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On the first- and second-order statistics of the capacity of N*Nakagami-m channels for applications in cooperative networks

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Abstract

This article deals with the derivation and analysis of the statistical properties of the instantaneous channel capacity[a] of *N**Nakagami-*m* channels, which has been recently introduced as a suitable stochastic model for multihop fading channels. We have derived exact analytical expressions for the probability density function (PDF), cumulative distribution function (CDF), level-crossing rate (LCR), and average duration of fades (ADF) of the instantaneous channel capacity of *N**Nakagami-*m* channels. For large number of hops, we have studied the first-order statistics of the instantaneous channel capacity by assuming that the fading amplitude of the channel can approximately be modeled as a lognormal process. Furthermore, an accurate closed-form approximation has been derived for the LCR of the instantaneous channel capacity. The results are studied for different values of the number of hops as well as for different values of the Nakagami parameters, controlling the severity of fading in different links of the multihop communication system. The results show that an increase in the number of hops or the severity of fading decreases the mean channel capacity, while the ADF of the instantaneous channel capacity increases. Moreover, an increase in the severity of fading or the number of hops decreases the LCR of the instantaneous channel capacity of *N**Nakagami-*m* channels at higher levels. The converse statement is true for lower levels. The presented results provide an insight into the influence of the number of hops and the severity of fading on the instantaneous channel capacity, and hence they are very useful for the design and performance analysis of multihop communication systems.

Keywords: multihop communication systems, cooperative networks, instantaneous channel capacity, probability density function, cumulative distribution function, level-crossing rate, average duration of fades

1 Introduction

Multihop communication systems fall under the category of cooperative diversity systems, in which the intermediate wireless network nodes assist each other by relaying the information from the source mobile station (SMS) to the destination mobile station (DMS) [1-3]. This kind of communication scheme promises an increased network coverage, enhanced mobility, and improved system performance. It has applications in wireless local area networks (WLANs) [4], cellular networks [5], ad-hoc networks [6,7], and hybrid networks [8]. Based on the amount of signal processing used for relaying the received signal, the relays can generally be classified into two types, namely amplify-and-forward (or non-regenerative) relays [9,10] and decode-and-forward (or regenerative) relays [9,11]. The relay nodes in multihop communication systems can further be categorized into channel state information (CSI) assisted relays [12], which employ the CSI to calculate the relay gains and blind relays with fixed relay gains [13].

In order to characterize the fading in the end-to-end link between the SMS and the DMS in a multihop communication system with N hops, the authors in [14] have proposed the N*Nakagami-m channel model, assuming that the fading in each link between the wireless nodes can be modeled by a Nakagami-m process. The second-order statistical properties of multihop

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Rayleigh fading channels have been studied in [15], while for dualhop Nakagami-m channels, the second-order statistics of the received signal envelope has been analyzed in [16]. Moreover, the performance analysis of multihop communication systems for different kinds of relaying can be found in [10,13,17] and the multiple references therein.

The statistical properties of the instantaneous capacity of different multiple-input multiple-output (MIMO) channels have been studied in several articles. For example, by assuming that the instantaneous channel capacity is a random variable, the PDF and the statistical moments of the instantaneous channel capacity have been derived in [18]. Moreover, by describing the instantaneous channel capacity as a discrete-time or a continuous-time stochastic process, the LCR and ADF of the instantaneous channel capacity have been studied in [19]. Furthermore, analytical expressions for the PDF, CDF, LCR, and ADF of the continuous-time instantaneous capacity of MIMO channels by using orthogonal space-time block codes have been derived in [20]. The temporal behavior of the instantaneous channel capacity can be studied with the help of the LCR and ADF of the channel capacity. The LCR of the instantaneous channel capacity describes the average rate of up-crossings (or down-crossings) of the instantaneous channel capacity through a certain threshold level. The ADF of the instantaneous channel capacity denotes the average duration of time over which the instantaneous channel capacity is below a given level [20,21]. In the literature, the analysis of the LCR and ADF has mostly been carried out for the received signal envelope, which provides useful information regarding the statistics of burst errors occurring in fading channels [22]. However, in [23], the channel capacity for systems employing multiple antennas has been proposed as a more pragmatic performance merit than the received signal envelope. Therein, the authors have used the LCR of the instantaneous channel capacity to improve the system performance. Hence, it is important to study the LCR and ADF in addition to the PDF and CDF of the instantaneous channel capacity in order to meet the increasing demand for high data rates in mobile communication systems[b]. In [24], the authors analyzed the statistical properties of the instantaneous capacity of dualhop Rice channels employing amplify-and-forward based blind relays. An extension of the work in [24] to the case of dualhop Nakagami-m channels has been presented in [25]. The ergodic capacity of generalized multihop fading channels has been studied in [26]. Though a lot of artilces have been published in the literature dealing with the performance and analysis of multihop communication systems, the statistical properties of the instantaneous capacity of N*Nakagami-m channels have not been investigated so far. The aim of this article is to fill in this gap of information.

In this article, the statistical properties of the instantaneous capacity[c] of N*Nakagami-m channels are analyzed. For example, we have derived exact analytical expressions for the PDF, CDF, LCR, and ADF of the channel capacity. The mean channel capacity (or the ergodic capacity) can be obtained from the PDF of the channel capacity [27], while the CDF of the channel capacity is helpful for the derivation of the outage capacity [27]. Both the mean channel capacity and outage capacity have widely been used in the literature due to their importance for the system design. The mean channel capacity is the ensemble average of the information rate over all realizations of the channel capacity [28]. The outage capacity is defined as the maximum information rate that can be transmitted over a channel with an outage probability corresponding to the probability that the transmission cannot be decoded with an arbitrarily small error probability [29]. In general, the mean channel capacity is less complicated to study analytically than the outage capacity [30]. Although the mean channel capacity and outage capacity are important quantities that describe the channel, they do not give any insight into the dynamic behavior of the channel capacity. For example, the outage capacity does not provide any information regarding the spread of the outage intervals or the rate of occurrence of these outage durations in the time domain. In [23], it has been demonstrated that the temporal behavior of the channel capacity is very useful for the improvement of the overall network performance.

The rest of the article is organized as follows. In Section 2, we briefly describe the N*Nakagami-m channel model and some of its statistical properties. Section 3 presents the statistical properties of the capacity of N*Nakagami-m channels. A study on the first-order statical properties of the channel capacity for a large number of hops N is presented in Section 4. The analysis of the obtained results is carried out in Section 5. The concluding remarks are finally stated in Section 6.

2 The N*Nakagami-m channel model

Amplify-and-forward relay-based multihop communication systems consist of an SMS, a DMS, and N - 1 blind mobile relays MR_n (n = 1, 2,..., N - 1), as depicted in Figure 1. In this article, we have assumed that the fading in the SMS-MR_1 link, MR_n-MR_{n+1} (n = 1, 2, ..., N - 2) links, and the MR_{N-1}-DMS link is characterized by independent but not necessarily identical Nakagami-m processes denoted by $\chi_1(t)$, $\chi_{n+1}(t)$ (n = 1, 2,..., N - 2), and $\chi_N(t)$, respectively. The received signal $r_n(t)$ at the nth

