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Performance analysis of analog network coding based two-way amplify-and-forward system in mixed Rician and Nakagami- m fading environment

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Abstract

This paper presents the performance analysis of analog network coding-based two-way amplify-and-forward relaying under mixed Rician and Nakagami- m fading environment. Closed form expressions for both the cumulative distribution function and probability distribution function of the instantaneous end-to-end SNR are derived. Using those, the closed form expressions for the first moment, second moment, and the symbol error rate (SER) for M -PSK modulated signals are obtained. The performance of the system is analyzed in terms of outage probability, average SER, and ergodic capacity. In addition, we investigate the outage probability for high SNR scenario to identify more details of the system performance in depth. Simulations are performed to verify the correctness of our theoretical analysis.

Keywords: Analog network coding (ANC), Amplify-and-forward (AF), Cumulative distribution function (CDF), Probability distribution function (PDF), Outage probability, Average symbol error rate (SER), Ergodic capacity

Introduction

Network coding (NC) was first introduced a decade ago [1], owing to its potential for improving the performance of a wireless network. In a wireless relay network, the application of network coding at the physical layer has been shown to increase the network throughput [2]. This network throughput is achieved by reducing the number of time slots required to exchange information between two source nodes S_1 and S_2 via a relay node R , from four to two. Four time slots have been used traditionally.

We can divide the prior related research into two main categories. The first category deals with the performance analysis over the symmetric fading channels for the relay networks (two-way, dual-hop, multi-hop, and multi-cast) [3-7]. The second category deals with the performance of relay networks over asymmetric fading channels [8,9].

In the past years, the performance of relay networks (two-way, dual-hop, multi-hop, and multi-cast) over symmetric fading channels is being carried out by many

researchers. Work in [3] presents the outage probability and average symbol error rate (SER) analysis for two-way amplify-and-forward relaying over the symmetric (Nakagami- m) fading channel and in [4,5] authors consider the symmetric (Rayleigh) fading channel. Along with network coding (NC) for two-way relaying the performance is studied in terms of outage probability and ergodic sum-rate in [6] over symmetric (Nakagami- m) fading channel, and in [7] the optimal transmission scheme analysis is carried out by computing the outage probability over symmetric (Rayleigh) fading channel.

In a relay network two sources are communicating with each other by using a single relay. Considering a practical scenario, relay network may have a line-of-sight (LOS) communication in one of the sides (source to relay link) and on the other side it may have only multipath communication (relay to destination link), and vice versa [8]. In such scenario using asymmetric fading channel is a promising solution. Recently, there is an increasing research interest on the performance of relay networks over asymmetric fading channels, and some recent examples of that are [8,9]. For the dual-hop

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amplify-and-forward relay transmission system, the performance in terms of deriving the closed form analytical expressions for cumulative distribution function (CDF) and probability density function (PDF) of end-end signal-to-noise-ratio (SNR) over asymmetric (Nakagami- m and Rician) fading channels was assumed in [8]. The asymmetric (Rayleigh and Rician) fading channels have been investigated in [9] by deriving the exact and lower bound expressions for the outage probability and average bit error probability. None of these works have focused on the ergodic capacity analysis.

To the best of our knowledge the performance analysis over mixed Rician and Nakagami- m asymmetric fading channels with analog network coding (ANC)-based two-way amplify-and-forward relay system has not been investigated before. Hence, in this paper we have adopted ANC-based two-way amplify-and-forward system in a mixed Rician and Nakagami- m asymmetric fading environment. We have assumed source nodes S_1 and S_2 to be a base station (BS) and a mobile station (MS), respectively. We have used Rician fading channel between BS and relay node R as they are assumed to have LOS communication and we have used Nakagami- m fading channel between MS and relay node R as they are considered to have only multipath communication. This particular setup is shown in Figure 1.

We will analyze the performance by evaluating the closed form expressions for outage probability, average SER (using the derived CDF), and ergodic capacity (using the derived PDF). We have plotted the analytical results using these derived close form expressions. Additionally, we investigate the outage probability for high SNR regime for a more comprehensive analysis of the system performance. At high SNR, the asymptotic outage probability that is obtained is very close to the exact outage probability. Remainder of the paper is organized as follows. Section 2 presents the system model. Performance and capacity analysis are discussed in Sections 3 and 4, respectively. Section 5 provides the asymptotic analysis. Results are given in Section 6, and Section 7 concludes the paper.

System Model

We consider a network coding-based two-way relaying system model in mixed Rician and Nakagami- m fading

environment as shown in Figure 1. Two source nodes S_1 and S_2 communicate through relay node R . The Rician fading (line-of-sight) environment is employed between the $S_1 \rightarrow R$ link, where as Nakagami- m fading environment is employed between the $S_2 \rightarrow R$ link. The communications between the nodes is accomplished in two time slots. During the first time slot, both source nodes S_1 and S_2 transmit simultaneously to relay node R . During the second time slot, relay node R normalizes the received signal-based on its transmit power constraint and broadcasts the combined signal to both source nodes. The source node knows its own information, thus the self information is discarded and the information of other source node is retrieved. We assume equal transmit power P_s at source nodes S_1, S_2 and relay node R and equal variance N_0 for the additive white Gaussian noise (AWGN) at the three nodes. The instantaneous end-to-end SNR at source node S_1 and source node S_2 during the second time slot, γ_{S_1} and γ_{S_2} , can be given by [3,4,6]

$$\gamma_{S_1} = \frac{\gamma_1 \gamma_2}{2\gamma_1 + \gamma_2} \tag{1}$$

$$\gamma_{S_2} = \frac{\gamma_1 \gamma_2}{\gamma_1 + 2\gamma_2}, \tag{2}$$

where $\gamma_1 = P_s |h_1|^2 / N_0$ and $\gamma_2 = P_s |h_2|^2 / N_0$ are the instantaneous SNRs of $S_1 \rightarrow R$ and $S_2 \rightarrow R$ links, respectively. The average transmit SNR is $\gamma_0 = P_s / N_0$ and $\Omega_1 = \mathbb{E}\{h_1^2\}$, $\Omega_2 = \mathbb{E}\{h_2^2\}$ are the respective variances of h_1 and h_2 , where $\mathbb{E}\{\cdot\}$ represents the statistical average operator. Since h_1 experiences Rician fading, the instantaneous SNR γ_1 is a noncentral- χ^2 distribution with PDF given by

$$f_{\gamma_1}(\gamma) = \frac{(K+1)e^{-K}}{\bar{\gamma}_1} e^{-\frac{(K+1)\gamma}{\bar{\gamma}_1}} I_0 \left(2\sqrt{\frac{K(K+1)\gamma}{\bar{\gamma}_1}} \right), \tag{3}$$

where K is a Rician K -factor defined as the ratio of the powers of the line-of-sight components to the scattered components, $\bar{\gamma}_1 = \Omega_1 \gamma_0$ is the average SNR of the $S_1 \rightarrow R$ link, and I_0 is the zeroth order modified Bessel function of the first kind. For $K = 0$, Rayleigh fading is experienced and $K = \infty$ gives the AWGN (no fading) situation. Also

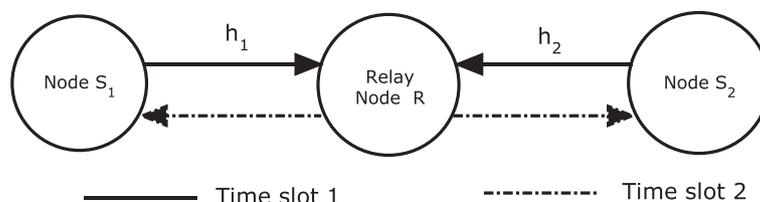


Figure 1 Block diagram of network coded-based two-way relay network.

