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# Optimization of transport capacity in wireless multihop networks

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# Abstract

Gamal et al. (IEEE Trans. Inform. Theory 52:2568–2592, 2006) showed that the end-to-end delay is *n* times the end-to-end throughput under centralized time division multiple access scheduling. In our other work (IEEE Trans. Mobile Computing, in press), it was proved that the relationship between the end-to-end throughput and the end-to-end delay of Gamal et al. still holds under the IEEE 802.11 distributed coordination function (DCF) when the carrier sensing range and the packet generation rate are jointly optimized. The main purpose of this study is to determine whether the result in our other work is achievable when the transmission range is adjusted instead of the carrier sensing range. To this end, we revise the transport capacity by reflecting a queue at each node and optimize the revised transport capacity by jointly controlling the transmission distance and the packet generation rate. Under our system model, it is shown that the end-to-end throughput and the end-to-end delay scale are  $\Theta\left(1/\sqrt{n \log n}\right)$ 

and  $\Theta\left(\sqrt{n/\log n}\right)$ , respectively, where *n* is the number of nodes in the network. That is to say, the result that the end-to-end delay is *n* times the end-to-end throughput under the DCF mode is also established while jointly optimizing the transmission range and packet generation rate.

**Keywords:** Wireless multihop networks, IEEE 802.11 DCF, Transport capacity, End-to-end throughput, End-to-end delay, Transmission range control, Scaling law

# 1 Introduction

Wireless multihop networks (WMNs) have received considerable attention because of their ability to enhance the spectrum and energy efficiency. To exploit such advantages, researchers have attempted to use various approaches at all levels of communication protocols. The key performance metric in a WMN is the end-to-end throughput, referring to the number of packets that can be transported successfully from a given source to its destination. As the node density increases, each node transmits packets to its nearest node. In the seminal work in this area [1], Gupta and Kumar proved that the maximum end-to-end throughput scales as  $\Theta \left(1/\sqrt{n \log n}\right)^a$ , where *n* is the number of nodes in the network.

The end-to-end delay, i.e., how fast a packet is transported from a given source to its destination, depends

Radio Resource Management and Optimization Laboratory, School of Electrical and Electronic Engineering, Yonsei University, 50 Yonsei-Ro, Seodaemun-Gu, Seoul 120-749, South Korea on the end-to-end throughput. Because a node delivers its packet to the nearest node, the number of hops to the destination increases. Thus, more transmissions are needed to transport one packet from a source to the destination. In [2], Gamal et al. showed that the end-to-end delay is *n* times the end-to-end throughput under centralized time division multiple access (TDMA) scheduling. In our other work [3], we proved that the relationship between the end-to-end throughput and the end-to-end delay [2] still holds when the IEEE 802.11 distributed coordination function (DCF), a representative practical medium access control (MAC), is utilized. We defined the delay-constrained capacity as the maximum end-to-end throughput satisfying the end-to-end delay requirement. Given the end-to-end delay requirement, we maximized the delay-constrained capacity by jointly controlling the carrier sensing range and the packet generation rate and proved that the delay-constrained capacity is the end-toend delay requirement divided by *n*.



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When multiple IEEE 802.11 networks coexist, controlling the transmission range is more secure than controlling the carrier sensing range. Tuning the carrier sensing range of one network may significantly degrade the performance of the other networks [4]. Provided that the carrier sensing range of one network decreases, the nodes in the network have more chances to access the wireless medium whereas the nodes in the other networks do not have. These nodes receive excessive interference because the distance to the nearest interfering node becomes shorter than they estimated. On the other hand, adjusting the transmission range of one network does not increase the interference temperature significantly because the carrier sensing range, which determines the transmission opportunity, is not changed. The objective of our study is to determine whether the relationship between the end-to-end throughput and the corresponding end-toend delay [3] is established when the *transmission range* is adjusted instead of the carrier sensing range. When controlling the transmission range, two different phenomena occur. As the transmission range increases, the number of transmissions to deliver one packet from a given source to its destination is reduced. On the other hand, the quality of each wireless link deteriorates. These phenomena can be explained by the transport capacity, which is defined as the product of the transmission rate and the transmission distance [1]. The original definition of the transport capacity does not contain the queuing delay at each node, which is a key factor to determine the performance of the WMN when the IEEE 802.11 DCF is used as a MAC protocol. We revise the transport capacity by incorporating a queuing delay at each node. This is referred to as the revised transport capacity throughout this paper. In [5], the authors defined the random access transport capacity to analyze the end-to-end throughput and the end-to-end delay of a WMN. However, the authors did not take into account the queue of each node.

In this paper, we start with maximizing the revised transport capacity of the WCM, where the IEEE 802.11 DCF MAC protocol is adopted. One way to increase the revised transport capacity is to minimize the number of hops by increasing the transmission distance. However, with a long transmission distance, the waiting time at the queue of each node will grow. When the transmission distance increases, fewer bits are transported to ensure reliable reception and more bits remain in the queue. Such a queuing delay affects the revised transport capacity as perceived by the receiver node. Reflecting such aspects, we optimize the revised transport capacity by jointly controlling the transmission distance and the packet generation rate. As a result, it is shown that under the IEEE 802.11 DCF, the end-to-end throughput and the end-toend delay scale are  $\Theta\left(1/\sqrt{n\log n}\right)$  and  $\Theta\left(\sqrt{n/\log n}\right)$ ,

respectively. This indicates that adjusting the transmission range and choosing the proper packet generation rate can establish the tradeoff relationship between the end-to-end throughput and the end-to-end delay [2].

The rest of this paper is organized as follows: In Section 2, we summarize related works. In Section 3, our system model is described. In Section 4, we introduce the revised transport capacity and optimize it with respect to the transmission range and the packet generation rate. In Section 5, we derive the scaling laws of the throughput and the end-to-end delay. Finally, we conclude the paper in Section 6.