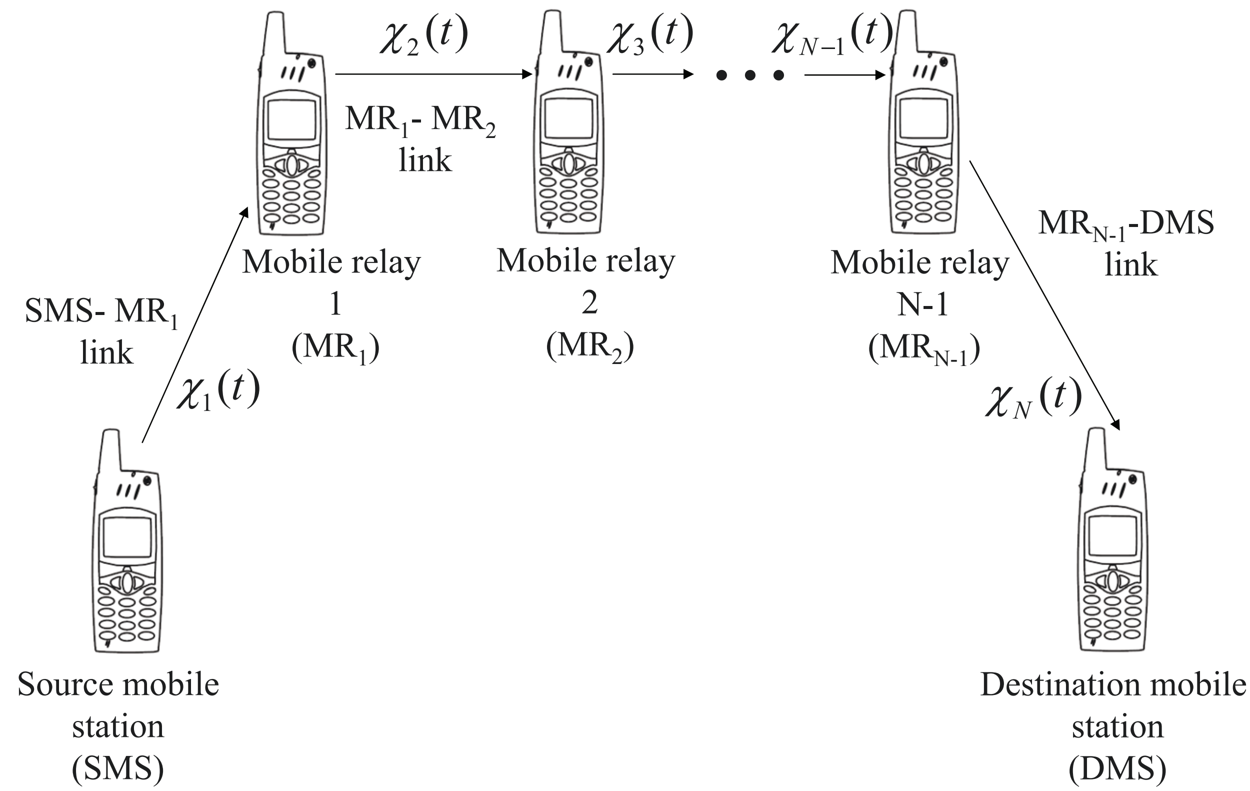

Figure 1 The propagation scenario describing *NNakagami-*m* fading channels**.

mobile relay MR$_n$ (n = 1, 2, ..., N - 1) or the DMS (n = N) can be expressed as [31]

$$r_n(t) = G_{n-1}\chi_n(t)r_{n-1}(t) + n_n(t) \tag{1}$$

where $n_n(t)$ is the additive white Gaussian noise (AWGN) at the nth relay or the DMS with zero mean and variance $N_{0,n}$, G_{n-1} denotes the gain of the (n - 1)th (n = 2, 3, ..., N) relay, $r_0(t)$ represents the signal transmitted from the SMS, and G_0 equals unity. The PDF $p_{\chi_n}(z)$ of the Nakagami-m process $\chi_n(t)$ (n = 1, 2, ..., N) is given by [32]

$$p_{\chi_n}(z) = \frac{2m_n^{m_n}z^{2m_n-1}}{\Gamma(m_n)\Omega_n^{m_n}}e^{-\frac{m_n z^2}{\Omega n}}, z \geq 0 \tag{2}$$

where $\Omega_n = E\{\chi_n^2(t)\}, m_n = \Omega_n^2/\mathrm{Var}\{\chi_n^2(t)\}$, and $\Gamma(\cdot)$ represents the gamma function [33]. The expectation and the variance operators are denoted by $E\{\cdot\}$ and Var$\{\cdot\}$, respectively. The parameter m_n controls the severity of the fading, associated with the nth link of the multihop communication system. Increasing the value of m_n decreases the severity of fading and vice versa. The overall fading channel describing the SMS-DMS link can be modeled as an N*Nakagami-m process given by [14,15]

$$\Xi(t) = \prod_{n=1}^{N} G_{n-1}\chi_n(t) = \prod_{n=1}^{N} \acute{\chi}_n(t) \tag{3}$$

where each of the processes $\acute{\chi}_n(t)(n = 1, 2, ..., N)$ follows the Nakagami-m distribution $p_{\acute{\chi}_n}(z)$ with parameters m_n and $\acute{\Omega}_n = G_{n-1}^2\Omega_n$. To gain an insight into the relationship between the relay gains G_n and the instantaneous signal-to-noise ratio (SNR) $\gamma(t)$ at the DMS, one can see the results presented in [[13], Equations (1)-(3)]. Therein, it can easily be observed that increasing the relay gains G_n increases the instantaneous SNR at the DMS for any arbitrary fixed values of the noise variances at the relays. However, at any instant of time t, the value of $\gamma(t)$ is always less than or equal to $\gamma_1(t)$, representing the instantaneous SNR at the first mobile relay In other words, as the value of G_n increases, the value of $\gamma(t)$ approaches $\gamma_1(t)$ for any value of t. It is worth mentioning that in general, the total noise at the DMS can be represented as a sum of products. Specifically, it is a sum of N terms, where except for one (which is the noise component of the final hop), all the other (N - 1) terms can be expressed as a product of the corresponding hop's noise component and the channel gains of all the pervious hops

[34]. However, we have assumed that each product term has Gaussian distribution and is independent from the others. Hence, the sum is also assumed to be Gaussian distributed, making the AWGN assumption valid at the DMS. In the following, for the sake of simplicity, we will assume a fixed noise power N_0 at the DMS. Hence, the instantaneous SNR at the DMS is given by $\gamma(t) = P_S(t)/N_0$. Here, $P_S(t)$ denotes the instantaneous signal power at the DMS and is expressed as $Ps(t) = \prod_{n=1}^{N} G_{n-1}^2 |\chi_n(t)|^2$.

For the calculation of the PDF of the capacity of N*Nakagami-m channels, we need to find the PDF $p_{\Xi^2}(z)$ of the squared N*Nakagami-m process $\Xi^2(t)$. Furthermore, for the calculation of the LCR and the ADF of the channel capacity, we need to find an expression for the joint PDF $p_{\Xi^2\dot{\Xi}^2}(z,\dot{z})$ of the squared process $\Xi^2(t)$ and its time derivative $\dot{\Xi}^2(t)$ at the same time t. By employing the relationship $p_{\Xi^2}(z) = p_{\Xi}\left(\sqrt{z}\right)/\left(2\sqrt{z}\right)$ [[35], Equations (5-22)], the PDF $p_{\Xi^2}(z)$ can be expressed in terms of the PDF $p_{\Xi}(z)$ of the N*Nakagami-m process $\Xi(t)$ in [[14], Equation (4)] as

$$p_{\Xi 2}(z) = \frac{1}{z\prod\limits_{n=1}^{N}\Gamma(m_i)} G_{0,N}^{N,0}\left[z\prod_{n=1}^{N}\left(\frac{m_n}{\acute{\Omega}_n}\right) \Bigg| \begin{matrix} - \\ m_1, m_2, .., m_N \end{matrix} \right], z \geq 0. \tag{4}$$

In (4), $G_{0,N}^{N,0}[\cdot]$ denotes the Meijer's G-function [[33], Equation (9.301)]. By following a similar procedure presented in [[15], Equations (12)-(15)] and by applying the concept of transformation of random variables [[35], Equations (7-8)], it can be shown that the expression for the joint PDF $p_{\Xi^2\dot{\Xi}^2}(z,\dot{z})$ can be written as

$$\begin{aligned} p_{\Xi^2\dot{\Xi}^2}(z,\dot{z}) &= \frac{1}{4z} p_{\Xi\dot{\Xi}}\left(\sqrt{z}, \frac{\dot{z}}{2\sqrt{z}}\right) \\ &= \frac{1}{4z}\int\limits_{x_1=0}^{\infty}\cdots\int\limits_{x_{N-1}=0}^{\infty} p_{\chi' N}\left(\sqrt{z}/\prod_{n=1}^{N-1}x_n\right) \\ &\quad \frac{p_{\frac{\dot{\Xi}}{2\sqrt{\Xi}}\sqrt{\Xi}_{\chi'1\cdots\chi'N-1}}\left(\frac{\dot{z}}{2\sqrt{z}}\,\Big|\sqrt{z}, x_1, ..., x_{N-1}\right)}{\prod\limits_{n=1}^{N-1}x_n} \\ &\quad \times p_{\chi'1}(x_1)\dots p_{\chi' N-1}(x_{N-1})dx_1\dots dx_{N-1} \end{aligned} \tag{5}$$

for $z \geq 0$ and $|\dot{z}| < \infty$, where

$$\begin{aligned} &p_{\frac{\dot{\Xi}}{2\sqrt{\Xi}}\sqrt{\Xi}_{\dot{\chi}1\cdots\dot{\chi}N-1}}\left(\frac{\dot{z}}{2\sqrt{z}}\,\Big|\sqrt{z}, x_1\dots, x_{N-1}\right) \\ &= \frac{1}{\sqrt{2\pi}}\frac{e^{-\frac{\dot{z}}{8zK^2(z,x1,\dots,xN-1)}}}{K(z,x_1,\cdots,x_{N-1})} \end{aligned} \tag{6}$$

and

$$K^2(z,x_1,\dots,x_{N-1}) = \beta_N\left[1+\frac{\sum_{n=1}^{N-1}\frac{\beta_n}{\beta_N x_n^2}}{\left(\prod_{n=1}^{N-1}x_n^2\right)}\right]\prod_{n=1}^{N-1}x_n^2, \tag{7a}$$

$$\beta_n = \frac{\acute{\Omega}_n\pi^2}{m_n}\left(f_{\max_n}^2+f_{\max_{n+1}}^2\right), \quad n = 1,2,\dots,N. \tag{7b}$$

Here, $f_{\max_1}$ and $f_{\max_{N+1}}$ represent the maximum Doppler frequencies of the SMS and DMS, respectively, while $f_{\max_{n+1}}$ denotes the maximum Doppler frequency of the nth mobile relay MR_n (n = 1, 2, ..., N - 1). It should be mentioned that the expression obtained in (7b) is only valid under isotropic scattering conditions [36,37].

