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On the achievable rates of a secondary link coexisting with a primary multiple access network

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Abstract

An achievable rate region for a primary multiple access network coexisting with a secondary link of one transmitter and a corresponding receiver is analyzed. The rate region depicts the sum primary rate versus the secondary rate and is established assuming that the secondary link performs rate splitting. The achievable rate region is the union of two types of rate regions. The first type is a rate region established assuming that the secondary receiver cannot decode any primary signal, whereas the second is established assuming that the secondary receiver can decode the signal of one primary link. The achievable rate region is determined first assuming discrete memoryless channel (DMC), then the results are applied to a Gaussian channel. In the Gaussian channel, the performance of rate splitting is characterized for the two types of rate regions. Moreover, a necessary and sufficient condition to determine which primary signal the secondary receiver can decode without degrading the range of primary achievable sum rates is provided. When this condition is satisfied by a certain primary user, the secondary receiver can decode its signal and achieve larger rates without reducing the sum of the primary achievable rates as compared to the case in which it does not decode any primary signal. It is also shown that the probability of having at least one primary user satisfying this condition grows with the primary signal-to-noise ratio.

Keywords: Rate splitting; Cognitive radios; Discrete memoryless channels

1 Introduction

A potential benefit of allowing secondary users to share primary bands is the enhancement of the spectrum utilization. As introduced in [1,2], cognitive radios, or secondary users, are frequency-agile devices that can utilize unused spectrum bands through dynamic spectrum access. In dynamic spectrum access, secondary users should sense the spectrum and identify unused bands or spectrum holes. If a band is sensed and found to be in low use by primary users, i.e., underutilized, a secondary user may opportunistically access this band by adjusting its transmit parameters to fully utilize this band without causing excessive interference on the primary users. However, a secondary user has to leave this band and switch to another if the demand by primary users increases.

The notion of dynamic spectrum access has opened research in different problems regarding the new functionalities that a secondary user should perform, e.g., spectrum sensing, spectrum sharing, spectrum mobility, and spectrum management [2,3]. Moreover, information theoretic bounds on potential achievable rates by cognitive radio networks are being investigated. In most of those works, cooperation between primary and secondary transmitters is considered. In [4], an achievable rate region of primary versus secondary users' rates is introduced when a cognitive transmitter has full knowledge of the primary message in a two-transmitter two-receiver interference channel and the primary user cooperates with the secondary link through rate splitting introduced in [5]. In [6,7], the notion of conferencing is introduced for the interference channel where the cognitive link is assumed to know part or all of the message of the primary transmitter.

In this paper, we consider a primary multiple access channel (MAC) that consists of two transmitters and a

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common receiver shared by a secondary link comprising a single transmitter and a corresponding receiver. The secondary transmitter is assumed to employ rate splitting by dividing its signal into two parts: one part is decodable by the secondary receiver and treated as noise by the primary receiver, whereas the other part is decodable at both receivers. Such rate splitting scheme has also been suggested in [8] for a partially connected interference multiple access channel, with all users belonging to the same class of quality of service (QoS). The scheme has been shown to achieve the semi-deterministic capacity of the addressed setup to within a quantifiable gap. In [9], interference mitigation for a similar setup of interfering MAC has been considered. Authors have shown that signal scale alignment can be achieved through layered lattice codes, which potentially reduces interference by a factor of half for linear deterministic channels.

While we conduct our analysis for the discrete memoryless channel (DMC), we will give particular focus on the Gaussian setup, which is in essence similar to that discussed in [10,11], with a primary multiple access network and a secondary transmitter-receiver pair. We investigate and characterize necessary and sufficient conditions under which interference cancellation (IC) at either primary or secondary users can strictly improve the performance of the achievable rates. Namely, we determine the case when the primary is able to cancel the interference of the secondary while not deteriorating the QoS for the secondary network. We also determine the case when the secondary can completely decode and cancel the interference of at least one primary transmitter while not hurting the primary achievable rates. In particular, we

- State the achievable rate region \mathcal{R}^o in the DMC assuming that all of the primary signals are treated as noise at the secondary receiver
- State the achievable rate region \mathcal{R}_i^r , where the signal of primary transmitter i is to be fully decodable at the secondary receiver besides being decodable at the primary receiver
- Show that there exists a case in which \mathcal{R}_i^r contains \mathcal{R}^o
- Analyze the effect of rate splitting in a Gaussian setup where a necessary and sufficient condition is determined so that the union of the above regions is obtained without rate splitting
- Derive a necessary and sufficient condition so that the secondary receiver can decode the signal of a primary user without affecting the range of achievable primary sum rates, but only enhances the range of achievable secondary rates. We call this condition *primary decodability condition for Gaussian (PDCG) channel*
- Show, numerically, that the probability of having at least one primary user satisfying PDCG

monotonically increases with the signal-to-noise ratio of the primary users

We conduct our analysis assuming a Gaussian communication channel as in [10], but for general channel gains, and adoption of rate splitting techniques. Some of the results in this paper have been presented in [11]. The introduced network model of a MAC primary network shared by secondary operations has been addressed in some resource allocation frameworks without rate splitting by secondary users [12-16]. Rate splitting by a secondary link, however, has been introduced in [17] where the secondary user is assumed to know the codebook of a primary transmitter and opportunistically splits its rate into two parts and decodes it in the following way. It decodes the first part treating both the primary signal and the second part as noise, decodes and cancels the primary signal, and then decodes the second part. This scheme is generalized in this paper as we consider the cases when the signal of one primary transmitter is decodable at the secondary receiver and when all the primary signals are treated as noise.

The rest of this paper is organized as follows. In Section 2, the DMC models are defined. In Section 3, the achievable rate regions are established for the defined DMC models. Then, obtained results are applied in a Gaussian channel setup in Section 4, and the paper is concluded in Section 5.

2 Channel model

In our formulation, we denote random variables by X, Y, \dots with realizations x, y, \dots from sets $\mathcal{X}, \mathcal{Y}, \dots$, respectively. The communication channel is considered to be discrete and memoryless.

2.1 Basic channel model

We consider a basic channel C_B defined by a tuple $(\mathcal{X}_1, \mathcal{X}_2, \mathcal{X}_s, \omega, \mathcal{Y}_p, \mathcal{Y}_s)$, where $\mathcal{X}_1, \mathcal{X}_2$ are two finite input alphabet sets of the primary transmitters and \mathcal{X}_s is a finite input alphabet set of the secondary transmitter. Sets \mathcal{Y}_p and \mathcal{Y}_s are two finite output alphabet sets at the primary and secondary receivers, respectively, and ω is a collection of conditional channel probabilities $\omega(y_p y_s | x_1 x_2 x_s)$ of $(y_p, y_s) \in \mathcal{Y}_p \times \mathcal{Y}_s$ given $(x_1, x_2, x_s) \in \mathcal{X}_1 \times \mathcal{X}_2 \times \mathcal{X}_s$, with marginal conditional distributions:

$$\omega_a(y_a | x_1 x_2 x_s) = \sum_{y_a^* \in \mathcal{Y}_a^*, a^* \neq a} \omega(y_p y_s | x_1 x_2 x_s), \quad a \in \{s, p\}.$$

Since the channel is memoryless, the conditional probability $\omega^n(y_p y_s | \mathbf{x}_1 \mathbf{x}_2 \mathbf{x}_s)$ is given by

$$\omega^n(y_p y_s | \mathbf{x}_1 \mathbf{x}_2 \mathbf{x}_s) = \prod_{t=1}^n \omega(y_p^{(t)} y_s^{(t)} | x_1^{(t)} x_2^{(t)} x_s^{(t)}),$$

where

$$\mathbf{x}_a = (x_a^{(1)}, \dots, x_a^{(n)}) \in \mathcal{X}_a^n, a = 1, 2, s,$$

$$\mathbf{y}_a = (y_a^{(1)}, \dots, y_a^{(n)}) \in \mathcal{Y}_a^n, a = p, s.$$

The same also holds for the marginal conditional distributions $\omega_p^n(\mathbf{y}_p | \mathbf{x}_1 \mathbf{x}_2 \mathbf{x}_s)$ and $\omega_s^n(\mathbf{y}_s | \mathbf{x}_1 \mathbf{x}_2 \mathbf{x}_s)$. Let $\mathcal{M}_1 = \{1, \dots, M_1\}$, $\mathcal{M}_2 = \{1, \dots, M_2\}$ be message sets for primary transmitters 1 and 2, respectively, and $\mathcal{M}_s = \{1, \dots, M_s\}$ be a message set for the secondary transmitter. A code $(n, M_1, M_2, M_s, \epsilon)$ is a collection of M_1, M_2 , and M_s codewords such that

1. Sender a , $a = 1, 2, s$, has an encoding function $\phi_a : i \rightarrow \mathbf{x}_{ai}$, $i \in \mathcal{M}_a$ and $\mathbf{x}_{ai} \in \mathcal{X}^n$
2. The primary receiver has $M_1 M_2$ disjoint decoding sets $\mathcal{D}_{pij} \subseteq \mathcal{Y}_p^n$, $ij \in \mathcal{M}_1 \times \mathcal{M}_2$ and a decoding function $\psi_p : \mathbf{y}_p \rightarrow ij$ if $\mathbf{y}_p \in \mathcal{D}_{pij}$, where $ij \in \mathcal{M}_1 \times \mathcal{M}_2$
3. The secondary receiver has M_s disjoint decoding sets $\mathcal{D}_{sk} \subseteq \mathcal{Y}_s^n$, $k \in \mathcal{M}_s$ and a decoding function $\psi_s : \mathbf{y}_s \rightarrow k$ if $\mathbf{y}_s \in \mathcal{D}_{sk}$, where $k \in \mathcal{M}_s$ (see Figure 1)

$$Pe_p = \frac{1}{M_1 M_2 M_s} \sum_{i,j,k} \omega_p^n(\mathbf{y}_p \notin \mathcal{D}_{pij} | \mathbf{x}_{1i} \mathbf{x}_{2j} \mathbf{x}_{sk}), \quad (1)$$

$$Pe_s = \frac{1}{M_1 M_2 M_s} \sum_{i,j,k} \omega_s^n(\mathbf{y}_s \notin \mathcal{D}_{sk} | \mathbf{x}_{1i} \mathbf{x}_{2j} \mathbf{x}_{sk}) \quad (2)$$

4. Probability of error for the primary network and the secondary link is less than ϵ , that is, $Pe_p \leq \epsilon$ and $Pe_s \leq \epsilon$, respectively, where

A rate tuple (R_1, R_2, R_s) of nonnegative real values is achievable if for any $\eta > 0$, $0 < \epsilon < 1$ there exists a code such that

$$\frac{1}{n} \log M_a \geq R_a - \eta, \quad a = 1, 2, s, \quad (3)$$

with sufficiently large n .

2.2 Rate splitting channel

Rate splitting channel, C_{RS} , is a modified version of the basic channel C_B , where C_{RS} is defined by a tuple

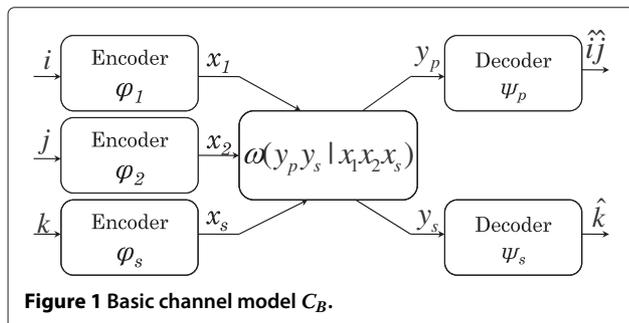


Figure 1 Basic channel model C_B .

$(\mathcal{X}_1, \mathcal{X}_2, \mathcal{X}_s, \omega, \mathcal{Y}_p, \mathcal{Y}_s)$ with its elements are as defined in C_B . Moreover, the input message sets for the primary transmitters are also \mathcal{M}_1 and \mathcal{M}_2 exactly as in C_B . However, the secondary user is assumed to have two finite message sets $\mathcal{L}_s = \{1, \dots, L_s\}$, $\mathcal{N}_s = \{1, \dots, N_s\}$. Hence, a code $(n, M_1, M_2, L_s, N_s, \epsilon)$ over the channel C_{RS} is a collection of $M_1, M_2, L_s N_s$ codewords such that

1. Primary transmitter a , $a = 1, 2$, has an encoding function $\phi_a : i \rightarrow \mathbf{x}_{ai}$, $i \in \mathcal{M}_a$, $\mathbf{x}_{ai} \in \mathcal{X}_a^n$
2. The secondary transmitter has an encoding function $\phi_s : kl \rightarrow \mathbf{x}_{skl}$, $kl \in \mathcal{L}_s \times \mathcal{N}_s$, $\mathbf{x}_{skl} \in \mathcal{X}_s^n$
3. The primary receiver has $M_1 M_2 N_s$ disjoint decoding sets $\mathcal{D}_{pijl} \subseteq \mathcal{Y}_p^n$, $ijl \in \mathcal{M}_1 \times \mathcal{M}_2 \times \mathcal{N}_s$ and a decoding function $\psi_p : \mathbf{y}_p \rightarrow ijl$ if $\mathbf{y}_p \in \mathcal{D}_{pijl}$, where $ijl \in \mathcal{M}_1 \times \mathcal{M}_2 \times \mathcal{N}_s$
4. The secondary receiver has $L_s N_s$ disjoint decoding sets $\mathcal{D}_{skl} \subseteq \mathcal{Y}_s^n$, $kl \in \mathcal{L}_s \times \mathcal{N}_s$ and a decoding function $\psi_s : \mathbf{y}_p \rightarrow kl$ if $\mathbf{y}_p \in \mathcal{D}_{skl}$, where $kl \in \mathcal{L}_s \times \mathcal{N}_s$ (see Figure 2)
5. Probability of error for primary network and secondary link is less than ϵ , that is, $Pe_p^o \leq \epsilon$ and $Pe_s^o \leq \epsilon$, respectively, where

$$Pe_p^o = \frac{1}{M_1 M_2 L_s N_s} \sum_{i,j,k,l} \omega_p^n(\mathbf{y}_p \notin \mathcal{D}_{pijl} | \mathbf{x}_{1i} \mathbf{x}_{2j} \mathbf{x}_{skl}), \quad (4)$$

$$Pe_s^o = \frac{1}{M_1 M_2 L_s N_s} \sum_{i,j,k,l} \omega_s^n(\mathbf{y}_s \notin \mathcal{D}_{skl} | \mathbf{x}_{1i} \mathbf{x}_{2j} \mathbf{x}_{skl}) \quad (5)$$

A rate tuple (R_1, R_2, S, T) of nonnegative real values is achievable over the channel C_{RS} if there exists a code $(n, M_1, M_2, L_s, N_s, \epsilon)$ such that for any arbitrary $0 < \epsilon < 1$ and $\eta > 0$

$$\frac{1}{n} \log M_a \geq R_a - \eta, \quad a \in \{1, 2\}, \quad (6)$$

$$\frac{1}{n} \log L_s \geq S - \eta, \quad (7)$$

$$\frac{1}{n} \log N_s \geq T - \eta, \quad (8)$$

with sufficiently large n .

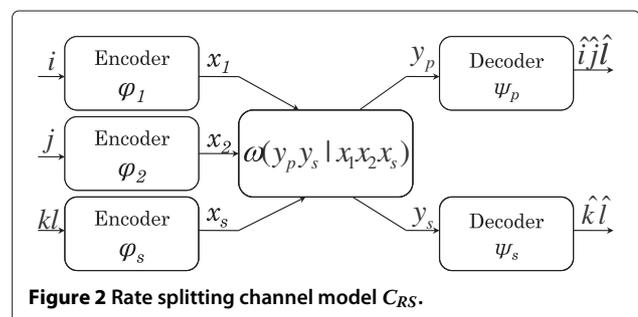


Figure 2 Rate splitting channel model C_{RS} .

Lemma 1. *If a rate tuple (R_1, R_2, S, T) is achievable for C_{RS} , then a rate tuple (R_1, R_2, R_s) where $R_s = S + T$ is achievable for C_B .*

Proof. It is sufficient to show that if $(n, M_1, M_2, L_s, N_s, \epsilon)$ is a code for C_{RS} , then $(n, M_1, M_2, L_s N_s, \epsilon)$ is a code for C_B . To do so, let $\mathcal{D}_{pij} = \cup_{l=1}^{N_s} \mathcal{D}_{pijl}$. Then,

$$\omega_p^n \left(\mathbf{y}_p \notin \mathcal{D}_{pij} | \mathbf{x}_1 \mathbf{x}_2 \mathbf{x}_{skl} \right) \leq \omega_p^n \left(\mathbf{y}_p \notin \mathcal{D}_{pijl} | \mathbf{x}_1 \mathbf{x}_2 \mathbf{x}_{skl} \right). \quad (9)$$

So, if $(n, M_1, M_2, L_s, N_s, \epsilon)$ is a code for C_{RS} , then $Pe_p^o \leq \epsilon$ and $Pe_s^o \leq \epsilon$; hence, from (9), $Pe_p \leq \epsilon$ and $Pe_s \leq \epsilon$ when k and M_s of (1) are replaced with kl and $L_s N_s$, respectively, meaning that $(n, M_1, M_2, L_s N_s, \epsilon)$ is a code for C_B . \square

2.3 Rate splitting channel with decodable primary signal at the secondary receiver

We introduce another channel, C_{RS}^p , in which the secondary user splits its set of messages into two sets, exactly as the case of C_{RS} . However, we assume that the signal of one primary transmitter is decodable at the secondary receiver. Without loss of generality, assume this is the first primary transmitter. Thus, C_{RS}^p is defined by a tuple $(\mathcal{X}_1, \mathcal{X}_2, \mathcal{X}_s, \omega, \mathcal{Y}_p, \mathcal{Y}_s)$ with its elements defined as in C_B and C_{RS} . A code for C_{RS}^p is the same as in C_{RS} , except that conditions 4 and 5 are replaced by

4. Secondary receiver has $M_1 L_s N_s$ disjoint decoding sets $\mathcal{D}_{sikl} \subseteq \mathcal{Y}_s^n$ and a decoding function $\psi_s : \mathbf{y}_s \rightarrow ikl$ if $\mathbf{y}_s \in \mathcal{D}_{sikl}$, where $ikl \in \mathcal{M}_1 \times \mathcal{L}_s \times \mathcal{N}_s$
5. Probability of error for the primary network and the secondary link is less than ϵ , that is, $Pe_p^r \leq \epsilon$ and $Pe_s^r \leq \epsilon$, respectively, where

$$Pe_p^r = \frac{1}{M_1 M_2 L_s N_s} \sum_{i,j,k,l} \omega_p^n \left(\mathbf{y}_p \notin \mathcal{D}_{pijl} | \mathbf{x}_1 \mathbf{x}_2 \mathbf{x}_{skl} \right) \quad (10)$$

$$Pe_s^r = \frac{1}{M_1 M_2 L_s N_s} \sum_{i,j,k,l} \omega_s^n \left(\mathbf{y}_s \notin \mathcal{D}_{sikl} | \mathbf{x}_1 \mathbf{x}_2 \mathbf{x}_{skl} \right). \quad (11)$$

A rate tuple (R_1, R_2, S, T) of nonnegative real values is achievable over the channel C_{RS}^p if for any arbitrary $\eta > 0$ and $0 < \epsilon < 1$, the inequalities (6) to (8) are satisfied for sufficiently large n .

