Jia et al. EURASIP Journal on Wireless Communications and Networking (2015) 2015:203
DOI 10.1186/s13638-015-0431-1

EURASIP Journal on Wireless Communications and Networking
a SpringerOpen Journal

RESEARCH Open Access

Performance analysis of cooperative cognitive MIMO multiuser downlink transmission with perfect and imperfect CSI over Rayleigh fading channels

Xiangdong Jia[1,2,3*], Dan Wang[1], Xiaochao Dang[1], Longxiang Yang[2,3] and Hongbo Zhu[2,3]

Abstract

In this paper, a comprehensive comparison analysis in terms of outage probability and average symbol error ratio (SER) is presented for cooperative cognitive multiple-input and multiple-output (CC-MIMO) multiuser systems with amplify-and-forward (AF) protocol. Specially, we consider two scenarios where the CC-MIMO multiuser systems have the perfect and imperfect channel state information (CSI). The CC-MIMO multiuser systems consist of one multi-antenna source, one single-antenna relay, and multiple multi-antenna destinations. At the secondary source and destinations, the maximal ratio transmission (MRT) and maximal ratio combining (MRC) are employed, respectively. For such CC-MIMO multiuser systems, we first obtain the exact closed-form expressions of outage probability under the two cases where the CC-MIMO multiuser systems have the perfect and imperfect CSI. Then, to reduce the implementation complexity, the tight lower bounds of outage probability and average SER are derived. Finally, to obtain insight, by using the high signal-to-noise ratio (SNR) approximation, the asymptotic estimations of outage probability are achieved. The numerical results show that the derivations are agreed with the simulations, which validate our derivations. At the same time, the results show that, for the systems without perfect CSI, the achievable diversity order reduces to one, regardless of the number of antennas at the cognitive source and destinations as well as the number of the cognitive destinations. Nevertheless, these key parameters affect the coding gain of the CC-MIMO multiuser systems. When the systems have the perfect CSI (or without feedback delay), the achievable diversity gain is determined by the minimum between the number of source's antennas and the product of the number of destinations and the number of destination's antennas. For the effect of PU's parameters, our results indicate that primary systems only affect the coding gain but not the diversity gain.

Keywords: Cognitive radio; Cooperative relaying; MIMO; User selection; Diversity and coding gains

1 Introduction

In recent years, due to the dramatic growth of wireless applications, the spectrum scarcity is becoming a major issue in future cellular mobile communications, which has drawn a lot of attention in academia and industry [1]. However, the traditional spectrum allocation policy is static and leads to spectrum congestion. On the other hand, the spectrum resources are not sufficiently utilized as reported by the Federal Communications Commission (FCC). To overcome these drawbacks, cognitive radio (CR) has been proposed by Mitola [2] as an effective dynamic spectrum allocation policy. There are three main CR paradigms: underlay, overlay, and interweave [3]. Among the three existing CR paradigms, the underlay paradigm, also known as spectrum sharing, has been extensively studied due to the higher spectrum efficiency. In the underlay CR, the secondary users (SUs) are allocated to utilize the spectrum of the primary users (PUs) as long as the interference caused by SUs at PUs is below a given interference threshold (also referred as interference

* Correspondence: jiaxd@nwnu.edu.cn
[1]College of Computer Science and Engineering, Northwest Normal University, Lanzhou 730070, China
[2]Wireless Communication Key Lab of Jiangsu Province, Nanjing University of Posts and Telecommunications, Nanjing 210003, China
Full list of author information is available at the end of the article

temperature). This means that the SUs' transmission power is often subject to various interference constraints given by PUs. This leads to a limitation in the wireless coverage of secondary networks. Therefore, a major problem in the underlay CR is how to guarantee quality of service (QoS) for SUs and to extend the wireless coverage range of secondary networks.

To extend the wireless coverage and guarantee the QoS of the secondary networks, the cooperative cognitive radio has been introduced as a promising solution. Specially, it has been shown in [4] that the wireless coverage range and link reliability can be enhanced significantly through the help of secondary relays. Inspired by the remarkable potential from combining the user cooperative diversity and CR, recently, one novel cognitive network architecture, referred as multiuser cognitive relaying scheme, has been proposed in [5–11]. In such multiuser cognitive relaying schemes, there are multiple cognitive sources or destinations. In a data transmission round, only one cognitive source or destination are selected for communications. As a result, the multiuser diversity gain is achieved [12]. The scheme enhances the reliability of data transmission due to the multiuser diversity gain. At the same time, the user cooperation also extends the wireless coverage range. Specially, in [8], one multiuser and multirelay CR network has been investigated in terms of outage probability. The result in [7] shown that the diversity order of the cognitive systems is determined by the summation of the numbers of relays and destinations. On the contrary, [9] focused on the cognitive system which has single relay and multiple destinations. The transmit power at the cognitive transmitters subject to the multiple PUs. For such multiuser cognitive systems, the outage performance is investigated over Nakagami-m fading channels. In [10], authors have investigated the performance of dual-hop decode-and-forward (DF) spectrum-sharing systems over Nakagami-m fading channels. The primary system consists of one transceiver pair, whereas the secondary system consists of one secondary source, one secondary relay, and multiple secondary destinations. In particular, authors have presented a detailed performance comparison for two different opportunistic scheduling algorithms, i.e., signal-to-noise ratio- (SNR) and signal-to-interference ratio- (SINR) based scheduling algorithms. It was achieved that the SINR-based algorithm always outperforms the SNR-based scheduling algorithm. However, when the number of destinations is very large, both schemes attain almost the same outage performance. This suggests that, in such scenarios, the SNR-based scheduling algorithm is preferred because it requires less channel state information (CSI). While the multiuser downlink cognitive relay schemes have been discussed in [8] and [10], the counterpart uplink ones have been investigated in [11]. In this work, authors have presented the difference between relay selection and user selection. For multiuser uplink cognitive relaying schemes, in [8], authors have presented an optimal solution for secondary multiuser scheduling by selecting the optimal secondary source which maximizes the received SNR at the secondary destination.

For such multiuser cognitive relaying scheme, though the works [8–11] are of significance, the key limitation of these works is that all terminals are equipped with single antenna. It is well known that, besides CR and relaying, the multiple-input and multiple-output (MIMO) is also a promising solution to improve the spectrum utilization and wireless coverage [13]. Therefore, in an effort to further improve the performance of CR networks, the combination of MIMO and CR (referred as MIMO CR) has been introduced as another promising method to improve the spectrum efficiency and throughout of wireless communications [14]. For example, the spectrum sharing multi-hop MIMO scheme has been investigated and an effective precoding scheme is presented in [15]. In [16], a cognitive radio-inspired asymmetric network coding scheme is proposed for MIMO systems, where the information exchange among users and base station broadcasting can be accomplished simultaneously. The wireless resource allocation in MIMO-OFDM cognitive radio systems has been investigated in [17]. In this work, to maximize the achievable data rate, authors jointly performed the optimal subcarrier paring, cooperation SU selection, as well as the determination of the optimal transmit covariance matrices. An interesting cognitive MIMO scheme, cognitive multiuser MIMO, has been investigated in [18]. For the multiuser CR relaying systems, the robust beamforming scheme has been presented. The results in [15–18] show that the utilization of MIMO techniques can significantly improve the performance of cognitive systems. However, it is also well known that the excellent performance of MIMO systems is achieved at the cost of implementation complexity due to the perfect precoding. Therefore, researchers devote to find a scheme with low implementation complexity for multiple antenna systems. We know that, in MIMO system, the classical combining technique is maximum ratio combining (MRC) [19], where the signals from the received antenna elements are weighted such that the output SNR is maximized. Besides MRC, the maximal ratio transmission (MRT) is also a powerful diversity technique. The joint utilization of MRT and MRC (or MRT/MRC) in multiple antenna systems has been shown to offer many benefits such as increasing reliability and low complexity. As a result, the MRT/MRC is a realistic candidate for MIMO systems and can provide a good trade-off between implementation complexity and good performance.

Inspired by the above literature review, in this work, we deal with a comprehensive performance analysis of cooperative cognitive MIMO (CC-MIMO) multiuser downlink systems. In the interested schemes, the four promising techniques, CR, MIMO, multi-user selection, and relaying, are integrated perfectly together. Specially, the utilization of multi-user selection is a realistic consideration with high energy efficiency. For example, in wireless local network (WLN), there are multiple access points of Internet. Before communications, the user in WLN first selects one best access point. Then, during the communications only the selected best access point is activated, and the others keep silence. As a result, the energy efficiency is improved greatly even if the spectrum efficiency is degraded. The secondary system consists of one secondary source, one secondary relay, and multiple secondary destinations. The secondary source and secondary destinations are equipped with multiple antennas, while the secondary relay is equipped with one single antenna. Note that the reason that we employ single antenna relay is that, compared with base station, the relays should have low implementation complexity and engineering cost. For example, in a large wireless network, we should arrange many relays. If all relays are equipped with multiple antennas, the total engineering cost may be very expensive. Moreover, the perfect synchronization of multiple antenna systems leads to high implication complexity. Besides this, our schemes can be extended easily to the systems in which the relays have multiple antennas, and the relays employ the antenna selection technique. The transmit power of the secondary transmitters is constrained by the interference threshold of primary user. One data transmission round consists of two phases. In the first phase, by using MRT, the secondary source transmits signal to the single antenna secondary relay. In the second phase, only one secondary destination is selected for receiving data from relay. Then, by using the amplify-and-forward (AF) relaying protocol, the secondary relay transmits the received signal to the selected best destination. At the selected secondary destination, the MRC is employed to process the received signal from relay. At the same time, in this paper, we consider the effect of feedback delay on MRT/MRC and user selection. This is due to the fact that, in beamforming and user selection schemes, the excellent performance is achieved only when the perfect CSI is achieved at transmitters. However, in practice, the available CSI at transmitters and the actual instant channels may be different due to the time-varying channels and feedback delay [20, 21]. At the same time, for comparison analysis, the CC-MIMO multiuser systems are also investigated under the case where the systems have the perfect CSI. For such CC-MIMO multiuser systems, the outage probability and average symbol error ratio (SER) are investigated over Rayleigh fading channels. Specially, we first derive the exact closed-form expressions of outage probability for the two scenarios with and without perfect CSI. Due to the fact that the exact closed-form expressions are very complicated, to reduce the implementation complexity, the tight lower bounds of outage probability and average SER are derived. Finally, to gain valuable insights and to highlight the effect of system's parameters on the CC-MIMO multiuser systems, we also present the asymptotic performance analysis by using high SNR approximation. The impact of different antenna configurations, feedback delay, and SNR imbalance on the performance is illustrated.

1.1 Notations

Throughout this work, we use bold lower case letters to denote vectors and lower case letters to denote scalars, respectively. The probability density function (PDF) and the cumulative distribution function (CDF) of a random variable (RV) X are denoted as $f_X(.)$ and $F_X(.)$, respectively. The symbol $\|.\|_F$ is the Frobenious norm, $E[.]$ stands for the expectation operator, $(.)^T$ denotes the transpose operator, and $(.)^H$ denotes the conjugate transpose operator. $\Gamma(.)$ and $\Upsilon(.,.)$ denote the Gamma and incomplete Gamma function $K_v(.)$ is the vth-order modified Bessel function of the second kind. $W_{\lambda,\mu}(z)$ is the Whittaker function $H_{p\ q}^{m\ n}[.|.]$ is the Fox's H-function. ${}_2F_1(.,.,.,.)$ is the Gaussian hypergeometric function. At all receivers, the Gaussian additive noise power $N_0 = 1$.

2 System model and assumptions

2.1 System models

We consider one CC-MIMO multiuser downlink dual-hop transmission system as illustrated in Fig. 1, where the secondary system is allowed to utilize the same spectrum licensed to the primary system through underlay paradigm. In this cognitive system, there exists one secondary source SS, one secondary relay SR, and M secondary destinations SD_m, $m = \{1, \ldots, M\}$. It is assumed that the secondary source SS is equipped with N_1 antennas, and all secondary destinations SD_m have the same N_2 antennas. At the same time, with the consideration of implementation complexity and engineering cost, we consider the case where the secondary relay SR is equipped with one single antenna. Thus, at the cognitive source SS and the cognitive destinations, the MRT and MRC are employed for signal transmission and receiving. Moreover, the primary system only includes one primary receiver PR that is equipped with one single antenna. There is no direct link between the secondary source and destinations. The communications between SR and SD_m is accomplished with the help of the secondary relay SR.

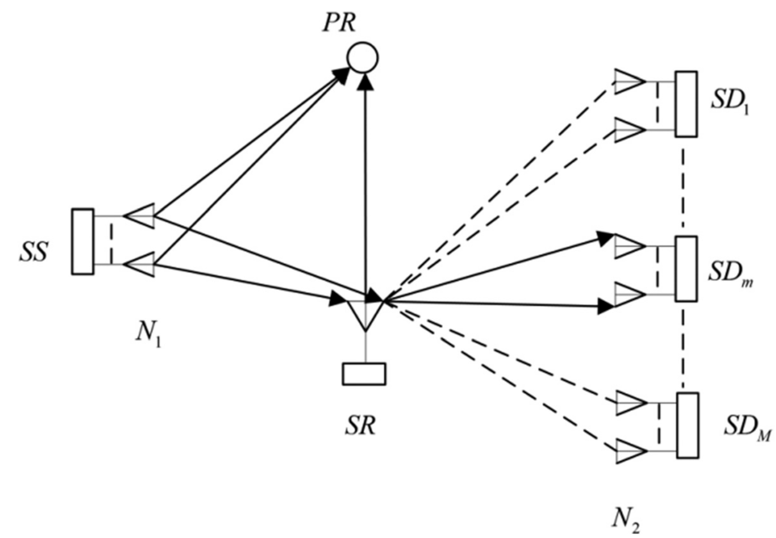


Fig. 1 CC-MIMO multiuser downlink model

Due to the fact that the secondary source SS and destinations SD_m have multiple antennas, the secondary system is equivalent to one cooperative MIMO system. With the consideration that the underlay paradigm is employed, the interference power at the primary receiver PR created by the secondary transmitters must be below a predefined interference threshold Q. We also assume that all links are subject to Rayleigh fading, and the received signals are corrupted by Gaussian additive noise with power N_0.

