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Optimizing distance, transmit power, and allocation time for reliable multi-hop relay system

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Abstract

In multi-hop relay systems, the end-to-end channel capacity is restricted by bottleneck node. In order to prevent some relay nodes from being the bottleneck of system and to guarantee the end-to-end channel capacity, the method of optimizing transmit power, distance and allocation time is proposed in this article. We show that the optimizing distance has more end-to-end channel capacity than the optimizing transmit power in case that both the distance and the transmit power are changeable. However, the optimizing transmit power can let the system reach high end-to-end channel capacity when the relay nodes have to shift from the desired location. We also propose the Markov Chain Monte Carlo method to optimize all transmit power, distance and allocation time simultaneously. The optimizing all transmit power, distance and allocation time is the most effective and achieves the highest channel capacity. Based on the average signal-to-noise-ratio, the average channel capacity is evaluated in this article.

1 Introduction

In the future, it is believed that the MIMO service area will become popular. Therefore, a MIMO relay system is considered. However, in a relay system with one relay, when the number of relay antenna elements is less than the number of transmitter and receiver antenna elements, the capacity of MIMO relay system is lower than that of the original MIMO system. In addition, when the number of relay antenna elements is equal to or more than the number of the transmitter and receiver antenna elements, a MIMO relay system can provide the same average capacity as an original MIMO system. In other words, although the number of relay antenna elements is larger than the transmitter and receiver antenna elements, the capacity of MIMO relay system cannot exceed the channel capacity of original MIMO system [1-3].

Therefore, a system with multi relays called multi-hop relay system was proposed and have been discussed in several literatures. The Gaussian MIMO relay channel with fixed channel condition has derived upper bounds and lower bounds that can be obtained numerically by convex programming [4-6]. Moreover, the capacity of a particular large Gaussian relay network is determined by the limit as the number of relays tends to infinity [7]. In addition, a multi-hop relay network with multi antenna terminals in a quasi-static slow fading environment also has been considered [8]. However, these researches assumed the signal-to-noise-ratio (SNR) at receiver(s) is fixed, the distance between the transceivers and the transmit power of transmitter(s) are not considered.

In multi-hop MIMO relay systems, when the distance between the base station (Tx) and the final receiver (Rx) is fixed, the distance between the Tx to a relay node (RS), an RS to an RS, an RS to the Rx called the distances between transceivers, is shorten. Consequently, the SNR and the channel capacity are increased. However, according to the number of the relay nodes, the location and the transmit power of each relay node; the channel capacity of each relay node is changed. In addition, the end-to-end channel capacity is limited by bottleneck node. Therefore, to obtain the upper bound of end-to-end channel capacity, the location of each relay node meaning the distance between the transceivers and the transmit power of each relay node need to be optimized. We have analyzed performance of multi-hop MIMO relay system with amplify-and-forward (AF) [9]. The distance between the transceivers is optimized when the transmit power of each relay node is assumed to be equal. However, the location of the relay nodes is not always changeable. Consequently, in order to obtain a certain value of end-to-end channel capacity, the distance and the transmit

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power need to be optimized. In this article, the distance and the transmit power are optimized separately or simultaneously to guarantee the end-to-end channel capacity in decode-and-forward scheme multi-hop relay systems. In addition, allocation time is optimized to guarantee and/or to obtain the higher end-to-end channel capacity. Moreover, the channel capacity that is mentioned in this article is average channel capacity. The rest of the article is as follows. After the introduction of the system model in Section 2, we propose the optimizing method of transmit power and distance in Section 3 and the optimizing method of allocation time in Section 4. The optimizing of all transmit power, distance and allocation time simultaneously is described in Section 5. Finally, Section 6 concludes the article.

2 Multi-hop MIMO relay system

2.1 System model

Figure 1 shows m relays intervened multi-hop MIMO relay system. Here, K_i ($i = 0, \ldots, m + 1$) denotes the number of the antenna elements at the Tx, the Rx and each relay node. d_i ($i = 0, \ldots, m$) represents the distance between the transceivers. The distance between the Tx and the Rx is fixed as d. The signal is transmitted from the Tx to the RS_1. At the RS_1, the signal is decoded, encoded and transmitted to the RS_2. Similarly, the signal is transmitted over and over until the signal reaches to the final receiver. We assumed the transmit power of the Tx (E_{Tx}) and the total transmit powers of relay node (E_{RS}) are fixed regardless of the number of the relay nodes and the number of antenna elements at each relay. The transmit power of each relay node is denoted by E_i. Moreover, the transmit power of each relay is equally divided into each antenna element. On the other hand, as described in Section 1, if the number of antenna elements at one relay is smaller than the other, this relay will be a bottleneck of the system and the end-to-end channel capacity will be restricted by this relay. Since in this article we consider the distance, transmit power and allocation time, the number of antenna elements at each relay is assumed to be the same as that of the Tx and the Rx and denoted by M. Moreover, we assume that the time-division-multiple-access (TDMA) algorithm is applied to control the transmission of each relay node. The allocation time of RS_i is denoted as t_i.

Let $\mathbf{H}_{ii+1}$ denotes a $K_{i+1} \times K_i$ channel matrix between the RS_i and the RS_{i+1}. Since the path loss is taken into consideration, H_{ii+1} is the composite matrix. We model $\mathbf{H}_{ii+1}$ as

$$\mathbf{H}_{ii+1} = \sqrt{l_{ii+1}}\mathbf{H}_{w_{ii+1}}, \quad i = 0, \ldots, m, \tag{1}$$

where $\mathbf{H}_{wii+1}$ is a matrix with independent and identically distributed (i.i.d.), zero mean, unit variance, circularly symmetric complex Gaussian entries, and l_{ii+1}

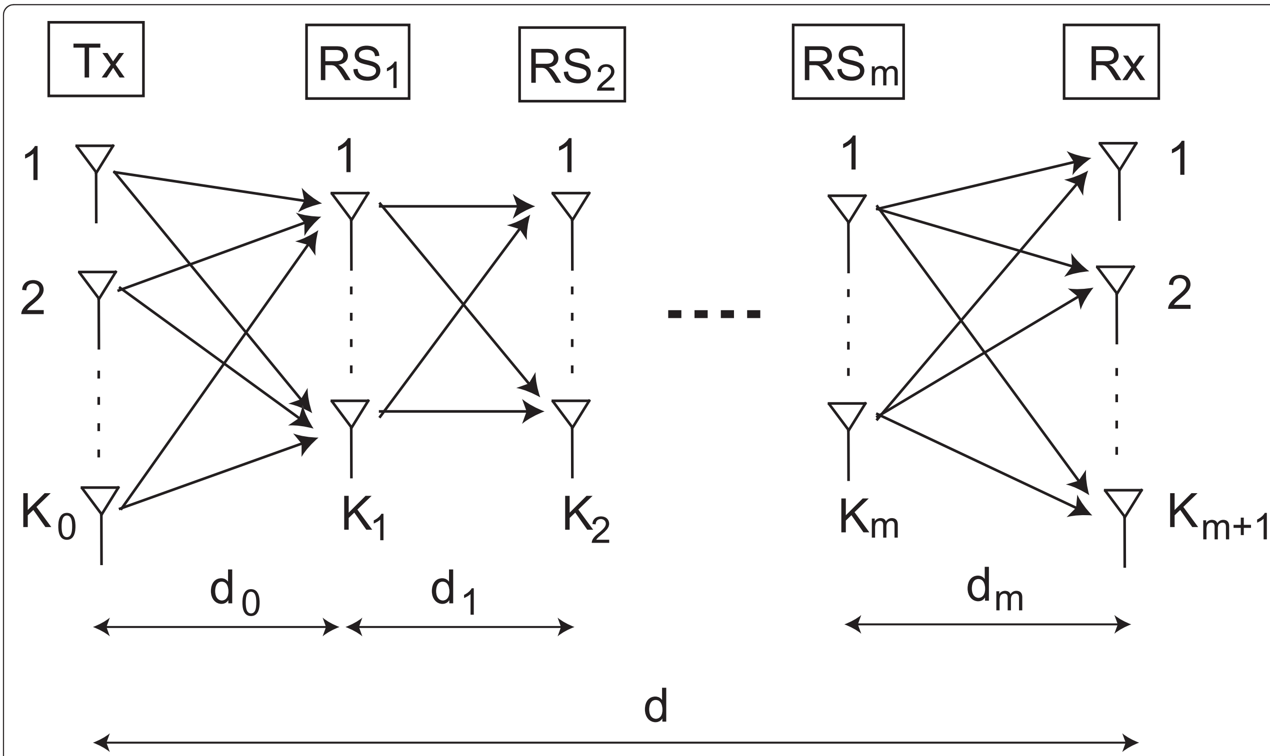

Figure 1 The channel model of multi-hop MIMO relay systems.

represents the path loss between the RS_i and the RS_{i+1}. The path loss is described in detail in the following section.

