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MRT precoding in downlink multi-user MIMO systems

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Abstract

This paper focuses on the design of maximum ratio transmission (MRT) precoding for multi-user multiple-input multiple-output (MU-MIMO) downlink transmission. Exiting block diagonalization (BD) precoding studies on MU-MIMO systems have the high complexity, because the transmitter precoding matrices constructed by singular value decomposition (SVD) are successively calculated twice. The MRT scheme construct precoding vectors aimed at each received antennas, respectively, so the signals of every antenna are independent. More spatial diversity gain can be obtained compared with BD precoding when MRT precoding and maximum ratio combining are employed. Simulations show that the proposed algorithm has many gains over the conventional BD precoding in various MU-MIMO systems.

Keywords: MU-MIMO, Precoding, Multi-user interference, Maximum ratio transmission

1 Introduction

The envisioned rapid increase of the wireless data traffic demand in the next years imposes rethinking current wireless cellular networks [1]. Multi-user (MU) multiple-input multiple-output (MIMO) systems have the potential of combining the high capacity by MIMO processing with the benefits of space-division multiple access (SDMA) [2]. In general, MU-MIMO systems not only suffer from the noise and the inner-antenna interference but are also affected by multi-user interference (MUI) during downlink transmission, which is means of channel-aware precoding methods implemented at the base station (BS). Precoding techniques for MIMO transmissions have recently gained increasing interest with the introduction of MU-MIMO, in which a large number of transmit antennas are used at the base station to simultaneously serve multiple receivers [3]. Nonlinear precoding methods such as dirty paper coding are performance achieving. However, these precoding are highly complex, thereby motivating the need for linear methods, which are computationally simpler. Channel inversion-based linear precoding algorithms such as zero-forcing channel inversion (ZF-CI) can still be used to cancel the MUI with the lower complexity. As the generalization of the ZF-CI precoding algorithm, the block diagonalization (BD) and regularized block diagonalization (RBD) precoding have been proposed for MU-MIMO systems in [4–6]. Singular value decomposition (SVD) operations are implemented twice for each user in BD precoding algorithm.

Over the last few years, many works have analyzed the zero-forcing beamforming for single-stream transmission per user and the zero-forcing precoding for multiple streams per user as the generalized single antenna [7–10]. As to the analysis of multiple streams, some researches for MU-MIMO focused on zero-forcing precoding, and it is noted that most of the previous works on BD precoding assumed designing the BD-type precoding schemes with less computational complexity. QR-decomposition-based BD (QR-BD), generalization ZF-CI (GZI), and lattice reduction (LR)-assisted precoding are proposed in [11–14]. For the multiple streams system, we must do power control or the adaptive modulation and coding to balance the effective channel gain for each stream [15, 16], which gets the same SNR for much channel with the geometric mean decomposition (GMD); otherwise, the performance of each user will suffer significant loss. However, the power control or adaptive modulation is hard to achieve especially in the multiple antennas systems, suffering from high complexity and large signaling overhead.

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In the following, we propose a new linear precoding scheme based on BD for a downlink MU-MIMO systems with multiple data streams per user. The design goal is simple and a low-complexity algorithm to compute the precoding matrix for each user without the power control. To do this, we introduce a slight relaxation for BD precoding with once SVD. Thereby, we obtain a general form which simultaneously diagonalizes covariance matrices through independently precoding.

The contribution of this work is that we develop a new BD precoding for downlink MU-MIMO system with multiple data streams per user to improve the BER performance without complex power, modulation, or coding. As detailed in the paper, the proposed algorithm is based on the multiple users and multiple streams MIMO systems which will be finally used for the derivation of the analytical system performance. A more thorough evaluation of proposed algorithm is confirmed via simulations. Furthermore, the new algorithm demonstrates a superiority performance.

The rest of the paper is organized as follows. Section 2 introduces the system model of MU-MIMO downlink transmissions. Then, the proposed maximum ratio transmission (MRT) precoding algorithm is detailed in Section 3. To examine the proposed transmission strategy in an efficient manner, a system-level simulator is designed. The benefits of the proposed transmission strategy are demonstrated through numerical simulations in Section 4. Finally, concluding remarks are drawn in Section 5.