h_2 experiences Nakagami- m fading, the PDF and the CDF of γ_2 are given by

$$f_{\gamma_2}(\gamma) = \frac{m^m}{\bar{\gamma}_2^m \Gamma(m)} \gamma^{m-1} e^{-\frac{m\gamma}{\bar{\gamma}_2}} \quad (4)$$

$$F_{\gamma_2}(\gamma) = 1 - \frac{\Gamma(m, (m/\bar{\gamma}_2)\gamma)}{\Gamma(m)}, \quad (5)$$

where m is the Nakagami- m fading parameter ranging from $\frac{1}{2}$ to ∞ , $\Gamma(\cdot)$ is the gamma function [10, Equation 8.310.1], $\Gamma(\cdot, \cdot)$ is the incomplete gamma function [10, Equation 8.350.2] and $\bar{\gamma}_2 = \Omega_2 \gamma_0$ is the average SNR of the $R \rightarrow S_2$ link. For $m = \frac{1}{2}$, it exhibits one-sided Gaussian distribution, for $m = 1$, it reduces to Rayleigh distribution and it converges to a nonfading AWGN channel as $m \rightarrow \infty$.

Performance analysis

Outage probability

The data is assumed to be correctly received at the source node S_1 when the equivalent receives SNR is greater than or equal to threshold SNR γ_{th} . When the received SNR at the source node S_1 is below the threshold SNR γ_{th} the receiver declares an outage. Thus the outage probability at the source node S_1 can be expressed as

$$P_{out, S_1}(\gamma_{th}) = \Pr(\gamma_{S_1} < \gamma_{th}) = F_{\gamma_{S_1}}(\gamma_{th}), \quad (6)$$

where $F_{\gamma_{S_1}}(\gamma_{th})$ is the CDF of the instantaneous SNR at source node S_1 , evaluated at $\gamma = \gamma_{th}$ as

$$\begin{aligned} F_{\gamma_{S_1}}(\gamma_{th}) &= \int_0^\infty \Pr\left(\frac{\gamma_1 \gamma_2}{2\gamma_1 + \gamma_2} \leq \gamma_{th} | \gamma_1\right) f_{\gamma_1}(\gamma_1) d\gamma_1 \\ &= \int_0^{\gamma_{th}} \Pr\left(\gamma_2 \geq \frac{2\gamma_{th}\gamma_1}{\gamma_1 - \gamma_{th}} | \gamma_1\right) f_{\gamma_1}(\gamma_1) d\gamma_1 \\ &\quad + \int_{\gamma_{th}}^\infty \Pr\left(\gamma_2 \leq \frac{2\gamma_{th}\gamma_1}{\gamma_1 - \gamma_{th}} | \gamma_1\right) f_{\gamma_1}(\gamma_1) d\gamma_1 \\ &= \int_0^{\gamma_{th}} f_{\gamma_1}(\gamma_1) d\gamma_1 + \int_{\gamma_{th}}^\infty F_{\gamma_2}\left(\frac{2\gamma_{th}\gamma_1}{\gamma_1 - \gamma_{th}}\right) f_{\gamma_1}(\gamma_1) d\gamma_1 \\ &= \frac{(K+1)e^{-K}}{\bar{\gamma}_1} \int_0^{\gamma_{th}} e^{-\frac{(K+1)\gamma_1}{\bar{\gamma}_1}} I_0\left(2\sqrt{\frac{K(K+1)\gamma_{th}}{\bar{\gamma}_1}}\right) d\gamma_1 \\ &\quad + \frac{(K+1)e^{-K}}{\bar{\gamma}_1} \int_{\gamma_{th}}^\infty e^{-\frac{(K+1)\gamma_1}{\bar{\gamma}_1}} I_0\left(2\sqrt{\frac{K(K+1)\gamma_{th}}{\bar{\gamma}_1}}\right) d\gamma_1 \\ &\quad - \frac{(K+1)e^{-K}}{\bar{\gamma}_1 \Gamma(m)} \int_{\gamma_{th}}^\infty e^{-\frac{(K+1)\gamma_1}{\bar{\gamma}_1}} \Gamma\left(m, \frac{2\gamma_1}{\gamma_1 - \gamma_{th}} \frac{m\gamma_{th}}{\bar{\gamma}_2}\right) \\ &\quad \times I_0\left(2\sqrt{\frac{K(K+1)\gamma_{th}}{\bar{\gamma}_1}}\right) d\gamma_1 \end{aligned} \quad (7)$$

After simplifying (7) using the identities [10, Equations 8.447.1, 3.351.1, 3.351.2, 8.352.1, and 8.352.2], we get

$$\begin{aligned} F_{\gamma_{S_1}}(\gamma_{th}) &= e^{-K} \sum_{g=0}^\infty \frac{K^g}{g!} - \frac{(K+1)e^{-K}}{\bar{\gamma}_1 \Gamma(m)} \sum_{g=0}^\infty \frac{K^g (K+1)^g}{(g!)^2 \bar{\gamma}_1^g} \\ &\quad \int_{\gamma_{th}}^\infty \gamma_1^g e^{-\frac{(K+1)\gamma_1}{\bar{\gamma}_1}} \Gamma\left(m, \frac{2\gamma_1}{\gamma_1 - \gamma_{th}} \frac{m\gamma_{th}}{\bar{\gamma}_2}\right) d\gamma_1 \end{aligned} \quad (8)$$

After applying the identity $e^K = \sum_{g=0}^\infty \frac{K^g}{g!}$ given in [10, Equation 1.211.1], the term $e^{-K} \sum_{g=0}^\infty \frac{K^g}{g!}$ in (8) reduces to 1 ($e^{-K} \sum_{g=0}^\infty \frac{K^g}{g!} = e^{-K} e^K = 1$), and also the integral term can be expressed in its general form as

$$\begin{aligned} F_{\gamma_{S_1}}(\gamma_{th}) &= 1 - \frac{(K+1)e^{-K}}{\bar{\gamma}_1 \Gamma(m)} \sum_{g=0}^\infty \frac{K^g (K+1)^g}{(g!)^2 \bar{\gamma}_1^g} \\ &\quad \times \int_{\gamma_{th}}^\infty x^g e^{-ax} \Gamma\left(c, 2b + \frac{2b}{x-y}\right) dx, \end{aligned} \quad (9)$$

where $a = (K+1)/\bar{\gamma}_1$, $b = m_2 \gamma_{th} / \bar{\gamma}_2$, $c = m_2$, $x = \gamma_1$, and $y = \gamma_{th}$. The incomplete gamma function of the integral in (9) can be written in the following form using identities [10, Equations 8.352.2, and 1.111]

$$\begin{aligned} \Gamma\left(c, 2b + \frac{2b}{x-y}\right) &= (c-1)! e^{-\left(2b + \frac{2b}{x-y}\right)} \sum_{q=0}^{c-1} \sum_{v=0}^q \frac{(2b)^q}{q!} \\ &\quad \times \binom{q}{v} \left(\frac{y}{x-y}\right)^v \end{aligned} \quad (10)$$