3 Statistical properties of the capacity of *N**Nakagami-*m* channels

The instantaneous channel capacity $C(t)$ is a time-varying process and evolves in time as a random process. Provided that the feedback channel is available, the transmitter can make use of the information regarding the statistics of the instantaneous channel capacity by choosing the right modulation, coding, transmission rate, and power to achieve the mean capacity (also known as the ergodic capacity) of the wireless channel [23,38,39]. However, in most cases only the receiver has the perfect CSI, while at the transmitter the CSI is either unavailable or is incorrect. In any case, it is not possible to design an efficient code having an appropriate length as well as able to cope with the fast variations of the instantaneous channel capacity In addition, since accurate CSI at the transmitter is also not possible to obtain in real time, the instantaneous channel capacity $C(t)$ cannot be reached by any proper coding schemes. It is due to these reasons, in practice the design of coding schemes is based on the mean channel capacity or the outage capacity [29]. Nevertheless, it has been demonstrated in [23] that a study of the temporal behavior of the channel capacity can be useful in designing a system that can adapt the transmission rate according to the capacity evolving process in order to improve the overall system performance and to transmit close to the ergodic capacity. Moreover, the importance of the statistical analysis of the channel capacity can also be witnessed in many other studies in the literature (see, e.g., [19,30,40]). As mentioned previously, the first-order statistical properties, such as the PDF, CDF, ergodic capacity, and the outage capacity, do not give any insight into the temporal behavior of the channel capacity. Therefore, it is very important to study the second-order statistical properties, such as the LCR and ADF of the channel capacity, in addition to the first-order statistical properties. In the following, we will study these aforementioned statistical properties of the instantaneous channel capacity. Firstly, the instantaneous channel capacity $C(t)$ of N*Nakagami-m channels is defined as

$$\begin{aligned} C(t) &= \frac{1}{N}\log_2\left(1+\gamma_s|\Xi(t)|^2\right) \\ &= \frac{1}{N}\log_2\left(1+\gamma_s\Xi^2(t)\right) \quad (\text{bits/s/Hz}) \end{aligned} \tag{8}$$

where $\gamma_s = 1/N_0$. The factor $1/N$ in (8) is due to the reason that the relays MRn (n = 1, 2, ..., N - 1) in Figure 1 operate in a half-duplex mode, and hence the signal transmitted from the SMS is received at the DMS in N time slots. We can consider (8) as a mapping of a random process $\Xi(t)$ to another random process $C(t)$. Therefore, the results for the statistical properties of the process $\Xi(t)$ can be used to obtain the expressions for the statistical properties of the channel capacity $C(t)$. Again, by applying the concept of transformation of random variables, the PDF $p_C(r)$ of the channel capacity $C(t)$ can be expressed in terms of the PDF $p_{\Xi^2}(z)$

$$\begin{aligned} pC(r) &= \left(\frac{N2^{Nr}\text{In}(2)}{\gamma_s}\right)p_{\Xi^2}\left(\frac{2^{Nr}-1}{\gamma_s}\right) \\ &= \frac{N2^{Nr}\text{In}(2)}{(2^{Nr}-1)\prod_{n=1}^{N}\Gamma(m_i)}G_{0,N}^{N,0} \\ &\left[\frac{2^{Nr}-1}{\gamma_s}\prod_{n=1}^{N}\left(\frac{m_n}{\Omega'_n}\right)\Bigg|\begin{matrix}-\\ m_1, m_2, \ldots, m_N\end{matrix}\right], r \geq 0. \end{aligned} \tag{9}$$

The mean channel capacity $E\{C(t)\} = \mu_C$ (or the ergodic capacity) and the variance $\text{Var}\{C(t)\} = \sigma_C^2$ of the channel capacity can be obtained using the PDF of the channel capacity [27]. Here, the mean channel capacity is of special interest to the researchers as it provides information regarding the average data rate offered by a wireless link with a negligible error probability (where the average is taken over all the realizations of the channel) [28,41]. The mean channel capacity is defined using the instantaneous channel capacity $C(t)$ as follows.

$$\begin{aligned} \mu_C &= E\left\{\frac{1}{N}\log_2\left(1+\gamma_s\left|\Xi(t)\right|^2\right)\right\} \\ &= \int_0^\infty \frac{1}{N}\log_2(1+\gamma_s x)p_{\Xi^2}(x)dx \\ &= \int_0^\infty z p_C(z)dz. \end{aligned} \tag{10}$$

Similar definition for the mean channel capacity can also be found in [27,29]. The variance of the channel capacity is a measure of the spread around the mean channel capacity. The variance of the channel capacity, denoted by σ_C^2, is defined as

$$\sigma_C^2 = \int_0^\infty (z-\mu_C)^2 p_C(z)dz. \tag{11}$$

The CDF $F_C(r)$ of the channel capacity $C(t)$ can be obtained by integrating the PDF $p_C(r)$ and making use of the relationships in [[33], Equation (9.34/3)] and [[42], Equation (26)] as

$$\begin{aligned} F_C(r) &= \int_0^r p_C(z)dz \\ &= \frac{1}{\prod_{n=1}^{N}\Gamma(m_i)}G_{1,N+1}^{N,1} \\ &\left[\frac{2^{Nr}-1}{\gamma_s}\prod_{N=1}^{N}\left(\frac{m_n}{\acute{\Omega}_n}\right)\Bigg|\begin{matrix}-\\ m_1, m_2, \cdots, m_N, 0\end{matrix}\right], \quad r \geq 0. \end{aligned} \tag{12}$$

The CDF of the channel capacity is helpful to study another important statistical quantity, known as the outage capacity, which determines the capacity (or the data rate) that is guaranteed with a certain level of reliability [28,41]. The ϵ-outage capacity C_ϵ, defined as the highest transmission rate R that keeps the outage probability under ϵ, can be expressed as $C_\epsilon = \max\{R: F_C(R) = \epsilon\}$. Using the CDF of the channel capacity in (12), the ϵ-outage capacity C_ϵ can be obtained by solving the following equation

$$F_C(C_\varepsilon) = \varepsilon. \tag{13}$$

Unfortunately, for N*Nakagami-m channels, closed-form analytical expressions for the mean channel capacity, variance of the channel capacity, and the outage capacity given by (10), (11), and (13), respectively, are very difficult to obtain. Nevertheless, these results can be obtained numerically, as will be presented in Section 5.

To find the LCR, denoted by $N_C(r)$, of the channel capacity $C(t)$, we first need to find the joint PDF $p_{C\dot{C}}(z,\dot{z})$ of $C(t)$ and its time derivative $\dot{C}(t)$. The joint PDF $p_{C\dot{C}}(z,\dot{z})$ can be found by using the joint PDF $p_{\Xi^2\dot{\Xi}^2}(z,\dot{z})$ given in (5) and by employing the relationship $p_{C\dot{C}}(z,\dot{z}) = \left(N2^{Nz}\ln(2)/\gamma_s\right)^2 \times p_{\Xi^2\dot{\Xi}^2}\left((2^{Nz}-1)/\gamma_s, N2^{Nz}\dot{z}\ln(2)/\gamma_s\right)$. Finally, the LCR $N_C(r)$ can be found as follows

$$N_C(r) = \int_0^\infty \dot{z} p_{C\dot{C}}(r, \dot{z}) d\dot{z}$$
$$= \frac{2^N \Phi}{\sqrt{2\pi}} \left(\frac{2^{Nr}-1}{\gamma_s} \right)^{m_N - \frac{1}{2}} \int_{x_1=0}^\infty \cdots \int_{x_{N-1}=0}^\infty \prod_{n=1}^{N-1} x_n^{2(m_n - m_N)-1} e^{-\sum_{n=1}^{N-1} \frac{m_n x_n^2}{\acute{\Omega}_n}} \quad (14)$$
$$\times e^{-\frac{m_N (2^{Nr}-1)}{\gamma_s \acute{\Omega}_N \prod_{n=1}^{N-1} x_n^2}} K\left(\frac{2^{Nr}-1}{\gamma_s}, x_1, ..., x_{N-1} \right) dx_1 \ldots dx_{N-1}, \quad r \geq 0$$

where $\Phi = \prod_{n=1}^N m_n^{m_n} / \left(\Gamma(m_n) \acute{\Omega}_n^{m_n} \right)$. The expression for the LCR $N_C(r)$ in (14) is mathematically very complex due to multiple integrals. However, by using the multivariate Laplace approximation theorem [43], it is shown in the Appendix that the LCR $N_C(r)$ of the channel capacity $C(t)$ can be approximated in a closed form as

$$N_C(r) \approx \frac{(2\pi)^{\frac{N}{2}} \Phi}{\pi \sqrt{N}} \left(\frac{2^{Nr}-1}{\gamma_s} \right)^{m_N - \frac{1}{2}} \left(e^{-N \frac{m_N (2^{Nr}-1)}{\tilde{\Phi}}} \right)^{\frac{1}{N}} \quad (15)$$

$$\times \left(\prod_{n=1}^{N-1} \frac{\tilde{x}_n^{2(m_n - m_N)-1}}{\sqrt{m_n / \acute{\Omega}_n}} \right) K\left(\frac{2^{Nr}-1}{\gamma_s}, \tilde{x}_1, ..., \tilde{x}_{N-1} \right), \quad r \geq 0 \quad (16)$$

where

$$\tilde{\Phi} = \gamma_s \acute{\Omega}_N \prod_{n=1}^{N-1} \frac{\acute{\Omega}_n}{m_n} \quad (17a)$$

and

$$\tilde{x}_n = \left(\frac{m_N (2^{Nr}-1)}{\tilde{\Phi} \left(m_n / \acute{\Omega}_n \right)^N} \right)^{\frac{1}{2N}}, n = 1, ..., N-1. \quad (17b)$$