# 2 Related works

In a random network, it is a critical problem as to how the end-to-end throughput and the end-to-end delay scale with the number of nodes, *n*. In [1], the authors showed that the end-to-end throughput scales as  $\Theta(1/\sqrt{n})$  or  $\Theta(1/\sqrt{n \log n})$ , depending on whether the network is arbitrary or random. The end-to-end throughput gap between an arbitrary and a random network is caused by the network connectivity characteristics. In a random network, there is the additional cost of tuning the transmission range, which degrades the performance on the order of  $\sqrt{\log n}$  as compared with an arbitrary network. When the end-to-end throughput is  $\Theta(1/\sqrt{n \log n})$ , the

corresponding end-to-end delay becomes  $\Theta\left(\sqrt{n/\log n}\right)$  [2]. On the other hand, a different study [6] derived the end-to-end throughput of  $\Theta(1/\sqrt{n})$  in a random network using what is known as percolation theory.

Supporting node mobility can improve the end-to-end throughput. In [7], it was claimed that when all nodes move around the network, the end-to-end throughput becomes independent of *n*, as described by the scaling law  $\Theta(1)$ . The mobile nodes communicate with each other only when they are very close. It was assumed that each node uniformly moves over the entire network area. In [8], the authors analyzed the end-to-end throughput scaling laws under various mobility models. Scaling laws in [7,8] are derived under a loose delay constraint, in which the speed of mobile movement is significantly slower than the delay requirement. In [2,9-11], the authors explained the tradeoff relationship between the end-to-end throughput and the end-to-end delay of mobile networks. In [12], it was proved that the average delay scales with  $\Theta(n)$ , for both the stationary and mobile nodes. Seol and Kim [13,14] claimed that controlling the mobility can decrease the end-to-end delay while maintaining the constant throughput scaling law.

Most previous works [1,2,6-14] tend to simplify the MAC influencing the end-to-end throughput and the end-to-end delay. Some studies have attempted to verify the

performance of a WMN when a practice MAC is utilized. Hwang and Kim [15] found that the result [1] is a reasonable throughput estimation for networks with ALOHA-like MAC. In [16,17], it was proved that IEEE 802.11 DCF cannot achieve the  $\Theta\left(1/\sqrt{n \log n}\right)$  end-toend throughput because of the randomness of the DCF. In [3], we jointly optimized the carrier sensing range and the packet generation rate and showed that the tradeoff relationship between the end-to-end throughput and the end-to-end delay occurs in spite of utilizing the DCF mode.

# 3 Preliminaries

# 3.1 Transmission rate and range

Consider a WMN in a finite area that adopts a common frequency channel of unit bandwidth. There are *n* nodes located randomly in the area. Each node is equipped with an omnidirectional single antenna and cannot transmit and receive packets simultaneously. A node transmits with constant power *P*. Given the distance between transmitter *i* and receiver *j* as  $d_{i,j}$ , the signal-to-interference ratio (SIR) at receiver *j*,  $\gamma_{i,j}$ , is

$$\gamma_{i,j} = \frac{Pd_{i,j}^{-\theta}}{\sum_{k \neq i} Pd_{k,j}^{-\theta}} = \frac{d_{i,j}^{-\theta}}{\sum_{k \neq i} d_{k,j}^{-\theta}},\tag{1}$$

where  $\theta$  (>2) denotes the path loss exponent. We neglect fast fading and only consider the path loss here. All the mathematical notations are summarized in Table 1.

The MAC protocol permits selected nodes to access the channel. The MAC protocol in this paper is IEEE 802.11 DCF [18], where carrier sense multiple access with collision avoidance (CSMA/CA) is adopted. Under CSMA/CA, two transmitters, between which the distance is less than the carrier sensing range *D*, cannot transmit simultaneously. Provided that the density of nodes is high, each distance to the nearest transmitting node becomes *D* and the location of the transmitting nodes is a honey-grid lattice, as shown in Figure 1. In this study, *R* is the singlehop transmission range ( $0 \le R \le \frac{D^{b}}{2}$ ). The SIR,  $\gamma_{i,j}$ , is then expressed in terms of *R* and *D* [19,20]:

$$\gamma_{i,j}(R,D) = \gamma_{i,j}^{\text{honey grid}}(R,D)$$

$$= \frac{R^{-\theta}}{6(D)^{-\theta} + 12(2D)^{-\theta} + 18(3D)^{-\theta} + \cdots}$$

$$= \frac{\left(\frac{D}{R}\right)^{\theta}}{6\zeta \ (\theta - 1)}.$$
(2)

Here,  $\zeta(s) \equiv \sum_{j=1}^{\infty} j^{-s}$  is the Riemann zeta function. Given that each node adopts capacity-achieving codes, we

# **Table 1 Notations**

Notations	Description
θ	Path loss exponent
n	Number of nodes
D	Carrier sensing range
R	Single-hop transmission range
C(R, D)	Transmission rate
m(R,D)	Maximum number of packets by single-hop transmission
М	Unit packet size
λ	Packet incoming rate
E[ H]	Average number of hops between a source-destination pair
Т	Time needed for one transmission
$E[d_q]$	Average queueing delay at each node
$v(R, \Lambda)$	Packet velocity
$d_e(R,\Lambda)$	End-to-end delay
Λ	Packet generating rate (end-to-end throughput when $d_e(R, \Lambda)$ is finite)
$\Lambda_{\max}(R)$	Throughput capacity
$C_T(R, \Lambda)$	Revised transport capacity
$R^*(\Lambda)$	Optimal transmission range when the packet generation rate is fixed to $\boldsymbol{\Lambda}$
$\Lambda^*(R)$	Optimal packet generation rate when the transmission range is fixed to ${\it R}$
$\Lambda_{ ext{opt}}$	Optimal packet generation rate (end-to-end throughput)

assume that the transmission rate C at receiver j follows the Shannon formula:

$$C(R,D) = \log_2\left(1 + \gamma_{ij}(R,D)\right) = \log_2\left(1 + \frac{\left(\frac{D}{R}\right)^{\theta}}{6\zeta \ (\theta - 1)}\right).$$
(3)

The maximum number of packets deliverable by a single-hop transmission, m(R, D), is determined by

$$m(R,D) = \left\lfloor \frac{C(R,D)}{M} \right\rfloor = \left\lfloor \log_2 \left( 1 + \frac{\left(\frac{D}{R}\right)^{\theta}}{6\zeta \left(\theta - 1\right)} \right) / M \right\rfloor,$$
(4)

where *M* denotes the unit packet size (in bits), and the function  $\lfloor x \rfloor$  indicates the largest integer not greater than *x*. It is understood that  $\left\lfloor \log_2 \left( 1 + \frac{\left(\frac{D}{R}\right)^{\theta}}{6\zeta(\theta-1)} \right) / M \right\rfloor$  packets are certainly delivered by a single transmission. As the transmission range *R* increases, *m*(*R*, *D*) decreases. There is a tradeoff between how many packets are *transmitted* and how far the packets are *transported*.