Lemma 2. *If a rate tuple (R_1, R_2, S, T) is achievable for C_{RS}^p , then a rate tuple (R_1, R_2, R_s) where $R_s = S + T$ is achievable for C_B .*

Proof. The proof follows exactly as the proof of Lemma 1 noting that if $\mathcal{D}_{skl} = \cup_{i=1}^{M_1} \mathcal{D}_{sikl}$, then

$$\omega_s^n \left(\mathbf{y}_s \notin \mathcal{D}_{skl} | \mathbf{x}_1 \mathbf{x}_2 \mathbf{x}_{skl} \right) \leq \omega_s^n \left(\mathbf{y}_s \notin \mathcal{D}_{sikl} | \mathbf{x}_1 \mathbf{x}_2 \mathbf{x}_{skl} \right). \quad (12)$$

\square

At the end of this section, it is worth noting that C_B furnishes a general structure for the communication setup of the system and does not explicitly pose any restrictions on the communication strategy used or limits the ability of certain receivers to decode the signals of non-corresponding transmitters. Yet, based on the primary-secondary nature of communication, we explicitly study special instances of C_B , in particular C_{RS} and C_{RS}^p , in which the secondary user is capable of employing rate splitting, and potentially decode the signal of one primary user. Hence, it follows clearly that achievable rates for C_{RS} and C_{RS}^p are also achievable for C_B as established in Lemmas 1 and 2.

3 Achievable rate region

In this section we investigate an achievable rate region for C_B . We first analyze two achievable rate regions, one for C_{RS} and another for C_{RS}^p , and then state the overall achievable rate region. The random variables U , W , and Q are defined over the finite sets \mathcal{U} , \mathcal{W} , and \mathcal{Q} , respectively, where Q is a time-sharing parameter. Let the set \mathcal{P}^* contain all $Z = QUWX_1 X_2 X_s Y_p Y_s$ such that

- X_1, X_2, U , and W are conditionally independent given Q
- $X_s = f(UW|Q)$

Since $X_s = f(UW|Q)$, then U and W can be considered as input sets to the channels C_{RS} and C_{RS}^p .

3.1 Achievable rate region for C_{RS}

Theorem 1. *For any $Z \in \mathcal{P}^*$, $\delta^o(Z)$ is the set of achievable rate tuples (R_1, R_2, S, T) for C_{RS} if the following inequalities are satisfied:*

$$R_1 \leq I(Y_p; X_1 | WX_2 Q), \quad (13)$$

$$R_2 \leq I(Y_p; X_2 | WX_1 Q), \quad (14)$$

$$T \leq I(Y_p; W | X_1 X_2 Q), \quad (15)$$

$$R_1 + R_2 \leq I(Y_p; X_1 X_2 | WQ), \quad (16)$$

$$T + R_1 \leq I(Y_p; WX_1 | X_2 Q), \quad (17)$$

$$T + R_2 \leq I(Y_p; WX_2 | X_1 Q), \quad (18)$$

$$T + R_1 + R_2 \leq I(Y_p; WX_1 X_2 | Q); \quad (19)$$

$$S \leq I(Y_s; U | WQ), \quad (20)$$

$$T \leq I(Y_s; W | UQ), \quad (21)$$

$$S + T \leq I(Y_s; UW | Q). \quad (22)$$

Proof. Please refer to Appendix 1. \square

Corollary 1. *For $\delta^o = \cup_{Z \in \mathcal{P}^*} \delta^o(Z)$, any rate tuple of δ^o is achievable.*

We focus on the achievable rates by the primary network $R_p = R_1 + R_2$ and the secondary link $R_s = S + T$. Let $\mathcal{R}^o(Z)$ be the set of all rate tuples (R_s, R_p) having (R_1, R_2, S, T) satisfy (13) to (22) for all $Z \in \mathcal{P}^*$, then the following theorem describes $\mathcal{R}^o(Z)$.

Theorem 2. For any $Z \in \mathcal{P}^*$, the achievable rate region $\mathcal{R}^o(Z)$ of the defined channel C_{RS} consists of all rate pairs (R_s, R_p) that satisfy

$$R_p \leq \rho_p^o, \quad R_s \leq \rho_s^o, \quad R_s + R_p \leq \rho_{sp}^o \quad (23)$$

where

$$\rho_p^o = I(Y_p; X_1 X_2 | WQ), \quad (24)$$

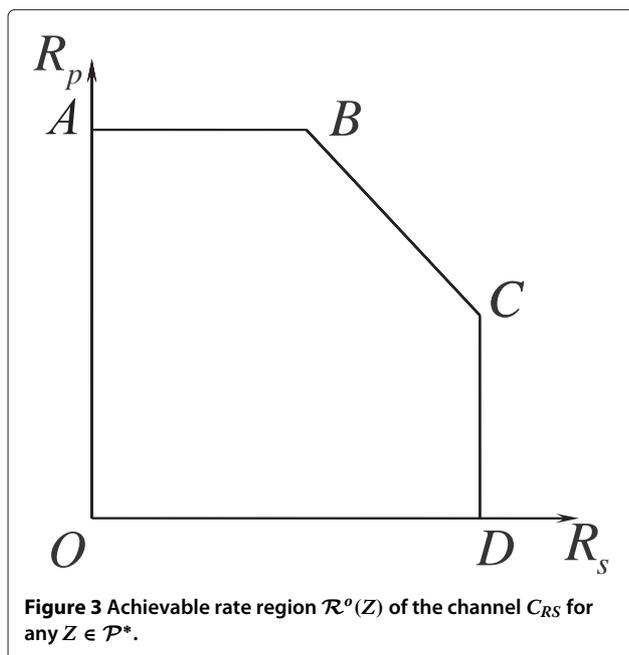
$$\rho_s^o = I(Y_s; U | WQ) + \sigma^*, \quad (25)$$

$$\rho_{sp}^o = \rho_p^o + I(Y_s; U | WQ) + \min \{I(Y_s; W | Q), I(Y_p, W | Q)\}, \quad (26)$$

and

$$\sigma^* = \min \{I(Y_p; W | X_1 X_2 Q), I(Y_s; W | Q)\}. \quad (27)$$

Proof. The proof can follow systematically using the Fourier-Motzkin elimination scheme, yet we use a different approach that determines the rate tuples (R_s, R_p) of the corner points of $\mathcal{R}^o(Z)$, which will essentially be utilized in the proofs of other statements in the rest of this work. To that end, we refer to Figure 3.



• *Point A:*

$R_s^A = 0$, i.e., $S^A = T^A = 0$. Thus, the maximum rate at which the primary network can operate is determined from (16) as

$$R_p^A = I(Y_p; X_1 X_2 | WQ) = \rho_p^o \quad (28)$$

• *Point B:*

At this point, we find the maximum possible rate at which the secondary user can transmit when the primary rate is $R_p^B = \rho_p^o$. In this case, the relations of (13) to (22) are reduced to

$$T \leq I(Y_p; W | Q), \quad (29)$$

$$\rho_p^o + T \leq I(Y_p; W X_1 X_2 | Q); \quad (30)$$

$$T \leq I(Y_s; W | UQ), \quad (31)$$

$$S \leq I(Y_s; U | WQ), \quad (32)$$

$$S + T \leq I(Y_s; UW | Q). \quad (33)$$

Since T is irrelevant in (32), then S can be set to

$$S^B = I(Y_s; U | WQ). \quad (34)$$

Hence, using chain rule in (30) and (33), the maximum value for T would be

$$T^B = \min \{I(Y_p; W | Q), I(Y_s; W | Q)\} \quad (35)$$

and $R_s^B = S^B + T^B$.

• *Point D:*

$R_1^D = R_2^D = R_p^D = 0$, then (13) to (22) are reduced to

$$T \leq I(Y_p; W | X_1 X_2 Q); \quad (36)$$

$$S \leq I(Y_s; U | WQ), \quad (37)$$

$$T \leq I(Y_s; W | UQ), \quad (38)$$

$$S + T \leq I(Y_s; UW | Q). \quad (39)$$

Since T is irrelevant in (37), S can be set to

$$S^D = I(Y_s; U | WQ). \quad (40)$$

Then,

$$T^D = \sigma^* = \min \{I(Y_s; W | Q), I(Y_p; W | X_1 X_2 Q)\} \quad (41)$$

and $R_s^D = S^D + T^D = \rho_s^o$.

• *Point C:*

At $R_s^C = \rho_s^o$, the maximum possible primary rate $R_p = R_1 + R_2$ has to satisfy

$$R_p \leq I(Y_p; X_1 X_2 | WQ), \quad (42)$$

$$R_p \leq I(Y_p; W X_1 X_2 | Q) - \sigma^*. \quad (43)$$

Using chain rule, (43) can be rewritten as

$$R_p \leq I(Y_p; X_1 X_2 | WQ) + I(Y_p; W | Q) - \sigma^*. \quad (44)$$

Thus, if $I(Y_p; W|Q) - \sigma^* > 0$, then (44) will be dominated by (42). Otherwise, (44) dominates (42). So, R_p^C will be given by

$$R_p^C = I(Y_p; X_1 X_2 | WQ) - [\sigma^* - I(Y_p; W|Q)]^+ \quad (45)$$

where $[x]^+ = \max\{0, x\}$. The following is to show that both points (R_s^B, R_p^B) and (R_s^C, R_p^C) lie on the line $R_s + R_p = \rho_{sp}^o$:

For Point B, using direct substitution with

$$R_s^B = I(Y_s; U|WQ) + \min\{I(Y_p; W|Q), I(Y_s; W|Q)\}$$

and

$$R_p^B = \rho_p^o,$$

it is clear that $R_s^B + R_p^B = \rho_{sp}^o$.

For Point C, we consider the following two possibilities:

- $\sigma^* \geq I(Y_p; W|Q)$:

$$\min\{I(Y_s; W|Q), I(Y_p; W|Q)\} = I(Y_p; W|Q).$$

Consequently,

$$\rho_{sp}^o = I(Y_s; U|WQ) + I(Y_p; WX_1 X_2 | Q)$$

and

$$R_s^C + R_p^C = I(Y_s; U|WQ) + I(Y_p; WX_1 X_2 | Q).$$

- $\sigma^* < I(Y_p; W|Q)$:

Since $I(Y_p; W|X_1 X_2 Q) \geq I(Y_p; W|Q)$, therefore $I(Y_s; W|Q) < I(Y_p; W|Q)$. Consequently,

$$\rho_{sp}^o = I(Y_s; UW|Q) + I(Y_p; X_1 X_2 | WQ)$$

and

$$R_s^C + R_p^C = I(Y_s; UW|Q) + I(Y_p; X_1 X_2 | WQ).$$

Therefore, both rate tuples (R_s^B, R_p^B) and (R_s^C, R_p^C) lie on the line $R_s + R_p = \rho_{sp}^o$. \square

Note that, in the appendix of Han and Kobayashi [5], they argued that part of the achievable rate region by their introduced scheme was bounded by lines of slopes -0.5 and -2 . Although from (13) to (22) reducing T by a value of r may result in increase of R_p by $2r$, the proof that point (R_s^C, R_p^C) lies on the line $R_s + R_p = \rho_{sp}^o$ means that a bound of slope -2 does not exist for $\mathcal{R}^o(Z)$.

Corollary 2. Any rate tuple (R_s, R_p) of the region

$$\mathcal{R}^o = \text{closure of } \bigcup_{Z \in \mathcal{P}^*} \mathcal{R}^o(Z) \quad (46)$$

is achievable.

3.2 Achievable rate region for C_{RS}^p

Since in C_{RS}^p the signal of one primary user has to be decodable at the secondary receiver, the model of C_{RS}^p can be considered as the modified interference channel model, C_m , introduced in [5]. The signals of the two primary users can be treated as if they are produced from a single source, splitting its signal into two parts and encoding each part separately such that one part is decodable at both receivers while the other is decodable only at the primary receiver. For this channel, we define the set $\delta_i^r(Z)$ as the set of all achievable rate tuples (R_1, R_2, S, T) when the signal of primary transmitter i , $i \in \{1, 2\}$, is decodable by the secondary receiver. Without loss of generality, we assume that $i = 1$. Hence, the achievable rate region for C_{RS}^p takes the following form.

Theorem 3. For any $Z \in \mathcal{P}^*$, $\delta_1^r(Z)$ is the set of achievable rate tuples (R_1, R_2, S, T) over the channel C_{RS}^p if the following inequalities are satisfied:

$$R_1 \leq I(Y_p; X_1 | WX_2 Q), \quad (47)$$

$$R_2 \leq I(Y_p; X_2 | WX_1 Q), \quad (48)$$

$$T \leq I(Y_p; W | X_1 X_2 Q), \quad (49)$$

$$R_1 + R_2 \leq I(Y_p; X_1 X_2 | WQ), \quad (50)$$

$$R_1 + T \leq I(Y_p; WX_1 | X_2 Q), \quad (51)$$

$$R_2 + T \leq I(Y_p; WX_2 | X_1 Q), \quad (52)$$

$$R_1 + R_2 + T \leq I(Y_p; WX_1 X_2 Q); \quad (53)$$

$$S \leq I(Y_s; U | WX_1 Q), \quad (54)$$

$$T \leq I(Y_s; W | UX_1 Q), \quad (55)$$

$$R_1 \leq I(Y_s; X_1 | UWQ), \quad (56)$$

$$S + T \leq I(Y_s; UW | X_1 Q), \quad (57)$$

$$R_1 + S \leq I(Y_s; UX_1 | WQ), \quad (58)$$

$$R_1 + T \leq I(Y_s; WX_1 | UQ), \quad (59)$$

$$R_1 + S + T \leq I(Y_s; UWX_1 | Q). \quad (60)$$

Proof. The proof follows exactly the proof of Theorem 3.1 in [5]. \square

Corollary 3. For $\delta_1^r = \bigcup_{Z \in \mathcal{P}^*} \delta_1^r(Z)$, any rate tuple of δ_1^r is achievable.

For C_{RS}^p , the region $\mathcal{R}_i^r(Z)$ is the set of rate tuples (R_s, R_p) where $R_s = S + T$, $R_p = R_1 + R_2$, and (R_1, R_2, S, T) is an element of $\delta_i^r(Z)$ for any $Z \in \mathcal{P}^*$, $i \in \{1, 2\}$.