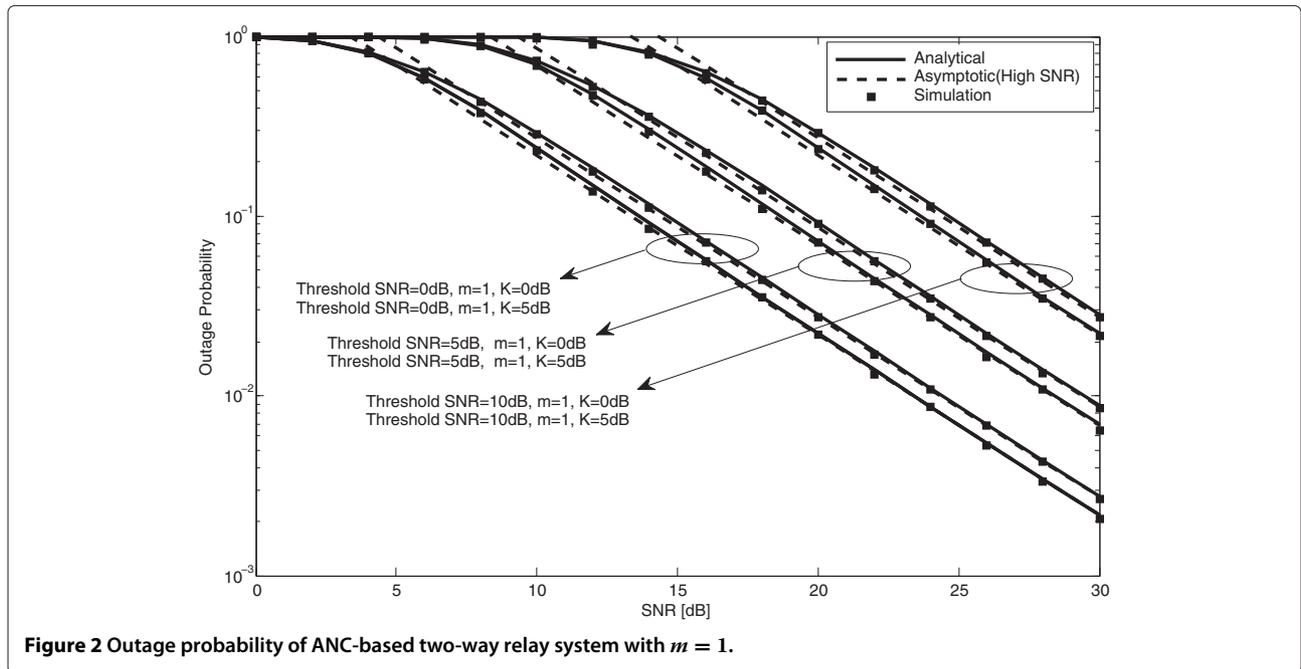
Using (10) in (9), we get

$$\begin{aligned} F_{\gamma_{S_1}}(\gamma_{th}) &= 1 - \frac{(K+1)e^{-K}}{\bar{\gamma}_1 \Gamma(m)} \sum_{g=0}^\infty \frac{K^g (K+1)^g}{(g!)^2 \bar{\gamma}_1^g} (c-1)! e^{-2b} \\ &\quad \sum_{q=0}^{c-1} \sum_{v=0}^q \frac{(2b)^q}{q!} \binom{q}{v} \int_{\gamma_{th}}^\infty x^g e^{-\left(ax + \frac{2b}{x-y}\right)} \left(\frac{y}{x-y}\right)^v dx \end{aligned} \quad (11)$$

The inner integral in (11) can be evaluated with the help of [10, Equation 3.471.9] yielding the compact closed-form CDF expression

$$\begin{aligned} F_{\gamma_{S_1}}(\gamma_{th}) &= 1 - \frac{2(K+1)(m-1)! \exp\left(-K - \left(\frac{2m}{\bar{\gamma}_2} + \frac{K+1}{\bar{\gamma}_1}\right)\gamma_{th}\right)}{\bar{\gamma}_1 \Gamma(m)} \\ &\quad \times \sum_{g=0}^\infty \sum_{q=0}^{m-1} \sum_{v=0}^q \sum_{l=0}^g \frac{1}{(g!)^2 q!} \binom{q}{v} \binom{g}{l} \left(\frac{K(K+1)}{\bar{\gamma}_1}\right)^g \\ &\quad \times \left(\frac{2m\gamma_{th}}{\bar{\gamma}_2}\right)^{q + \frac{l-v+1}{2}} \left(\frac{\bar{\gamma}_1}{K+1}\right)^{\frac{l-v+1}{2}} \\ &\quad \times \gamma_{th}^{g + \frac{v-l+1}{2}} \mathbf{K}_{l-v+1}\left(2\gamma_{th} \sqrt{\frac{2m(K+1)}{\bar{\gamma}_2 \bar{\gamma}_1}}\right), \end{aligned} \quad (12)$$

where $\mathbf{K}_u(\cdot)$ is the u -th order modified Bessel function of the second kind defined in [10, Equation 8.432.6]. Using the closed form CDF expression (12) in (6), the analytical probability of outage for two-way AF relaying network over asymmetric (Rician and Nakagami- m) fading



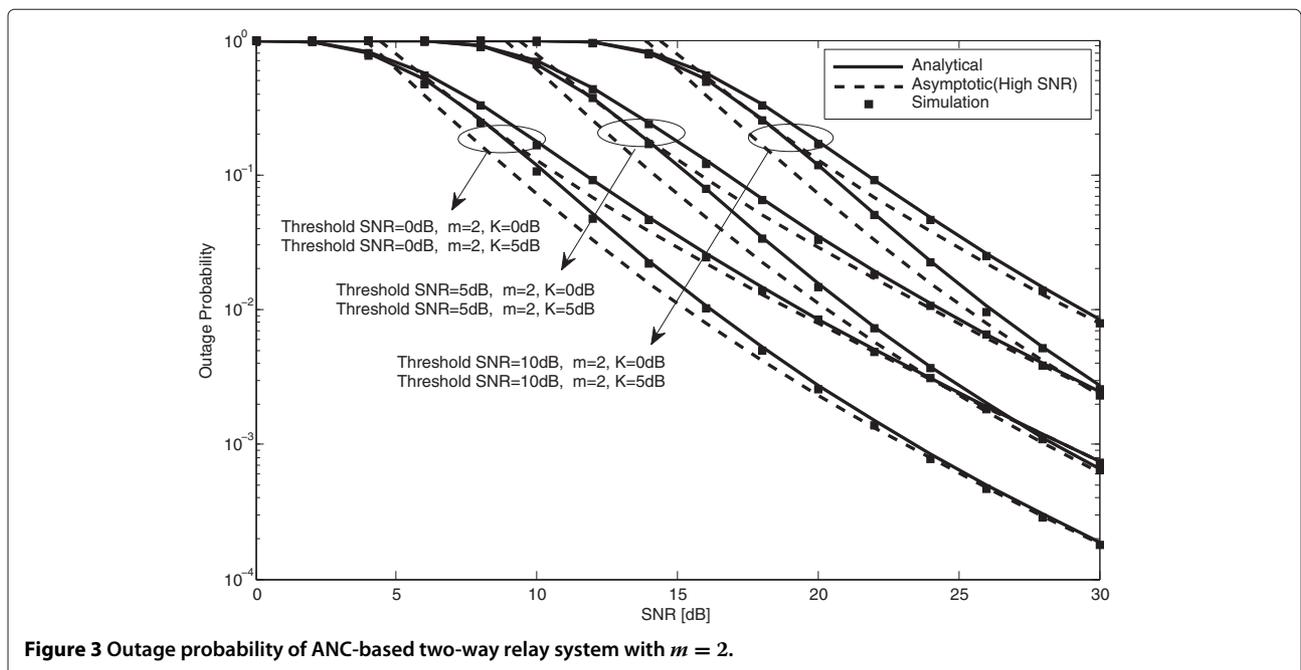
channels can be easily expressed in the closed form. The analytical outage probability versus SNR curves are plotted in Figures 2, 3 and 4 for different values of threshold SNR (γ_{th}), K , and m . As can be seen from (12), the fading parameter m and K affects the outage probability. When $m = 1$, Rayleigh fading is experienced as a special case. In Section 5, we compared both the analytical and simulation results of the outage probability with the derived asymptotic outage probability expression to better understand the system performance at high SNR regions.