2.2 Path loss

Since there are a lot of obstacles, such as huge buildings, in propagation environment, the path loss is necessary to consider being attenuated by the reflection. The power of the signal is reduced when the reflection occurs. An amount of the reduction by one time of reflection is called reflection factor. It is natural that the reflection factor is changed according to the matter of the obstacles, the angle of reflections and so on. However, in this article, the reflection factor of all reflections is assumed to be the same and denoted by a. The path loss between the transceivers is described in Figure 2. The path loss in this case is expressed as [10]

$$l_i = \left(\frac{\lambda a^{\mathrm{ref_i}}}{4\pi d_\mathrm{i}}\right)^2, \tag{2}$$

where ref_i is the number of reflection while a signal is transmitted between RS_i and RS_{i+1}. In addition, in order to obtain the number of reflection, the propagation environment coefficient W_i is defined as the average distance from a reflection point to the next reflection point. In other words, it is the average of line-of-sight (LOS) distance between RS_i and RS_{i+1}. Therefore, the number of reflection between the transceivers can be expressed as $\mathrm{ref_i} = \frac{d_\mathrm{i}}{W_\mathrm{i}}$. Consequently, the path loss in (2) can be rewritten as

$$l_i = \left(\frac{\lambda a^{\frac{d_\mathrm{i}}{W_\mathrm{i}}}}{4\pi d_\mathrm{i}}\right)^2. \tag{3}$$

2.3 Channel capacity

Let the channel capacity between the RS_i and the RS_{i+1} is C_i, and C denotes the end-to-end channel capacity of the multi-hop MIMO relay system (Figure 2).

The transmission in multi-hop relay system is assumed to be controlled accurately. There-fore, when the signal $\mathbf{S}_{\text{i-1}}$ is transmitted from the $RS_{i\text{-}1}$, the received signal at the RS_i is expressed as cite9,

$$\mathbf{S}_\mathrm{i} = H_{\mathrm{i-1i}}\sqrt{P_{\mathrm{i-1}}}S_{\mathrm{i-1}} + n_\mathrm{i}. \tag{4}$$

Here, $\mathbf{P}_\mathrm{i} = \mathrm{diag}(p_{\mathrm{i1}}, p_{\mathrm{i2}}, \ldots, p_{\mathrm{iK_i}})$ is the transmit power matrix and is assumed to be subject to a constraint

$$Tr(\mathbf{P}_\mathrm{i}) = \begin{cases} E_{\mathrm{Tx}} & (i = 0), \\ E_\mathrm{i} & (i \neq 0), \end{cases}$$

where $Tr(\cdot)$ and p_{ij} denote the trace and transmit power of j^{th} antenna element of RS_i, respectively. $\mathbf{n}_\mathrm{i}$ is the noise vector with i.i.d., zero mean, σ^2 variance. The transmit power of every antenna in the same relay node was assumed to be equal.

$$p_{\mathrm{i1}} = p_{\mathrm{i2}} = \cdots = p_{\mathrm{iK_i}} = \frac{E_i}{M}.$$

Moreover, the channel capacity is represented as follows.

$$C_\mathrm{i} = \log_2\left(\det\left(I_{\mathrm{K_{i+1}}} + \frac{SNR_{i+1}}{K_{\mathrm{i+1}}} H_{\mathrm{w_{ii+1}}} H^{\mathrm{H}}_{\mathrm{w_{ii+1}}}\right)\right), \quad \text{for } i = 0, \ldots, m, \tag{5}$$

where $I_{\mathrm{K_i}}$ is $K_\mathrm{i} \times K_\mathrm{i}$ unit matrix. Since the channel capacity of each relay node is independent from each other, the end-to-end channel capacity is equal to channel capacity of bottleneck relay node.

$$C = \min(t_0 C_0,\ t_1 C_1,\ \ldots,\ t_m C_\mathrm{m}). \tag{6}$$

3 Optimizing transmit power and distance

In order to explain the optimizing of the distance and transmit power clearly, the allocation time of each relay node is assumed to be the same, $t_i = \frac{1}{m+1}$. In addition, $\mathbf{H}_{\mathrm{w_{ii+1}}}\mathbf{H}^{\mathrm{H}}_{\mathrm{w_{ii+1}}}$ $(i = 0, \ldots, m)$ is independent from the distance and the transmit power. Consequently, the SNR can be examined instead of a channel capacity. Hence, in order to avoid that some nodes become the bottleneck

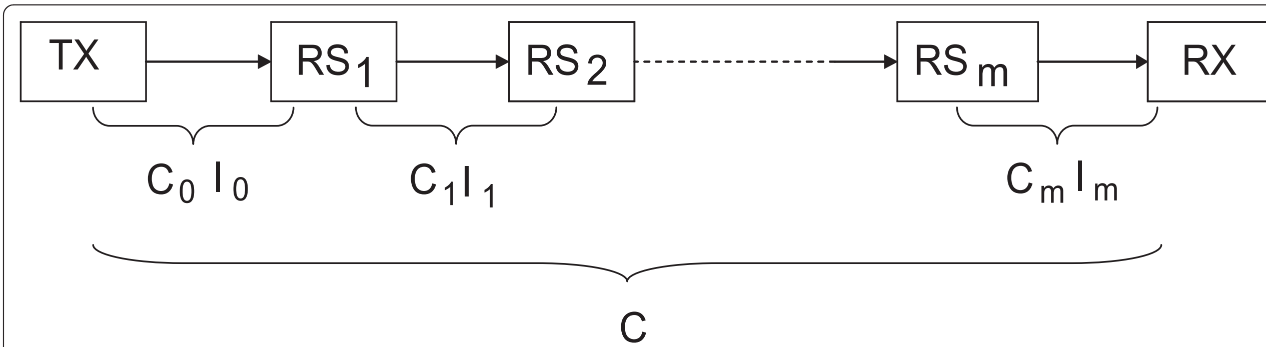

Figure 2 Channel capacity and path loss of multi-hop MIMO relay system.

and to obtain high channel capacity, the channel capacity of each node should be equal. Consequently, the received SNR of each node is necessary to be equal.

$$SNR_i = SNR_j, \quad \text{for } i \neq j,\ i,j = 0, \ldots, m, \tag{7}$$

here,

$$SNR_0 = \frac{E_{Tx} l_0}{M\sigma^2},$$
$$SNR_i = \frac{E_i l_i}{M\sigma^2}, \quad \text{with } i = 1, \ldots, m.$$

3.1 Optimizing transmit power

3.1.1 Optimization method

When the location of the relay nodes is not changeable, the transmit power of each relay node should be optimized to increase the channel capacity of bottleneck relay node. Consequently, the necessary transmit power can be obtained from (7).

$$\frac{E_i l_i}{M\sigma^2} = \frac{E_j l_j}{M\sigma^2}, \quad \text{for } i \neq j,\ i,j = 1, \ldots, m. \tag{8}$$

Note that total transmit power of relay node is fixed.

$$\sum_{i=1}^{m} E_i = E_{RS}. \tag{9}$$

From (8) we have

$$E_i = \frac{l_1}{l_i} E_1, \quad \text{for all } i = 2, \ldots, m. \tag{10}$$

By substituting each E_i to (9), it can be rewritten as

$$E_1 + \sum_{i=2}^{m} \frac{l_1}{l_i} E_1 = E_{RS}. \tag{11}$$

Consequently, the necessary transmit power of RS_1 is obtained.

$$E_1 = \frac{E_{RS}}{l_1} \frac{1}{\sum_{i=1}^{m} \frac{1}{l_i}}. \tag{12}$$

Similarly, the transmit power of each relay node is obtained by,

$$E_j = \frac{E_{RS}}{l_j} \frac{1}{\sum_{i=1}^{m} \frac{1}{l_i}}, \quad \text{for all } j = 1, \ldots, m. \tag{13}$$

3.1.2 Numerical evaluation of optimizing transmit power

The system parameters summarized in Table 1 are used as an example for evaluating the optimizing method mentioned above. Let the distance between the transceivers be random. The channel capacity, in case the number of the relay nodes are 3, 6, 9, are described in Figure 3, here the average propagation environment coefficient W, $W = \frac{\sum_{i=0}^{m} W_i}{m+1}$, meaning the average of LOS distance between the Tx and Rx, is set as 500 m.