The ADF, denoted by $T_C(r)$, of the channel capacity can be expressed as [20]

$$T_C(r) = \frac{F_C(r)}{N_C(r)} \quad (18)$$

where $F_C(r)$ and $N_C(r)$ are given by (4) and (8), respectively

4 Asymptotic analysis

In this section, we will study the PDF, CDF, mean, and variance of the channel capacity when the number of hops N is large. Similarly to [14], we will apply the central limit theorem of products [35] to obtain an accurate approximation for the PDF of the N*Nakagami-m process in (3). In the case when $N \to \infty$, we will denote the N*Nakagami-m process $\Xi(t)$ by $\Xi_\infty(t)$. From [14], it follows that the PDF of $\Xi_\infty(t)$ is lognormal distributed and can be expressed as

$$p_{\Xi_\infty}(z) = \frac{1}{\sqrt{2\pi}\sigma_\infty z} e^{-\frac{1}{2\sigma_\infty^2}(\ln z - \mu_\infty)^2}, \quad z \geq 0 \quad (19)$$

where

$$\mu_\infty = \lim_{N \to \infty} \frac{1}{2} \sum_{n=1}^N \left[\Psi(m_n) - \ln\left(\frac{m_n}{\acute{\Omega}_n} \right) \right] \quad (20)$$

and

$$\sigma_\infty^2 = \lim_{N \to \infty} \frac{1}{4} \sum_{n=1}^N \Psi^{(1)}(m_n). \quad (21)$$

Here, $\Psi^{(1)}(\cdot)$ is the first derivative of the Digamma function $\Psi(\cdot)$ [[33], Equation (8.360)]. In order to derive the PDF of the capacity of N*Nakagami-m channels, we need to find the PDF $p_{\Xi_\infty^2}(z)$ of the squared N*Nakagami-m process $\Xi_\infty^2(t)$. Again, by employing the relationship $p_{\Xi_\infty^2}(z) = p_{\Xi_\infty}(\sqrt{z}) / (2\sqrt{z})$, the PDF $p_{\Xi_\infty^2}(z)$ can be obtained as

$$p_{\Xi_\infty^2}(z) = \frac{1}{2\sqrt{\pi}\sigma_\infty z} e^{-\frac{1}{2\sigma_\infty^2}(\ln\sqrt{z} - \mu_\infty)^2}, z \geq 0. \quad (22)$$

Hence, by using (22) and applying the same transformation technique presented in Section 3, the PDF $p_C(t)$ of the channel capacity $C(t)$ can be approximated as

$$p_C(r) \approx \frac{N 2^{Nr} \ln 2}{2\sqrt{2\pi}(2^{Nr}-1)\sigma_N} e^{-\frac{1}{2\sigma_N^2}\left(\ln\sqrt{\frac{2^{Nr}-1}{\gamma_s}} - \mu_N \right)^2}, r \geq 0 \quad (23)$$

where μ_N and σ_N^2 are obtained from (20) and (21), respectively, by using a finite number of hops N. Furthermore, by integrating the PDF $p_C(r)$ in (23), the CDF $F_C(r)$ can be expressed as

$$F_C(r) = \int_0^r p_C(z) dz \approx \frac{N \ln 2}{2\sqrt{2\pi}\sigma_N} \int_0^r \frac{2^{Nz}}{(2^{Nz}-1)} e^{-\frac{1}{2\sigma_N^2}\left(\ln\sqrt{\frac{2^{Nz}-1}{\gamma_s}} - \mu_N \right)^2} dz. \quad (24)$$

Finally, the mean μ_C and the variance σ_C^2 of $C(t)$ can now be easily obtained as

$$\mu_C = \int_0^\infty z p_C(z) dz \approx \frac{N \ln 2}{2\sqrt{2\pi}\sigma_N} \int_0^\infty \frac{2^{Nz} z}{2^{Nz}-1} e^{-\frac{1}{2\sigma_N^2}\left(\ln\sqrt{\frac{2^{Nz}-1}{\gamma_s}} - \mu_N\right)^2} dz \tag{25}$$

and

$$\sigma_C^2 = \int_0^\infty (z-\mu_C)^2 p_C(z) dz \approx \frac{N \ln 2}{2\sqrt{2\pi}\sigma_N} \int_0^\infty \frac{2^{Nz}(z-\mu_C)^2}{2^{Nz}-1} e^{-\frac{1}{2\sigma_N^2}\left(\ln\sqrt{\frac{2^{Nz}-1}{\gamma_s}} - \mu_N\right)^2} dz, \tag{26}$$

respectively. In the next section, it will be shown by simulations that the approximations obtained in (23)-(26) perform well even for a small number of hops N.

5 Numerical results

In this section, we will discuss the analytical results obtained in the previous sections. The validity of the theoretical results is confirmed with the help of simulations. For comparison purposes, we have also shown the results for Rayleigh channels ($m_n = 1$; $n = 1, 2, ..., N$). By doing some mathematical manipulations, it can be shown that the obtained results for the statistical properties of the capacity of N*Nakagami-m channels reduce to the special cases of double Nakagami-m (for $N = 2$) and double Rayleigh (for $N = 2$ and $m_n = 1$) channels presented in [24,25], respectively. In order to generate Nakagami-m processes $\chi_n(t)$ for natural values of $2m_n$, the following relationship can be used [36]

$$\chi_n(t) = \sqrt{\sum_{l=1}^{2\times m_n} \mu_{n,l}^2(t)} \tag{27}$$

where $\mu_{n,l}(t)$ ($l = 1, 2, ..., 2m_n$; $n = 1, 2, ..., N$) are the underlying independent and identically distributed (i.i.d.) Gaussian processes, and m_n is the parameter of the Nakagami-m distribution associated with the nth link of the multihop communication systems. The Gaussian processes $\mu_{n,l}(t)$, each with zero mean and variances $m_n\sigma_0^2$, were simulated using the sum-of-sinusoids model [37]. The model parameters were computed using the generalized method of exact Doppler spread (GMEDS_1) [44]. The number of sinusoids for the generation of Gaussian processes $\mu_{n,l}(t)$ was chosen to be 20. The parameter Ω_n was chosen to be equal to 2 $(m_n\sigma_0)^2$, the values of the maximum Doppler frequencies $f_{\max_n}$ were set to be equal to 125 Hz, and the quantity γ_s was equal to 15 dB. The parameters G_{n-1} ($n = 1, 2, ..., N$) and σ_0 were chosen to be unity. The simulation time for the channel realizations was set set to be 250 s with sampling duration of 50 μs. Finally, using (3), (8), and (27), the simulation results for the statistical properties of the channel capacity were found[d]. For analytical illustrations, the Meijer's G-function as well as the multifold integrals can be numerically evaluated using the existing built-in functions of the numerical computation tools, such as MATLAB or MATHEMATICA.