Average symbol error rate (SER)

In this subsection we derive the closed form expression of average SER. For a wide range of modulation schemes the formula for average SER as given in [11]

$$SER = aE_{\gamma} [Q(\sqrt{2b\gamma})], \tag{13}$$

where $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-\frac{y^2}{2}} dy$ and $[a, b > 0]$, which defines the parameters for the modulation schemes. For



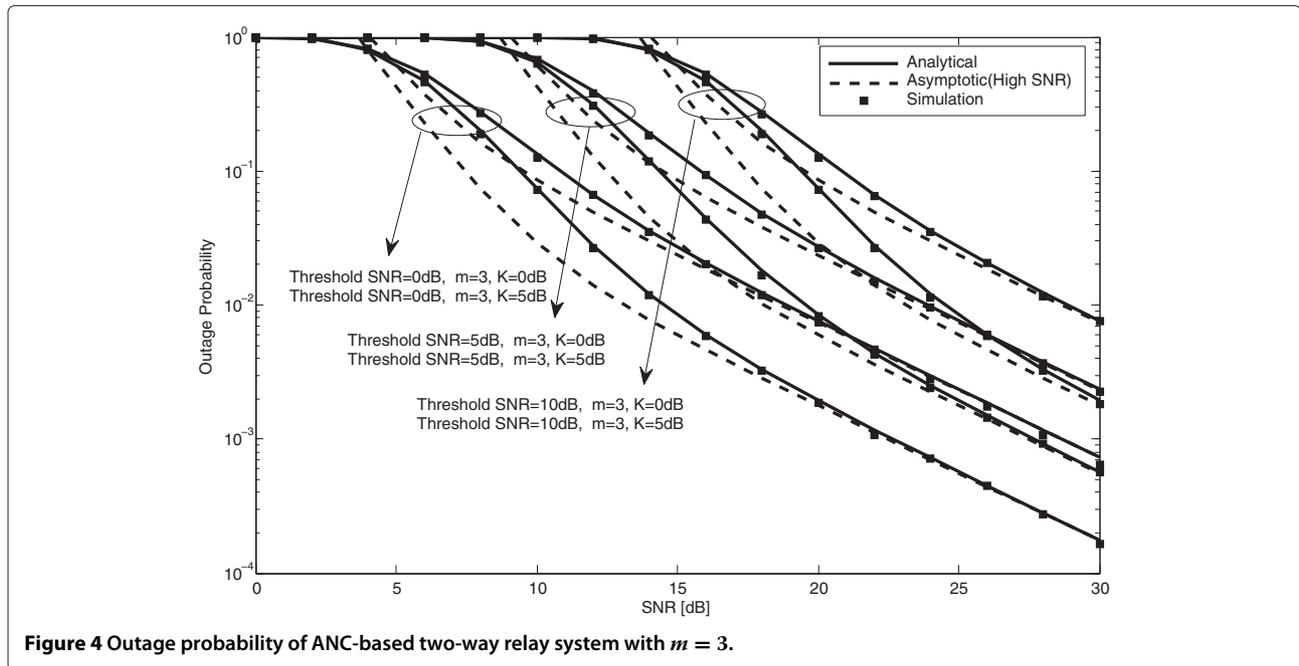


Figure 4 Outage probability of ANC-based two-way relay system with $m = 3$.

example for BPSK $a = 1$ and $b = 1$, and for QPSK $a = 1$ and $b = 0.5$. Integration by parts of (13) leads to

$$SER = \frac{a\sqrt{b}}{2\sqrt{\pi}} \int_0^\infty \frac{F_{\gamma_{th}}(z)}{\sqrt{z}} e^{-bz} dz \quad (14)$$

Substituting (12) in (14), we get

$$SER = \frac{a\sqrt{b}}{2\sqrt{\pi}} \int_0^\infty \frac{e^{-b\gamma_{th}}}{\sqrt{\gamma_{th}}} d\gamma_{th} - \frac{a\sqrt{b}}{\sqrt{\pi}} \frac{(K+1)(m-1)!e^{-K}}{\gamma_1 \Gamma(m)} \sum_{g=0}^\infty \sum_{q=0}^{m-1} \sum_{v=0}^q \sum_{l=0}^g \frac{1}{(g!)^2 q!} \binom{q}{v} \binom{g}{l} \left(\frac{K(K+1)}{\gamma_1}\right)^g \left(\frac{\gamma_1}{K+1}\right)^{\frac{l-v+1}{2}} \left(\frac{2m}{\gamma_2}\right)^{q+\frac{l-v+1}{2}} \int_0^\infty \gamma_{th}^{q+g+\frac{3}{2}-1} \exp\left(-\left(b+\frac{2m}{\gamma_2}+\frac{K+1}{\gamma_1}\right)\gamma_{th}\right) \mathbf{K}_{l-v+1}\left(2\gamma_{th}\sqrt{\frac{2m(K+1)}{\gamma_2\gamma_1}}\right) d\gamma_{th} \quad (15)$$

The inner integrals in (15) can be evaluated using the identities [10, Equations 3.361.2 and 6.621.3] to yield the closed-form SER expression

$$SER = \frac{a}{2} - \frac{a\sqrt{b}e^{-K}(K+1)(m-1)!}{\gamma_1 \Gamma(m)} \sum_{g=0}^\infty \sum_{q=0}^{m-1} \sum_{v=0}^q \sum_{l=0}^g \frac{1}{(g!)^2 q!} \binom{q}{v} \binom{g}{l} \left(\frac{K(K+1)}{\gamma_1}\right)^g \left(\frac{2m}{\gamma_2}\right)^{q+\frac{l-v+1}{2}} \left(\frac{\gamma_1}{K+1}\right)^{\frac{l-v+1}{2}} \frac{\left(4\sqrt{\frac{2m(K+1)}{\gamma_2\gamma_1}}\right)^{l-v+1} \Gamma\left(q+g+l-v+\frac{5}{2}\right)\Gamma\left(q+g-l+v+\frac{1}{2}\right)}{\left(b+\frac{2m}{\gamma_2}+\frac{K+1}{\gamma_1}+2\sqrt{\frac{2m(K+1)}{\gamma_2\gamma_1}}\right)^{q+g+l-v+\frac{5}{2}} \Gamma(q+g+2)} \mathbf{F}\left(q+g+l-v+\frac{5}{2}, l-v+\frac{3}{2}; q+g+2; \frac{b+\frac{2m}{\gamma_2}+\frac{K+1}{\gamma_1}-2\sqrt{\frac{2m(K+1)}{\gamma_2\gamma_1}}}{b+\frac{2m}{\gamma_2}+\frac{K+1}{\gamma_1}+2\sqrt{\frac{2m(K+1)}{\gamma_2\gamma_1}}}\right) \quad (16)$$

where $F(\cdot, \cdot; \cdot; \cdot)$ is the hypergeometric function defined in [10, Equation 9.111], $\Gamma(\cdot)$ is the gamma function defined in [10, Equation 8.310.1], a and b defines the parameters for the modulation schemes. The closed form average SER expression in (16) provides the analytical results over asymmetric (Rician and Nakagami- m) fading channels. For different values of K and m , the analytical average SER versus SNR curves are plotted in Figures 5, 6 and 7 for M -ary modulation such as M -PSK (BPSK and QPSK). It can be seen from (16), that by fixing value of m at the MS and changing the value of K at the BS, we can achieve a significant performance. For instance, at $m = 3$, we can observe the best case scenario.

