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# Macro-pico amplitude-space sharing with layered interference alignment

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## Abstract

Inter-cell interference leads to severe performance degradation in cellular networks, and the study of multi-user interference channel is the corner stone for solving this problem. Amplitude-space layered interference alignment (IA), as an effective complementation to the vector-space IA, is a promising method to increase the data rate in static interference channels. However, recent studies of layered IA has been focused on analyzing the degrees of freedom (DoF) or the achievable rate under specific channel constraints. In this paper, we propose a layered IA scheme that can work with arbitrary channel coefficients. We develop a layer partitioning method and optimize the active layer assignment through linear programming. An implementation scheme is then introduced with multi-level nested lattice codes where the signal and interference are nested in amplitude space, and the interference from different users is nestedly aligned. The performance of the proposed scheme is finally evaluated in homogeneous and heterogeneous cellular networks with practical settings.

**Keywords:** Amplitude-space sharing; Heterogeneous network; Interference alignment; Nested lattice code

## 1 Introduction

Multi-tier heterogeneous networks can achieve significant areal capacity gain by intensifying spatial reuse of the spectrum [1, 2]. While the high-power macro-cell base station (BS) provides basic coverage and supports high mobility, the low-power nodes like pico-cell BSs support high-capacity transmission for hotspot zones. Accordingly, the data rate requirements of macro-cell user and pico-cell user may have large difference.

Operated in the same frequency band, the interference in heterogeneous networks is complicated, including not only the co-tier interference among macro-cells and among pico-cells but also the cross-tier interference between the macro-cell and pico-cells [3]. One pico-user may encounter interference from the macro-BS and the pico-BSs in the same macro-cell and also from the macro-BSs in adjacent macro-cells. The pico-cells might be deployed at any places in the macro-cell, and the transmit power of the macro-BS is much stronger than the pico-BS.

Conventional methods to mitigate inter-cell interference mainly involve various orthogonalization-based processing. For example, almost blank subframe (ABS) is a time orthogonal interference coordination scheme; fractional frequency reuse (FFR) is a frequency orthogonal interference coordination scheme; coordinated beamforming (CB) is a spatial orthogonal interference coordination scheme [4–7]. The vector-space interference alignment (IA) scheme is an advanced orthogonalization-based interference coordination scheme, where multiple interference are aligned in a subspace while leave the orthogonal signal subspace interference-free [8, 9]. However, the orthogonalization-based processing schemes are only efficient when the interference has similar strength with the desired signal. If the interference is very weak or very strong, it is a waste of resource to provide orthogonal subspaces for each interference. In fact, for very weak interference channels, treating the interference as noise is optimal [10–13]. For strong interference channels, interference cancelation can achieve the capacity [14–16].

In multi-tier heterogeneous networks with different interference strengths in various scenarios, amplitude-space sharing of the signal and interferences is an effective way to complement the weakness of the orthogonalization-based processing. In [17], an

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amplitude-space sharing scheme in a two-cell network was proposed, where each BS schedules one user and the two users simultaneously scheduled in two overlapping cells consist a two-user interference channel. In different interference scenarios, the opportunities for interference cancellation are proactively created and the network sum rates are maximized through the optimized Han-Kobayashi coding [18, 19]. In multi-cell networks, Han-Kobayashi coding is no longer applicable [20], we need to study amplitude-space sharing scheme that can accommodate multiple interference. In this case, the desired signal and interference should occupy different layers in the amplitude space, and multiple interference may be aligned in one layer to compress their occupied spaces.

Layered interference alignment had been studied in multi-user interference channels, but the existing results are obtained under special channel conditions and are not applicable in practical systems. In [21], the degrees of freedom (DoF) of the fully connected  $K$ -user Gaussian interference channel were proved to be arbitrarily close to  $K/2$ , but the channel coefficients should take specialized forms that the inputs are shifted by an even amount on the desired links and by an odd amount on the interference links (or vice versa). In [22], it was shown that  $K/2$  DoF are achievable when the cross-link channel gains corresponding to the interferers are rational, whereas the direct-link channel gains corresponding to the intended signal are irrational algebraic; if the direct-link channel gains are rational as well, the DoF of the channel are strictly smaller than  $K/2$ . Later, a general formula for the DoF of real interference channels was derived in [23] by maximizing the Rényi information dimension. Recently, the authors of [24] proved that the DoF of the static interference channel are  $K/2$  for almost all sets of channel gains through the real interference alignment. In [25], incorporating both vector and real interference alignment, the total DoF of  $2KM/(K+1)$  are characterized in the  $K \times 2$  and  $2 \times K$ ,  $M$  antennas  $\times$  channels, for almost all channel realizations. The achievability schemes in [22, 24, 25] rely on some results from the field of Diophantine approximation in number theory, especially the lower bounds on the approximability of irrational algebraic numbers by rationals [26–32]. However, these results also imply that in asymptotic high signal-to-noise ratio (SNR) region the capacity characterization of the interference channel is extremely sensitive to slight variations of the channel gains.

The generalized degrees of freedom (GDoF) of the symmetric Gaussian  $K$ -user interference channel were studied in [33], where all direct links have the same SNR and all cross-links have the same interference-to-noise ratio (INR). The authors found that the GDoF per user do not depend on the number of users, so that

the characterization is identical to the two-user interference channel with the exception of a singularity when the INR equals to the SNR. The achievable rates in moderate SNR values were obtained in [34, 35], where the interference alignment scheme is suitable for a class of integer-interference channel, where all cross-link channel gains are integer or rational.

In this paper, we first study the  $K$ -user layered interference alignment scheme with arbitrary channel coefficients and then apply the scheme to multi-cell heterogeneous cellular networks with practical SNR values. The main contributions of this paper are as follows. 1) With arbitrary channel coefficients, a novel layer partitioning method is designed based on the power level intersections between the signals and interference. 2) To maximize the network sum rate, a linear programming method was proposed to optimize the assignment of active transmit layers. 3) A nested lattice coding scheme is developed to implement the encoding and decoding when the power levels of each interference are not exactly aligned. 4) The performance of the proposed scheme applied in cellular networks is evaluated, and the affecting factors in different network scenarios are analyzed.

The rest of this paper is organized as follows: In Section 2, we first study the optimal active layer assignment under deterministic interference channel models. Then, based on the obtained insight, we study  $K$ -user Gaussian interference channel with arbitrary coefficients and study the layer partitioning and active layer assignment methods in Section 3. The encoding and decoding schemes are developed in Section 4, and the performance in cellular networks is evaluated in Section 5. Finally, Section 6 concludes the paper.

## 2 Interference alignment in deterministic channels

### 2.1 Deterministic channel model

It is very hard to directly study the interference network problem under Gaussian channels. In this paper, we first resort to the deterministic channel model to gain some insights on the layered interference alignment problem and then extend the idea to general Gaussian interference channels. The deterministic channel model is an approximation methodology developed to solve network information theory problems [36, 37]. Its general principle is that the coding problem in a noisy channel is first approximated by a noiseless problem, then analyze the simplified problem, and use insights obtained from the simplified problem to find new achievable schemes and/or outer bounds of the original problem [38]. The deterministic channel model approximates the Gaussian channel as a discrete set of parallel noiseless channels.