The PDF $p_C(r)$ and the CDF $F_C(r)$ of the capacity $C(t)$ of N*Nakagami-m channels are presented in Figures 2 and 3, respectively. Also, the approximation results obtained in (23) and (24) are shown in Figures 2 and 3, respectively. Specifically, for $N = 6$ and $N = 8$, the approximation results are in a reasonable agreement with the exact results. Furthermore, it can be observed in both figures that an increase in the severity of fading (i.e., decreasing the value of the fading parameter m_n) decreases the mean channel capacity. Similarly, as the number of hops N in N*Nakagami-m channels increases, the mean channel capacity decreases. The influence of the severity of fading and the number of hops N in N*Nakagami-m channels on the mean channel capacity is specifically studied in Figure 4. It can also be observed that the mean capacity of multihop Rayleigh channels ($m_n = 1$; $n = 1, 2, ..., N$) is lower as compared to that of N*Nakagami-m channels ($m_n = 2$; $n = 1, 2, ..., N$). Moreover, it can also be observed from Figures 2 and 3 that an increase in the value of the fading parameter m_n or the number of hops N in N*Nakagami-m channels results in a decrease in the variance of the channel capacity. This result can easily be observed in Figure 5, where the variance of the capacity of N*Nakagami-m channels is studied for different values of the fading parameter m_n and the number of hops N in N*Nakagami-m channels. In Figures 4 and 5, we have also included the approximations obtained in (25) and (26), respectively. The illustrations show that as the number of hops N increases the approximation results show close correspondence to the exact results. In addition, a careful study of Figures 2, 3, 4, and 5 also reveals that the approximation results given by Equations (23)-(26) are more closely fitted to the exact results for larger values of m_n, e.g., $m_n = 2$ ($n = 1, 2, ..., N$). Figure 6 illustrates the influence of the number of hops N and the SNR on the outage capacity C_ϵ of N*Nakagami-m channels for $\epsilon = 0.01$. The results show that at low SNR, systems with a larger number of hops N show improved performance than the ones with a lower number of hops. However, the converse statement is true at high SNR.

Figure 7 presents the LCR $N_C(r)$ of the capacity $C(t)$ of N*Nakagami-m channels. It can be observed that at lower levels r, the LCR $N_C(r)$ of the capacity of N*Nakagami-m channels with lower values of the fading parameter m_n is lower as compared to that of the channels

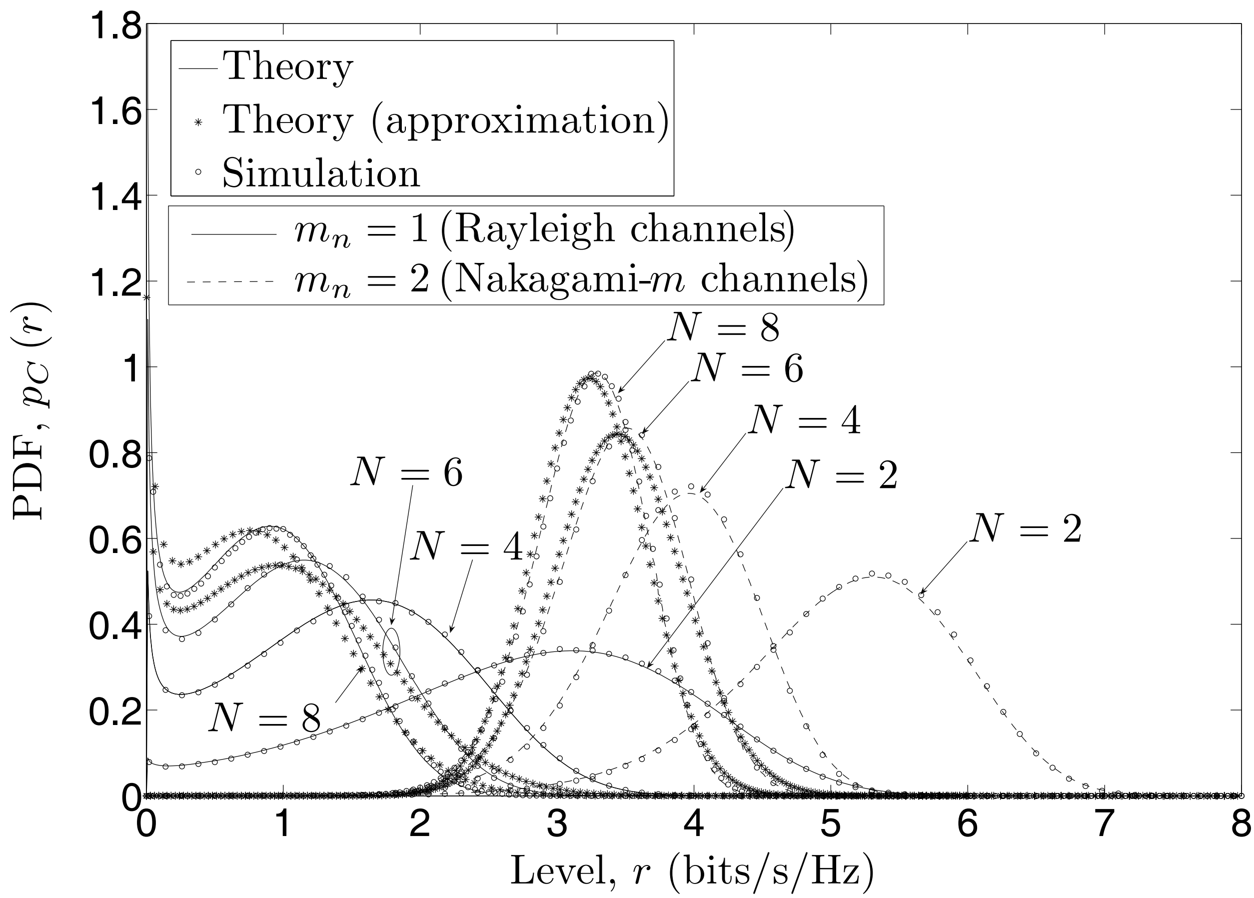

Figure 2 The PDF of the capacity of *NNakagami-*m* channels**.

with higher values of the fading parameter m_n. However, the converse statement is true for lower levels r. On the other hand, an increase in the number of hops N has an opposite influence on the LCR of the channel capacity as compared to the fading parameter m_n. Furthermore, Figure 7 illustrates the approximated LCR $N_C(r)$ of the channel capacity $C(t)$ given by (16). It is observed that as the number of hops N increases, the approximated LCR fits quite closely to the exact results. Specifically for $N \geq 4$, a very good fitting between the exact and the approximation results is observed. The ADF $T_C(r)$ of the capacity $C(t)$ of N*Nakagami-m channels is studied in Figure 8 for different values of the number of hops N and the fading parameter m_n. It is observed that an increase in the severity of fading or the number of hops N in N*Nakagami-m channels increases the ADF $T_C(r)$ of the channel capacity.

6 Conclusion

In this article, we have presented a statistical analysis of the capacity of N*Nakagami-m channels. Specifically, we have studied the influence of the severity of fading and the number of hops on the PDF, CDF, LCR, and ADF of the channel capacity. We have derived an accurate closed-form approximation for the LCR of the channel capacity. For a large number of hops N, we have investigated the suitability of the assumption that the N*Nakagami fading distribution can be approximated by the lognormal distribution. The findings of this article show that an increase in the number of hops N or the severity of fading decreases the mean channel capacity, while it results in an increase in the ADF of the channel capacity. Moreover, at higher levels r, the LCR $N_C(r)$ of the capacity of N*Nakagami-m channels decreases with an increase in severity of fading or the number of hops N. However, the converse statement is true for lower levels r. Furthermore, the variance of the channel capacity decreases by increasing the number of hops, while increase in the severity of fading has an opposite influence on the variance of the channel capacity. It is also observed that increasing the relay gains increases the received SNR at the DMS, however the received SNR at the DMS is always less than or equal to the SNR at