Theorem 4. For any $Z \in \mathcal{P}^*$, the achievable rate region $\mathcal{R}_1^r(Z)$ for the channel C_{RS}^p consists of all rate pairs (R_s, R_p) that satisfy

$$\begin{aligned} R_s &\leq \rho_s^r, & R_p &\leq \rho_p^r, & R_s + R_p &\leq \rho_{sp}^r, \\ 2R_s + R_p &\leq \rho_{2p}^r, & R_s + 2R_p &\leq \rho_{s2}^r \end{aligned} \quad (61)$$

where

$$\rho_s^r = I(Y_s; U|WX_1Q) + \sigma_s^*, \quad (62)$$

$$\rho_p^r = I(Y_p; X_2|WX_1Q) + \sigma_p^*, \quad (63)$$

$$\begin{aligned} \rho_{sp}^r &= I(Y_s; U|WX_1Q) + I(Y_p; X_2|WX_1Q) \\ &+ \min \{ I(Y_p; WX_1|Q), I(Y_s; WX_1|Q), \\ &I(Y_p; W|X_1Q) + I(Y_s; X_1|WQ), I(Y_p; X_1|WQ) \\ &+ I(Y_s; W|X_1Q) \}, \end{aligned} \quad (64)$$

$$\begin{aligned} \rho_{2p}^r &= 2I(Y_s; U|WX_1Q) + 2\sigma_s^* + I(Y_p; X_2|WX_1Q) \\ &- [\sigma_s^* - I(Y_p; W|X_1Q)]^+ + \min \{ I(Y_s; X_1|WQ), \\ &I(Y_s; WX_1|Q) - \sigma_s^*, I(Y_p; X_1|Q) + \\ &[I(Y_p; W|X_1Q) - \sigma_s^*]^+, I(Y_p; X_1|WQ) \}, \end{aligned} \quad (65)$$

$$\begin{aligned} \rho_{s2}^r &= 2I(Y_p; X_2|WX_1Q) + 2\sigma_p^* + I(Y_s; U|WX_1Q) - \\ &[\sigma_p^* - I(Y_s; X_1|WQ)]^+ + \min \{ I(Y_p; W|X_1Q), \\ &I(Y_p; WX_1|Q) - \sigma_p^*, I(Y_s; W|Q) + \\ &[I(Y_s; X_1|WQ) - \sigma_p^*]^+, I(Y_s; W|X_1Q) \}, \end{aligned} \quad (66)$$

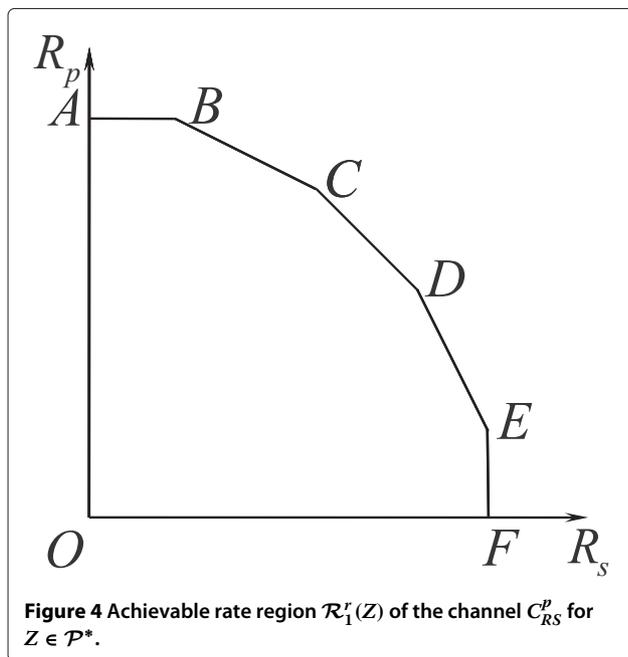
and

$$\sigma_s^* = \min \{ I(Y_s; W|X_1Q), I(Y_p; W|X_1X_2Q) \}, \quad (67)$$

$$\sigma_p^* = \min \{ I(Y_p; X_1|WQ), I(Y_s; X_1|UWQ) \} \quad (68)$$

as shown in Figure 4.

Proof. From the similarity between C_{RS}^p and the modified interference channel of Han and Kobayashi [5], the



derivation of the achievable rate region can be found in the appendix of [5]. The analysis goes as that done for $\mathcal{R}^o(Z)$ in C_{RS} . Hence, the corner points of the $\mathcal{R}_1^r(Z)$ are shown in Figure 4 and are given as follows.

- Point A:

$$R_s^A = 0, \quad (69)$$

$$R_p^A = \rho_p^r = I(Y_p; X_2|X_1WQ) + \sigma_p^*. \quad (70)$$

- Point B:

$$\begin{aligned} R_s^B &= I(Y_s; U|WX_1Q) - [\sigma_p^* - I(Y_s; X_1|WQ)]^+ \\ &+ \min \{ I(Y_p; W|X_1Q), I(Y_p; WX_1|Q) - \sigma_p^*, \\ &I(Y_s; W|Q) + [I(Y_s; X_1|WQ) - \sigma_p^*]^+, \\ &I(Y_s; W|X_1Q) \}, \end{aligned} \quad (71)$$

$$R_p^B = \rho_p^r = I(Y_p; X_2|X_1WQ) + \sigma_p^*. \quad (72)$$

- Point C:

$$R_s^C = 2\rho_{sp}^r - \rho_{s2}^r, \quad (73)$$

$$R_p^C = \rho_{s2}^r - \rho_{sp}^r. \quad (74)$$

- Point D:

$$R_s^D = \rho_{2p}^r - \rho_{sp}^r, \quad (75)$$

$$R_p^D = 2\rho_{sp}^r - \rho_{s2}^r. \quad (76)$$

- Point E:

$$R_s^E = I(Y_s; U|WX_1Q) + \sigma_s^*, \quad (77)$$

$$\begin{aligned} R_p^E &= I(Y_p; X_2|WX_1Q) - [\sigma_s^* - I(Y_p; W|X_1Q)]^+ \\ &+ \min \{ I(Y_s; X_1|WQ), I(Y_s; WX_1|Q) - \sigma_s^*, \\ &I(Y_p; X_1|Q) + [I(Y_p; W|X_1Q) - \sigma_s^*]^+, \\ &I(Y_p; X_1|WQ) \}. \end{aligned} \quad (78)$$

- Point F:

$$R_s^F = \rho_s^r = I(Y_s; U|WX_1Q) + \sigma_s^*, \quad (79)$$

$$R_p^F = 0. \quad (80)$$

□

Corollary 4. Any rate tuple (R_s, R_p) of the regions

$$\mathcal{R}_i^r = \text{closure} \bigcup_{Z \in \mathcal{P}^*} \mathcal{R}_i^r(Z), \quad i \in \{1, 2\}, \quad (81)$$

is achievable.

Constraining the signal of one primary user to be decodable at the secondary receiver might result in a degradation in the achievable primary rate especially when the

secondary rate is small. In general, \mathcal{R}^o and \mathcal{R}_i^r do not necessarily contain one another; however, there exists a case in which $\mathcal{R}^o(Z) \subseteq \mathcal{R}_i^r(Z)$. The following theorem characterizes that case.

Theorem 5. For a given $Z \in \mathcal{P}^*$, $\mathcal{R}^o(Z) \subseteq \mathcal{R}_i^r(Z)$ if and only if

$$I(Y_p; X_i | WQ) \leq I(Y_s; X_i | UWQ). \quad (82)$$

Proof. Please refer to Appendix 2. □

Corollary 5. If for all $Z \in \mathcal{P}^*$ condition (82) is satisfied, then $\mathcal{R}^o \subseteq \mathcal{R}_i^r$, where $\mathcal{R}_i^r = \cup_{Z \in \mathcal{P}^*} \mathcal{R}_i^r(Z)$.

Theorem 5 shows that when a primary user encodes its messages at a rate decodable at both receivers, the primary network may achieve the same rate range when none of the signals of its users is decodable at the secondary receiver. Moreover, at every primary rate, the secondary rate is enhanced (see Figure 5).

Consequently, if for any $Z \in \mathcal{P}^*$ condition (82) is satisfied, then allowing the secondary receiver to decode the signal of primary user i at this Z enhances the range of the secondary achievable rates without reducing the range of the achievable primary sum rates.

We call Corollary 5 the *primary decodability condition (PDC)*.

3.3 Achievable rate region for the channel C_B

From C_{RS} and C_{RS}^p , we define

$$\mathcal{R}_i(Z) = \mathcal{R}^o(Z) \cup \mathcal{R}_i^r(Z), \quad Z \in \mathcal{P}^*, i \in \{1, 2\}, \quad (83)$$

and

$$\mathcal{R}_i = \text{closure} \bigcup_{Z \in \mathcal{P}^*} \mathcal{R}_i(Z), \quad i \in \{1, 2\}. \quad (84)$$

Hence, an achievable rate region for the channel C_B

$$\mathcal{R} = \mathcal{R}_1 \cup \mathcal{R}_2, \quad (85)$$

or equivalently,

$$\mathcal{R} = \mathcal{R}^o \cup \mathcal{R}_1^r \cup \mathcal{R}_2^r. \quad (86)$$

At this point, it is worth reflecting the resulting achievable rate region \mathcal{R} on the Han-Kobayashi region derived for the 2×2 interference channel, denoted \mathcal{R}_{HK} , especially with the adopted channel model C_B which is well related to the interference channel C in [5]. In the light of the considered communication setup and adopted rate splitting communication scheme, we can note that the two primary transmitters of our setup C_B can be viewed as a common transmitter in C splitting its signal into X_1, X_2 .

However, since the transmitters are sending independent messages and having no control over the codebook of each other, their transmit strategies adopted in \mathcal{R} can be considered as *only* three realizations of the possible rate splitting strategies for the common transmitter in C . Thus, we can note that $\mathcal{R} \subseteq \mathcal{R}_{HK}$. In particular, for

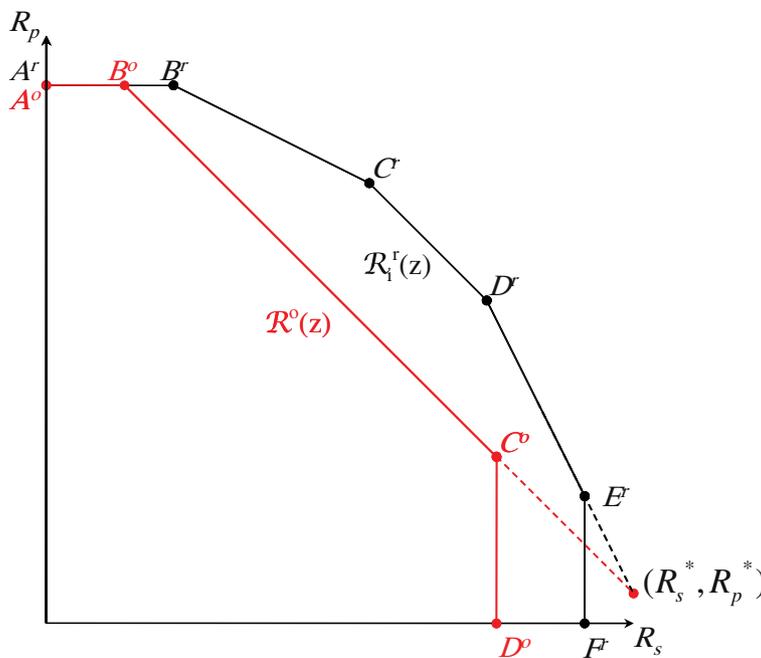


Figure 5 Regions $\mathcal{R}_i^r(Z)$ and $\mathcal{R}^o(Z)$ when $I(Y_p; X_i | WQ) \leq I(Y_s; X_i | UWQ)$.

the two primary transmitters of C_B behaving as a common transmitter in \mathbf{C} , rate region \mathcal{R} spans only the rate splitting strategies of such common transmitter when the secondary receiver (1) cannot decode any primary signal, (2) can only decode the whole signal of user 1, and (3) can only decode the whole signal of user 2.

Note that, inequalities (15) and (49) used in $\delta^o(Z)$ and $\delta_1^r(Z)$, respectively, to limit the error in decoding the public part of the secondary signal at the primary receiver while the primary signals are decoded successfully. In fact, the primary receiver may not be interested in limiting the probability of such error event. Similarly, inequality (56) in $\delta_1^r(Z)$ may not be relevant as the secondary receiver is not interested in limiting the probability of error in decoding the primary signal when the two parts of its signal are decoded successfully. However, removing (15) from the definition of $\delta^o(Z)$ and (49) and (56) from the definition of $\delta_1^r(Z)$ does not enhance the achievable rate region \mathcal{R} .

To demonstrate this fact, we define $\delta^{'o}(Z)$ exactly as $\delta^o(Z)$ but without the constraint of (15), and $\delta_1^{'r}(Z)$ exactly as $\delta_1^r(Z)$ but without the constraints (49) and (56). Let $\mathcal{R}^{'o}(Z)$ and $\mathcal{R}_1^{'r}(Z)$ be two sets of rate tuples (R_s, R_p) such that $R_s = S + T$ and $R_p = R_1 + R_2$, and the rate tuple (R_1, R_2, S, T) is an element of $\delta^{'o}(Z)$ and $\delta_1^{'r}(Z)$, respectively. Also, we define

$$\mathcal{R}'_1(Z) = \mathcal{R}^{'o}(Z) \cup \mathcal{R}_1^{'r}(Z).$$

Theorem 6. *If $\mathcal{R}'_1 = \bigcup_{Z \in \mathcal{P}^*} \mathcal{R}'_1(Z)$, then $\mathcal{R}'_1 = \mathcal{R}_1$.*

Proof. Please refer to Appendix 3. □

Corollary 6. *For*

$$\mathcal{R}' = \text{closure of } \mathcal{R}'_1 \cup \mathcal{R}'_2,$$

then

$$\mathcal{R}' = \mathcal{R}.$$

4 Gaussian channel

In this section, we quantify the obtained achievable rate regions in a Gaussian channel model. A memoryless Gaussian channel of the introduced system is defined by a tuple $(\mathcal{X}_1, \mathcal{X}_2, \mathcal{X}_s, \omega, \mathcal{Y}_p, \mathcal{Y}_s)$ with $\mathcal{X}_1 = \mathcal{X}_2 = \mathcal{X}_s = \mathcal{Y}_p = \mathcal{Y}_s = \Re$ (the field of real numbers), and a channel probability ω specified by

$$y_p = \sqrt{g_1^p} x_1 + \sqrt{g_2^p} x_2 + \sqrt{g_s^p} x_s + n_p, \quad (87)$$

$$y_s = \sqrt{g_1^s} x_1 + \sqrt{g_2^s} x_2 + \sqrt{g_s^s} x_s + n_s \quad (88)$$

for $x_1 \in \mathcal{X}_1$, $x_2 \in \mathcal{X}_2$, $x_s \in \mathcal{X}_s$, $y_p \in \mathcal{Y}_p$, and $y_s \in \mathcal{Y}_s$, where n_p and n_s are independent Gaussian additive noise samples with zero mean and variance N_0 , and $g_1^p, g_2^p, g_s^p, g_1^s, g_2^s$, and g_s^s are the channel power gains. Power constraints

are imposed on codewords $\mathbf{x}_1(i)$, $\mathbf{x}_2(j)$, $\mathbf{x}_s(k)$ ($i \in \mathcal{M}_1, j \in \mathcal{M}_2, k \in \mathcal{M}_s$):

$$\frac{1}{n} \sum_{t=1}^n \left(x_1(i)^{(t)} \right)^2 = P_1, \quad (89)$$

$$\frac{1}{n} \sum_{t=1}^n \left(x_2(j)^{(t)} \right)^2 = P_2, \quad (90)$$

$$\frac{1}{n} \sum_{t=1}^n \left(x_s(k)^{(t)} \right)^2 = P_s. \quad (91)$$

Also, we define a subclass $\mathcal{G}(P_1, P_2, P_s)$ of \mathcal{P}^* as follows: $Z = \phi U W X_1 X_2 X_s Y_p Y_s \in \mathcal{G}(P_1, P_2, P_s)$ if and only if $Z \in \mathcal{P}^*$, $\sigma^2(X_1) = P_1$, $\sigma^2(X_2) = P_2$, and $\sigma^2(X_s) = P_s$, with X_1, X_2, U , and W being zero mean Gaussian and $X_s = U + W$. Hence, we have the following rate regions achievable:

$$\mathcal{R}_g^o = \text{closure of } \bigcup_{Z \in \mathcal{G}(P_1, P_2, P_s)} \mathcal{R}^o(Z), \quad (92)$$

$$\mathcal{R}_{ig}^r = \text{closure of } \bigcup_{Z \in \mathcal{G}(P_1, P_2, P_s)} \mathcal{R}_i^r(Z), i \in \{1, 2\}, \quad (93)$$

$$\mathcal{R}_{ig} = \text{closure of } \bigcup_{Z \in \mathcal{G}(P_1, P_2, P_s)} \mathcal{R}_i(Z), i \in \{1, 2\}, \quad (94)$$

$$\mathcal{R}_g = \mathcal{R}_g^o \bigcup \left(\bigcup_{i \in \{1, 2\}} \mathcal{R}_{ig}^r \right) = \mathcal{R}_{1g} \bigcup \mathcal{R}_{2g}. \quad (95)$$

Assume the secondary user splits its power into λP_s and $\bar{\lambda} P_s$ such that $0 \leq \lambda \leq 1$ and $\lambda + \bar{\lambda} = 1$. The part of secondary signal decodable at the primary and secondary receivers is encoded with power $\bar{\lambda} P_s$ where the other part is encoded with power λP_s . Let $\tau(x) = 0.5 \log_2(1+x)$, and the relevant quantities in Theorems 2 and 4 will be given by

$$I(Y_p; X_1 X_2 | W) = \tau \left(\frac{g_1^p P_1 + g_2^p P_2}{g_s^p \lambda P_s + N_0} \right),$$

$$I(Y_p; X_1 X_2) = \tau \left(\frac{g_1^p P_1 + g_2^p P_2}{g_s^p P_s + N_0} \right),$$

$$I(Y_p; X_2 | W X_1) = \tau \left(\frac{g_2^p P_2}{g_s^p \lambda P_s + N_0} \right),$$

$$I(Y_p; X_1 | W) = \tau \left(\frac{g_1^p P_1}{g_s^p \lambda P_s + g_2^p P_2 + N_0} \right),$$

$$I(Y_p; W | X_1 X_2) = \tau \left(\frac{g_s^p \bar{\lambda} P_s}{g_s^p \lambda P_s + N_0} \right),$$

$$I(Y_p; W | X_1) = \tau \left(\frac{g_s^p \bar{\lambda} P_s}{g_s^p \lambda P_s + g_2^p P_2 + N_0} \right),$$

$$\begin{aligned}
 I(Y_p; WX_1) &= \tau \left(\frac{g_1^p P_1 + g_s^p \bar{\lambda} P_s}{g_s^p \lambda P_s + g_2^p P_2 + N_0} \right), \\
 I(Y_p; W) &= \tau \left(\frac{g_s^p \bar{\lambda} P_s}{g_s^p \lambda P_s + g_1^p P_1 + g_2^p P_2 + N_0} \right) \\
 I(Y_p; X_1) &= \tau \left(\frac{g_1^p P_1}{g_s^p P_s + g_2^p P_2 + N_0} \right); \\
 I(Y_s; U|WX_1) &= \tau \left(\frac{g_s^s \lambda P_s}{g_2^s P_2 + N_0} \right), \\
 I(Y_s; U|W) &= \tau \left(\frac{g_s^s \lambda P_s}{g_1^s P_1 + g_2^s P_2 + N_0} \right), \\
 I(Y_s; W|X_1) &= \tau \left(\frac{g_s^s \bar{\lambda} P_s}{g_s^s \lambda P_s + g_2^s P_2 + N_0} \right), \\
 I(Y_s; WX_1) &= \tau \left(\frac{g_s^s \bar{\lambda} P_s + g_1^s P_1}{g_s^s \lambda P_s + g_2^s P_2 + N_0} \right), \\
 I(Y_s; W) &= \tau \left(\frac{g_s^s \bar{\lambda} P_s}{g_s^s \lambda P_s + g_1^s P_1 + g_2^s P_2 + N_0} \right), \\
 I(Y_s; X_1|W) &= \tau \left(\frac{g_1^s P_1}{g_s^s \lambda P_s + g_2^s P_2 + N_0} \right), \\
 I(Y_s; X_1|UW) &= \tau \left(\frac{g_1^s P_1}{g_2^s P_2 + N_0} \right).
 \end{aligned}$$

4.1 Performance of rate splitting

We study the effect of rate splitting by the secondary link on the achievable rate regions \mathcal{R}_g^o and \mathcal{R}_{ig}^r , $i \in \{1, 2\}$ and hence \mathcal{R}_{ig} . For each region, there exists a case for which no rate splitting determines the overall region, i.e., each achievable rate region is obtained at $\lambda = 0$ or $\lambda = 1$. We say that rate splitting does not affect an achievable rate region \mathcal{A} if $\mathcal{A}(Z)$ coincides on \mathcal{A} at $\lambda = 0$ or $\lambda = 1$, $Z \in \mathcal{G}(P_1, P_2, P_s)$, where $\mathcal{A} = \bigcup_{Z \in \mathcal{G}(P_1, P_2, P_s)} \mathcal{A}(Z)$, meaning that either decoding the whole secondary signal at the primary receiver or not decoding it at all determines \mathcal{A} .