An entire communication round between the secondary SS and SD_m consists of two orthogonal phases. In the first phase, the secondary source SS transmits its signal x_S with power P_S, where $E[|x_S|^2] = 1$. To maximize the instantaneous received SNR at the secondary relay SR, the secondary source SS employs the MRT principle and steers the signal along the direction matching the first hop channel. Due to the fact that the cognitive source has N_1 antennas and the cognitive relay has single antenna, the channel between the secondary source and relay can be modeled as one $1 \times N_1$ vector $\mathbf{h}_1$. It is assumed that the elements h_{1j} of the channel vector $\mathbf{h}_1$ follow the independent and identically distributed (i.i.d) complex Gaussian distribution with zero mean and variance ω_1, i.e., $h_{1j} \sim CN(0, \omega_1)$, $j = \{1, \ldots, N_1\}$. Therefore, with the beamforming vector $\mathbf{W}_1$, the received signal at SR is written as

$$y_R = \sqrt{P_S}\mathbf{h}_1\mathbf{W}_1^{\mathrm{H}}x_S + n_R \tag{1}$$

where n_R is the Gaussian additive noise at the secondary relay SR, defined as $n_R \sim CN(0, N_0)$, and the beamforming vector is defined as

$$\mathbf{W}_1 = \frac{\mathbf{h}_1}{\|\mathbf{h}_1\|_F} \tag{2}$$

At the second phase, by using AF relaying protocol and multiplying an amplification fact G, the secondary relay SR forwards the received signal to the secondary destinations. Thus, for a given secondary destination SD_m, $m \in \{1, \ldots, M\}$, the received signal is given by

$$\mathbf{y}_{Dm}^{\mathrm{T}} = G\mathbf{h}_{2m}^{\mathrm{T}}\left[\sqrt{P_S}\mathbf{h}_1\mathbf{W}_1^{\mathrm{H}}x_S + n_R\right] + \mathbf{n}_D^{\mathrm{T}} \tag{3}$$

where $\mathbf{n}_D$ is the $1 \times N_2$ noise vector at the secondary destination, whose elements follow $CN(0, N_0)$, $\mathbf{h}_{2m}$ is the $1 \times N_2$ channel vector of the link from SR to SD_m, and its elements follow i.i.d complex Gaussian distribution with zero mean and variance ω_2, i.e., $h_{2mi} \sim CN(0, \omega_2)$, $i = \{1, \ldots, N_2\}$. Furthermore, according to (1), the amplification factor is given by

$$G^2 = \frac{P_R}{P_S\left|\mathbf{h}_1\mathbf{W}_1^{\mathrm{H}}\right|^2} \tag{4}$$

where P_R is the transmit power at the secondary relay SR. Then, at the secondary destination, the MRC is applied by multiplying the received signal $\mathbf{y}_{Dm}^{\mathrm{T}}$ with the beamforming vector $\mathbf{W}_2$. Therefore, after MRC processing, the received signal at the mth secondary destination SD_m is given by

$$y_{Dm} = G\mathbf{W}_2\mathbf{h}_{2m}^{\mathrm{T}}\left[\sqrt{P_S}\mathbf{h}_1\mathbf{W}_1^{\mathrm{H}}x_S + n_R\right] + \mathbf{W}_2\mathbf{n}_D^{\mathrm{T}} \tag{5}$$

Therefore, from (5), the instantaneous end-to-end SNR at the mth destination SD_m is given by

$$\gamma_{Dm} = \frac{P_S \left|\mathbf{h}_1 \mathbf{W}_1^{\mathrm{H}}\right|^2 \left\|\mathbf{h}_{2m}^{\mathrm{T}}\right\|_F^2}{N_0 \left\|\mathbf{h}_{2m}^{\mathrm{T}}\right\|_F^2 + N_0/G^2} \tag{6}$$

At the same time, due to the interference constraint Q at primary user, both the secondary source SS and relay SR must control their transmit powers P_S and P_R to meet the interference constraint at primary receiver, i.e., the transmit powers P_S and P_R constrained by [22–25].

$$P_S = \frac{Q}{\|\mathbf{h}_3\|_F^2} \quad P_R = \frac{Q}{|h_4|^2} \tag{7}$$

where $\mathbf{h}_3$ is the $1 \times N_1$ channel vector of the link from SS to PR with all i.i.d complex Gaussian RV entries, $h_{3i} \sim CN(0, \Omega_3)$, and h_4 is the channel coefficient of the link from SR to PR whose distribution is $CN(0, \Omega_4)$. Note that, in (7), only the PUs' interference power constraint is considered. Indeed, in realistic engineering implementation, the transmit power of SUs is also limited by the available maximum power level of SUs. Specially, when the value of the channel gains from SUs to PUs is very small, the effect of the available maximum power level must be considered. However, for simplicity, as an approximation, we can only consider the PUs' interference constraint as in [22–25].

Finally, combining (7), (6), and (4), the end-to-end SNR at the mth secondary destination SD_m is written as

$$\gamma_{Dm} = \frac{1}{N_0} \frac{\frac{Q\left|\mathbf{h}_1\mathbf{W}_1^{\mathrm{H}}\right|^2}{\|\mathbf{h}_3\|_F^2} \frac{Q\left\|\mathbf{h}_{2m}^{\mathrm{T}}\right\|_F^2}{|h_4|^2}}{\frac{Q\left\|\mathbf{h}_{2m}^{\mathrm{T}}\right\|_F^2}{|h_4|^2} + \frac{Q\left|\mathbf{h}_1\mathbf{W}_1^{\mathrm{H}}\right|^2}{\|\mathbf{h}_3\|_F^2}} = \frac{1}{N_0} \frac{\gamma_1 \gamma_{2m}}{\gamma_1 \gamma_4 + \gamma_{2m} \gamma_3} \tag{8}$$

where we define $\gamma_1 = Q\left|\mathbf{h}_1\mathbf{W}_1^{\mathrm{H}}\right|^2$, $\gamma_{2m} = Q\left\|\mathbf{h}_{2m}^{\mathrm{T}}\right\|_F^2$, $\gamma_3 = \|\mathbf{h}_3\|_F^2$, and $\gamma_4 = |h_4|^2$. Since $h_{1j} \sim CN(0, \omega_1)$, $j = \{1, \ldots, N_1\}$, it is achieved that the RV $\sqrt{Q}h_{1j} \sim CN(0, \Omega_1)$, where $\Omega_1 = Q\omega_1$. Similarly, we have that the RV $\sqrt{Q}h_{2mi} \sim CN(0, \Omega_2)$, $i = \{1, \ldots, N_2\}$, where $\Omega_2 = Q\omega_2$.

To exploit the multiuser diversity, the user selection is employed in our interested CC-MIMO multiuser systems. In this case, the secondary relay first identifies and selects the best relay-destination link out of all achievable relay-destination links and then feeds back the index of the selected best destination to the secondary source. Thus, the equivalent instantaneous SNR from the secondary relay to the selected best destination is given by

$$\gamma_2 = \underbrace{\max}_{m=\{1,\ldots,M\}} \left(\gamma_{2m}\right) = \underbrace{\max}_{m=\{1,\ldots,M\}} \left(Q\left\|\mathbf{h}_{2m}^{\mathrm{T}}\right\|_F^2\right) \tag{9}$$

Therefore, by using the above discussion, (8) and (9), and the assumption $N_0 = 1$, the equivalent instantaneous end-to-end SNR for the CC-MIMO multiuser system with the best destination user selection can be formulated as

$$\gamma_D = \frac{\gamma_1 \gamma_2}{\gamma_1 \gamma_4 + \gamma_2 \gamma_3} \tag{10}$$

2.2 Outdate CSI

It is well known, in practical implementation, due to channel estimation error, mobility, feedback delay, limited feedback, or feedback quantization, obtaining the full CSI is difficult, and often only partial CSI information can achieved. Thus, the analysis for imperfect CSI channel knowledge is also very beneficial for the design of systems. In our work, we only consider the imperfect CSI of secondary link, i.e., $\mathbf{h}_1$ and $\mathbf{h}_2$. This yields that in (8), (9), and (10), $\mathbf{h}_i$ are the delay version of the channel $\tilde{\mathbf{h}}_i$, $i = \{1, 2\}$. Specially, with feedback delay, the design of the beamforming vectors $\mathbf{W}_1$ and $\mathbf{W}_2$ and the user selection are based on the delay version. According to [26, 27], the relationship between $\tilde{\mathbf{h}}_i$ and $\mathbf{h}_i$ is expressed as the following time-varying channel model

$$\mathbf{h}_i = \rho_i \tilde{\mathbf{h}}_i + \sqrt{1-\rho_i^2}\mathbf{e}_i \quad i = \{1, 2\} \tag{11}$$

where ρ_i stands for the normalized correlation coefficient between $\mathbf{h}_i$ and $\tilde{\mathbf{h}}_\mathbf{i}$. According to the Jakes' autocorrelation mode [28, 29], we have $\rho_i = J_0(2\pi f_{di}\tau_i)$, where $J_0(.)$ is the zeroth-order Bessel function of the first kind, τ_i is the time delay, and f_{di} is the maximum Doppler frequency. Furthermore, $\mathbf{e}_i$ is an $1 \times N_i$ error vector whose elements follow the complex Gaussian distribution $CN(0, \omega_i)$. Obviously, under the case where the feedback delay exists, according to (8) and (9), we have the definitions $\tilde{\gamma}_1 = Q\left|\tilde{\mathbf{h}}_1\tilde{\mathbf{W}}_1^{\mathrm{H}}\right|^2$, $\tilde{\gamma}_{2m} = Q\left\|\tilde{\mathbf{h}}_{2m}^{\mathrm{T}}\right\|_F^2$, and $\tilde{\gamma}_2 = \underbrace{\max}_{m=\{1,\ldots,M\}} \left(\tilde{\gamma}_{2m}\right)$.

3 Statistical descriptions of RVS γ_1, γ_2, and γ_3

To obtain the outage performance and average SER of the interested CC-MIMO multiuser downlink transmission systems, (10) indicates that the statistical descriptions of the RVs γ_1, γ_2, and γ_3 are required firstly [30–33]. Therefore, in this section, we first present the statistical descriptions for the RVs γ_1, γ_2, and γ_3 as well as the detailed proof, which will be frequently utilized in the sequent derivations. Note that, although the statistical description of γ_1 has been presented in [30], it is also given here for the purpose of improving the readability of this paper.

Proposition 1: In the presence of feedback delay ($0 \le \rho_1 < 1$), the exact closed-form expressions for the PDF $f_{\gamma_1}(x)$ of the RV γ_1 is given by

$$f_{\gamma_1}(x) = \sum_{i=0}^{N_1-1} \frac{C_i^{N_1-1}}{\Omega_1^{N_1-i}} \frac{\rho_1^{N_1-i-1}(1-\rho_1)^i}{(N_1-i-1)!} x^{N_1-i-1} \exp\left(-\frac{x}{\Omega_1}\right) \tag{12}$$

$$F_{\gamma_1}(x) = 1-\sum_{i=0}^{N_1-1} C_i^{N_1-1}\rho_1^{N_1-i-1}(1-\rho_1)^i \sum_{p=0}^{N_1-i-1} \frac{1}{p!\Omega_1^p} x^p \exp \times \left(-\frac{x}{\Omega_1}\right) \tag{13}$$

where $C_n^N = N!/(n!(N-n)!)$ is the binomial coefficient. Δ

Proof: Since $h_{1j} \sim CN(0, \omega_1)$, where $j = \{1, \dots, N_1\}$, we have $\sqrt{Q}h_{1j} \sim CN(0, \Omega_1)$, where $\Omega_1 = Q\omega_1$. Thus, using the definition $\gamma_1 = Q\left|\mathbf{h}_1(t)\mathbf{W}_S^{\mathrm{H}}(t)\right|^2$ and the similar method as the one presented in [30], (12) and (13) can be achieved.

Δ

Proposition 2: In the presence of feedback delay ($0 \le \rho_2 < 1$), the exact closed-form expressions for the PDF $f_{\gamma_2}(x)$of the RV γ_2 are given by

$$f_{\gamma_2}(x) = \frac{M}{\Gamma(N_2)} \widehat{\sum} \Phi x^{N_2+kk-1} \exp\left(-\frac{(1+m)x}{(1+m(1-\rho_2))\Omega_2}\right) \tag{14}$$

where we define

$$\widehat{\sum} = \sum_{m=0}^{M-1} C_m^{M-1}(-1)^m \sum_{a_0+\ldots+a_{N_2-1}=m} \binom{m}{a_0, \dots, a_{N_2-1}} \prod_{0 \le t \le N_2-1} \left(\frac{1}{t!}\right)^{a_t} \tag{15}$$

$$\Phi = \sum_{kk=0}^{\sum_{t=0}^{N_2-1} a_t t} C_k^{\sum_{t=0}^{N_2-1} a_t t} \frac{\rho_2^{kk}(1-\rho_2)^{\left(\sum_{t=0}^{N_2-1} a_t t-kk\right)}}{(\Omega_2(1+m(1-\rho_2)))^{N_2+kk}(1+m(1-\rho_2))^{\sum_{t=0}^{N_2-1} a_t t}} \frac{\Gamma\left(\sum_{t=0}^{N_2-1} a_t t + N_2\right)}{\Gamma(N_2+kk)} \tag{16}$$

In (15), $\binom{m}{a_0, \dots, a_{N_2-1}} = \frac{m!}{a_0! \dots (a_{N_2-1})!}$ is a multinomial coefficient.

Proof: See Appendix A.

Δ

Proposition 3: In the presence of feedback delay ($0 \le \rho_2 < 1$), the exact closed-form expression for the CDF $F_{\gamma_2}(x)$ of the RV γ_2 is given by

$$F_{\gamma_2}(x) = 1-\frac{M}{\Gamma(N_2)} \widehat{\sum} \Phi \sum_{qq=0}^{N_2+kk-1} \frac{\Gamma(N_2+kk)}{(qq)!} \times \left(\frac{(1+m)x}{(1+m(1-\rho_2))\Omega_2}\right)^{qq-N_2-kk} \exp \times \left(-\frac{(1+m)x}{(1+m(1-\rho_2))\Omega_2}\right) x^{qq} \tag{17}$$

where $\widehat{\sum}$ and Φ are defined by (15) and (16), respectively.

Δ

Proof: By taking the integral of (14) with respect to γ_2 and using (3.351.1) in [34], we have

$$F_{\gamma_2}(x) = \frac{M}{\Gamma(N_2)} \widehat{\sum} \Phi \left(\frac{(1+m)}{(1+m(1-\rho_2))\Omega_2}\right)^{-(N_2+kk)} \Upsilon \times \left(N_2+kk, \frac{(1+m)x}{(1+m(1-\rho_2))\Omega_2}\right) \tag{18}$$

Then, using the series representation of the incomplete Gamma function $\Upsilon(.,.)$ leads to the result (17).

Proposition 4: If $\rho_2 = 1$, the PDF and CDF of the RV γ_2 defined by (9) are

$$f_{\gamma_2}(y) = \frac{M}{\Gamma(N_2)} \widehat{\sum} \left(\frac{1}{\Omega_2}\right)^{\sum_{t=0}^{N_2-1} a_t t+N_2} y^{\sum_{t=0}^{N_2-1} a_t t+N_2-1} \exp\left(-\frac{(m+1)y}{\Omega_2}\right) \tag{19}$$

$$F_{\gamma_2}(y) = \tilde{\sum} \exp\left(-\frac{m}{\Omega_2}y\right) \left(\frac{y}{\Omega_2}\right)^{\sum_{t=0}^{N_2-1} a_t t} \tag{20}$$

Δ

where $\widehat{\Sigma}$ is defined by (15).

Proof: Using the definition γ_2 defined by (9) and $F_{\gamma_{2m}}(y) = 1-\sum_{i=0}^{N_2-1}\frac{1}{i!}\left(\frac{y}{\Omega_2}\right)^i\exp\left(-\frac{y}{\Omega_2}\right)$, it is easy to see that the CDF $F_{\gamma_2}(y)$ is given by

$$F_{\gamma_2}(y) = \left(1-\sum_{i=0}^{N_2-1}\frac{1}{i!}\left(\frac{y}{\Omega_2}\right)^i\exp\left(-\frac{y}{\Omega_2}\right)\right)^M \tag{21}$$

By using the similar method as the one employed in Propositions 2 and 3, the results (19) and (20) are achieved.

