac{\theta_1 T\overline{P}_1}{N_0\log_e 2}z_1}\right\}\right)^2 \mathbb{E}_{z_1}\left\{e^{-\frac{\theta_1 T\overline{P}_1}{N_0\log_e 2}z_1}\right\}}{\overline{P}_1\overline{P}_2\mathbb{E}_{\mathbf{z}}\left\{e^{-\frac{\theta_1 T\overline{P}_1}{N_0\log_e 2}z_1} z_1 z_2\right\}}\left(\frac{1}{\mathcal{S}_1} - \frac{1}{\mathcal{S}_1^{\mathrm{up}}}\right)\end{aligned} \tag{95}$$

and

$$\begin{aligned}&\frac{\int_0^\infty\int_{g(0)}^\infty e^{-\frac{\theta_2 T\overline{P}_2}{N_0\log_e 2}z_2} z_1 z_2 p(z_1, z_2)dz_2 dz_1}{\mathbb{E}_{\mathbf{z}}\left\{e^{-\frac{\theta_2 T\overline{P}_2}{N_0\log_e 2}z_2} z_1 z_2\right\}} \\ &= \left(\frac{N_0\log_e 2}{\theta_2 T}\right)^2 \frac{\left(\log_e \mathbb{E}_{z_2}\left\{e^{-\frac{\theta_2 T\overline{P}_2}{N_0\log_e 2}z_2}\right\}\right)^2 \mathbb{E}_{z_1}\left\{e^{-\frac{\theta_2 T\overline{P}_2}{N_0\log_e 2}z_1}\right\}}{\overline{P}_1\overline{P}_2\mathbb{E}_{\mathbf{z}}\left\{e^{-\frac{\theta_2 T\overline{P}_2}{N_0\log_e 2}z_2} z_1 z_2\right\}}\left(\frac{1}{\mathcal{S}_2} - \frac{1}{\mathcal{S}_2^{\mathrm{up}}}\right)\end{aligned} \tag{96}$$

Denote

$$\gamma_1 = \frac{\int_0^\infty\int_0^{g(0)} e^{-\frac{\theta_1 T\overline{P}_1}{N_0\log_e 2}z_1} z_1 z_2 p(z_1, z_2)dz_2 dz_1}{\mathbb{E}_{\mathbf{z}}\left\{e^{-\frac{\theta_1 T\overline{P}_1}{N_0\log_e 2}z_1} z_1 z_2\right\}} \tag{97}$$

$$\gamma_2 = \frac{\int_0^\infty\int_{g(0)}^\infty e^{-\frac{\theta_2 T\overline{P}_2}{N_0\log_e 2}z_2} z_1 z_2 p(z_1, z_2)dz_2 dz_1}{\mathbb{E}_{\mathbf{z}}\left\{e^{-\frac{\theta_2 T\overline{P}_2}{N_0\log_e 2}z_2} z_1 z_2\right\}} \tag{98}$$

We know that $0 \le \gamma_1 \le 1$ and $0 \le \gamma_2 \le 1$ vary with different $g(0)$. Substitute (95) and (96) into the third condition of (36), we can obtain

$$\begin{aligned}&\left(\frac{N_0\log_e 2}{\theta_1 T}\right)^2 \frac{\left(\log_e \mathbb{E}_{z_1}\left\{e^{-\frac{\theta_1 T\overline{P}_1}{N_0\log_e 2}z_1}\right\}\right)^2 \mathbb{E}_{z_1}\left\{e^{-\frac{\theta_1 T\overline{P}_1}{N_0\log_e 2}z_1}\right\}}{\overline{P}_1\overline{P}_2\mathbb{E}_{\mathbf{z}}\left\{e^{-\frac{\theta_1 T\overline{P}_1}{N_0\log_e 2}z_1} z_1 z_2\right\}}\left(\frac{1}{\mathcal{S}_1} - \frac{1}{\mathcal{S}_1^{\mathrm{up}}}\right) \\ &+\left(\frac{N_0\log_e 2}{\theta_2 T}\right)^2 \frac{\left(\log_e \mathbb{E}_{z_2}\left\{e^{-\frac{\theta_2 T\overline{P}_2}{N_0\log_e 2}z_2}\right\}\right)^2 \mathbb{E}_{z_1}\left\{e^{-\frac{\theta_2 T\overline{P}_2}{N_0\log_e 2}z_1}\right\}}{\overline{P}_1\overline{P}_2\mathbb{E}_{\mathbf{z}}\left\{e^{-\frac{\theta_2 T\overline{P}_2}{N_0\log_e 2}z_2} z_1 z_2\right\}}\left(\frac{1}{\mathcal{S}_2} - \frac{1}{\mathcal{S}_2^{\mathrm{up}}}\right) \\ &= \gamma_1 + \gamma_2\end{aligned} \tag{99}$$

Following similar steps, we can get from (93) and (94)

$$\begin{aligned}&\left(\frac{N_0\log_e 2}{\theta_1 T}\right)^2 \frac{\left(\log_e \mathbb{E}_{z_1}\left\{e^{-\frac{\theta_1 T\overline{P}_1}{N_0\log_e 2}z_1}\right\}\right)^2 \mathbb{E}_{z_1}\left\{e^{-\frac{\theta_1 T\overline{P}_1}{N_0\log_e 2}z_1}\right\}}{\overline{P}_1\overline{P}_2\mathbb{E}_{\mathbf{z}}\left\{e^{-\frac{\theta_1 T\overline{P}_1}{N_0\log_e 2}z_1}z_1 z_2\right\}}\left(\frac{1}{\mathcal{S}_1}-\frac{1}{\mathcal{S}_1^{\mathrm{up}}}\right)\\ &+\left(\frac{N_0\log_e 2}{\theta_2 T}\right)^2 \frac{\left(\log_e \mathbb{E}_{z_2}\left\{e^{-\frac{\theta_2 T\overline{P}_2}{N_0\log_e 2}z_2}\right\}\right)^2 \mathbb{E}_{z_1}\left\{e^{-\frac{\theta_2 T\overline{P}_2}{N_0\log_e 2}z_1}\right\}}{\overline{P}_1\overline{P}_2\mathbb{E}_{\mathbf{z}}\left\{e^{-\frac{\theta_2 T\overline{P}_2}{N_0\log_e 2}z_2}z_1 z_2\right\}}\left(\frac{1}{\mathcal{S}_2}-\frac{1}{\mathcal{S}_2^{\mathrm{up}}}\right)\\ &= 2-\gamma_1-\gamma_2\end{aligned} \tag{100}$$

Considering (99) and (100), we know that either $\gamma_1 + \gamma_2$ or 2 - γ_1 - γ_2 must be less than 1, which implies that variable decoding order achieves points outside the region attained with fixed decoding order, proving the theorem. □

Endnotes

[a]We note that the expressions in (9) and (11) differ from those in [1] by a constant factor due to the fact that we assume that the units of C_E is bits/s/Hz rather than nats/s/Hz. [b]The partition function can in general be a function of z_1 as well, i.e., $g(\mathrm{SNR}_1) = g(z_1, \mathrm{SNR}_1)$. [c]As discussed in [13,14], wideband and low-power regimes are equivalent if rich multipath fading is experienced. Hence, in such a case, the same minimum bit energy and wideband slope expressions are obtained in both regimes.

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