#### 3.2 Queuing delay in each node

A pair of source and destination nodes is given randomly. Each node generates  $\Lambda$  packets per second. Let us define  $\lambda$  as the rate of incoming traffic including the self-generated  $\Lambda$ . It is assumed that there are on an average E[H] hops per routing path. Thus, there are E[H] - 1 relay nodes between a source-destination pair. Given that there are n nodes in the network, the probability that one node becomes a relay node for a source-destination pair is (E[H] - 1)/(n - 1) and the expected amount of relay traffic from a source node is  $\Lambda(E[H] - 1)/(n - 1)$ . Because any node can be the relay for n - 1 other nodes, the expected amount of the total relay traffic from other nodes is  $\Lambda(E[H] - 1)$  [19]. Therefore, the packet incoming rate  $\lambda$  is expressed as the sum of  $\Lambda$  and  $\Lambda(E[H] - 1)$ :

$$\lambda = \Lambda + \Lambda(E[H] - 1) = \Lambda E[H].$$
(5)

Let us assume that the density of nodes is significantly high. When a shortest-path routing is utilized, the average number of hop counts, E[H], is L/R, where L is the average distance between a source and a destination. Therefore, we have

$$\lambda = \Lambda E[H] = \Lambda \frac{L}{R}.$$
(6)

Under the DCF, a typical protocol that utilizes carrier sensing, the average time needed for one transmission T is composed of the random back-off time, the frozen

duration<sup>c</sup>, and the actual transmission time. In [16], the author analyzed T as follows:

$$T = \frac{\frac{1}{\xi} + T_t}{1 - n\pi D^2 \lambda T_t}.$$
(7)

In the equation,  $\frac{1}{\xi}$  is the average duration of the back-off timer, and  $T_t$  is the fixed time required to transmit. With the number of packets in the single-hop transmission m(R, D), the average service time for one packet is

$$\frac{1}{\mu} = \frac{T}{m(R,D)} = \frac{M \cdot T}{\log_2(1 + \gamma_{i,j}(R,D))}.$$
(8)

Assuming that the queue of each node is modeled as M/M/1, the average waiting time at the queue,  $E[d_q]$ , is given as

$$E[d_q] = \frac{1}{\mu - \lambda} = \left(\frac{\log_2(1 + \gamma_{i,j}(R, D))}{M \cdot T} - \Lambda \cdot \frac{L}{R}\right)^{-1}.$$
(9)

The average queuing delay  $(E[d_q])$  is dependent on the number of nodes (n), the sensing range (D), the transmission range (R), and the packet generation rate  $(\Lambda)$ . The average queuing delay  $E[d_q]$  is a key factor that is used to express the performance of the IEEE 802.11 DCF. In the following section, we will define some performance metrics in terms of  $E[d_q]$  and show how these metrics are affected by n, D, R, and  $\Lambda$ .

# 4 Optimization of the transport capacity

4.1 Revised transport capacity of WMN

In [3], we defined the following terms:

**Definition 1.** (Packet velocity) The packet velocity  $\nu$  is the average speed of the packet as it moves from its source to its destination:

$$\nu(R,\Lambda) = \frac{R}{E[d_q]} = R \cdot \left(\frac{\log_2(1+\gamma_{i,j}(R))}{M \cdot T} - \Lambda \cdot \frac{L}{R}\right).$$
(10)

**Definition 2.** (End-to-end delay) The end-to-end delay of a packet  $d_e$  is the expected time required to deliver it from its source to its destination:

$$d_e(R,\Lambda) = \frac{L}{\nu(R,\Lambda)} = \frac{L}{R} E[d_q] = E[H] E[d_q].$$
(11)

**Definition 3.** (Feasible end-to-end throughput) The end-to-end throughput of  $\Lambda$  packets per second for a node is feasible when the end-to-end delay is finite.

Note that the packet generation rate  $\Lambda$  is equivalent to the end-to-end throughput when its corresponding end-to-end delay is finite. In other words, the destination node can receive packets with rate  $\Lambda$ .

**Definition 4.** (Throughput capacity) For a given *R*, the throughput capacity  $\Lambda_{max}$  is the *supremum* of the feasible end-to-end throughput:

$$\Lambda_{\max}(R) = \sup \left\{ \Lambda : d_{e}(R, \Lambda) < \infty \right\}$$
  
= sup {\Lambda : \nu(R, \Lambda) > 0}  
= sup \left\{\Lambda : R \cdot \left( \frac{\log\_2(1 + \gamma\_{i,j}(R))}{M \cdot T} - \Lambda \cdot \frac{L}{R} \right) > 0 \right\}. (12)

**Example 1.** Let us calculate the packet velocity and the throughput capacity when the transmission range is fixed to D/2. From (2), (7), and (10), the packet velocity of the network is

$$\begin{split} \nu(D/2,\Lambda) &= \frac{D}{2E\left[d_q\right]} \\ &= \frac{D}{2} \left\{ \frac{\log_2\left(1 + \frac{2^{\theta}}{6\zeta(\theta - 1)}\right)}{M \cdot \left(\frac{1}{\xi} + T_t\right)} \left(1 - n\pi D^2 \Lambda \frac{L}{D/2} T_t\right) - \Lambda \frac{L}{D/2} \right\} \\ &= \frac{\log_2\left(1 + \frac{2^{\theta}}{6\zeta(\theta - 1)}\right)}{M \cdot \left(\frac{1}{\xi} + T_t\right)} \left(\frac{D}{2} - n\pi D^2 \Lambda L T_t\right) - \Lambda \cdot L \\ &= \frac{D \cdot \log_2\left(1 + \frac{2^{\theta}}{6\zeta(\theta - 1)}\right)}{2 \cdot M \cdot \left(\frac{1}{\xi} + T_t\right)} \\ &- \left\{\frac{\log_2\left(1 + \frac{2^{\theta}}{6\zeta(\theta - 1)}\right)n\pi D^2 L T_t}{M \cdot \left(\frac{1}{\xi} + T_t\right)} + L\right\} \Lambda. \end{split}$$
(13)