4.1.1 For \mathcal{R}_g^o

The region \mathcal{R}_g^o is obtained when the secondary receiver is assumed to treat the primary interference as noise. The following theorem determines the effect of rate splitting on \mathcal{R}_g^o .

Theorem 7. For $Z \in \mathcal{G}(P_1, P_2, P_s)$, an achievable rate region $\mathcal{R}^o(Z)$ can only coincide on \mathcal{R}_g^o at $\lambda = 0$, if and only if

$$I(Y_s; W) \leq I(Y_p; W|X_1 X_2) \quad (96)$$

or equivalently,

$$g_s^s N_0 \leq g_s^p (g_1^s P_1 + g_2^s P_2 + N_0). \quad (97)$$

Proof. Please refer to Appendix 4. \square

Theorem 7 shows that rate splitting does not affect the achievable rate region \mathcal{R}_g^o when inequality (97) is satisfied. Hence, a primary receiver decoding all the secondary signal is preferable at this case. Figure 6 depicts this case for different values of λ . It is clear that $\mathcal{R}^o(Z)$ at smaller λ contains $\mathcal{R}^o(Z)$ at larger λ . This figure was obtained at $g_1^p = 2.5664$, $g_2^p = 3.7653$, $g_1^s = 0.1812$, $g_2^s = 0.1784$, $g_s^p = 2.3620$, and $g_s^s = 8.6065$ and at the following power setup. The noise variance $N_0 = 1$ unit power and $\frac{P_1}{N_0} = \frac{P_2}{N_0} = \text{SNR}_p = 10$ dB and $\frac{P_s}{N_0} = \text{SNR}_s = 10$ dB. Note that in this case, the maximum secondary throughput does not depend on λ , so the best performance from the primary rate point of view is to decode all the secondary signal by setting $\lambda = 0$.

Moreover, when inequality (97) is not satisfied, rate splitting affects \mathcal{R}_g^o as for any two different values of λ the corresponding $\mathcal{R}^o(Z)$ s do not contain one another. Hence, \mathcal{R}_g^o is obtained by varying λ from 0 to 1. Figure 7 represents the case when (97) is not satisfied for the following parameters. $g_1^p = 1.5066$, $g_2^p = 0.8290$, $g_1^s = 0.1902$, $g_2^s = 0.0122$, $g_s^p = 1.1953$, and $g_s^s = 10.3229$ with the same power setup of Figure 6.

Also, it is shown in [11] that when (97) is not satisfied, the sum throughput of the whole network, i.e., $R_s + R_p$, increases with λ . That is, as λ increases, the primary sum rate decreases but the secondary rate gains an increase larger than the decrease in rate encountered by the primary network. Figure 8 depicts $R_s + R_p$ for the same simulation parameters of Figure 7. It is clear that the increase in the total sum rate, $R_s + R_p$, is accompanied by a decrease in the sum primary rate R_p . Hence, the sum primary rate has to be protected above a minimum limit.

4.1.2 For \mathcal{R}_{ig}^r , $i \in \{1, 2\}$

The region \mathcal{R}_{ig}^r is obtained when the secondary receiver can decode the signal of primary user i . The rate splitting effect on this region is determined in the following theorem.

Theorem 8. For $Z \in \mathcal{G}(P_1, P_2, P_s)$ and $i \in \{1, 2\}$, an achievable rate region $\mathcal{R}_i^r(Z)$ can only coincide on \mathcal{R}_{ig}^r at $\lambda = 0$ if and only if

$$I(Y_s; W|X_i) \leq I(Y_p; W|X_1 X_2) \quad (98)$$

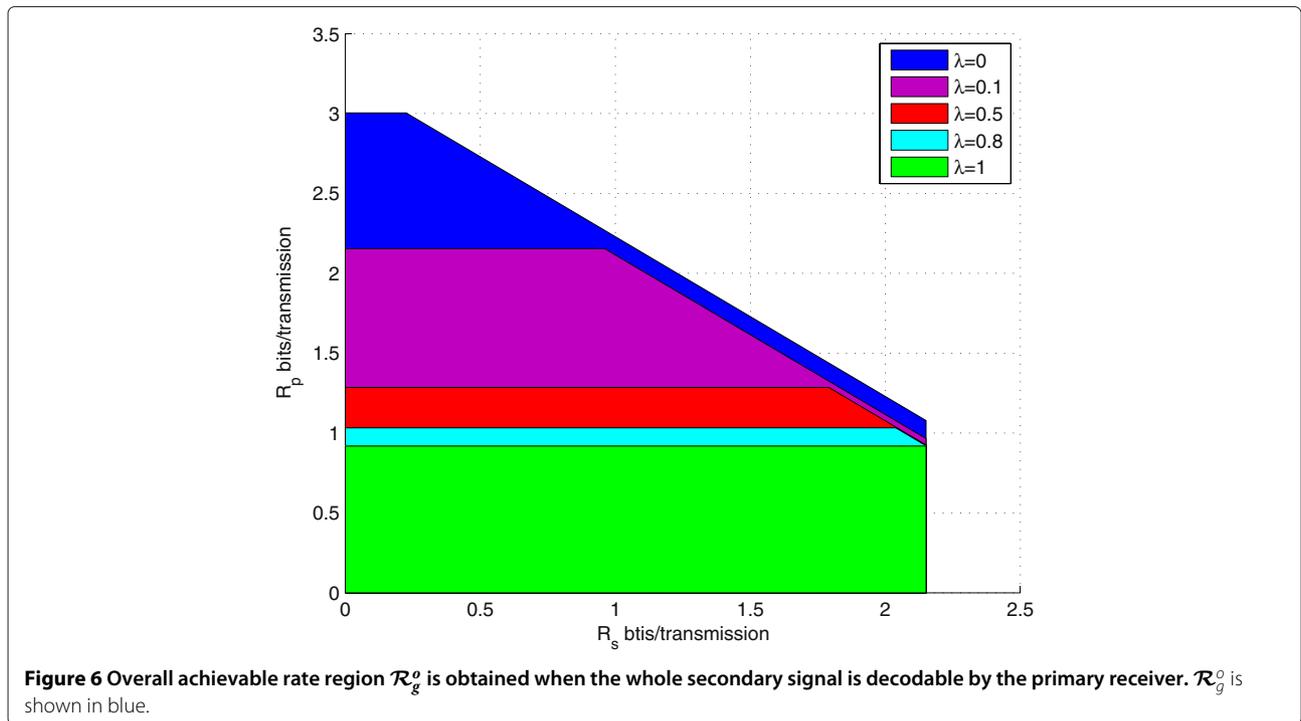
or equivalently,

$$g_s^s N_0 \leq g_s^p (g_j^s P_j + N_0), \quad j \in \{1, 2\}, j \neq i. \quad (99)$$

Proof. Please refer to Appendix 5. \square

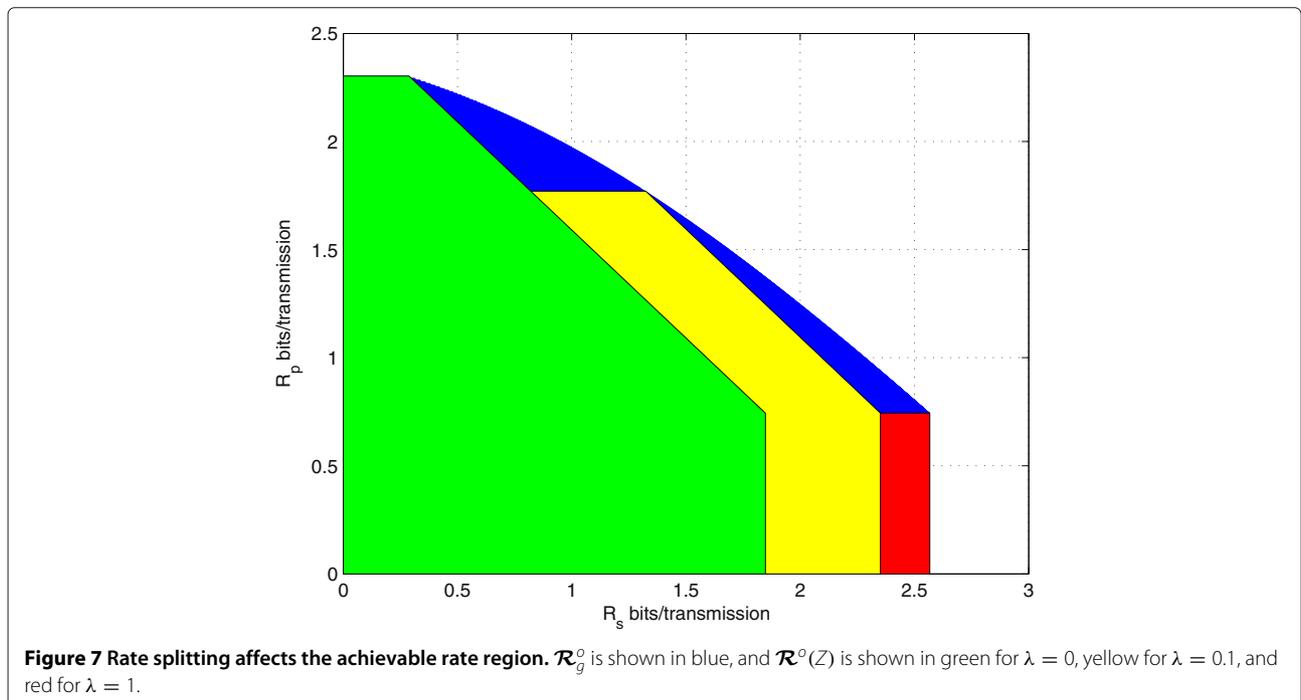
Hence, if inequality (99) is satisfied, \mathcal{R}_{ig}^r is obtained without rate splitting, specifically, when $\lambda = 0$.

Figures 9 and 10 show the performance of rate splitting under same power setup used with Figures 6 and 7 where



it is assumed that the secondary receiver can decode the signal of primary user 1. In Figure 9, the achievable rate region \mathcal{R}_{1g}^r coincides on $\mathcal{R}_1^r(Z)$ when inequality (99) is satisfied. The parameters for this scenario are $g_1^p = 5.5303$, $g_2^p = 4.2865$, $g_1^s = 0.6542$, $g_2^s = 0.8121$, $g_s^p = 3.9334$, and $g_s^s = 8.1575$.

In Figure 10, the opposite scenario is considered where inequality (99) is not satisfied. It is obvious that the overall rate region \mathcal{R}_{1g}^r is obtained by varying λ from 0 to 1 as a consequence of the fact that rate regions corresponding to different values of λ do not include one another if inequality (99) is not satisfied. The channel gains for Figure 10



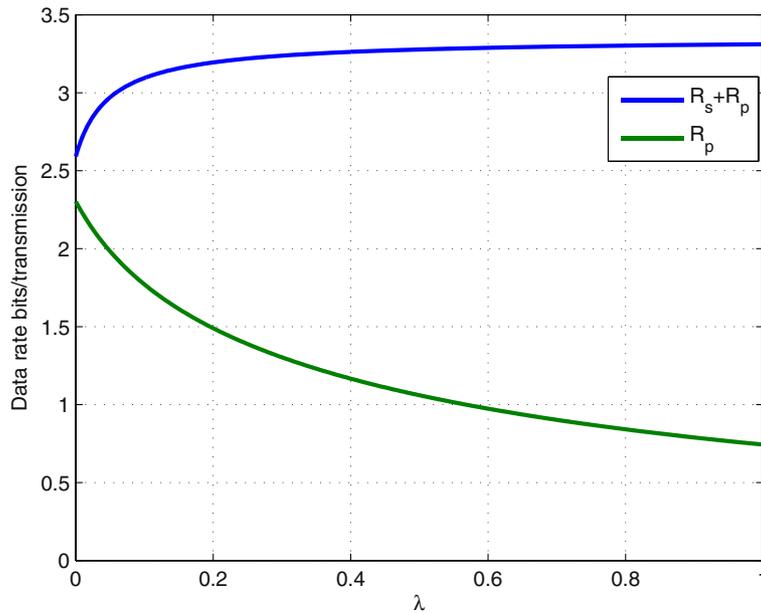


Figure 8 Increase in the sum rate of the whole network when inequality (97) is not satisfied.

are $g_1^p = 9.566$, $g_2^p = 14.5045$, $g_1^s = 0.0808$, $g_2^s = 0.2894$, $g_s^p = 0.7032$, and $g_s^s = 16.6226$.

Consequently, the achievable rate region \mathcal{R}_{ig} coincides on $\mathcal{R}_{ig}(Z)$ at $\lambda = 0$ if and only if (99) is satisfied.

4.2 Decoding one primary signal

In Section 3.2, we have discussed the achievable rate region in the DMC case assuming that the signal of one

primary transmitter has to be reliably decoded by the secondary receiver. Although this may impose a constraint on the range of achievable sum rates by the primary network, we showed in Theorem 5 and Corollary 5 that there exists a condition for which this constraint only enhances the achievable rates for the secondary link without degrading the range of achievable rates by the primary network. This condition is called PDC. When applying this condition to

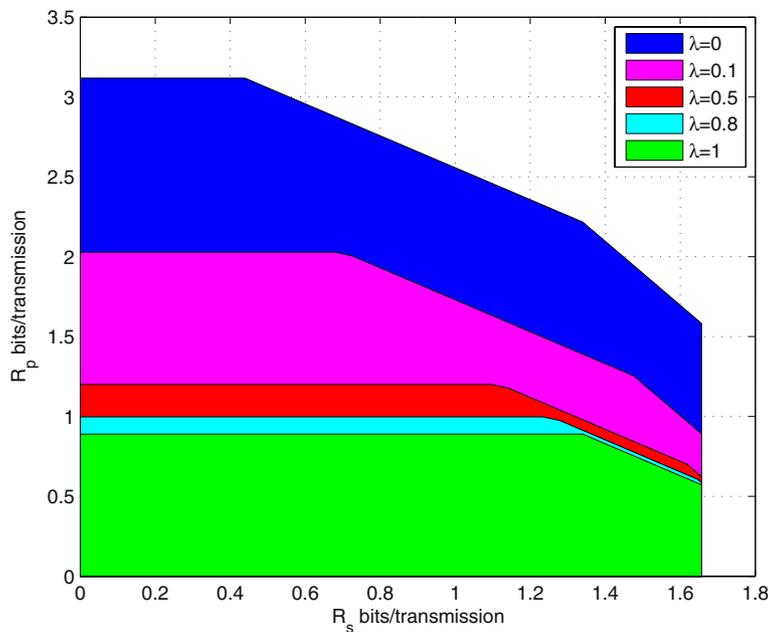


Figure 9 Overall achievable rate region \mathcal{R}_{1g}^r is obtained when the whole secondary signal is decodable by the primary receiver. \mathcal{R}_{1g}^r is shown in blue.