Capacity analysis

Ergodic capacity

The ergodic capacity is defined in [12] as the expectation of the mutual information rate over the channel distribution between the source and the destination as

$$C_{erg} = E\left[\frac{1}{2}\log_2(1 + \gamma_{S_1})\right] = \log_2 e \cdot E\left[\frac{1}{2}\ln(1 + \gamma_{S_1})\right] \quad (17)$$

We obtain the exact closed form of approximate ergodic capacity expression of ANC-based two-way AF relaying system in a mixed Rician and Nakagami- m fading channels using the Taylor expansion of $\ln(1 + \gamma_{S_1})$, involving

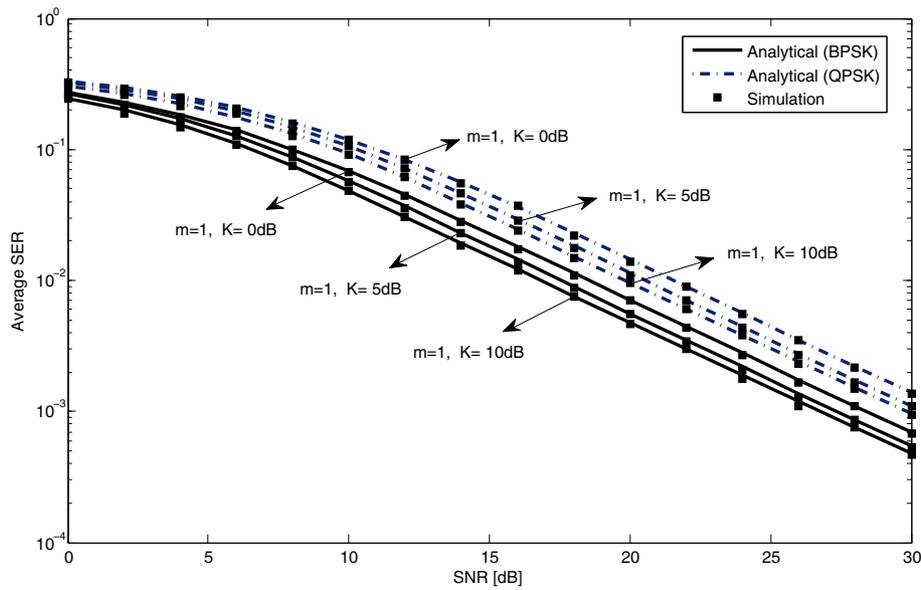


Figure 5 Average SER of ANC-based two-way relay system with $m = 1$.

the expectation $E\{\gamma_{th}\}$ and the power $E\{\gamma_{th}^2\}$ to obtain the second-order approximation for C_{erg} given in [12] as

$$C_{erg} \approx \frac{1}{2} \log_2 e \cdot \left[\ln(1 + E\{\gamma_{th}\}) - \frac{E\{\gamma_{th}^2\} - (E\{\gamma_{th}\})^2}{2(1 + E\{\gamma_{th}\})^2} \right] \quad (18)$$

In order to obtain the first moment (mean) $E\{\gamma_{th}\}$ and second moment $E\{\gamma_{th}^2\}$, we need first to get the probability density function (PDF) of the instantaneous SNR at

source node S_1 . This PDF can be found by taking the first derivative of (12) with respect of γ_{th} , yielding

$$f_{\gamma_s}(\gamma_{th}) = -\frac{2(K+1)(m-1)!e^{-K}}{\gamma_1 \Gamma(m)} \sum_{g=0}^{\infty} \sum_{q=0}^{m-1} \sum_{v=0}^g \frac{1}{(g!)^2 q!} \binom{q}{v} \binom{g}{l} \left(\frac{K(K+1)}{\gamma_1} \right)^g \left(\frac{2m\gamma_{th}}{\gamma_2} \right)^{q+\frac{l-v+1}{2}} \left(\frac{\gamma_1}{K+1} \right)^{\frac{l-v+1}{2}} \underbrace{\left[U_1'(\gamma_{th})U_2(\gamma_{th}) + U_1(\gamma_{th})U_2'(\gamma_{th}) \right]}_x, \quad (19)$$