In the single-user Gaussian channel, a real-valued input  $x$  generates a real-valued output  $y$  that is degraded by

Gaussian noise  $z$ , that is

$$y = hx + z, \quad (1)$$

where  $h$  is the channel coefficient,  $E[x^2] = P$ , and the variance of  $z$  is  $N_0$ . The SNR is thus defined as  $h^2P/N_0 = \gamma$ . If  $x$  and  $z$  are normalized, then the effective channel gain is  $\sqrt{\gamma}$ .

To transform the Gaussian channel to a deterministic channel, we first represent the normalized  $x$  in a base-2 notation as

$$\bar{x} = 0.b_1b_2b_3b_4b_5\dots, \quad (2)$$

where each bit  $b_i$  can be interpreted as occupying a signal level, and the most significant bit coincides with the highest level. Since  $\bar{x}$  is a normalized value,  $\bar{x} < 1$ , all the bits  $b_i$  are in fractional part.

Given the SNR  $\gamma$ , the output of the deterministic channel model is

$$\bar{y} = \lfloor 2^\alpha \bar{x} \rfloor = b_1b_2\dots b_\alpha, \quad (3)$$

where  $\alpha = \lfloor \log_2 \gamma \rfloor$  is the largest integer not exceeding  $\log_2 \gamma$  and  $b_\alpha$  is the smallest signal level containing a transmit power larger than  $1/2^\alpha$ . In other words, the input bit sequence is shifted by  $\alpha$  positions and the remained part after  $b_\alpha$  is truncated due to the degradation of noise.

In the  $K$ -user Gaussian interference channels, the inputs of  $K$  users form a vector  $\mathbf{x}$ , and the output vector is

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{z}, \quad (4)$$

where the entry of channel matrix  $H_{i,j}$  stands for the channel gain from transmitter  $j$  to receiver  $i$ . The noise of different users is assumed to be independent and identically distributed (i.i.d.), and  $E[\mathbf{z}\mathbf{z}^H] = N_0\mathbf{I}$ . The SNRs depend on the direct channel gains and are defined as  $\gamma_{k,k} = H_{k,k}^2P_k/N_0$ , and the INRs depend on the cross-channel gains and are defined as  $\gamma_{i,j} = H_{i,j}^2P_j/N_0$ . These SNRs and INRs compose a link quality matrix  $\mathbf{\Gamma}$ , whose  $(i,j)$ -th entry is  $\gamma_{i,j}$ .

Defining  $\alpha_{ij} = \lfloor \log_2 \gamma_{i,j} \rfloor$ , and applying (3) to every direct and cross-links of (4), we obtain the deterministic model of interference channel. The input-output relationship is shown in the left part of Fig. 1, for the convenience of demonstration; the input levels are also shifted by  $\alpha_{k,k}$  positions so that the inputs and outputs of the direct links are at the same levels. For the cross-link from transmitter  $j$  to receiver  $i$ , the shifting is thus changed to  $\alpha_{i,j} - \alpha_{j,j}$  instead of  $\alpha_{i,j}$ . At each receiver, the direct-link outputs (signal) and the cross-link outputs (interference) are added together when they are on the same level, and the outputs below the noise level are discarded. Rather than the normal integer addition, modulo-2 addition is chosen here to avoid interaction between different levels. This is a simplified operation only applied in the deterministic model. In Gaussian channels, the addition of signals and

interference will produce carry-over problem, which will be addressed in Section 3.

## 2.2 Alignment in amplitude space

From the left part of Fig. 1, we can see that, if all the transmit levels are active, there must be collisions between signals and interference. To avoid collision, only part of the transmit levels can be active. Therefore, we need to study an optimal active level assignment algorithm to avoid collision and simultaneously maximize the network sum rate.

In the  $K$ -user deterministic interference channel, each active level conveys one bit. To avoid collision, each level at the receiver side should be occupied either by a signal bit or by an interference bit. Actually, this interference bit can be a modulo-2 addition of several interference bits, since it is not necessary to decode each of these interference bits separately. In order to leave as many receive levels as possible "free" for reception of signal bits, it will be advantageous if multiple interference bits fall on one receive level. This is the principle of interference alignment in amplitude space: the alignment of interference saves amplitude levels for desired signals and thus the throughput can be increased.

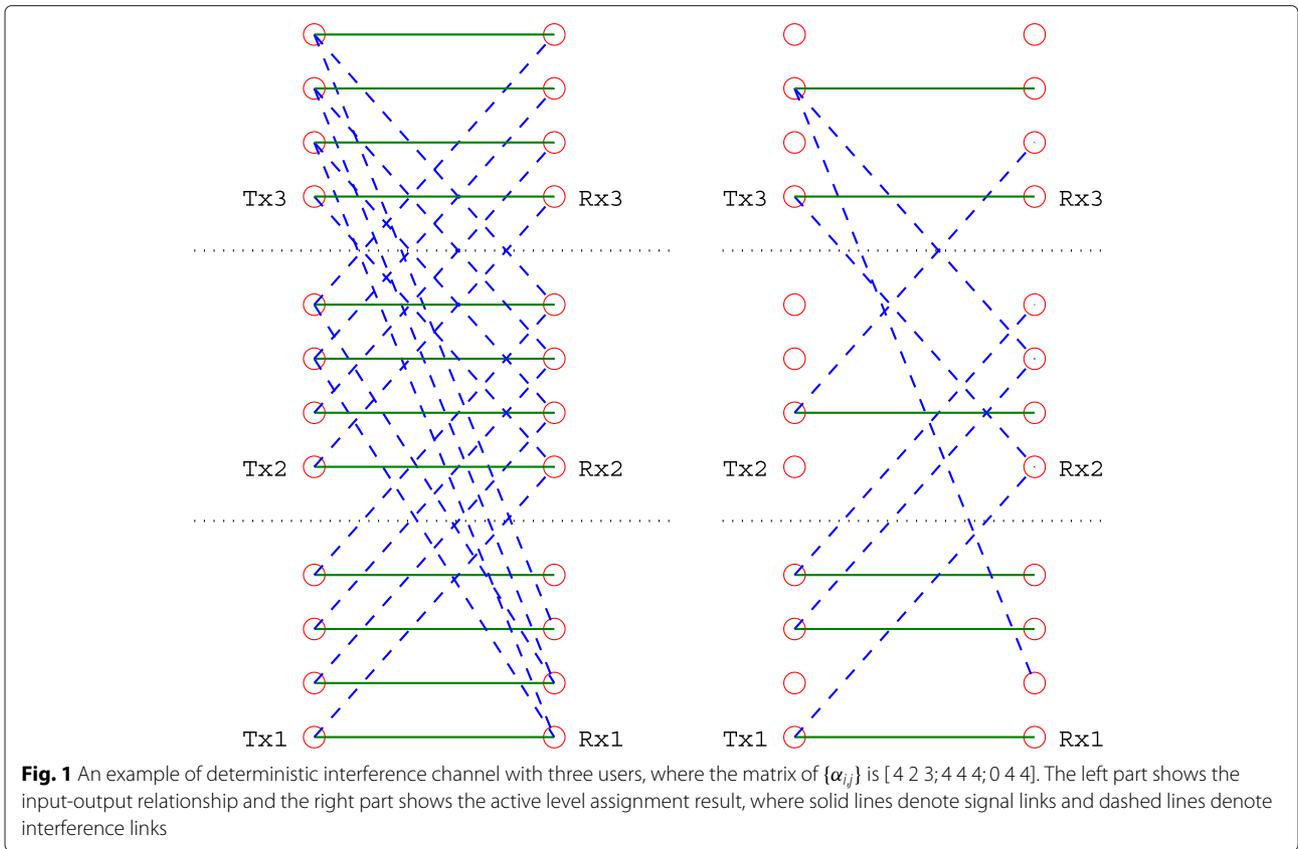
The optimal active level assignment problem can be formulated as