4 Exact outage performance

Based on the statistical descriptions of the RVs γ_1, γ_2, and γ_3 presented in previous section, in this section, we investigate the outage probability of the CC-MIMO multiuser downlink transmission systems, which is an effective approach to quantify the system performance. Specially, by using appropriate mathematical proof, the closed-form expressions of outage probability would be achieved for the two cases where the CC-MIMO multiuser systems have the perfect and imperfect CSI. It is well known that the outage probability is defined as the probability that the instantaneous end-to-end SNR falls below a predefined outage threshold γ_{th} that is determined by the spectrum efficiency R_0 (bit/s/Hz). Mathematically, by using (10), the outage probability $P_{\text{Out}}(\gamma_{\text{th}})$ can be formulated as [35]

$$P_{\text{Out}}(\gamma_{\text{th}}) = \Pr\{\gamma_D < \gamma_{\text{th}}\} = F_{\gamma_D}(\gamma_{\text{th}}) \tag{22}$$

Since we consider two systems with and without perfect CSI, the corresponding outage probabilities in (22) are denoted by $P_{\text{Out}}^{\text{Exa-CSI}}(\gamma_{\text{th}})$ and $P_{\text{Out}}^{\text{Exa-ImCSI}}(\gamma_{\text{th}})$, respectively. Therefore, we have Theorem 1 that presents the exact closed-form expression of the outage probability for the CC-MIMO multiuser systems with imperfect CSI.

Theorem 1: For the CC-MIMO multiuser downlink transmission systems with imperfect CSI ($0 \le \rho_1 < 1$, $0 \le \rho_2 < 1$), the exact closed-form expression of outage probability is

$$\begin{aligned}
P_{\text{Out}}^{\text{Exa-ImCSI}}(\gamma_{\text{th}}) &= \Pr\{\gamma_D < \gamma_{\text{th}}\} = F_{\gamma_D}(\gamma_{\text{th}}) = 1-M\sum_{i=0}^{N_1-1}C_i^{N_1-1}\rho_1^{N_1-i-1}(1-\rho_1)^i\sum_{p=0}^{N_1-i-1}\sum_{q=0}^{p}\frac{C_q^p}{p!}\widehat{\sum}\Phi\sum_{tt=0}^{N_2+kk-1}C_{tt}^{N_2+kk-1}\\
&\times\frac{\Gamma(N_1+p+tt-q+1)\Gamma(N_1+p)}{\Gamma(N_2)\Gamma(N_1)}\frac{\Gamma(N_2+kk+1)\Gamma(N_2+kk-tt+q)}{\Gamma(N_2+N_1+kk+p+1)}\\
&\times\left(\frac{(1+m(1-\rho_2))}{(1+m)}\right)^{(N_2+kk+1)}\frac{\Omega_1^{N_1}\Omega_3^{p+tt-q+1}\Omega_4^{(N_2+kk)}\Omega_2^{[N_2+kk+1]}\gamma_{\text{th}}{}^{N_2+kk+p+tt-q+1}}{(\Omega_1+\gamma_{\text{th}}\Omega_3)^{N_1+p+tt-q+1}\left(\gamma_{\text{th}}\Omega_4+\frac{(1+m(1-\rho_2))\Omega_2}{(1+m)}\right)^{(N_2+kk+1)}}\\
&\times{}_2F_1\left(N_2+kk+1,N_1+p+tt-q+1,N_2+N_1+kk+p+1,\frac{(1+m)\gamma_{\text{th}}\Omega_1\Omega_4+(1+m(1-\rho_2))(\Omega_2\Omega_1+\gamma_{\text{th}}\Omega_2\Omega_3)}{(\Omega_1+\gamma_{\text{th}}\Omega_3)((1+m)\gamma_{\text{th}}\Omega_4+(1+m(1-\rho_2))\Omega_2)}\right)
\end{aligned} \tag{23}$$

where $\widehat{\sum}$ and Φ are defined by (15) and (16), respectively.

Proof: See Appendix B

Δ

In Theorem 1, the exact closed-form expression of outage probability (or the CDF of the equivalent end-to-end SNR) is presented for the CC-MIMO multiuser systems without the perfect CSI. Using the obtained result and the similar method, we also can obtain the exact outage probability for the CC-MIMO multiuser systems with the perfect CSI, i.e., $\rho_1 = \rho_2 = 1$. We have Corollary 1.

Corollary 1: Under the case where the CC-MIMO multiuser systems have the perfect CSI ($\rho_1 = \rho_2 = 1$), the exact outage probability is given by

$$\begin{aligned}
P_{\text{Out}}^{\text{Exa-CSI}}(\gamma_{\text{th}}) &= 1-M\sum_{p=0}^{N_1-1}\sum_{q=0}^{p}\frac{C_q^p}{p!}\widehat{\sum}\sum_{tt=0}^{N_2+\varphi-1}C_{tt}^{N_2+\varphi-1}\\
&\times\frac{\Gamma(N_1+p+tt-q+1)\Gamma(N_1+p)}{\Gamma(N_2)\Gamma(N_1)}\\
&\times\frac{\Gamma(N_2+\varphi+1)\Gamma(N_2+\varphi-tt+q)}{\Gamma(N_2+N_1+\varphi+p+1)}\times\left(\frac{1}{1+m}\right)^{(N_2+\varphi+1)}\\
&\times\frac{\Omega_1^{N_1}\Omega_2\Omega_3^{p+tt-q+1}\Omega_4^{N_2+\varphi}\gamma_{th}{}^{N_2+\varphi+p+tt-q+1}}{(\Omega_1+\gamma_{\text{th}}\Omega_3)^{N_1+p+tt-q+1}\left(\gamma_{\text{th}}\Omega_4+\frac{\Omega_2}{1+m}\right)^{N_2+\varphi+1}}\\
&\times{}_2F_1\left(N_2+\varphi+1,N_1+p+tt-q+1,N_2+N_1+\varphi\right.\\
&\left.+p+1,\frac{(1+m)\gamma_{\text{th}}\Omega_1\Omega_4+(\Omega_2\Omega_1+\gamma_{\text{th}}\Omega_2\Omega_3)}{(\Omega_1+\gamma_{\text{th}}\Omega_3)((1+m)\gamma_{\text{th}}\Omega_4+\Omega_2)}\right)
\end{aligned} \tag{24}$$

Proof: See Appendix C

Δ

5 Tight lower bounds of outage probability and average SER

In Section 4, we obtain the exact closed-form expressions of the outage probability (or the CDF of the equivalent end-to-end SNR). In practice, besides the outage performance metric, there are some other performance metrics that are very important, such as diversity and coding gain, average SER, ergodic capacity, and so on. With these performance metrics, we can obtain the insight and highlight the effect of system parameters on performance of the interested CC-MIMO multiuser systems. However, the derivations in Section 4 show that the exact expressions of outage probability (or CDF) are very complicated. This yields that it is intractable to obtain the insight. An alternative is to obtain the performance bounds and high SNR approximation. The performance bounds are usually used to obtain the approximated estimation, while the high SNR approximation is used for the investigation of diversity and coding gains. Therefore, in this section, we investigate the tight lower bounds of outage probability and average SER for the systems with and without perfect CSI. For the high SNR approximations and the investigation on diversity and coding gains, it would be presented in Section 6.

5.1 Tight lower bound of outage probability

It is well known that the instantaneous end-to-end SNR (10) can be upper bounded by [25]

$$\gamma_D \le \gamma_{\mathrm{Up}} = \min\left\{\frac{\gamma_1}{\gamma_3}, \frac{\gamma_2}{\gamma_4}\right\} \tag{25}$$

Since γ_{Up} is the upper bound of γ_D, the corresponding outage probability is lower bound of the exact probability, defined as $P_{\mathrm{Out}}^{\mathrm{Low}}(\gamma_{\mathrm{th}})$. Thus, we have

$$P_{\mathrm{Out}}^{\mathrm{Low}}(\gamma_{\mathrm{th}}) = \Pr\left\{\gamma_{\mathrm{Up}} \le \gamma_{\mathrm{th}}\right\} = F_{\gamma_{\mathrm{Up}}}(\gamma_{\mathrm{th}}) \tag{26}$$

Due to the independence of random variables γ_1, γ_2, γ_3, and γ_4, using the order statistics [33] in (26), the CDF $F_{\gamma_{\mathrm{Up}}}(\gamma_{\mathrm{th}})$ can be expressed as

$$\begin{aligned} F_{\gamma_{\mathrm{Up}}}(\gamma_{\mathrm{th}}) &= \Pr\left\{\min\left\{\frac{\gamma_1}{\gamma_3}, \frac{\gamma_2}{\gamma_4}\right\} \le \gamma\right\} \\ &= 1 - \left(1 - F_{\gamma_{13}}(\gamma)\right)\left(1 - F_{\gamma_{24}}(\gamma)\right) \end{aligned} \tag{27}$$

where we define $\gamma_{13} = \frac{\gamma_1}{\gamma_3}$ and $\gamma_{24} = \frac{\gamma_2}{\gamma_4}$. Eqs. (26) and (27) indicate that, to obtain the tight lower bound $P_{\mathrm{Out}}^{\mathrm{Low}}(\gamma_{\mathrm{th}})$, the CDF $F_{\gamma_{\mathrm{Up}}}(\gamma)$ is required. Therefore, we first consider the CDF $F_{\gamma_{\mathrm{Up}}}(\gamma)$ of the RV γ_{Up}.

For the CC-MIMO multiuser systems without perfect CSI, we denote the outage probability $P_{\mathrm{Out}}^{\mathrm{Low}}(\gamma_{\mathrm{th}})$ and CDF $F_{\gamma_{\mathrm{Up}}}(\gamma_{\mathrm{th}})$ in (26) as $P_{\mathrm{Out}}^{\text{Low-ImCSI}}(\gamma_{\mathrm{th}})$ and $F_{\gamma_{\mathrm{Up}}^{\mathrm{ImCSI}}}(\gamma_{\mathrm{th}})$, respectively. Thus, with the definition $\gamma_{13} = \frac{\gamma_1}{\gamma_3}$, in (27), the CDF $F_{\gamma_{13}}(\gamma)$ is expressed as

$$F_{\gamma_{13}}(\gamma) = \int_0^\infty F_{\gamma_1}(\gamma_3\gamma) f_{\gamma_3}(\gamma_3)\, d\gamma_3 \tag{28}$$

The CDF $F_{\gamma_1}(.)$ is given by (13), and PDF $f_{\gamma_3}(.)$ is

$$f_{\gamma_3}(\gamma_3) = \frac{1}{\Gamma(N_1)} \frac{1}{\Omega_3^{N_1}} \gamma_3^{N_1-1} \exp\left(-\frac{\gamma_3}{\Omega_3}\right) \tag{29}$$

Then, using (3.351.3) in [34], after some algebra manipulation, we have

$$F_{\gamma_{13}}(\gamma) = 1 - \sum_{F_{13}} \frac{1}{(\Omega_1 + \Omega_3\gamma)^{(N_1+p)}} \gamma^p \tag{30}$$

Where we define

$$\sum_{F_{13}} = \sum_{i=0}^{N_1-1} C_i^{N_1-1} \rho_1^{N_1-i-1} (1-\rho_1)^i \sum_{p=0}^{N_1-i-1} \frac{\Omega_1^{N_1}\Omega_3^p}{p!} \frac{\Gamma(N_1+p)}{\Gamma(N_1)} \tag{31}$$

Similarly, with $\gamma_{24} = \frac{\gamma_2}{\gamma_4}$, the CDF $F_{\gamma_{24}}(\gamma)$ in (27) is written as

$$F_{\gamma_{24}}(\gamma) = \int_0^\infty F_{\gamma_2}(\gamma\gamma_4) f_{\gamma_4}(\gamma_4)\, d\gamma_4 \tag{32}$$

Substituting the CDF $F_{\gamma_2}(.)$ given by (17) and $f_{\gamma_4}(\gamma_4) = \frac{1}{\Omega_4}\exp\left(-\frac{\gamma_4}{\Omega_4}\right)$ into (32), we have

$$F_{\gamma_{24}}(\gamma) = 1 - \sum_{F_{24}} \left(\frac{(1+m)\gamma}{(1+m(1-\rho_2))\Omega_2} + \frac{1}{\Omega_4}\right)^{-qq-1} (\gamma)^{qq} \tag{33}$$

where we define

$$\sum_{F_{24}} = \frac{M}{\Gamma(N_2)} \widehat{\sum\Phi} \sum_{qq=0}^{N_2+kk-1} \frac{\Gamma(N_2+kk)}{\Omega_4} \left(\frac{(1+m)}{(1+m(1-\rho_2))\Omega_2}\right)^{qq-N_2-kk} \tag{34}$$

Therefore, combining (26), (27), (30), and (33), we have Theorem 2 that presents that tight lower bound of outage probability for the CC-MIMO multiuser systems without perfect CSI.

Theorem 2: For the CC-MIMO multiuser downlink transmission systems with imperfect CSI, the exact outage probability is lower bounded by

$$P_{\mathrm{Out}}^{\text{Low-ImCSI}}(\gamma_{\mathrm{th}}) = \Pr\left\{\gamma_{\mathrm{Up}} < \gamma_{\mathrm{th}}\right\} = F_{\gamma_{\mathrm{Up}}^{\mathrm{ImCSI}}}(\gamma_{\mathrm{th}}) \tag{35}$$

where the CDF $F_{\gamma_{\mathrm{Up}}^{\mathrm{ImCSI}}}(\gamma_{\mathrm{th}})$ is given by

$$F_{\gamma_{\mathrm{Up}}^{\mathrm{ImCSI}}}(\gamma_{\mathrm{th}}) = 1 - \sum_{F_{13}} \sum_{F_{24}} \left(\frac{(1+m)\gamma_{\mathrm{th}}}{(1+m(1-\rho_2))\Omega_2} + \frac{1}{\Omega_4} \right)^{-qq-1} \times (\Omega_1 + \Omega_3 \gamma_{\mathrm{th}})^{-(N_1+p)} (\gamma_{\mathrm{th}})^{p+qq} \tag{36}$$

where $\sum_{F_{13}}$ and $\sum_{F_{24}}$ are defined by (31) and (34), respectively.

Δ

From Theorem 2, we can obtain the tight lower bound of outage probability for the CC-MIMO systems with perfect CSI, which is given by Corollary 2. Note that, in this case where the systems have the perfect CSI, we denote the outage probability $P_{\mathrm{Out}}^{\mathrm{Low}}(\gamma_{\mathrm{th}})$ and CDF $F_{\gamma_{\mathrm{Up}}}(\gamma_{\mathrm{th}})$ in (26) as $P_{\mathrm{Out}}^{\mathrm{Low\text{-}CSI}}(\gamma_{\mathrm{th}})$ and $F_{\gamma_{\mathrm{Up}}^{\mathrm{CSI}}}(\gamma_{\mathrm{th}})$, respectively.

Corollary 2: Under the case where the CC-MIMO multiuser downlink transmission systems have the perfect CSI ($\rho_1 = \rho_2 = 1$), the tight lower bound of outage probability is given by

$$P_{\mathrm{Out}}^{\mathrm{Low\text{-}CSI}}(\gamma_{\mathrm{th}}) = F_{\gamma_{\mathrm{Up}}^{\mathrm{CSI}}}(\gamma_{\mathrm{th}}) \tag{37}$$

where the CDF $F_{\gamma_{\mathrm{Up}}^{\mathrm{CSI}}}(\gamma_{\mathrm{th}})$ is given by

$$F_{\gamma_{\mathrm{Up}}^{\mathrm{CSI}}}(\gamma_{\mathrm{th}}) = 1 - \tilde{\sum} \sum_{i=0}^{N_1-1} \frac{1}{i!} \frac{\Gamma(N_1+i)\Gamma\left(\sum_{t=0}^{N_2-1} a_t t + 1\right)}{\Gamma(N_1)} \times \left(\frac{\Omega_3}{\Omega_1}\right)^i \left(\frac{\Omega_4}{\Omega_2}\right)^{\sum_{t=0}^{N_2-1} a_t t} \left(1 + \frac{\Omega_3}{\Omega_1}\gamma_{\mathrm{th}}\right)^{-N_1-i} \times \left(1 + \frac{m\Omega_4 r_{\mathrm{th}}}{\Omega_2}\right)^{-\sum_{t=0}^{N_2-1} a_t t - 1} \gamma_{\mathrm{th}}^{\sum_{t=0}^{N_2-1} a_t t + i} \tag{38}$$

where $\tilde{\sum}$ is defined by (15).