The throughput capacity  $\Lambda_{\max}(D/2)$  is derived by finding the packet generation rate  $\Lambda$  that makes the packet velocity (13) zero:

$$\Lambda_{\max}(D/2) = \frac{D \cdot \log_2 \left(1 + \frac{1}{6\zeta(\theta - 1)} 2^{\theta}\right)}{2\left(\log_2 \left(1 + \frac{1}{6\zeta(\theta - 1)} 2^{\theta}\right) n\pi D^2 L T_t + L M\left(\frac{1}{\xi} + T_t\right)\right)}.$$
(14)

Throughput capacity  $\Lambda_{max}(D/2)$  (14) is achieved by forcing the packet velocity to be very close to zero. However, this is accomplished at the cost of the infinite delay. In [1], the authors defined the transport capacity as the product of the transmission rate and the transmission distance of a single hop. The main motivation is to grasp the end-to-end throughput and the end-to-end delay within one framework. In a WMN, one node processes multiple source-destination pairs in a WMN and the corresponding queuing delay appears. Under the DCF mode, the queuing delay is an important factor to determine the end-to-end throughput and the end-to-end delay owing to its randomness property. The original transport capacity did not take into account the queuing delay. Therefore, we revise the transport capacity to consider the queuing delay in each node. We divide the transport capacity by the average queuing delay,  $E[d_q]$  at each node, as follows:

**Definition 5.** (Revised transport capacity) The revised transport capacity  $C_T(R, \Lambda)$  is the product of the end-toend throughput and the packet velocity:

$$C_{T}(R,\Lambda) = M \cdot \Lambda \cdot \nu(R,\Lambda)$$
  
=  $M \cdot \Lambda \cdot \frac{R}{E[d_{q}]}$   
=  $M \cdot \Lambda \cdot R\left(\frac{\log_{2}(1+\gamma_{i,j}(R))}{M \cdot T} - \Lambda \cdot \frac{L}{R}\right)$   
(15)

From (13), the revised transport capacity of the network with the transmission range of D/2 is as follows:

$$C_{T}(D/2,\Lambda) = M \cdot \Lambda \cdot \left(\frac{D \cdot \log_{2}\left(1 + \frac{1}{6\zeta(\theta - 1)}2^{\theta}\right)}{2 \cdot M \cdot \left(\frac{1}{\xi} + T_{t}\right)} - \left(\frac{\log_{2}\left(1 + \frac{1}{6\zeta(\theta - 1)}2^{\theta}\right)n\pi D^{2}LT_{t}}{M \cdot \left(\frac{1}{\xi} + T_{t}\right)} + L\right)\Lambda\right).$$
(16)

The optimal packet generation rate of  $\Lambda^*(D/2)$  maximizing the revised transport capacity (16) can be derived by differentiating (16) and finding  $\Lambda$  that makes it zero:

$$\Lambda^{*}(D/2) = \frac{D \cdot \log_{2}\left(1 + \frac{1}{6\zeta(\theta - 1)}2^{\theta}\right)}{4\left(\log_{2}\left(1 + \frac{1}{6\zeta(\theta - 1)}2^{\theta}\right)n\pi D^{2}LT_{t} + LM\left(\frac{1}{\xi} + T_{t}\right)\right)},$$
(17)

and the corresponding packet velocity  $\nu(D/2, \Lambda^*(D/2))$  is given by

$$\nu(D/2, \Lambda^*(D/2)) = \frac{D \cdot \log_2\left(1 + \frac{1}{6\zeta(\theta - 1)}2^\theta\right)}{4 \cdot M \cdot \left(\frac{1}{\xi} + T_t\right)}.$$
 (18)

# 4.2 Cross-layer optimization for maximizing the revised transport capacity

Our objective in this subsection is to determine the optimal transmission range and packet generation rate that maximize the revised transport capacity  $C_T(R, \Lambda)$ . We begin by finding the optimal transmission range  $R^*(\Lambda)$ that maximizes  $C_T(R, \Lambda)$  (maximizing the packet velocity  $\nu$ ) when  $\Lambda$  is fixed. The number of nodes that have at least one packet to transmit  $(nD^2\lambda = nD^2\Lambda \frac{L}{R})$  depends on transmission range R, as given in (6). One way to decrease the transmission time *T* of (7) is to increase *R* and thus reduce the number of incoming packets  $\lambda$  (see also (6)). On the other hand, an excessive increase in the transmission range *R* may increase  $E[d_q]$  by reducing the number of packets, m(R, D), as noted in (4) and (9).

**Proposition 1.** For a given packet generation rate (endto-end throughput)  $\Lambda$ , the optimal transmission range,  $R^*(\Lambda)$  that maximizes the revised transport capacity is a function of *n*, *D*, and  $\Lambda$ :

$$R^*(\Lambda) = \min\left[\frac{D}{2}, \frac{\frac{D}{2} - A_1}{A_2} \cdot \Lambda + A_2\right],$$
(19)

where

$$A_1 = \lim_{\Lambda \to 0} R^*(\Lambda) = D \cdot \sqrt[\theta]{\frac{1}{6\zeta \ (\theta - 1) \ (e^{\theta} - 1)}}$$
(20)

$$A_{2} = \inf \left\{ \Lambda > 0 : R^{*}(\Lambda) = \frac{D}{2} \right\}$$
$$= \frac{\frac{2^{\theta} \cdot \theta}{6\zeta(\theta - 1)} - \ln \left(1 + \frac{2^{\theta}}{6\zeta(\theta - 1)}\right) \left(1 + \frac{2^{\theta}}{6\zeta(\theta - 1)}\right)}{2n\pi DLT_{t} \cdot \frac{2^{\theta}\theta}{6\zeta(\theta - 1)}}.$$
 (21)