$$\begin{aligned} & \max \sum_{k=1}^K \sum_{l=1}^{\alpha_{k,k}} b_{k,l}, \\ & \text{s.t. } b_{k,l} \in \{0, 1\}, \\ & \quad b_{k,l} + b_{j,l'} \leq 1, \quad \forall j, k, l \text{ and } j \neq k \end{aligned} \quad (5)$$

where  $b_{k,l}$  indicates whether the  $l$ -th transmit level of user  $k$  is active or not, and  $l = l' + (\alpha_{k,j} - \alpha_{j,j})$ . When the level is active,  $b_{k,l} = 1$ , otherwise  $b_{k,l} = 0$ . The second constraint means that, if the  $l'$ -th level of user  $j$  interferes with the  $l$ -th level of user  $k$  through the cross-link,  $b_{k,l}$  and  $b_{j,l'}$  can not be "1" simultaneously; otherwise, the received signal is not decodable. Since the transmit levels of user  $j$  are shifted by  $\alpha_{k,j} - \alpha_{j,j}$  positions when they arrive at receiver  $k$ , the  $l'$ -th transmit level of user  $j$  will fall on level  $l' + (\alpha_{k,j} - \alpha_{j,j})$  at receiver  $k$ . Thus, when  $l = l' + (\alpha_{k,j} - \alpha_{j,j})$ ,  $b_{j,l'}$  is an interference to  $b_{k,l}$ , and these two bits are not allowed to be "1" simultaneously.

If there is another user  $i$  and its  $l''$ -th level  $b_{i,l''}$  also interfere with  $b_{k,l}$ , similarly there will be a constraint that  $b_{k,l} + b_{i,l''} \leq 1$ . However, we do not restrict the summation of the interference bits, i.e.,  $b_{j,l'} + b_{i,l''}$  can be any value even when they fall on the same level at receiver  $k$ . If both  $b_{i,l''}$  and  $b_{j,l'}$  are active, where  $l'' + (\alpha_{k,i} - \alpha_{i,i}) = l' + (\alpha_{k,j} - \alpha_{j,j}) = l$ , the constraint can still be satisfied as long as  $b_{k,l} = 0$ .

The optimization problem in (5) is a binary integer programming problem, which can be solved efficiently by a



linear programming (LP)-based branch-and-bound algorithm [39]. The algorithm searches for an optimal solution to the binary integer programming problem by solving a series of LP relaxation problems, in which the binary integer requirement on the variables is replaced by the weaker constraint  $0 \leq b_{k,l} \leq 1$ .

One example of the optimized results is shown in the right part of Fig. 1. We can see the features of an amplitude-aligned interference network. From the transmitter side, each user has different active level assignment, and the active levels may not be contiguous. At the receiver side, several interference links might be aligned on one level, and the interference levels might be above, below, or interlaced with the signal levels.

### 3 Interference alignment in Gaussian channels

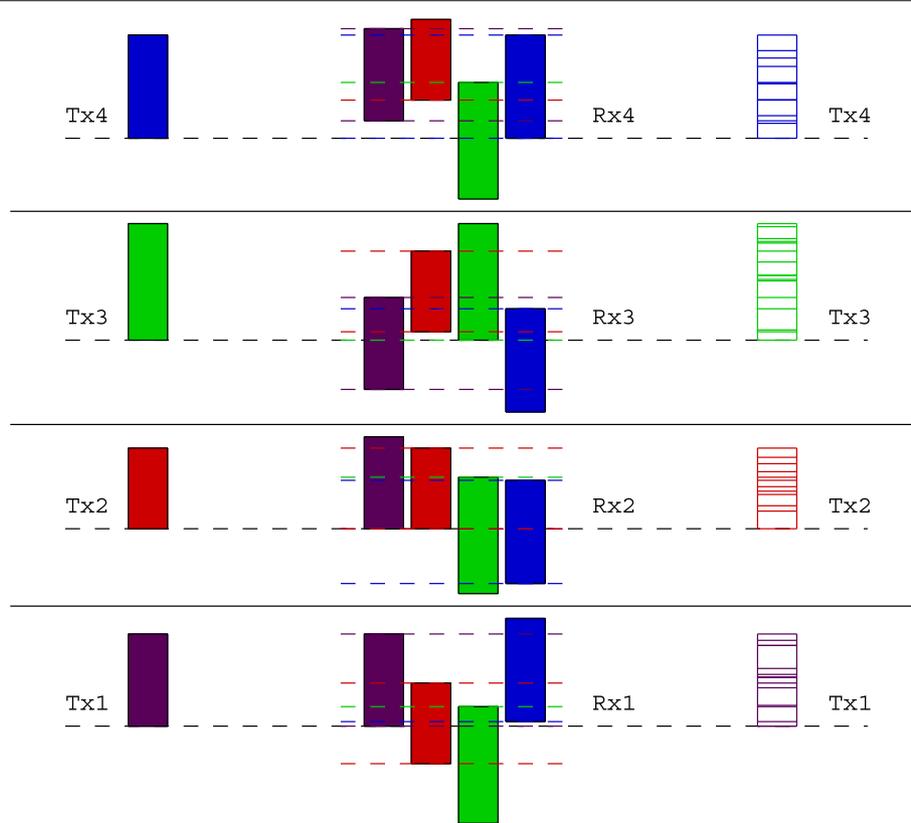
The deterministic channel model described above assumes that all SNRs and INRs are integer on the log scale, i.e., the signals fall exactly on the evenly spaced “levels” at the receiver. In Gaussian channels, however, this assumption is not valid. In the following, we will address the interference alignment problem taking into account arbitrary SNR and INR values.

Given the link quality matrix  $\Gamma$ , at each receiver, there is one collision pattern in amplitude space, i.e., part of

the desired signal may collide with part of the interference. The collision pattern changes at different receivers because they experience different SNR and INRs. The layer partitioning requires to find all the possible collision areas in the amplitude space between the signals and interference.