Δ

Proof: In the case $\rho_1 = 1$, using (29) and $F_{\gamma_1}(\gamma_1) = 1 - \sum_{i=0}^{N_1-1} \frac{1}{i!}\left(\frac{\gamma_1}{\Omega_1}\right)^i \exp\left(-\frac{\gamma_1}{\Omega_1}\right)$, it is easy to see that, in (27), $F_{\gamma_{13}}(\gamma)$ can be formulated as

$$F_{\gamma_{13}}(\gamma) = 1 - \sum_{i=0}^{N_1-1} \frac{1}{i!} \frac{\Gamma(N_1+i)\Omega_3^i \gamma^i \Omega_1^{N_1}}{\Gamma(N_1)(\gamma\Omega_3 + \Omega_1)^{N_1+i}} \tag{39}$$

Furthermore, in the case $\rho_2 = 1$, using (20) leads to $F_{\gamma_{24}}(\gamma)$ in (27) given by

$$F_{\gamma_{24}}(\gamma) = \int_0^\infty F_{\gamma_2}(\gamma\gamma_4) f_{\gamma_4}(\gamma_4) d\gamma_4 = 1 - \tilde{\sum} \times \left(\sum_{t=0}^{N_2-1} a_t t\right)! \frac{1}{\Omega_4} \left(\frac{\gamma}{\Omega_2}\right)^{\sum_{t=0}^{N_2-1} a_t t} \times \left(\frac{1}{\Omega_4} + \frac{\mathrm{mr}}{\Omega_2}\right)^{-\sum_{t=0}^{N_2-1} a_t t - 1} \tag{40}$$

Finally, using the fact $P_{\mathrm{Out}}^{\mathrm{Low\text{-}CSI}}(\gamma_{\mathrm{th}}) = 1 - (1 - F_{\gamma_{13}}(\gamma_{\mathrm{th}}))(1 - F_{\gamma_{24}}(\gamma_{\mathrm{th}}))$, we have Corollary 2.

Δ

5.2 Tight lower bound of average SER

For a broad variety of modulations, the average SER can be expressed approximately as $\bar{P}_E = aE\left[Q\sqrt{2b\gamma_{\mathrm{Up}}}\right]$, where $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty \exp\left(-\frac{y^2}{2}\right) d\gamma$ is the Gaussian Q-function, where the parameters a and b depend on the type of modulation. Specially, $a = 2(N-1)/N$, $b = 3/(N^2-1)$ for rectangular N-PAM, $a = 2$, $b = \sin^2(\pi/N)$ for N-PSK ($N \geq 4$), and $a = b = 1$ for phase-shift keying (BPSK). After taking the integrating by parts, the average SER can be alternatively re-expressed by

$$\bar{P}_E = \frac{a}{2}\sqrt{\frac{b}{\pi}} \int_0^\infty \frac{F_{\gamma_{\mathrm{Up}}}(\gamma)}{\sqrt{\gamma}} \exp(-b\gamma) d\gamma \tag{41}$$

In (41), for the two systems with and without perfect CSI, the $F_{\gamma_{\mathrm{Up}}}(\gamma)$ is given by (36) and (38), respectively. Therefore, for the CC-MIMO multiuser systems without perfect CSI, by substituting the CDF $F_{\gamma_{\mathrm{Up}}^{\mathrm{ImCSI}}}(\gamma_{\mathrm{th}})$ given by (36) into (41), we have Theorem 3.

Theorem 3: For the CC-MIMO multiuser downlink transmission systems with imperfect CSI, the lower bound of average SER is given by

$$\bar{P}_E^{\mathrm{Low\text{-}ImCSI}} = \frac{a}{2} - \frac{a}{2}\sqrt{\frac{b}{\pi}} \sum_{F_{13}} \sum_{F_{24}} \frac{\Omega_1^{-(N_1+p)} \Omega_4^{qq+1}}{\Gamma(qq+1)\Gamma(N_1+p)} b^{-p-qq-\frac{1}{2}} H^{1,\ 1,\ 1,\ 1}_{1,\ [1:1],0,[1:1]} \times \left[\begin{array}{c} \frac{\Omega_3}{\Omega_1 b} \\ \frac{(1+m)\Omega_4}{(1+m(1-\rho_2))\Omega_2 b} \end{array} \middle| \begin{array}{cc} \multicolumn{2}{c}{\left(1+qq+p-\frac{1}{2}, 1\right)} \\ (1-p-N_1, 1); & (qq, 1) \\ \multicolumn{2}{c}{----} \\ (0,1); & (0,1) \end{array} \right] \tag{42}$$

where $H^{K,N,N',M,M'}_{E,[A:C],F,[B:D]}[.|.]$ is the generalized Fox's H-function [36].

Δ

Proof: Combining (36) and (41), we have the average SER formulated by

$$\bar{P}_E^{\text{Low-ImCSI}} = \underbrace{\frac{a}{2}\sqrt{\frac{b}{\pi}}\int_0^\infty \frac{1}{\sqrt{\gamma}}\exp(-b\gamma)d\gamma}_{E_1} - \frac{a}{2}\sqrt{\frac{b}{\pi}}\sum_{F_{13}}\sum_{F_{24}}\underbrace{\int_0^\infty \left(\frac{(1+m)\gamma}{(1+m(1-\rho_2))\Omega_2}+\frac{1}{\Omega_4}\right)^{-qq-1}(\Omega_1+\Omega_3\gamma)^{-(N_1+p)}\gamma^{p+qq-\frac{1}{2}}\exp(-b\gamma)\,d\gamma}_{E_2} \tag{43}$$

With the help of (3.361.2) in [34], the integral E_1 is given by

$$\int_0^\infty \frac{1}{\sqrt{\gamma}}\exp(-b\gamma)d\gamma = \frac{1}{\sqrt{b}}\Gamma\left(\frac{1}{2}\right) \tag{44}$$

Using (1.7.1) in [36], $(1+cx)^{-b} = \frac{1}{\Gamma(b)}H_{11}^{11}\left[cx\left|\begin{matrix}(1-b,1)\\(0,1)\end{matrix}\right.\right]$. Thus, in (43), the integral term E_2 is written as

$$\begin{aligned}E_2 = {} & \frac{\Omega_1^{-(N_1+p)}\Omega_4^{qq+1}}{\Gamma(qq+1)\Gamma(N_1+p)}\int_0^\infty H_{1\ 1}^{1\ 1} \\ & \times\left[\frac{(1+m)\Omega_4\gamma}{(1+m(1-\rho_2))\Omega_2}\left|\begin{matrix}1-(qq+1)\\0\end{matrix}\right.\right]H_{1\ 1}^{1\ 1} \\ & \times\left[\frac{\Omega_3}{\Omega_1}\gamma\left|\begin{matrix}1-(N_1+p)\\0\end{matrix}\right.\right]\gamma^{p+qq-\frac{1}{2}}\exp(-b\gamma)dr\end{aligned} \tag{45}$$

Using (2.6.2) in [36], we can obtain the closed-form expression of E_2 given by

$$\begin{aligned}E_2 = {} & \frac{\Omega_1^{-(N_1+p)}\Omega_4^{qq+1}}{\Gamma(qq+1)\Gamma(N_1+p)}\int_0^\infty H_{1\ 1}^{1\ 1}\left[\frac{\Omega_3}{\Omega_1}\gamma\left|\begin{matrix}1-(N_1+p)\\0\end{matrix}\right.\right]H_{1\ 1}^{1\ 1} \\ & \times\left[\frac{(1+m)\Omega_4\gamma}{(1+m(1-\rho_2))\Omega_2}\left|\begin{matrix}1-(qq+1)\\0\end{matrix}\right.\right]\gamma^{p+qq-\frac{1}{2}}\exp(-b\gamma)dr \\ = {} & \frac{\Omega_1^{-(N_1+p)}\Omega_4^{qq+1}}{\Gamma(qq+1)\Gamma(N_1+p)}b^{-p-qq-\frac{1}{2}}H_{1,[1:1],0,[1:1]}^{1,\ 1,\ 1,\ 1} \\ & \times\left[\begin{matrix}\dfrac{\Omega_3}{\Omega_1 b}\\ \dfrac{(1+m)\Omega_4}{(1+m(1-\rho_2))\Omega_2 b}\end{matrix}\left|\begin{matrix}\left(1+qq+p-\frac{1}{2},1\right)\\(1-p-N_1,1);(qq,1))\\----\\(0,1);\qquad(0,1)\end{matrix}\right.\right]\end{aligned} \tag{46}$$

Substituting (46) and (44) into (43), the result (42) is proved.

Δ

By using the similar approach and the result in (38), we can obtain the average SER of the CC-MIMO multiuser systems with perfect CSI for all links, which is given by Corollary 3.

Corollary 3: Under the case where the CC-MIMO multiuser downlink transmission systems have the perfect CSI, the lower bound of average SER is given by

$$\begin{aligned}\bar{P}_E^{\text{Low-CSI}} = {} & \frac{a}{2}-\frac{a}{2}\sqrt{\frac{b}{\pi}}\tilde{\sum}\sum_{i=0}^{N_1-1}\frac{1}{i!}\frac{1}{\Gamma(N_1)}\left(\frac{\Omega_3}{\Omega_1}\right)^i \\ & \times\left(\frac{\Omega_4}{\Omega_2}\right)^{\sum_{t=0}^{N_2-1}a_t t}b^{-\sum_{t=0}^{N_2-1}a_t t-i-1}H_{1,\ [1:1],0,[1:1]}^{1,\ 1,\ 1,\ 1} \\ & \times\left[\begin{matrix}\dfrac{\Omega_3}{\Omega_1 b}\\ \dfrac{m\Omega_4}{\Omega_2 b}\end{matrix}\left|\begin{matrix}\left(1+\sum_{t=0}^{N_2-1}a_t t+i,1\right)\\(1-N_1-i_1,1);(1-\sum_{t=0}^{N_2-1}a_t t-1,1)\\----\\(0,1);\qquad(0,1))\end{matrix}\right.\right]\end{aligned} \tag{47}$$

where $\tilde{\Sigma}$ is defined by (15).

Δ

6 Asymptotic performance in high SNR

To obtain the insight and highlight the effect of system parameters on performance of the interested CC-MIMO multiuser systems, in this section, we present the asymptotic expressions of outage probability in high SNR for the systems with and without perfect CSI.

Without loss of generality, we define SNR = Q/N_0 and still employ (25), (26), and (27) to obtain the asymptotic expressions of outage probability in high SNR. Due to the fact the noise power $N_0 = 1$, we have SNR = Q. We first consider the case where the systems does not have the perfect CSI, i.e., $0 \le \rho_1 < 1$, $0 \le \rho_2 < 1$. Therefore, according to (13), in high SNR, the CDF $F_{\gamma_1}(x)$ can be given approximately by

$$F_{\gamma_1}(\gamma) \approx (1-\rho_1)^{N_1-1} \frac{\gamma}{\Omega_1} \tag{48}$$

With the PDF $f_{\gamma_3}(\gamma_3)$ given by $f_{\gamma_3}(\gamma_3) = \frac{1}{\Gamma(N_1)} \frac{1}{\Omega_3^{N_1}} \gamma_3^{N_1-1} \exp\left(-\frac{\gamma_3}{\Omega_3}\right)$ and the definition $\gamma_{13} = \frac{\gamma_1}{\gamma_3}$, in (27), the CDF $F_{\gamma_{13}}(\gamma)$ is given approximately by

$$F_{\gamma_{13}}(\gamma) \approx (1-\rho_1)^{N_1-1} N_1 \frac{\Omega_3}{\Omega_1} \gamma \tag{49}$$

At the same time, according to (14), (15), and (16), in high SNR, the PDF $f_{\gamma_2}(\gamma)$ and the CDF $F_{\gamma_2}(\gamma)$ of the RV γ_2 can be expressed approximately by

$$f_{\gamma_2}(\gamma) \approx \frac{M}{\Gamma(N_2)} \widehat{\sum} \Xi \gamma^{N_2-1} \tag{50}$$

$$F_{\gamma_2}(\gamma) \approx \frac{M}{\Gamma(N_2+1)\Omega_2^{N_2}} \widehat{\sum} \Xi \gamma^{N_2} \tag{51}$$

where $\widehat{\sum}$ is defined by (15), and Ξ is given by

$$\Xi = \frac{(1-\rho_2)^{\left(\sum\limits_{t=0}^{N_2-1} a_t t\right)}}{(1+m(1-\rho_2))^{\sum\limits_{t=0}^{N_2-1} a_{t} t+N_2}} \frac{\Gamma\left(\sum\limits_{t=0}^{N_2-1} a_t t + N_2\right)}{\Gamma(N_2)} \tag{52}$$

With $\gamma_{24} = \frac{\gamma_2}{\gamma_4}$ and $f_{\gamma_4}(\gamma_4) = \frac{1}{\Omega_4} \exp\left(-\frac{\gamma_4}{\Omega_4}\right)$, in high SRN, the CDF $F_{\gamma_{24}}(\gamma)$ in (27) can be expressed as

$$F_{\gamma_{24}}(\gamma) = \int_0^\infty F_{\gamma_2}(\gamma\gamma_4) f_{\gamma_4}(\gamma_4) d\gamma_4 \approx \widehat{\sum} \frac{\Xi M}{\Omega_2^{N_2}} \left(\frac{1}{\Omega_4}\right)^{-N_2} \gamma^{N_2} \tag{53}$$

Therefore, according to (27) and (26), and using (49), (53), and $\Omega_1 = Q\omega_1$, we have Theorem 4.

Theorem 4: For the CC-MIMO multiuser downlink transmission systems with imperfect CSI, in high SNR, the outage probability is approximately given by

$$P_{\text{Out}}^{\text{Asy-ImCSI}}(\gamma_{\text{th}}) \approx \begin{cases} (1-\rho_1)^{N_1-1} N_1 \frac{\Omega_3}{\omega_1} \frac{\gamma_{\text{th}}}{Q} & \text{if } N_2 > 1 \\ \left((1-\rho_1)^{N_1-1} N_1 \frac{\Omega_3}{\omega_1} + \widehat{\sum} \frac{\Xi M}{\omega_2} \Omega_4\right) \frac{\gamma_{\text{th}}}{Q} & f \;\; N_2 = 1 \end{cases} \tag{54}$$

Δ

Similarly, the asymptotic outage probability for the systems with perfect CSI is achieved, and given by Corollary 4.

Corollary 4: Considering the case where the systems have the perfect CSI, i.e., $\rho_1 = \rho_2 = 1$, in high SNR, the asymptotic outage probability is given approximately by

$$P_{\text{Out}}^{\text{Asy-CSI}}(\gamma_{\text{th}}) \approx \begin{cases} \frac{(MN_2)!}{(N_2!)^M} \left(\frac{\Omega_4}{\omega_2}\right)^{MN_2} \left(\frac{\gamma_{\text{th}}}{Q}\right)^{MN_2} & f \;\; MN_2 < N_1 \\ \left(\frac{\Gamma(2N_1)}{\Gamma(N_1-1)\Gamma(N_1)} \left(\frac{\Omega_3}{\omega_1}\right)^{N_1} + \frac{(MN_2)!}{(N_2!)^M} \left(\frac{\Omega_4}{\omega_2}\right)^{MN_2}\right) \left(\frac{\gamma_{\text{th}}}{Q}\right)^{N_1} & f \;\; MN_2 < N_1 \\ \frac{\Gamma(2N_1)}{\Gamma(N_1-1)\Gamma(N_1)} \left(\frac{\Omega_3}{\omega_1}\right)^{N_1} \left(\frac{\gamma_{\text{th}}}{Q}\right)^{N_1} & f \;\; MN_2 > N_1 \end{cases} \tag{55}$$

The proof of Corollary 4 is presented in Appendix D.