#### Proof 1. Appendix.

Figure 2 shows how the optimal transmission range  $R^*(\Lambda)$  varies according to the packet generation rate (end-to-end throughput)  $\Lambda$ . The solid and dotted lines depict

the numerical search and our analytic results from Proposition 1, respectively. Note that the optimal transmission range  $R^*(\Lambda)$  should be smaller than D/2 to avoid the hidden node problem in CSMA. The optimal transmission range increases almost linearly until  $\Lambda$  satisfies the following inequality:

Note that (23) is derived from Proposition 1 (19), where the optimal transmission range becomes D/2 when (23) is satisfied with equality. The linear increase in  $R^*(\Lambda)$  is attributed to the packet incoming rate  $\lambda$ . From (7), (9), and (10),  $\lambda$  is related to the packet velocity. According to (7), if  $\lambda$  becomes greater than  $\frac{1}{n\pi D^2 T_t}$ , the time required to transmit one packet, *T*, becomes negative. This makes the packet velocity negative, and the end-to-end throughput  $\Lambda$  is then infeasible. Remind that the packet incoming rate  $\lambda$  is a function of  $\Lambda$  and *R* (see (5)). Therefore, the optimal transmission range  $R^*(\Lambda)$  should be bounded by the following equation:

$$\Lambda \frac{L}{R^*(\Lambda)} < \frac{1}{n\pi D^2 T_t}$$

$$R^*(\Lambda) > \Lambda n\pi D^2 T_t L.$$
(22)

As  $\Lambda$  increases, the optimal transmission range  $R^*(\Lambda)$  should linearly increase to maintain the constraint (22).

Figure 3 shows the relationship between the packet velocity  $\nu(R^*(\Lambda), \Lambda)$  and the packet generation rate (end-to-end throughput)  $\Lambda$  when the optimal transmission range of Proposition 1 is utilized. These curves determine





the boundaries of the packet velocity (i.e., the inverse of the end-to-end delay) and the end-to-end throughput. The points at which the packet velocity is zero correspond to the end-to-end throughput capacity  $\Lambda_{\max}(R^*(\Lambda))$  of Definition 4. As the packet velocity increases, the endto-end throughput decreases. An interesting feature is that  $\Lambda_{\max}(R^*(\Lambda))$  increases as the path loss exponent increases. This is attributed to the fact that the interference from neighbors is filtered by the high path loss, as well as to the fact that the single-hop transmission rate increases.

$$\Lambda \leq \frac{\frac{2^{\theta}\theta}{6\zeta(\theta-1)} - \ln\left(1 + \frac{2^{\theta}}{6\zeta(\theta-1)}\right)\left(1 + \frac{2^{\theta}}{6\zeta(\theta-1)}\right)}{2n\pi DLT_t \cdot \frac{2^{\theta}\theta}{6\zeta(\theta-1)}}.$$
 (23)

Our next question is how to find the optimal packet generation rate  $\Lambda$  when the optimal transmission range  $R^*(\Lambda)$  is utilized. Reminding that the packet generation rate is equivalent to the end-to-end throughput unless the end-to-end delay is infinite, finding the optimal packet generation rate is the same as deriving the optimal end-toend throughput. Figure 4 shows how the revised transport capacity  $C_T(R^*(\Lambda), \Lambda)$  varies with the packet velocity  $\nu(R^*(\Lambda), \Lambda)$ . Noting that the revised transport capacity is a product of  $\Lambda$  and  $\nu(R^*(\Lambda), \Lambda)$ , Figure 4 is directly obtained from Figure 3. There is an optimal point maximizing  $C_T(R^*(\Lambda), \Lambda)$ , which is given as follows:



**Proposition 2.** The optimal end-to-end throughput (packet generation rate),  $\Lambda_{opt}$  maximizing the revised transport capacity is

$$\Lambda_{\text{opt}} = \frac{\frac{-A_1B_1}{A_2} - B_2B_3 + L\left(\frac{1}{\xi} + T_t\right) - \sqrt{-\frac{3A_1B_1B_2B_3}{A_2} + \left(\frac{A_1B_1}{A_2} + B_2B_3 - L\left(\frac{1}{\xi} + T_t\right)\right)^2}}{\frac{3B_1B_3}{A_2}},$$
(24)

where

$$B_1 = m\left(\frac{D}{2}, D\right) - m(A_1, D)$$
$$= \frac{\log_2\left(1 + \frac{2^{\theta}}{6\zeta(\theta - 1)}\right)}{M} - \frac{\theta}{\ln 2 \cdot M},$$
(25)

$$B_2 = m(A_1, D) + c = \frac{\theta}{\ln 2M} + c,$$
 (26)

$$B_{3} = \frac{\frac{D}{2} - A_{1}}{A_{2}} - n\pi D^{2}LT_{t}$$

$$= \frac{\left(\frac{D}{2} - D\sqrt[\theta]{\frac{1}{6\zeta(\theta-1)(e^{\theta}-1)}}\right)2n\pi DLT_{t}\frac{2^{\theta}\theta}{6\zeta(\theta-1)}}{\frac{2^{\theta}\cdot\theta}{6\zeta(\theta-1)} - \ln\left(1 + \frac{2^{\theta}}{6\zeta(\theta-1)}\right)\left(1 + \frac{2^{\theta}}{6\zeta(\theta-1)}\right)} - n\pi D^{2}LT_{t},$$
(27)

and *c* is an arbitrary constant value.

### Proof 2. Appendix.

Figure 5 shows the relationship between the end-to-end throughput, the packet velocity, and the revised transport



capacity. The solid and dotted lines represent cases in which the optimal transmission range  $R^*(\Lambda)$  is used and where the transmission range R is fixed at D/2, respectively. A higher end-to-end throughput consequently makes the packet velocity slower. The regions under the curves denote the set of feasible end-to-end throughput for both cases. When the packet velocity is low, there is no gap between the two regions. This is explained by the fact that the optimal transmission range  $R^*(\Lambda)$  is D/2when the end-to-end throughput  $\Lambda$  is high (see Figure 5). As the packet velocity increases, on the other hand, the gap becomes larger. We conclude that there is end-to-end throughput enhancement caused by the optimally controlled transmission range  $R^*(\Lambda)$  (Proposition 1) as the packet velocity becomes faster, and the end-to-end delay becomes shorter.