Table 1 Numerical parameters

Antenna elements at *Tx*, *Rx*, *RS*	4
Transmit power of *Tx* (mW)	100
Total transmit power of *RS* (mW)	100
Noise power (mW)	6.12e-011
Reflection factor	0.38
Distance between *Tx-Rx* (m)	3000
Average LOS *W* (m)	500

As shown in Figure 3, in case the number of the relay nodes is 9, the channel capacity of bottleneck node is improved and the end-to-end channel capacity is increased. However, since the transmit power of Tx is assumed to be constant, the channel capacity of RS_1 is fixed. Therefore RS_1 becomes a bottleneck node if d_0 is large, such as the number of the relay nodes is 3. In this case, the end-to-end channel capacity can not be improved. Moreover, SNR is increased by transmit power to the 1st power and decreased by distance to the 2nd power. Hence, in order to increase the channel capacity, the huge transmit power needs to be provided when the distance is large. Under the assumption that total transmit power of relay node is fixed; the channel capacity is not considerably improved. It is the reason why the end-to-end channel capacity of the system with 6 relay nodes is low.

3.2 Optimizing distance between transceivers

3.2.1 Optimization method

Since the optimizing of transmit power remains some drawbacks as mentioned above, the distance between the transceivers needs to be considered. In order to analyze the distance more easily, the transmit power of each relay node is assumed to be equal. Therefore, the channel capacity only depends on the distance.

By solving (7), the optimized distance becomes as follows.

$$l_i = l_j, \quad \text{for } i \neq j,\ i,j = 1, \ldots, m,$$

$$E_{Tx} l_0 = \frac{E_{RS}}{m} l_i.$$

Firstly, at scheme 1, we assume that all channel models between the transceivers are the same (W). Therefore, by substituting the path loss of (3) in (7), this equation becomes,

$$\sqrt{\frac{mE_{Tx}}{E_{RS}}} d_i = (d - md_i)\, a^{\frac{(m+1)d_i - d}{W}}. \tag{14}$$

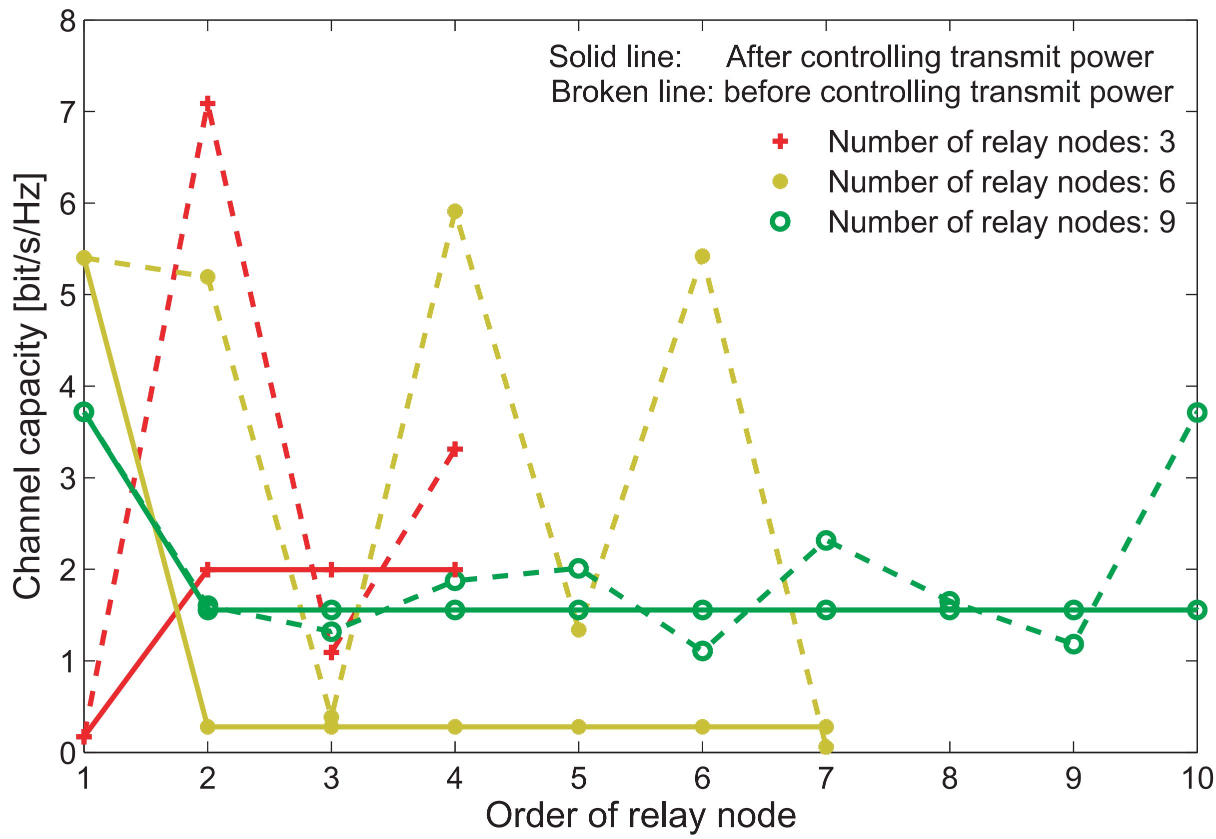

Figure 3 Channel capacity of optimizing the transmit power where the number of the relay nodes are 3, 6, 9.

Let

$$g\left(d_{\mathrm{i}}\right) = a^{\frac{(m+1)\mathrm{d_i}-\mathrm{d}}{W}}.$$

The Taylor expansion is approximate to $g(d_i)$, and (14) can be expressed as

$$b_4 d_{\mathrm{i}}^4 + b_3 d_{\mathrm{i}}^3 + b_2 d_{\mathrm{i}}^2 + b_1 d_{\mathrm{i}} + b_0 = 0, \quad (15)$$

here,

$$b_0 = d,$$

$$b_1 = \frac{d\,(m+1)\ln(a)}{W} - m - \sqrt{\frac{mP_{\mathrm{Tx}}}{P_{\mathrm{RS}}}a^{\frac{d}{W}}},$$

$$b_2 = \frac{d}{2}\left(\frac{m+1}{W}\ln(a)\right)^2 - \frac{m\,(m+1)\ln(a)}{W},$$

$$b_3 = \frac{d}{6}\left(\frac{m+1}{W}\ln(a)\right)^3 - \frac{m}{2}\left(\frac{m+1}{W}\ln(a)\right)^2,$$

$$b_4 = -\frac{m}{6}\left(\frac{m+1}{W}\ln(a)\right)^3.$$

Let

$$x = d_i - \frac{b_3}{4b_4}.$$

(15) is rewritten as

$$x^4 + px^2 + qx + r = 0, \quad (16)$$

here

$$p = \frac{b_2}{b_4} - 6\left(\frac{b_3}{4b_4}\right)^2,\ q = \frac{b_1}{b_4} - 2\frac{b_2}{b_4}\frac{b_3}{4b_4} + 8\left(\frac{b_3}{4b_4}\right)^3,\ r = \frac{b_0}{b_4} - \frac{b_1}{b_4}\frac{b_3}{4b_4} + \frac{b_2}{b_4}\left(\frac{b_3}{4b_4}\right)^2 - 3\left(\frac{b_3}{4b_4}\right)^4.$$