The layer partitioning procedure is shown in Fig. 2. For each user  $k$ , the SNR of the direct channel is  $\gamma_{k,k}$ , which is represented by a bar with a height  $\log \gamma_{k,k}$  at the transmitter side. At receiver  $k$ , there are  $K$  bars representing the  $K$  received signals. Their relative positions in the amplitude space are affected by the corresponding SNRs and INRs. For user  $i$ 's bar, the upper and lower boundaries are  $\log \gamma_{k,i}$  and  $\log \gamma_{k,i} - \log \gamma_{i,i}$ , respectively. If the bar of user  $j, j \neq k$ , overlaps with the bar of user  $k$ , both bars are split (separated into different layers) at the intersecting boundary positions. Repeating these processes at every receiver, we will obtain at most  $2K(K - 1)$  layers for each transmit signal. The results shown in Fig. 2 are obtained when a group of random values are set for  $\Gamma$ , i.e.,

$$10 \log_{10} \Gamma = \begin{bmatrix} 7.43 & 3.47 & 1.57 & 8.69 \\ 7.42 & 6.51 & 4.16 & 3.92 \\ 3.45 & 7.18 & 9.40 & 2.53 \\ 8.84 & 9.58 & 4.50 & 8.33 \end{bmatrix} \text{ dB.} \quad (6)$$



**Fig. 2** From left to right: the transmit signal before layer partitioning, the received signals of different users, and the layer partitioning results

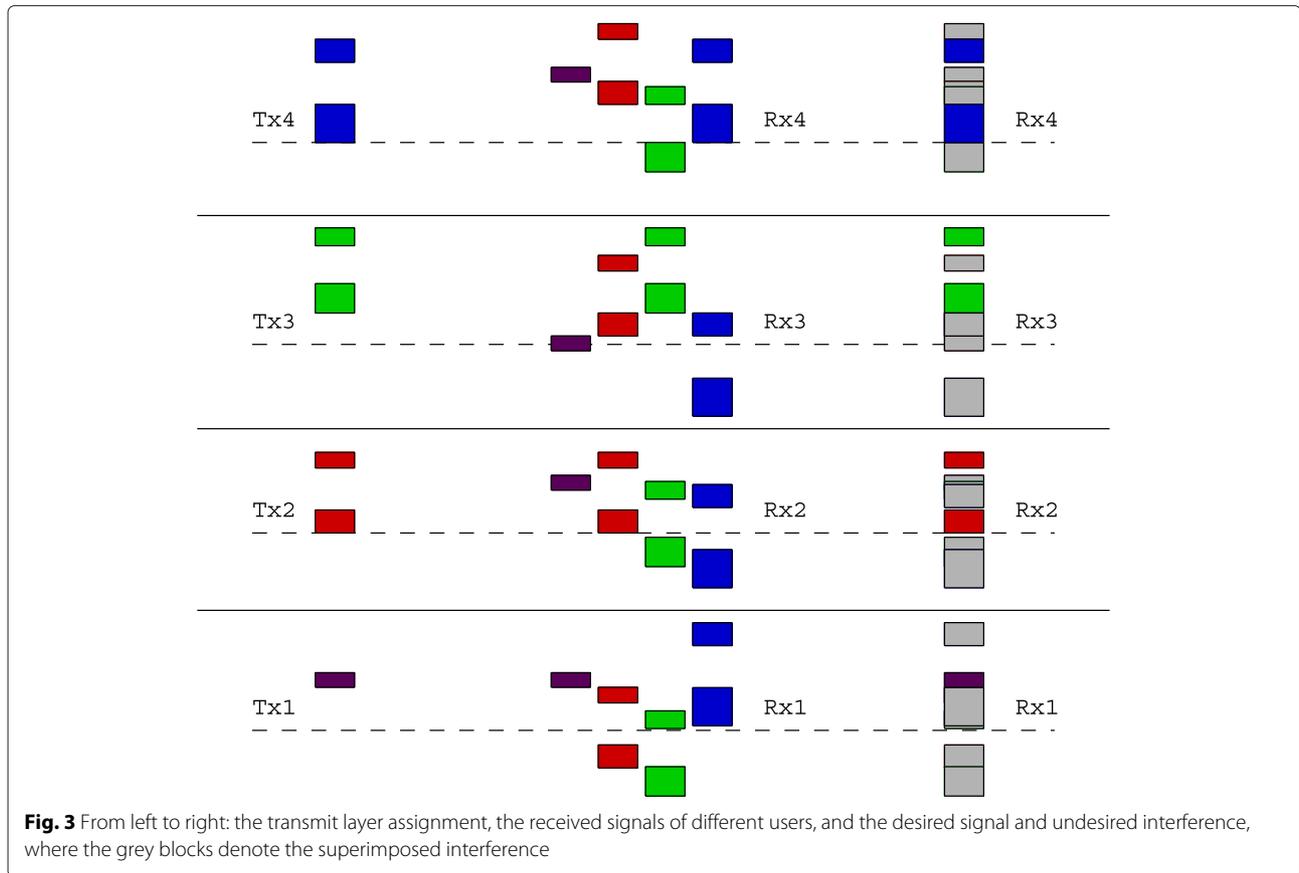
After layer partitioning, user  $k$  has  $L_k$  layers in the transmitter side, and the upper and lower boundaries of the  $l$ -th layer are respectively  $P_{k,l}$  and  $P_{k,l-1}$ . Thus, the  $l$ -th layer has an amplitude space represented by  $\rho_{k,l} = P_{k,l}/P_{k,l-1}$ . Assume now that we have a layered transmission scheme where the bits in the highest layers are decoded first. Thus, the decisions of the current layer can at most be disturbed by as-yet undecoded layers, i.e., the layers with a lower power than the current one. Thus,  $\rho_{k,l}$  can be viewed as a signal-to-interference ratio (SIR) of this layer. If Shannon capacity achieved transmission were used,  $R_{k,l} = 1/2 \log(1 + \rho_{k,l})$  bits could be transmitted in this layer. Considering the encoding and decoding methods to be introduced in Section 4, which have a rate loss of at most 0.5 bit, we use  $R_{k,l} = 1/2 \log(\rho_{k,l})$ .

As in the deterministic model of Section 2, some layers must be inactive to avoid collision between the layered signals of different users. We can still apply the binary integer programming algorithm to search the optimal active layer assignment, so that the total throughput can be maximized. In Gaussian channels, since each layer has different amplitude space, the objective function should be weighted by the transmission capability of each layer, i.e.,  $R_{k,l}$ . The optimization problem is therefore formulated as

$$\begin{aligned} \max \quad & \sum_{k=1}^K \sum_{l=1}^{L_k} R_{k,l} b_{k,l} \\ \text{s.t.} \quad & b_{k,l} \in \{0, 1\}, \\ & b_{k,l} + b_{j,l'} \leq 1, \quad \forall j, k, l \text{ and } j \neq k \end{aligned} \quad (7)$$

where  $b_{k,l}$  denotes the transmit state of layer  $l$  of user  $k$ . When this layer is active,  $b_{k,l} = 1$ , otherwise  $b_{k,l} = 0$ . The second constraint is to avoid collision between the desired signal and interferences. If the  $l'$ -th layer of user  $j$  interferes with the  $l$ -th layer of user  $k$  through the cross-link,  $b_{k,l}$  and  $b_{j,l'}$  can not be "1" simultaneously. Unlike in the deterministic model, the relation between  $l$  and  $l'$  does not have an explicit expression here.