The optimal end-to-end throughput  $\Lambda_{opt}$  (Proposition 2) exists on the solid curve. To express the meaning of optimal packet generating point  $\Lambda_{opt}$ , let us denote two arbitrary points, (x, y) and (x', y'), in the feasible throughput region. Point (x, y) is a reference coordinate. We can define the gain of the packet velocity and end-to-end throughput at point (x', y') as

$$G_v = \frac{x'}{x}, \qquad G_\Lambda = \frac{y'}{y}.$$

The total gain *G* is defined as a product of  $G_{\nu}$  and  $G_{\Lambda}$ :

$$G = G_{\nu} \cdot G_{\Lambda}$$
$$= \frac{x'y'}{xy}$$

The total gain *G* of point ( $\nu(R^*(\Lambda_{opt}), \Lambda_{opt}), \Lambda_{opt}$ ) is always higher than any reference coordinate in the feasible region.

# 5 Scaling laws of end-to-end throughput and delay

In the case of a WMN, we expect that the end-toend throughput decreases and that the end-to-end delay increases as the node density increases. An important problem is how to *quantify* these relationships.

**Proposition 3.** The scaling laws of the end-to-end throughput and the corresponding packet velocity are

$$\Lambda_{\rm opt} = \Theta\left(\frac{1}{\sqrt{n\log n}}\right),\tag{28}$$

$$\nu(R^*(\Lambda_{\text{opt}}), \Lambda_{\text{opt}}) = \Theta\left(\sqrt{\frac{\log n}{n}}\right).$$
(29)

Proof 3. Appendix.

The results of Proposition 3 are identical to those of the random network when centralized TDMA scheduling is utilized [2]. Let us recall the results of Example 1, where the transmission range is D/2. When the sensing range D decreases at the rate of  $\sqrt{\frac{\log n}{n}}^d$ , the scaling laws of the optimal end-to-end throughput  $\Lambda^*(D/2)$  (17) and the corresponding packet velocity  $v(D/2, \Lambda^*(D/2))$  (18) are

$$\Lambda^*(D/2)$$

$$= \frac{\sqrt{\frac{\log n}{n}} \cdot \log_2\left(1 + \frac{2^{\theta}}{6\zeta(\theta-1)}\right)}{4\left(\log_2\left(1 + \frac{2^{\theta}}{6\zeta(\theta-1)}\right)n\pi\left(\sqrt{\frac{\log n}{n}}\right)^2 LT_t + LM\left(\frac{1}{\xi} + T_t\right)\right)}$$
$$= o\left(\frac{1}{\sqrt{n\log n}}\right), \tag{30}$$

$$\nu(D/2, \Lambda^*(D/2)) = \frac{\sqrt{\frac{\log n}{n}} \cdot \log_2\left(1 + \frac{2^{\theta}}{6\zeta(\theta - 1)}\right)}{4 \cdot M \cdot \left(\frac{1}{\xi} + T_t\right)}$$
$$= \Theta\left(\sqrt{\frac{\log n}{n}}\right). \tag{31}$$

Note that when the transmission range is fixed to D/2, the scaling law of the end-to-end throughput (30) is lower than the results in [2]. This occurs as a result of the difference between centralized TDMA scheduling and distributed MAC (IEEE 802.11 DCF). However, controlling the transmission range and the packet generation rate fully compensates for the gap between the two. Figures 6 and 7 prove that Proposition 3 is well matched.

# 6 Conclusions

In [2], Gamal et al. proved that the end-to-end delay is *n* times the end-to-end throughput by controlling the carrier sensing range and the packet generation rate under centralized TDMA scheduling. In this paper, we verified whether IEEE 802.11 DCF can establish this tradeoff relationship by jointly controlling the transmission range and the packet generation rate. To this end, we redefined the transport capacity in [1] by considering the average queuing time at each node. There are interdependencies among the transmission range, the maximum data rate by a single-hop transmission, and the average waiting time in the node queue. Increasing the transmission range plays a key role in decreasing the hop count, which in turn shortens the packet delay. On the other hand, increasing the transmission range reduces the number of packets that can be transmitted by a singlehop transmission, which therefore increases the average packet waiting time. Therefore, there exists an optimal transmission range suitable to minimizing the end-toend delay. In this study, we analyzed the optimal revised





transport capacity by controlling the transmission range and by choosing the appropriate packet generating rate. Finally, it is proved that the scaling laws of the end-toend throughput and end-to-end delay are  $\Theta\left(1/\sqrt{n\log n}\right)$ and  $\Theta\left(\sqrt{n/\log n}\right)$ , respectively, which are equivalent to the results in [2]. Note that when the transmission range is fixed, the scaling law of the end-to-end throughput is  $o\left(1/\sqrt{n\log n}\right)$ . This means that the performance degradation by using IEEE 802.11 DCF instead of centralized TDMA scheduling is mitigated by adjusting the transmission range and the corresponding packet generating rate.

# Endnotes

<sup>a</sup>We recall the following notations:

- $f(n) = \Theta(g(n)) \Rightarrow \exists c_1, c_2, n_0 > 0$  s.t.  $c_1g(n) \le f(n) \le c_2g(n), \forall n \ge n_0.$
- $f(n) = O(g(n)) \Rightarrow \exists c, n_0 > 0 \text{ s.t. } 0 \le f(n) \le cg(n),$  $\forall n \ge n_0.$
- $f(n) = o(g(n)) \Rightarrow \exists c, n_0 > 0 \text{ s.t. } 0 \le f(n) < cg(n),$  $\forall n \ge n_0.$

<sup>b</sup>The transmission range (R) should be less than D/2 to avoid the hidden node problem [16].

<sup>c</sup>The back-off timer stops if any node in the sensing range is transmitting.

<sup>d</sup> D cannot shrink faster than  $\sqrt{\frac{\log n}{n}}$  to maintain the connectivity of the nodes [1].