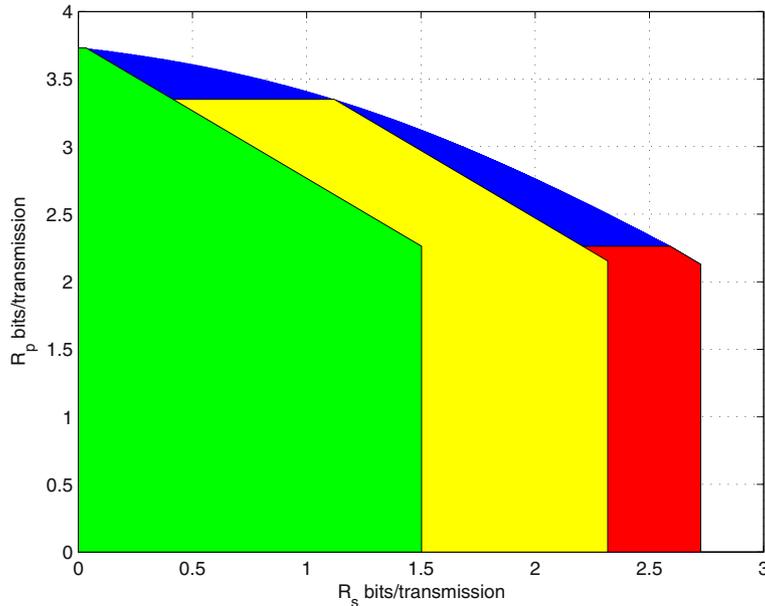


Figure 10 Rate splitting affects the achievable rate region. \mathcal{R}_g^r is shown in blue, and $\mathcal{R}_1^r(Z)$ is shown in green for $\lambda = 0$, yellow for $\lambda = 0.1$, and red for $\lambda = 1$.

the given Gaussian channel, the PDC would be as follows: If for all $Z \in \mathcal{G}(P_1, P_2, P_s)$ $I(Y_p; X_i|W) \leq I(Y_s; X_i|UW)$, then $\mathcal{R}_g^o \subseteq \mathcal{R}_{ig}^r$. Equivalently, the following inequality must hold:

$$\tau \left(\frac{g_i^p P_i}{g_s^p \lambda P_s + g_j^p P_j + N_0} \right) \leq \tau \left(\frac{g_i^s P_i}{g_j^s P_j + N_0} \right), \quad \forall \lambda: 0 \leq \lambda \leq 1, j \neq i, i, j \in \{1, 2\}. \quad (100)$$

But since $I(Y_s; X_i|UW)$ does not depend on λ , then a necessary and sufficient condition to have (100) satisfied is

$$\frac{g_i^p}{g_j^p P_j + N_0} \leq \frac{g_i^s}{g_j^s P_j + N_0}, \quad j \neq i, \quad i, j \in \{1, 2\}. \quad (101)$$

We call inequality (101) primary decodability condition for Gaussian channel (PDCG).

Figure 11 shows a scenario for which three rate regions are obtained: \mathcal{R}_g^o , \mathcal{R}_{1g}^r , and \mathcal{R}_{2g}^r . It is clear that $\mathcal{R}_g^o \subseteq \mathcal{R}_{1g}^r$ meaning that primary user 1 satisfies the PDCG described in (101), whereas primary user 2 does not. By decoding the signal of primary user 1 at the secondary receiver, the range of achievable primary rates in \mathcal{R}_g^o remains the same for \mathcal{R}_{1g}^r while the secondary link can achieve higher rate at a given primary rate in \mathcal{R}_{1g}^r than in \mathcal{R}_g^o . The power setup used to produce this figure is the same as that of Figure 6, and the channel gains are $g_1^p = 0.3413$, $g_2^p = 10.2047$, $g_1^s = 0.2821$, $g_2^s = 0.3782$, $g_1^p = 0.2495$, and $g_s^s = 6.3337$.

Note that a primary user that satisfies PDCG does not always exist, so we evaluate the probability of PDCG as

the probability of finding at least one primary user satisfying (101). We assume $N_0 = 1$ unit power and g_1^s and g_2^s are i.i.d. exponentially distributed with mean μ_s , whereas g_1^p and g_2^p are i.i.d. exponentially distributed with mean μ_p , where g_1^s , g_2^s , g_1^p , and g_2^p are mutually independent. A closed form formula for the probability of PDCG is difficult to obtain, so we evaluate it numerically by generating 10^7 different values for each channel gain element and calculating the average number of times at which neither primary user satisfies (101) at a given P_1 and P_2 ; then by subtracting it from 1, we get a numerical estimate for the probability of PDCG. A simulation has been done in which we assume that $\frac{P_1}{N_0} = \frac{P_2}{N_0} = SNR_p$. We vary SNR_p and evaluate the corresponding probability of PDCG. This simulation is done for the following pairs of (μ_p, μ_s) : (1, 1), (1, 5), (5, 1), and (5, 5). The result is shown in Figure 12, where it is obvious that the probability of PDCG increases with SNR_p and that the increase in μ_s yields more increase in probability of PDCG. The monotonic increase of such probability with SNR_p can be seen by explicitly expressing the probability of event (101) as $P \left(\frac{g_j^s SNR_p + 1}{g_j^p SNR_p + 1} \leq \frac{g_i^s}{g_i^p}, \text{ for some } i, j \in \{1, 2\} \right)$, which is essentially monotonically increasing in SNR_p and approaches 1 as SNR_p goes to ∞ . While it is hard to mathematically show the dependence of the probability of PDCG on μ_s and μ_p , we can justify the increase of such probability with μ_s relative to μ_p because it statistically implies higher quality of the channel to the secondary receiver than that to the primary, hence more chances of (101).

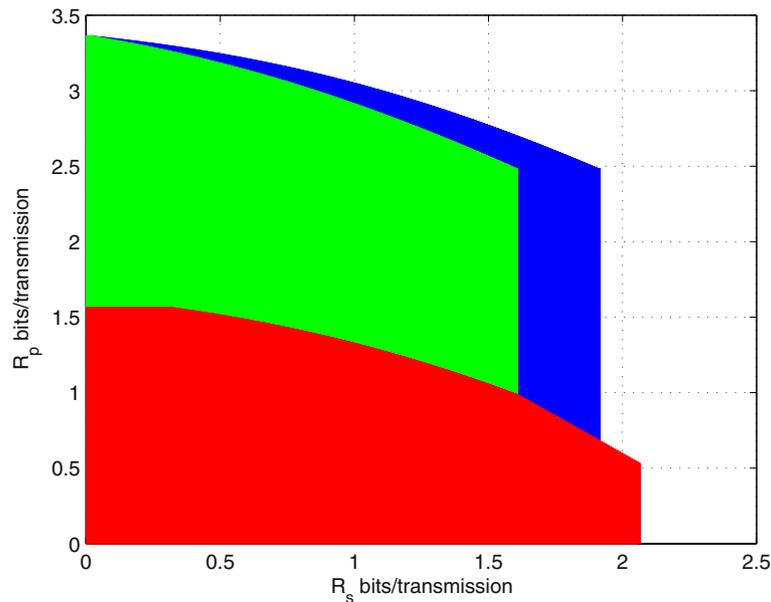


Figure 11 Achievable rate regions for the Gaussian channel. \mathcal{R}_g^o is shown in green, \mathcal{R}_{1g}^f in blue, and \mathcal{R}_{2g}^f in red.

5 Conclusions

In this work, we have analyzed an achievable rate region for a primary multiple access network coexisting with a secondary link that comprises one transmitter and a corresponding receiver. The achievable rate regions depict the sum primary rate versus the secondary rate. We have considered DMC where the secondary link employs rate splitting and investigated two types of achievable rate regions: the first is when the secondary receiver treats the

primary signal as noise, whereas the second is when the secondary is able to decode the signal of only one primary transmitter. An overall achievable rate region is the union of those two regions. Moreover, we have shown that there exists a case for which allowing the secondary receiver to decode a primary signal results in an achievable rate region that includes the achievable rate region obtained when the secondary receiver does not decode the primary signal. Subsequently, we have investigated

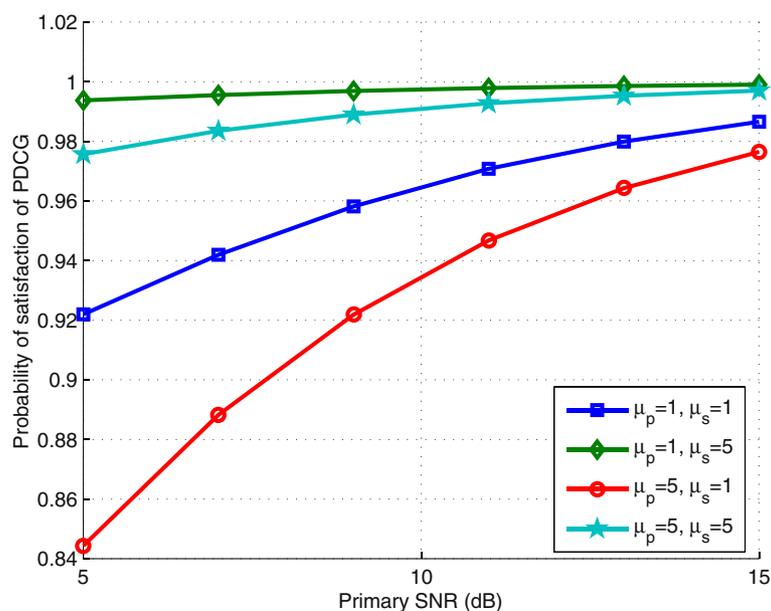


Figure 12 Probability of finding at least one primary user that satisfies the PDCG.

the performance of rate splitting in the Gaussian channel where it has been found that rate splitting by the secondary user is *useless* when the channel between the secondary transmitter and the primary receiver supports larger rate than the channel between the two secondary nodes. Furthermore, on decoding the signal of a primary transmitter at the secondary receiver, a necessary and sufficient condition has been provided to allow the secondary user to decode the primary signal without reducing the range of achievable primary sum rates; in fact, it can only increase the range of achievable secondary rates. Finally, we have shown numerically that the probability of finding at least one primary user that satisfies this condition increases with the signal-to-noise ratio of the primary users.

Appendix 1 - Proof of Theorem 1

It is sufficient to show that there exists at least one code for which if the rate tuple (R_1, R_2, S, T) satisfies (13) to (22), then the rate tuple is achievable. We use the following random code.

Random code generation

A random code \mathcal{C} is generated as follows. Let $\mathbf{q} = (q^{(1)}, \dots, q^{(n)})$ be a random i.i.d sequence of \mathcal{Q}^n , $\mathbf{u}_k = (u_k^{(1)}, \dots, u_k^{(n)})$, $k \in \mathcal{L}_s$ a sequence of random variables of \mathcal{U}^n that are i.i.d given \mathbf{q} . Moreover, \mathbf{u}_k and $\mathbf{u}_{k'}$ are independent $\forall k \neq k', k, k' \in \mathcal{L}_s$. Similarly, generate \mathbf{w}_l , $l \in \mathcal{N}_s$, \mathbf{x}_{1i} , $i \in \mathcal{M}_1$ and \mathbf{x}_{2j} , $j \in \mathcal{M}_2$, which are respectively i.i.d. given \mathbf{q} .

Encoding

For primary user 1 to send a message $i \in \mathcal{M}_1$, it sends \mathbf{x}_{1i} . Similarly, for primary user 2 to send a message $j \in \mathcal{M}_2$, it sends \mathbf{x}_{2j} . For the secondary user to send a message $kl \in \mathcal{L}_s \times \mathcal{N}_s$, it sends $f^n(\mathbf{u}_k \mathbf{w}_l | \mathbf{q}) = (f^{(1)}(u_k^{(1)} w_l^{(1)} | q^{(1)}), \dots, f^{(n)}(u_k^{(n)} w_l^{(n)} | q^{(n)}))$, where \mathbf{q} is known at the transmitters.

Decoding: jointly typical decoding

We use the concept of jointly typical sequences and the properties of typical sets introduced in Chapter 15 of [18] to implement the decoding functions. Let $A_\epsilon^{(n)}$ denote the set of typical $(\mathbf{q}, \mathbf{x}_1, \mathbf{x}_2, \mathbf{w}_l, \mathbf{y}_p)$ sequences, then the primary receiver decides ijl if $(\mathbf{q}, \mathbf{x}_{1i}, \mathbf{x}_{2j}, \mathbf{w}_l, \mathbf{y}_p) \in A_\epsilon^{(n)}$. Also, let $B_\epsilon^{(n)}$ denote the set of typical $(\mathbf{q}, \mathbf{u}, \mathbf{w}, \mathbf{y}_s)$ sequences, then secondary receiver decides kl if $(\mathbf{q}, \mathbf{u}_k, \mathbf{w}_l, \mathbf{y}_s) \in B_\epsilon^{(n)}$.

Probability of error analysis

By the symmetry of the random code generation, the conditional probability of error does not depend on the

transmitted messages. Hence, the conditional probability of error is the same as the average probability of error. So, let $ijkl = 1111$ be sent. An error occurs if the transmitted codewords are not typical with the received sequences.

For the primary receiver

Let the event

$$E_p(ijl) = \left\{ (\mathbf{q}, \mathbf{x}_{1i}, \mathbf{x}_{2j}, \mathbf{w}_l, \mathbf{y}_p) \in A_\epsilon^{(n)} \right\};$$

hence, the probability of error averaged over the random code \mathcal{C} is

$$\bar{P}e_p^o = P\left(E_p^c(111) \cup \bigcup_{ijl \neq 111} E_p(ijl)\right),$$

where $E_p^c(111)$ denotes the complement of $E_p(111)$. Using union bound, we have

$$\begin{aligned} \bar{P}e_p^o &\leq P\left(E_p^c(111)\right) + P\left(\bigcup_{ijl \neq 111} E_p(ijl)\right) \\ &\leq P\left(E_p^c(111)\right) + (M_1 - 1) P\left(E_p(211)\right) \\ &\quad + (M_2 - 1) P\left(E_p(121)\right) + (N_s - 1) P\left(E_p(112)\right) \\ &\quad + (M_1 - 1)(M_2 - 1) P\left(E_p(221)\right) \\ &\quad + (M_1 - 1)(N_s - 1) P\left(E_p(212)\right) \\ &\quad + (M_2 - 1)(N_s - 1) P\left(E_p(122)\right) \\ &\quad + (M_1 - 1)(M_2 - 1)(N_s - 1) P\left(E_p(222)\right). \end{aligned}$$

From the properties of jointly typical sequences [18], $P\left(E_p^c(111)\right) \rightarrow 0$ as $n \rightarrow \infty$, and

$$\begin{aligned} P\left(E_p(211)\right) &= 2^{-n(H(X_1|Q) - H(X_1|X_2 W Y_p Q)) + 6\epsilon} \\ &= 2^{-n(I(X_1; X_2 W Y_p | Q)) + 6\epsilon} \\ &= 2^{-n(I(Y_p; X_1 | W X_2 Q)) + 6\epsilon}, \end{aligned}$$

where the last equality holds from the assumption that X_1, X_2, U , and W are conditionally independent given Q . Similarly, for other $E_p(ijl \neq 111)$ and applying Equations (6) to (8), we get

$$\begin{aligned} \bar{P}e_p^o &\leq 2^{-n(I(Y_p; X_1 | W X_2 Q) - R_1 + \eta - 6\epsilon)} \\ &\quad + 2^{-n(I(Y_p; X_2 | W X_1 Q) - R_2 + \eta - 6\epsilon)} \\ &\quad + 2^{-n(I(Y_p; W | X_1 X_2 Q) - T + \eta - 6\epsilon)} \\ &\quad + 2^{-n(I(Y_p; X_1 X_2 | W Q) - (R_1 + R_2) + \eta - 6\epsilon)} \\ &\quad + 2^{-n(I(Y_p; W X_1 | X_2 Q) - (T + R_1) + \eta - 6\epsilon)} \\ &\quad + 2^{-n(I(Y_p; W X_2 | X_1 Q) - (T + R_2) + \eta - 6\epsilon)} \\ &\quad + 2^{-n(I(Y_p; X_1 X_2 W | Q) - (T + R_1 + R_2) + \eta - 6\epsilon)}. \end{aligned}$$

Thus, if (13) to (19) are satisfied, $\bar{P}e_p^o \rightarrow \epsilon$ as $n \rightarrow \infty$.

For the secondary receiver

Let the event

$$E_s(kl) = \left\{ (\mathbf{q}, \mathbf{u}_k, \mathbf{w}_l, \mathbf{y}_s) \in B_\epsilon^{(n)} \right\};$$

hence, the probability of decoding error averaged over the random code \mathcal{C} is

$$\bar{P}e_s^o = P\left(E_s^c(11) \bigcup \bigcup_{kl \neq 11} E_p(kl)\right),$$

where $E_s^c(11)$ denotes the complement of $E_s(11)$. Using union bound, we have

$$\begin{aligned} \bar{P}e_s^o &\leq P(E_s^c(11)) + (L_s - 1)P(E_s(21)) \\ &\quad + (N_s - 1)P(E_s(12)) \\ &\quad + (L_s - 1)(N_s - 1)P(E_s(22)). \end{aligned}$$

Since $P(E_s^c(11)) \rightarrow \epsilon$ as $n \rightarrow \infty$, then

$$\begin{aligned} \bar{P}e_s^o &\leq 2^{-n(I(Y_s; U|WQ) - S + \eta - 6\epsilon)} \\ &\quad + 2^{-n(I(Y_s; W|UQ) - T + \eta - 6\epsilon)} \\ &\quad + 2^{-n(I(Y_s; UW|Q) - (S+T) + \eta - 6\epsilon)}. \end{aligned}$$

So, if (20) to (22) are satisfied, $\bar{P}e_s^o \rightarrow \epsilon$ as $n \rightarrow \infty$. This concludes the proof.

Appendix 2 - Proof of Theorem 5

Sufficiency part

Suppose (82) is satisfied, we use Figure 5 to prove that $\mathcal{R}^o(Z) \subseteq \mathcal{R}_i^r(Z)$. It is sufficient to show that $R_p^{A^o} = R_p^{A^r}$, $R_s^{B^o} \leq R_s^{B^r}$, $R_s^{D^o} \leq R_s^{E^r}$ and that lines $2R_s + R_p = \rho_{2p}^r$ and $R_s + R_p = \rho_{sp}^o$ intersect at a point (R_s^*, R_p^*) for which $R_s^* \geq R_s^{D^o}$, i.e., the intersection between the two lines is outside $\mathcal{R}^o(Z)$. Consider that the primary user whose signal is not decodable at the secondary receiver is indexed by j , $j \in \{1, 2\}$ and $i \neq j$.