Δ

Theorem 4 and Corollary 4 present a clear insight about the joint effect of the primary and secondary system parameters on the performance of the CC-MIMO multiuser systems. Firstly, Theorem 4 shows that, in the presence of feedback delay (imperfect CSI), the achievable diversity gain of the cognitive systems reduces to one, regardless the number of antennas at the cognitive source and destinations as well as the number of the cognitive destinations. However, the key parameters of the cognitive systems will still affect the coding gain. Secondly, Corollary 4 indicates that the diversity order is $\min\{N_1, MN_2\}$ under the case where the cognitive systems have the perfect CSI. Specially, when $N_1 < MN_2$, the outage performance is dominated by the first hop, or in another word, the contribution of multiuser diversity is negligible. This is because the performance of a two-hop communication system is limited by the weakest bottleneck link. On the contrary, when $N_1 > MN_2$, the second hop dominates the outage performance. The multiuser diversity has the dominated effect on the diversity order. In this case, the increasing number of the secondary destinations can improve the CC-MIMO multiuser systems and can enhance the reliability of wireless communications. Only when $N_1 = MN_2$, the outage performance is determined by both the hops. Finally, from Theorem 4 and Corollary 4, it is observed that the CSI of primary systems only affect the coding gain but not the diversity gain.

7 Numerical results

In previous sections, we obtain the closed-form expression of the exact outage probability as well as the lower bound and high SNR approximation for the interested

CC-MIMO multiuser downlink transmission systems with and without perfect CSI. Based on these derivations, in this section, the simulations and numerical results are presented, which is used to validate the derivations and to obtain the insight about the mutual effect of system parameters on system performance. In all cases, the channels are generated by using MATLAB toolbox "Rayleighchan," which models a Rayleigh fading channel. Throughout the analysis, we take the outage threshold $\gamma_{th} = 1$ dB. For simplicity, we also take the correlation coefficients $\rho_1 = \rho_2$.

By taking $M = 2, N_1 = N_2 = 2, \omega_1 = \omega_2 = 1$, and $\Omega_3 = \Omega_4 = 2$, in Fig. 2, we first present the comparison analysis of the exact outage probability versus the PUs' outage constraint Q (or SNR) as well as the lower bound and the high SNR approximation under different values of correlation coefficients ρ_1 and ρ_2. For the convenience of comparison analysis, the outage probabilities corresponding to the different correlation coefficients are presented in separated figures. Obviously, for the case $\rho_1 = \rho_2 = 1$, it represents that the system has the perfect CSI. From Fig. 2a–d, it is clearly observed that the exact derivations match well the simulations for all realizations, which validates the obtained closed-form analysis solutions given by Theorem 1 and Corollary 1. It is also seen that the values of the correlation coefficient have very severe impact on outage performance. The figures show that the outage probability is decreasing with the increase of the correlation coefficients ρ_1 and ρ_2. When the CC-MIMO multiuser system has the perfect CSI, $\rho_1 = \rho_2 = 1$, the outage performance is optimal.

In Fig. 2, compared with the outage probability of the systems with $\rho_1 = \rho_2 = 0.3$, when we take the correlation coefficients $\rho_1 = \rho_2 = 0.5,\ 0.8,\ 1$, at 10^{-2} of outage probability, the achieved SNR gains by the CC-MIMO multiuser system are 1.5, 5, and 9.1 dB, approximately. The result indicates that the design of feedback links with small delay is a very important topic. The favorable selection is to reduce the feedback delay as possible as. For the lower bound, we find that the lower bound is

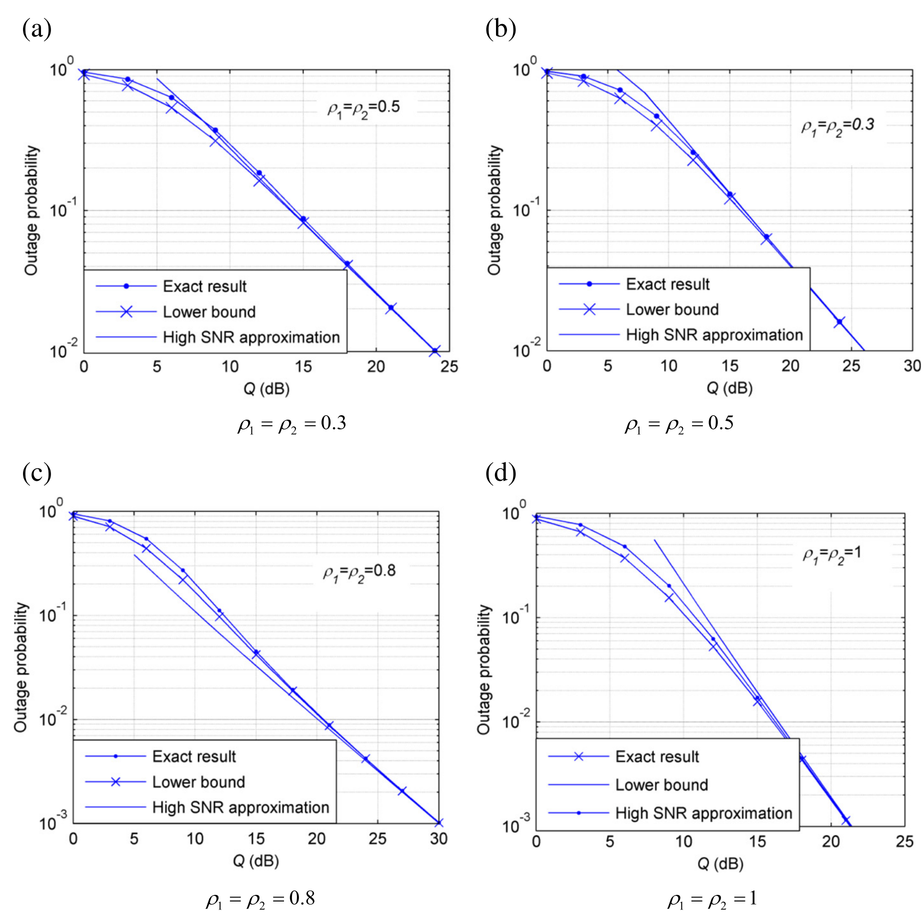


Fig. 2 Outage performance comparison among the exact result, lower bound, and high SNR approximation with different correlation coefficients ($M = 2,\ N_1 = N_2 = 2,\ \omega_1 = \omega_2 = 1,\ \Omega_3 = \Omega_4 = 2$) (**a**) $\rho_1 = \rho_2 = 0.3$; (**b**) $\rho_1 = \rho_2 = 0.5$; (**c**) $\rho_1 = \rho_2 = 0.8$; (**d**) $\rho_1 = \rho_2 = 1$

tight sufficiently, especially in high SNR (or Q) region. Though in low SNR (or Q) the lower bound does not match the exact estimation, the gap between the two results is very small. Therefore, in realistic implementation, the lower bound is a favorable alternative for the evaluation of the exact solution due to its low implementation complexity. In the sequent discussion, the lower bound is employed to investigate corresponding performance such as outage probability and average SER. At the same time, from the figure we can find, in high SNR (or Q), the exact result, lower bound, and the high SNR approximation of outage probability are agreed completely. Moreover, the SNR region in which these results are agreed completely is increasing with the decrease of the correlation coefficients. From the results presented in Theorem 4 and Corollary 4, it is observed that the high SNR approximation expressions are simpler than the lower bound given by Theorem 2 and Corollary 2. This observation indicates that, in high SNR region, we can employ the high SNR approximation not only to estimate the performance of the CC-MIMO multiuser systems but also to obtain the insight about the effect of system's parameters on the CC-MIMO multiuser systems.

By employing the lower bound and taking $\rho_1 = \rho_2 = 1$, in Fig. 3, we investigate the effect of the parameters of secondary systems on outage probability. In Fig. 3a, the effect of channel powers ω_1 and ω_2 is considered. It is easy to say that the outage probability is decreasing with ω_1 and ω_2. Especially, when ω_1 and ω_2 are greater than 2, the outage probability is less than 0.1 over the entire values of Q. At the same time, Fig. 3a indicates that the slopes of all curves are the same. This is due to the fact that in Fig. 3a, the values of M, N_1, and N_2 are fixed. As a result, the systems have the fixed diversity order. In Fig. 3b, the effect of N_1 is investigated. We say that the increase of N_1 can improve the outage performance greatly. Moreover, the diversity order is increasing with the increase of N_1.

In Fig. 4, by employing the lower bound and taking the correlation coefficients $\rho_1 = \rho_2 < 1$, we investigate the effect of the secondary system's parameters on the diversity and coding gains for the two CC-MIMO multiuser systems where the values of N_1 and N_2 are $N_1 = 12$, $N_2 = 3$, and $N_1 = 2$, $N_2 = 6$, respectively. To illustrate the effect of the first and second hops on the total outage performance, in Fig. 4, we also present the outage probabilities of the first and second hops. It is observed that, for the two cases, the total outage probability is dominated by the first hop. Besides this result, we also find the result that, in the two figures, the slopes of curves for high SNR approximation are the same. The observation indicates that the diversity gains are the same. This is to say, when the systems have the imperfect CSI, the achievable diversity gain is not affected by the number of antennas at the cognitive source and destinations as well as the number of the cognitive destinations. At the same time, for the coding gain, in Fig. 4a, b, we can find the total outage probabilities are different. This is due to the fact that in the two figures, the CC-MIMO multiuser systems have a different number of antennas at the cognitive source and destinations as well as the number of the cognitive destinations. These observations can be explained by the using the results given by Theorem 4. It is found that in Theorem 4, for the systems without perfect CSI, the achievable diversity order reduces to one, regardless the number of antennas at the cognitive source and destinations as well as the number of the cognitive destinations. Nevertheless, these key parameters will affect the coding gain.

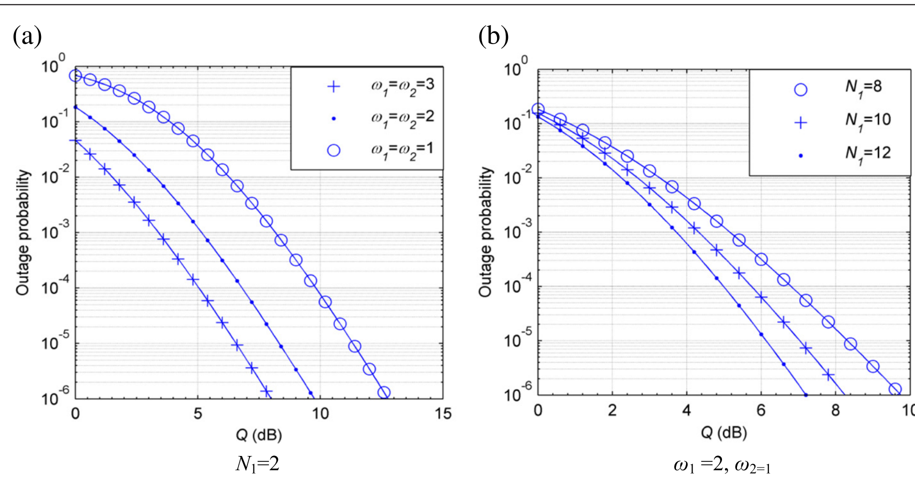


Fig. 3 Effect of the parameters of secondary systems on outage probability ($\rho_1 = \rho_2 = 1$, $M = 2$, $\Omega_3 = \Omega_4 = 2$, $N_2 = 5$) (**a**) $N_1 = 2$; (**b**) $\omega_1 = 2$, $\omega_2 = 1$

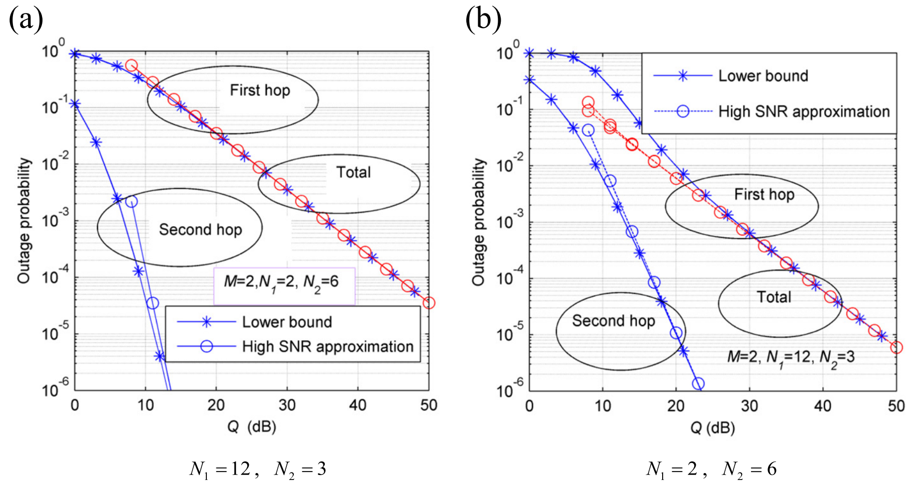


Fig. 4 Diversity and coding gains comparison for the CC-MIMO multiuser systems with imperfect CSI under different antenna equipment ($\rho_1 = \rho_2 < 1$, $M = 2$, $\omega_1 = \omega_2 = 1$, $\Omega_3 = \Omega_4 = 2$) (**a**) N_1 = 12, N_2 = 3; (**b**) N_1 = 2, N_2 = 6

With the given systems' parameters presented in Fig. 4, we find that the second link outperforms the first link, and the total outage performance is dominated by the second link. Indeed, the performance of the two hopes is greatly dependent on the systems' parameters. In certain case, the first link would outperform the second link. Therefore, the diversity and coding gains are further investigated in Fig. 5, where the corresponding system parameters are presented. Due to the fact that when $\rho_i < 1$, $i = 1, 2$, the multiuser selection diversity disappears, in Fig. 5, we take $M = 1$. It is easy to see that with the given system's parameters, the first link outperforms the second link in the interested regime of Q. The total outage performance is dominated by the second link. At the same time, we can find that in Fig. 5a, the slopes of the outage probability of the two hops are the same, while in Fig. 5b, they are different. The reason is that when $\rho_i < 1$, $i = 1, 2$, the diversity order of the first hop is one, while the one of the second hop is N_2. In Fig. 5a, we take $N_2 = 1$, and in Fig. 5b, $N_2 = 2$. Nevertheless, the total diversity order of the considered systems is one when $\rho_i < 1$, $i = 1, 2$. The above observations further validate the derivations.