# Appendix

# **Proof of Proposition 1**

Packet velocity  $v(\Lambda, R)$  (10) is expressed as follows:

$$\nu(\Lambda, R) = \frac{R \cdot \log_2\left(1 + \frac{1}{6\zeta(\theta - 1)} \left(\frac{D}{R}\right)^{\theta}\right)}{M\left(\frac{1}{\xi} + T_t\right)} \left(1 - n\pi D^2 \Lambda \frac{L}{R} T_t\right) - \Lambda L$$
$$= \frac{\log_2\left(1 + \frac{1}{6\zeta(\theta - 1)} \left(\frac{D}{R}\right)^{\theta}\right)}{M\left(\frac{1}{\xi} + T_t\right)} (R - n\pi D^2 \Lambda L T_t) - \Lambda L.$$
(32)

In order to find the optimal  $R^*(\Lambda)$ , we formulate a differential equation by partially differentiating packet velocity  $v(\Lambda, R)$  (32) by *R*:

$$\frac{\partial}{\partial R} \nu(\Lambda, R) \equiv \frac{\partial}{\partial R} \log_2 \left( 1 + \frac{1}{6\zeta(\theta - 1)} \left( \frac{D}{R} \right)^{\theta} \right) (R - n\pi D^2 \Lambda L T_t)$$
$$= -\frac{\frac{1}{6\zeta(\theta - 1)} \left( \frac{D}{R} \right)^{\theta} \cdot \theta \cdot (R - n\pi D^2 \Lambda L T_t)}{R \cdot \left( 1 + \frac{1}{6\zeta(\theta - 1)} \left( \frac{D}{R} \right)^{\theta} \right)}$$
$$+ \frac{\log_2 \left( 1 + \frac{1}{6\zeta(\theta - 1)} \left( \frac{D}{R} \right)^{\theta} \right)}{\ln 2} = 0.$$
(33)

The closed-form solution of the differential equation (33) cannot be found. Hence, we use a linear interpolation. When  $\Lambda$  is zero, the corresponding *R* that satisfies the differential equation (33) is

$$R = D \cdot \left(\frac{1}{6\zeta \ (\theta - 1) \ (\exp(\theta) - 1)}\right)^{1/\theta} \equiv A_1. \tag{34}$$

Next, we substitute D/2 for *R* in the differential equation (33). Then, we can find the corresponding  $\Lambda$  as follows:

$$\Lambda = \frac{\frac{2^{\theta} \cdot \theta}{6\zeta(\theta - 1)} - \ln\left(1 + \frac{2^{\theta}}{6\zeta(\theta - 1)}\right)\left(1 + \frac{2^{\theta}}{6\zeta(\theta - 1)}\right)}{2n\pi DLT_t \cdot \frac{2^{\theta}\theta}{6\zeta(\theta - 1)}} \equiv A_2.$$
(35)

The optimal transmission range  $R^*(\Lambda)$  passes from point ( $\Lambda = 0, R^* = A_1$ ) to point ( $\Lambda = A_2, R^* = D/2$ ). We linearly interpolate two points. This line is described by the following equation:

$$R^*(\Lambda) = \frac{D/2 - A_1}{A_2} \Lambda + A_1,$$
(36)

which is equivalent to the result of Proposition 1.

#### Proof of Proposition 2

When the optimal transmission range  $R^*(\Lambda)$  (19) is adopted, the revised transport capacity of the WMN is as follows:

$$C_{T}(R^{*}(\Lambda),\Lambda) = \Lambda \left( \frac{m(R^{*}(\Lambda),D)}{\left(\frac{1}{\xi} + T_{t}\right)} \left( R^{*}(\Lambda) - n\pi D^{2}\Lambda LT_{t} \right) - \Lambda L \right)$$
$$= \Lambda \left\{ \frac{m(R^{*}(\Lambda),D)}{\left(\frac{1}{\xi} + T_{t}\right)} \left( \frac{D/2 - a}{b} \Lambda + a - n\pi D^{2}\Lambda LT_{t} \right) - \Lambda L \right\}.$$
(37)

To find the optimal packet generation rate  $\Lambda_{opt}$  that maximizes the above equation, we need to use a polynomial form. For this purpose,  $m(R^*(\Lambda), D)$  should be approximated to a polynomial form. When  $\Lambda$  is 0 and  $A_2$  (35),  $m(R^*(\Lambda), D)$  becomes  $\theta / \ln 2 \cdot M$  and  $\frac{\log_2(1 + \frac{2^{\theta}}{6\zeta(\theta-1)})}{M}$ , respectively. We approximate  $m(R^*(\Lambda), D)$  into the linear equation of  $m(R^*(\Lambda), D)$ :

$$m(R^{*}(\Lambda), D) = \frac{\frac{\log_{2}\left(1 + \frac{2^{\theta}}{6\zeta(\theta-1)}\right)}{M} - \frac{\theta}{\ln 2M}}{b} \Lambda + \frac{\theta}{\ln 2M} + c.$$
(38)

Here, *c* is a parameter that minimizes the mean square error (we set *c* to 0.3 when  $\theta$  is 4). We rearrange the revised transport capacity by inserting  $m(R^*(\Lambda), D)$  instead of  $m(R^*(\Lambda), D)$ :

$$C_T(\mathcal{R}^*(\Lambda),\Lambda) \approx \Lambda \left\{ \frac{\frac{\frac{\log_2(1+\frac{1}{6\xi(\theta-1)}2^{\theta})}{M} - \frac{\theta}{\ln 2M} \Lambda + \frac{\theta}{\ln 2M} + c}{(\frac{1}{\xi} + T_t)} \left( \frac{D/2 - a}{b} \Lambda + a - n\pi D^2 \Lambda L T_t \right) - \Lambda L \right\},$$
(39)

which is a quadratic equation. The solution of Equation 39 is identical to the result of Proposition 2.

# **Proof of Proposition 3**

Network connectivity is a critical problem in a WMN because nodes are randomly distributed. In our system model, sensing range *D* is a critical parameter in the determination of the network connectivity. Let us define  $D_c(n)$  as the sensing range that guarantees network connectivity. We use  $D_c(n)$  instead of *D*.

