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On the detection probability of the standard condition number detector in finite-dimensional cognitive radio context

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Abstract

Standard condition number (SCN) detector is an efficient detector in multi-dimensional cognitive radio systems since no a priori knowledge is needed. The earlier studies usually assume a large number of dimensions and a large number of samples per dimension and use random matrix theory (RMT) to derive asymptotic distributions of the SCN metric. In practice, the number of dimensions may not be large enough for the SCN distribution to be well approximated by the asymptotic ones. In this context, the false alarm probability is considered in literature and formulas for 2D, 3D, and infinite-dimensional systems have been derived. However, the detection probability, which is of great importance in cognitive radio, has not been well discussed in literature. In this paper, we discuss, analytically, the detection probability of the SCN detector. Since the probability of detection is totally related to the SCN distribution, we derive new results on the joint ordered eigenvalues and SCN distributions for central semi-correlated Wishart matrices. These results are used to approximate the detection probability by the non-central/central approximation. We consider systems with three or more dimensions, and we give an approximated form of the detection probability. The analytical results of this paper on probability of detection along with those on probability of false alarm present a complete performance analysis and are validated through simulations. We show that the proposed analytical expressions provide high accuracy and that the SCN detector outperforms the well-known energy detector and the largest eigenvalue detector even with a small number of dimensions and low noise uncertainty environments.

Keywords: Cognitive radio, Joint distribution, Spectrum sensing, Standard condition number, Wishart matrix

1 Introduction

Cognitive radio (CR) is considered as a promising solution for the scarcity and inefficient use of the spectrum [1, 2]. It allows the secondary (unlicensed) user (SU) to use the spectrum in an opportunistic way so that it does not cause harmful interference to the primary (licensed) user (PU). One possibility is to detect whether the PU is present or not to avoid any interference. Therefore, spectrum sensing has a key functionality in CR as it allows the CR to differentiate between the spectrum being used and the spectrum holes.

Several spectrum sensing techniques were proposed in the last decade [3]. Energy detector (ED) is among the most popular sensing techniques; it is simple, non-coherent, and needs no prior knowledge about the PU's signal. However, ED is very sensitive to noise variations and requires perfect knowledge of the noise power. Thus, the performance of ED suffers from a high degradation under low signal-to-noise ratio (SNR) and noise uncertainty conditions [4, 5].

Recently, eigenvalue-based detector (EBD) has been shown to overcome noise uncertainty challenges and performs adequately even in low SNR conditions. EBD is based on the eigenvalues of the received signal covariance matrix. In this context, the multi-dimensional aspect of a cognitive radio could be (i) multi-antenna CR [6, 7], (ii) cooperative scenario between several CR nodes [8], (iii) oversampling within the CR node [9], or any combination of them. If the entries within these dimensions are considered Gaussian, then the receiver's covariance matrix is known as a Wishart matrix in random matrix

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theory (RMT) [10]. Results from RMT have been used to study the EBD algorithms which include the largest eigenvalue (LE) detector proposed in [9, 11], the scaled largest eigenvalue (SLE) detector [12, 13], and the standard condition number (SCN) detector [9, 14–18]. The LE is a semi-blind detector since it requires the knowledge of the noise power whereas the SLE and SCN are blind detectors that do not require this knowledge.

If we denote the maximum and the minimum eigenvalues of the receiver's covariance matrix by $\lambda_{\max}$ and $\lambda_{\min}$, respectively, then the standard condition number (SCN) is defined as the ratio between $\lambda_{\max}$ and $\lambda_{\min}$ ($\lambda_{\max}/\lambda_{\min}$). In multiple-input multiple-output (MIMO) systems, the SCN has an effective connection with MIMO receiver performance in spatial multiplexing systems [19–21] and it indicates the multipath richness of the MIMO channel [22, 23]. In CR systems, the SCN detector compares this ratio to a threshold to decide whether the PU exists or not. The performance of a detection algorithm is, usually, characterized by two types of probabilities: the false alarm probability (P_{fa}) and the detection probability (P_{d}). The latter depends on the decision threshold and are related to the statistics of the detector metric. Thus, the characteristics of the SCN are of great importance in both MIMO and CR systems.

The SCN metric was well studied in the recent years; for example, this metric was studied asymptotically in [14] and the threshold was presented according to Marchenko-Pastur (MP) law [24]. If the size of the receiver's matrix is $K \times N$, then MP law proves that the largest ($\lambda_{\max}$) and the smallest ($\lambda_{\min}$) eigenvalues of the receiver's covariance matrix converge to constants as $(K, N) \to \infty$ with $c = {}^{K}/_{N}$ is a constant ratio [25]. These constants are simply determined by c and the powers of the signal and the noise.

In [9], the authors improved the accuracy of the asymptotic statistical distribution of the SCN by using the Tracy-Widom (TW) distribution to model the largest eigenvalue (numerator of the SCN) [26] while maintaining the MP representation of the smallest one. TW distribution is a limiting distribution of the largest eigenvalue of a central Wishart matrix as $(K, N) \to \infty$ [27]. This approximation of the SCN, by TW distribution for the numerator and MP law for the denominator, results in an approximated relation between the decision threshold and the P_{fa}.

This work was further extended in [15, 16] by using the Curtiss formula for the distribution of the ratio of random variables (i.e., SCN) [28] where both the largest and the smallest eigenvalues converge to Tracy-Widom distributions when $(K, N) \to \infty$ as shown in [26, 29]. Moreover, by the exploitation of the normal and TW distributions and using the Curtiss formula, the authors in [17] provided a form for the probability of missed detection ($P_{\mathrm{md}} = 1 - P_{\mathrm{d}}$) for sufficiently large values of K and

N. However, all these expressions include TW distribution and Curtiss formula that are hard to evaluate numerically online in addition to the considered asymptotic assumptions which are not necessarily practical.

By considering finite dimensions, the authors in [30] provided the exact generic form of the distribution of the SCN based on the joint distribution of the ordered eigenvalues of Wishart matrix. Consequently, [18] provides the exact forms of the P_{fa} of the SCN detector for CR with two dimensions and [31] provides P_{fa} form for three dimensions. In addition, the P_{d} for two-dimensional systems was approximated using the non-central/central approximation and provided in [18] following the results in [30]. When $K \geq 3$ (for K-dimensional system), the form of the P_{d} could be derived using the results in [30] as long as the non-centrality matrix Ω_K is full rank. Moreover, the non-central/central approximation is used to approximate the distribution of the non-central uncorrelated Wishart matrix by the distribution of the central semi-correlated Wishart matrix. However, using this method to approximate the distribution of the SCN requires normalization and the behavior of this approximation could be affected especially at high SNR.