We briefly summarize the notations used in this paper. Uppercase boldfaced letters are used to denote matrices and lowercase boldfaced letters for vectors. The superscripts $(\bullet)^{\mathrm{T}}$, $(\bullet)^{*}$, $(\bullet)^{\mathrm{H}}$ note the transpose, conjugate, and conjugate transpose, respectively.

2 System model

2.1 System model

Consider downlink MU-MIMO system with one BS equipped with M transmits antennas and N received antennas (as shown in Fig. 1). Each BS simultaneously serves K number of l_k-antennas users. Without loss of generality, we assume that the received antenna number l_k for each user is the same. We consider a flat fading channel, we further assume that the channel state information (CSI) is perfectly known at the transmitter, and synchronization between the BS and the users is assumed.

The channel matrix coupling the BS to the user is of the kth user set and modeled as the flat Rayleigh fading MIMO channel:

$$\boldsymbol{H}_k = \begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1M} \\ h_{21} & h_{22} & \cdots & h_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ h_{l_k 1} & h_{l_k 2} & \cdots & h_{l_k M} \end{bmatrix} \quad (1)$$

where $\boldsymbol{H}_k \in \mathbb{C}^{l_k \times M}$ is MIMO channel matrix for the user k and the element $h_{u,v}$ indicates the channel impulse response coupling the k transmit antenna to the received antenna. The system channel matrix is as follows:

$$\boldsymbol{H} = \left[\boldsymbol{H}_1^{\mathrm{T}}\ \boldsymbol{H}_2^{\mathrm{T}}\ \cdots\ \boldsymbol{H}_k^{\mathrm{T}}\right]^{\mathrm{T}} \quad (2)$$

The received signal y_k for user k is given by:

$$y_k = \boldsymbol{H}_k \boldsymbol{P}_k \boldsymbol{s}_k + \boldsymbol{H}_k \sum_{i=1, i\neq k}^{K} \boldsymbol{P}_i \boldsymbol{s}_i + n_k \quad (3)$$

where $\boldsymbol{P}_k$ is the precoding matrix for user k. $\boldsymbol{s}_k = \left[s_k^1\ s_k^2\ \cdots\ s_k^{r_k}\right]$ is the transmission symbol vector for the kth user set, $r_k \le l_k$. n_k is the kth user's additive white Gaussian noise (AWGN) with zero-mean and σ^2 variance, that is, CN $(0, \sigma^2)$ for all user terminals.

$$\boldsymbol{P} = [\boldsymbol{P}_1\ \ \boldsymbol{P}_2\ \ \cdots \boldsymbol{P}_K] \quad (4)$$

$$\boldsymbol{S} = \left[\boldsymbol{s}_1^T\ \ \boldsymbol{s}_2^T\ \ \cdots \boldsymbol{s}_K^T\right]^T \quad (5)$$

where $\boldsymbol{P} \in \mathbb{C}^{M \times t}$ and S are the precoding matrix and the

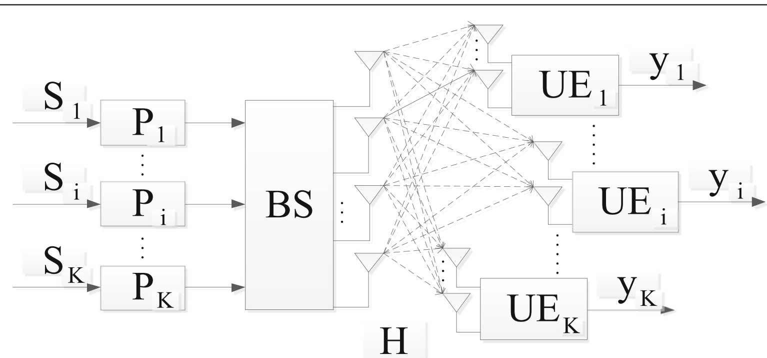

Fig. 1 MU-MIMO system model

transmit signal for K users, respectively. The total data stream for K users is t, and $t \le$ rank (H).

2.2 MUI as interference

We can consider the MUI as an additive interference, affecting the useful signal. The useful symbol is weighted by the equivalent channel frequency response, which is the sum of random variables of channels. In Eq. (3), $\boldsymbol{H}_k \sum_{i=1,i\neq k}^{K} p_i s_i$ is the MUI contribution. In this paper, the interference models based on classical approximation or on other distributions are assumed to be independent from the useful term.