Using the similar system parameters as in Fig. 4, the systems with perfect CSI ($\rho_1 = \rho_2 = 1$) have been investigated in Fig. 6. In the figure, we compare the effect of the first and second hops on the total outage performance, especially on the diversity and coding gains. Comparing Figs. 4, 5, and 6, we can find that

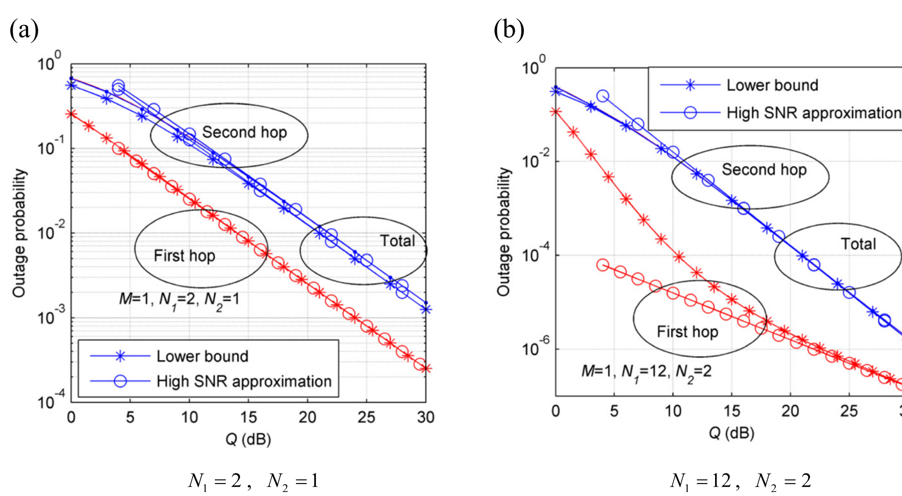


Fig. 5 Diversity and coding gains comparison for the CC-MIMO multiuser systems with imperfect CSI under different antenna equipment ($\rho_1 = \rho_2 < 1$, $\omega_1 = 2$, $\omega_2 = 1$, $\Omega_3 = 0.5$, $\Omega_4 = 1$) (**a**) N_1 = 2, N_2 = 1; (**b**) N_1 = 12, N_2 = 2

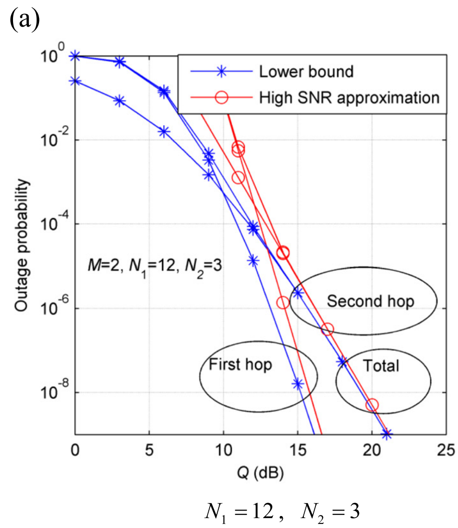


Fig. 6 Diversity and coding gains comparison for the CC-MIMO multiuser systems with perfect CSI under different antenna equipment ($\rho_1 = \rho_2 = 1$, $M = 2$, $\omega_1 = \omega_2 = 1$, $\Omega_3 = \Omega_4 = 2$) (**a**) $N_1 = 12$, $N_2 = 3$; (**b**) $N_1 = 2$, $N_2 = 6$

the channel correlation coefficients have very severe effect on the diversity and coding gains. Figures 4 and 5 show that when the CC-MIMO systems does not have the perfect CSI ($\rho_1 = \rho_2 < 1$), the total diversity gain is not affected by the number of antennas at the cognitive source and destinations as well as the number of the cognitive destinations. However, Fig. 6 shows that when the systems have perfect CSI, the values of N_1, M, and N_2 not only affect coding gain but also diversity gain, which is different from the one in Figs. 4 and 5. Specially, the diversity gain is dominated by MN_2 when $MN_2 < N_1$. On the contrary, i.e., $MN_2 > N_1$, it is dominated by N_1. The observations from Figs. 4, 5, and 6 are agreed complete with the results given by Theorem 4 and Corollary 4, which validate our derivations once again.

In Figs. 2, 3, 4, 5, and 6, we investigate the effect of the secondary system's parameters on system performance. Besides the secondary system's parameters, the ones of primary systems also affect the system performance. Thus, in Fig. 7, the effect of primary system's parameters on the outage performance is investigated for the two

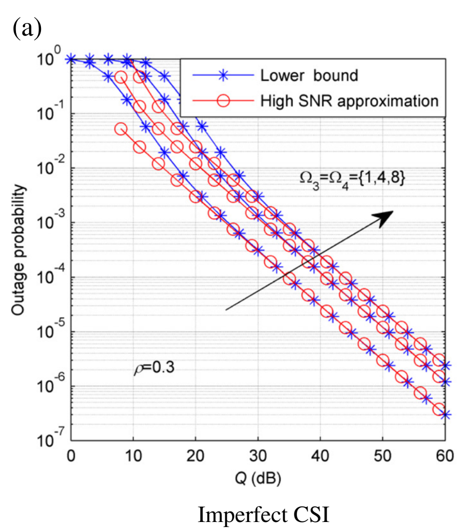


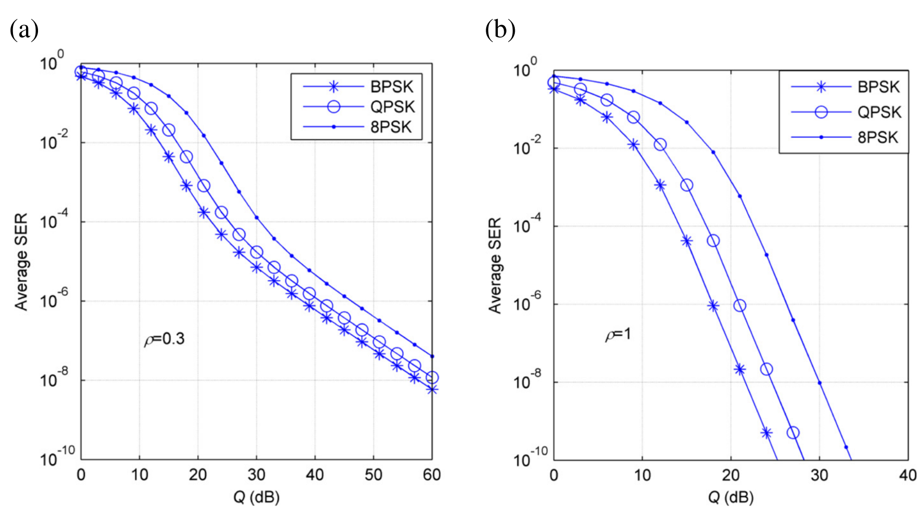
Fig. 7 The effect of primary system's parameters on outage performance ($\omega_1 = \omega_2 = 1$, $M = 2$, $N_1 = 12$, $N_2 = 3$) (**a**) Imperfect CSI; (**b**) Perfect CSI

CC-MIMO multiuser systems with and without perfect CSI. Figure 7a is for the system without perfect CSI, while Fig. 7b is for the one with perfect CSI. From Fig. 7a, b, it is clearly found that the slopes of lower bounds are the same over the entire values of Q even if the primary system's parameters Ω_3 and Ω_4 are different. However, the outage probabilities are changing with the values of Ω_3 and Ω_4. Theses observations indicate that, in cognitive systems, the diversity gain is solely determined by the parameters of the secondary systems. The ones of primary systems only affect the coding gain but not the diversity gain. At the same time, it is also observed that the slopes in Fig. 7a, b are different even if the other system parameters are the same (besides the correlation coefficients). Moreover, the diversity order in Fig. 7a is less greatly than the one in Fig. 7b. The observations illustrate clearly the effect of feedback delay on diversity gain. As a result, for the CC-MIMO multiuser systems with imperfect CSI, the diversity order tends to one.

Besides the outage performance, in this paper, we also obtain the closed-form expressions for average SER. Similar to the investigation on outage performance, using the derivations, we can investigate the effect of system parameters on the average SER. However, due to the limitation of space, we only present the lower bound of the average SER for the two systems with and without perfect CSI in Fig. 8. For other system realizations, the average SERs can be achieved by using the similar methods as in Figs. 2, 3, 4, 5, 6, and 7. Figure 8 clearly shows the effect of ρ_1 and ρ_2 on the average SER. As expected, when the systems have the perfect CSI, the increase of Q can improve the average SER greatly. On the contrary, when $\rho_1 = \rho_2 < 1$, the change of average SER is very small in high SNR.

8 Conclusions

In this paper, the CC-MIMO multiuser downlink transmission systems are investigated in terms of outage probability and average SER under the idea case (without feedback delay) and the actual implementation case (in the presence of feedback delay). By using the approximate mathematical proof, the exact closed-form expressions of outage probability are obtained firstly. Secondly, to reduce the implementation complexity, we achieve the lower bounds of outage probability and average SER. Finally, we consider the high SNR approximation of outage performance to obtain the diversity and coding gains. Based on these derivations, the simulations are presented. The presented numerical results show that the simulations match well with the obtained exact solutions, which validate the derivations. For the systems without perfect CSI, the achievable diversity order reduces to one, regardless the number of antennas at the cognitive source and destinations as well as the number of the cognitive destinations. These key parameters only affect the coding gain of the CC-MIMO multiuser systems. On the contrary, for the case where the CC-MIMO systems have the perfect CSI, the diversity gain is dominated by N_1 if $MN_2 > N_1$; otherwise it is determined by MN_2. For the effect of PU's parameters on system's diversity and coding gains, it is achieved that, in cognitive systems, the diversity gain is solely determined by the parameters of the secondary systems. The ones of primary systems only affect the coding gain but not the diversity gain.



$\rho_1 = \rho_2 < 1$ $\rho_1 = \rho_2 = 1$

Fig. 8 The lower bound of average SER ($M = 2$, $N_1 = 12$, $N_2 = 3$, $\omega_1 = \omega_2 = 1$, $\Omega_3 = \Omega_4 = 2$) (**a**) $\rho_1 = \rho_2 < 1$; (**b**) $\rho_1 = \rho_2 = 1$

9 Appendix A

9.1 Proof of Proposition 2

When the feedback delay is considered, with (11) we have γ_2 is the delay version of $\tilde{\gamma}_2$. Therefore, according to the principle of concomitants or induced order statistics, the PDF of γ_2 is formulated as

$$f_{\gamma_2}(x) = \int_0^\infty f_{\gamma_2|\tilde{\gamma}_2}(x|y) f_{\tilde{\gamma}_2}(y) dy \quad (56)$$

where $f_{\gamma_2|\tilde{\gamma}_2}(x|y)$ is the PDF of the RV γ_2 conditioned on $\tilde{\gamma}_2$. Due to the fact that all entries in γ_{2m}, $m = \{1, \ldots, M\}$, as well as the ones in $\tilde{\gamma}_{2m}$, are i.i.d RVs, the conditioned PDFs $f_{\gamma_{2m}|\tilde{\gamma}_{2m}}(x|y)$ are identical for $m = \{1, \ldots, M\}$. Moreover, it follows from the principle of concomitants of order statistics that the conditional PDF $f_{\gamma_2|\tilde{\gamma}_2}(x|y)$ is identical to $f_{\gamma_{2m}|\tilde{\gamma}_{2m}}(x|y)$, yielding [33]

$$f_{\gamma_2|\tilde{\gamma}_2}(x|y) = f_{\gamma_{2m}|\tilde{\gamma}_{2m}}(x|y) = \frac{f_{\gamma_{2m},\tilde{\gamma}_{2m}}(x, y)}{f_{\tilde{\gamma}_{2m}}(y)} \quad (57)$$

where $f_{\gamma_{2m},\tilde{\gamma}_{2m}}(x, y)$ is the joint PDF of the RVs γ_{2m} and $\tilde{\gamma}_{2m}$. Eq. (57) is very important for the derivation of $f_{\gamma_2}(x)$. Since $\tilde{h}_{2mi} \sim CN(0, \omega_2)$, $i = \{1, \ldots, N_2\}$, we have $\sqrt{Q}\tilde{h}_{2mi} \sim CN(0, \Omega_2)$, where $\Omega_2 = Q\omega_2$. According to [29], the joint PDF $f_{\gamma_{2m},\tilde{\gamma}_{2m}}(x, y)$ is given by

$$f_{\gamma_{2m},\tilde{\gamma}_{2m}}(x, y) = \frac{1}{\Omega_2^{N_2+1}} \frac{(xy/\rho_2)^{\frac{N_2-1}{2}}}{(N_2-1)!(1-\rho_2)} \exp \times \left(-\frac{x+y}{(1-\rho_2)\Omega_2}\right) I_{N_2-1}\left(\frac{2\sqrt{\rho_2 xy}}{(1-\rho_2)\Omega_2}\right) \quad (58)$$

where $I_n(.)$ is the n-th order modified Bessel function of the first kind defined by (8.406.1) in [34]. At the same time, the PDF $f_{\tilde{\gamma}_{2m}}(y)$ is given by $f_{\tilde{\gamma}_{2m}}(y) = \frac{1}{\Gamma(N_2)} \frac{1}{\Omega_2^{N_2}} y^{N_2-1} \exp\left(-\frac{y}{\Omega_2}\right)$. Therefore, $f_{\gamma_2|\tilde{\gamma}_2}(x|y)$ is given by

$$f_{\gamma_2|\tilde{\gamma}_2}(x|y) = \frac{1}{\Omega_2} \frac{(x/\rho_2)^{\frac{N_2-1}{2}}}{(1-\rho_2)} y^{-\frac{N_2-1}{2}} \exp\left(-\frac{x+\rho_2 y}{(1-\rho_2)\Omega_2}\right) I_{N_2-1}\left(\frac{2\sqrt{\rho_2 xy}}{(1-\rho_2)\Omega_2}\right) \quad (59)$$

For the multiuser diversity systems, according to (9) we have $\tilde{\gamma}_2 = \underbrace{\max}_{m=\{1,\ldots,M\}} (\tilde{\gamma}_{2m})$. That is to say, the selection of best user is based on outdated CSI $\tilde{\gamma}_{2m}$. Using the order statistics [33], the PDF $f_{\tilde{\gamma}_2}(y)$ of the RV $\tilde{\gamma}_2$ is given by

$$f_{\tilde{\gamma}_2}(y) = M\left(F_{\tilde{\gamma}_{2m}}(y)\right)^{M-1} f_{\tilde{\gamma}_{2m}}(y) \quad (60)$$

where the CDF $F_{\tilde{\gamma}_{2m}}(y)$ of the RV $\tilde{\gamma}_{2m}$ is given by $F_{\tilde{\gamma}_{2m}}(y) = 1 - \sum_{i=0}^{N_2-1} \frac{1}{i!}\left(\frac{\gamma_2}{\Omega_2}\right)^i \exp\left(-\frac{\gamma_2}{\Omega_2}\right)$. By using binomial expansion theorem, the middle term $\left(F_{\tilde{\gamma}_{2m}}(y)\right)^{M-1}$ in (60) is written as

$$\left(F_{\tilde{\gamma}_{2m}}(y)\right)^{M-1} = \sum_{m=0}^{M-1} C_m^{M-1} (-1)^m \exp\left(-\frac{my}{\Omega_2}\right) \times \left(\sum_{k=0}^{N_2-1} \frac{1}{k!}\left(\frac{y}{\Omega_2}\right)^k\right)^m \quad (61)$$

Furthermore, using multinomial expansion leads to

$$\left(F_{\tilde{\gamma}_{2m}}(y)\right)^{M-1} = \sum_{m=0}^{M-1} C_m^{M-1}(-1)^m \exp\left(-\frac{my}{\Omega_2}\right) \sum_{a_0+\ldots+a_{N_2-1}} \times \binom{m}{a_0, \ldots, a_{N_2-1}} \prod_{0 \le t \le N_2-1} \left(\frac{1}{t!}\right)^{a_t} \left(\frac{y}{\Omega_2}\right)^{\sum_{t=0}^{N_2-1} a_t t} \quad (62)$$

Substituting (62) and $f_{\tilde{\gamma}_{2m}}(y) = \frac{1}{\Gamma(N_2)} \frac{1}{\Omega_2^{N_2}} y^{N_2-1} \exp\left(-\frac{y}{\Omega_2}\right)$ into (60), the PDF $f_{\tilde{\gamma}_2}(y)$ is given by

$$f_{\tilde{\gamma}_2}(y) = \frac{M}{\Gamma(N_2)} \widehat{\sum} \left(\frac{1}{\Omega_2}\right)^{\sum_{t=0}^{N_2-1} a_t t + N_2} y^{\sum_{t=0}^{N_2-1} a_t t + N_2 - 1} \exp \times \left(-\frac{(m+1)y}{\Omega_2}\right) \quad (63)$$