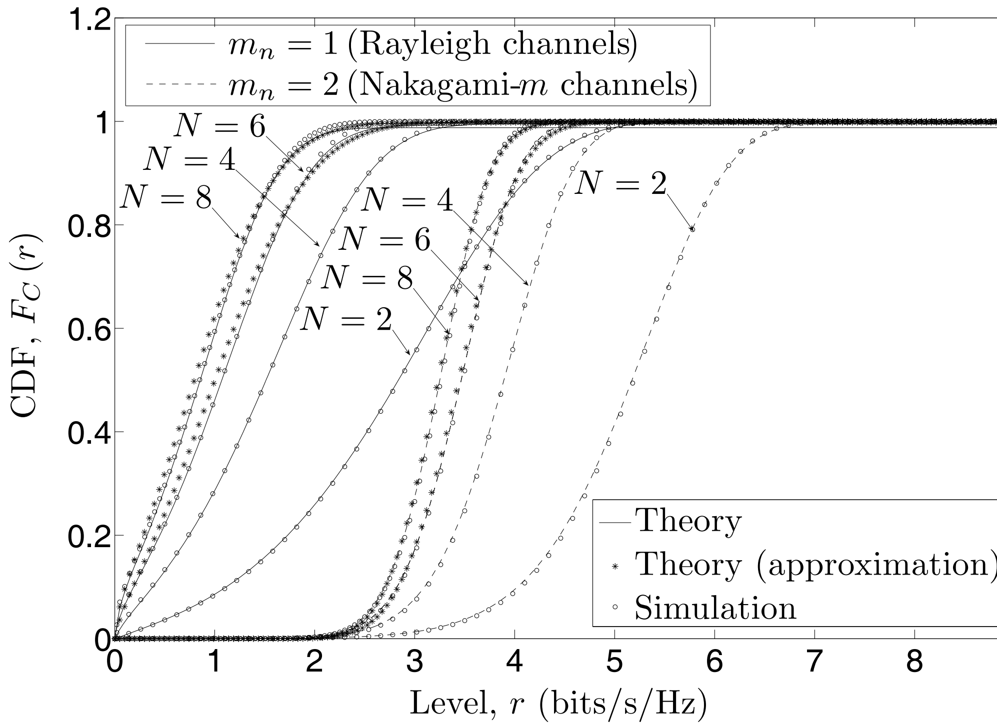

Figure 3 The CDF of the capacity of *NNakagami-*m* channels.**

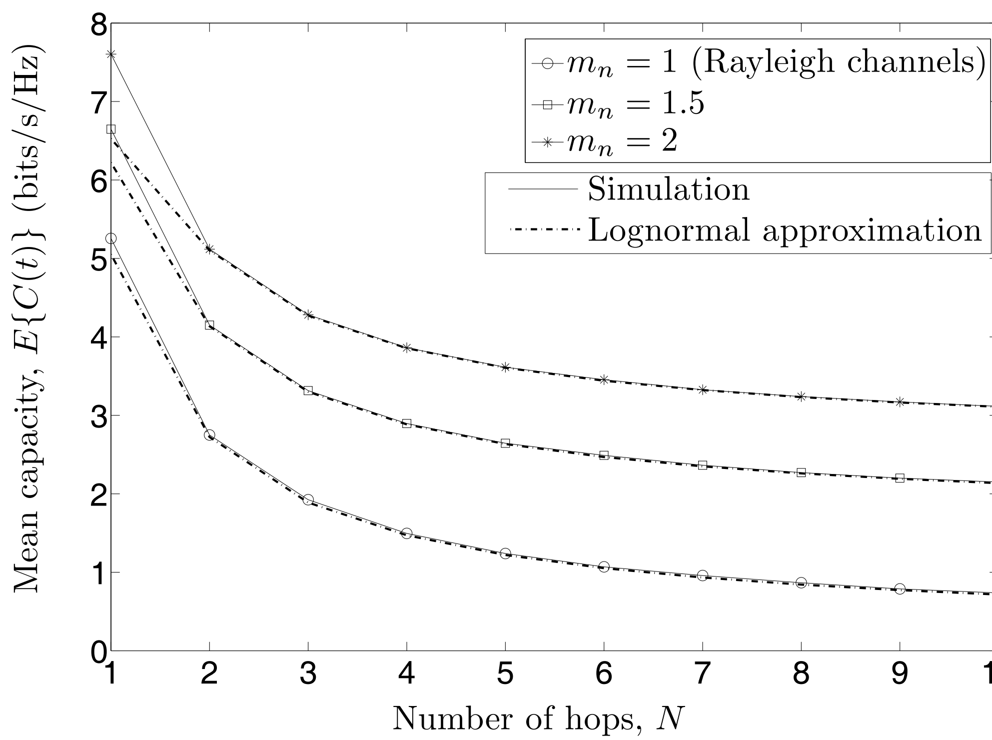

Figure 4 The mean channel capacity of *NNakagami-*m* channels.**

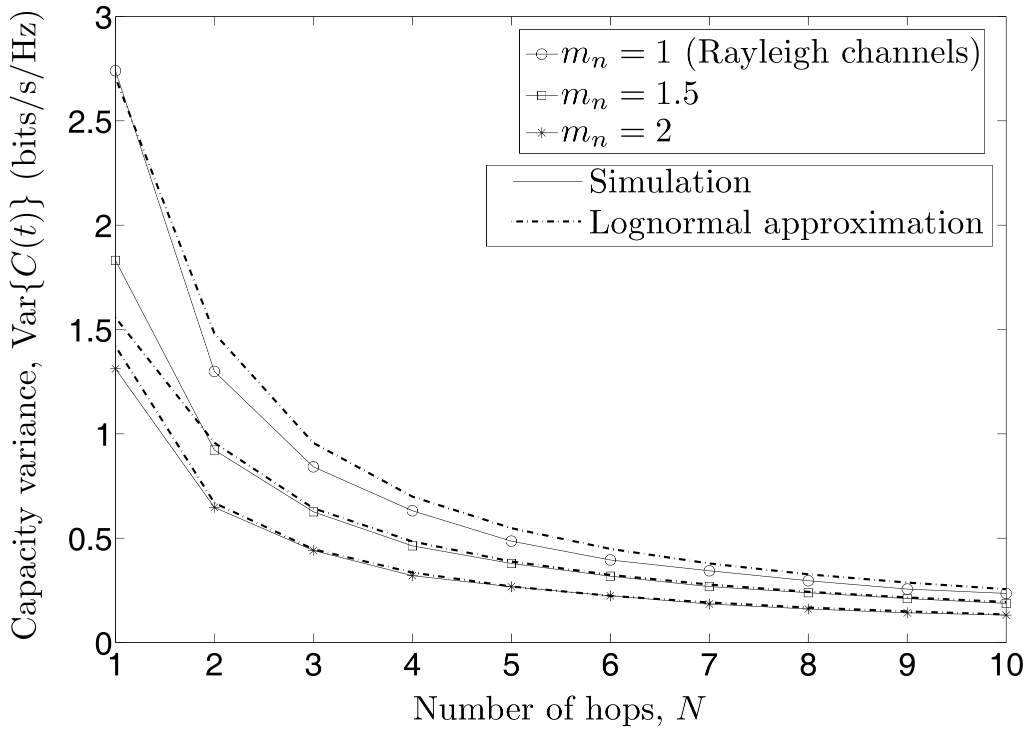

Figure 5 The variance of the capacity of *NNakagami-*m* channels**.

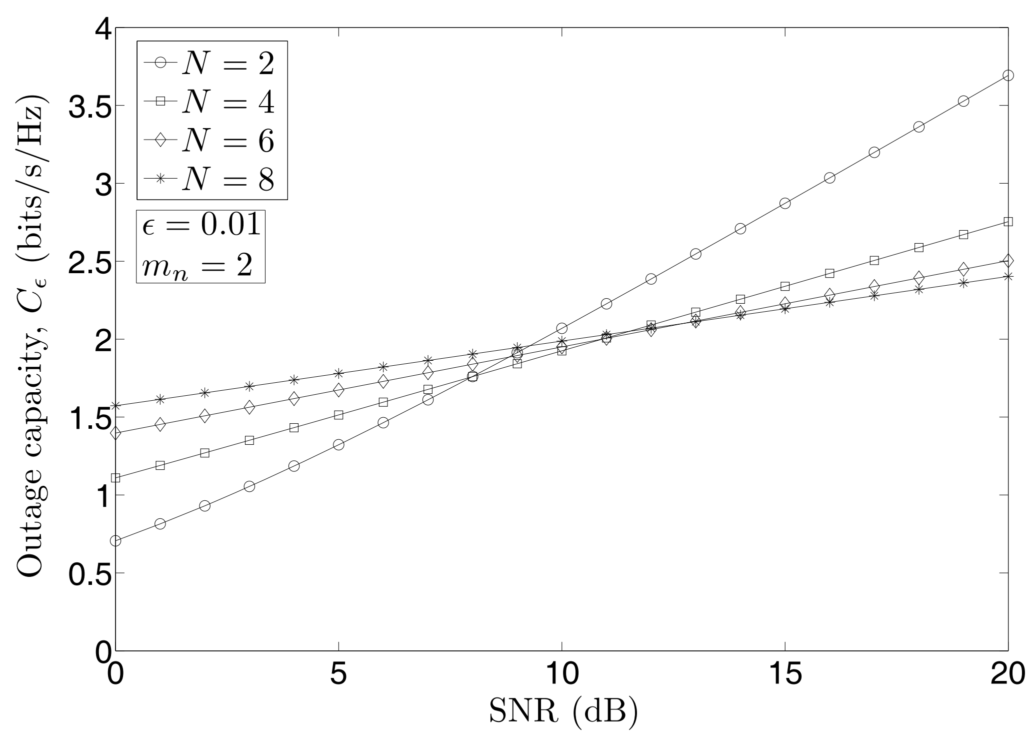

Figure 6 The outage capacity C_ϵ of *NNakagami-*m* channels for ϵ = 0.01 and m_n = 2**.