3 Precoding scheme

3.1 BD precoding algorithms

As the generalization of the ZF precoding algorithm, the design of BD-based precoding algorithms is performed in two steps [4, 17]. Two SVD operations are implemented for each user in the BD precoding algorithm. The first SVD eliminate completely the MUI from other users for MU-MIMO channel. Thus, the MU-MIMO channel is decomposed into multiple parallel single-user MIMO (SU-MIMO) channels. The second SVD operation is implemented to parallelize each user's streams and obtain maximum precoding gain for each sub-stream to further improve the performance. Precoding matrix for the system is the product of the matrixes of the two steps.

The interference channel matrix for the kth user.

$$\tilde{\boldsymbol{H}}_{\mathrm{k}} = \left[\boldsymbol{H}_1^{\mathrm{T}} \cdots \boldsymbol{H}_{\mathrm{k}-1}^{\mathrm{T}} \ \boldsymbol{H}_{\mathrm{k}+1}^{\mathrm{T}} \cdots \boldsymbol{H}_{\mathrm{K}}^{\mathrm{T}}\right]^{\mathrm{T}} \tag{6}$$

In order to eliminate completely the MUI, that is $\boldsymbol{H}_i \boldsymbol{P}_k = 0$, $i \neq k$, $\boldsymbol{P}_k$ in the zero space of $\tilde{\boldsymbol{H}}_k$.

SVD decomposition for $\tilde{\boldsymbol{H}}_k$.

$$\tilde{\boldsymbol{H}}_{\mathrm{k}} = \tilde{\boldsymbol{U}}_{\mathrm{k}} \tilde{\boldsymbol{\Sigma}}_{\mathrm{k}} \left[\tilde{\boldsymbol{V}}_{\mathrm{k}}^{(1)} \ \ \tilde{\boldsymbol{V}}_{\mathrm{k}}^{(0)}\right]^{\mathrm{H}} \tag{7}$$

where $\tilde{\boldsymbol{V}}_{\mathrm{k}}^{(1)}$ is the right-singular matrix with non-zero singular value and $\tilde{\boldsymbol{V}}_k^{(0)}$ is the right-singular matrix with zero singular value. When the BD algorithm is employed as the precoding technique, the equivalent single-user channel matrix after the MUI elimination is shown. Therefore, $\tilde{\boldsymbol{V}}_{\mathrm{k}}^{(0)}$ is the zero space of $\tilde{\boldsymbol{H}}_{\mathrm{k}}$.

The equivalent channel of the kth user can be given as:

$$\boldsymbol{H}_{\mathrm{effk}} = \boldsymbol{H}_{\mathrm{k}} \tilde{\boldsymbol{V}}_{\mathrm{k}}^{(0)} \tag{8}$$

When the BD algorithm is employed, the norm of the equivalent channel after precoding equals the norm of the projected channel.

$$\left\|\boldsymbol{H}_i \tilde{\boldsymbol{V}}_i\right\|_F^2 = \left\|\boldsymbol{H}_{\mathrm{i}} \tilde{\boldsymbol{P}}_i\right\|_{\mathrm{F}}^2 \ i = 1, \cdots K \tag{9}$$

where $\tilde{\boldsymbol{P}}_i$ is a projection matrix that serves to the channel matrix of the user of i.

The SVD decomposition for $\boldsymbol{H}_{\mathrm{eff}k}$ is:

$$\boldsymbol{H}_{\mathrm{effk}} = \boldsymbol{H}_{\mathrm{k}} \tilde{\boldsymbol{V}}_{\mathrm{k}}^{(0)} = \boldsymbol{U}_{\mathrm{k}} [\boldsymbol{\Sigma}_{\mathrm{k}} \ \ \boldsymbol{0}] \left[\boldsymbol{V}_{\mathrm{k}}^{(1)} \ \ \boldsymbol{V}_{\mathrm{k}}^{(0)}\right]^{\mathrm{H}} \tag{10}$$

in the same way, where the right-singular matrix with non-zero singular value is $\boldsymbol{V}_{\mathrm{k}}^{(1)}$ and $\boldsymbol{V}_{\mathrm{k}}^{(0)}$ is the right-singular matrix with zero singular value. The precoding vector can be written as:

$$\boldsymbol{P} = \left[\tilde{\boldsymbol{V}}_1^{(0)} \boldsymbol{V}_1^{(1)} \ \cdots \ \tilde{\boldsymbol{V}}_K^{(0)} \boldsymbol{V}_K^{(1)}\right] \tag{11}$$