The function y is added to this equation.

$$x^4 + \left(p + y\right)x^2 + r = yx^2 - qx. \quad (17)$$

Hence,

$$4y\left(x^2 + \frac{p+y}{2}\right)^2 - y(p+y)^2 + 4yr = 4y^2\left(x - \frac{q}{2u}\right)^2 - q^2. \quad (18)$$

Moreover, we can describe as

$$x^4 + px^2 + qx + r = \left(x^2 + c_1 x + c_0\right)\left(x^2 + d_1 x + d_0\right). \quad (19)$$

Therefore,

$$c_1 + d_1 = 0,\ c_0 + d_0 + c_1 d_1 = p,\ c_1 d_0 + c_0 d_1 = q,\ c_0 d_0 = r.$$

From these equations, the resolvent cubic equation can be obtained.

$$c_1^2\left(c_1^2 + p\right)^2 - q^2 = 4c_1^2 r. \tag{20}$$

Let $c_1^2 = y$, the resolvent cubic equation can be rewritten as

$$y\left(y + p\right)^2 - q^2 = 4yr. \tag{21}$$

From this equation, function y is obtained.

$$y = \frac{1}{6}\left(-4p + \sqrt[3]{4\left(z_2 + 3\sqrt{3z_1}\right)} + \sqrt[3]{4\left(z_2 - 3\sqrt{3z_1}\right)}\right), \tag{22}$$

here

$$z_1 = 27q^4 + 36pq^2\left(p^2 - 4r\right) + 4\left(p^2 - 4r\right)^3 - 8p^3q^2 - 4p^2\left(p^2 - 4r\right)^2,$$
$$z_2 = 27q^2 + 18p\left(p^2 - 4r\right)16p^3.$$

Additionally, by applying the resolvent cubic equation to (18), we have

$$\left(x^2 + \frac{p + y}{2}\right)^2 = y\left(x - \frac{q}{2y}\right)^2. \tag{23}$$

Thus,

$$x^2 + \frac{p + y}{2} = \pm\sqrt{y}\left(x - \frac{q}{2y}\right). \tag{24}$$

Consequently, the function x can be obtained.

$$x_{1,2} = \frac{-\sqrt{y} \pm \sqrt{-2p - y + \frac{2q}{\sqrt{y}}}}{2}, \quad x_{3,4} = \frac{\sqrt{y} \pm \sqrt{-2p - y - \frac{2q}{\sqrt{y}}}}{2}. \tag{25}$$

As a result, the optimized d_i can be obtained by $d_i = x - \frac{b_3}{4b_4}$ with the condition that d_i is a real number within (0,d). For the system parameter described in the next section, only $d_i = \frac{\sqrt{y} + \sqrt{-2p - y - \frac{2q}{\sqrt{y}}}}{2} - \frac{b_3}{4b_4}$ is satisfied.

In analyzing the performance of the system that has all channel models between the transceivers which are different, (W_i) is similar. The system in this case is indicated for scheme 2. The Taylor expansion is approximately used for a term $a^{\frac{-2d_i}{W_i}}$. Then, the partial differential equation with respect to each d_i is obtained, and each d_i can be obtained similarly to be mentioned above. However, in this case, we have made the Taylor expansion, solving partial differential $m + 1$ times to obtain each d_i.

3.2.2 Numerical evaluation of optimizing distance

The system model is the same as mentioned above, and the system parameters are summarized in Table 1. Figure 4 shows the end-to-end channel capacity responded to each number of the relay nodes, i.e., 3, 6, 9.

By contrast to the optimizing of the transmit power, the optimizing of the distance can change the channel capacity of RS_1 and improve the channel capacity of all bottleneck nodes. The comparison in channel capacity of optimizing the distance and optimizing the transmit power is shown in Figure 5. It is clear that in the case of optimizing the distance, the channel capacity of all relay nodes is the same and relatively higher than the end-to-end channel capacity in the case of optimizing the transmit power. The optimizing of the distance is effective; however, the relay node can not always be set at the desired location. For example, the desired location is on roads, in rivers, and so on. Thus, the relay node needs to be shifted in the front or the rear of the desired location. As a result, the channel capacity is decreased. In order to remain high channel capacity, the transmit power is adjusted after shifting distance by method of optimizing transmit power mentioned above. Figures 6 and 7 show the end-to-end channel capacity before and after adjusting the transmit power. In this scenario, the channel capacity of all relay nodes after adjusting the transmit power is almost the same, and achieves the end-to-end channel capacity of the system without shifting the location of the relay nodes. The optimizing of the transmit power in this scenario is much more effective.

In order to compare the end-to-end channel capacity when the number of the relay nodes is changed, the average end-to-end channel capacity from 1, 000× of calculation is shown in Figure 8. The end-to-end channel capacity is increased when the number of the relay nodes is small. Moreover, when the number of the relay nodes exceeds a certain value, the end-to-end channel capacity is decreased. The reason is that the SNR increases when the number of the relay nodes increases, however the allocation time of each relay node reduces rapidly. Therefore, although the SNR is high, the end-to-end channel capacity is decreased. As a result, there is the optimum number of the relay nodes for maximum end-to-end channel capacity.

As shown in Figure 8, we can confirm that after shifting the location of the relay nodes, the end-to-end channel capacity is rapidly decreased, especially when the shifted distance is large and/or the number of the relay nodes increases. However, the optimizing of the transmit power is quite effective in this case, and the end-to-end channel capacity after adjusting the transmit power

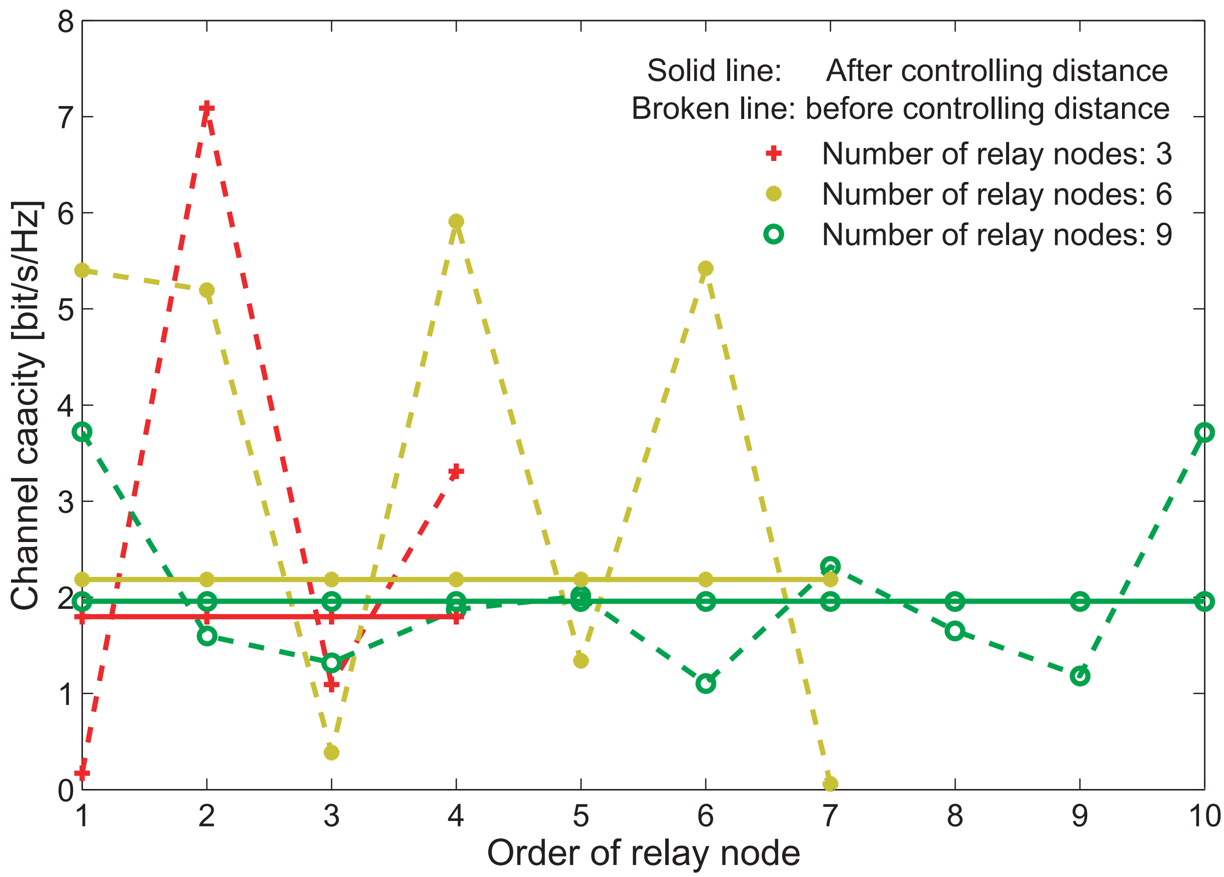

Figure 4 Channel capacity of optimizing the distance where the number of relay nodes are 3, 6, 9.

is approximate to the end-to-end channel capacity of the system without shifting the location of the relay nodes.