In this paper, the non-centrality matrix (Ω_K) is not a full-rank matrix as considered in previous studies [18, 30]. This case includes, for example, but is not limited to, a single PU transmitting orthogonal frequency-division multiplexing (OFDM) signal. Accordingly, results derived in literature may not be applicable since they are derived for the full rank case. Thus, by using the non-central/central approximation, we provide the new form of the P_{d} of the SCN metric for general finite K-dimensional systems and we give the form in the particular case when $K = 3$. Also, we study the impact of the system parameters (K, N, and SNR) on the approximation accuracy and on the detection probability. The main contribution of this paper could be summarized as follows:

- Using the non-central/central approximation, the eigenvalues of the correlation matrix of the central semi-correlated Wishart matrix may coincide. For this purpose, we generalize the joint distribution of the ordered eigenvalues of the central semi-correlated Wishart matrix. In our derivations, the eigenvalues of the correlation matrix can be equal or distinct.
- The authors in [30] consider the case where all the eigenvalues of the correlation matrix are distinct. In this paper, we show that the generic form of the cumulative distribution function (CDF) of the SCN given in [30] is still valid if the eigenvalues are not distinct. However, the form of the parameters differ where we give the new forms.
- We derive the analytical forms of the CDF and the probability density function (PDF) of the SCN of

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three-dimensional systems when Ω_K is not a full-rank matrix.

- For the sake of completeness, we also derive the CDF and the PDF of the SCN of three-dimensional systems when Ω_K is a full-rank matrix. Thus, the results of this paper combined with results in [31] form a complete closed-form system for the three-dimensional cognitive radio.
- Since the previous studies consider the non-central/central approximation in low SNR regions [18], we study the impact of the system parameters on the accuracy of the non-central/central approximation. This impact is given in terms of P_d as it is an important target in the general CR context.

The rest of this paper is organized as follows. In Section 2, the system model is introduced and discussed. In Section 3, the non-central/central approximation is recalled and analyzed. In Section 4, we give the joint ordered eigenvalue distribution with a generalization to equal eigenvalues of the correlation matrix. In Section 5, we emphasize on the general form of the CDF of the SCN for the central semi-correlated Wishart matrix when L eigenvalues of the correlation matrix are equal. The exact CDF and PDF of this SCN for three-dimensional systems are derived as well. Simulation results and approximation accuracy are discussed in Section 6, and we conclude in Section 7.

Notations: Vectors and matrices are represented, respectively, by lowercase and uppercase boldface. The symbols |.| and tr(.) represent, respectively, the determinant and trace of a matrix while $(.)^{1/2}$, $(.)^{\mathrm{T}}$, and $(.)^{\dagger}$ are the square root, transpose, and Hermitian symbols, respectively. $\mathbf{I}_n$ is the $n \times n$ identity matrix. Symbol $\|.\|^2$ stands for the norm, $E[.]$ stands for the expected value, and $\sim$ stands for distributed as.

2 System model

We consider a K-dimensional CR system, and we aim to detect the presence/absence of a single PU during a sensing period which corresponds to N samples. The K-dimensions might be K collaborating SUs, K antennas, an oversampling by a factor of K, or any combination of them. For this detection problem, there are two hypotheses: $\mathcal{H}_0$ corresponds to the absence of PU transmission (i.e., spectrum hole) and $\mathcal{H}_1$ corresponds to the presence of the PU transmission (i.e., spectrum being used). The received vector, at instant n, under each hypothesis is given by

$$\mathcal{H}_0 : \boldsymbol{y}(n) = \boldsymbol{\eta}(n), \tag{1}$$

$$\mathcal{H}_1 : \boldsymbol{y}(n) = \boldsymbol{h}(n)s(n) + \boldsymbol{\eta}(n), \tag{2}$$

where $\boldsymbol{y}(n) = [y_1(n), \cdots, y_K(n)]^{\mathrm{T}}$ is the observed $K \times 1$ complex samples across all the dimensions at instant n.

$\boldsymbol{\eta}(n)$ is a $K \times 1$ complex circular white Gaussian noise. $\boldsymbol{h}(n)$ is a $K \times 1$ complex vector that represents the channel coefficients between the PU and each of the CR dimensions, and $s(n)$ stands for the primary signal sample at instant n.

After collecting N samples from each dimension, the received signal matrix $\boldsymbol{Y}$ is written as follows:

$$\boldsymbol{Y} = \begin{pmatrix} y_1(1) & y_1(2) & \cdots & y_1(N) \\ y_2(1) & y_2(2) & \cdots & y_2(N) \\ \vdots & \vdots & \ddots & \vdots \\ y_K(1) & y_K(2) & \cdots & y_K(N) \end{pmatrix}. \tag{3}$$

Without loss of generality, we suppose that $K \leq N$ and we define the received sample covariance matrix as $\boldsymbol{W} = \boldsymbol{Y}\boldsymbol{Y}^{\dagger}$.

Under $\mathcal{H}_0$, the input of the matrix $\boldsymbol{Y}$ is a complex circular white Gaussian noise with zero mean and unknown variance σ_η^2, then $\boldsymbol{W}$ is well known as a central uncorrelated complex Wishart matrix and is denoted by $\boldsymbol{W} \sim \mathcal{CW}_K(N, \sigma_\eta^2 \mathbf{I}_K)$ where K is the size of the matrix, N is the number of degrees of freedom (DoF), and $\sigma_\eta^2 \mathbf{I}_K$ is the correlation matrix.

Under $\mathcal{H}_1$, we suppose that the channel remains constant during the sensing time and we have a single PU whose signal is drawn independently from Gaussian process for every sample. Consequently, $\boldsymbol{W}$ follows a non-central uncorrelated complex Wishart distribution which is denoted as $\boldsymbol{W} \sim \mathcal{CW}_K(N, \sigma_\eta^2 \mathbf{I}_K, \Omega_K)$, where Ω_K is a rank 1 non-centrality matrix [32].

Let us denote by $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_K > 0$ the eigenvalues of $\boldsymbol{W}$, then the SCN metric is given by
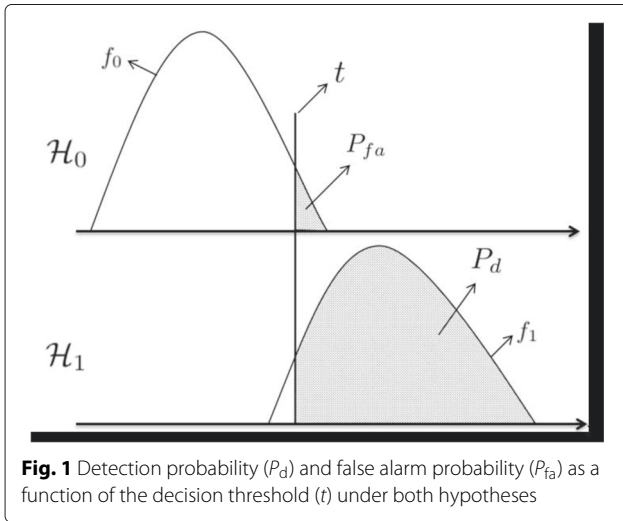
$$\mathrm{SCN} = \frac{\lambda_1}{\lambda_K}. \tag{4}$$