This optimization problem can also be solved by the LP-based branch-and-bound algorithm. The search result is shown in Fig. 3, where the contiguous active layers are combined. We can see that some interference layers overlap and occupy the same part of the amplitude space, but they may not be strictly aligned with respect to the upper and lower boundaries. Since we need to decode the superimposed interference, this kind of alignment complicates the encoding designs. We will discuss in detail the encoding and decoding schemes in Section 4.



Different from the deterministic channels, the addition of two layers of interference has a higher signal level than any of the two layers, i.e., the carry-over problem must be considered. The active layers assigned through (7) might be tightly connected, which means that at one receiver the lower boundary of an upper signal layer might be the upper boundary of a lower interference layer. If the interference layer is superimposed from two or more users, the carry-over part might collide with the upper signal layer. Therefore, at the bottom of the signal layer, we need to reserve some amplitude space for the carry-over interference. In practice, this can be done by retaining the transmit power and reducing the data rate of the upper signal layer.

For example, at receiver  $k$ , suppose there are two interference layers below the signal layer  $l$ . One is from user  $j$ , and the upper boundary is  $h_{k,j}^2 P_{j,l'}$ ; the other is from user  $i$ , and the upper boundary is  $h_{k,i}^2 P_{i,l''}$ . Thus, the lower boundary of the signal layer  $l$  at the transmitter side is changed into

$$\tilde{P}_{k,l-1} = \max \left\{ P_{k,l-1}, \frac{h_{k,j}^2 P_{j,l'} + h_{k,i}^2 P_{i,l''}}{h_{k,k}^2} \right\}, \quad (8)$$

where the first term inside the maximum operation is the upper boundary of the  $(l - 1)$ -th layer of user  $k$ , and the second term is the sum power of two layers of interference divided by the square of the direct-link channel gain. If the second term is larger, at receiver  $k$ , the lower boundary of the  $l$ -th layer of user  $k$  is changed from  $h_{k,k}^2 P_{k,l-1}$  to  $h_{k,j}^2 P_{j,l'} + h_{k,i}^2 P_{i,l''}$ . The reserved space is at most 3 dB and thus the data rate loss is at most 0.5 bit. When the lower layer is occupied by the superimposed interference from  $K$  users, the reserved space is at most  $10 \log_{10}(K)$  dB and the data rate loss is at most  $1/2 \log(K)$  bits. This data rate loss can be neglected compared to the increasing sum rate when SNR goes to infinity.

#### 4 Implementation by multi-level nested lattice codes

In the layered interference alignment scheme, since the superimposed interference layers need to be decoded, random coding is no longer applicable. We thus present a coding scheme with multi-level nested lattice codes in this section. Lattice coding is a structural coding; if there are two codewords that are selected from a lattice, their sum and difference are also within the same lattice. Thus, we can directly decode the superposition of the aligned

interferences, instead of decoding the interference signals one by one.

A lattice  $\Lambda$  is an  $n$ -dimensional discrete subgroup of the Euclidean space  $\mathbb{R}^n$  under vector addition. Thus, if  $\lambda_1$  and  $\lambda_2$  are in  $\Lambda$ , their sum and difference are also in  $\Lambda$ . A lattice  $\Lambda_2$  is said to be nested in a lattice  $\Lambda_1$  if  $\Lambda_2 \subseteq \Lambda_1$ . The lattice  $\Lambda_1$  is often referred to as a fine lattice and  $\Lambda_2$  as a coarse lattice. A nested lattice code  $\mathcal{L}$  is the set of all points of the fine lattice that are within the fundamental Voronoi region  $\mathcal{V}_2$  of the coarse lattice, i.e.,  $\mathcal{L} = \Lambda_1 \cap \mathcal{V}_2$ .

For any two powers  $P_a \geq P_b \geq 0$ , [40] shows that there exist nested  $n$ -dimensional lattices  $\Lambda_2 \subseteq \Lambda_1$ , as  $n \rightarrow \infty$ , their second moment  $\sigma^2(\Lambda_2) = P_a$ ,  $\sigma^2(\Lambda_1) = P_b$ , and the coding rate of  $\mathcal{L}$  satisfies

$$R = \frac{1}{n} \log |\mathcal{L}| = \frac{1}{n} \log \frac{\text{Vol}(\mathcal{V}_2)}{\text{Vol}(\mathcal{V}_1)} = \frac{1}{2} \log \left( \frac{P_a}{P_b} \right), \quad (9)$$

where  $|\mathcal{L}|$  is the cardinality of set  $\mathcal{L}$  and  $\text{Vol}(\mathcal{V}_i)$  is the volume of Voronoi region  $\mathcal{V}_i$ .

In [41], it is shown that nested lattice codes can achieve the capacity of point-to-point AWGN channels. In [42], a doubly nested lattice coding scheme was provided which can approach the capacity region of a two-way relay channel within 0.5 bit. Reference [43] provides a practical implementation scheme for nested lattice coding, where turbo coding and trellis shaping (multidimensional quantization) are involved. Due to space limitation, we refer the interested reader to [44, 45] for the detailed definitions and general construction methods of lattice codes.

For the transmitter  $k$ , define a sequence of nested lattice  $\Lambda_{k,0}, \Lambda_{k,1}, \dots, \Lambda_{k,L_k}$ , where  $\Lambda_{k,L_k} \subseteq \dots \subseteq \Lambda_{k,1} \subseteq \Lambda_{k,0}$ . The second moment of  $\Lambda_{k,l}$  is  $\sigma^2(\Lambda_{k,l}) = P_{k,l}$ .

The codebook used by level  $l$  is a nested lattice code  $\mathcal{C}_{k,l} = \Lambda_{k,l-1} \cap \mathcal{V}_{k,l}$ , and the codeword  $\mathbf{c}_{k,l} \in \mathcal{C}_{k,l}$ . The rate of this code is

$$R_{k,l} = \frac{1}{n} \log \frac{\text{Vol}(\mathcal{V}_{k,l})}{\text{Vol}(\mathcal{V}_{k,l-1})} = \frac{1}{2} \log \left( \frac{P_{k,l}}{P_{k,l-1}} \right). \quad (10)$$

The transmitted signal of user  $k$  is the summation of all the codewords where the corresponding layers are active, i.e.,

$$\mathbf{s}_k = \sum_{l=1}^{L_k} b_{k,l} \mathbf{c}_{k,l}. \quad (11)$$

The received signal at user  $k$  is the superposition of the signals from all transmitted users, which is

$$\mathbf{r}_k = \sum_{j=1}^K h_{k,j} \mathbf{s}_j + \mathbf{v}_k. \quad (12)$$

However, it has decoding problem if we just use  $L_k$  layers of nested lattice code for each user  $k$ . As shown in Fig. 3, the received signals and interference are in layers and might be interlaced. If an interference layer is

above a signal layer, we need first to decode and cancel the interference layer before decoding the signal layer. If the interference from two or more users exist above a signal layer, we need to decode the superimposed interference irrespective of whether the boundaries of interference layers are aligned or not.