Therefore, the relation (3) will be rewritten.

$$\begin{aligned} \boldsymbol{y} = \boldsymbol{HPx} + \boldsymbol{n} &= \begin{pmatrix} \boldsymbol{H}_1\boldsymbol{P}_1 & \cdots & \boldsymbol{H}_1\boldsymbol{P}_K \\ \vdots & \ddots & \vdots \\ \boldsymbol{H}_K\boldsymbol{P}_1 & \cdots & \boldsymbol{H}_K\boldsymbol{P}_K \end{pmatrix} \boldsymbol{x} + \boldsymbol{n} \\ &= \begin{pmatrix} \boldsymbol{H}_1\boldsymbol{P}_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \boldsymbol{H}_K\boldsymbol{P}_K \end{pmatrix} \boldsymbol{x} + \boldsymbol{n} \end{aligned} \tag{12}$$

The receive vector of the kth user can be given as:

$$\boldsymbol{y}_{\mathrm{k}} = \boldsymbol{H}_{\mathrm{k}} \boldsymbol{P}_{\mathrm{k}} \boldsymbol{x}_{\mathrm{k}} + \sum_{\mathrm{i}\neq\mathrm{k}}^{\mathrm{K}} \boldsymbol{H}_{\mathrm{i}} \boldsymbol{P}_{\mathrm{i}} \boldsymbol{x}_{\mathrm{i}} + \boldsymbol{n}_{\mathrm{k}} \tag{13}$$

Thus, from (13), the interference is eliminated completely. The kth user's decoding matrix is expressed as:

$$\boldsymbol{D}_{\mathrm{k}} = \boldsymbol{U}_{\mathrm{k}}^{\mathrm{H}} \tag{14}$$

BD precoding provides better performance due to the unitary precoding. However, the drawback of such precoding scheme-based BD is that the effective channel gain for each stream is severely destroyed by the interference. It is known that the overall performance of a user with multiple streams is dominated by the stream with the worst channel condition. Hence, interference would lead to poor overall error performance for a user. In the next section, we design a new precoding scheme so as to overcome this drawback.

3.2 Proposed precoding

For the conventional BD precoding, the interferences are involved in the received signals. This will be a key aspect and have to be specifically studied in our study. In the following, we drive a new BD precoding by constructing the interference matrix independently in a MU-MIMO system using QPSK modulation. We focus on the two-user and three-user cases to provide analysis. The further study will give the new information about the arbitrary number of antennas.

Firstly, calculate the precoding vector of each receiving antenna by the first SVD. Then, the precoding matrix of the kth user denoted $\boldsymbol{P}_k$, which each column is calculated separately for each receiving antenna.

Let us define $\overline{\boldsymbol{H}}_k^{(i)}$ to be the interference channel matrix of the ith antenna at the kth user.

$$\overline{\boldsymbol{H}}_k^{(i)} = \begin{pmatrix} \boldsymbol{H}_{k(\bar{i})} \\ \boldsymbol{H}_1 \\ \vdots \\ \boldsymbol{H}_{k-1} \\ \boldsymbol{H}_{k+1} \\ \vdots \\ \boldsymbol{H}_K \end{pmatrix} \tag{15}$$

where $\overline{\boldsymbol{H}}_k^{(i)}$ is the kth channel matrix that removed the ith line. $\overline{\boldsymbol{H}}_k^{(i)}$ can be expressed through SVD and described by:

$$\overline{\boldsymbol{H}}_k^{(i)} = \overline{\boldsymbol{U}}_k^{(i)} \overline{\boldsymbol{\Sigma}}_k^{(i)} \left[\overline{\boldsymbol{V}}_k^{(i)(1)} \quad \overline{\boldsymbol{V}}_k^{(i)(0)} \right]^{\mathrm{H}} \tag{16}$$

where $\overline{\boldsymbol{V}}_k^{(i)(1)}$ is the right-singular matrix with non-zero singular value and $\overline{\boldsymbol{V}}_k^{(i)(0)}$ is the right-singular matrix with non-zero singular value. Therefore, $\overline{\boldsymbol{V}}_k^{(i)(0)}$ is the zero space of $\overline{\boldsymbol{H}}_k^{(i)}$. For the kth user, MRT precoding is given by:

$$\overline{\boldsymbol{P}}_k = \left[\overline{\boldsymbol{P}}_k^{(1)}, \overline{\boldsymbol{P}}_k^{(2)}, \ldots, \overline{\boldsymbol{P}}_k^{(l_k)} \right] \tag{17}$$