Denoting by t the decision threshold, then the detection probability (P_d), defined as the probability of correctly detecting the presence of PU, and the false alarm probability (P_{fa}), defined as the probability of detecting the presence of PU while it does not exist, are, respectively, given by (5) and (6) and shown in Fig. 1.

$$P_d = P(\mathrm{SCN} \geq t / \mathcal{H}_1) \tag{5}$$

$$P_{fa} = P(\mathrm{SCN} \geq t / \mathcal{H}_0). \tag{6}$$

These probabilities depend on the threshold (t) being used. However, if the expressions of the P_{fa} and P_d are previously known, then a threshold could be set according to a required error constraint. Then, it is clear that these probabilities depend on the distribution of the SCN metric. If we denote the CDF and PDF of SCN, respectively, by $F_i(.)$ and $f_i(.)$ with index $i \in \{0, 1\}$ indicating the considered hypothesis, then we can write

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Fig. 1 Detection probability (P_d) and false alarm probability (P_{fa}) as a function of the decision threshold (t) under both hypotheses

$$P_{\text{fa}} = 1 - F_0(t) \tag{7}$$

$$P_{\text{d}} = 1 - F_1(t) \tag{8}$$

The exact general form of the distribution of SCN under $\mathcal{H}_0$ ($F_0(t)$) has been derived in [30]. Two particular cases have been considered in [18, 31] when $K = 2$ and 3, respectively. Asymptotically, this distribution could be found in [9, 14, 15]. Under $\mathcal{H}_1$, an asymptotic distribution of the SCN, $F_1(t)$, could be found in [17]. In the finite case, the exact generic form of $F_1(t)$ is derived in [30]; however, further numerical evaluation would either require Nuttall-Q function which could be replaced by Marcum Q-function and a finite weighted sum of Bessel functions [33] or by hypergeometric functions that could be expanded to an infinite sum (see, for example, [34] for $K = 2$ and $N = 2$). Thus, and since both solutions are difficult to manipulate,[1] a third solution is to use the non-central/central approximation that approximates the distribution of the non-central uncorrelated Wishart matrix by the distribution of the central semi-correlated Wishart matrix. This approximation is presented in the next section. The exact general form of the distribution of the SCN of central semi-correlated Wishart matrix is provided by [30] and is used to approximate $F_1(t)$ for $K = 2$ in [18, 30].

3 Non-central/central approximation and parameter analysis

The non-central/central approximation is recalled in Lemma 1.

Lemma 1 ([35]). *The complex non-central Wishart matrix $\boldsymbol{W} \sim \mathcal{CW}_K(N, \kappa^2 \mathbf{I}_K, \Omega_K)$ and the complex semi-correlated Wishart matrix $\boldsymbol{W} \sim \mathcal{CW}_K(N, \widehat{\Sigma}_K)$ with effective correlation matrix $\widehat{\Sigma}_K = \kappa^2 I_K + {}^{\Omega_K}/_N$ have the same first- and second-order moments differing by ${}^{\Omega_K}/_N$.*

Using Lemma 1, the distribution of the SCN under $\mathcal{H}_1$ hypothesis could hence be approximated by the distribution of the SCN of complex semi-correlated Wishart matrix with a proper correlation matrix. It is worth noting that this approximation for the SCN distribution also requires a normalization by κ^2 ($\mathbf{W}_n = \kappa^{-2}\mathbf{W}$) which gives the same SCN as $\mathbf{W}$. Therefore, following (8), the detection probability is, accordingly, approximated.

Under hypothesis $\mathcal{H}_1$ and following our assumptions, the non-centrality matrix $-\Omega_K$ is a rank 1 matrix and the only non-zero eigenvalue of Ω_K is denoted by ω_1. Consequently, and after applying normalization of Lemma 1, the effective correlation matrix, $\widehat{\Sigma}_K$, has coincidence in its eigenvalues. Let $\boldsymbol{\sigma} = [\sigma_1, \sigma_2, \cdots, \sigma_K]^{\text{T}}$ be the vector of ordered eigenvalues of $\widehat{\Sigma}_K = I_K + {}^{\Omega_K}/_N$, then all but one eigenvalue of $\widehat{\Sigma}_K$ are still equal to 1 (eigenvalues of I_K) while σ_1 is given by

$$\sigma_1 = 1 + \frac{\omega_1}{N}. \tag{9}$$

The average SNR, under $\mathcal{H}_1$, is defined by

$$\rho = \frac{\sigma_s^2 \sigma_h^2}{\sigma_\eta^2}, \tag{10}$$

where the PU signal power, σ_s^2, could be estimated by[2] ($\|\boldsymbol{s}\|^2/N$), and the channel power $\sigma_h^2 = (\|\boldsymbol{h}\|^2/K)$. The non-centrality matrix is given by

$$\Omega_K = \Sigma_K^{-1} MM^\dagger = \frac{1}{\sigma_\eta^2} \|\boldsymbol{s}\|^2 \boldsymbol{h}\boldsymbol{h}^\dagger, \tag{11}$$

where Σ_K is the covariance matrix of $\boldsymbol{Y}$, defined as $\Sigma_K = E[(\boldsymbol{Y} - \boldsymbol{M})(\boldsymbol{Y} - \boldsymbol{M})^\dagger] = \sigma_\eta^2 I_K$, and $\boldsymbol{M}$ is the mean of $\boldsymbol{Y}$ defined as $\boldsymbol{M} = E[\boldsymbol{Y}] = \boldsymbol{h}\boldsymbol{s}^{\text{T}}$ with $\boldsymbol{h} = [h_1 h_2 \cdots h_K]^{\text{T}}$ and $\boldsymbol{s} = [s(1)s(2) \cdots s(N)]^{\text{T}}$. It is worth mentioning that (11) will not be changed after normalization.

As a result, and by using the property that the trace of a matrix equals the sum of its eigenvalues, then ω_1 could be written as

$$\omega_1 = \text{tr}(\Omega_K) = \frac{1}{\sigma_\eta^2} \|\boldsymbol{s}\|^2 \text{tr}\left(\boldsymbol{h}\boldsymbol{h}^\dagger\right) = \frac{1}{\sigma_\eta^2} \|\boldsymbol{s}\|^2 \|\boldsymbol{h}\|^2$$

$$= NK\rho. \tag{12}$$

Thus, Ω_K is not full rank and the eigenvalues of $\widehat{\Sigma}_K$ are not distinct. It is worth noting that for the exact form of P_{d}, it would be useful to refer to the joint distribution of the ordered eigenvalues of non-central uncorrelated Wishart matrix with arbitrary non-centrality matrix rank derived in [36, App. I] and consider rank 1 case. However, as discussed before, it is hard to derive a tractable form for the SCN distribution since it requires Nuttall-Q functions or hypergiometric functions. Therefore, by considering the non-central/central approximation, results based on assuming all correlation eigenvalues distinct could not be

used and new results must be derived. Accordingly, an approximated and simpler form for the P_d will be derived.