For example, at receiver  $k$ , above the  $l$ -th signal layer there is a superimposed interference layer that is added by the  $l'$ -th layer of user  $j$  and the  $l''$ -th layer of user  $i$ . The upper boundary of the signal layer is  $h_{k,k}^2 P_{k,l}$ , and the lower boundaries of two interference layers are respectively  $h_{k,j}^2 P_{j,l'-1}$  and  $h_{k,i}^2 P_{i,l''-1}$ , where we have  $h_{k,j}^2 P_{j,l'-1} \geq h_{k,k}^2 P_{k,l}$  and  $h_{k,i}^2 P_{i,l''-1} \geq h_{k,k}^2 P_{k,l}$ .

The received signal codeword is  $h_{k,k} \mathbf{c}_{k,l} \in h_{k,k} \Lambda_{k,l-1}$ , and the received interference codewords are  $h_{k,j} \mathbf{c}_{j,l'} \in h_{k,j} \Lambda_{j,l'-1}$  and  $h_{k,i} \mathbf{c}_{i,l''} \in h_{k,i} \Lambda_{i,l''-1}$ . To decode the superimposed interference  $h_{k,j} \mathbf{c}_{j,l'} + h_{k,i} \mathbf{c}_{i,l''}$ , we require that the lattices  $h_{k,j} \Lambda_{j,l'-1}$  and  $h_{k,i} \Lambda_{i,l''-1}$  are aligned or nested. Through adjusting the transmit powers of related layers, we can make  $h_{k,j} \Lambda_{j,l'-1}$  and  $h_{k,i} \Lambda_{i,l''-1}$  aligned at receiver  $k$ , but it is impossible to simultaneously make  $h_{k,j} \Lambda_{j,l'-1}$  and  $h_{k',i} \Lambda_{i,l''-1}$  aligned at another receiver  $k'$ .

To solve this problem, we just require a relationship that  $h_{k,j} \Lambda_{j,l'-1}$  and  $h_{k,i} \Lambda_{i,l''-1}$  are nested at receiver  $k$ . Since  $h_{k,j}^2 P_{j,l'-1} \geq h_{k,k}^2 P_{k,l}$  and  $h_{k,i}^2 P_{i,l''-1} \geq h_{k,k}^2 P_{k,l}$ , we can add a level of nested lattice  $\Lambda_{j,m}$  with second moment  $h_{k,k}^2 / h_{k,j}^2 P_{k,l}$  at transmitter  $j$ , and add a level of nested lattice  $\Lambda_{i,m}$  with second moment  $h_{k,k}^2 / h_{k,i}^2 P_{k,l}$  at transmitter  $i$ . The lattice nesting relation becomes  $\Lambda_{j,l'-1} \subseteq \Lambda_{j,m} \subseteq \Lambda_{j,l'-2}$  and  $\Lambda_{i,l''-1} \subseteq \Lambda_{i,m} \subseteq \Lambda_{i,l''-2}$ . Thus, both  $h_{k,j} \mathbf{c}_{j,l'}$  and  $h_{k,i} \mathbf{c}_{i,l''}$  are nested in  $h_{k,k} \Lambda_{k,l}$ . Obviously, their sum is also nested in  $h_{k,k} \Lambda_{k,l}$ . The superimposed interferences can then be decoded. Similarly, at receiver  $k'$ , if there are interference layers above the signal layers and the interference layer is superimposed from multiple users, we can add more levels of nested lattice at the transmitter side to make the received interference codewords nested.

After these level insertion operations, the nested lattice levels increase to  $L_k^+$  for user  $k$ , but the number of active signal layers does not change. The decoded codeword of the  $l$ -th signal layer is

$$\hat{\mathbf{c}}_{k,l} = [Q_{\Lambda_{k,l-1}}(\mathbf{r}_k)] \bmod \Lambda_{k,l}, \quad (13)$$

i.e., the received signal  $\mathbf{r}_k$  is first quantized to lattice  $\Lambda_{k,l-1}$  and then taken a modulus operation relative to lattice  $\Lambda_{k,l}$ . The quantization operation is to mitigate the interference of lower layers, and the modulus operation can mitigate the interference of the upper layers by decoding and cancellation.

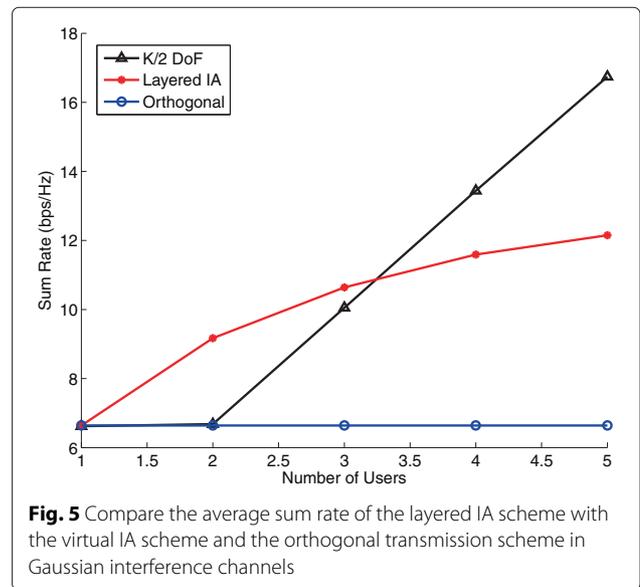
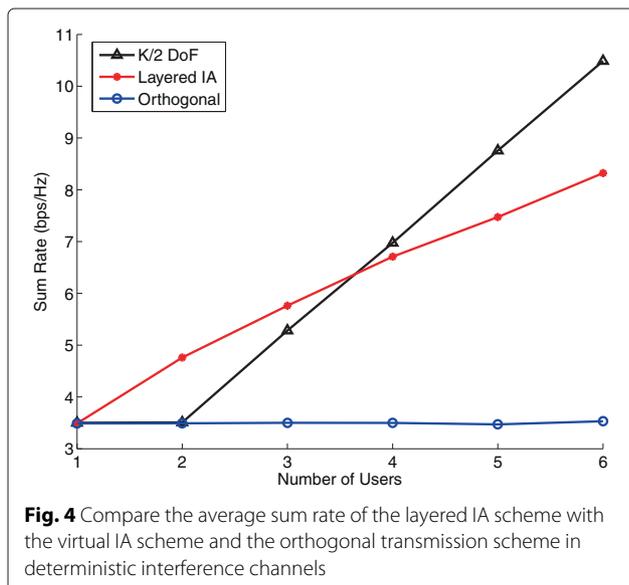
The developed implementation scheme is based on the idea of nested alignment and can guarantee the superimposed interference decodable at every receiver with any kind of power relationships.

### 5 Simulation results

We first study the performance of the layered IA scheme in deterministic interference channels and compare it with the orthogonal transmission scheme, and a virtual IA scheme where each user can achieve 1/2 DoF of the channel.