The total number of the receiver antennas is l_k, the new precoding matrix of the ith receiver antenna at the kth user (M dimension column) denotes $\overline{\boldsymbol{P}}_{\mathrm{k}}^{(i)} = \overline{\boldsymbol{V}}_{\mathrm{k}}^{(i)(0)}(l)$, and $\overline{\boldsymbol{P}_k}$ is the precoding matrix for the kth user ($l_k \times M$ dimension). Further, the receiver vector at user k is written as:

$$\begin{aligned} \overline{\boldsymbol{y}}_k &= \boldsymbol{H}_k \overline{\boldsymbol{P}_k} \boldsymbol{x}_k + \sum_{\mathrm{i}=1, \mathrm{i} \neq k}^{\mathrm{K}} \boldsymbol{H}_k \overline{\boldsymbol{P}_k} \boldsymbol{x}_{\mathrm{i}} + \boldsymbol{n}_k \\ &= \Psi_k \boldsymbol{x}_k + \boldsymbol{n}_k \end{aligned} \tag{18}$$

where the first part is the coefficient of the receiver antenna at user k, $\Psi_k = \boldsymbol{H}_k \overline{\boldsymbol{P}_k}$ (l_k is the dimension column), the second part is the MUI, and $\boldsymbol{H}_k \overline{\boldsymbol{P}_i}$ is zero obviously. So, the interference of the antenna of equivalent channel is eliminated.

Then, the maximal ratio combination (MRC) for the received symbols for the entire antenna at user k is written as:

$$\overline{\overline{\boldsymbol{y}}}_k = \sum_{i=1}^{l_k} \overline{\boldsymbol{y}}_k(i) \cdot \Psi_k^*(i) \tag{19}$$

Finally, the estimated result at user k is get, through the hard decision for the received symbol $\overline{\overline{\boldsymbol{y}}}_{\mathrm{k}}$.

4 Simulation results

In this section, we investigate the performance of the proposed scheme for MU-MIMO downlink system by means of Monte-Carlo simulations, comparing it with the original proposed BD precoding. A spatially uncorrelated flat Rayleigh fading of the wireless channel is assumed, and the noise distributed complex Gaussian variables with zero-mean and unit-variance, considering quadrature phase shift keying (QPSK) modulation.

For simplicity but without loss of generality, we consider that the number of received antennas for each user is the same, which equal to the total data streams of the transmit user. Then, we study the performance of the MU-MIMO system for the different antenna configuration by simulations. We compare the simulated bit error rate (BER) in a MU-MIMO system with different system transmission and the antenna configuration.

Figure 2 compares the BD precoding with two streams and every stream using SVD decomposition. The BD precoding scheme operate two SVD for each user and the interference is introduced, so the performance highly affected by the worst channel conditions for multi-streams transmission. Because BD precoding algorithm effective channel gains unbalance between different data flows, it leads to the poor performance of the system. It can be observed that as SNR increased, the performance of BER improved obviously for the SVD decomposition of each stream. This shows that the interference is influenced by the system performance even with a low SNR, then the performance would be worse than the once SVD decomposition.

By comparing the performance of BD precoding of two streams, we can summarize the conclusion, and a significant interference is introduced in the BD precoding. It is observed in simulation conditions. So, it can be concluded that BD precoding is not a suitable approach for MUI scenarios and dense network. It is important to recall that gains are obtained even in SVD decomposition only once and as SNR is increased, higher impact of interference on performance is observed. But in MU-MIMO systems, the MUI is also important.

As is shown in Fig. 3, we compare the BER for every stream when an MRT is used for transmission from BS to the UEs. The BER curves are plotted versus the transmit SNR with the antenna configuration for $\{2, 2\} \times 4$, $M = 4$, $n_k = 2$, and $K = 2$. We compared the BER for the conventional BD and the proposed MRT precoding schemes with the same antenna configuration, using the multiplexing strategy. As expected, the proposed MRT precoding scheme performs better BER than the conventional BD precoding algorithm. Because the BD precoding scheme operate two SVD and the interference is introduced, therefore, the performance of single data stream is better than multiple data streams for BD precoding scheme. The MRT precoding SVD operation only once and therefore reduces the interference and computational complexity.