4 Joint ordered eigenvalues distribution generalization

As discussed in the previous section, all but one of the eigenvalues of the correlation matrix are equal. In this section, and since the distribution of the SCN metric is derived using the joint distribution of the eigenvalues of the Wishart matrix, we generalize the joint distribution of the ordered eigenvalues of the central semi-correlated Wishart matrix. This new form of the joint distribution covers any number (L) of equal eigenvalues positioned anywhere in vector σ and is given in Theorem 1 below.

In addition, Theorem 1 is also useful in MIMO systems as it could be used to derive marginal and joint distributions in correlated Rayleigh channels when the correlation eigenvalues may coincide. Moreover, these derivations could, also, approximate that of the Rician channels when the mean channel matrix is of rank 1; this could be satisfied when all channel vectors have the same mean value or in some other practical scenarios where the line-of-sight (LOS) component of the channel matrix approaches the scaled all-ones matrix [37].

Theorem 1 (Central semi-correlated Wishart when Σ_K is full rank with L equal eigenvalues). *Let W be a central semi-correlated Wishart matrix $W \sim \mathcal{CW}_K(N, \Sigma_K)$ and denote by $\sigma_1 > \sigma_2 > \cdots > \sigma_p = \cdots = \sigma_q > \cdots > \sigma_K > 0$ the ordered eigenvalues of Σ_K assumed full rank with L equal eigenvalues (from p to q with $q = p + L - 1$), then the joint PDF of λ could be written as follows:*

$$f(\lambda) = C|\Phi(\lambda)| \times |\Psi(\lambda)| \prod_{i=1}^{K} \xi(\lambda_i) \tag{13}$$

with the associated parameters defined as

- $\Phi(\lambda)$ *is a $K \times K$ Vandermonde matrix of entries λ_j^{i-1}.*
- $\Psi(\lambda)$ *is a $K \times K$ matrix of the form $\mathbf{E}(\lambda, \sigma)$ given, for all $1 \leq j \leq K$, by*

$$\{\mathbf{E}(\lambda, \sigma)\}_{i,j} = \begin{cases} \dfrac{\partial^{q-i}(e^{-\frac{\lambda_j}{\sigma_p}})}{\partial \sigma_p^{q-i}} & p \leq i \leq q \\ e^{-\frac{\lambda_j}{\sigma_i}} & otherwise \end{cases} \tag{14}$$

- $\xi(\lambda_i)$ *is defined as $\xi(\lambda_i) = \lambda_i^{N-K}$.*
- *The normalization constant is given by*

$$C = \frac{\prod_{i<j}^{K} \sigma_i \sigma_j}{\prod_{i=1}^{K}(N-i)! \sigma_i^N \prod_{\substack{i<j \\ \sigma_i \neq \sigma_j}}^{K}(\sigma_j - \sigma_i)\,\Gamma_L(L)}, \tag{15}$$

where $\Gamma_s(l) = \prod_{i=1}^{s}(l-i)!$ and

$$\frac{\partial^n(e^{-\frac{\lambda_j}{\sigma_p}})}{\partial \sigma_p^n} = \begin{cases} e^{-\frac{\lambda_j}{\sigma_p}} \sum_{k=1}^{n} \dfrac{(-1)^{k+n} \cdot \mathcal{L}(n,k) \cdot \lambda_j^k}{\sigma_p^{n+k}} & if\, n > 0 \\ e^{-\frac{\lambda_j}{\sigma_p}} & if\, n = 0 \end{cases} \tag{16}$$

with $\mathcal{L}(n,k)$ is the Lah number defined by

$$\mathcal{L}(n,k) = \binom{n}{k}\binom{n-1}{k-1}(n-k)! \tag{17}$$

Proof. See Appendix 1.   □

It can be easily shown that by taking $L = 1$, the parameters in Theorem 1 will be equivalent to the case of all distinct eigenvalues in the literature [30, Table I].

5 SCN distribution

In this section, we will derive the general form of the CDF of the SCN of central semi-correlated Wishart matrix for arbitrary K and N. Then, we emphasize by considering the three-dimensional case ($K = 3$). Given (8) and the results of this section, P_d can then be directly computed.

5.1 The general form

The CDF, $F(x)$, with $x \geq 1$, of the SCN of the central semi-correlated Wishart matrix $W \sim \mathcal{CW}_K(N, \Sigma_K)$ with distinct correlation eigenvalues, denoted by $\sigma_1 > \sigma_2 > \cdots > \sigma_K > 0$, is given by [30]

$$F(x) = C \sum_{l=1}^{K} \int_0^{\infty} \left| \begin{bmatrix} \int_{\lambda_K}^{x\lambda_K} \phi_i(u)\psi_j(u)\xi(u)du, & i \neq l \\ \phi_i(\lambda_K)\psi_j(\lambda_K)\xi(\lambda_K), & i = l \end{bmatrix} \right| d\lambda_K, \tag{18}$$

where $\phi_i(\lambda_j) = [\Phi(\lambda)]_{i,j}$ and $\psi_i(\lambda_j) = [\Psi(\lambda)]_{i,j}$ with $\Phi(\lambda)$, $\Psi(\lambda)$, $\xi(\lambda_i)$, and C are defined in [30, Table I].

If we have L equal correlation eigenvalues (say from p to q with $q = p + L - 1$), then (18) is still valid by considering the parameters $\Phi(\lambda)$, $\Psi(\lambda)$, $\xi(\lambda_i)$, and C that are defined in Theorem 1. This could be easily shown by comparing the joint distribution parameters in Theorem 1 and Table I of [30] and noting that they have the same general form used in deriving the SCN CDF in (18).

It is important to note that all the integrals inside the determinant in (18), $I_{i,j} = \int_{\lambda_K}^{x\lambda_K} \phi_i(u)\psi_j(u)\xi(u)du$, still admit a closed-form solution given, directly after integrating using [38, Eq. (3.351.1)], by

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$$I_{i,j} = \begin{cases} \left[\gamma\left(l, \frac{x\lambda_K}{\sigma_j}\right) - \gamma\left(l, \frac{\lambda_K}{\sigma_j}\right)\right] \cdot \sigma_j^l & i < p \text{ or } i \geq q \\ \sum_{k=1}^{q-j} \left[(-1)^{k+q-j}\mathcal{L}(q-j,k) \cdot \left[\gamma\left(l+k, \frac{x\lambda_K}{\sigma_p}\right) - \gamma\left(l+k, \frac{\lambda_K}{\sigma_p}\right)\right]\right]\sigma_p^{l+j-q} & p \leq i < q \end{cases}, \tag{19}$$

where $\gamma(.,.)$ is the lower incomplete gamma function [38, Eq. (8.350.1)], the constant $l = N - K + i$, and $1 \leq i \leq K$.

5.2 SCN distribution for three-dimensional systems

In the previous sections, we have considered an arbitrary number (L) of equal eigenvalues positioned anywhere in vector $\boldsymbol{\sigma}$. Considering our assumptions that Ω_K is a rank 1 matrix reveals, from Lemma 1, that the eigenvalues of the effective correlation matrix of the central semi-correlated Wishart matrix has $(K-1)$-equal eigenvalues such that $\sigma_1 > \sigma_2 = \cdots = \sigma_K > 0$.