4 Optimizing allocation time

4.1 Optimization method of allocation time

Similar to transmit power and distance, in order to guarantee the end-to-end channel capacity, the allocation time of each relay needs to be optimized. To explain the optimizing of the allocation time, we assume all distance and transmit power to be fixed. When every relay transmits the signal in its allocation time, the end-to-end channel capacity is as follows.

$$C = \min (t_i C_i), \quad (i = 0, \dots, m). \tag{26}$$

The practicable channel capacity of the system is guaranteed when

$$t_i C_i = t_j C_j, \quad (i \neq j).$$

As a result,

$$\frac{t_i}{t_0} = \frac{C_0}{C_i}, \qquad \frac{\sum_{i=1}^{m} t_i}{t_0} = C_0 \sum_{j=1}^{m} \frac{1}{C_j}. \tag{27}$$

Therefore, the optimized allocation time is expressed as

$$t_0 = \frac{1}{1 + C_0 \sum_{j=1}^{m} \frac{1}{C_j}}, \tag{28}$$

$$t_i = \frac{\prod^{j \neq i+1} C_j}{\sum_{j=1}^{m} \prod^{k \neq j} C_k}, \quad (i = 1, \dots, m). \tag{29}$$

Consequently, the channel capacity of each relay node is the same and the end-to-end channel capacity is expressed as follows.

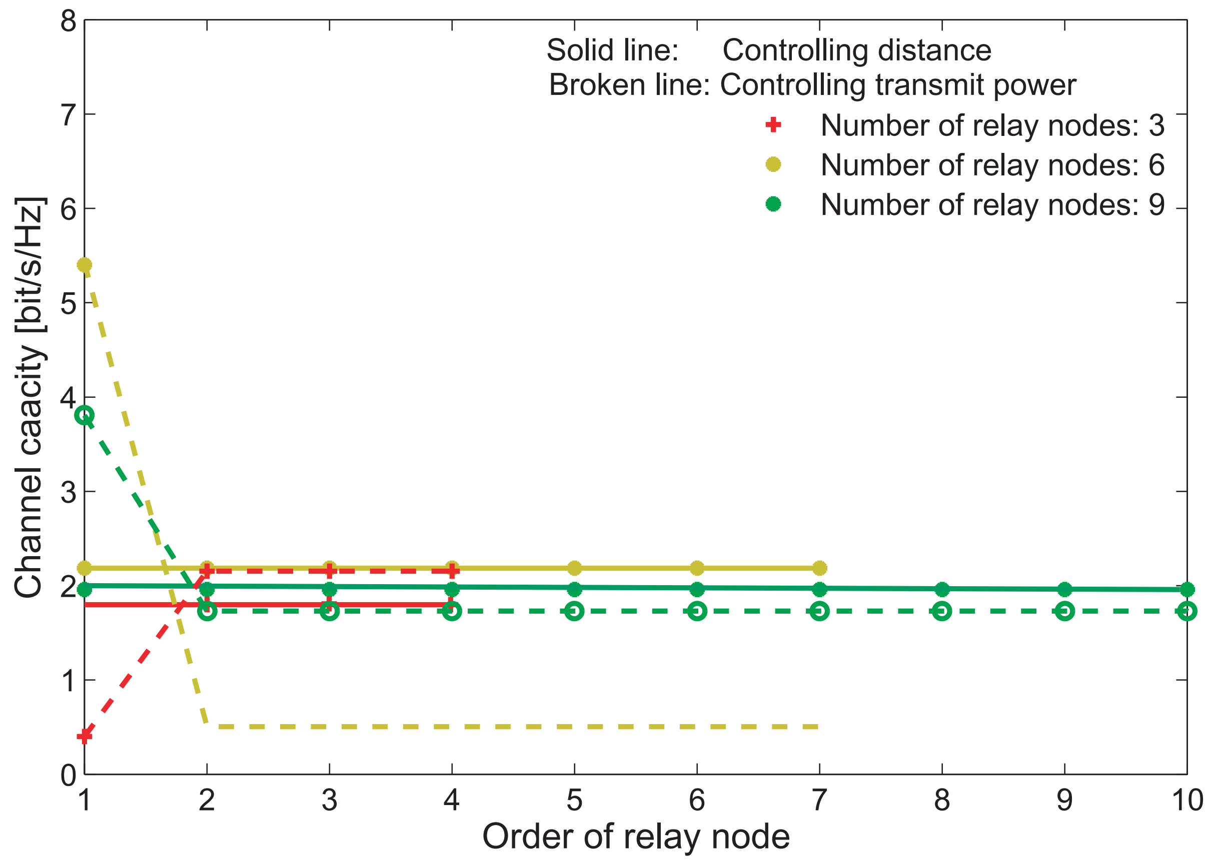

Figure 5 Comparing channel capacity of optimizing the transmit power and optimizing the distance where the number of the relay nodes are 3, 6, 9.

$$C = \frac{1}{\sum_{i=0}^{m} \frac{1}{C_i}}. \tag{30}$$

From (30), we have

$$C \geq \frac{1}{\sum_{i=0}^{m} \frac{1}{\min(C_i)}} = \frac{1}{m+1} \min(C_i). \tag{31}$$

It means that the end-to-end channel capacity of the system after optimizing allocation time is higher than that of the system with equal allocation time.

4.2 Numerical evaluation of optimizing allocation time

In this section, the optimizing of the distance, the transmit power and the allocation time by mathematical method at scheme 1 is compared. The channel model is the same as mentioned above. Figure 9 shows the channel capacity of each relay node in case the number of the relay nodes are 3 and 9. In the case of 3 relay nodes, the difference of channel capacity before and after optimizing the allocation time is small. It can be explained that if there is a relay node that has the channel capacity much smaller than that of another, the end-to-end channel capacity before and after optimizing allocation time is restricted by this relay node. In the case of optimizing allocation time, let's assume that the channel capacity of RS_k is the lowest ($C_k \ll C_i$, $i \neq k$,). It means $\frac{1}{C_k} \gg \frac{1}{C_i}$. Consequently, the end-to-end channel capacity in (30) can be changed as follows.

$$C \approx \frac{1}{\frac{1}{C_k}} = C_k. \tag{32}$$

However, as mentioned in the previous section, the end-to-end channel capacity after optimizing allocation time is higher than that before the optimizing. Additionally, the end-to-end channel capacity of optimizing allocation time is lower than that of optimizing distance, but higher than that of optimizing transmit power (in the case of 9 relay nodes in Figure 9). We can confirm the relation between optimization results to Figure 10. Figure 10 shows the average from 1, 000 × calculation