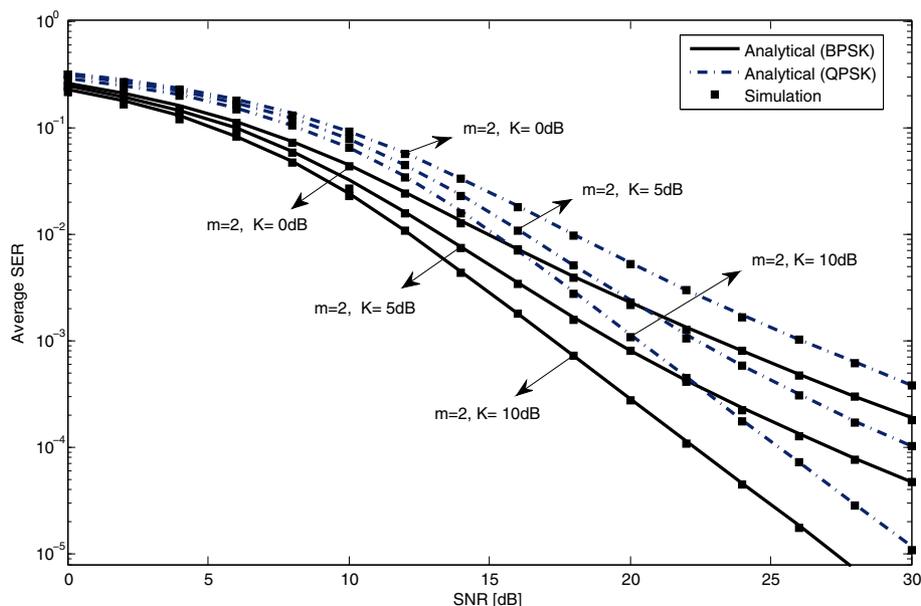
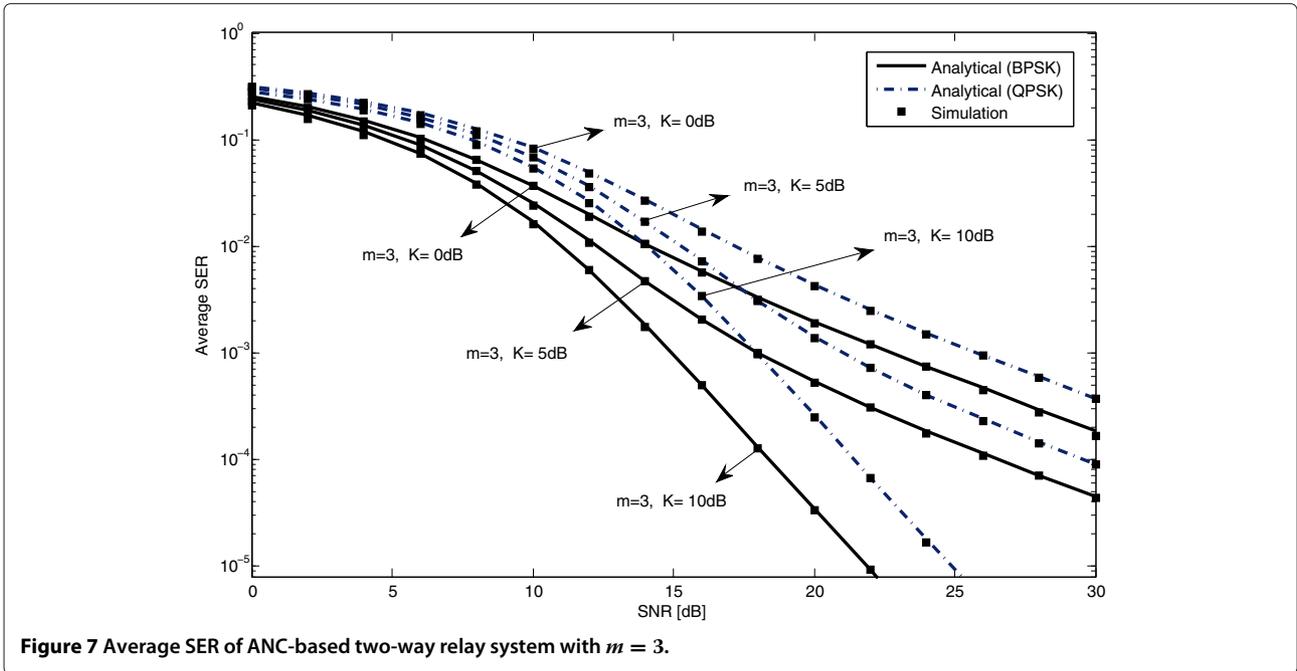
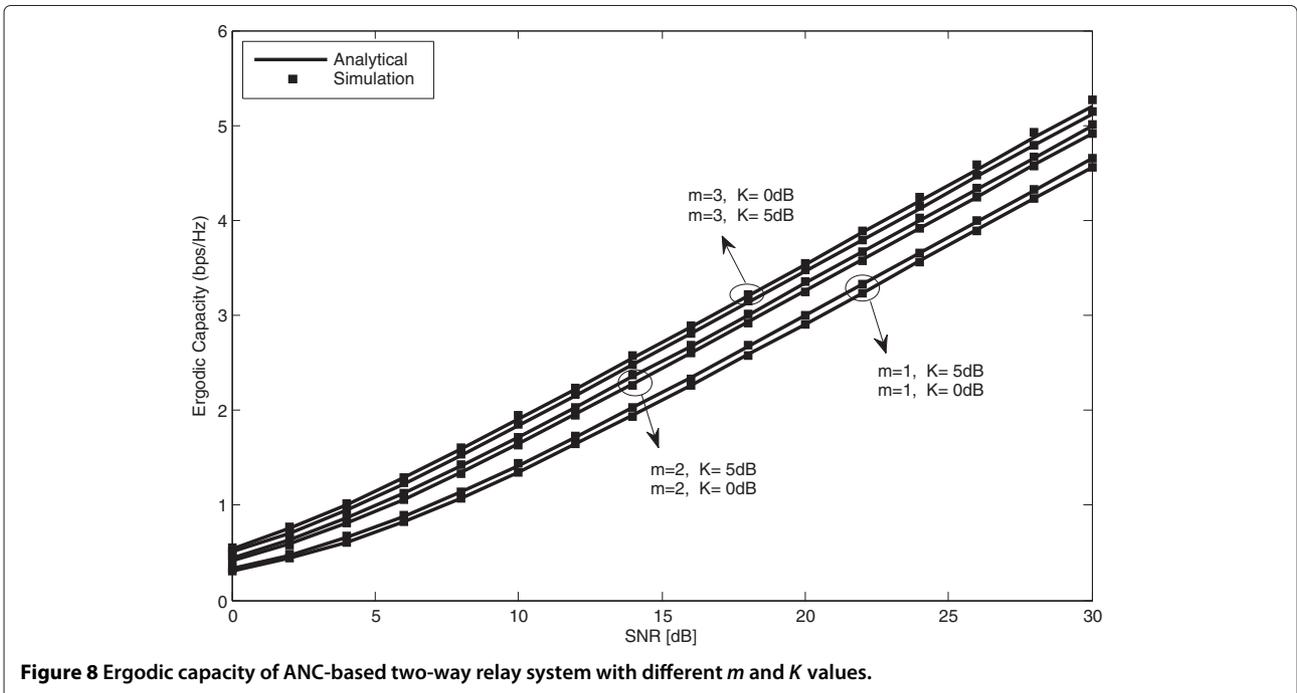


Figure 6 Average SER of ANC-based two-way relay system with $m = 2$.



where $U_1(\gamma_{th}) = e^{-\left(\frac{2m}{\gamma_2} + \frac{K+1}{\gamma_1}\right)\gamma_{th}}$, $U_2(\gamma_{th}) = \gamma_{th}^{q+g+1} \mathbf{K}_{l-v+1}\left(2\gamma_{th}\sqrt{\frac{2m(K+1)}{\gamma_2\gamma_1}}\right)$ and $U'_i(\gamma_{th})$ with $i = 1, 2$ is the first derivative of $U_i(\gamma_{th})$. The derivatives in (19) can be evaluated with the help of [10, Equation 8.486.12], yielding

$$\begin{aligned} \chi &= \left[U'_1(\gamma_{th})U_2(\gamma_{th}) + U_1(\gamma_{th})U'_2(\gamma_{th}) \right] \\ &= -\left(\frac{2m}{\gamma_2} + \frac{K+1}{\gamma_1}\right)\gamma_{th}^{q+g+1} e^{-\left(\frac{2m}{\gamma_2} + \frac{K+1}{\gamma_1}\right)\gamma_{th}} \mathbf{K}_{l-v+1}\left(2\gamma_{th}\sqrt{\frac{2m(K+1)}{\gamma_2\gamma_1}}\right) + \gamma_{th}^{q+g} \\ &\quad e^{-\left(\frac{2m}{\gamma_2} + \frac{K+1}{\gamma_1}\right)\gamma_{th}} \left[(q+g-r+v) \mathbf{K}_{l-v+1}\left(2\gamma_{th}\sqrt{\frac{2m(K+1)}{\gamma_2\gamma_1}}\right) - 2\gamma_{th} \right. \\ &\quad \left. \sqrt{\frac{2m(K+1)}{\gamma_2\gamma_1}} \mathbf{K}_{l-v}\left(2\gamma_{th}\sqrt{\frac{2m(K+1)}{\gamma_2\gamma_1}}\right) \right] \end{aligned} \tag{20}$$