Now, if we consider the case $K = 3$, then $\widehat{\Sigma}_K$ has two equal eigenvalues ($\sigma_1 > \sigma_2 = \sigma_3 > 0$). In this case, the CDF and PDF of the SCN are given in Theorem 2 and Corollary 1, respectively.

Theorem 2 (CDF of SCN of central semi-correlated Wishart matrix when Σ_3 has two equal eigenvalues). *The CDF of the SCN of a 3×3 central semi-correlated Wishart matrix* $\mathbf{W} \sim \mathcal{CW}_K(N, \Sigma_3)$ *whose Σ_3 has two equal eigenvalues ($\sigma_1 > \sigma_2 = \sigma_3$) is given by*

$$F_1(x) = C_{sc}[R_1(N,x) - G_1(N,x)], \tag{20}$$

where $R_1(N,x)$ is defined in (21), $G_1(N,x)$ in (22), $S_1(r,s,t,\mu,\nu,x)$ in (23), $\Delta_1(r,s,t,\mu,\nu,\varepsilon,x)$ in (24), and C_{sc} in (25).

$$R_i(N,x) = S_i\left(N-3, N, N-2, \sigma_1^{-1}, \sigma_2^{-1}, x\right)$$
$$+ S_i\left(N-2, N, N-1, \sigma_1^{-1}, \sigma_2^{-1}, x\right)$$
$$+ S_i\left(N-1, N-1, N-3, \sigma_1^{-1}, \sigma_2^{-1}, x\right), i = 1, 2 \tag{21}$$

$$G_i(N,x) = \Delta_i\left(N-2, N-1, N-2, \sigma_2^{-1}, \sigma_2^{-1}, \sigma_1^{-1}, x\right)$$
$$+ \Delta_i\left(N-2, N, N-3, \sigma_2^{-1}, \sigma_2^{-1}, \sigma_1^{-1}, x\right)$$
$$+ \Delta_i\left(N-3, N-1, N-1, \sigma_2^{-1}, \sigma_2^{-1}, \sigma_1^{-1}, x\right), i = 1, 2 \tag{22}$$

$$S_i(r,s,t,\mu,\nu,x) = \Delta_i(r,s,t,\nu,\nu,\mu,x) + \Delta_i(t,r,s,\nu,\mu,\nu,x)$$
$$- \Delta_i(r,t,s,\nu,\mu,\nu,x) - \Delta_i(r,s,t,\nu,\mu,\nu,x)$$
$$+ \Delta_i(t,s,r,\nu,\mu,\nu,x), \quad i = 1, 2 \tag{23}$$

$$\Delta_1(r,s,t,\mu,\nu,\varepsilon,x) = -\mu^2 \Delta_3(r,s,t,\nu,\varepsilon,\mu,x) \tag{24}$$

$$C_{sc} = \frac{(\sigma_2 - \sigma_1)^{-2}}{(N-1)!\,(N-2)!\,(N-3)!\,\sigma_1^{N-2}\sigma_2^{2(N-2)}} \tag{25}$$

with Δ_3 defined in (35) in Appendix 2, Theorem 3.

Proof. The proof could be summarized as follows:

1. Considering (18) and setting $K = 3$.
2. Substituting the parameters from Theorem 1.
3. Expanding the summation and using (19).
4. Expanding the determinant and integrating using Eqs. (3.351.1) and (3.351.3) of [38].

Then, the result comes after simplification. □

Corollary 1. *The PDF of the SCN of a 3×3 central semi-correlated Wishart matrix* $\mathbf{W} \sim \mathcal{CW}_K(N, \Sigma_3)$ *whose Σ_3 has two equal eigenvalues ($\sigma_1 > \sigma_2 = \sigma_3$) is given by*

$$f_1(x) = C_{sc} \cdot [R_2(N,x) - G_2(N,x)], \tag{26}$$

where $R_2(N,x)$ is defined in (21), $G_2(N,x)$ in (22), $S_2(r,s,t,\mu,\nu,x)$ in (23), C_{sc} in (25), and $\Delta_2(r,s,t,\mu,\nu,\varepsilon,x)$ in (27) below.

$$\Delta_2(r,s,t,\mu,\nu,\varepsilon,x) = -\mu^2 \Delta_4(r,s,t,\nu,\varepsilon,\mu,x) \tag{27}$$

with Δ_4 defined in (38) in Appendix 2, Corollary 2.

Proof. Obtained by differentiating Eq. (20). □

For the sake of completeness, the reader could find, in Appendix 2, the CDF and PDF of the SCN of the central semi-correlated Wishart matrix when all the correlation eigenvalues are distinct.

6 Simulation and discussion

In this section, we discuss the analytical results through Monte Carlo simulations. We, firstly, validate the theoretical analysis presented in Sections 4 and 5. Given the non-central/central approximation in Lemma 1, we study the approximation accuracy as well as the impact of the SNR on this approximation. Since the probability of detection is the main target, we study the effect of the approximation accuracy and the impact of system parameters on P_d. Finally, we show that the SCN detector outperforms the ED and the LE detector if noise power is not perfectly known.

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For this purpose, we, first, generate 10^5 central semi-correlated Wishart matrices. This type of matrices can be easily generated under $\mathcal{H}_0$ by considering a certain correlation between different dimensions of the CR system. The received matrix, under $\mathcal{H}_0$ hypothesis, is then written as
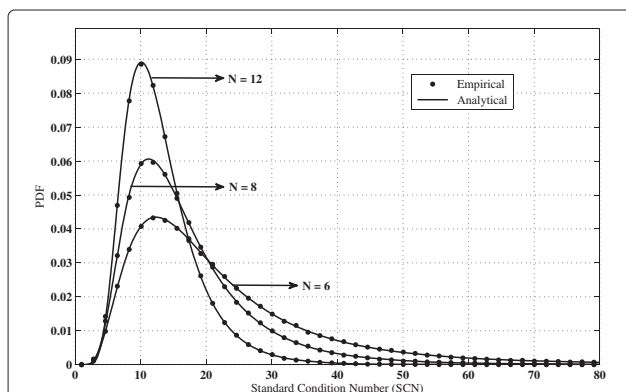
$$\mathbf{Y} = \mathbf{\Sigma}_K^{1/2} \mathbf{G}, \tag{28}$$

where $\mathbf{G}$ is the Gaussian noise matrix and $\mathbf{\Sigma}_K$ is the correlation matrix.

Figures 2 and 3 validate the analytical form of the distribution of the SCN of the central semi-correlated Wishart matrix derived in Section 5. A three-dimensional system is considered, and the receiver matrix is given in (28). The correlation matrix, $\mathbf{\Sigma}_K$, is modeled according to the normalization of Lemma 1; thus, $\sigma_1 = 1 + K\rho$ and $\sigma_2 = \sigma_3 = 1$. Figure 2 shows the impact of varying the number of received samples (N) while the SNR (ρ) variation is considered in Fig. 3. Results show a perfect match between the empirical SCN distribution of the central semi-correlated Wishart matrix and the analytical form in Section 5. Moreover, these figures show a validation of the joint distribution given in Theorem 1.