Where we define

$$\widehat{\sum} = \sum_{m=0}^{M-1} C_m^{M-1}(-1)^m \sum_{a_0+\ldots+a_{N_2-1}} \binom{m}{a_0, \ldots, a_{N_2-1}} \prod_{0 \le t \le N_2-1} \left(\frac{1}{t!}\right)^{a_t} \quad (64)$$

Thus, substituting (63) and (59) into (56), the PDF $f_{\gamma_2}(x)$ is given by

$$f_{\gamma_2}(x) = \frac{M}{\Gamma(N_2)} \frac{(x/\rho_2)^{\frac{N_2-1}{2}}}{(1-\rho_2)} \exp\left(-\frac{x}{(1-\rho_2)\Omega_2}\right) \widehat{\sum} \times \left(\frac{1}{\Omega_2}\right)^{\sum_{t=0}^{N_2-1} a_t t + N_2 + 1} \times \int_0^\infty y^{\sum_{t=0}^{N_2-1} a_t t + \frac{N_2-1}{2}} \exp \times \left(-\frac{y}{\Omega_2}\left(m + \frac{1}{1-\rho_2}\right)\right) I_{N_2-1}\left(\frac{2\sqrt{\rho_2 xy}}{(1-\rho_2)\Omega_2}\right) dy \quad (65)$$

Using (6.643.2) in [34], the integral term in (65) is given by

$$\Delta = \frac{1}{\Gamma(N_2)}\Gamma\left(\sum_{t=0}^{N_2-1} a_t t + N_2\right)\frac{(1-\rho_2)\Omega_2}{\sqrt{\rho_2 x}} \times \left(\frac{1+m(1-\rho_2)}{(1-\rho_2)\Omega_2}\right)^{-\left(\sum_{t=0}^{N_2-1} a_t t + \frac{N_2}{2}\right)} \times \exp \times \left(\frac{\rho_2 x}{2(1-\rho_2)\Omega_2(1+m(1-\rho_2))}\right) M_{-\left(\sum_{t=0}^{N_2-1} a_t t + \frac{N_2}{2}\right), \frac{N_2-1}{2}} \times \left(\frac{\rho_2 x}{(1-\rho_2)\Omega_2(1+m(1-\rho_2))}\right) \tag{66}$$

where $M_{a,b}(.)$ is the Whittaker-M function defined by (9.220.2) in [34]. Furthermore, we resort to (14) in [12] to re-express the Whittaker-M function in term of the polynomial as

$$M_{a,b}(z) = \exp\left(\frac{z}{2}\right)\sum_{k=0}^{-b-a-1/2} C_k^{-b-a-1/2} z^{b+k+1/2}\frac{\Gamma(2b+1)}{\Gamma(2b+k+1)} \tag{67}$$

Therefore, the integral term Δ is given by

$$\Delta = \frac{1}{\Gamma(N_2)}\Gamma\left(\sum_{t=0}^{N_2-1} a_t t + N_2\right)\frac{(1-\rho_2)\Omega_2}{\sqrt{\rho_2 x}} \times \left(\frac{1+m(1-\rho_2)}{(1-\rho_2)\Omega_2}\right)^{-\left(\sum_{t=0}^{N_2-1} a_t t + \frac{N_2}{2}\right)} \times \exp\left(\frac{\rho_2 x}{(1-\rho_2)\Omega_2(1+m((1-\rho_2)))}\right)\sum_{kk=0}^{\sum_{t=0}^{N_2-1} a_t t} C_k^{\sum_{t=0}^{N_2-1} a_t t} \times \left(\frac{\rho_2 x}{(1-\rho_2)\Omega_2(1+m(1-\rho_2))}\right)^{\frac{N_2}{2}+kk}\frac{\Gamma(N_2)}{\Gamma(N_2+kk)} \tag{68}$$

Finally, substituting (68) into (65), Proposition 2 is proved.

Δ

10 Appendix B

10.1 Proof of Theorem 1

Proof: According to (22), we first derive the exact expression for the CDF $F_{\gamma_D}(\gamma)$ of RV γ_D. Then using the definition of outage probability in (22), the exact closed-form expression of the outage probability can be achieved. With the consideration that the RVs γ_1, γ_2, γ_3, and γ_4 are mutual independent [35], we have

$$\begin{aligned} F_{\gamma_D}(\gamma) &= \Pr\left\{\frac{\gamma_1\gamma_2}{\gamma_1\gamma_4+\gamma_2\gamma_3} < \gamma\right\} \\ &= \underbrace{\int_0^\infty\int_0^\infty\underbrace{\int_0^\infty \Pr\left\{\frac{\gamma_1\gamma_2}{\gamma_1\gamma_4+\gamma_2\gamma_3} < \gamma \middle| \gamma_2,\gamma_3,\gamma_4\right\} f_{\gamma_2}[\gamma_2]d\gamma_2}_{I_1} f_{\gamma_3}(\gamma_3)d\gamma_3}_{I_2} f_{\gamma_4}(\gamma_4)d\gamma_4 \end{aligned} \tag{69}$$

For the convenience of derivation, we first consider the innermost integral in term of γ_2, which can be formulated as the following form

$$\begin{aligned} I_1 &= \int_0^\infty \Pr\left\{\frac{\gamma_1\gamma_2}{\gamma_1\gamma_4+\gamma_2\gamma_3} < \gamma \middle| \gamma_2,\gamma_3,\gamma_4\right\} f_{\gamma_2}(\gamma_2)d\gamma_2 \\ &= \underbrace{\int_0^{\gamma\gamma_4} \Pr\left\{\gamma_1 \geq \frac{\gamma_2\gamma_3\gamma}{\gamma_2-\gamma\gamma_4} \middle| \gamma_2,\gamma_3,\gamma_4\right\} f_{\gamma_2}(\gamma_2)d\gamma_2}_{I_{11}} \\ &\quad + \underbrace{\int_{\gamma\gamma_4}^\infty \Pr\left\{\gamma_1 < \frac{\gamma_2\gamma_3\gamma}{\gamma_2-\gamma\gamma_4} \middle| \gamma_2,\gamma_3,\gamma_4\right\} f_{\gamma_2}(\gamma_2)d\gamma_2}_{I_{12}} \end{aligned} \tag{70}$$

It is easy to see that the part I_{11} in (70) can be written as

$$I_{11} = F_{\gamma_2}(\gamma\gamma_4) \tag{71}$$

This is due to the fact that $\Pr\left\{\gamma_1 \geq \frac{\gamma_2\gamma_3\gamma}{\gamma_2-\gamma\gamma_4} \middle| \gamma_2,\gamma_3,\gamma_4\right\} = 1$ as $0 \leq \gamma_2 \leq \gamma\gamma_4$. Moreover, by taking the variable change $x = \gamma_2 - \gamma\gamma_4$, the part I_{12} is written as

$$I_{12} = \int_0^\infty F_{\gamma_1}\left(\gamma_3\gamma + \frac{\gamma_3\gamma_4\gamma^2}{x} \middle| \gamma_2,\gamma_3,\gamma_4\right) f_{\gamma_2}(x+\gamma\gamma_4)dx \tag{72}$$

For the CC-MIMO multiuser system without perfect CSI, using the CDF $F_{\gamma_1}(x)$ given in (13), the integral (72) is given by

$$\begin{aligned} I_{12} &= 1 - F_{\gamma_2}(\gamma\gamma_4) - \sum_{i=0}^{N_1-1} C_i^{N_1-1}\rho_1^{N_1-i-1}(1-\rho_1)^i \sum_{p=0}^{N_1-i-1}\frac{1}{p!\Omega_1^p}\int_0^\infty \\ &\quad \times \left(\gamma_3\gamma + \frac{\gamma_3\gamma_4\gamma^2}{x}\right)^p \exp\left(-\frac{1}{\Omega_1}\left(\gamma_3\gamma + \frac{\gamma_3\gamma_4\gamma^2}{x}\right)\right) f_{\gamma_2} \\ &\quad \times (x+\gamma\gamma_4)dx \end{aligned} \tag{73}$$

Therefore, combining (73) and (71) results in (70) given by

$$I_1 = 1-\sum_{i=0}^{N_1-1} C_i^{N_1-1}\rho_1^{N_1-i-1}(1-\rho_1)^i \sum_{p=0}^{N_1-i-1} \frac{1}{p!\Omega_1^p}\int_0^\infty \times\left(\gamma_3\gamma+\frac{\gamma_3\gamma_4\gamma^2}{x}\right)^p \exp\left(-\frac{1}{\Omega_1}\left(\gamma_3\gamma+\frac{\gamma_3\gamma_4\gamma^2}{x}\right)\right)f_{\gamma_2} \times(x+\gamma\gamma_4)dx \tag{74}$$

With binomial theorem, having

$$I_1 = 1-\sum_{i=0}^{N_1-1} C_i^{N_1-1}\rho_1^{N_1-i-1}(1-\rho_1)^i \sum_{p=0}^{N_1-i-1}\sum_{q=0}^{p}\frac{C_q^p}{p!\Omega_1^p}\gamma^{p+q}\gamma_3{}^p\gamma_4^p \exp\left(-\frac{1}{\Omega_1}\gamma_3\gamma\right)\int_0^\infty \exp\left(-\frac{1}{\Omega_1}\frac{\gamma_3\gamma_4\gamma^2}{x}\right)x^{-q}f_{\gamma_2} \times(x+\gamma\gamma_4)dx \tag{75}$$

Using (14), I_1 is expressed as

$$I_1 = 1-\frac{M}{\Gamma(N_2)}\sum_{i=0}^{N_1-1}C_i^{N_1-1}\rho_1^{N_1-i-1}(1-\rho_1)^i\sum_{p=0}^{N_1-i-1}\sum_{q=0}^{p}\frac{C_q^p}{p!}\widehat{\sum}\Phi\sum_{tt=0}^{N_2+kk-1}C_{tt}^{N_2+kk-1} \times\frac{\gamma^{N_2+kk-1-tt+p+q}\gamma_3{}^p\gamma_4{}^{N_2+kk-1-tt+q}}{\Omega_1^p}\exp\left(-\gamma\left(\frac{(1+m)\gamma_4}{(1+m(1-\rho_2))\Omega_2}+\frac{\gamma_3}{\Omega_1}\right)\right)\int_0^\infty x^{tt-p}\exp\left(-\frac{(1+m)x}{(1+m(1-\rho_2))\Omega_2}-\frac{1}{\Omega_1}\frac{\gamma_3\gamma_4\gamma^2}{x}\right)dx \tag{76}$$

Using (3.471.9) in [34], we have

$$I_1 = 1-\frac{2M}{\Gamma(N_2)}\sum_{i=0}^{N_1-1}C_i^{N_1-1}\rho_1^{N_1-i-1}(1-\rho_1)^i\sum_{p=0}^{N_1-i-1}\sum_{q=0}^{p}\frac{C_q^p}{p!}\widehat{\sum}\Phi\sum_{tt=0}^{N_2+kk-1}C_{tt}^{N_2+kk-1} \times\frac{\gamma^{N_2+kk+p}\gamma_3^{\frac{2p+tt-q+1}{2}}\gamma_4^{\frac{2N_2+2kk-1-tt+q}{2}}}{\Omega_1^{\frac{2p+tt-q+1}{2}}\left(\frac{(1+m(1-\rho_2))\Omega_2}{(1+m)}\right)^{-\frac{tt-q+1}{2}}}\exp\left(-\gamma\left(\frac{(1+m)\gamma_4}{(1+m(1-\rho_2))\Omega_2}+\frac{\gamma_3}{\Omega_1}\right)\right)K_{tt-q+1}\left(2\sqrt{\frac{(1+m)\gamma_3\gamma_4\gamma^2}{(1+m(1-\rho_2))\Omega_2\Omega_1}}\right) \tag{77}$$

Therefore, combining (77) and (69), the middle integral I_2 in (69) can be written as

$$I_2 = \int_0^\infty I_1 f_{\gamma_3}(\gamma_3)d\gamma_3 \tag{78}$$

Substituting (77) and $f_{\tilde{\gamma}_{2m}}(y)=\frac{1}{\Gamma(N_2)}\frac{1}{\Omega_2^{N_2}}y^{N_2-1}\exp\left(-\frac{y}{\Omega_2}\right)$ into (78), the integral I_2 is given by

$$I_2 = 1-\frac{2M}{\Gamma(N_2)\Gamma(N_1)\Omega_3^{N_1}}\sum_{i=0}^{N_1-1}C_i^{N_1-1}\rho_1^{N_1-i-1}\times(1-\rho_1)^i\sum_{p=0}^{N_1-i-1}\sum_{q=0}^{p}\frac{C_q^p}{p!}\widehat{\sum}\Phi\sum_{tt=0}^{N_2+kk-1} \times\frac{C_{tt}^{N_2+kk-1}\gamma^{N_2+kk+p}\gamma_4^{\frac{2N_2+2kk-1-tt+q}{2}}}{\Omega_1^{\frac{2p+tt-q+1}{2}}\left(\frac{(1+m(1-\rho_2))\Omega_2}{(1+m)}\right)^{-\frac{tt-q+1}{2}}}\exp\left(-\frac{(1+m)\gamma_4\gamma}{(1+m(1-\rho_2))\Omega_2}\right)\int_0^\infty\gamma_3^{\frac{2N_1+2p+tt-q-1}{2}}\exp\left(-\gamma_3\left(\frac{1}{\Omega_3}+\frac{\gamma}{\Omega_1}\right)\right)K_{tt-q+1}\left(2\sqrt{\frac{(1+m)\gamma_3\gamma_4\gamma^2}{(1+m(1-\rho_2))\Omega_2\Omega_1}}\right)d\gamma_3 \tag{79}$$

Using (6.643.3) in [34], we have

$$I_2 = 1-M\sum_{i=0}^{N_1-1}C_i^{N_1-1}\rho_1^{N_1-i-1}(1-\rho_1)^i\sum_{p=0}^{N_1-i-1}\sum_{q=0}^{p}\frac{C_q^p}{p!}\widehat{\sum}\Phi\sum_{tt=0}^{N_2+kk-1}C_{tt}^{N_2+kk-1} \times\frac{\Gamma(N_1+p+tt-q+1)\Gamma(N_1+p)}{\Gamma(N_2)\Gamma(N_1)}\frac{\Omega_1^{N_1}\Omega_3^{\frac{2p+tt-q}{2}}}{\left(\frac{(1+m(1-\rho_2))\Omega_2}{(1+m)}\right)^{-\frac{tt-q+2}{2}}} \times\frac{\gamma^{N_2+kk+p-1}\gamma_4^{\frac{2N_2+2kk-2-tt+q}{2}}}{(\Omega_1+\gamma\Omega_3)^{\frac{2N_1+2p+tt-q}{2}}}\times\exp\left(-\gamma_4\left(\frac{(1+m)\gamma}{(1+m(1-\rho_2))\Omega_2}-\frac{(1+m)\Omega_3\gamma^2}{2(\Omega_1+\gamma\Omega_3)(1+m(1-\rho_2))\Omega_2}\right)\right)W_{-\frac{2N_1+2p+tt-q}{2},\frac{tt-q+1}{2}}\left(\frac{(1+m)\Omega_3\gamma^2\gamma_4}{\Omega_2(\Omega_1+\gamma\Omega_3)(1+m(1-\rho_2))}\right) \tag{80}$$

According to (69), by taking the integral of I_2 with respect to γ_4 the closed-form expression of $F_{\gamma_D}(\gamma)$ can be achieved, and is given by

$$F_{\gamma_D}(\gamma) = \int_0^\infty I_2 f_{\gamma_4}(\gamma_4)d\gamma_4 \tag{81}$$

With (80), we assume

$$I_\Delta = \frac{1}{\Omega_4}\int_0^\infty \gamma_4^{\frac{2N_2+2kk-2-tt+q}{2}} \times \exp\left(-\gamma_4\left(\frac{(1+m)\gamma}{(1+m(1-\rho_2))\Omega_2}\left(\frac{2\Omega_1+\gamma\Omega_3}{2(\Omega_1+\gamma\Omega_3)}\right)+\frac{1}{\Omega_4}\right)\right) W_{-\frac{2N_1+2p+tt-q}{2},\frac{tt-q+1}{2}} \times \left(\frac{(1+m)\Omega_3\gamma^2\gamma_4}{\Omega_2(\Omega_1+\gamma\Omega_3)(1+m(1-\rho_2))}\right) d\gamma_4 \tag{82}$$

Using (7.621.3) in [34], we have

$$I_\Delta = \frac{1}{\Omega_4}\frac{\Gamma(N_2+kk+1)\Gamma(N_2+kk-tt+q)}{\Gamma(N_2+N_1+kk+p+1)} \times \left(\frac{(1+m)\Omega_3\gamma^2}{\Omega_2(\Omega_1+\gamma\Omega_3)(1+m(1-\rho_2))}\right)^{\frac{tt-q}{2}+1} \times \left(\frac{((1+m)\gamma\Omega_4+(1+m(1-\rho_2))\Omega_2)}{\Omega_2\Omega_4(1+m(1-\rho_2))}\right)^{-(N_2+kk+1)} {}_2F_1 \times \left(N_2+kk+1, N_1+p+tt-q+1, N_2+N_1+kk \times +p+1, \frac{(1+m)\gamma\Omega_1\Omega_4+(1+m(1-\rho_2))(\Omega_2\Omega_1+\gamma\Omega_2\Omega_3)}{(\Omega_1+\gamma\Omega_3)((1+m)\gamma\Omega_4+(1+m(1-\rho_2))\Omega_2)}\right) \tag{83}$$

Finally, combining (83), (81), and (80), Theorem 1 is proved.