After substituting (20) in (19), we get the closed form PDF expression as follows

$$f_{\gamma_{s_i}}(\gamma_{th}) = \frac{2(K+1)(m-1)e^{-K}}{\gamma_1 \Gamma(m)} \sum_{g=0}^{\infty} \sum_{q=0}^{m-1} \sum_{v=0}^q \sum_{l=0}^g \frac{1}{(g!)^2 q!} \binom{q}{v} \binom{g}{l} \left(\frac{K(K+1)}{\gamma_1}\right)^g \left(\frac{2m\gamma_{th}}{\gamma_2}\right)^{q+l-\frac{v+1}{2}} \left(\frac{\gamma_1}{K+1}\right)^{\frac{l-v+1}{2}} \gamma_{th}^{q+g} e^{-(\frac{2m}{\gamma_2} + \frac{K+1}{\gamma_1})\gamma_{th}} \left[\mathbf{K}_{l-v+1}\left(2\gamma_{th}\sqrt{\frac{2m(K+1)}{\gamma_2\gamma_1}}\right) \left(\gamma_{th}\left(\frac{2m}{\gamma_2} + \frac{K+1}{\gamma_1}\right) - (g+g-r+v)\right) + 2\gamma_{th}\sqrt{\frac{2m(K+1)}{\gamma_2\gamma_1}} \mathbf{K}_{l-v}\left(2\gamma_{th}\sqrt{\frac{2m(K+1)}{\gamma_2\gamma_1}}\right)\right] \quad (21)$$

Thus, the first moment (mean) $E\{\gamma_{th}\}$ and second moment $E\{\gamma_{th}^2\}$ can be obtained by using the closed form PDF expression in (21) and the identity [10, Equation 6.621.3] as

$$E\{\gamma_{th}\} = \int_0^{\infty} \gamma_{th} f_{\gamma_{s_i}}(\gamma_{th}) d\gamma_{th} = \frac{2e^{-K}(K+1)(m-1)!}{\gamma_1 \Gamma(m)} \sum_{g=0}^{\infty} \sum_{q=0}^{m-1} \sum_{v=0}^q \sum_{l=0}^g \frac{1}{(g!)^2 q!} \binom{q}{v} \binom{g}{l} \left(\frac{K(K+1)}{\gamma_1}\right)^g \left(\frac{\gamma_1}{K+1}\right)^{\frac{l-v+1}{2}} \left(\frac{2m}{\gamma_2}\right)^{q+l-\frac{v+1}{2}} \frac{\sqrt{\pi}\left(4\sqrt{\frac{2m(K+1)}{\gamma_2\gamma_1}}\right)^{l-v+1}}{\left(\frac{2m}{\gamma_2} + \frac{K+1}{\gamma_1}\right) 2\sqrt{\frac{2m(K+1)}{\gamma_2\gamma_1}})^{q+g+l-v+3}} \left(\frac{\frac{2m}{\gamma_2} + \frac{K+1}{\gamma_1}}{\frac{2m}{\gamma_2} + \frac{K+1}{\gamma_1} + 2\sqrt{\frac{2m(K+1)}{\gamma_2\gamma_1}}}\right) \frac{\Gamma(q+g+l-v+4)\Gamma(q+g-l+v+2)}{\Gamma(q+g+\frac{3}{2})} \mathbf{F}\left(q+g+l-v+4, l-v+\frac{3}{2}; q+g+\frac{7}{2}; \frac{2m}{\gamma_2} + \frac{K+1}{\gamma_1} - 2\sqrt{\frac{2m(K+1)}{\gamma_2\gamma_1}}; \frac{2m}{\gamma_2} + \frac{K+1}{\gamma_1} + 2\sqrt{\frac{2m(K+1)}{\gamma_2\gamma_1}}\right) - (q+g-l+v) \frac{\Gamma(q+g+l-v+3)\Gamma(q+g-l+v+1)}{\Gamma(q+g+\frac{3}{2})} \mathbf{F}\left(q+g+l-v+3, l-v+\frac{3}{2}; q+g+\frac{5}{2}; \frac{2m}{\gamma_2} + \frac{K+1}{\gamma_1} - 2\sqrt{\frac{2m(K+1)}{\gamma_2\gamma_1}}; \frac{2m}{\gamma_2} + \frac{K+1}{\gamma_1} + 2\sqrt{\frac{2m(K+1)}{\gamma_2\gamma_1}}\right) + 2 \frac{\Gamma(q+g+l-v+3)\Gamma(q+g-l+v+3)}{\Gamma(q+g+\frac{3}{2})} \mathbf{F}\left(q+g+l-v+3, l-v+\frac{1}{2}; q+g+\frac{7}{2}; \frac{2m}{\gamma_2} + \frac{K+1}{\gamma_1} - 2\sqrt{\frac{2m(K+1)}{\gamma_2\gamma_1}}; \frac{2m}{\gamma_2} + \frac{K+1}{\gamma_1} + 2\sqrt{\frac{2m(K+1)}{\gamma_2\gamma_1}}\right) \quad (22)$$

and

$$E\{\gamma_{th}^2\} = \int_0^{\infty} \gamma_{th}^2 f_{\gamma_{s_i}}(\gamma_{th}) d\gamma_{th} = \frac{2e^{-K}(K+1)(m-1)!}{\gamma_1 \Gamma(m)} \sum_{g=0}^{\infty} \sum_{q=0}^{m-1} \sum_{v=0}^q \sum_{l=0}^g \frac{1}{(g!)^2 q!} \binom{q}{v} \binom{g}{l} \left(\frac{K(K+1)}{\gamma_1}\right)^g \left(\frac{\gamma_1}{K+1}\right)^{\frac{l-v+1}{2}} \left(\frac{2m}{\gamma_2}\right)^{q+l-\frac{v+1}{2}} \frac{\sqrt{\pi}\left(4\sqrt{\frac{2m(K+1)}{\gamma_2\gamma_1}}\right)^{l-v+1}}{\left(\frac{2m}{\gamma_2} + \frac{K+1}{\gamma_1}\right) 2\sqrt{\frac{2m(K+1)}{\gamma_2\gamma_1}})^{q+g+l-v+4}} \left(\frac{\frac{2m}{\gamma_2} + \frac{K+1}{\gamma_1}}{\frac{2m}{\gamma_2} + \frac{K+1}{\gamma_1} + 2\sqrt{\frac{2m(K+1)}{\gamma_2\gamma_1}}}\right) \frac{\Gamma(q+g+l-v+5)\Gamma(q+g-l+v+3)}{\Gamma(q+g+\frac{3}{2})} \mathbf{F}\left(q+g+l-v+5, l-v+\frac{3}{2}; q+g+\frac{9}{2}; \frac{2m}{\gamma_2} + \frac{K+1}{\gamma_1} - 2\sqrt{\frac{2m(K+1)}{\gamma_2\gamma_1}}; \frac{2m}{\gamma_2} + \frac{K+1}{\gamma_1} + 2\sqrt{\frac{2m(K+1)}{\gamma_2\gamma_1}}\right) - (q+g-l+v) \frac{\Gamma(q+g+l-v+4)\Gamma(q+g-l+v+2)}{\Gamma(q+g+\frac{3}{2})} \mathbf{F}\left(q+g+l-v+4, l-v+\frac{3}{2}; q+g+\frac{7}{2}; \frac{2m}{\gamma_2} + \frac{K+1}{\gamma_1} - 2\sqrt{\frac{2m(K+1)}{\gamma_2\gamma_1}}; \frac{2m}{\gamma_2} + \frac{K+1}{\gamma_1} + 2\sqrt{\frac{2m(K+1)}{\gamma_2\gamma_1}}\right) + 2 \frac{\Gamma(q+g+l-v+4)\Gamma(q+g-l+v+4)}{\Gamma(q+g+\frac{3}{2})} \mathbf{F}\left(q+g+l-v+4, l-v+\frac{1}{2}; q+g+\frac{9}{2}; \frac{2m}{\gamma_2} + \frac{K+1}{\gamma_1} - 2\sqrt{\frac{2m(K+1)}{\gamma_2\gamma_1}}; \frac{2m}{\gamma_2} + \frac{K+1}{\gamma_1} + 2\sqrt{\frac{2m(K+1)}{\gamma_2\gamma_1}}\right) \quad (23)$$

respectively. Thus, by substituting (22) and (23) in (21), we obtained the second-order approximated ergodic capacity for ANC-based two-way AF relaying system in a mixed Rician and Nakagami- m fading channels Figure 8.