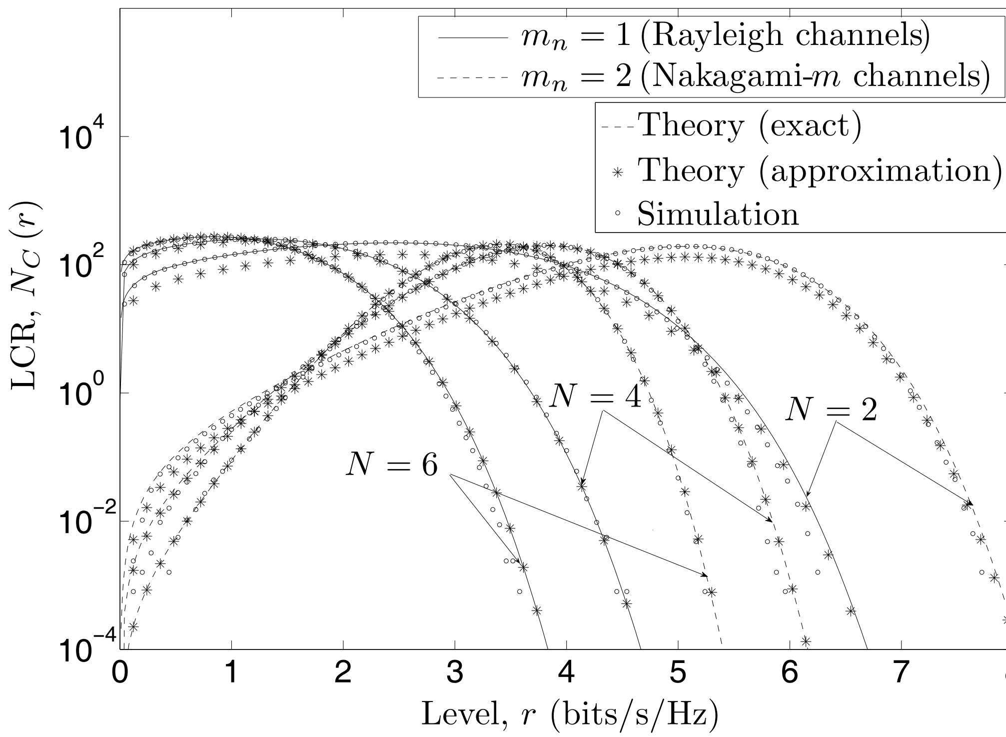

Figure 7 The LCR of the capacity of *N**Nakagami-*m* channels.

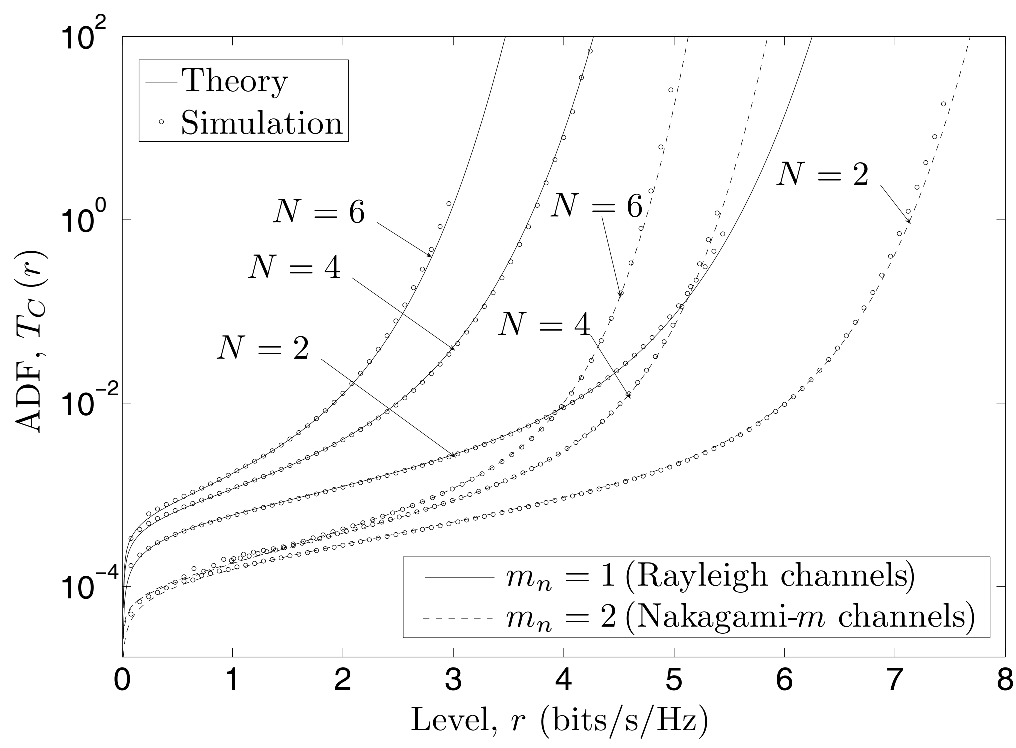

Figure 8 The ADF of the capacity of *N**Nakagami-*m* channels.

the first mobile relay MR_1. The analytical results are verified by simulations, whereby a very good fitting is observed.

Appendix

We can obtain an approximate closed-form expression for (14) by applying a similar technique as presented in [15]. By employing the result given by [[15], Equation (A.3)], the LCR $N_c(r)$ can be approximated as

$$N_C(r) \approx \frac{2^N \Phi}{\sqrt{2\pi}} \left(\frac{2^{Nr} - 1}{\gamma_s} \right)^{m_N - \frac{1}{2}} (2\pi)^{(N-1)/2} \frac{u(\tilde{\mathbf{x}})}{\sqrt{\alpha}} e^{-h(\tilde{\mathbf{x}})}, \quad r \geq 0 \tag{28}$$

where

$$u(\tilde{\mathbf{x}}) = \left(\prod_{n=1}^{N-1} \tilde{x}_n^{2(m_n - m_N) - 1} \right) K \left(\frac{2^{Nr} - 1}{\gamma_s}, \tilde{x}_1, \ldots, \tilde{x}_{N-1} \right), \tag{29}$$

$$h(\tilde{\mathbf{x}}) = \sum_{n=1}^{N-1} \frac{m_n \tilde{x}_n^2}{\acute{\Omega}_n} + \frac{m_n (2^{Nr} - 1)}{\gamma_s \acute{\Omega}_N \prod_{n=1}^{N-1} \tilde{x}_n^2}, \tag{30}$$

and $\tilde{\mathbf{x}} = [\tilde{x}_1, \ldots, \tilde{x}_{N-1}]$. Moreover, the values of the parameters $\tilde{x}_1, \ldots, \tilde{x}_{N-1}$ presented in (17b) can be obtained by using [[15], Equation (25)]. Furthermore, with the help of [[15], Equation (30)], we can easily show that the quantity α in (28) is given by

$$\alpha = N 2^{2(N-1)} \prod_{n=1}^{N-1} \frac{m_n}{\acute{\Omega}_n}. \tag{31}$$

Finally, by substituting (30), (30), (31), and (17b) in (28), we obtain the approximate closed-form expression for the LCR $N_C(r)$ of the channel capacity $C(t)$ given by (16).

Endnotes

[a]By instantaneous channel capacity we mean the time-variant channel capacity [45,46]. In the literature, the instantaneous channel capacity is also referred to as the mutual information [47-49].

[b]The scope of this article is limited only to the derivation and analysis of the statistical properties of the instantaneous channel capacity. However, a detailed discussion regarding the use of statistical properties of the channel capacity for the improvement of the system performance can be found in, e.g., [23,38,39] and the references therein.

[c]Henceforth, for ease of brevity, we will call the instantaneous channel capacity [45,46] simply as the channel capacity, which has also been done in [19,50,51].

[d]For further details, the interested reader is referred to [37], where MATLAB source codes are provided for simulating different channel realizations as well as the corresponding statistical properties (such as the PDF, CDF, LCR, and ADF) for a variety of propagation scenarios.

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References

1. A Sendonaris, E Erkip, B Aazhang, User cooperation diversity–Part I: system description. IEEE Trans Commun. **51**(11), 1927–1938 (2003). doi:10.1109/TCOMM.2003.818096
2. H Yanikomeröglu, Fixed and mobile relaying technologies for cellular networks, in *Proc 2nd Workshop on Applications and Services in Wireless Networks (ASWN'02)*, Paris, France, pp. 75–81 (July 2002)
3. JN Laneman, DNC Tse, GW Wornell, Cooperative diversity in wireless networks: efficient protocols and outage behavior. IEEE Trans Inf Theory. **50**(12), 3062–3080 (2004). doi:10.1109/TIT.2004.838089
4. TJ Harrold, AR Nix, Intelligent relaying for future personal communications systems, in *Proc IEE Colloq Capacity and Range Enhancement Techniques for Third Generation Mobile Communication and Beyond*, London, UK, pp. 9/1–9/5 (Feb 2000)
5. V Sreng, H Yanikomeroglu, D Falconer, Coverage enhancement through two-hop relaying in cellular radio systems, in *Proc IEEE Wireless Communications and Networking Conf (WCNC 2002)*, vol 2, Orlando, USA, pp. 881–885 (Mar 2002)
6. Haas ZJ, et al (eds.), Special issue on *wireless ad hoc networks*. J IEEE Sel Areas Commun. **17**(8), 1329–1531 (1999)
7. Vaidya NH, Kermani P (eds.), Special issue on *advances in mobile ad hoc networking*. IEEE Pers Commun. **8**(1), 72–80 (2001)
8. O Dousse, P Thiran, M Hasler, Connectivity in ad-hoc and hybrid networks, in *Proc 21st Annual Joint Conference of the IEEE Computer and Communications Societes (INFOCOM 2002)*, vol 2, New York, USA, pp. 1079–1088 (June 2002)
9. A Nosratinia, TE Hunter, A Hedayat, Cooperative communication in wireless networks. IEEE Commun Mag. **42**(10), 74–80 (2004). doi:10.1109/MCOM.2004.1341264
10. MO Hasna, MS Alouini, End-to-end Performance of transmission systems with relays over Rayleigh-fading channels. IEEE Trans Wirel Commun. **2**(6), 1126–1131 (2003). doi:10.1109/TWC.2003.819030
11. G Kramer, M Gastpar, P Gupta, Cooperative strategies and capacity theorems for relay networks. IEEE Trans Inf Theory. **51**(9), 3037–3063 (2005). doi:10.1109/TIT.2005.853304
12. PA Anghel, M Kaveh, Exact symbol error probability of a cooperative network in a Rayleigh-fading environment. IEEE Trans Wirel Commun. **3**(5), 1416–1421 (2004). doi:10.1109/TWC.2004.833431
13. GK Karagiannidis, Performance bounds of multihop wireless communications with blind relays over generalized fading channels. IEEE Trans Wirel Commun. **5**(3), 498–502 (2006)
14. GK Karagiannidis, NC Sagias, PT Mathiopoulos, N*Nakagami, A novel stochastic model for cascaded fading channels. IEEE Trans Commun. **55**(8), 1453–1458 (2007)