Δ

11 Appendix C

11.1 Proof of Corollary 1

Proof: When the systems have the perfect CSI, the CDF $F_{\gamma_1}(\gamma_1)$ is given by

$$F_{\gamma_1}(\gamma_1) = 1-\sum_{i=0}^{N_1-1}\frac{1}{i!}\left(\frac{\gamma_1}{\Omega_1}\right)^i \exp\left(-\frac{\gamma_1}{\Omega_1}\right) \tag{84}$$

The CDF of the equivalent end-to-end SNR can also be formulated by (69). Similar to (70), (71), (72), (73), using (10) the integral in (69) I_1 is given by

$$I_1 = 1-\sum_{p=0}^{N_1-1}\sum_{q=0}^{p}\frac{C_q^p}{p!\Omega_1^p}\gamma^{p+q}{\gamma_3}^p\gamma_4^p \exp\left(-\frac{1}{\Omega_1}\gamma_3\gamma\right)\int_0^\infty \exp \times \left(-\frac{1}{\Omega_1}\frac{\gamma_3\gamma_4\gamma^2}{x}\right)x^{-q} f_{\tilde{\gamma}_2}(x+\gamma\gamma_4)dx \tag{85}$$

With the consideration that the two RVs $\tilde{\gamma}_2$ and γ_2 have the same statistical properties when $\rho_2 = 1$, by substituting (19) into (85), after some algebraic operation we have

$$I_1 = 1-\frac{2M}{\Gamma(N_2)}\sum_{p=0}^{N_1-1}\sum_{q=0}^{p}\frac{C_q^p}{p!}\widehat{\sum}\sum_{tt=0}^{N_2+\varphi-1} C_{tt}^{N_2+\varphi-1} \times \left(\frac{1}{\Omega_2}\right)^{\varphi+N_2} \frac{\gamma^{N_2+\varphi+p}\gamma_3^{\frac{2p+tt-q+1}{2}}\gamma_4^{\frac{2N_2+2\varphi-1-tt+q}{2}}}{\Omega_1^{\frac{2p+tt-q+1}{2}}\left(\frac{\Omega_2}{(1+m)}\right)^{-\frac{tt-q+1}{2}}} \times \exp\left(-\gamma\left(\frac{(1+m)\gamma_4}{\Omega_2}+\frac{\gamma_3}{\Omega_1}\right)\right)K_{tt-q+1}\left(2\sqrt{\frac{(1+m)\gamma_3\gamma_4\gamma^2}{\Omega_2\Omega_1}}\right) \tag{86}$$

Then, similar to (78), (79), (80), (81), (82), (83), the result (24) is proved.

Δ

12 Appendix D

12.1 Proof of Corollary 4

With perfect CSI (ρ_1 =1), in high SNR the CDF $F_{\gamma_1}\pi(\gamma)$ is expressed as $F_{\gamma_1}(\gamma)\approx\frac{1}{N_1!}\left(\frac{\gamma}{\Omega_1}\right)^{N_1}$. Then, similar to (49), the CDF $F_{\gamma_{13}}(\gamma)$

$$F_{\gamma_{13}}(\gamma) = \int_0^\infty F_{\gamma_1}(\gamma_3\gamma)f_{\gamma_3}(\gamma_3)d\gamma_3 \approx \frac{\Gamma(2N_1)}{\Gamma(N_1-1)\Gamma(N_1)} \times \left(\frac{\Omega_3}{\Omega_1}\right)^{N_1}\gamma^{N_1} \tag{87}$$

With $\rho_2 = 1$, in high SNR the PDF $f_{\gamma_{2m}}(\gamma_2)\approx\frac{\gamma_2^{N_2-1}}{(N_2-1)!\Omega_2^{N_2}}$, and CDF $F_{\gamma_{2m}}(\gamma_2)\approx\frac{1}{N_2!}\left(\frac{\gamma_2}{\Omega_2}\right)^{N_2}$. This leads to $F_{\tilde{\gamma}_2}(\gamma_2) = \left(F_{\gamma_{2m}}(\gamma_2)\right)^M\approx\frac{1}{(N_2!)^M}\left(\frac{\gamma_2}{\Omega_2}\right)^{MN_2}$. Thus, having

$$F_{\gamma_{24}}(\gamma) = \int_0^\infty F_{\tilde{\gamma}_2}(\gamma\gamma_4)f_{\gamma_4}(\gamma_4)d\gamma_4 \approx \frac{(MN_2)!}{(N_2!)^M}\left(\frac{\Omega_4\gamma}{\Omega_2}\right)^{MN_2} \tag{88}$$

Combining (87) and (88), Corollary 4 is proved. *Δ*

Competing interests

The authors declare that they have no competing interests.

Acknowledgements

The authors would like to thank the editors and the anonymous reviewers for their constructive comments and suggestions, which helped to improve the quality of this paper. This work was supported by the Natural Science Foundation of China under Grant 61261015, the 973 project 2013CB329104, the Natural Science Foundation of China under Grant 61372124, 61363059, and 61302100, the projects BK2011027, the Natural Science Foundation of Gansu Province for Distinguished Young Scholars (1308RJDA007), and by the project "Massive MIMO channels modeling and estimation over millimeter wave band for 5G."

Author details
[1]College of Computer Science and Engineering, Northwest Normal University, Lanzhou 730070, China. [2]Wireless Communication Key Lab of Jiangsu Province, Nanjing University of Posts and Telecommunications, Nanjing 210003, China. [3]Key Lab of Broadband Wireless Communication and Sensor Network Technique of Ministry of Education, Nanjing University of Posts and Telecommunications, Nanjing 210003, China.

Received: 27 July 2014 Accepted: 4 August 2015
Published online: 16 August 2015

References

1. P Mugen, Y Changqing, Z Zhongyuan, W Wenbo, C Hsiao-Hwa, Cooperative network coding in relay-based IMT-advanced systems. Communications Magazine, IEEE **50**(4), 76–84 (2012)
2. J Mitola, GQ Maguire, Cognitive radio: making software radios more personal. Personal Communications, IEEE **6**(4), 13–18 (1999)
3. JM Peha, Approaches to spectrum sharing. IEEE Commun. Mag **43**(2), 10–12 (2005)
4. J Xiangdong, Y Longxiang, Z Hongbo, Cognitive opportunistic relaying systems with mobile nodes: average outage rates and outage durations. Communications, IET **8**(6), 789–799 (2014)
5. H Yuzhen, F Al-Qahtani, W Qihui, Z Caijun, W Jinlong, H Alnuweir, Outage analysis of spectrum sharing relay systems with multiple secondary destinations under primary user's interference. Vehicular Technology, IEEE Transactions on **63**(7), 3456–3464 (2014)
6. J Sydir, R Taori, An evolved cellular system architecture incorporating relay stations. Communications Magazine, IEEE **47**(6), 115–121 (2009)
7. SW Peters, RW Heath, The future of WiMAX: multihop relaying with IEEE 802.16j. Communications Magazine, IEEE **47**(1), 104–111 (2009)
8. F Guimaraes, C D-Da, T Tsiftsis, C Cavalcante, G Karagiannidis, Multi-user and multi-relay cognitive radio networks under spectrum sharing constraints. Vehicular Technology, IEEE Transactions on **63**(1), 433–439 (2014)
9. DB da Costa, M Elkashlan, Y Phee Lep, Y Nan, MD Yacoub, Dual-hop cooperative spectrum sharing systems with multi-primary users and multi-secondary destinations over Nakagami-m fading, in *PROC. Of Personal Indoor and Mobile Radio Communications (PIMRC), 2012 IEEE 23rd International Symposium on*, 2012, pp. 1577–1581
10. H Yuzhen, F Al-Qahtani, Z Caijun, W Qihui, Outage analysis of spectrum sharing relay systems with multi-secondary destinations in the presence of primary user's interference, in *Proc. Of Wireless Communications & Signal Processing (WCSP), 2013 International Conference on*, 2013, pp. 1–6
11. L Fan, X Lei, T Duong, R Hu, M Elkashlan, Multiuser cognitive relay networks: joint impact of direct and relay communications. Wireless Communications, IEEE Transactions on **13**(9), 5043–5055 (2014)
12. Y Nan, M Elkashlan, PL Yeoh, Y Jinhong, Multiuser MIMO relay networks in Nakagami-*m* fading channels. Communications, IEEE Transactions on **60**(11), 3298–3310 (2012)
13. P Mugen, Z Xiang, W Wenbo, C Hsiao-Hwa, Performance of dual-polarized MIMO for TD-HSPA Evolution Systems. Systems Journal, IEEE **5**(3), 406–416 (2011)
14. H Yuzhen, FS Al-Qahtani, C Zhong, W Qihui, W Jinlong, HM Alnuweiri, Cognitive MIMO relaying networks with primary user's interference and outdated channel state information. Communications. IEEE Transactions on. **62**(12), 4241–4254 (2014)
15. L Quanzhong, F Renhai, Q Jiayin, Optimal relay precoding for spectrum sharing multi-hop MIMO cognitive radio networks. Electronics Letters. **48**(24), 1562–1564 (2012)
16. Z Zhao, Z Ding, M Peng, W Wang, J Thompson, On the design of cognitive radio inspired asymmetric network coding transmissions in MIMO systems. Vehicular Technology, IEEE Transactions on **PP**(99), 1–1 (2014)
17. M Ghamari Adian, H Aghaeinia, Resource allocation in MIMO-OFDM based cooperative cognitive radio networks. Communications, IEEE Transactions on **62**(7), 2224–2235 (2014)
18. L Tianxiang, G Feifei, Z Xian-Da, JCF Li, L Ming, Robust beamforming for relay-aided multiuser MIMO cognitive radio networks, in *Proc of Wireless Communications and Networking Conference (WCNC), 2013 IEEE*, 2013, pp. 2473–2477
19. TKY Lo, Maximum ratio transmission. Communications. IEEE Transactions on **47**(10), 1458–1461 (1999)
20. H Jianli, L Quanzhong, Z Qi, Z Guangchi, Q Jiayin, Relay beamforming for amplify-and-forward multi-antenna relay networks with energy harvesting constraint. Signal Processing Letters, IEEE **21**(4), 454–458 (2014)
21. HA Suraweera, TA Tsiftsis, GK Karagiannidis, A Nallanathan, Effect of feedback delay on amplify-and-forward relay networks with beamforming. Vehicular Technology, IEEE Transactions on **60**(3), 1265–1271 (2011)
22. S JiangBo, L Zan, H HaiYan, C JunJie, G Rui, Capacity analysis of cognitive relay networks with the PU's interference. Communications Letters, IEEE **16**(12), 2020–2023 (2012)
23. I Trigui, I Mechmeche, S Affes, A Stephenne, Capacity and error rate analysis of cognitive MIMO AF relaying systems. Wireless Communications Letters, IEEE **3**(6), 665–668 (2014)
24. H HaiYan, L Zan, S JiangBo, G Rui, Outage analysis of underlay cognitive multiple relays networks with a direct link. Communications Letters, IEEE **17**(8), 1600–1603 (2013)
25. B Vo Nguyen Quoc, TQ Duong, DB da Costa, GC Alexandropoulos, A Nallanathan, Cognitive amplify-and-forward relaying with best relay selection in non-identical Rayleigh fading. Communications Letters, IEEE **17**(3), 475–478 (2013)
26. P Hoc, TQ Duong, M Elkashlan, H Zepernick, Beamforming amplify-and-forward relay networks with feedback delay and interference. Signal Processing Letters, IEEE **19**(1), 16–19 (2012)
27. C Hongyu, Z Rongqing, S Lingyang, J BingLi, Capacity analysis of bidirectional AF relay selection with imperfect channel state information. Wireless Communications Letters, IEEE **2**(3), 255–258 (2013)
28. H Yuzhen, L Cong, Z Caijun, W Jinlong, C Yunpeng, W Qihui, On the capacity of dual-hop multiple antenna AF relaying systems with feedback delay and CCI. Communications Letters, IEEE **17**(6), 1200–1203 (2013)
29. MK Simon, MS Alouini, *Digital communications over fading channels: a unified approach to performance analysis*, 2nd edn. (John Wiley, New York, 2000)
30. M Yao, Z Dongbo, A Leith, W Zhengdao, Error performance of transmit beamforming with delayed and limited feedback. Wireless Communications, IEEE Transactions on **8**(3), 1164–1170 (2009)
31. PL Yeoh, M Elkashlan, Y Nan, DB da Costa, TQ Duong, Unified analysis of transmit antenna selection in MIMO multirelay networks. Vehicular Technology, IEEE Transactions on **62**(2), 933–939 (2013)
32. A Yilmaz, O Kucur, Performances of transmit antenna selection, receive antenna selection, and maximal-ratio-combining-based hybrid techniques in the presence of feedback errors. Vehicular Technology, IEEE Transactions on **63**(4), 1976–1982 (2014)
33. HA David, HN Nagaraja, *Order statistics*, 3rd edn. (John Wiley, New York, 2003)
34. IS Gradshteyn, IM Ryzhik, *Table of integrals, series, and products*, 7th edn. (Acdemic, San Diego, CA, 2007)
35. A Papoulis, *Probability, random variables, and stochastic processes*, 3rd edn. (McGraw-Hill, New York, 1991)
36. AM Mathai, RK Saxena, *The H-function with application in statistics and other disciplines* (Wiley Eastern, New Delhi, and Halsted Press, New York, 1978)