Proof of $R_p^{A^o} = R_p^{A^r}$

From the analysis of the channels C_{RS} and C_{RS}^p in Section 3, we have

$$\begin{aligned} R_p^{A^o} &= I(Y_p; X_1 X_2 | WQ), \\ R_p^{A^r} &= I(Y_p; X_j | WX_i Q) + \sigma_p^*. \end{aligned}$$

From (82), $\sigma_p^* = I(Y_p, X_i | WQ)$. Therefore,

$$R_p^{A^r} = I(Y_p; X_1 X_2 | WQ) = R_p^{A^o}.$$

Proof of $R_s^{B^o} \leq R_s^{B^r}$

From the proof of Theorem 2,

$$R_s^{B^o} = I(Y_s; U|WQ) + \min \left\{ \overbrace{I(Y_p; W|Q)}^{o_1}, \overbrace{I(Y_s; W|Q)}^{o_2} \right\}, \quad (102)$$

and from the proof of Theorem 4,

$$\begin{aligned} R_s^{B^r} &= I(Y_s; U|WX_i Q) - [I(Y_p; X_i | WQ) \\ &\quad - I(Y_s; X_i | WQ)]^+ + \min \{I(Y_p; W|Q), \\ &\quad I(Y_s; W|Q) + [I(Y_s; X_i | WQ) - I(Y_p; X_i | WQ)]^+, \\ &\quad I(Y_s; W|X_i Q)\}. \end{aligned}$$

Case I: If $I(Y_p; X_i | WQ) \leq I(Y_s; X_i | WQ)$

$$\begin{aligned} R_s^{B^r} &= I(Y_s; U|WX_i Q) + \min \left\{ \overbrace{I(Y_p; W|Q)}^{v_1}, \right. \\ &\quad \left. \overbrace{I(Y_s; W|Q) + I(Y_s; X_i | WQ) - I(Y_p; X_i | WQ)}^{v_2}, \right. \\ &\quad \left. \overbrace{I(Y_s; W|X_i Q)}^{v_3} \right\}. \end{aligned} \quad (103)$$

Note that, $v_1 = o_1$.

- If $o_1 \leq o_2$ in (102)

$$\begin{aligned} R_s^{B^o} &= I(Y_s; U|WQ) + I(Y_p; W|Q), \\ R_s^{B^r} &= I(Y_s; U|WX_i Q) + I(Y_p; W|Q) \\ &\geq R_s^{B^o}. \end{aligned}$$

- If $o_2 \leq o_1$ in (102)

$$\begin{aligned} R_s^{B^o} &= I(Y_s; U|WQ) + I(Y_s; W|Q) \\ &= I(Y_s; UW|Q). \end{aligned}$$

When $v_1 = \min\{v_1, v_2, v_3\}$ in (103), then

$$\begin{aligned} R_s^{B^r} &= I(Y_s; U|WX_i Q) + \overbrace{I(Y_p; W|Q)}^{\geq o_2} \\ &\geq R_s^{B^o}. \end{aligned}$$

When $v_2 = \min\{v_1, v_2, v_3\}$ in (103), then

$$\begin{aligned} R_s^{B^r} &= I(Y_s; U|WX_i Q) + I(Y_s; W|Q) + I(Y_s; X_i | WQ) \\ &\quad - I(Y_p; X_i | WQ) \\ &\geq I(Y_s; U|WX_i Q) + I(Y_s; W|Q) \\ &\geq R_s^{B^o}. \end{aligned}$$

When $v_3 = \min\{v_1, v_2, v_3\}$ in (103), then

$$\begin{aligned} R_s^{B^r} &= I(Y_s; U|WX_i Q) + I(Y_s; W|X_i Q) \\ &= I(Y_s; UW|X_i Q) \\ &\geq R_s^{B^o}. \end{aligned}$$

Case II: If $I(Y_S; X_i|WQ) \leq I(Y_p; X_i|WQ)$

$$R_S^{Br} = I(Y_S; U|WX_iQ) + I(Y_S; X_i|WQ) - I(Y_p; X_i|WQ) + \min \left\{ \overbrace{I(Y_p; W|Q)}^{\nu_4}, \overbrace{I(Y_S; W|Q)}^{\nu_5} \right\}. \quad (104)$$

Note that $o_1 = \nu_4$ and $o_2 = \nu_5$.

- If $o_1 \leq o_2$ in (102)

$$R_S^{Bo} = I(Y_S; U|WQ) + I(Y_p; W|Q),$$

$$\begin{aligned} R_S^{Br} &= I(Y_S; UX_i|WQ) - I(Y_p; X_i|WQ) + I(Y_p; W|Q) \\ &= I(Y_S; U|WQ) + I(Y_p; W|Q) \\ &\quad \geq 0 \text{ from (82)} \\ &\quad + \overbrace{I(Y_S; X_i|UWQ) - I(Y_p; X_i|WQ)} \\ &\geq R_S^{Bo}. \end{aligned}$$

- If $o_2 \leq o_1$ in (102)

The proof follows exactly as the case of $o_1 \leq o_2$.

Proof of $R_S^{Fr} \geq R_S^{Do}$

$$R_S^{Fr} = I(Y_S; U|WX_iQ) + \min\{I(Y_S; W|X_iQ), I(Y_p; W|X_1X_2Q)\}.$$

$$R_S^{Do} = I(Y_S; U|WQ) + \min\{I(Y_S; W|Q), I(Y_p; W|X_1X_2Q)\}.$$

It is obvious that each term in R_S^{Fr} is greater than or equal to its corresponding term in R_S^{Do} . Hence, $R_S^{Fr} \geq R_S^{Do}$.

Proof of the intersection point between the two lines

$2R_s + R_p = \rho_{2p}^r$ and $R_s + R_p = \rho_{sp}^o$ occurs at a point

(R_s^*, R_p^*) where $R_s^* \geq R_s^{Do}$

The secondary rate of the intersection point is $R_s^* = \rho_{2p}^r - \rho_{sp}^o$. From Theorems 2 and 4,

$$R_S^{Do} = I(Y_S; U|WQ) + \sigma^*, \quad (105)$$

$$\begin{aligned} R_S^* &= 2I(Y_S; U|WX_iQ) + 2\sigma_s^* + I(Y_p; X_j|WX_iQ) \\ &\quad - [\sigma_s^* - I(Y_p; W|X_iQ)]^+ + \min \left\{ I(Y_S; X_i|WQ), \right. \\ &\quad I(Y_S; WX_i|Q) - \sigma_s^*, I(Y_p; X_i|Q) + [I(Y_p; W|X_iQ) \\ &\quad \left. - \sigma_s^*]^+, I(Y_p; X_i|WQ) \right\} - I(Y_p; X_1X_2|WQ) \\ &\quad - I(Y_S; U|WQ) - \min \{I(Y_S; W|Q), I(Y_p; W|Q)\}. \end{aligned} \quad (106)$$

Hence, it is required to show that $R_S^* \geq R_S^{Do}$.

If $\sigma_s^* = I(Y_S; W|X_iQ) \leq I(Y_p; W|X_1X_2Q)$

- If $I(Y_S; W|X_iQ) \leq I(Y_p; W|X_iQ)$, from (105) and (106), we have

$$R_S^{Do} = I(Y_S; U|WQ) + I(Y_S; W|Q) = I(Y_S; UW|Q), \quad (107)$$

$$\begin{aligned} R_S^* &= 2I(Y_S; U|WX_iQ) + 2I(Y_S; W|X_iQ) - I(Y_S; U|WQ) \\ &\quad + I(Y_p; X_j|WX_iQ) + \min \left\{ \overbrace{I(Y_S; X_i|Q)}^{\nu_6}, \right. \\ &\quad \overbrace{I(Y_p; X_i|Q) + I(Y_p; W|X_iQ) - I(Y_S; W|X_iQ)}^{\nu_7}, \\ &\quad \left. \overbrace{I(Y_p; X_i|WQ)}^{\nu_8} \right\} - I(Y_p; X_1X_2|WQ) - \min\{\nu_4, \nu_5\}. \end{aligned} \quad (108)$$

When $\nu_6 = \min\{\nu_6, \nu_7, \nu_8\}$ in (108), then

$$\begin{aligned} R_S^* &= I(Y_S; U|WX_iQ) - I(Y_S; U|WQ) + \overbrace{I(Y_S; UW|Q)}^{=R_S^{Do}} \\ &\quad + I(Y_S; W|X_iQ) - \min\{\nu_4, \nu_5\} + I(Y_p; X_j|WX_iQ) \\ &\quad \geq I(Y_p; X_i|WQ) \text{ from (82)} \\ &\quad + \overbrace{I(Y_S; X_i|UWQ)} - I(Y_p; X_1X_2|WQ) \\ &\geq R_S^{Do}. \end{aligned}$$

When $\nu_7 = \min\{\nu_6, \nu_7, \nu_8\}$ in (108), then

$$\begin{aligned} R_S^* &= I(Y_S; U|WX_iQ) - I(Y_S; U|WQ) + \overbrace{I(Y_S; UW|X_iQ)}^{\geq R_S^{Do}} \\ &\quad + \nu_4 - \min\{\nu_4, \nu_5\} \\ &\geq R_S^{Do}. \end{aligned}$$

When $\nu_8 = \min\{\nu_6, \nu_7, \nu_8\}$ in (108), then

$$\begin{aligned} R_S^* &= I(Y_S; U|WX_iQ) - I(Y_S; U|WQ) + \overbrace{I(Y_S; UW|X_iQ)}^{\geq R_S^{Do}} \\ &\quad + I(Y_S; W|X_iQ) - \min\{\nu_4, \nu_5\} \\ &\geq R_S^{Do}. \end{aligned}$$

- If $I(Y_S; W|X_iQ) \geq I(Y_p; W|X_iQ)$,

R_S^{Do} will remain the same as (107) and R_S^* will be given by

$$\begin{aligned} R_S^* &= 2I(Y_S; U|WX_iQ) + 2I(Y_S; W|X_iQ) - I(Y_S; U|WQ) \\ &\quad + I(Y_p; X_j|WX_iQ) + I(Y_p; W|X_iQ) \\ &\quad - I(Y_S; W|X_iQ) + \min \left\{ \overbrace{I(Y_S; X_i|Q)}^{\nu_9}, \overbrace{I(Y_p; X_i|Q)}^{\nu_{10}} \right\} \\ &\quad - I(Y_p; X_1X_2|WQ) - \min\{\nu_4, \nu_5\}. \end{aligned} \quad (109)$$

When $v_9 = \min\{v_9, v_{10}\}$ in (109), then

$$\begin{aligned} R_s^* &= I(Y_s; U|WX_iQ) - I(Y_s; U|WQ) + \overbrace{I(Y_s; UW|Q)}^{=R_s^{D^o}} \\ &\quad + I(Y_p; W|X_iQ) - \min\{v_4, v_5\} + I(Y_s; X_i|UWQ) \\ &\quad + I(Y_p; X_j|WX_iQ) - I(Y_p; X_1X_2|WQ) \\ &\geq R_s^{D^o}. \end{aligned}$$

When $v_{10} = \min\{v_9, v_{10}\}$ in (109), then

$$\begin{aligned} R_s^* &= I(Y_s; U|WX_iQ) - I(Y_s; U|WQ) + \overbrace{I(Y_s; UW|X_iQ)}^{\geq R_s^{D^o}} \\ &\quad + I(Y_p; W|X_iQ) - \min\{v_4, v_5\} \\ &\geq R_s^{D^o} \end{aligned}$$

If $\sigma_s^* = I(Y_p; W|X_1X_2Q) \leq I(Y_s; W|X_iQ)$ from (105) and (106), we have

$$R_s^{D^o} = I(Y_s; U|WQ) + \min \left\{ \overbrace{I(Y_s; W|Q)}^{o_2}, \overbrace{I(Y_p; W|X_1X_2Q)}^{o_3} \right\}, \quad (110)$$

$$\begin{aligned} R_s^* &= 2I(Y_s; U|WX_iQ) - I(Y_s; U|WQ) + I(Y_p; W|X_iQ) \\ &\quad + I(Y_p; X_j|WX_iQ) + I(Y_p; W|X_1X_2Q) \\ &\quad + \min \left\{ \overbrace{I(Y_p; X_i|Q)}^{v_{10}}, \overbrace{I(Y_s; X_i|WQ)}^{v_{11}}, \right. \\ &\quad \left. \overbrace{I(Y_s; WX_i|Q) - I(Y_p; W|X_1X_2Q)}^{v_{12}} \right\} - \min\{v_4, v_5\} \\ &\quad - I(Y_p; X_1X_2|WQ) \end{aligned} \quad (111)$$

- If $o_2 \leq o_3$ in (110),

$$R_s^{D^o} = I(Y_s; U|WQ) + I(Y_s; W|Q) = I(Y_s; UW|Q).$$

When $v_{10} = \min\{v_{10}, v_{11}, v_{12}\}$ in (111), then

$$\begin{aligned} R_s^* &= I(Y_s; U|WX_iQ) - I(Y_s; U|WQ) + v_4 \\ &\quad - \min\{v_4, v_5\} + \overbrace{I(Y_s; UW|X_iQ)}^{\geq R_s^{D^o}} + o_3. \end{aligned}$$

Since $o_2 \leq o_3$, then v_{11} cannot be smaller than v_{12} .

When $v_{12} = \min\{v_{10}, v_{11}, v_{12}\}$, then

$$\begin{aligned} R_s^* &= I(Y_s; U|WX_iQ) - I(Y_s; U|WQ) + I(Y_p; W|X_iQ) \\ &\quad - \min\{v_4, v_5\} + \overbrace{I(Y_s; UW|Q)}^{=R_s^{D^o}} + I(Y_s; X_i|UWQ) \\ &\quad + I(Y_p; X_j|WX_iQ) - I(Y_p; X_1X_2|WQ) \\ &\geq R_s^{D^o}. \end{aligned}$$

- If $o_2 \geq o_3$

$$R_s^{D^o} = I(Y_s; U|WQ) + I(Y_p; W|X_1X_2Q).$$

When $v_{10} = \min\{v_{10}, v_{11}, v_{12}\}$ in (111), then

$$\begin{aligned} R_s^* &= I(Y_s; U|WX_iQ) - I(Y_s; U|WQ) \\ &\quad + \overbrace{I(Y_s; UW|X_iQ) + I(Y_p; W|X_1X_2Q)}^{\geq R_s^{D^o}} \\ &\geq R_s^{D^o}. \end{aligned}$$

When $v_{11} = \min\{v_{10}, v_{11}, v_{12}\}$ in (111), then

$$\begin{aligned} R_s^* &= I(Y_s; U|WX_iQ) - I(Y_s; U|WQ) + I(Y_s; X_i|UWQ) \\ &\quad + I(Y_p; X_j|WX_iQ) - I(Y_p; X_1X_2|WQ) - v_4 \\ &\quad + I(Y_p; W|X_iQ) + \overbrace{I(Y_s; UW|Q) + I(Y_p; W|X_1X_2Q)}^{=R_s^{D^o}} \\ &\geq R_s^{D^o}. \end{aligned}$$

Since $o_2 \geq o_3$, then v_{12} cannot be smaller than v_{11} .

Necessity part

Suppose $\mathcal{R}^o(Z) \subseteq \mathcal{R}_i^r(Z)$ then $R_p^{A^o}$ must be not larger than $R_p^{A^r}$ which necessitates the satisfaction of (82).

This concludes the proof.

Appendix 3 - Proof of Theorem 6

From the definition of $\delta^o(Z)$ and $\delta_1^r(Z)$, it is clear that $\delta^o(Z) \subseteq \delta^o(Z)$ and $\delta_1^r(Z) \subseteq \delta_1^r(Z)$. Consequently, $\mathcal{R}^o(Z) \subseteq \mathcal{R}^o(Z)$, $\mathcal{R}_1^r \subseteq \mathcal{R}_1^r(Z)$, and $\mathcal{R}_1^r(Z) \subseteq \mathcal{R}_1^r(Z)$. However, we show that if there exists $Z \in \mathcal{P}^*$ such that a rate tuple (R_s, R_p) belongs to $\mathcal{R}^o(Z)$ but does not belong to $\mathcal{R}^o(Z)$, then there exists another $Z' \in \mathcal{P}^*$ for which (R_s, R_p) belongs to $\mathcal{R}^o(Z')$. And we do the same for $\mathcal{R}_1^r(Z)$, $\mathcal{R}_1^r(Z)$.