15. ZH Velkov, N Zlatanov, GK Karagiannidis, On the second order statistics of the multihop Rayleigh fading channel. IEEE Trans Commun. **57**(06), 1815–1823 (2009)
16. N Zlatanov, ZH Velkov, GK Karagiannidis, Level crossing rate and average fade duration of the double Nakagami-m random process and application in MIMO keyhole fading channels. IEEE Commun Lett. **12**(11), 822–824 (2008)
17. J Boyer, DD Falconer, H Yanikomeröglu, Multihop diversity in wireless relaying channels. IEEE Trans Commun. **52**(10), 1820–1830 (2004). doi:10.1109/TCOMM.2004.836447
18. H Ge, KD Wong, M Barton, JC Linerti, Statistical characterization of multiple-input multiple-output (MIMO) channel capacity, in *Proc IEEE Wireless Communications and Networking Conf, (WCNC 2002)*, vol 2, Florida, USA, pp. 789–793 (Mar 2002)
19. A Giorgetti, PJ Smith, M Shafi, M Chiani, MIMO capacity, level crossing rates and fades: the impact of spatial/temporal channel correlation. J Commun Netw. **5**(2), 104–115 (2003)
20. BO Hogstad, M Pätzold, N Youssef, V Kontorovitch, Exact closed-form expressions for the distribution, level-crossing rate, and average duration of fades of the capacity of OSTBC-MIMO channels. IEEE Trans Veh Technol. **58**(2), 1011–1016 (2009)
21. BO Hogstad, M Pätzold, Capacity studies of MIMO models based on the geometrical one-ring scattering model, in *Proc 15th IEEE Int Symp on Personal, Indoor and Mobile Radio Communications (PIMRC 2004)*, vol 3, Barcelona, Spain, pp. 1613–1617 (Sep 2004)
22. K Ohtani, K Daikoku, H Omori, Burst error performance encountered in digital land mobile radio channel. IEEE Trans Veh Technol. **30**(4), 156–160 (1981)
23. M Luccini, A Shami, S Primak, Cross-layer optimisation of network performance over multiple-input multiple-output wireless mobile channels. IET Commun. **4**(6), 683–696 (2010). doi:10.1049/iet-com.2009.0140
24. G Rafiq, M Pätzold, On the statistical properties of the capacity of amplify-and-forward channels under LOS conditions, in *Proc IEEE 11th Int Conf Communication Systems (ICCS 2008)*, Guangzhou, China, pp. 1614–1619 (Nov 2008)
25. G Rafiq, BO Hogstad, M Pätzold, Statistical properties of the capacity of double Nakagami-m channels, in *Proc IEEE 5th Int Symposium on Wireless Pervasive Computing, (ISWPC 2010)*, Modena, Italy, pp. 39–44 (May 2010)
26. O Waqar, DC McLernon, M Ghogho, Exact evaluation of ergodic capacity for multihop variable-gain relay networks: A unified framework for generalized fading channels. IEEE Trans Veh Technol. **59**(8), 4181–4187 (2010)
27. J Proakis, M Salehi, *Digital Communications*, 5th edn. (McGraw-Hill, New York, 2008)
28. AJ Paulraj, RU Nabar, DA Gore, *Introduction to Space-Time Wireless Communications*, (Cambridge University Press, Cambridge, 2003)
29. A Goldsmith, *Wireless Communications*, (Cambridge University Press, Cambridge, 2005)
30. GJ Byers, F Takawira, Spatially and temporally correlated MIMO channels: Modeling and capacity analysis. IEEE Trans Veh Technol. **53**(3), 634–643 (2004). doi:10.1109/TVT.2004.825766
31. S Yeh, O Leveque, Asymptotic capacity of multi-level amplify-and-forward relay networks, in *Proc IEEE International Symposium on Information Theory (ISIT 2007)*, Nice, France, pp. 1436–1440 (June 2007)
32. M Nakagami, The *m*-distribution: A general formula of intensity distribution of rapid fading, in *Statistical Methods in Radio Wave Propagation*, ed. by WG Hoffman (Pergamon Press, Oxford, 1960)
33. IS Gradshteyn, IM Ryzhik, *Table of Integrals, Series, and Products*, 6th edn. (Academic Press, San Diego, 2000)
34. MO Hasna, MS Alouini, Outage probability of multihop transmission over Nakagami fading channels. IEEE Commun Lett. **7**(5), 216–218 (2003). doi:10.1109/LCOMM.2003.812178
35. A Papoulis, SU Pillai, *Probability, Random Variables and Stochastic Processes*, 4th edn. (McGraw-Hill, New York, 2002)
36. MD Yacoub, JEV Bautista, LG de Rezende Guedes, On higher order statistics of the Nakagami-*m* distribution. IEEE Trans Veh Technol. **48**(3), 790–794 (1999)
37. M Pätzold, *Mobile Fading Channels*, (Wiley, Chichester, 2002)
38. AJ Goldsmith, SG Chua, Variable-rate variable-power MQAM for fading channels. IEEE Trans Commun. **45**(10), 1218–1230 (1997). doi:10.1109/26.634685
39. AJ Goldsmith, P Varaiya, Capacity of fading channels with channel side information. IEEE Trans Inf Theory. **43**(6), 1986–1992 (1997). doi:10.1109/18.641562
40. AG Zajic, GL Stuber, Three-dimensional modeling, simulation, and capacity analysis of space time correlated mobile-to-mobile channels. IEEE Trans Veh Technol. **57**(4), 2042–2054 (2008)
41. D Gesbert, J Akhtar, Breaking the barriers of Shannon's capacity: An overview of MIMO wireless systems. Telektronikk Telenor's J Inf Theory Appl.**1**, 53–64. 2002 (2002)
42. VS Adamchik, OI Marichev, The algorithm for calculating integrals of hypergeometric type functions and its realization in REDUCE system, in *Proc Int Symp Symbolic and Algebraic Computation (ISSAC '90)*, Tokyo, Japan, pp. 212–224 (August 1990)
43. LC Hsu, A theorem on the asymptotic behavior of a multiple integral. Duke Math J. **15**(3), 623–632 (1948). doi:10.1215/S0012-7094-48-01554-3
44. M Pätzold, CX Wang, B Hogstad, Two new methods for the efficient generation of multiple uncorrelated Rayleigh fading waveforms. IEEE Trans Wirel Commun. **8**(6), 3122–3131 (2009)
45. G Fraidenraich, O Leveque, JM Cioffi, On the MIMO channel capacity for the Nakagami-*m* channel. IEEE Trans Inf Theory. **54**(8), 3752–3757 (2008)
46. N Costa, S Haykin, *Multiple-Input, Multiple-Output Channel Models: Theory and Practice*, (Wiley, New Jersey, 2010)
47. IE Telatar, Capacity of multi-antenna Gaussian channels. Eur Trans Telecommun Rel Technol. **10**(6), 585–595 (1999)
48. A Tulino, A Lozano, S Verdu, Impact of antenna correlation on the capacity of multiantenna channels. IEEE Trans Inf Theory. **51**(7), 2491–2509 (2005). doi:10.1109/TIT.2005.850094
49. M Kang, SM Alouini, Capacity of MIMO Rician channels. IEEE Trans Wirel Commun. **5**, 112–122 (2006)
50. GJ Foschini, MJ Gans, On limits of wireless communications in a fading environment when using multiple antennas. Wirel Pers Commun. **6**, 311–335 (1998). doi:10.1023/A:1008889222784
51. PJ Smith, LM Garth, S Loyka, Exact capacity distributions for MIMO systems with small numbers of antennas. IEEE Commun Lett. **7**(10), 481–483 (2003). doi:10.1109/LCOMM.2003.817318

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