The virtual IA scheme demonstrated here is not a real transmission scheme. Since currently there is only DoF result for the layered IA transmission with arbitrary channel coefficients, we use this result to provide a performance benchmark of the achievable sum rate. In the virtual IA scheme, we compute the achievable rate of each user based on its SNR and without taking into account the impact of interference, and the sum rate of  $K$  users is obtained by the summation of each user’s achievable rate and divided by 2. This method reflects the fact that each user can exploit 1/2 DoF of the channel. If there is only one user, this user will use the full DoF of the channel. The sum rate obtained in this way is labeled as “ $K/2$  DoF” in Figs. 4 and 5. When SNR approaches infinity, the DoF result can reflect the upper bound of channel capacity. However, in moderate SNR values the achieved rate obtained from the DoF may have large difference with the capacity.

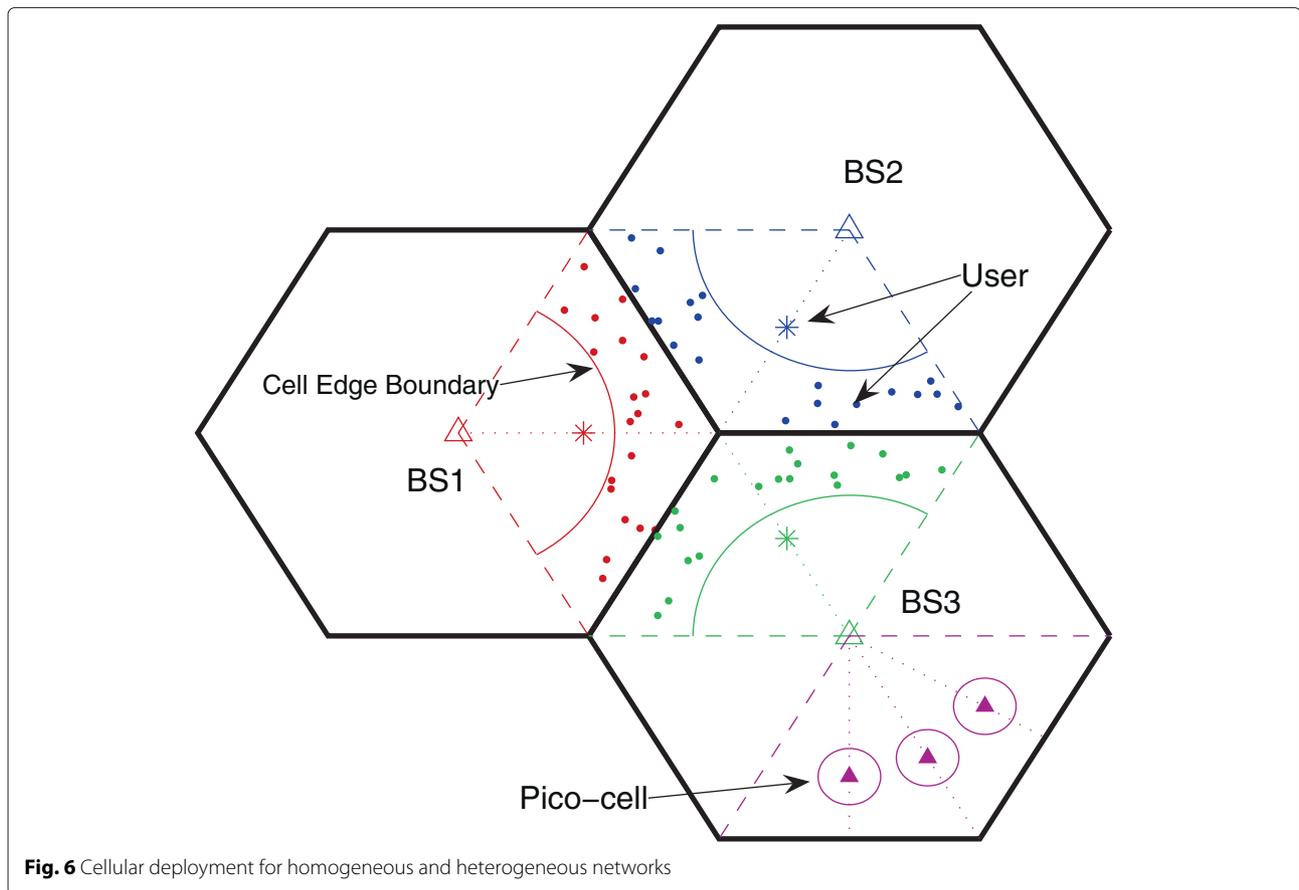
In deterministic models, the logarithmic channel gains are integer. We set the logarithmic SNRs and INRs as random integer selected from 1~6. In  $K$ -user orthogonal transmission scheme, each user occupies 1/ $K$  of the time or frequency resources no matter what the SNRs and INRs are. The corresponding results are shown in Fig. 4. We can see that the sum rate of the virtual IA scheme linearly increases with the number of users when  $K \geq 2$ , but its performance is not the best for all cases. In two-user or three-user interference channels, the proposed



layered IA scheme can achieve higher sum rate. According to the optimization process in Section 2, the active level assignment is not an equal distribution among users. By contrast, the user with better channel conditions (e.g., large SNR and small INR) might be assigned more active levels to maximize the sum rate. Hence, when the number of users is not too much, the optimized level assignment scheme outperforms the scheme that each user exploits one half of the channel resource. In interference channels with more than three users, the virtual IA scheme will gradually dominate the performance due to the advantage that each user can exploit 1/2 DoF of the channel. No matter how many users exist, the achieved sum rate of the orthogonal transmission scheme always keeps unchanged.

The comparisons of the layered IA scheme and other two schemes in Gaussian channels are shown in Fig. 5, where the SNRs and INRs are all randomly selected from 0~40 dB. Because of the carry-over effect, in moderate SNR levels, the achieved sum rate of the layered IA scheme grows not as fast as in deterministic channels. Yet with two and three users, the layered IA scheme still outperforms the virtual IA scheme. Similarly, with more users the virtual IA scheme will dominate the performance again since its achieved sum-rate keeps linearly increased.

We then investigate the performance of the layered IA scheme in cellular systems. Two network topologies, i.e., homogeneous networks and heterogeneous networks, are respectively studied. In homogeneous networks, every cell has similar coverage and the user is probably interfered by all other adjacent BSs. In heterogeneous networks, the macro-BS may interfere with a lot of pico-cell users, but the pico-BS only interferes with a few macro-cell users. Both kinds of cellular deployment are shown in Fig. 6.