Central/non-central approximation accuracy is shown in Fig. 4. In this case, we consider a BPSK modulated signal and a complex Gaussian noise with unit variance ($\sigma_\eta^2 = 1$). Since we have considered a flat fading channel, we suppose that the signal will experience a unity fading magnitude. Figure 4 gives the empirical distribution of the SCN of non-central uncorrelated Wishart matrices (under $\mathcal{H}_1$ hypothesis) and its corresponding normalized analytical approximation using Lemma 1 for different SNR values (ρ). Results show a perfect match between the empirical results and the analytical approximation at low SNR values, however, approximation accuracy degrades as SNR increases ($\rho > -2\,\mathrm{dB}$).

Figures 5 and 6 present the impact of the system parameters (K, N, and ρ) on the approximation in terms of

Fig. 3 Empirical and analytical probability density function of the SCN of the central semi-correlated Wishart matrices as ρ varies

the performance of the SCN detector for a preset target P_{fa}. Figure 5 shows the P_{d} of the SCN detector of three-dimensional CR as a function of SNR for different number of samples ($N = \{10, 30, 50, 100\}$) while in Fig. 6, we vary K ($K = \{2, 3\}$) with the false alarm being set to to $P_{\mathrm{fa}} = 0.1$. Both figures show perfect accuracy for low SNR values ($\rho < -2$ dB); however, as SNR increases the approximation starts to show dissimilarity with the empirical results until both probabilities reach 1 (both distributions, the empirical and the analytical, are totally to the right of the considered threshold).

Figure 5 shows that the dissimilarity between the curves decreases as N increases and becomes very close to zero at $N = 100$. This is related to the threshold selection criteria. In CR, the threshold is—usually—selected according to a target P_{fa} (i.e., the distribution of the SCN under $\mathcal{H}_0$ as shown in Fig. 1). As N increases, the detection probability is improved as the part of the distribution under $\mathcal{H}_1$ to the right of the threshold increases. This reveals that,
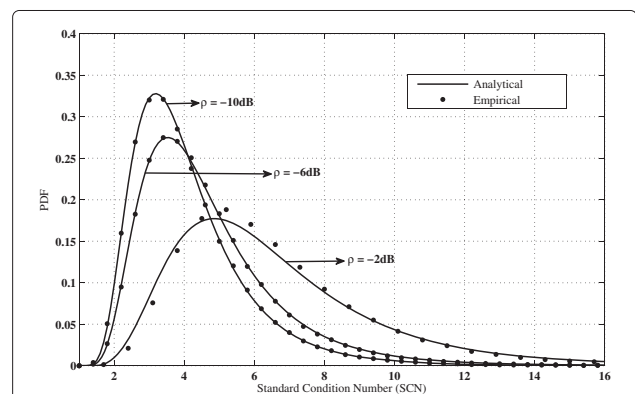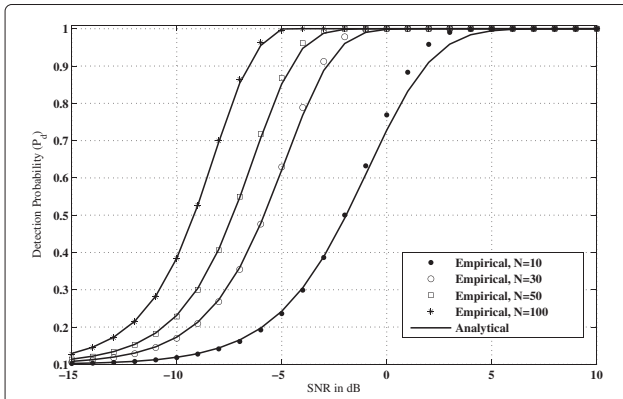
Fig. 2 Empirical and analytical probability density functions of the SCN of the central semi-correlated Wishart matrices as N varies

Fig. 4 Empirical probability density function of the SCN of non-central uncorrelated Wishart matrices and the corresponding analytical probability density function of central semi-correlated Wishart matrices using normalized non-central/central approximation

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Fig. 5 Empirical detection probability of the SCN metric and its corresponding analytical detection probability using normalized non-central/central approximation as a function of SNR for $K = 3$ and $P_{fa} = 0.1$
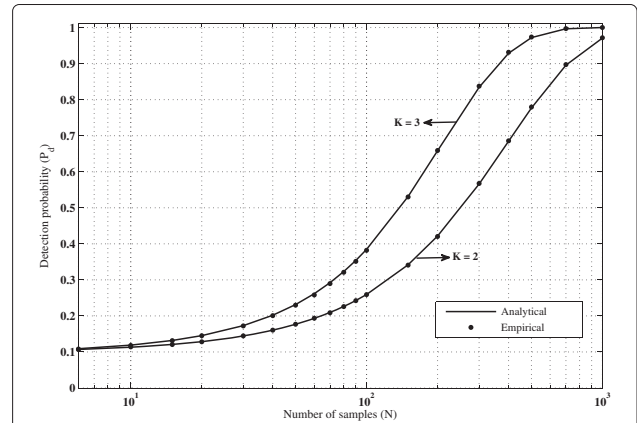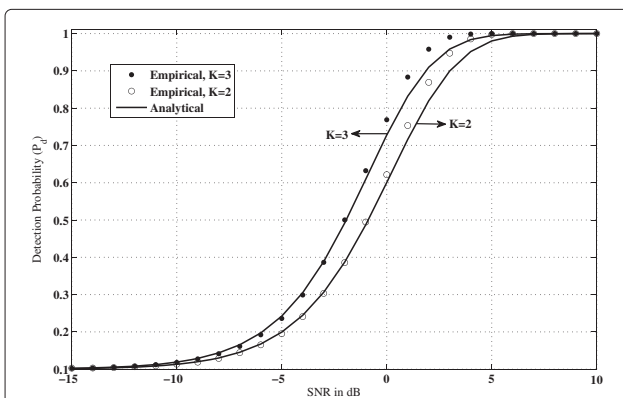

Fig. 7 Empirical detection probability of the SCN metric and its corresponding analytical detection probability using normalized non-central/central approximation as a function of the number of received samples for $\rho = -10$ dB, and $P_{fa} = 0.1$

in the sense of detection performance, the normalized non-central/central approximation could be considered as a good fit for the empirical distribution of the SCN for sufficiently large values of N. Indeed, both distributions perfectly match at low SNR values and exceed the threshold for higher SNR values where the approximation accuracy degrades. Moreover, the same result could be deduced from Fig. 6 where the performance of the detector increases as K increases.