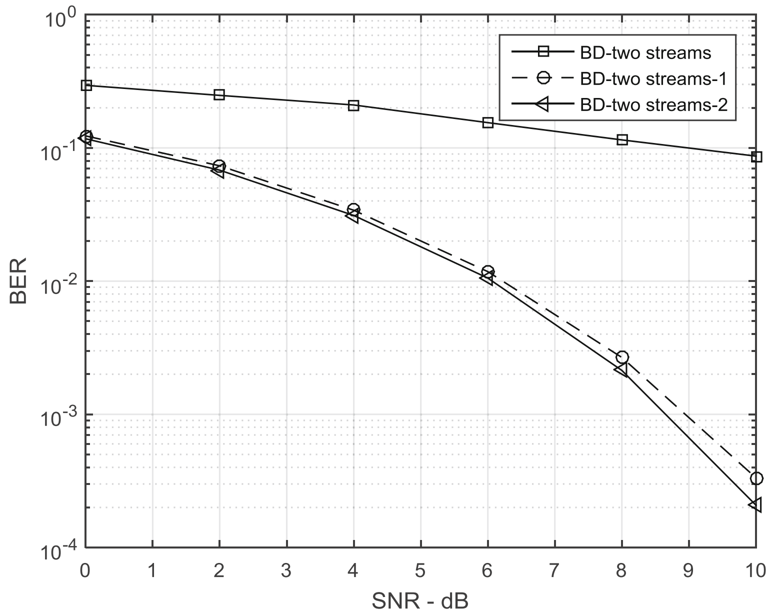

Fig. 2 Comparison of the BER performance for BD two streams and the every stream using SVD decomposition

In Fig. 4, the BER curves versus the transmit SNR are plotted. We compared the BER for the conventional BD, ZF, and the proposed MRT precoding schemes with the same antenna configuration {2, 2} × 4, using the diversity strategy. The MRT precoding and MRC for BD is operated at the transmitter and receiver, respectively, when the multiple data stream is sent. Compared with Fig. 2, the BER has declined significantly, since the multi-way spatial diversity was obtained. So it can be concluded that MRT is a suitable approach for interference limited scenarios and dense networks. It is important to include that large gains are obtained and that no additional overhead is needed for MRT if we use UL transmission.

To further understand the proposed scheme, Fig. 5 shows the result of the MU-MIMO system with the space diversity for the antenna configuration of $M = 6$, $n_k = 3$, and $K = 3$. It can be observed that we can get the similar performance conclusions as in Fig. 4, and the

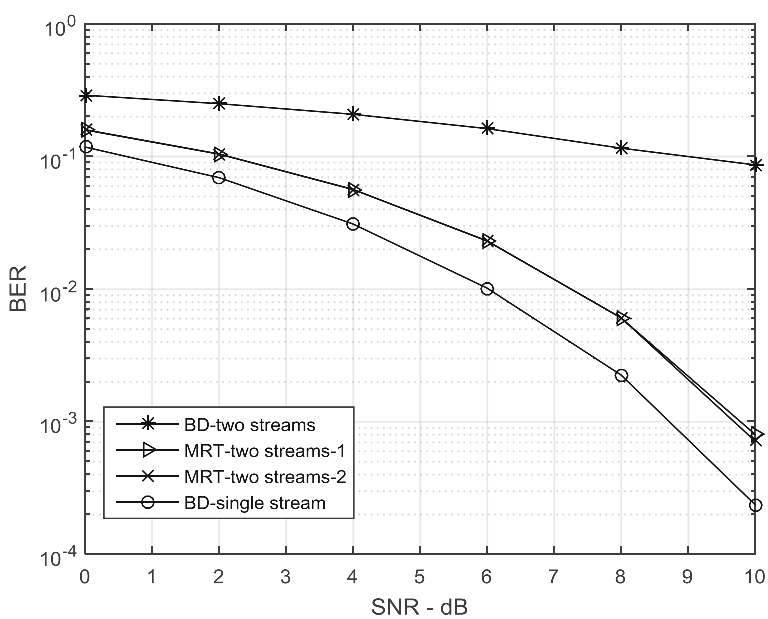

Fig. 3 Comparison of the BER performance for BD and MRT algorithm with multiplexing strategy, where the antenna {2, 2} × 4