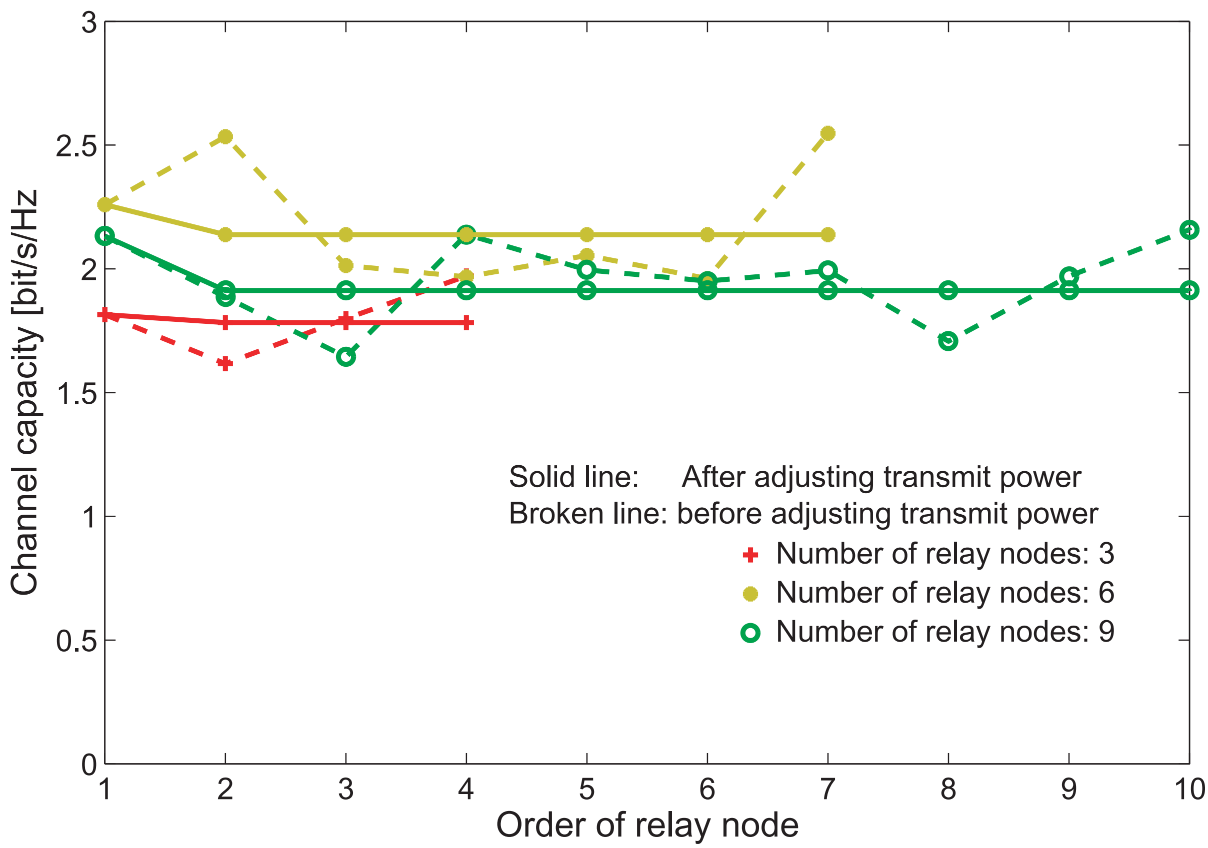

Figure 6 **Channel capacity after shifting distance within 100 m and adjusting transmit power**.

of the end-to-end channel capacity when the number of the relay nodes is changed from 1 to 10.

5 Optimized transmit power, distance, and allocation time simultaneously

Till now, we explained the method of optimizing transmit power, distance and allocation time separately in Sections 3 and 4. Each method is effective. However, the optimizing of the transmit power, the distance and the allocation time simultaneously is expected to achieve higher channel capacity than optimizing each one separately.

5.1 Mathematical method

In order to optimize the transmit power, the distance and the allocation time, firstly let's fix the distance, equalize the allocation time and optimize the transmit power. Therefore, the optimization method and the result in Section 3 can be applied and (7) can be rewritten by

$$E_{Tx} l_0 = \frac{E_{RS}}{\sum_{i=1}^{m} \frac{1}{l_i}}. \tag{33}$$

To solve this equation, each path loss should be expressed by Taylor expansion. It is relatively complex, especially at scheme 2. Consequently, in order to optimize the distance, the transmit power and the allocation time more easily in any channel model, the Markov Chain Monte Carlo (MCMC) method is proposed in the following section. Moreover, from (13), the SNR_i can be expressed as follows.

$$\begin{aligned} SNR_{\mathrm{i}} &= \frac{E_{\mathrm{i}} l_i}{M\sigma^2}, \\ &= \frac{E_{RS}}{M\sigma^2} \frac{1}{\sum_{i=1}^{m} \frac{1}{l_i}}, \\ &\leq \frac{E_{RS}}{M\sigma^2} \frac{m}{\left(\sum_{i=1}^{m} \frac{4\pi d_{\mathrm{i}}}{\lambda a^{\frac{d_{\mathrm{i}}}{W_{\mathrm{i}}}}} \right)^2}. \end{aligned} \tag{34}$$

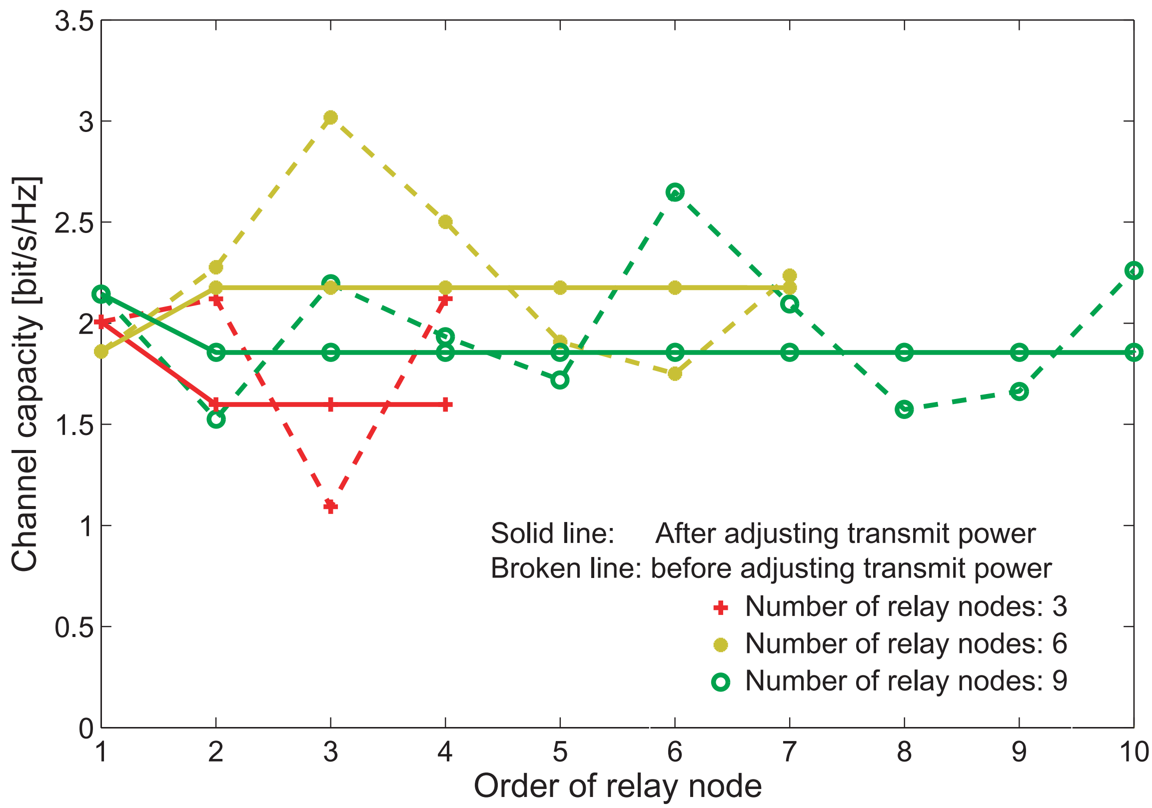

Figure 7 Channel capacity after shifting distance within 200 m and adjusting transmit power.

Thus, when W_i and d_i, $i = 1, \ldots, m$ are equal, respectively (the optimum distance of scheme 1), SNR_i becomes maximum. In this case, the equal allocation time is also the optimum solution. In other words, the optimized distance, transmit power and allocation time at scheme 1 is one of the optimal solutions for maximal end-to-end channel capacity of any channel model. Consequently, the optimizing of the distance, the transmit power and the allocation time lets the end-to-end channel capacity reach to this maximum.