To stand on the previous results and to validate the approximation accuracy obtained at low SNR for different values of N and K, we present in Fig. 7 the P_d of SCN metric for different N and K with SNR fixed at $\rho = -10$ dB and $P_{fa} = 0.1$. The figure shows a perfect match between the empirical and the analytical results for all values of K and N. As it could be seen from the figure, the distribution under $\mathcal{H}_1$ hypothesis partially exceeds the threshold
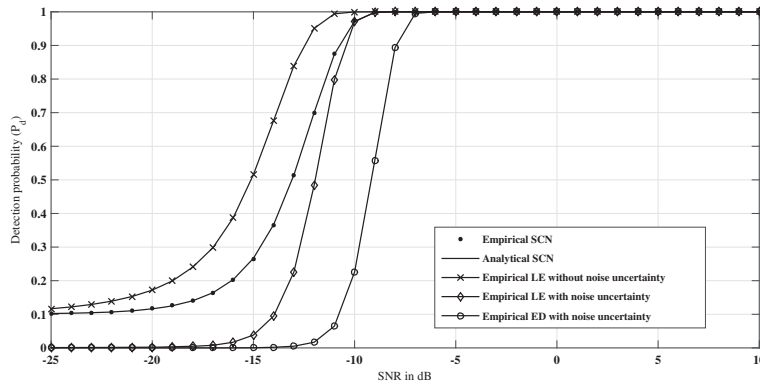

Fig. 6 Empirical detection probability of the SCN metric and its corresponding analytical detection probability using normalized non-central/central approximation as a function of SNR for $N = 10$ and $P_{fa} = 0.1$

($P_d < 1$), therefore, the approximation perfectly fits the empirical distribution. Thus, at an acceptable SNR value, the approximation does not show any degradation as N and K values change.

The performance comparison between the SCN detector, the LE detector, and the ED [5] is illustrated in Fig. 8. In this case, we set $K = 3$, $N = 500$, and $P_{fa} = 0.1$. If noise variance is perfectly known, results show that the LE detector outperforms the SCN detector as it is the optimal detector under this condition. However, noise variance is not known in practice. Accordingly, we considered a 0.3 dB noise uncertainty and the results are shown in Fig. 8. Results show that the SCN detector outperforms both the LE and ED as noise uncertainty is considered.

7 Conclusions
In this paper, we have considered the detection probability of the SCN detector. We have provided new statistics for the SCN metric, particularly for a K-dimensional cognitive radio system, by generalizing the form of the joint distribution of the ordered eigenvalues of central semi-correlated Wishart matrices. Then, we have considered a three-dimensional system and have derived the approximated detection probability by the use of normalized non-central/central approximation. The analytical results are validated through Monte Carlo simulations in different environments and with different system parameters. Results show that the approximation perfectly fits the empirical results at low SNR values; however, the approximation accuracy degrades as SNR increases. For a target false alarm probability, results prove that the probability of detection could be approximated using normalized non-central/central approximation for all SNR values when the values of N and K are large enough. In addition, and by considering the noise uncertainty problem, results show

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Fig. 8 Probability of detection of the SCN metric, LE, and energy detector with 0.3 dB noise uncertainty as a function of SNR for $K = 3$, $N = 500$, and $P_{\text{fa}} = 0.1$

that the SCN detector outperforms the LE detector and the ED even with very small noise uncertainty value.

Endnotes

[1] The infinite sum of the hypergeometric function could be truncated to a finite sum with good accuracy if the K, N, and SNR are small. However, for a CR system, N is usually large.

[2] The norm of the PU signal, $\|s\|^2$, is still a random variable; however, this randomness decreases fast as N increases and ($\|s\|^2/N$) can be well approximated by σ_s^2 for sufficient value of N.

Appendices

Appendix 1: proof of Theorem 1.

Before we start the proof, it is required to give the following Lemma presenting the nth derivative of any function of the form $e^{-\frac{a}{x}}$ with respect to the variable x.

Lemma A.1. *Let us define the function $f(x) = e^{-\frac{a}{x}}$, then the nth derivative of $f(x)$ is given by*

$$f^{(n)}(x) = \frac{a}{x^{2n}} \cdot e^{-\frac{a}{x}} \cdot \left[\sum_{k=1}^{n} (-1)^{k+n} \mathcal{L}(n,k) x^{n-k} a^{k-1} \right]$$
(29)

with $\mathcal{L}(n,k)$ the Lah number defined in (17).

Proof. It should be noted that, for any value of n, the nth derivative of $e^{-\frac{a}{x}}$ will result into the function $e^{-\frac{a}{x}}$ multiplied by sum of (a/x) with different powers for a and x. Regardless of the sign, it is found that the number multiplied by each component of this sum is exactly equals to the Lah number.

Following the pattern seen by calculating the derivative of $f(x)$ for small values of n, and matching the numbers at each derivative level, "n," with the Lah number $\mathcal{L}(n,k)$, it

can be proved, by recurrence, that the nth derivative of the function $f(x)$ of the form $e^{-\frac{a}{x}}$ is given by (29). $\square$

Now, we proceed in proving Theorem 1. If we have L equal eigenvalues ($\sigma_p = \cdots = \sigma_q$), we must generalize the result of central semi-correlated Wishart with distinct correlation eigenvalues for arbitrary number of coincident eigenvalues by taking the limit of joint distribution given by Eq. 6 of [30] as follows:

$$\lim_{\sigma_{p,q} \to \sigma_p} f(\lambda) = |\Phi(\lambda)| \prod_{i=1}^{K} \xi(\lambda_i) \lim_{\sigma_{p,q} \to \sigma_p} (C \times |E(\lambda, \sigma)|)$$

$$= |\Phi(\lambda)| \prod_{i=1}^{K} \xi(\lambda_i) \lim_{\sigma_{p,q} \to \sigma_p} \left(\frac{\prod_{i<j}^{K} \sigma_i \sigma_j}{\prod_{i=1}^{K} \sigma_i^N (N-i)!} \cdot \frac{|E(\lambda, \sigma)|}{\prod_{i<j}^{K} \sigma_j - \sigma_i} \right)$$
(30)

with $\sigma_{p,q} = [\sigma_p, \cdots, \sigma_q]$, and C the normalization constant, $\Phi(\lambda)$ is a Vandermonde matrix of entries λ_j^{i-1}, $E(\lambda, \sigma)$ is matrix of entries $e^{\frac{-\lambda_j}{\sigma_i}}$, and $\xi(\lambda_i) = \lambda_i^{N-K}$ [30, Table I]. To evaluate the limit in (30), we apply Lemma 2 from [39] to obtain

$$\lim_{\sigma_{p,q} \to \sigma_p} \frac{|E(\lambda, \sigma)|}{\prod\limits_{i<j}^{K} \sigma_j - \sigma_i} = \frac{\begin{vmatrix} e(\lambda_1, \sigma_1) & \cdots\cdots & e(\lambda_K, \sigma_1) \\ \vdots & \ddots\ddots & \vdots \\ e(\lambda_1, \sigma_{p-1}) & \cdots\cdots & e(\lambda_K, \sigma_{p-1}) \\ e^{(L-1)}(\lambda_1, \sigma_p) & \cdots\cdots & e^{(L-1)}(\lambda_K, \sigma_p) \\ \vdots & \ddots\ddots & \vdots \\ e^{(0)}(\lambda_1, \sigma_p) & \cdots\cdots & e^{(0)}(\lambda_K, \sigma_p) \\ e(\lambda_1, \sigma_{q+1}) & \cdots\cdots & e(\lambda_K, \sigma_{q+1}) \\ \vdots & \ddots\ddots & \vdots \\ e(\lambda_1, \sigma_K) & \cdots\cdots & e(\lambda_K, \sigma_K) \end{vmatrix}}{\prod\limits_{\substack{i<j \\ \sigma_i \neq \sigma_j}}^{K} (\sigma_j - \sigma_i) \cdot \Gamma_L(L)}$$
(31)