For $\mathcal{R}^o(Z)$

Following a similar procedure to that used in the proof of Theorem 2, the region $\mathcal{R}^o(Z)$ is defined by

$$R_p \leq I(Y_p; X_1X_2|WQ), \quad (112)$$

$$\begin{aligned} R_s &\leq I(Y_s; U|WQ) + \min\{I(Y_s; W|Q), \\ &\quad I(Y_p; WX_1|X_2Q), I(Y_p; WX_2|X_1Q)\}, \end{aligned} \quad (113)$$

$$\begin{aligned} R_s + R_p &\leq I(Y_s; U|WQ) + I(Y_p; X_1X_2|WQ) \\ &\quad + \min\{I(Y_s; W|Q), I(Y_p; W|Q)\}. \end{aligned} \quad (114)$$

Suppose that at a certain $Z \in \mathcal{P}^*$, $R_s' > I(Y_s; U|WQ) + I(Y_p; W|X_1X_2Q)$, hence, the rate tuple $(R_s', R_p') \in \mathcal{R}^o(Z)$ but $(R_s', R_p') \notin \mathcal{R}^o(Z)$. From (112) to (114), (R_s', R_p') has to satisfy

$$R_s \leq I(Y_s; UW|Q) = I(Y_s; X_s|Q), \quad (115)$$

$$R_p < I(Y_p; X_1X_2|Q). \quad (116)$$

Now, assume another $Z' \in \mathcal{P}^*$ such that $W = \phi$, i.e., no rate splitting. At this Z' , $\mathcal{R}^o(Z')$ is given by

$$R_s \leq I(Y_s; X_s|Q), \quad (117)$$

$$R_p \leq I(Y_p; X_1 X_2|Q). \quad (118)$$

Then it is clear that $(R'_s, R'_p) \in \mathcal{R}^o(Z')$. Thus,

$$\mathcal{R}'^o(Z) \subseteq \mathcal{R}^o(Z) \cup \mathcal{R}^o(Z').$$

For $\mathcal{R}'^r_1(Z)$

First, for a point (R''_s, R''_p) such that $R''_s > I(Y_s; U|WQ) + I(Y_p; W|X_1 X_2 Q)$ at a specific $Z \in \mathcal{P}^*$, a similar argument as in the above subsection ('For $\mathcal{R}^o(Z)$ '), or in Lemma 2 of [19], can show that there exists $Z'' \in \mathcal{P}^*$ such that $(R''_s, R''_p) \in \mathcal{R}'^r_1(Z'')$.

Second, for another point (R^{**}_s, R^{**}_p) such that $R^{**}_p > I(Y_p; X_2|WX_1 Q) + I(Y_s; X_1|UWQ)$, or in other words $R^{**}_1 > I(Y_s; X_1|UWQ)$, in this case, $\delta'^r_1(Z) \subset \delta'^o(Z)$. And since $\mathcal{R}'^o(Z)$ is the set of (R_s, R_p) corresponding to $\delta'^o(Z)$ for which $R_s = S + T$ and $R_p = R_1 + R_2$, then $\mathcal{R}'^r_1(Z) \subset \mathcal{R}'^o(Z)$. Moreover, it has been shown in the above subsection ('For $\mathcal{R}^o(Z)$ ') that $\mathcal{R}'^o(Z) \subseteq \mathcal{R}^o(Z) \cup \mathcal{R}^o(Z')$. Therefore,

$$\mathcal{R}'^r_1(Z) \subseteq \mathcal{R}'^r_1(Z) \cup \mathcal{R}'^r_1(Z'') \cup \mathcal{R}^o(Z) \cup \mathcal{R}^o(Z').$$

Consequently,

$$\mathcal{R}'_1 = \mathcal{R}_1.$$

Appendix 4 - Proof of Theorem 7

Sufficiency part

We refer to Figure 3 to determine the effect of varying λ on $\mathcal{R}^o(Z)$ where $Z \in \mathcal{G}(P_1, P_2, P_s)$.

- Point A:

$$R_p^A = \rho_p^o = \tau \left(\frac{g_1^p P_1 + g_2^p P_2}{g_s^p \lambda P_s + N_0} \right)$$

- Point D:

$$R_s^D = \rho_s^o = \tau \left(\frac{P_s g_s^s}{g_1^s P_1 + g_2^s P_2 + N_0} \right)$$

- $R_s + R_p$: We can move $\rho_p^o + I(Y_s; U|W)$ inside the $\min\{\}$ to have

$$\rho_{sp}^o = \min \left\{ \tau \left(\frac{g_1^p P_1 + g_2^p P_2}{g_s^p \lambda P_s + N_0} \right) + \tau \left(\frac{g_s^s P_s}{P_1 g_1^s + P_2 g_2^s + N_0} \right), \right. \\ \left. \tau \left(\frac{\lambda P_s g_s^s}{g_1^s P_1 + g_2^s P_2 + N_0} \right) + \tau \left(\frac{g_1^p P_1 + g_2^p P_2 + \lambda g_s^p P_s}{\lambda g_s^p P_s} \right) \right\} +.$$

Clearly, the first argument of the $\min\{\}$ is monotonically decreasing with λ . By taking the first derivative of the second argument with respect to λ , it turns out that such

second argument is also monotonically decreasing with λ if $g_s^s N_0 \leq g_s^p (g_1^s P_1 + g_2^s P_2 + N_0)$. It is therefore obvious that if (97) is satisfied, ρ_{sp}^o increase as λ decreases. Consequently, $\mathcal{R}^o(Z)$ at $\lambda = 0$ includes all other $\mathcal{R}^o(Z)$ obtained at $0 < \lambda \leq 1$. Hence, $\mathcal{R}^o(Z)$ coincides on \mathcal{R}_g^o at $\lambda = 0$.

Necessity part

Here we prove that the condition in (97) is necessary for $\mathcal{R}^o(Z)$ to coincide on \mathcal{R}_g^o at $\lambda = 0$ and $Z \in \mathcal{G}(P_1, P_2, P_s)$. We do so by showing that if (97) is not satisfied, then for any two different values of λ , the corresponding rate regions do not contain one another. Assume that (97) is not satisfied, then by referring to Figure 3 we have

- Point A:

$$R_p^A = \tau \left(\frac{g_1^p P_1 + g_2^p P_2}{g_s^p \lambda P_s + N_0} \right)$$

i.e., the R_p^A decreases as λ increases.

- Point D:

$$R_s^D = \tau \left(\frac{g_s^s \lambda P_s}{g_1^s P_1 + g_2^s P_2 + N_0} \right) + \tau \left(\frac{g_s^p \bar{\lambda} P_s}{g_s^p \lambda P_s + N_0} \right)$$

Then by substituting with $\bar{\lambda} = 1 - \lambda$ and differentiating R_s^D with respect to λ , we get

$$\frac{\partial R_s^D}{\partial \lambda} = \frac{\frac{1}{2 \ln(2)} P_s (g_s^s N_0 - g_s^p (P_1 g_1^s + P_2 g_2^s + N_0))}{(\lambda P_s g_s^p + N_0) (P_1 g_1^s + P_2 g_2^s + \lambda P_s g_s^s + N_0)}, \quad (119)$$

and since the condition (97) is not satisfied, the numerator of (119) is always positive; therefore, R_s^D increases as λ increases.

Since R_p^A decreases and R_s^D increases as λ increases, then for any two different values of λ the corresponding rate regions will never contain one another. Hence, the overall rate region \mathcal{R}_g^o does not coincide on a certain $\mathcal{R}^o(Z)$ at a certain λ . This concludes the proof.

Appendix 5 - Proof of Theorem 8

For the proof, we consider $i = 1$, i.e., the secondary user is assumed to be able to decode the signal of primary user 1.

Sufficiency part

In this part, we show that if inequality (99) is satisfied, then \mathcal{R}'_{1g} coincides on $\mathcal{R}'_1(Z)$ at $\lambda = 0$. We refer to Figure 4 and determine the effect of varying λ on $\mathcal{R}'_1(Z)$, $Z \in \mathcal{G}(P_1, P_2, P_s)$ as follows.

At point A

$$R_p^{rA} = \tau \left(\frac{g_2^p P_2}{g_s^p \lambda P_s + N_0} \right) + \min \left\{ \tau \left(\frac{g_1^s P_1}{g_2^s P_2 + N_0} \right), \tau \left(\frac{g_1^p P_1}{g_s^p \lambda P_s + g_2^p P_2 + N_0} \right) \right\}.$$

Therefore, R_p^{rA} increases as λ decreases.

At point F

$$R_s^{rF} = \tau \left(\frac{g_s^s P_s}{g_2^s P_2 + N_0} \right).$$

Hence, R_s^{rF} does not depend on λ .

$$R_s^r + R_p^r = \rho_{sp}^r$$

$$\begin{aligned} \rho_{sp}^r &= \tau \left(\frac{g_s^s \lambda P_s}{g_2^s P_2 + N_0} \right) + \tau \left(\frac{g_2^p P_2}{g_s^p \lambda P_s + N_0} \right) \\ &+ \min \left\{ \overbrace{\tau \left(\frac{g_s^p \bar{\lambda} P_s + g_1^p P_1}{g_s^p \lambda P_s + g_2^p P_2 + N_0} \right)}^{\mu_1}, \overbrace{\tau \left(\frac{g_s^s \bar{\lambda} P_s + g_1^s P_1}{g_s^s \lambda P_s + g_2^s P_2 + N_0} \right)}^{\mu_2} \right\} \\ &\underbrace{\tau \left(\frac{g_s^p \bar{\lambda} P_s}{g_s^p \lambda P_s + g_2^p P_2 + N_0} \right) + \tau \left(\frac{g_1^s P_1}{g_s^s \lambda P_s g_2^s + P_2 + N_0} \right)}^{\mu_3}, \\ &\underbrace{\tau \left(\frac{g_1^p P_1}{g_s^p \lambda P_s + g_2^p P_2 + N_0} \right) + \tau \left(\frac{g_s^s \bar{\lambda} P_s}{g_s^s \lambda P_s + g_2^s P_2 + N_0} \right)}^{\mu_4} \end{aligned} \quad (120)$$

When $\mu_1 = \min\{\mu_1, \mu_2, \mu_3, \mu_4\}$ in (120)

$$\rho_{sp}^r = \tau \left(\frac{g_s^s \lambda P_s}{g_2^s P_2 + N_0} \right) + \tau \left(\frac{g_s^p \bar{\lambda} P_s + g_1^p P_1 + g_2^p P_2}{g_s^p \lambda P_s + N_0} \right).$$

$$\frac{\partial \rho_{sp}^r}{\partial \lambda} = - \frac{0.5 P_s (g_s^p g_2^s P_2 + g_s^p N_0 - g_s^s N_0)}{\ln 2 (g_s^p \lambda P_s + N_0) (g_s^s \lambda P_s + g_2^s P_2 + N_0)} \leq 0 \quad \text{from (99)}.$$

Hence, ρ_{sp}^r decreases with λ . Note that, $\bar{\lambda} = 1 - \lambda$.

When $\mu_2 = \min\{\mu_1, \mu_2, \mu_3, \mu_4\}$ in (120)

$$\tau \left(\frac{g_s^s P_s + g_1^s P_1}{g_2^s P_2 + N_0} \right) + \tau \left(\frac{g_2^p P_2}{g_s^p \lambda P_s + N_0} \right),$$

i.e., ρ_{sp}^r decreases with λ .

When $\mu_3 = \min\{\mu_1, \mu_2, \mu_3, \mu_4\}$ in (120)

$$\rho_{sp}^r = \tau \left(\frac{g_s^s \lambda P_s + g_1^s P_1}{g_s^s P_2 + N_0} \right) + \tau \left(\frac{g_s^p \bar{\lambda} P_s + g_2^p P_2}{g_s^p \lambda P_s + N_0} \right).$$

$$\frac{\partial \rho_{sp}^r}{\partial \lambda} = - \frac{0.5 P_s (g_s^p g_2^s P_2 + g_s^p g_1^s P_1 + g_s^p N_0 - g_s^s N_0)}{\ln 2 (g_s^p \lambda P_s + N_0) (g_s^s \lambda P_s + g_2^s P_2 + g_1^s P_1 + N_0)} \leq 0 \quad \text{from (99)}.$$

Thus, ρ_{sp}^r decreases with λ .

When $\mu_4 = \min\{\mu_1, \mu_2, \mu_3, \mu_4\}$ in (120)

$$\rho_{sp}^r = \tau \left(\frac{g_s^s P_s}{g_s^p P_2 + N_0} \right) + \tau \left(\frac{g_1^p P_1 + g_2^p P_2}{g_s^p \lambda P_s + N_0} \right).$$

Therefore, ρ_{sp}^r decreases with λ .

$$R_s^r + 2R_p^r = \rho_{s2}^r$$

$$\begin{aligned} \rho_{s2}^r &= 2\tau \left(\frac{g_2^p P_2}{g_s^p \lambda P_s + N_0} \right) + 2\sigma_p^* + \tau \left(\frac{g_s^s \lambda P_s}{g_2^s P_2 + N_0} \right) \\ &- \left[\sigma_p^* - \tau \left(\frac{g_1^s P_1}{g_s^s \lambda P_s + g_2^s P_2 + N_0} \right) \right]^+ + \min \left\{ \tau \left(\frac{g_s^p \bar{\lambda} P_s}{g_s^p \lambda P_s + g_2^p P_2 + N_0} \right), \tau \left(\frac{g_s^s \bar{\lambda} P_s + g_1^s P_1}{g_s^p \lambda P_s + g_2^p P_2 + N_0} \right) - \sigma_p^*, \tau \left(\frac{g_s^s \bar{\lambda} P_s}{g_s^s \lambda P_s + g_2^s P_2 + N_0} \right), \tau \left(\frac{g_s^s \bar{\lambda} P_s}{g_s^s \lambda P_s + g_1^s P_1 + g_2^p P_2 + N_0} \right) + \left[\tau \left(\frac{g_1^p P_1}{g_s^p \lambda P_s + g_2^p P_2 + N_0} \right) - \sigma_p^* \right]^+ \right\}, \\ \sigma_p^* &= \min \left\{ \tau \left(\frac{g_1^p P_1}{g_s^p \lambda P_s + g_2^p P_2 + N_0} \right), \tau \left(\frac{g_1^s P_1}{g_2^s P_2 + N_0} \right) \right\}. \end{aligned}$$

$$\text{At } \sigma_p^* = \tau \left(\frac{g_1^p P_1}{g_s^p \lambda P_s + g_2^p P_2 + N_0} \right) \leq \tau \left(\frac{g_1^s P_1}{g_2^s P_2 + N_0} \right)$$

$$\begin{aligned} \rho_{s2}^r &= 2\tau \left(\frac{g_1^p P_1 + g_2^p P_2}{g_s^p \lambda P_s + N_0} \right) + \tau \left(\frac{g_s^s \lambda P_s}{g_2^s P_2 + N_0} \right) \\ &- \left[\tau \left(\frac{g_1^p P_1}{g_s^p \lambda P_s + g_2^p P_2 + N_0} \right) - \tau \left(\frac{g_1^s P_1}{g_s^s \lambda P_s + g_2^s P_2 + N_0} \right) \right]^+ \\ &+ \min \left\{ \tau \left(\frac{g_s^p \bar{\lambda} P_s}{g_s^p \lambda P_s + g_1^p P_1 + g_2^p P_2 + N_0} \right), \tau \left(\frac{g_s^s \bar{\lambda} P_s}{g_s^s \lambda P_s + g_1^s P_1 + g_2^p P_2 + N_0} \right) + \left[\tau \left(\frac{g_1^p P_1}{g_s^p \lambda P_s + g_2^p P_2 + N_0} \right) - \tau \left(\frac{g_1^s P_1}{g_s^s \lambda P_s + g_2^s P_2 + N_0} \right) \right]^+, \tau \left(\frac{g_s^s \bar{\lambda} P_s}{g_s^s \lambda P_s + g_2^s P_2 + N_0} \right) \right\}. \end{aligned}$$

- If $\tau \left(\frac{g_1^p P_1}{g_s^p \lambda P_s + g_2^p P_2 + N_0} \right) \leq \tau \left(\frac{g_1^s P_1}{g_s^s \lambda P_s + g_2^s P_2 + N_0} \right)$

$$\rho_{s2}^r = 2\tau \left(\frac{g_1^p P_1 + g_2^p P_2}{g_s^p \lambda P_s + N_0} \right) + \tau \left(\frac{g_s^s \lambda P_s}{g_2^s P_2 + N_0} \right) + \min \left\{ \begin{array}{l} \tau \left(\frac{g_s^p \bar{\lambda} P_s}{g_s^p \lambda P_s + g_1^p P_1 + g_2^p P_2 + N_0} \right), \tau \left(\frac{g_s^s \bar{\lambda} P_s}{g_2^s \lambda P_s + g_2^p P_2 + N_0} \right), \\ \tau \left(\frac{g_s^s \bar{\lambda} P_s + g_1^s P_1}{g_s^s \lambda P_s + g_2^p P_2 + N_0} \right) - \tau \left(\frac{g_1^p P_1}{g_s^p \lambda P_s + g_2^p P_2 + N_0} \right) \end{array} \right\}. \quad (121)$$

When $\mu_5 = \min\{\mu_5, \mu_6, \mu_7\}$ in (121) we have

$$\rho_{s2}^r = \tau \left(\frac{g_s^p \bar{\lambda} P_s + g_1^p P_1 + g_2^p P_2}{g_s^p \lambda P_s + N_0} \right) + \tau \left(\frac{g_s^s \lambda P_s}{g_2^s P_2 + N_0} \right) + \tau \left(\frac{g_1^p P_1 + g_2^p P_2}{g_s^p \lambda P_s + N_0} \right). \quad (122)$$

Note that the third term in (122) is decreasing with λ , and the first derivative of the first two terms with respect to λ is given by

$$-\frac{0.5P_s (g_s^p g_2^s P_2 + g_s^p N_0 - g_s^s N_0)}{\ln 2 (g_s^p \lambda P_s + N_0) (g_s^s \lambda P_s + g_2^s P_2 + N_0)} - \frac{0.5g_s^p P_s (g_2^p P_2 + g_1^p P_1)}{\ln 2 (g_s^p \lambda P_s + N_0) (g_s^p \lambda P_s + g_1^p P_1 + g_2^p P_2 + N_0)}.$$

Since inequality (99) is satisfied for user 1, then the derivative is negative and consequently ρ_{s2}^r is decreasing with λ .

When $\mu_6 = \min\{\mu_5, \mu_6, \mu_7\}$ in (121), we have

$$\rho_{s2}^r = 2\tau \left(\frac{g_1^p P_1 + g_2^p P_2}{g_s^p \lambda P_s + N_0} \right) + \tau \left(\frac{g_s^s P_s}{g_2^s P_2 + N_0} \right),$$

i.e., ρ_{s2}^r is decreasing with λ .