Asymptotic analysis

In this section, we derive the asymptotic expression for the outage probability, which provides the insight view of the system performance and the diversity order. To obtain the asymptotic results for high SNR, we change some variables as, $z_1 = \frac{\gamma_{th}}{\gamma_1}$ and $z_2 = \frac{\gamma_{th}}{\gamma_2}$. For high SNR analysis, the CDF of γ_{s_i} can be expressed as given in [13]

$$F_{\gamma_{s_i}}(z) = 1 - [1 - F_{\gamma_1}(z_1)] [1 - F_{\gamma_2}(z_2)] \quad (24)$$

we can rewrite the CDF given in (5) for higher SNR analysis, by using the identity [10, Equation 8.354.2]

$$\Gamma(a, x) = \Gamma(a) - \sum_{n=0}^{\infty} \frac{(-1)^n x^{a+n}}{n!(a+n)} = \Gamma(a) - x^a \sum_{n=0}^{\infty} \frac{(-x)^n}{\Gamma(n+1)(a+n)} \quad (25)$$

After applying the asymptotic expansion identity given in [14, Equation 14], (25) becomes

$$\Gamma(a, x) = \Gamma(a) - \frac{x^a}{a} \quad (26)$$

Using the result of (26) in (5), we get the CDF $F_{\gamma_2}(z_2)$ for high SNR as

$$F_{\gamma_2}(z_2) = 1 - \left[\frac{\Gamma(m) - m^m z_2^m / m}{\Gamma(m)}\right] = \frac{m^m z_2^m}{m \Gamma(m)} + o(z_2^m) \quad (27)$$

Also the CDF $F_{\gamma_1}(z_1)$ can be expressed by following the same procedure as in [15, Equation 15].

$$F_{\gamma_1}(z_1) = e^{-K} (1 + K) z_1 + o(z_1) \quad (28)$$

Hence, after some mathematical simplification and using (27), (28) in (24), we can express the outage probability of γ_{s_i} at the high SNR as,

$$F_{\gamma_{s_i}}(z) = \begin{cases} \frac{m^m z_2^m}{m \Gamma(m)} + e^{-K} (1 + K) z_1 + o(z_1) & m = 1 \\ \frac{(m+1)^m z_2^m}{m \Gamma(m)} + e^{-K} (1 + K) z_1 + o(z_2^m) & m > 1 \end{cases} \quad (29)$$

where $o(x) = \lim_{x \rightarrow 0} \frac{f(x)}{x}$ is defined in [11]. It is clear from (29) that the diversity order of the system is $\min[m, K]$. The diversity order will be verified in the Section 6.

Simulation results and discussion

In this section, we analyze the performance of the analog network coded two-way relay cooperative network by plotting the analytical curves along with the simulation results and comparing them over mixed Rician and Nakagami- m fading channels. The performance of the system is analyzed by plotting the curves in terms of outage probability, average SER, ergodic capacity, and

asymptotic outage probability for high SNR of the transmitted signal. We use Mathematica 7 to validate our close form analytical expressions and we build Monte-Carlo simulation using MATLAB for the system shown in Figure 1.

Figures 2, 3 and 4 show the outage probability performance of an ANC-based two-way relay network with different combinations of Nakagami- m factor values (1, 2, and 3), Rician K factor values (0, and 5 dB), and threshold SNR γ_{th} values (0, 5, and 10 dB). It can be observed from the figures that the increase in threshold SNR γ_{th} degrades the performance of the system. For increasing values of K and m the performance of the system gets improved. The analytical and simulation results are in close proximity and validate the accuracy of analytical closed form expression. In high SNR region the asymptotic curves coincide with the analytical curves.

Figures 5, 6 and 7 present the average SER analysis over the mixed Rician and Nakagami- m fading channels for BPSK and QPSK modulation schemes with different Nakagami- m factor values (1, 2, and 3, respectively). The parameters for the modulation scheme are $a = 1$, $b = 1$ for BPSK and $a = 1$, $b = 0.5$ for QPSK. Analytical results are obtained by substituting these parameters in (16). The results show that the performance improves with the increasing values of Rician K factor and Nakagami fading parameter m . From the examination of the slopes of SER curves, it is seen that, if we fix some value of m at the MS and change the value of K at the BS, then we can achieve a significant performance gain. In Figure 5 at average SER of 10^{-2} , the system with $m = 1$, $K = 0$ dB provides a performance gain of 1 dB over the system with $m = 1$, $K = 5$ dB for BPSK constellation. Furthermore, the system with $m = 1$, $K = 10$ dB is generating the same performance gain over the system with $m = 1$, $K = 5$ dB at the average SER of 10^{-3} . It can be shown that the diversity of system depends upon the channel conditions between the link S_1 and R . Here, we have improved the quality of the link between S_1 and R by increasing the value of Nakagami fading parameter m from 1 to 2 as shown in Figure 6. It is observed that the system is now exhibiting a larger performance gain at high SNR regions for different values of Rician factor K . For example, when average SER = 10^{-3} , a system with $m = 2$, $K = 0$ dB provides a performance gain of 4 dB over the system with $m = 2$, $K = 5$ dB and a performance gain of 2 dB is achieved between $m = 3$, $K = 5$ dB and $m = 3$, $K = 10$ dB for BPSK modulation scheme. The same is true for $m = 3$ as shown in Figure 7, we achieved the performance gain of 5 dB between $m = 2$, $K = 0$ dB and $m = 2$ and $K = 5$ dB and a performance gain of 3 dB is achieved for the system with $m = 3$, $K = 5$ dB over the system with $m = 3$, $K = 10$ dB for both BPSK and QPSK modulation schemes. Moreover, the analytical closed form average SER expression is verified with Monte Carlo simulations.

Figure 8 shows the ergodic capacity for the different m and K values. The correctness of the analytical closed form expression for ergodic capacity is validated with Monte Carlo simulations. It is observed that increasing the Rician K factor or Nakagami parameter m , improves the capacity of the system.

Conclusion

We analyzed the performance of network coding-based two-way relay network over mixed Rician and Nakagami- m asymmetric fading channels. We derived the closed-form expressions for the outage probability, average SER using the derived CDF, and ergodic capacity using the derived PDF to predict the performance of the proposed system. The outage probability for different values of threshold SNR γ_{th} over mixed Rician and Nakagami- m fading channels was calculated, which gives a better performance with lower amount of fading (high m and K values). The average SER analysis was carried out for BPSK, and QPSK modulation schemes. It is clear from the simulation results that for higher values of m and K , better performance is achieved. In addition to that, the asymptotic outage probability for high SNR has been obtained, which provides a better understanding of the diversity order of the system.

Competing interests

Both authors declare that they have no competing interests.

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