5.2 Markov chain Monte Carlo method

The MCMC method is constructed to find the optimal state of transmit power, distance and allocation time that has the end-to-end channel capacity close to the maximal channel capacity. The algorithm is explained as follows.

Calculate $W = \frac{1}{(m+1)} \sum_{i=0}^{m} (W_i)$ and maximal channel capacity $C_{\max}$ (optimize transmit power, distance and allocation time at scheme 1).

Step 1: Create d_i, E_i and t_i randomly. Subject to

$$\sum_{i=0}^{m} (d_\mathrm{i}) = d, \quad \sum_{i=1}^{m} (E_\mathrm{i}) = E_{\mathrm{RS}}, \quad \sum_{i=0}^{m} (t_\mathrm{i}) = 1.$$

Step 2: Calculate all channel capacities C_i, and soften them from high to low. Adjust the distance, the transmit power and the allocation time to make all channel capacity to be almost the same.

$$d_\mathrm{i} = d_\mathrm{i} + (d_{\mathrm{m-i}} - d_\mathrm{i})\, rand1, \quad d_{\mathrm{m-i}} = d_{\mathrm{m-i}} - (d_{\mathrm{m-i}} - d_\mathrm{i})\, rand1, \tag{35}$$

$$t_\mathrm{i} = t_\mathrm{i} - (t_\mathrm{i} - t_{\mathrm{m-i}})\, rand2, \quad t_{\mathrm{m-i}} = t_{\mathrm{m-i}} + (t_\mathrm{i} - t_{\mathrm{m-i}})\, rand2, \tag{36}$$

$$E_\mathrm{i} = E_\mathrm{i} - (E_\mathrm{i} - E_{\mathrm{m-i}})\, rand3, \quad E_{\mathrm{m-i}} = E_{\mathrm{m-i}} + (E_\mathrm{i} - E_{\mathrm{m-i}})\, rand3, \quad (except\ E_{\mathrm{TX}}), \tag{37}$$

here, rand1, rand2, rand3 are random value within (0,1). Iterate step 2 until standard deviation of all channel capacities is smaller than sigma (σ).

Step 3: If end-to-end channel capacity of scheme 2 is close to maximal channel capacity $\left(\frac{C_{\max}-C}{C_{\max}} \leq \alpha\right)$, the algorithm is finished. Otherwise, return to step 1.

Compare to the mathematical method, MCMC algorithm is easier to optimize the distance, the transmit power and the allocation time simultaneously at any channel model. However, the MCMC algorithm requests running in the computer and the convergence of this algorithm should be discussed. The convergence is dependent on σ and α, if σ is not tight enough, the

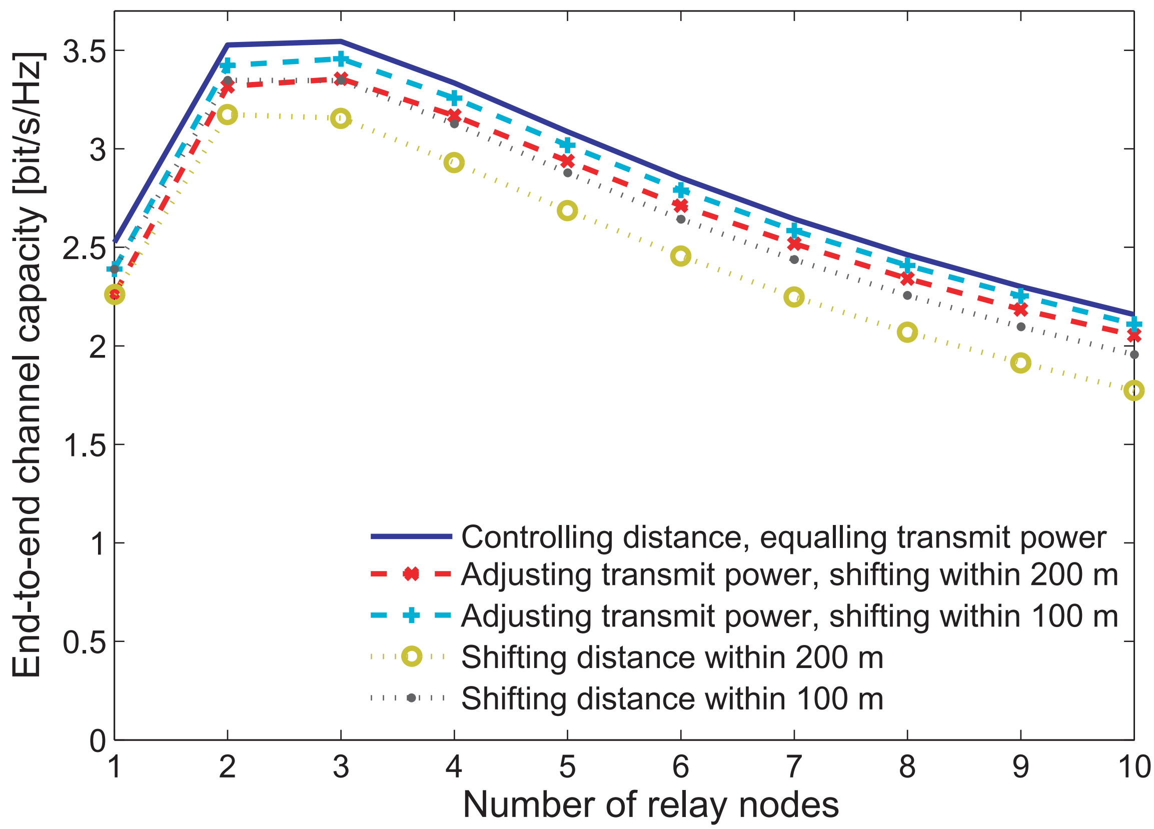

Figure 8 The end-to-end channel capacity responded to each number of relay node where W is 500 m.

algorithm doesn't converge. On the other hand, if σ is too tight, the algorithm takes a long time for convergence. Hence, for each α, the suitable σ needs to be considered.

Let's denote the average of all channel capacities to be $\bar{C}$. When the end-to-end channel capacity ($C = \min(C_i)$) is approximate to the maximal channel capacity ($C_{\max}$), we have $\bar{C} \approx C_{\max}$ and $|\max(C_i) - \bar{C}| \approx |\min(C_i) - \bar{C}|$. Thus, σ can be described by

$$\begin{aligned}\sigma^2 &= \frac{1}{m+1}\sum_{i=0}^{m}(C_i - \bar{C})^2,\\ &\le \frac{1}{2}\left((\max(C_i) - \bar{C})^2 + (\min(C_i) - \bar{C})^2\right),\\ &\le \frac{C_{\max}^2}{2}\left(\left(\frac{\max(C_i) - \bar{C}}{C_{\max}}\right)^2 + \left(\frac{\min(C_i) - \bar{C}}{C_{\max}}\right)^2\right),\\ &\le \frac{C_{\max}^2}{2}\left(\left(\frac{\max(C_i) - C_{\max}}{C_{\max}}\right)^2 + \left(\frac{\min(C_i) - C_{\max}}{C_{\max}}\right)^2\right),\\ &\le \frac{C_{\max}^2}{2}((\alpha)^2 + (\alpha)^2),\\ &\le C_{\max}^2\alpha^2.\end{aligned} \quad (38)$$

As a result, $\sigma = C_{\max}\alpha$ is the suitable value. Since $C_{\max}$ changes when the number of the relay nodes changes. σ is changed for each number of the relay nodes and α. Figure 11 shows the end-to-end channel capacity in case σ is changed, i.e., 1%, 5%, and 10%. Here, let W_i be random within (0, 1000) and satisfy $\frac{1}{m+1}\sum_{i=0}^{m} W_i = 500$. According to α, the end-to-end channel capacity of MCMC method is different. However, with small *alpha*, MCMC method optimizes the transmit power, the distance, the allocation time simultaneously and achieves the maximal channel capacity in any channel model.