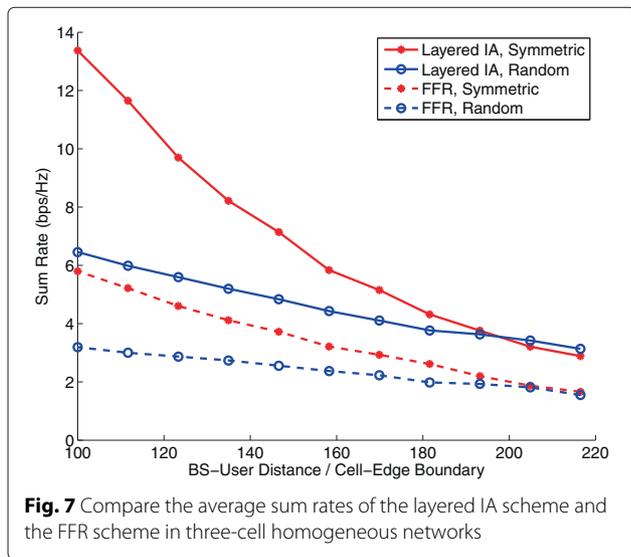
No matter in homogeneous or heterogeneous network deployments, we consider single-antenna configurations for BSs and users. This is to show the performance gain purely provided by the amplitude-space sharing scheme. The inter-site distance of the macro-BSs is 500 m, and the cell radius of pico-cell is 30 m. The transmit powers of macro-BS and pico-BS are 46 and 30 dBm, respectively. The path loss models for macro-BS to users and for pico-BS to users are chosen according to the 3GPP channel model [46]

$$\begin{aligned} PL_1 &= 15.3 + 37.6 \log_{10}(D), \\ PL_2 &= 30.6 + 36.7 \log_{10}(D), \end{aligned}$$

where  $D$  is the distance between the BS and user. The cell-edge SNR is set as 5 dB, and small-scale Rayleigh fading is also considered.

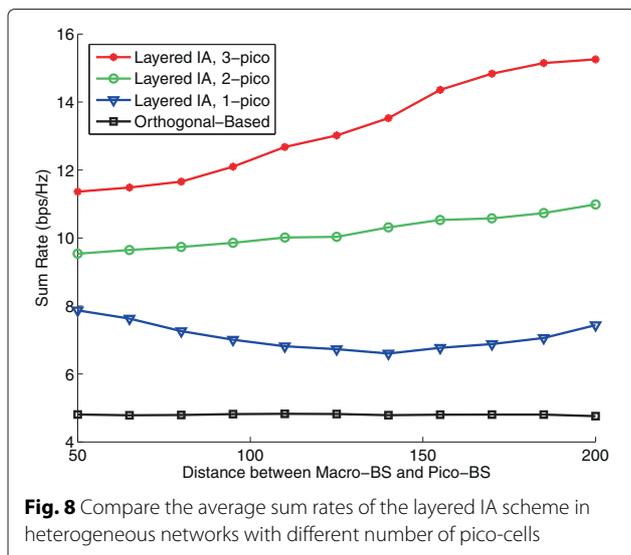
We first observe the performance of layered IA scheme in three-cell homogeneous networks. Although the proposed scheme can be applied with any number of cells, for the homogeneous deployment, the interferences to one cell are mainly from the adjacent two cells. The outer cells with further distance only contribute noisy interference and will not affect the layer participation and alignment

scheme. In this configuration, two kinds of user distribution are simulated. The first is symmetric distribution, where the macro-users are moved along the line connecting the macro-BS and the central vertex of the three cells, as the asterisks (“\*”) shown in Fig. 6, and their distances to macro-BSs are the same. The second is random distribution, where the macro-users are randomly distributed in the cell-edge areas, as the dots (“.”) shown in Fig. 6, and the cell-edge boundaries are changed symmetrically. Conventionally, a FFR scheme is used to avoid inter-cell interference in the cell edge areas, where each cell uses 1/3 of the available bandwidth. In the layered IA scheme, we use frequency reuse factor of one, i.e., every cell uses all the bandwidth, and the interferences are coordinated in the amplitude space. The sum rate performances are shown in Fig. 7. We can see that the layered IA scheme has approximately two times of the data rate over the FFR scheme both for the symmetric and random user distribution scenarios. The performance gain increases as the users approach the BSs. Although with small-scale fading, when the users are close to the BSs, the average SNRs are higher and the average INRs are lower; thus, there are more opportunities to assign active layers. The random distribution scenario is more practical; when the cell-edge boundary moves far away from the BS, the average sum



rate achieved in this scenario can be greater than that in symmetric distribution scenario. The reason is that random user distribution diverges the SNR and INR values, which also creates more opportunities to assign active layers.

The performance of the layered IA scheme in heterogeneous networks is evaluated in Fig. 8, where 1~3 pico-cells coexisting with one macro-cell are considered. In this kind of deployment, the macro-user is randomly distributed in one sector of the macro-cell, as shown in Fig. 6, and there is also one pico-user randomly distributed in each pico-cell. The layered IA scheme demonstrates great potential in this scenario, and the performance gain keeps increasing along with more coexisting pico-cells. When the distance between the pico-BS and macro-BS changes,



the sum rate varies in different patterns with different number of pico-cells. With only one pico-cell coexists with the macro-cell, it constitutes a two-user interference channel. Although the position of the macro-user is random, the interference scenarios that the pico-user experienced have certain rules to follow. When the pico-cell moves from cell center to cell edge, the interference caused by the macro-BS to the pico-user will change from strong to weak. In this kind of two-user interference channel, the sum rate is higher both in strong interference or weak interference scenarios and is lower when the interference has a similar strength with the signal. From the explanation in Section 3, we also know that the strong interference can share the upper amplitude space above the signal layer, and the weak interference can share the lower amplitude space below the signal layer. If the interference has a similar strength as the signal, both the signal and the interference will reduce their occupied amplitude space for coexisting. In the simulation results of Fig. 8, the average sum rate for one pico-cell configuration is a concave curve, which subsidizes our analysis.

When the number of pico-cells increases to two and three, the sum rate curves are monotonically increasing. The reason behind this phenomenon is the interference among the pico-cells. In the simulations, the SNRs and INRs of all links are computed from the transmit power and the simulated channel gains, which include the large-scale path loss and small-scale Rayleigh fading. When the pico-cells are close to the macro-BS, as can be seen from Fig. 6, the distances between these pico-cells are shorter. In this scenario, not only the interference from macro-BS to pico-users is stronger but also the interference from pico-BSs to pico-users. When the pico-cells move away from the macro-BS, the interference from macro-BS and pico-BS are all weaker. As observed in Section 3, the layer partitioning and assignment are complicated in the multi-user case. Although we do not provide an explicit analysis, the simulation results show that the average sum rate will increase when we move the pico-cells towards the macro-cell edge.

## 6 Conclusions

In this paper, we proposed a layered interference alignment scheme for Gaussian interference networks with arbitrary channel coefficients and applied the idea of amplitude-space sharing to homogeneous and heterogeneous cellular networks. We introduced a layer partitioning method and optimized the active layer assignment based on the insights obtained in deterministic channels. The transmission is implemented by multiple-level nested lattice codes, where the encoding method is judiciously designed to guarantee the superimposed interference layers keep aligned at all receivers. Simulation results show that in Gaussian interference channels the achieved sum

rate grows with the number of users, and with two or three users, the proposed scheme outperforms the virtual scheme that each user occupies half of the channel resource. In practical cellular systems, the layered IA scheme provided evident rate gain over the orthogonal-based transmission schemes and showed great potential to mitigate the complicated co-tier and cross-tier interferences in heterogeneous networks.

#### Competing interests

The authors declare that they have no competing interests.

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