The scaling laws of the related components,  $A_1$  (20),  $A_2$  (21),  $B_1$  (26),  $B_2$  (27), and  $B_3$  (25) are

$$A_{1} = \Theta (D_{c}(n))$$

$$A_{2} = \Theta \left(\frac{1}{nD_{c}(n)}\right)$$

$$B_{1} = \Theta(1)$$

$$B_{2} = \Theta(1)$$

$$B_{3} = \Theta (nD_{c}(n)).$$

Therefore, the scaling law of  $\Lambda_{opt}$  (24) is

$$\Lambda_{\text{opt}} = \frac{\Theta\left(nD_{c}(n)^{2}\right) + \Theta\left(nD_{c}(n)^{2}\right) + \Theta(1) + \sqrt{\Theta(n^{2}D_{c}(n)^{4}) + \Theta(n^{2}D_{c}(n)^{4})}}{\Theta(n^{2}D_{c}(n)^{3})}$$
$$= \Theta\left(\frac{1}{nD_{c}(n)}\right),$$
(40)

and the corresponding transmission range  $R^*(\Lambda_{opt})$  and packet velocity  $\nu(R^*(\Lambda_{opt}), \Lambda_{opt})$  are

$$R^{*}(\Lambda_{opt}) = \Theta(nD_{c}(n)^{2}\Lambda_{opt} + D_{c}(n)) = \Theta(D_{c}(n))$$

$$\nu(R^{*}(\Lambda_{opt}), \Lambda_{opt}) = \frac{m(R^{*}(\Lambda_{opt}), D_{c}(n))}{\frac{1}{\xi} + T_{t}} \left(R^{*}(\Lambda_{opt}) - n\pi D_{c}(n)^{2}\Lambda_{opt}LT_{t}\right)$$

$$-\Lambda_{opt}L$$

$$= \frac{m(R^{*}(\Lambda_{opt}), D_{c}(n))}{\frac{1}{\xi} + T_{t}} \cdot R^{*}(\Lambda_{opt})$$

$$-\Lambda_{opt}L \left(n\pi D_{c}(n)^{2}T_{t}\frac{m(R^{*}(\Lambda_{opt}), D_{c}(n))}{\frac{1}{\xi} + T_{t}} + 1\right)$$

$$= \Theta(1) \cdot \Theta(D_{c}(n)) - \Theta\left(\frac{1}{nD_{c}(n)}\right) \cdot \Theta\left(nD_{c}(n)^{2}\right)$$

$$= \Theta(D_{c}(n)). \tag{41}$$

Because  $D_c(n)$  cannot shrink faster than  $\sqrt{\frac{\log n}{n}}$  to maintain the connectivity of the nodes [1], (40) and (41) are reduced to Proposition 3.

#### **Competing interests**

The authors declare that they have no competing interests.

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#### References

- P Gupta, PR Kumar, The capacity of wireless networks. IEEE Trans. Inform. Theory. 46, 388–404 (2000)
- AE Gamal, J Mammen, B Prabhakar, D Shah, Optimal throughput-delay scaling in wireless networks - part I: the fluid model. IEEE Trans. Inform. Theory. 52, 2568–2592 (2006)
- SW Ko, SL Kim, Delay-constrained capacity of the IEEE 802.11 DCF in wireless multihop networks. IEEE Trans. Mobile Computing. submitted
- KJ Park, J Choi, JC Hou, YC Hu, H Lim, Optimal physical carrier sense in wireless networks. Ad Hoc Netw. 9, 16–27 (2011)
- JG Andrews, F Baccelli, M Kountouris, M Haenggi, Random access transport capacity. IEEE Trans. Wireless Commun. 9, 2101–2111 (2010)
- M Franceschetti, O Douse, DNC Tse, P Thiran, Closing the gap in the capacity of wireless networks via percolation theory. IEEE/ACM Trans. Netw. 53, 1009–1018 (2007)
- M Grossglauser, DNC Tse, Mobility increases the capacity of ad hoc wireless networks. IEEE/ACM Trans. Netw. 10, 477–486 (2002)
- M Garetto, P Giancone, E Leonardi, Capacity scaling in ad hoc networks with heterogenous mobile nodes: the super critical regime. IEEE/ACM Trans. Netw. 17 (2009)
- MJ Neely, E Modiano, Capacity and delay tradeoffs for ad-hoc mobile networks. IEEE Trans. Inform. Theory. 51, 1917–1936 (2005)
- G Sharma, RR Mazumdar, in Proceedings of the 2004 IEEE International Conference on Communications Paris, 20–24 June. Scaling laws for capacity and delay in wireless ad hoc networks with random mobility, (2004), pp. 3869–3873
- G Sharma, RR Mazumdar, NB Shroff, Delay and capacity tradeoffs in mobile ad hoc networks: a global perspective. IEEE Trans. Netw. 15, 981–992 (2007)
- 12. SM Yu, SL Kim, End-to-end delay in wireless random networks. IEEE Commun. Lett. 14, 109–111 (2010)
- JY Seol, SL Kim, in Proceedings of the 2009 IFAC/IEEE Workshop on Networked Robotics Golden, 6–8 Oct. Node mobility control and capacity in wireless ad hoc networks, (2009)
- 14. JY Seol, SL Kim, Node mobility and capacity in wireless controllable ad hoc networks. Comput. Commun. **35**, 1345–1354 (2012)
- YJ Hwang, SL Kim, The capacity of random wireless networks. IEEE Trans. Wireless Comm. 7, 4968–4975 (2008)
- N Bisnik, AA Abouzeid, Queueing network models for delay analysis of multihop wireless ad hoc networks. Ad Hoc Netw. 10, 79–97 (2009)
- CK Chau, M Chen, SC Liew, in Proceedings of the ACM 15th Annual International Conference on Mobile Computing and Networking Beijing, 20–25, Sept. Capacity of large-scale CSMA wireless networks, (2009), pp. 97–108
- IEEE Standards Association, IEEE Standard for Wireless LAN Medium Access Control (MAC) and Physical Layer (PHY) Specifications, P802.11. (IEEE, Piscataway, 1997)
- R Hekmat, PV Mieghem, Interference in wireless multi-hop ad-hoc networks and its effect on network capacity. Wireless Netw. 10, 389–399 (2004)
- TS Kim, H Lim, JC Hou, in Proceedings of the ACM 12th Annual International Conference on Mobile Computing and Networking Los Angeles, 24–29 Sept. Improving spatial reuse through transmit power, carrier sense threshold and data rate in multihop wireless networks, (2006), pp. 366–377

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