6 Conclusion

In this article, we examined the performance of multi-hop relay systems with decode-and-forward method. The optimizing of the transmit power, the distance and the allocation time is effective in preventing some relay nodes from becoming the bottleneck of the system and in guaranteeing the end-to-end channel capacity. However, the optimizing of the distance is the most effective and the optimizing of the transmit power is the least effective. The optimizing of the transmit power is effective when the location of the relay nodes is shifted within a short range from the desired location. The

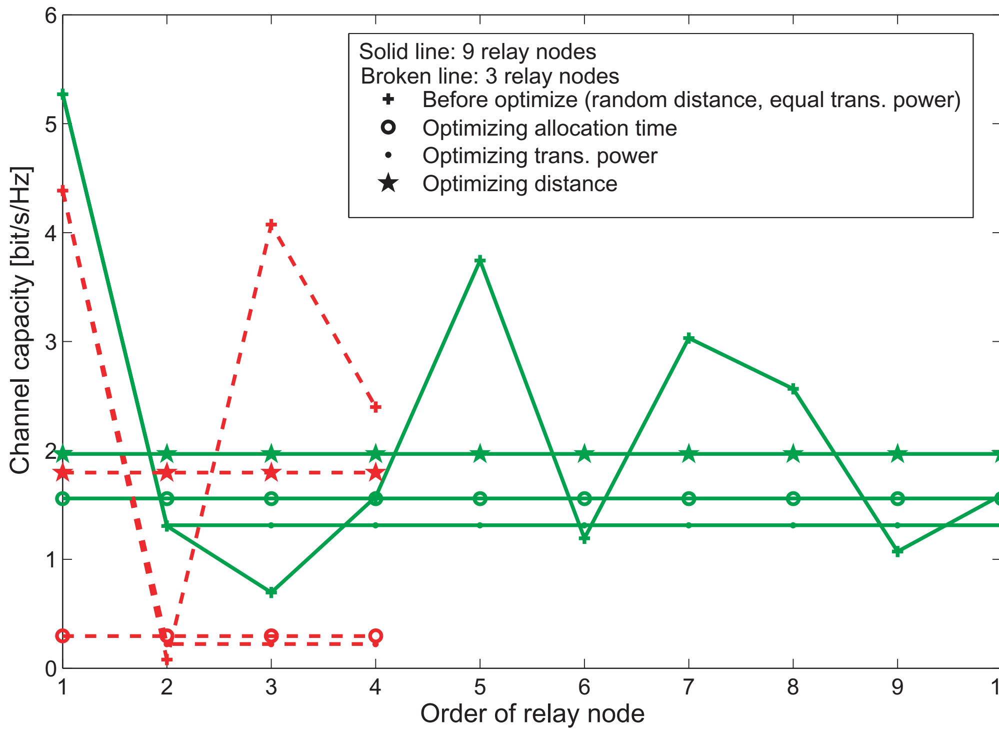

Figure 9 Channel capacity of optimizing the transmit power, the distance and the allocation time when the number of relay nodes are 3 and 9.

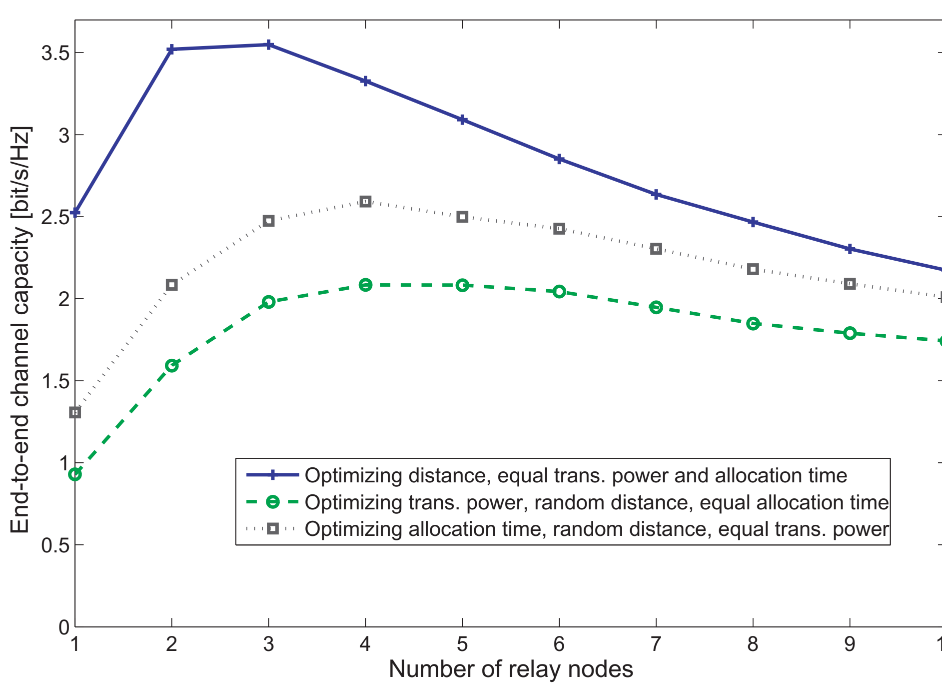

Figure 10 Comparing the end-to-end channel capacities of optimizing the transmit power, the distance and the allocation time.

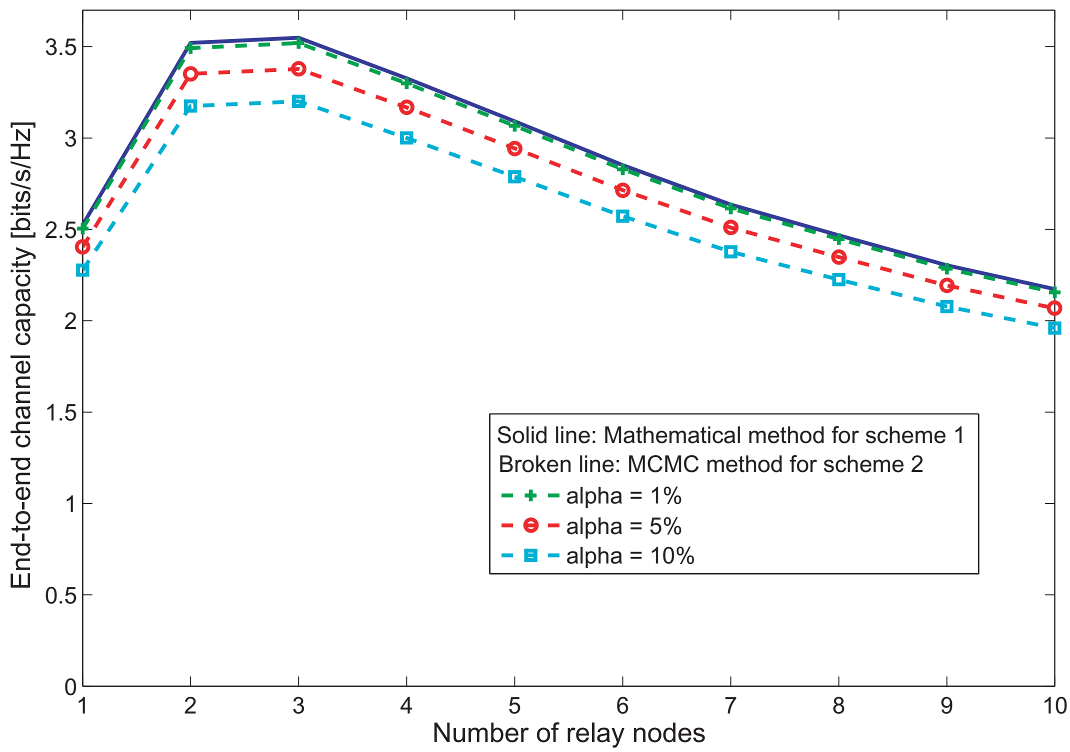

Figure 11 The end-to-end channel capacity obtained by MCMC method in some cases of alpha.

MCMC algorithm was proposed to optimize all transmit power, distance and allocation time simultaneously. MCMC method can achieve the maximal channel capacity.

In this article, in order to simplify the analysis, we have analyzed the system under Gaussian channel model. However, the performance of this system needs to be analyzed under the channel model which is close to the practice. Additionally, in order to apply the optimization method to any channel model, more general path loss functions needs to be considered. We considered the transmit power, the distance and the allocation time to guarantee the end-to-end channel capacity. The other method, such as the changing of modulation and/or coding is left for future studies.

Competing interests
The authors declare that they have no competing interests.

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