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with $e(\lambda_i, \sigma_j) = e^{-\frac{\lambda_i}{\sigma_j}}$, the required derivatives are evaluated using (29) and $\Gamma_s(l) = \prod_{i=1}^{s}(l-i)!$. Then, the result is obtained by substituting (29) and (31) into (30) and simplifying.

In the case, we have several coincidences (e.g., $\sigma_{p,q} = \sigma_p$, $\sigma_{t,s} = \sigma_s$), the same analysis could be applied and the limit must be taken over all coincidence cases.

Appendix 2: the CDF and PDF of SCN

In the case where Ω_3 is not a rank 1 matrix (we have more than one PU), then the eigenvalues of Σ_3 are not equal anymore. For sake of completeness of three-dimensional system case, we give in Theorem 3 and Corollary 2 the CDF and PDF of the SCN metric when all the correlation eigenvalues are distinct.

Theorem 3 (CDF of SCN of Central Semi-Correlated Wishart Matrix when Σ_3 has distinct eigenvalues). *The CDF of the SCN of 3×3 central semi-correlated Wishart matrix* $\mathbf{W} \sim \mathcal{CW}_K(N, \Sigma_3)$ *when eigenvalues of Σ_3 are distinct is given by*

$$F_1(x) = C_s \cdot R_3(N, x), \quad z \geq 1, \tag{32}$$

where $R_3(N, x)$ is defined in (33), $S_3(r, s, t, x)$ in (34), $\Delta_3(r, s, t, \mu, \nu, \varepsilon, x)$ in (35), and C_s in (36).

$$
\begin{aligned}
R_i(N, x) = {} & S_i(N-2, N-1, N-3, x) \\
& - S_i(N-3, N-1, N-2, x) \\
& + S_i(N-3, N-2, N-1, x), \ i = 3, 4
\end{aligned} \tag{33}
$$

$$
\begin{aligned}
S_i(r, s, t, x) = {} & \Delta_i\left(r, s, t, \sigma_2^{-1}, \sigma_1^{-1}, \sigma_3^{-1}, x\right) \\
& - \Delta_i\left(r, s, t, \sigma_3^{-1}, \sigma_1^{-1}, \sigma_2^{-1}, x\right) \\
& - \Delta_i\left(r, s, t, \sigma_1^{-1}, \sigma_2^{-1}, \sigma_3^{-1}, x\right) \\
& + \Delta_i\left(r, s, t, \sigma_3^{-1}, \sigma_2^{-1}, \sigma_1^{-1}, x\right) \\
& + \Delta_i\left(r, s, t, \sigma_1^{-1}, \sigma_3^{-1}, \sigma_2^{-1}, x\right) \\
& - \Delta_i\left(r, s, t, \sigma_2^{-1}, \sigma_3^{-1}, \sigma_1^{-1}, x\right), i = 3, 4
\end{aligned} \tag{34}
$$

$$
\begin{aligned}
\Delta_3(r, s, t, \mu, \nu, \varepsilon, x) = {} & \left(\frac{r! s!}{\mu^{r+1}\varepsilon^{s+1}}\right) \sum_{k=0}^{r}\sum_{u=0}^{s}\left[\frac{(k+u+t)!}{k! u! \mu^{-k}\varepsilon^{-u}} \cdot \right. \\
& \times \left(\frac{1}{(\mu+\varepsilon+\nu)^{k+u+t+1}}\right. \\
& - \frac{x^u}{(\mu+\varepsilon x+\nu)^{k+u+t+1}} \\
& - \frac{x^k}{(\mu x+\varepsilon+\nu)^{k+u+t+1}} \\
& \left.\left. + \frac{x^{k+u}}{(\mu x+\varepsilon x+\nu)^{k+u+t+1}}\right)\right]
\end{aligned} \tag{35}
$$

$$
C_s = \frac{[(\sigma_2 - \sigma_1)(\sigma_3 - \sigma_1)(\sigma_3 - \sigma_2)]^{-1}}{(N-1)!(N-2)!(N-3)!\sigma_1^{N-2}\sigma_2^{N-2}\sigma_3^{N-2}}. \tag{36}
$$

Proof. The proof could be summarized as follows:

1. Considering (18) and setting $K = 3$.
2. Substituting the parameters from Theorem 1 with $L = 1$.
3. Expanding the summation and using (19) with $p = q$.
4. Expanding the determinant and integrate using Eqs. (3.351.1) and (3.351.3) of [38].

Then, the result comes after simplification. $\square$

Corollary 2. *The PDF of the SCN of 3×3 central semi-correlated Wishart matrix* $\mathbf{W} \sim \mathcal{CW}_K(N, \Sigma_3)$ *when eigenvalues of Σ_3 are distinct is given by*

$$f_1(x) = C_s \cdot R_4(N, x), \quad z \geq 1, \tag{37}$$

where $R_4(N, x)$ defined in (33), $S_4(r, s, t, x)$ in (34), C_s in (36), and $\Delta_4(r, s, t, \mu, \nu, \varepsilon, x)$ in (38).

$$
\begin{aligned}
\Delta_4(r, s, t, \mu, \nu, \varepsilon, x) = {} & \left(\frac{r! s!}{\mu^{r+1}\varepsilon^{s+1}}\right) \sum_{k=0}^{r}\sum_{u=0}^{s}\left[\left(\frac{(k+u+t)!}{k! u! \mu^{-k}\varepsilon^{-u}}\right)\cdot \right. \\
& \times \left(\frac{x^{k+u-1}(\nu(k+u) - x(t+1)(\mu+\varepsilon))}{(\mu x+\varepsilon x+\nu)^{k+u+t+2}} \right. \\
& - \frac{x^{u-1}(u(\mu+\nu) - \varepsilon x(k+t+1))}{(\mu+\varepsilon x+\nu)^{k+u+t+2}} \\
& \left.\left. - \frac{x^{k-1}(k(\varepsilon+\nu) - \mu x(u+t+1))}{(\mu x+\varepsilon+\nu)^{k+u+t+2}}\right)\right].
\end{aligned} \tag{38}
$$

Proof. Obtained by differentiating Eq. (32). $\square$

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