When $\mu_7 = \min\{\mu_5, \mu_6, \mu_7\}$ in (121), we have

$$\rho_{s2}^r = 2\tau \left(\frac{g_1^p P_1 + g_2^p P_2}{g_s^p \lambda P_s + N_0} \right) - \tau \left(\frac{g_1^p P_1}{g_s^p \lambda P_s + g_2^p P_2 + N_0} \right) + \tau \left(\frac{g_s^s P_s + g_1^s P_1}{g_2^s P_2 + N_0} \right).$$

Hence, ρ_{s2}^r is decreasing with λ .

- If $\tau \left(\frac{g_1^s P_1}{g_s^s \lambda P_s + g_2^p P_2 + N_0} \right) \leq \tau \left(\frac{g_1^p P_1}{g_s^p \lambda P_s + g_2^p P_2 + N_0} \right)$

$$\rho_{s2}^r = 2\tau \left(\frac{g_1^p P_1 + g_2^p P_2}{g_s^p \lambda P_s + N_0} \right) - \tau \left(\frac{g_1^p P_1}{g_s^p \lambda P_s + g_2^p P_2 + N_0} \right) + \min \left\{ \begin{array}{l} \tau \left(\frac{g_s^p \bar{\lambda} P_s}{g_s^p \lambda P_s + g_1^p P_1 + g_2^p P_2 + N_0} \right), \\ \tau \left(\frac{g_s^s \bar{\lambda} P_s}{g_s^s \lambda P_s + g_1^p P_1 + g_2^p P_2 + N_0} \right) \end{array} \right\} + \tau \left(\frac{g_s^s \lambda P_s + g_1^s P_1}{g_2^s P_2 + N_0} \right) \quad (123)$$

When $\mu_5 = \min\{\mu_5, \mu_8\}$ in (123), then

$$\rho_{s2}^r = \tau \left(\frac{g_1^p P_1 + g_2^p P_2}{g_s^p \lambda P_s + N_0} \right) - \tau \left(\frac{g_1^p P_1}{g_s^p \lambda P_s + g_2^p P_2 + N_0} \right) + \tau \left(\frac{g_1^p P_1 + g_2^p P_2}{g_s^p \lambda P_s + N_0} \right) + \tau \left(\frac{g_s^s \lambda P_s + g_1^s P_1}{g_2^s P_2 + N_0} \right) + \tau \left(\frac{g_s^p \bar{\lambda} P_s}{g_s^p \lambda P_s + g_1^p P_1 + g_2^p P_2 + N_0} \right). \quad (124)$$

For all values of $0 \leq \lambda \leq 1$, the difference between the first two terms in (124) is always positive and decreasing as λ increases. To see this, we can write such difference as $I(Y_p; X_1 X_2 | W) - I(Y_p, X_1 | W) = I(Y_p; X_2 | W) = \tau \left(\frac{g_2^p P_2}{\lambda g_s^p P_s} \right)$. The first derivative of the last three terms in (124) with respect to λ is given by

$$-\frac{0.5P_s (g_s^p g_2^s P_2 + g_s^p g_1^s P_1 + g_s^p N_0 - g_s^s N_0)}{\ln 2 (g_s^p \lambda P_s + N_0) (g_s^s \lambda P_s + g_2^s P_2 + g_1^s P_1 + N_0)} \leq 0 \quad \text{from (99)}.$$

Therefore, ρ_{s2}^r is decreasing with λ .

When $\mu_8 = \min\{\mu_5, \mu_8\}$ in (123), then

$$\rho_{s2}^r = 2\tau \left(\frac{g_1^p P_1 + g_2^p P_2}{g_s^p \lambda P_s + N_0} \right) - \tau \left(\frac{g_1^p P_1}{g_s^p \lambda P_s + g_2^p P_2 + N_0} \right) + \tau \left(\frac{g_s^s P_s + g_1^s P_1}{g_2^s P_2 + N_0} \right).$$

In the above formula, the difference between the first two terms is always positive and decreasing as λ increases.

The third term does not depend on λ . Hence, ρ_{s2}^r is decreasing with λ .

$$\begin{aligned} \text{At } \sigma_p^* &= \tau \left(\frac{g_1^s P_1}{g_2^s P_2 + N_0} \right) \leq \tau \left(\frac{g_1^p P_1}{g_s^p \lambda P_s + g_2^s P_2 + N_0} \right) \\ \rho_{s2}^r &= 2\tau \left(\frac{g_2^p P_2}{g_s^p \lambda P_s + N_0} \right) + \tau \left(\frac{g_1^s P_1}{g_2^s P_2 + N_0} \right) + \tau \left(\frac{g_s^s \lambda P_s + g_1^s P_1}{g_2^s P_2 + N_0} \right) \\ &+ \min \left\{ \overbrace{\tau \left(\frac{g_s^s \bar{\lambda} P_s}{g_s^s \lambda P_s + g_1^s P_1 + g_2^s P_2 + N_0} \right)}^{\mu_8}, \right. \\ &\left. \overbrace{\tau \left(\frac{g_s^p \bar{\lambda} P_s}{g_s^p \lambda P_s + g_2^p P_2 + N_0} \right)}^{\mu_9}, \right. \\ &\left. \overbrace{\tau \left(\frac{g_s^p \bar{\lambda} P_s + g_1^p P_1}{g_s^p \lambda P_s + g_2^p P_2 + N_0} \right) - \tau \left(\frac{g_1^s P_1}{g_2^s P_2 + N_0} \right)}^{\mu_{10}} \right\}. \end{aligned} \quad (125)$$

When $\mu_8 = \min\{\mu_8, \mu_9, \mu_{10}\}$ in (125), we have

$$\begin{aligned} \rho_{s2}^r &= 2\tau \left(\frac{g_2^p P_2}{g_s^p \lambda P_s + N_0} \right) + \tau \left(\frac{g_1^s P_1}{g_2^s P_2 + N_0} \right) \\ &+ \tau \left(\frac{g_s^s P_s + g_1^s P_1}{g_2^s P_2 + N_0} \right). \end{aligned}$$

That is, ρ_{s2}^r is decreasing with λ .

When $\mu_9 = \min\{\mu_8, \mu_9, \mu_{10}\}$ in (125), we have

$$\begin{aligned} \rho_{s2}^r &= \tau \left(\frac{g_2^p P_2}{g_s^p \lambda P_s + N_0} \right) + \tau \left(\frac{g_1^s P_1}{g_2^s P_2 + N_0} \right) \\ &+ \tau \left(\frac{g_s^s \lambda P_s + g_1^s P_1}{g_2^s P_2 + N_0} \right) + \tau \left(\frac{g_s^p \bar{\lambda} P_s + g_2^p P_2}{g_s^p \lambda P_s + N_0} \right). \end{aligned} \quad (126)$$

The first term in (126) is decreasing with λ for all values of λ . The first derivative of the other terms with respect to λ is given by

$$\begin{aligned} &-\frac{0.5P_s (g_s^p g_s^s P_2 + g_s^p g_1^s P_1 + g_s^p N_0 - g_s^s N_0)}{\ln 2 (g_s^p \lambda P_s + N_0) (g_s^s \lambda P_s + g_2^s P_2 + g_1^s P_1 + N_0)} \\ &\leq 0 \quad \text{from (99)}. \end{aligned}$$

Hence, ρ_{s2}^r is decreasing with λ .

When $\mu_{10} = \min\{\mu_8, \mu_9, \mu_{10}\}$ in (125), we have

$$\begin{aligned} \rho_{s2}^r &= \tau \left(\frac{g_2^p P_2}{g_s^p \lambda P_s + N_0} \right) + \tau \left(\frac{g_s^p \bar{\lambda} P_s + g_1^p P_1 + g_2^p P_2}{g_s^p \lambda P_s + N_0} \right) \\ &+ \tau \left(\frac{g_s^s \lambda P_s + g_1^s P_1}{g_2^s P_2 + N_0} \right). \end{aligned} \quad (127)$$

The first term in (127) is decreasing with λ , and the first derivative of the other three terms with respect to λ is given by

$$\begin{aligned} &-\frac{0.5P_s (g_s^p g_2^s P_2 + g_s^p g_1^s P_1 + g_s^p N_0 - g_s^s N_0)}{\ln 2 (g_s^p \lambda P_s + N_0) (g_s^s \lambda P_s + g_1^s P_1 + g_2^s P_2 + N_0)} \\ &\leq 0 \quad \text{from (99)}. \end{aligned}$$

Thus, ρ_{s2}^r is decreasing with λ .

$$2R_s^r + R_p^r = \rho_{2p}^r$$

From (99),

$$\sigma_s^* = \tau \left(\frac{g_s^s \bar{\lambda} P_s}{g_s^s \lambda P_s + g_2^s P_2 + N_0} \right).$$

$$\begin{aligned} \rho_{2p}^r &= 2\tau \left(\frac{g_s^s P_s}{g_2^s P_2 + N_0} \right) + \tau \left(\frac{g_2^p P_2}{g_s^p \lambda P_s + N_0} \right) \\ &- \left[\tau \left(\frac{g_s^s \bar{\lambda} P_s}{g_s^s \lambda P_s + g_2^s P_2 + N_0} \right) - \tau \left(\frac{g_s^p \bar{\lambda} P_s}{g_s^p \lambda P_s + g_2^p P_2 + N_0} \right) \right]^+ \\ &+ \min \left\{ \tau \left(\frac{g_1^s P_1}{g_s^s P_s + g_2^s P_2 + N_0} \right), \tau \left(\frac{g_1^p P_1}{g_s^p P_s + g_2^p P_2 + N_0} \right) \right. \\ &+ \left. \left[\tau \left(\frac{g_s^p \bar{\lambda} P_s}{g_s^p \lambda P_s + g_2^p P_2 + N_0} \right) - \tau \left(\frac{g_s^s \bar{\lambda} P_s}{g_s^s \lambda P_s + g_2^s P_2 + N_0} \right) \right]^+ \right. \\ &\left. \tau \left(\frac{g_1^p P_1}{g_s^p \lambda P_s + g_2^p P_2 + N_0} \right) \right\}. \end{aligned}$$

$$\text{If } \tau \left(\frac{g_s^s \bar{\lambda} P_s}{g_s^s \lambda P_s + g_2^s P_2 + N_0} \right) \leq \tau \left(\frac{g_s^p \bar{\lambda} P_s}{g_s^p \lambda P_s + g_2^p P_2 + N_0} \right)$$

$$\begin{aligned} \rho_{2p}^r &= 2\tau \left(\frac{g_s^s P_s}{g_2^s P_2 + N_0} \right) + \tau \left(\frac{g_2^p P_2}{g_s^p \lambda P_s + N_0} \right) \\ &+ \min \left\{ \overbrace{\tau \left(\frac{g_1^s P_1}{g_s^s \lambda P_s + g_2^s P_2 + N_0} \right)}^{\mu_{11}}, \overbrace{\tau \left(\frac{g_1^p P_1}{g_s^p \lambda P_s + g_2^p P_2 + N_0} \right)}^{\mu_{12}}, \right. \\ &\left. \overbrace{\tau \left(\frac{g_1^p P_1 + g_s^p \bar{\lambda} P_s}{g_s^p \lambda P_s + g_2^p P_2 + N_0} \right) - \tau \left(\frac{g_s^s \bar{\lambda} P_s}{g_s^s \lambda P_s + g_2^s P_2 + N_0} \right)}^{\mu_{13}} \right\}. \end{aligned} \quad (128)$$

• When $\mu_{11} = \min\{\mu_{11}, \mu_{12}, \mu_{13}\}$ in (128), then

$$\begin{aligned} \rho_{2p}^r &= 2\tau \left(\frac{g_s^s P_s}{g_2^s P_2 + N_0} \right) + \tau \left(\frac{g_2^p P_2}{g_s^p \lambda P_s + N_0} \right) \\ &+ \tau \left(\frac{g_1^s P_1}{g_s^s \lambda P_s + g_2^s P_2 + N_0} \right). \end{aligned}$$

It is clear that ρ_{2p}^r is decreasing with λ .

- When $\mu_{12} = \min\{\mu_{11}, \mu_{12}, \mu_{13}\}$ in (128), then

$$\rho_{2p}^r = 2\tau \left(\frac{g_s^s P_s}{g_2^s P_2 + N_0} \right) + \tau \left(\frac{g_2^p P_2}{g_s^p \lambda P_s + N_0} \right) + \tau \left(\frac{g_1^p P_1}{g_s^p \lambda P_s + g_2^p P_2 + N_0} \right).$$

It is also clear that ρ_{2p}^r is decreasing with λ .

- When $\mu_{13} = \min\{\mu_{11}, \mu_{12}, \mu_{13}\}$ in (128), then

$$\rho_{2p}^r = 2\tau \left(\frac{g_s^s P_s}{g_2^s P_2 + N_0} \right) + \tau \left(\frac{g_1^p P_1}{g_s^p P_s + g_2^p P_2 + N_0} \right) + \tau \left(\frac{g_s^s \bar{\lambda} P_s + g_2^p P_2}{g_s^p \lambda P_s + N_0} \right) - \tau \left(\frac{g_s^s \bar{\lambda} P_s}{g_s^s \lambda P_s + g_2^s P_2 + N_0} \right).$$

$$\frac{\partial \rho_{2p}^r}{\partial \lambda} = - \frac{0.5P_s (g_s^p g_2^s P_2 + g_s^p N_0 - g_s^s N_0)}{\ln 2 (g_s^p \lambda P_s + N_0) (g_s^s \lambda P_s + g_2^s P_2 + N_0)} \leq 0 \quad \text{from (99)}.$$

Thus, ρ_{2p}^r is decreasing with λ .

$$\text{If } \tau \left(\frac{g_s^s \bar{\lambda} P_s}{g_s^p \lambda P_s + g_2^p P_2 + N_0} \right) \leq \tau \left(\frac{g_s^s \bar{\lambda} P_s}{g_s^s \lambda P_s + g_2^s P_2 + N_0} \right)$$

$$\rho_{2p}^r = 2\tau \left(\frac{g_s^s P_s}{g_2^s P_2 + N_0} \right) + \tau \left(\frac{g_s^p \bar{\lambda} P_s + g_2^p P_2}{g_s^p \lambda P_s + N_0} \right) - \tau \left(\frac{g_s^s \bar{\lambda} P_s}{g_s^s \lambda P_s + g_2^s P_2 + N_0} \right) + \min \left\{ \tau \left(\frac{g_1^p P_1}{g_s^p P_s + g_2^p P_2 + N_0} \right), \tau \left(\frac{g_1^p P_1}{g_s^p P_s + g_2^p P_2 + N_0} \right) \right\}.$$

$$\frac{\partial \rho_{2p}^r}{\partial \lambda} = - \frac{0.5P_s (g_s^p g_2^s P_2 + g_s^p N_0 - g_s^s N_0)}{\ln 2 (g_s^p \lambda P_s + N_0) (g_s^s \lambda P_s + g_2^s P_2 + N_0)} \leq 0 \quad \text{from (99)}.$$

Therefore, ρ_{2p}^r is decreasing with λ .

Thus, since we showed that if (99) is satisfied, assuming that the secondary receiver can decode the signal of primary user 1, then ρ_p^r , ρ_{sp}^r , ρ_{s2}^r , and ρ_{2p}^r decrease with λ , whereas ρ_s^r does not depend on λ ; hence, $\mathcal{R}_1^r(Z)$ at $\lambda = 0$ coincides on \mathcal{R}_{1g}^r . And for any λ_1 and λ_2 such that $\lambda_1 > \lambda_2$, $\mathcal{R}_1^r(Z)$ at λ_1 is a subset of $\mathcal{R}_1^r(Z)$ at λ_2 .

Necessity part

In this part of the proof, we show that if condition (99) is not satisfied, then \mathcal{R}_{1g}^r does not coincide on any $\mathcal{R}_1^r(Z)$ for all values of λ . So, assume that (99) is not satisfied, i.e.,

$$N_0 g_s^s > g_s^p g_2^s P_2 + g_s^p N_0. \quad (129)$$

By referring to Figure 4, the effect of λ on $\mathcal{R}_1^r(Z)$ at points *A* and *F* is determined as follows.

At point A

$$R_p^{rA} = \tau \left(\frac{g_2^p P_2}{g_s^p \lambda P_s + N_0} \right) + \min \left\{ \tau \left(\frac{g_1^s P_1}{g_2^s P_2 + N_0} \right), \tau \left(\frac{g_1^p P_1}{g_s^p \lambda P_s + g_2^p P_2 + N_0} \right) \right\}.$$

It is clear that R_p^{rA} is decreasing with λ .

At point F

$$R_s^{rF} = \tau \left(\frac{g_s^s \lambda P_s}{g_2^s P_2 + N_0} \right) + \tau \left(\frac{g_s^p \bar{\lambda} P_s}{g_s^p \lambda P_s + N_0} \right).$$

$$\frac{\partial R_s^{rF}}{\partial \lambda} = \frac{0.5P_s (g_s^s N_0 - (g_s^p g_2^s P_2 + g_s^p N_0))}{\ln 2 (g_s^p \lambda P_s + N_0) (g_s^s \lambda P_s + g_2^s P_2 + N_0)} > 0 \quad \text{from (129)}.$$

Consequently, R_s^{rF} is increasing with λ .

So, for any two different values of λ , the corresponding rate regions $\mathcal{R}_1^r(Z)$ do not include one another; thus, \mathcal{R}_{1g}^r does not coincide on $\mathcal{R}_1^r(Z)$ at any value of λ .

Competing interests

The authors declare that they have no competing interests.

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