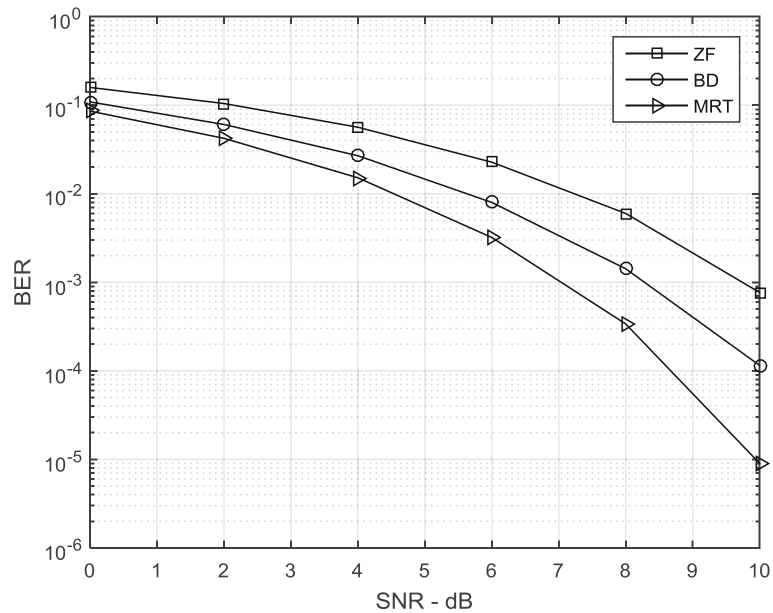

Fig. 4 Comparison of the BER performance for BD and MRT algorithm with diversity strategy, where the antenna {2, 2} × 4

proposed MRT precoding scheme has better performance than the conventional ZF and BD precoding algorithm. As the SNR increased, higher impact of the interference on performance and more gains obtained are observed.

5 Conclusions

In this paper, a low-complexity linear precoding scheme named MRT precoding is proposed in downlink MU-MIMO systems. The MRT precoding scheme introduces a designed precoding matrix operation to mitigate the interference and then to obtain the gain with the MRC. In contrast to the existing BD-type precoding, the main focus of this work is motivated by the low complexity and the better performance. Our algorithm was shown to be valid for multiple data streams, which reduces the interference among the antennas. The theoretical derivation and simulation show that the MRT precoding can achieve better BER and reliability but requires the lower computational complexity.

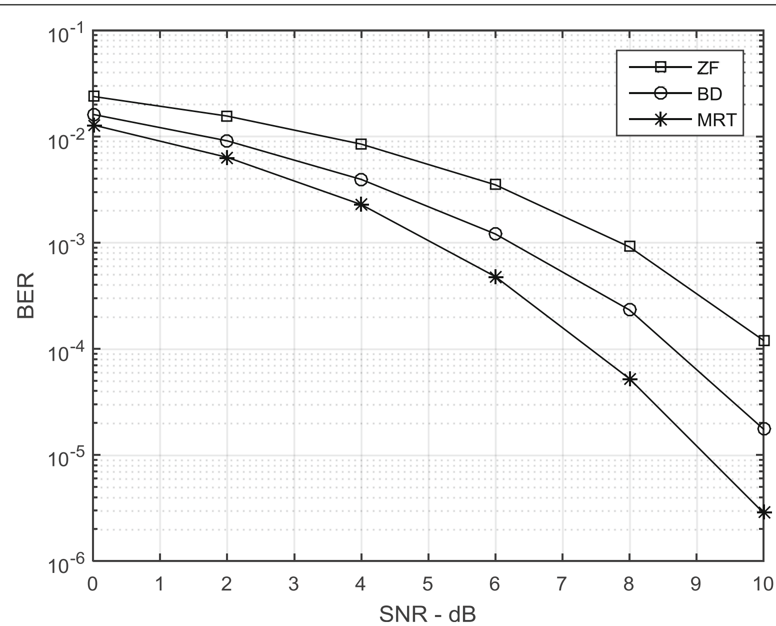

Fig. 5 Comparison of the BER performance for BD and MRT algorithm with the antenna {3, 3} × 6

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Competing interests
The authors declare that they have no competing interests.

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